Letter | Editors' Suggestion

Bell inequality is violated in $B^0 \to J/\psi K^*(892)^0$ decays

M. Fabbrichesi,[1] R. Floreanini,[1] E. Gabrielli,[2,1,3,4] and L. Marzola[4]

[1]*INFN, Sezione di Trieste, Via Valerio 2, I-34127 Trieste, Italy*
[2]*Physics Department, University of Trieste, Strada Costiera 11, I-34151 Trieste, Italy*
[3]*CERN, Theoretical Physics Department, Geneva, Switzerland*
[4]*Laboratory of High-Energy and Computational Physics, NICPB, Rävala 10, 10143 Tallinn, Estonia*

(Received 5 June 2023; accepted 24 January 2024; published 23 February 2024)

The violation of the Bell inequality is one of the hallmarks of quantum mechanics and can be used to rule out local deterministic alternative descriptions. We utilize the data analysis published by the LHCb Collaboration on the helicity amplitudes for the decay $B^0 \to J/\psi K^*(892)^0$ to compute the entanglement among the polarizations of the final vector mesons and the violation of the Bell inequality that it entails. We find that quantum entanglement can be detected with a significance well above 5σ (nominally 84σ) and Bell inequality is violated with a significance well above 5σ (nominally 36σ)—thereby firmly establishing these distinguishing feature of quantum mechanics at high energies in a collider setting and in the presence of strong and weak interactions. Entanglement is also present and the Bell inequality is violated in other decays of the B mesons into vector mesons, but with lesser significance.

DOI: 10.1103/PhysRevD.109.L031104

Introduction. The violation of the Bell inequality [1] is a phenomenon that shows that quantum mechanics cannot be explained by any local hidden variable theory, which assumes that physical systems have definite properties independent of measurement. It has been verified experimentally with the polarizations of low-energy (that is, few eV) photons in [2,3]: two photons are prepared into a singlet state and their polarizations measured along different directions to verify their entanglement [4] and the violation of the Bell inequality.

Verifying quantum entanglement and the violation of the Bell inequality in the presence of strong and weak interactions would tell us whether these fundamental forces of nature exhibit quantum correlations and nonlocality, which would have profound implications for our understanding of reality. In order to test the inequality at higher energies, we need a sufficiently heavy scalar (or pseudoscalar) particle decaying into two spin-1/2 or spin-1 states. While we do not know of any data in the case of fermions, these are available for the two final states being massive vectorlike particles. The setup in the latter case closely resembles that in which the polarizations of two photons prepared into a singlet state are measured—except that the photon polarizations are described by a two-value quantum state, or qubit, while those of the massive spin-1 state have three values and are described as *qutrits*.

The most promising examples can be found among B mesons decaying into comparatively heavy final states with approximately equal shares of longitudinal and transverse polarizations. In addition, larger branching fractions make for better statistics. These requirements single out the decay $B^0 \to J/\psi K^*(892)^0$ as the best candidate. The data analysis of the LHCb collaboration for this decay [5] provides the helicity amplitudes necessary for the test. They make it possible to extend the testing of the violation of Bell identities to energies of the order of 5 GeV—which are a billion times larger than those utilized in [2,3,6]. The same decay has previously been studied by the experiments CLEO [7], CDF [8], Belle [9], *BABAR* [10], and D0 [11]. We only use the most recent analysis because it is the most precise.

In this Letter, we explain why the $B^0 \to J/\psi K^*(892)^0$ decay provides a most favorable setting, introduce two operators to quantify entanglement and violation of the Bell inequality for a two-qutrit system, compute the expectation values of these two operators using the polarization amplitudes provided in [5], and show that quantum entanglement is present with a significance well above 5σ (nominally 84σ) and the Bell inequality is violated with a significance well above 5σ (nominally 36σ). This result firmly establishes this quantum mechanical hallmark for a system of two qutrits, and it does it at high energies and in the presence of strong and weak interactions—thereby extending what is known to be true for qubits, at low energies and for electromagnetic interactions.

We also analyze the decays $B^0 \to \phi K^*(892)^0$, $B^0 \to \rho K^*(892)^0$, $B_s \to J/\psi\phi$, and $B_s \to \phi\phi$, which have sizable transverse polarizations, and find that quantum entanglement is present with a significance of 5.35σ, 5.84σ, 22.8σ, and 19.8σ, respectively, and they violate the Bell inequality with a significance of 1.1σ, 1.4σ, 5.8σ, and 8.2σ, respectively.

Previous inquiries about Bell inequality violations with data from heavy-particle physics have been presented for kaons [12] and for the B^0-$\bar{B}^0$ system [13]. Both of these examples, though providing important clues, are indirect tests: the first relies on the measure of the CP violating parameter ε'/ε, the second on oscillations in flavor space.

Materials. The analysis of the decay $B^0 \to J/\psi K^*(892)^0$ in [5] is based on the data sample collected in pp collisions at 7 TeV (part of run 1 of the LHC) with the LHCb detector and corresponds to an integrated luminosity of 1 fb^{-1}. The branching fraction for this decay is $(1.27 \pm 0.05) \times 10^{-3}$ [14].

The selection of $B^0 \to J/\psi K^*(892)^0$ events, as explained in [5], is based upon the combined decays of the $J/\psi \to \mu^+\mu^-$ and $K^*(892)^0 \to K^+\pi^-$ final states. The muons, as they leave two oppositely charged tracks originating from a common vertex, are selected by taking their transverse momentum $p_T > 500$ MeV/c. The invariant mass of this pair of muons is required to be in the range between 3030 and 3150 MeV/c^2. The kaon and the pion leave two oppositely charged tracks that originate from the same vertex. It is required that the $K^*(892)^0$ has transverse momentum $p_T > 2$ GeV/c and invariant mass in the range 826–966 MeV/c^2. The B^0 are reconstructed from the J/ψ and $K^*(892)^0$ candidates, with the invariant mass of the $\mu^+\mu^-$ pair constrained to the J/ψ mass. The resulting B^0 candidates are required to have an invariant mass of the system $J/\psi K^+\pi^-$ in the range 5150–5400 MeV/c^2.

The polarizations of the spin-1 massive particles J/ψ and $K^*(892)^0$ can be reconstructed using the momenta of the final charged mesons and leptons in which they decay. The differential decay rate is described in terms of three angles: two angles are defined by the direction of the μ^+ momentum with respect to the z and x axes in the J/ψ rest frame, and one by the direction of the momentum of the K^+ with respect to the opposite direction of the momentum of the J/ψ in the $K^*(892)^0 \to K^+\pi^-$ rest frame, as shown in Fig. 2 of [5]. The longitudinal polarization amplitudes A_0 and the two transverse amplitudes $A_\perp$ and $A_\parallel$ are found as coefficients of combinations of trigonometric functions of these three angles [15].

The analysis in [5] gives the two complex polarization amplitudes $A_\parallel$ and $A_\perp$ as well as the nonresonant amplitude A_s. We need only the former two and take the following values for the squared moduli and phases of these polarization amplitudes:

$$\begin{aligned} |A_\parallel|^2 &= 0.227 \pm 0.004(\text{stat}) \pm 0.011(\text{syst}), \\ |A_\perp|^2 &= 0.201 \pm 0.004(\text{stat}) \pm 0.008(\text{syst}), \\ \delta_\parallel[\text{rad}] &= -2.94 \pm 0.02(\text{stat}) \pm 0.03(\text{syst}), \\ \delta_\perp[\text{rad}] &= 2.94 \pm 0.02(\text{stat}) \pm 0.02(\text{syst}), \end{aligned} \quad (1)$$

with $|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2 = 1$, and we can take $\delta_0 = 0$ because there are only two physical phases. The correlations among the amplitude and phase uncertainties are also provided in [5]. The polarization amplitudes are complex mostly because of the final-state interactions (see, for instance, [16]). The values in Eq. (1) have errors that are 2 or 3 times smaller than those of the previous analyses [7–11].

The decays of the J/ψ and K^* take place well outside of the range of the strong interactions ongoing at the time of their production (which is due to gluons exchange and is about 3×10^{-5} fm [17]) as well as of the final-state interactions. The distance between the two mesons, at the time they both have decayed, can be estimated to be $d \simeq 1.1 \times 10^3$ fm. This distance must be compared with the typical range of the virtual meson exchange, that is, at most equal to $\lambda_\pi = 1.5$ fm. We thus obtain that $d/\lambda_\pi \simeq 750$, indicating the impossibility of any strong interaction exchange between the two decaying particles. About the same distance is found for the decay into $J/\psi\phi$, while values of d between 100 and 10 are found for the other decays, namely $\phi\phi$, ϕK^*, and, with the least separation, ρK^*.

Methods. There are three helicity amplitudes for the decay of a scalar, or pseudo-scalar, into two massive spin-1 particles:

$$h_\lambda = \langle V_1(\lambda) V_2(-\lambda) | \mathcal{H} | B \rangle \quad \text{with} \quad \lambda = (+, 0, -), \quad (2)$$

and $\mathcal{H}$ is the interaction Hamiltonian giving rise to the decay. For the spin quantization axis ($\hat{z}$) we use the direction of the momenta of the decay products in the B^0 rest frame. Helicities are here defined with respect to the $\hat{z}$ direction in the rest frame of one of the two spin-1 particles and $(+, 0, -)$ is a shorthand for $(+1, 0, -1)$.

The polarizations in the decay are described by a quantum state that is pure for any values of the helicity amplitudes [22,23]. This state can be written as

$$|\Psi\rangle = \frac{1}{\sqrt{|H|^2}}\big[h_+|V_1(+)V_2(-)\rangle + h_0|V_1(0)V_2(0)\rangle + h_-|V_1(-)V_2(+)\rangle\big], \quad (3)$$

with

$$|H|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2. \quad (4)$$

The relative weight of the transverse components $|V_1(+)V_2(-)\rangle$ and $|V_1(-)V_2(+)\rangle$ with respect to the longitudinal one $|V_1(0)V_2(0)\rangle$ is controlled by the conservation of angular momentum. In general, only the helicity is conserved and the state in Eq. (3) belongs to the $J_z = 0$ component of the $S = 0$, 1, or 2 state.

The polarization density matrix $\rho = |\Psi\rangle\langle\Psi|$ can be written in terms of the helicity amplitudes as

$$\rho = \frac{1}{|H|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+h_+^* & 0 & h_+h_0^* & 0 & h_+h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_+^* & 0 & h_0h_0^* & 0 & h_0h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_-h_+^* & 0 & h_-h_0^* & 0 & h_-h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (5)$$

on the basis given by the tensor product of the polarizations $(+, 0, -)$ of the produced spin-1 particles.

The helicity amplitudes are mapped into the polarization amplitudes used in Eq. (1) by the correspondence

$$\frac{h_0}{|H|} = A_0, \qquad \frac{h_+}{|H|} = \frac{A_\parallel + A_\perp}{\sqrt{2}}, \qquad \frac{h_-}{|H|} = \frac{A_\parallel - A_\perp}{\sqrt{2}}. \quad (6)$$

Having written the density matrix, we can study the entanglement among the polarizations of the two massive vector particles by means of a simple observable. For a bipartite pure state, like the one in Eq. (5), the von Neumann entropy [4]

$$\mathcal{E} = -\mathrm{Tr}\big[\rho_{S_A}\ln\rho_{S_A}\big] = -\mathrm{Tr}\big[\rho_{S_B}\ln\rho_{S_B}\big] \quad (7)$$

quantifies entanglement; in Eq. (7), ρ_{S_A} and ρ_{S_B} are the reduced density matrices for the two subsystems S_A and S_B, which are the two spin-1 mesons in the decay under consideration. The von Neumann entropy of a two-qutrit system satisfies $0 \le \mathcal{E} \le \ln 3$. The first equality is true if and only if the bipartite state is separable, the second if the bipartite state is maximally entangled.

The optimal generalization of the Bell inequality in the case of a bipartite system made of two qutrits is the Collins, Gisin, Linden, Massar, and Popescu (CGLMP) inequality [24,25]. In order to explicitly write this condition, consider again the components S_A and S_B of the bipartite qutrit system. For the qutrit S_A, select two spin measurement settings, $\hat{S}_{A_1}$ and $\hat{S}_{A_2}$, which correspond to the projective measurement of two spin-1 observables having each three possible outcomes $\{0, 1, 2\}$—that, in our case, take values in $\{+1, 0, -1\}$. Similarly, the measurement settings and corresponding observables for the other qutrit S_B are $\hat{S}_{B_1}$ and $\hat{S}_{B_2}$. Then, denote by $P(A_i = B_j + k)$ the probability that the outcome S_{A_i} for the measurement of $\hat{S}_{A_1}$ and S_{B_j} for the measurement of $\hat{S}_{B_j}$, with i, j either 1 or 2, differ by k modulo 3. One can then construct the combination:

$$\begin{aligned}\mathcal{I}_3 = {} & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) \\ & + P(B_2 = A_1) - P(A_1 = B_1 - 1) - P(A_1 = B_2) \\ & - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1). \end{aligned} \quad (8)$$

For deterministic local models, this quantity satisfies the generalized Bell inequality

$$\mathcal{I}_3 \le 2, \quad (9)$$

which instead can be violated by computing the above joint probabilities using the rules of quantum mechanics. In quantum mechanics, $\mathcal{I}_3$ in Eq. (8) can be expressed as an expectation value of a suitable Bell operator $\mathcal{B}$ as

$$\mathcal{I}_3 = \mathrm{Tr}[\rho\mathcal{B}]. \quad (10)$$

The explicit form of $\mathcal{B}$ depends on the choice of the four measured operators $\hat{A}_i$ and $\hat{B}_i$. Hence, given the two-qutrit state ρ, it is possible to enhance the violation of the Bell inequality (9) through a specific choice of these operators. The numerical value of the observable $\mathcal{I}_3$ is bound to be less than or equal to 4. For the case of the maximally entangled state, the problem of finding an optimal choice of measurements has been solved [24]. By working in the single spin-1 basis formed by the eigenstates of the spin operator in the direction $\hat{z}$ with eigenvalues $\{+1, 0, -1\}$, the Bell operator takes a particular simple form:

$$\mathcal{B}=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (11)$$

after rotating it into the helicity basis from the so-called computational basis employed in [26].

Within the choice of measurements leading to the Bell operator, there is still the freedom of modifying the measured observables through local unitary transformations, which effectively corresponds to local changes of basis in the measurement of the polarizations. Correspondingly, the Bell operator undergoes the change:

$$\mathcal{B} \to (U \otimes V)^{\dagger} \cdot \mathcal{B} \cdot (U \otimes V), \quad (12)$$

where U and V are independent 3×3 unitary matrices. In the following we make use of this freedom to maximize the value of $\mathcal{I}_3$.

Results. Our results can now be given in a very concise form. The polarization amplitudes in Eq. (1) determine the polarization density matrix in Eq. (5) for the decay $B^0 \to J/\psi K^*(892)^0$. The density matrix makes it possible to estimate the observables in which we are interested.

We determine the rotation matrices U and V in the optimization procedure of Eq. (12) by means of the central values in Eq. (1).

We propagate the uncertainties in the polarization amplitudes in Eq. (1), taking into account also their correlations. We find that the entropy of entanglement among the polarizations of the final mesons for the decay $B^0 \to J/\psi K^*(892)^0$ is given by

$$\mathcal{E} = 0.756 \pm 0.009. \quad (13)$$

The result in Eq. (13) represents a detection of the presence of quantum entanglement with a significance well above 5σ (nominally 84σ).

Propagating the uncertainties through the expectation value of the Bell operator, while keeping the two matrices U and V fixed, we determine that $\mathcal{I}_3$ for the decay $B^0 \to J/\psi K^*(892)^0$ has expectation value

TABLE I. Entanglement and Bell operator $\mathcal{I}_3$ for some B-meson decays. An asterisk indicates that the correlations in the uncertainties of the polarization amplitudes are not given in the corresponding reference and therefore only an upper bound on the propagated uncertainty can be computed.

	$\mathcal{E}$	$\mathcal{I}_3$
• $B^0 \to J/\psi K^*(892)^0$ [5]	0.756 ± 0.009	2.548 ± 0.015
• $B^0 \to \phi K^*(892)^0$ [18]	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$
• $B^0 \to \rho K^*(892)^0$ [19]	$0.450 \pm 0.077^*$	$2.208 \pm 0.151^*$
• $B_s \to \phi\phi$ [20]	0.734 ± 0.037	2.525 ± 0.064
• $B_s \to J/\psi \phi$ [21]	0.731 ± 0.032	2.462 ± 0.080

$$\mathcal{I}_3 = 2.548 \pm 0.015, \quad (14)$$

and therefore the CGLMP inequality $\mathcal{I}_3 < 2$ is violated with a significance well above 5σ (nominally 36σ).

Other decays of B mesons provide polarization amplitudes that can be used in similar fashion to test the Bell inequality. We list in Table I the values for the entanglement $\mathcal{E}$ and the Bell operator $\mathcal{I}_3$ for some of the decays we have considered. Specifically, $\mathcal{I}_3 < 2$ is violated with a significance of more than 5σ in the decays $B_s \to \phi\phi$ and $B_s \to J/\psi\phi$.

Outlook. We have shown that quantum entanglement is present and the Bell inequality is violated by the data on the polarization amplitudes in the decay $B^0 \to J/\psi K^*(892)^0$, and other similar decays. The presence of entanglement and the Bell inequality violation have very large significance and establish these property of quantum mechanics at high energies in a collider setting and in a system in which all standard model interactions are involved. It is the first time that the violation of the inequality is shown to take place in a system of two qutrits and between two different particles.

We are aware that potential loopholes are present in any test of the Bell inequality. These loopholes have been closed in low-energy tests with photons [6,27] and in atomic physics [28].

To close the *locality loophole*—which exploits events not separated by a spacelike interval, as is the case for the $J/\psi K^*$ decays—one must consider decays in which the produced particles are identical, as in the $B_s \to \phi\phi$ decay, and therefore their lifetimes are also the same. The actual decays take place with an exponential spread, with, in the $\phi\phi$ case, more than 90% of the events being separated by a spacelike interval.

The presence and relevance of other possible loopholes is still an open question in the high-energy setting and is beyond the scope of the present Letter.

Acknowledgments. We thank M. Dorigo for discussions on various aspects of B-meson physics. L. M. is supported by Estonian Research Council Grant No. PRG356.

[1] J. S. Bell, Phys. Phys. Fiz. **1**, 195 (1964); *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, England, 1987–2004).
[2] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).
[3] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **81**, 5039 (1998).
[4] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[5] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. D **88**, 052002 (2013).
[6] B. Hensen, H. Bernien, A. E. Dreau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenberg, R. F. L. Vermeulen, R. N. Schouten, C. Abellan *et al.*, Nature (London) **526**, 682 (2015).
[7] C. P. Jessop *et al.* (CLEO Collaboration), Phys. Rev. Lett. **79**, 4533 (1997).
[8] T. Affolder *et al.* (CDF Collaboration), Phys. Rev. Lett. **85**, 4668 (2000).
[9] R. Itoh *et al.* (Belle Collaboration), Phys. Rev. Lett. **95**, 091601 (2005).
[10] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **76**, 031102 (2007).
[11] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **102**, 032001 (2009).
[12] F. Benatti and R. Floreanini, Phys. Rev. D **57**, 1332 (1998); Mod. Phys. Lett. A **14**, 1519 (1999); Eur. Phys. J. C **13**, 267 (2000).
[13] A. Go (Belle Collaboration), J. Mod. Opt. **51**, 991 (2004).
[14] R. L. Workman *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
[15] A. S. Dighe, I. Dunietz, H. J. Lipkin, and J. L. Rosner, Phys. Lett. B **369**, 144 (1996).
[16] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999).
[17] A. Yamamoto and H. Suganuma, Phys. Rev. D **79**, 054504 (2009).
[18] K. F. Chen *et al.* (Belle Collaboration), Phys. Rev. Lett. **94**, 221804 (2005).
[19] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 05 (2019) 026.
[20] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **131**, 171802 (2023).
[21] G. Aad *et al.* (ATLAS Collaboration), Eur. Phys. J. C **81**, 342 (2021).
[22] M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, Eur. Phys. J. C **83**, 823 (2023).
[23] M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, J. High Energy Phys. 09 (2023) 195.
[24] D.Collins, N. Gisin, N. Linden, S. Massar, and S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).
[25] D. Kaszlikowski, L. C. Kwek, J. L. Chen, M. Zukowski, and C. H. Oh, Phys. Rev. A **65**, 032118 (2002).
[26] A. Acin, T. Durt, N. Gisin, and J. I. Latorre, Phys. Rev. A **65**, 052325 (2002).
[27] M. Giustina *et al.*, Phys. Rev. Lett. **115**, 250401 (2015); L. K. Shalm *et al.*, Phys. Rev. Lett. **115**, 250402 (2015).
[28] W. Rosenfeld, D. Burchardt, R. Garthoff, K. Redeker, N. Ortegel, M. Rau, and H. Weinfurter, Phys. Rev. Lett. **119**, 010402 (2017).