

## Testing the nonunitarity of the leptonic mixing matrix at $\text{FASER}\nu$ and $\text{FASER}\nu 2$

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The  $\text{FASER}\nu$  experiment has detected the first neutrino events coming from LHC. Near future high-statistic neutrino samples will allow us to search for new physics within the neutrino sector. Motivated by the forthcoming promising  $\text{FASER}\nu$  neutrino data, and its successor,  $\text{FASER}\nu 2$ , we study its potential for testing the unitarity of the neutrino lepton mixing matrix. Although it would be challenging for  $\text{FASER}\nu$  and  $\text{FASER}\nu 2$  to have strong constraints on this kind of new physics, we discuss its role in contributing to a future improved global analysis.

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*Introduction.* The neutrino oscillations discovery tells us that neutrinos have a small mass. Compared with other fundamental particles, the nonzero neutrino mass and its smallness strongly suggest that the Standard Model (SM) needs an extension to describe the neutrino oscillation picture. Also, the SM needs a new mechanism or an explanation for the mass degeneracy of the active neutrinos. An attempt to describe the mass generation of neutrinos is the seesaw mechanism [1–5]. The type-I seesaw mechanism uses neutral heavy leptons (NHL), with Majorana mass, as a messenger to transport mass to the light neutrinos. Due to their heavy mass, the NHLs do not oscillate to active neutrinos. However, their effects are contained in a submatrix of the full  $N \times N$  lepton-mixing matrix, with  $N$  the number of light plus heavy neutrino species. As a consequence, the  $3 \times 3$  mixing matrix of the light neutrino is nonunitary. In recent years, many experiments have been used to test non-unitarity effects [6–14]. Among the new experiments expected to give further information about neutrino interactions, we can consider the case of  $\text{FASER}$ , and more especially  $\text{FASER}\nu$ , which will measure the neutrino cross section in a new energy

window, making this experiment an exciting place to study either Standard Model physics or beyond.

In this work, we will explore the nonunitarity sensitivity in the  $\text{FASER}\nu$  and  $\text{FASER}\nu 2$  experiments. With different neutrino channels measured at high energies,  $\text{FASER}\nu$  will test nonunitarity effects in an experimental setup different from any other experiment. Therefore, this makes the study of a future nonunitary test at  $\text{FASER}\nu$  interesting.  $\text{FASER}\nu$  experiment works at 100–1000 GeV [15], and the momentum transfer for this fixed target detector will be around  $Q^2 \sim (10 \text{ GeV})^2$  [15]. At this energy, it might be possible to generate NHL for specific theories, like a linear or inverse seesaw below the electroweak energy, for example, in the mass range of GeV, as was studied in [16]. In this work, we will focus on a model-independent formalism for nonunitarity [17], valid for neutrino mass eigenstates at high mass scales, above hundreds of GeV, and show the sensitivity to the corresponding parameters. A different study for the nonunitary case was done previously [18]. However, their approach is different in terms of the theoretical description of nonunitarity as well as in the study of other neutrino observables in their analysis.

The paper structure is: in Sec. II we briefly review the nonunitarity formalism and the zero-distance approximation used in this work. In Sec. III we show the statistical procedure that we follow to obtain the sensitivity in the nonunitarity formalism. The  $\chi^2$  analysis and the sensitivity of the nonunitarity parameters are discussed in Sec. IV. Finally, in Sec. V, we talk about the conclusions and perspectives.

*Nonunitarity.* Any model with additional neutrino species implies the nonunitarity of the standard leptonic mixing

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matrix for the three oscillation neutrino picture. In this scenario, the three times three mixing matrix is a block of the complete mixing matrix  $U^{n \times n}$ , with  $n$  the total number of neutrino eigenstates. Studies on the implications of the nonunitarity can be found in the literature [5–8], as well as constraints from either neutrino experiments or those coming from charge leptons [9–11]. Recent constraints from a combined analysis of short and long-baseline experiments are reported in Ref. [12].

For the general case of 3 active neutrinos and  $n - 3$  heavy neutrino states, we can define the matrix  $U^{n \times n}$  as compose of four submatrices

$$U^{n \times n} = \begin{pmatrix} N & S \\ V & T \end{pmatrix}, \quad (1)$$

where  $N$  is the  $3 \times 3$  matrix in the light-active neutrino sector, and  $S$  describes the contribution of the extra isosinglets states to the three active neutrinos.

The neutral heavy leptons effects in the active neutrino oscillation can be factorized into the  $N$  matrix as follows [17]:

$$N = N^{\text{NP}} U = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U, \quad (2)$$

where  $U$  is the usual leptonic mixing matrix, and  $N^{\text{NP}}$  is the matrix characterizing the unitary violation that arises when new heavy neutrino states are introduced.

Clearly, the  $3 \times 3$   $N$  matrix is not unitary and, in this case,  $NN^\dagger = I - SS^\dagger = N^{\text{NP}} U U^\dagger N^{\text{NP}\dagger}$

$$= \begin{pmatrix} \alpha_{11}^2 & \alpha_{11}\alpha_{21}^* & \alpha_{11}\alpha_{31}^* \\ \alpha_{11}\alpha_{21} & \alpha_{22}^2 + |\alpha_{21}|^2 & \alpha_{22}\alpha_{32}^* + \alpha_{21}\alpha_{31}^* \\ \alpha_{11}\alpha_{31} & \alpha_{22}\alpha_{32} + \alpha_{31}\alpha_{21}^* & \alpha_{33}^2 + |\alpha_{31}|^2 + |\alpha_{32}|^2 \end{pmatrix}. \quad (3)$$

The  $\alpha$  parameters are related to the mixings  $\cos \theta_{ij}$  and  $\sin \theta_{ij}$  as [9]

$$\begin{aligned} \alpha_{11} &= c_{1n} c_{1n-1} c_{1n-2} \cdots c_{14}, \\ \alpha_{22} &= c_{2n} c_{2n-1} c_{2n-2} \cdots c_{24}, \\ \alpha_{33} &= c_{3n} c_{3n-1} c_{3n-2} \cdots c_{34}, \end{aligned} \quad (4)$$

where  $\theta$  is the oscillation angle,  $c_{ij} = \cos \theta_{ij}$ . The non-diagonal terms are [9]

$$\begin{aligned} \alpha_{21} &= c_{2n} c_{2n-1} \cdots c_{25} \eta_{24} \bar{\eta}_{14} + c_{2n} \cdots c_{26} \eta_{25} \bar{\eta}_{15} c_{14} + \cdots \\ &\quad + \eta_{2n} \bar{\eta}_{1n} c_{1n-1} c_{1n-2} \cdots c_{14}, \\ \alpha_{32} &= c_{3n} c_{3n-1} \cdots c_{35} \eta_{34} \bar{\eta}_{24} + c_{3n} \cdots c_{36} \eta_{35} \bar{\eta}_{25} c_{24} + \cdots \\ &\quad + \eta_{3n} \bar{\eta}_{2n} c_{2n-1} c_{2n-2} \cdots c_{24}, \\ \alpha_{31} &= c_{3n} c_{3n-1} \cdots c_{35} \eta_{34} \bar{\eta}_{14} c_{24} + c_{3n} \cdots c_{36} \eta_{35} c_{25} \bar{\eta}_{15} c_{14} \\ &\quad + \cdots + \eta_{3n} c_{2n} \bar{\eta}_{1n} c_{1n-1} c_{1n-2} \cdots c_{14}, \end{aligned} \quad (5)$$

with  $\eta_{ij} = \sin \theta_{ij} e^{-i\delta_{ij}}$ , where  $\delta_{ij}$  is the  $CP$  phase associated to  $\theta_{ij}$ . Also, the nondiagonal parameters are related with the diagonal ones through the triangle inequality [9]:

$$\alpha_{ij} \leq \sqrt{(1 - \alpha_{ii}^2)(1 - \alpha_{jj}^2)}. \quad (6)$$

As a consequence of the nonunitarity, the oscillation probability will change. The new oscillation probability is [17,19]:

$$\begin{aligned} P_{\alpha\beta} &= \sum_{i,j}^3 N_{\alpha i}^* N_{\beta i} N_{\alpha j} N_{\beta j}^* \\ &\quad - 4 \sum_{j>i}^3 \text{Re}[N_{\alpha j}^* N_{\beta j} N_{\alpha i} N_{\beta i}^*] \sin^2 \left( \frac{\Delta m_{ji}^2 L}{4E_\nu} \right) \\ &\quad + 2 \sum_{j>i}^3 \text{Im}[N_{\alpha j}^* N_{\beta j} N_{\alpha i} N_{\beta i}^*] \sin \left( \frac{\Delta m_{ji}^2 L}{2E_\nu} \right). \end{aligned} \quad (7)$$

In this work, we focus on the analysis of nonunitarity formalism in the FASER experiment. In FASER, the typical energy is in the range of 100–1000 GeV and the distance between the source and the detector is  $L = 480$  m. Therefore, the wavelength is enough to consider that our results are in the regime of a short baseline. Also, as a good approximation, we can work in the so-called zero-distance approximation. The oscillation probability in this zero-distance case is

$$P_{\alpha\beta} = \sum_{i,j}^3 N_{\alpha i}^* N_{\beta i} N_{\alpha j} N_{\beta j}^*, \quad (8)$$

where Greek letters refers to the lepton-flavor index and Latin letters denote the mass state index. In terms of the nonunitary parameters  $\alpha_{ij}$  from Eq. (2), the oscillation probabilities in this approximation are

$$\begin{aligned} P_{\mu e} &= \alpha_{11}^2 |\alpha_{21}|^2, \\ P_{e\tau} &= \alpha_{11}^2 |\alpha_{31}|^2, \\ P_{\mu\tau} &\approx \alpha_{22}^2 |\alpha_{32}|^2, \\ P_{ee} &= \alpha_{11}^4, \\ P_{\mu\mu} &= (|\alpha_{21}|^2 + \alpha_{22}^2)^2, \\ P_{\tau\tau} &= (|\alpha_{31}|^2 + \alpha_{32}^2 + \alpha_{33}^2)^2. \end{aligned} \quad (9)$$

*Experiment description and analysis procedures.* FASER $\nu$  experiment will provide an abundant neutrino flux with thousands of expected events in the very near future. The first few neutrino events have already been recorded by FASER $\nu$  [20]. This new experiment at the LHC opens a new opportunity to study the nonunitarity of the neutrino oscillation matrix in the search for heavy neutrino states due to the high statistics for neutrino events. Being a high energy neutrino flux that spans from 100 GeV to 1 TeV [15], the momentum transfer in this fix target experiment is expected to be in the range of  $Q^2 \sim (10 \text{ GeV})^2$ , allowing an

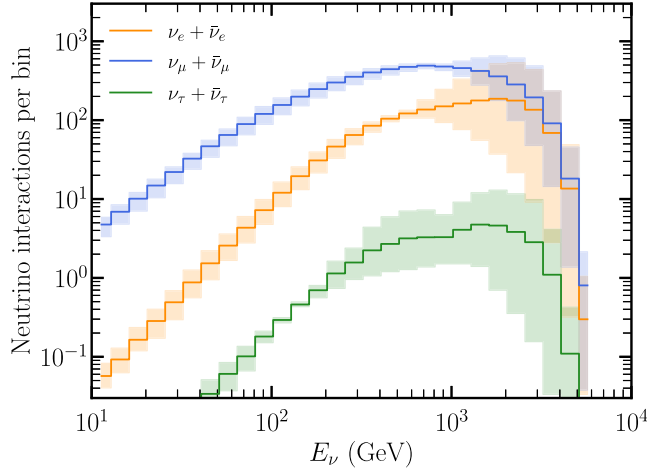


FIG. 1. Expected charged-current neutrino interactions at the FASER $\nu$  detector with  $150 \text{ fb}^{-1}$  integrated luminosity, as a function of neutrino energy (recomputed from Ref. [22]).

indirect test for new heavy states through the nonunitarity of the leptonic mixing matrix. The FASER $\nu$  experiment has a total tungsten target mass of 1.2 tons and a baseline of 480 m. The FASER $\nu$  collaboration measures these events by measuring the charged current (CC) with high accuracy. The neutral-current (NC) events are more complicated to measure due to the absence of charged leptons in the final states, although there are some attempts to describe the neutral-current (NC) interactions [21]. Although the FASER $\nu$  collaboration has estimated the number of SM neutrino interactions at the detector [15], a more recent prediction of these events and their uncertainties using various event generators has been made [22]. Figure 1 shows the expected neutrino interactions at the FASER $\nu$  detector for each flavor. This figure was recomputed<sup>1</sup> using the information given in Ref. [22] and coincides with the corresponding figure of this reference. We will use this prediction in the following analyses on nonunitarity.

An upgrade plan for the detection of collider neutrinos in the high luminosity era of the LHC is the FASER $\nu$ 2 detector [23]. With a mass of 20 tonnes and 20 times the luminosity of its predecessor, it will be able to detect two orders of magnitude more events than FASER $\nu$ . Reference [22] has also estimated the number of interactions at this detector (footnote 1).

In this work, we will use the zero-distance approximation to have a forecast on the sensitivity of FASER $\nu$  and FASER $\nu$ 2 to nonunitary  $\alpha$  parameters. In this analysis, we will use 3 observables: electron, muon, and tau neutrino events. The SM expected events can be computed as

$$N_{\alpha}^{\text{SM}} = \varepsilon_{\alpha} N_T \int f(E_{\text{reco}}) R(E_{\text{reco}}, E_{\nu}) \sigma_{\alpha}(E_{\nu}) \phi_{\alpha} dE_{\nu} dE_{\text{reco}}, \quad (10)$$

<sup>1</sup><https://github.com/KlingFelix/FastNeutrinoFluxSimulation>.

TABLE I. Expected number of events and systematic uncertainties used in the analysis, for different neutrino energy ranges. For the projections of FASER $\nu$ 2, we will consider two systematic uncertainty scenarios, 5% and 10% (see text for details).

Lepton flavor	FASER $\nu$		FASER $\nu$ 2	
	$10^2$ – $10^4$ GeV	100–600 GeV	$10^2$ – $10^4$ GeV	100–600 GeV
$e$	$1095 \pm 937$	$307 \pm 101$	44230	20775
$\mu$	$2807 \pm 909$	$1163 \pm 190$	193630	85044
$\tau$	$19 \pm 19$	$6 \pm 4$	767	314

where  $\phi_{\alpha}$  is the expected flux at the detector,  $\sigma_{\alpha}$  is the neutrino-nucleus DIS cross section,  $R(E_{\text{reco}}, E_{\nu})$  is a Gaussian smearing function of width  $0.3E_{\nu}$ ,  $f(E_{\text{reco}})$  is the vertex reconstruction efficiency (taken from Fig. 9 of Ref. [15]),  $\varepsilon_{\alpha}$  is the charged-lepton identification efficiency ( $\varepsilon_e = 100\%$ ,  $\varepsilon_{\mu} = 86\%$ ,  $\varepsilon_{\tau} = 76\%$ ), and  $N_T$  is the number of targets in the detector. To estimate the number of events at both FASER $\nu$  and FASER $\nu$ 2 detectors, we take the estimated interactions from Ref. [22] and apply smearing, vertex reconstruction and charged-lepton identification efficiencies. Our estimated number of events for the complete neutrino energy range ( $10^2$ – $10^4$  GeV), along with their uncertainties, are shown in Table I. As can be seen from Fig. 1, the uncertainties on the number of interactions at the detector are high, and come mostly from flux estimations. We propose a scenario where only interactions between 100–600 GeV are taken into account in order to reduce the systematic error significantly. The expected number of events in this energy regime is also shown in Table I.

We compute the expected sensitivity through a  $\chi^2$  analysis:

$$\chi^2 = \sum_{\alpha=e}^{\tau} \frac{(N_{\alpha}^{\text{NU}} - N_{\alpha}^{\text{exp}})^2}{\sigma_{\alpha}^2} + \sum_{ij} \frac{(\alpha_{ij} - \delta_{ij})^2}{\sigma_{ij}^2}, \quad (11)$$

where  $N_{\alpha}^{\text{exp}}$  is the expected measured number of events per neutrino flavor,  $N_{\alpha}^{\text{NU}}$  is the events number computed when nonunitarity is present,  $\alpha$  refers to the lepton flavor, and  $\sigma_{\alpha}$  is the total expected error (statistical and systematic). Regarding the systematic uncertainties, for FASER $\nu$ , we symmetrized this error as an approximation, whereas for FASER $\nu$ 2 we consider two scenarios, 5% and 10%, motivated by the expected improvement in the flux estimation by the time the HL-LHC starts taking data. To make a complete analysis considering the three observables that FASER $\nu$  expects to measure, we have to consider both appearance and disappearance channels. Therefore, the complete theoretical prediction will depend on six different nonunitary parameters. Therefore, the complete expressions will have more parameters than FASER $\nu$  observables, and we need to consider priors to perform our analysis. In Eq. (11), we have included priors to the values of  $\alpha_{ij}$  that

will be marginalized in our fit, using as errors,  $\sigma_{ij}$ , the constraints reported in Ref. [12]. Notice that these constraints were presented at 90% C.L., then our results can be considered as conservative.

The theoretical expected number of events will be then expressed as

$$N_{\alpha}^{\text{NU}} = \frac{1}{\alpha_{11}^2 (\alpha_{22}^2 + |\alpha_{21}|^2)} \left( N_{\alpha}^{\text{SM}} P_{\alpha\alpha} + \sum_{\beta \neq \alpha} P_{\alpha\beta} N_{\beta}^{\text{SM}} \right), \quad (12)$$

where  $P_{\alpha\alpha}$  and  $P_{\alpha\beta}$  are defined in Eq. (9) and depend on the nonunitary parameters,  $N_{\alpha}^{\text{SM}}$  is the standard model predicted number of events for the flavor  $\alpha$ , and there is no sum over  $\alpha$ . The prefactor in the right-hand side of this equation corresponds to the correction due to the measurement of the Fermi constant,  $G_F$  [7,8,24], that comes from muon decay and in the case of nonunitary must be considered as  $G_F = G_{\mu} / \sqrt{\alpha_{11}^2 (\alpha_{22}^2 + |\alpha_{21}|^2)}$ , with  $G_{\mu}$  as the Fermi constant measured in muon decay.

Since we consider several nonunitary parameters at a time, it may happen that the disappearance events compensate for the appearance events and no effect would be visible. Therefore, to take into account the combined effect of the six different parameters, considering that we have only three observables (the three neutrino flavors), we will consider only one free parameter at a time by marginalizing over the other five parameters. It is important to remark that in the computation of  $\chi^2$ , we take into account the triangle inequality condition for all the off-diagonal  $\alpha_{ij}$  parameters given in Eq. (6). In other words, we have included three triangle inequality conditions.

*Results.* We will show in this section the results of computing the expected sensitivity for FASER $\nu$  and FASER $\nu 2$  in two different energy windows. In Fig. 2 we illustrate the expected FASER $\nu$  sensitivity for the two energy regimes already mentioned. As discussed in the previous sections, the energy range from 100–600 GeV has smaller uncertainties. Therefore, besides the case with the full energy range, we also consider this reduced region. However, for FASER $\nu 2$ , the analysis has been performed in the energy range of 100–600 GeV because we consider systematic uncertainties very significant beyond this energy range. Results are shown in Fig. 3. We must remember that this analysis considers every appearance and disappearance channel for all neutrino flavors, as well as priors from the current limits on  $\alpha_{ij}$ , hence it provides a realistic and useful projection of the FASER $\nu$  and FASER $\nu 2$  capabilities to constrain the nonunitary parameters. A summary of the expected 90% C.L. sensitivity to each parameter is shown in Table II. From this table, and from Fig. 2, we can notice that it is not expected that FASER $\nu$  could improve the current limits on nonunitary parameters. Nevertheless, for FASER $\nu 2$  we found competitive results, mainly for  $\alpha_{11}$  and  $\alpha_{33}$ . On the other hand, for the case of FASER $\nu 2$ , we illustrate in Fig. 3 how the sensitivity to nonunitarity can play a role in future global analysis, especially if we restrict ourselves to the preferable energy window that goes from 100–600 GeV and if FASER $\nu 2$  can keep under control its systematic uncertainties. Since the experiments plans to collect high statistics events (below 1%) it is reasonable to expect an important campaign to reduce systematic effects. From Table II we can see that the most promising

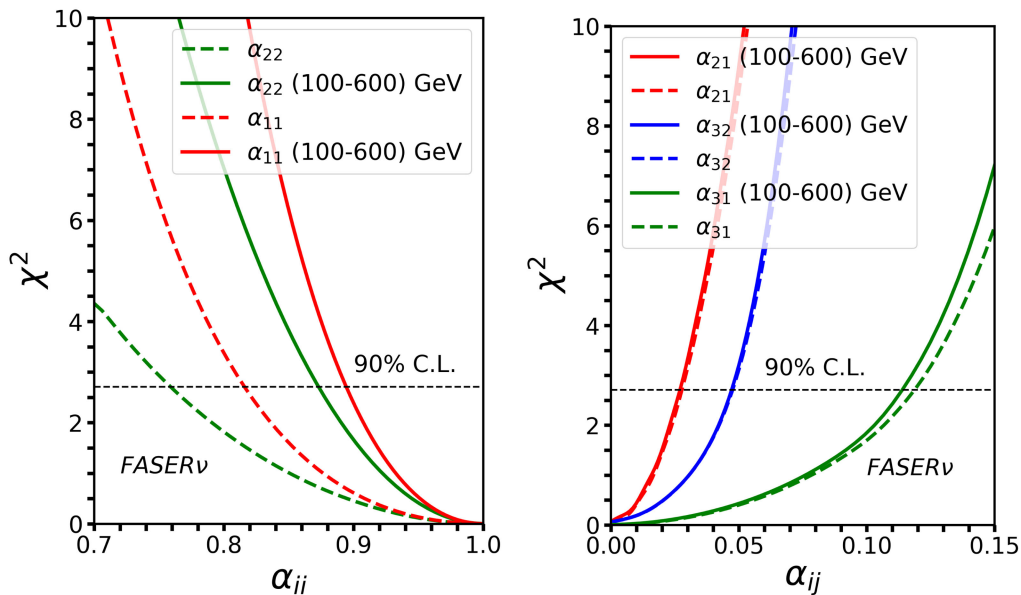


FIG. 2. Sensitivity to one at a time diagonal (left panel) and nondiagonal (right panel) nonunitarity parameters for FASER $\nu$ . The dashed curves represent the sensitivity for the full FASER $\nu$  energy regime, while the solid curves represent the scenario with events only between 100–600 GeV. The horizontal line shows the 90% C.L. Besides marginalization over the other nonunitary parameters, the triangle inequality conditions have also been taken into account.

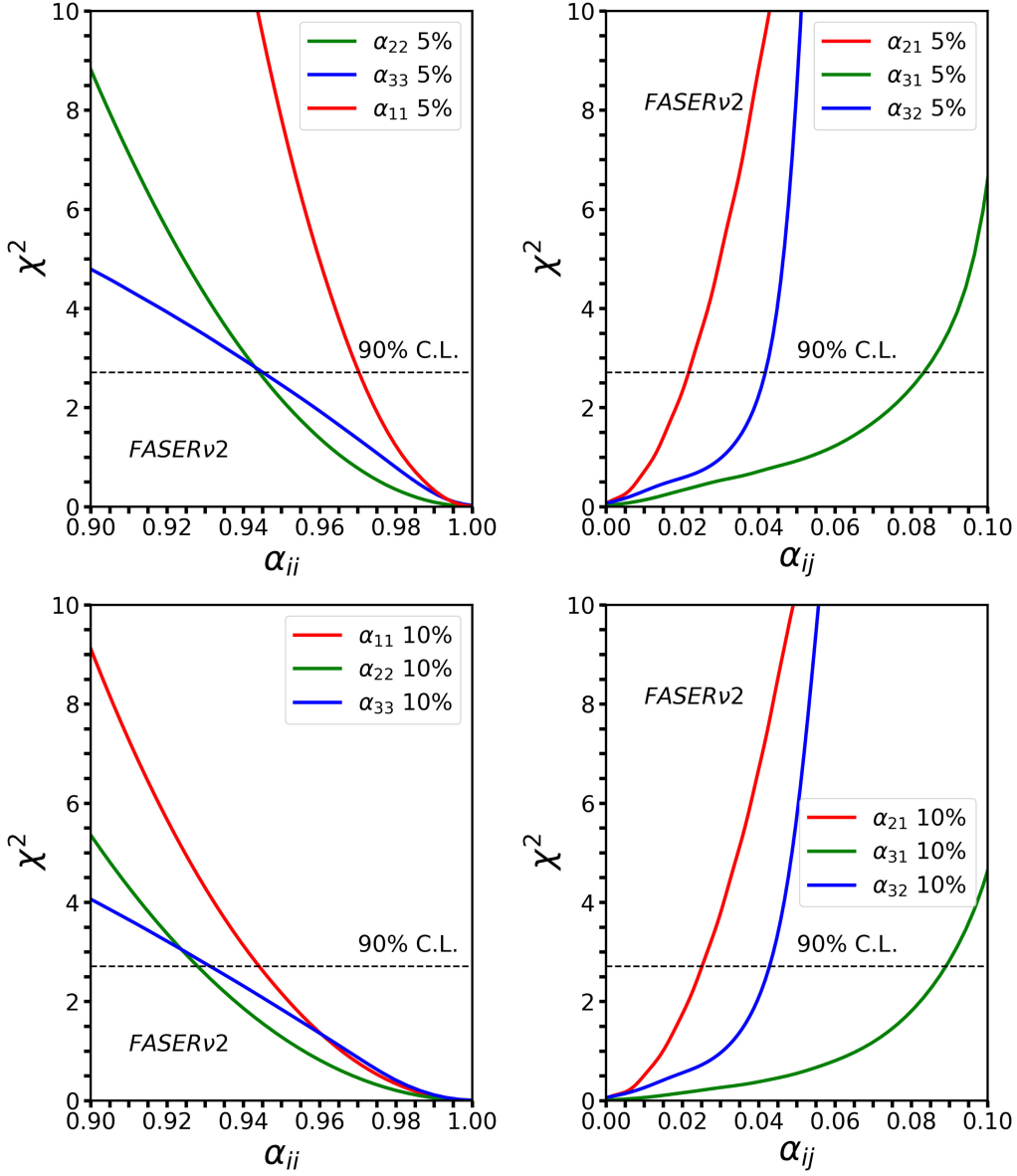


FIG. 3. Sensitivity to one at a time diagonal (left) and nondiagonal (right) nonunitarity parameters for FASER $\nu$ 2. The upper (lower) panels correspond to the case where the systematic uncertainty is taken as 5% (10%). The horizontal line shows the 90% C.L. Besides marginalization over the other nonunitary parameters, the triangle inequality conditions have also been taken into account.

TABLE II. Expected sensitivities at 90% C.L. for all the nonunitary  $\alpha_{ij}$  parameters for the FASER $\nu$  and FASER $\nu$ 2 experiments, for different energy ranges and systematic uncertainties. For FASER $\nu$ , there is no constraint on  $\alpha_{33}$ . In the last column we show the current global limits from Ref. [12].

Parameter	FASER $\nu$		FASER $\nu$ 2		Current limit
	10 <sup>2</sup> –10 <sup>4</sup> GeV	100–600 GeV	100–600 GeV (5%)	100–600 GeV (10%)	
$\alpha_{11} \geq$	0.818	0.894	0.970	0.944	0.969
$\alpha_{22} \geq$	0.760	0.873	0.944	0.928	0.995
$\alpha_{33} \geq$	...	...	0.945	0.932	0.890
$\alpha_{21} \leq$	0.028	0.027	0.022	0.025	0.013
$\alpha_{31} \leq$	0.118	0.114	0.083	0.089	0.033
$\alpha_{32} \leq$	0.048	0.048	0.042	0.043	0.009

sensitivities are expected for the case of  $\alpha_{11}$  and, especially,  $\alpha_{33}$ , where the tau neutrino FASER $\nu$ 2 events will represent a window of opportunity to shed light on this parameter.

*Conclusions.* In this work, we analyzed the nonunitary effects in the context of the FASER $\nu$  and FASER $\nu$ 2 experiments. We used the approximation of zero distance and performed an analysis of the expected sensitivity for all the nonunitary parameters. We find that the expected FASER $\nu$  sensitivity to nonunitarity test is in general poor, being best sensitive to the  $\alpha_{21}$  parameter that might give a complementary information, useful perhaps in a global analysis. On the other hand, for the FASER $\nu$ 2 case, the perspectives are much better and the sensitivity to the  $\alpha_{33}$  parameter could be quite competitive with current restrictions, thanks to the relatively large number of tau neutrino events that are expected in this detector.

Besides, FASER $\nu$ 2 also has the possibility to give a competitive constraint on the  $\alpha_{11}$  parameter. Since the expected statistic in FASER $\nu$ 2 is high, the main challenge rest in reducing the systematic uncertainties. In summary, future measurements at FASER $\nu$ 2 may test the nonunitarity of the leptonic mixing angle in a different neutrino channel and energy region and may have competitive sensitivities for some of the nonunitary parameters.

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