Letter

Precision unification and the scale of supersymmetry

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In this paper, we study the implications of precise gauge coupling unification on supersymmetric particle masses. We argue that precise unification favors the superpartner masses that are in the range of several TeV and well beyond. We demonstrate this in the minimal supersymmetric theory with a common sparticle mass threshold, and two simple high-scale scenarios; minimal supergravity and minimal anomaly-mediated supersymmetry. We also identify candidate models with a Higgsino or a wino dark matter candidate. Finally, the analysis shows unambiguously that unless one takes foggy naturalness notions too seriously, the lack of direct superpartner discoveries at the LHC has not diminished the viability of supersymmetric unified theories in general nor even precision unification in particular.

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Introduction. Although the data at the Large Hadron Collider (LHC) is consistent with the Standard Model (SM) and there are no unambiguous signs of new physics up to the TeV range, supersymmetry still remains a plausible extension to the SM for several reasons. For example, it can explain the origin of the weak scale [1], provide for the unification of gauge interactions [2], and accommodate a viable dark matter (DM) candidate [3,4].

Both the discovery of SM-like Higgs boson with $m_h \sim$ 125 GeV and the lack of evidence for superpartners at the LHC (albeit under simplifying assumptions) suggest that supersymmetry exists above the TeV scale, if it exists at all. It must be kept in mind that all the undiscovered superpartners, unlike the SM fermions and the massive gauge bosons, can get their masses entirely from supersymmetry-breaking terms and therefore in principle can be much heavier than the weak scale. Moreover, supersymmetric theories obey decoupling as the supersymmetry-breaking scale is raised. Some phenomenologically viable scenarios where the gauginos are somewhat above the TeV scale and the sfermions are much heavier at order 100 TeV and beyond can be found, e.g., in Refs. [5–8].

In this letter, we will explore the implications of the precision unification conjecture; supersymmetry is a correct principle of nature, and the gauge couplings unify at a high scale with high-scale threshold corrections much smaller in magnitude than naive expectations from grand unified theories (GUTs).

The reason for this conjecture is that within the minimal supersymmetric theory the three gauge couplings, when renormalization-group flowed to the high scale, meet much more closely than typical threshold corrections of a grand unification group from its remnants of high-dimensional representations [9,10]. The natural implication of this has long been to put more stock in the idea of supersymmetry. But the extraordinary confluence of gauge couplings, which will be demonstrated below, may be giving a message stronger than that; the high-scale threshold corrections are highly suppressed.

One could speculate on reasons for suppressed threshold corrections. Perhaps the precision unification comes from an orbifold GUT that projects out extraneous representations, which may even arise from string compactification [11,12]. We are agnostic about any given precise underlying reason, but we take seriously the hint of precision gauge coupling unification as embodied in the conjecture above.

Supersymmetric frameworks. Precision unification has been studied in the past. For example, it has been recognized within TeV-scale minimal supersymmetric Standard Model (MSSM) that precise unification can follow from small μ-term and disengaging the gluino mass from the normal assumptions of "GUT-normalized" gaugino mass hierarchies [12,13]. Such ideas remain valid. However, in the present day we also know that the LHC has not found superpartners and that the Higgs boson mass has a rather high value of 125 GeV, consistent with heavy sparticle mass spectrum. This allows for a much heavier superpartner spectrum and the prospect of finding precision unification even within the more standard supersymmetric frameworks. The two straightforward approaches to supersymmetry that we employ to investigate the consequences of our precision unification conjecture are minimal supergravity with its GUT-normalized gaugino mass hierarchy (mSUGRA) and minimal anomaly mediation (mAMSB) with its special anomaly-mediated gaugino mass hierarchy. Both of these scenarios are reviewed in detail in [14].

To be more precise, we first study these implications for the MSSM with a common superpartner mass threshold. After that, we consider mSUGRA and mAMSB rigorously. The MSSM particle spectrum can be determined by just a few parameters, and we use well-tested computational tools in the literature to do multiloop renormalization group flow and mass determinations.

Supersymmetry-breaking is gravity mediated in mSUGRA, and all superpartner masses are somewhat similar in mass. On the other hand, gaugino masses are mediated via the superconformal anomaly in mAMSB, giving gaugino masses in a distinctive hierarchy and one-loop order lower than scalar masses [15–19].

The Higgs boson mass is an output in the supersymmetric theories, being a function of other masses and couplings already specified by the model. When identifying models with exact gauge unification we require the lightest CP-even neutral Higgs boson mass to be ∼125 GeV within uncertainties of the calculation.

Furthermore, we also identify regions of parameter space where the lightest neutralino is the lightest supersymmetric particle (LSP), and can generate the required thermal abundance of Higgsino or wino DM assuming R-parity is conserved [5,7,20–33].

MSSM with a common threshold. It is well-known that, unlike in the SM, the gauge couplings approximately unify in the MSSM with supersymmetric particle masses roughly around the TeV scale. As a measure of unification of the gauge couplings, we define

$$\frac{\rho_\lambda}{48\pi^2} \equiv \sqrt{\sum_{i\neq j}\left(\frac{1}{g_i^2}-\frac{1}{g_j^2}\right)^2}, \tag{1}$$

with i, $j = 1$, 2, 3. Here, g_i are the gauge couplings with the usual GUT normalization. The minimum value of ρ_λ, obtained at the scale $\mu_\star$, is denoted by $\rho_\lambda^{\rm min}$ [$\equiv \rho_\lambda(\mu_\star)$].

Within standard grand unified theories $\rho_\lambda^{\rm min}$ is a weighted logarithmic mass sum of remnant high-scale representations. Specifically, assuming degenerate masses within an irreducible representation, we have [9,34]

$$\begin{aligned}\frac{1}{g_i^2(\mu_\star)}-\frac{1}{g_j^2(\mu_\star)} &= (I_j^{V_n}-I_i^{V_n})\left(1+21\ln\frac{\mu_\star}{M_{V_n}}\right)\\ &\quad-\left(I_j^{S_n}-I_i^{S_n}\right)\ln\frac{\mu_\star}{M_{S_n}}\\ &\quad-8\left(I_j^{F_n}-I_i^{F_n}\right)\ln\frac{\mu_\star}{M_{F_n}},\end{aligned} \tag{2}$$

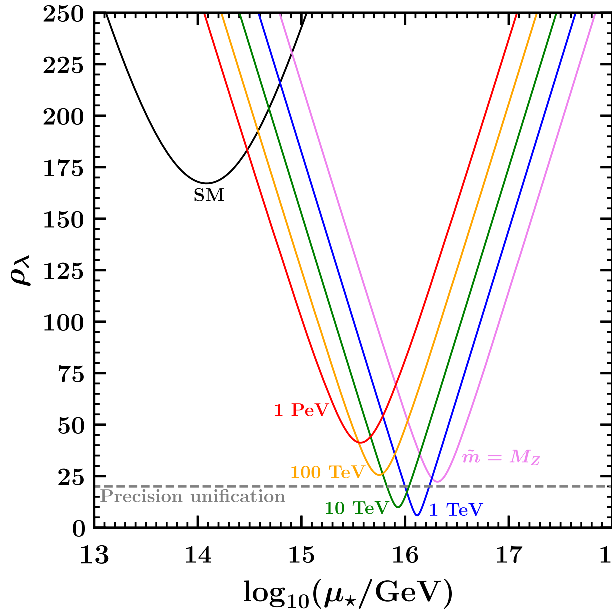

FIG. 1. ρ_λ as a function of the putative unification scale in the SM (black line) and in the MSSM with a common supersymmetric particle mass threshold $\tilde m$ (various colored lines for various $\tilde m$ as labeled).

with an implicit sum over n different particles for each of the vectors V_n, scalars S_n, and fermions F_n. Here, I_i^X are the Dynkin indices of the representation of X under $(SU(3)_c, SU(2)_L, U(1)_Y)$ for $i = (3, 2, 1)$, respectively. In typical supersymmetric GUT theories, such as those discussed in [35], one expects to have values of $\rho_\lambda^{\rm min}$ roughly of order hundreds.

For precision unification, on the other hand, we require $\rho_\lambda^{\rm min} < 20$ which roughly corresponds to 3σ deviation from exact gauge coupling unification. By that we mean that we allow a factor of three higher correction than what might arise from naive Planck scale corrections; $(\alpha_i^{-1}-\alpha_j^{-1})/4\pi \sim \mu_\star/M_P$ (see also Refs. [9,10]). Such a threshold should not be taken too seriously. There are potential reasons for raising the allowed $\rho_\lambda^{\rm min}$ and for lowering it somewhat to define "precision unification", but to be concrete we choose $\rho_\lambda^{\rm min} < 20$.

Figure 1 shows ρ_λ as a function of the putative unification scale in the SM and MSSM with various choices for the common superpartner threshold $\tilde m$. We performed the renormalization group evolution (RGE) of the gauge couplings in the (MS)SM at two-loop (along with one-loop running of the third generation Yukawa couplings) [36]. It is evident from the figure that precision unification is achieved in the MSSM with a common threshold if $\tilde m$ is roughly in the 1–10 TeV range.

High scale scenarios. We now turn to the implications of exact gauge unification in mSUGRA and mAMSB frameworks. Both frameworks have three common parameters;

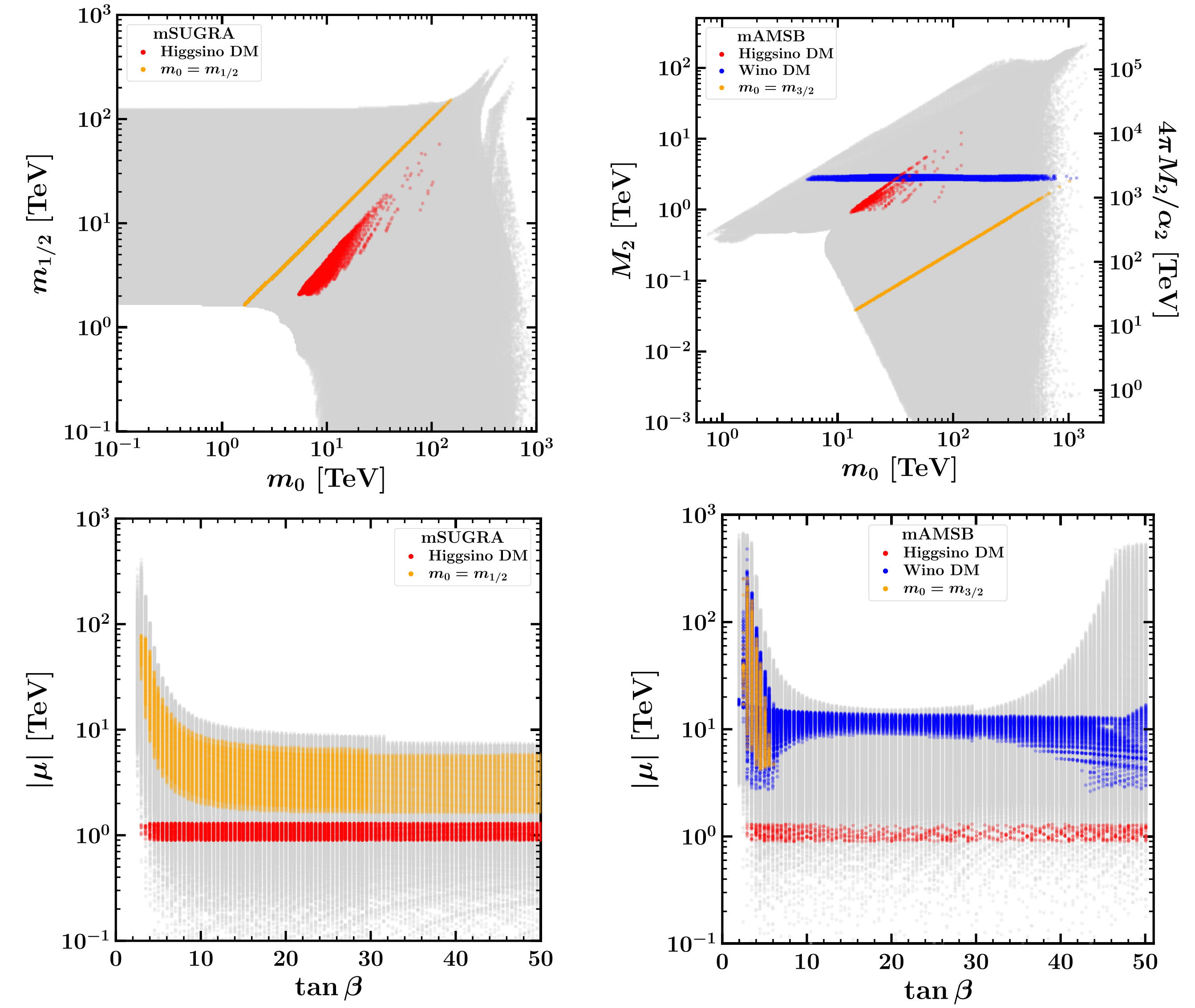

FIG. 2. Parameter space (gray points) where precise gauge unification ($\rho_\lambda^{\min} < 20$) can be achieved in addition to satisfying the observed Higgs mass constraint ($m_h = 125.25 \pm 3$ GeV). Various colored points correspond to various special cases as labeled. Top panels show $m_{1/2}$ in mSUGRA (top-left) and the wino mass M_2 in mAMSB (top-right) plotted against m_0. The right vertical axis in the top-right panel shows a rough estimate of $m_{3/2} \sim 4\pi M_2/\alpha_2$ with $\alpha_2^{-1} \simeq 25$ in the mAMSB scenario. Bottom panels show the $|\mu|$ term plotted against $\tan\beta$ in the mSUGRA (bottom-left) and the mAMSB (bottom-right) scenarios.

the unified scalar mass m_0 at the GUT scale,[1] the ratio of the vacuum expectation values of the two Higgs doublets $\tan\beta$ (at M_Z), and the sign of the Higgsino mass parameter μ. mSUGRA has two additional parameters, namely, the unified gaugino mass $m_{1/2}$ at the GUT scale and the universal scalar trilinear coupling A_0 at the GUT scale. On the other hand, mAMSB has one additional parameter; the gravitino mass $m_{3/2}$ at the GUT scale. In mSUGRA (mAMSB), bino (wino) is the lightest gaugino. Therefore, the LSP can be binolike (winolike) or Higgsino-like in mSUGRA (mAMSB), depending on μ and $\tan\beta$.

We used SPHENO [37,38] for generating the MSSM particle spectrum, which implements two-loop supersymmetric RGEs (and three-loop SM RGEs) with improved prediction of the Higgs boson mass [39]. To find the parameter space of interest in these model frameworks, we independently varied $(m_0, m_{1/2})$ and $(m_0, m_{3/2})$ in mSUGRA and mAMSB scenarios, respectively, from 10^2 GeV to 10^8 GeV in 500 evenly spaced steps on a log scale. We simultaneously also varied $\tan\beta$ from 2 to 50 in steps of 0.5 for both $\mu \gtrless 0$. In mSUGRA, we found that varying A_0 did not have much impact on our results,

[1] Supersymmetric spectra generators, including SPHENO [37,38], which we have employed, commonly define the GUT scale as the scale where the gauge couplings g_1 and g_2 unify.

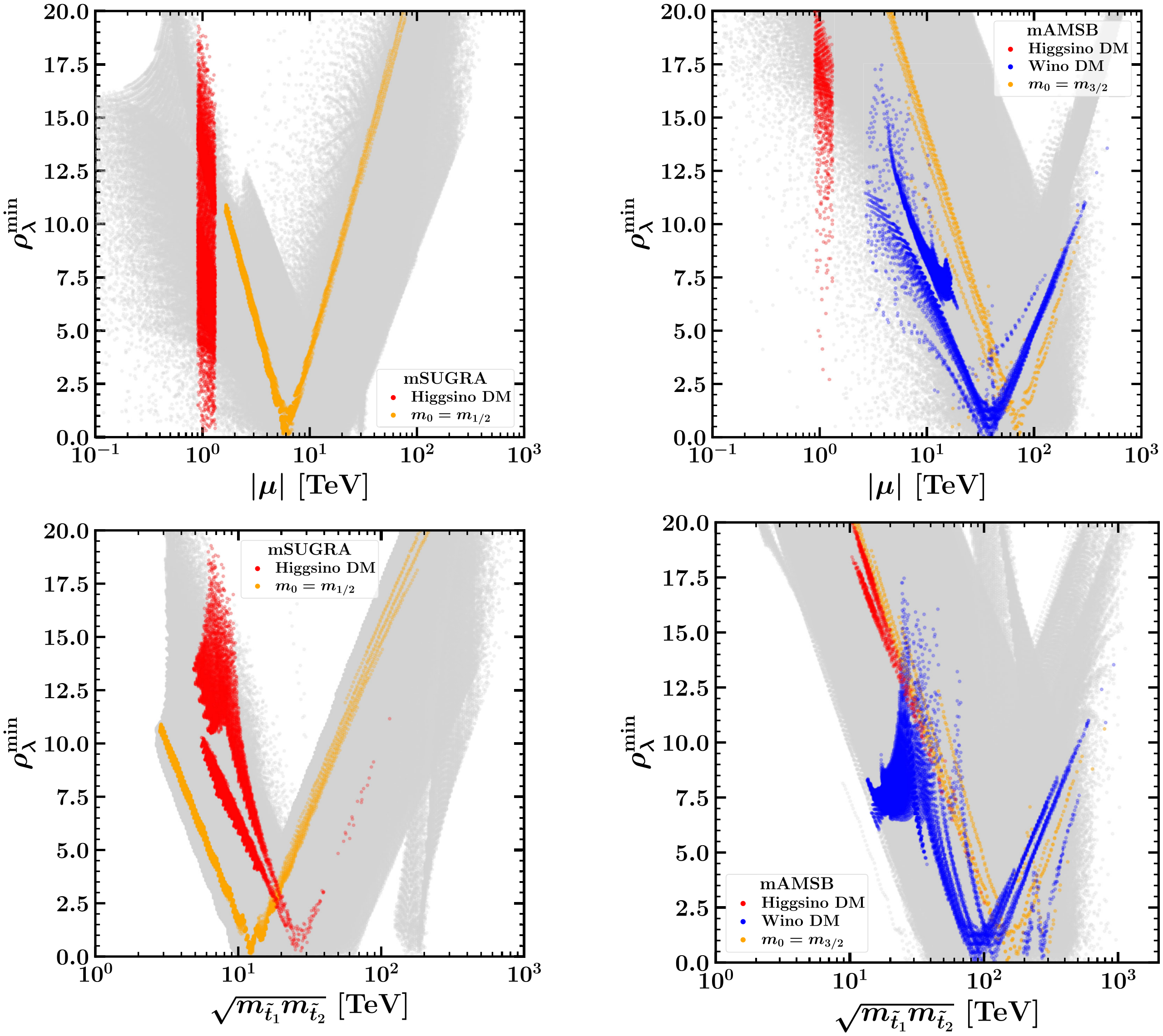

FIG. 3. $\rho_\lambda^{\min}$ plotted against the absolute value of the μ term (top panels) and the geometric mean of the top squark masses (bottom panels) in mSUGRA (left) and mAMSB (right) scenarios. The gray points correspond to all models with precise gauge coupling unification ($\rho_\lambda^{\min} < 20$) and the Higgs mass m_h within 3 GeV of 125.25 GeV. Various colored points correspond to various special cases as labeled.

especially when the scale of supersymmetry is much larger than M_Z. Therefore, we have simply taken $A_0 = 0$. In order to impose the observed Higgs boson mass constraint, we required that $m_h = 125.25 \pm 3$ GeV for the theory calculation. Requiring that the models reproduce the observed Higgs boson mass is a nontrivial constraint and significantly restricts the parameter space. For precision unification, we impose $\rho_\lambda^{\min} < 20$. Moreover, we also identify models where the required thermal abundance is generated by Higgsino or wino DM. In particular, we identify models with a neutralino LSP and require $\mu = 1.1 \pm 0.2$ TeV (with $|\mu| < M_1, M_2$) [20–22,26] for Higgsino DM, and $M_2 = 2.8 \pm 0.2$ TeV (with $M_2 < M_1, |\mu|$) [24–26] for wino DM.

In Fig. 2, we show the parameter space (gray points) in both mSUGRA (left panels) and mAMSB (right panels) frameworks with precise gauge coupling unification ($\rho_\lambda^{\min} < 20$) and the observed Higgs boson mass within 3 GeV of 125.25 GeV. Specifically, m_0 vs $m_{1/2}$ (top-left panel) and $|\mu|$ vs $\tan\beta$ (bottom-left panel) scatter plots in mSUGRA. And m_0 vs M_2 (or equivalently $m_{3/2} \sim 4\pi M_2/\alpha_2$) (top-right panel) and $|\mu|$ vs $\tan\beta$ (bottom-right panel) scatter plots in mAMSB. Figure 2 also shows the parameter space where the neutralino LSP, that can reproduce the required thermal DM abundance, is Higgsino-like (red points) or winolike (blue points). In mSUGRA, since the neutralino LSP cannot be winolike, there are no cases with wino DM. Also shown in the figure are the cases (orange points) where $m_0 = m_{1/2}$ in mSUGRA (left panels) and $m_0 = m_{3/2}$ in mAMSB (right panels).

Finally, Fig. 3 shows $\rho_\lambda^{\rm min}$ plotted as a function of the absolute value of the μ term (top panels) and the geometric mean of the stop masses $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ (bottom panels) in mSUGRA (left panels) and mAMSB (right panels) scenarios. It is apparent from the figure that in the cases that satisfy the observed Higgs boson mass constraint with (near) perfect gauge-coupling unification, the $|\mu|$ term is nearly in the range of one to a few hundred TeV. On the other hand, the stop masses, which can be taken as a rough proxy of the scale of supersymmetry, is in the range of a few TeV to PeV. In mSUGRA (mAMSB), these quantities are slightly towards the lower (upper) end of the aforementioned ranges.

Several comments about the viability of the parameter space with precise gauge unification are in order. First, it is important to emphasize that the majority of this parameter space is unexplored by the LHC or lies well-beyond its reach. Moreover, in some cases, to prevent the overproduction of gravitinos in the early Universe, the reheat temperature may need to be less than about $m_{3/2}/20$, ensuring sufficient Boltzmann suppression of gravitino production [40]. In addition, due to the direct detection constraints, the neutralino LSP for Higgsino DM should be an extremely pure Higgsino [31,41]. And, due to indirect detection constraints, wino DM seems to be experimentally less viable [28,29].

Conclusion. We have investigated the implications of precise gauge coupling unification on the supersymmetric particle masses. We considered the minimal supersymmetric Standard Model with a common superpartner mass, the minimal supergravity model, and the minimal anomaly-mediated supersymmetry-breaking model. We found that the superpartner masses are typically in the range of a few TeV and (well) beyond in order to achieve (near) perfect gauge-coupling unification and to also obtain the correct observed Higgs boson mass. Even after requiring the Higgs mass conform with experiment and that gauge-coupling unification is (nearly) exact, we can still identify large regions of parameter space where a Higgsino or a wino can reproduce the thermal dark matter abundance. Finally, as the analysis has made clear, unless one implements numerically too precisely and too aggressively the qualitative notions of naturalness and fine-tuning [42], the LHC results have had essentially no impact on the viability of supersymmetric unified theories, even precision unification. This assessment could have come out differently, depending on the gauge coupling measurements and the Higgs boson mass measurement.

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