Emergent 1-form symmetries

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We explore the necessary conditions for 1-form symmetries to emerge in the long-distance limit when they are explicitly broken at short distances. A minimal requirement is that there exist operators which become topological at long distances and that these operators have nontrivial correlation functions. These criteria are obeyed when the would-be emergent symmetry is spontaneously broken, or is involved in 't Hooft anomalies. On the other hand, confinement, i.e. a phase with unbroken 1-form symmetry, is nearly incompatible with the emergence of 1-form symmetries. We comment on some implications of our results for QCD as well as the idea of Higgs-confinement continuity.

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I. INTRODUCTION

The global symmetries we encounter in nature are generally explicitly broken at short distances, but they often emerge in long-distance QFT descriptions. In recent years, it has become appreciated that relativistic QFTs can have generalized global symmetries [1], such as higher-form symmetries that act on nonlocal operators like lines and surfaces. For example, 1-form symmetries can act on Wilson loops in gauge theories, and have led to a helpful new perspective on color confinement, among many other results [2,3]. In this paper we explore how 1-form symmetries can emerge in long-distance limits even when they are explicitly broken at short distances.

Standard "0-form" symmetries act on local operators and can be explicitly broken by adding an appropriate charged local operator to a Lagrangian density. The fate of the symmetry at long distances is then determined by the scaling dimension of the perturbing operator. It is difficult (though not impossible, see Ref. [4]) to use similar renormalization-group-style ideas to study the long-distance fate of explicitly broken 1-form symmetries, because the charged objects are line operators. In practice, explicit breaking of one-form symmetries in gauge theories arises due to couplings to dynamical charged matter fields [1]. Then there are two basic ways a 1-form symmetry can emerge. First, it can appear upon taking a limit in the space of QFTs. If we consider a sequence of field theories where the mass m of the symmetry-breaking fields increases with all other physical parameters held fixed, then a 1-form symmetry must certainly emerge in the limit $m \to \infty$. Second, a 1-form symmetry might emerge in a fixed QFT (in this context, with fixed m) in the long-distance limit. How and when this happens is more subtle, and such "infrared-emergent 1-form symmetries" will be our main focus in this paper.

Below we will state a condition that must be satisfied for a QFT to have a nontrivial emergent 1-form symmetry in the long-distance limit. We then analyze how this condition is satisfied (or not) in some simple examples. Along the way we will make contact with other discussions of emergent and approximate 1-form symmetries in the recent literature [4–10] and discuss the role of 't Hooft anomalies in the emergence of 1-form symmetries. We also explain why there is no emergent 1-form symmetry in QCD even when all of the quarks have large finite masses, and implications for Higgs-confinement continuity with fundamental matter.

II. EXACT 1-FORM SYMMETRY

A modern definition of exact symmetries in relativistic QFTs is via the existence of topological operators along with data on how they act in correlation functions [1]. These operators can be thought of as charges which generate the symmetry. We focus on invertible 1-form symmetries in relativistic QFTs in d-dimensional Euclidean spacetime. The associated topological operators $U_\alpha(M_{d-2})$ live on codimension-2 manifolds M_{d-2} and are labeled by an element α of the symmetry group G. Their correlation functions have a purely topological dependence on M_{d-2},

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and they satisfy fusion rules given by group composition in G. The charged operators are line operators $W_R(C)$ where C is a closed loop and R contains information about the charge of the operator.[1] For example, in $U(1)$ Maxwell theory with a 1-form gauge field $a = a_\mu dx^\mu$, there is a $U(1)$ 1-form "electric" symmetry generated by Gukov-Witten operators which prescribe the holonomy of a on infinitesimal circles C linking M_{d-2} to be $e^{i\alpha}$. In this simple example one can also give the less abstract definition

$$U_\alpha(M_{d-2}) \sim e^{i\alpha \int_{M_{d-2}} \star j}, \tag{1}$$

where $j = -\frac{i}{g^2} da$ is the electric 2-form conserved current, d is the exterior derivative, $\star$ is the Hodge star operator, and g is the gauge coupling.

The charged objects are electric Wilson loops $W_n(C) = \exp\left(in \oint_C a\right)$, and

$$\frac{\langle U_\alpha(M_{d-2}) W_n(C)\rangle}{\langle U_\alpha(M_{d-2})\rangle \langle W_n(C)\rangle} = e^{in\alpha \mathrm{Link}(M_{d-2}, C)}. \tag{2}$$

As with any exact grouplike symmetry, invertible 1-form symmetries lead to selection rules on finite-volume correlation functions [1]. In the infinite-volume limit they can be spontaneously broken, which is signaled by a perimeter-law behavior for Wilson loops, and is interpreted as deconfinement of test charges. For example in 4D QED $\langle W_1(C)\rangle \to e^{-\mu P(C)}$ when C is large, $P(C)$ is the perimeter of C and μ is an energy scale that depends on the choice of renormalization scheme. One can choose a counterterm localized on C to set $\mu = 0$, and then $\langle W_1(C)\rangle \neq 0$ for large Wilson loops. If instead $\langle W_1(C)\rangle$ goes to zero faster than a perimeter law for large C, then $\lim_{|C|\to\infty} \langle W_1(C)\rangle = 0$ regardless of the choice of scheme, and the symmetry is not spontaneously broken. This signals charge confinement.

III. EMERGENT 1-FORM SYMMETRY

If a QFT contains dynamical minimal-charge matter fields, then there are no topological codimension-2 operators that satisfy Eq. (2), and hence no exact 1-form symmetry. The absence of such operators can be established from the existence of nontrivial open Wilson lines. The issue is that different ways of "shrinking" a symmetry generator in the presence of an open Wilson line give manifestly different results, which at the same time must be identical if the generator is topological [12].

Heuristically, an infrared-emergent 1-form symmetry should be associated with the existence of operators $U_\alpha(\Sigma_{d-2})$ which are only topological in the long distance limit. Then the existence of open Wilson lines is not an issue, since the $U_\alpha(\Sigma_{d-2})$ operators only behave topologically when they are large, and cannot be freely shrunk on open lines.

We define an infrared-emergent 1-form symmetry as

(ES) Existence of a set of operators $U_\alpha(\Sigma_{d-2})$ defined on codimension-2 manifolds Σ_{d-2} with correlation functions that are topological and nontrivial in the long-distance limit.

This differs from the definition of an exact 1-form symmetry in three ways: it involves a long-distance limit, it does not explicitly refer to the action of $U_\alpha(\Sigma_{d-2})$ on line operators, and it does not assume that Σ_{d-2} is closed. The nature of the correlation functions of U_α that remain nontrivial in the long-distance limit can be quite subtle, and they do not always involve genuine line operators [13] or U_α operators defined on closed manifolds.

In what follows we explore the emergence of 1-form symmetries in a number of simple examples, which are all variants of 3D scalar QED. Specifically, we consider parity-invariant $U(1)$ gauge theory with or without magnetic monopoles coupled to matter fields with various charges. We will see that whether our definition of emergent 1-form symmetry (ES) is satisfied can depend on the realization of the symmetry when we consider a limit in the space of theories where it becomes exact, and whether the symmetry is involved in 't Hooft anomalies in that limit.

IV. CONFINEMENT VERSUS EMERGENCE

Consider scalar QED with a charge-1 scalar field ϕ with mass m,

$$S = \int_{M_3} \left(\frac{1}{2g^2} (da)^2 + |d\phi - ia\phi|^2 + \ldots \right), \tag{3}$$

with a short-distance definition that allows finite-action magnetic monopole-instantons with unit charge to contribute to the path integral. The model has Gukov-Witten operators $U_\alpha(\Gamma)$ whose correlators depend on the geometry of Γ at finite m, but become topological in the limit $m \to \infty$, where they generate an exact $U(1)$ 1-form global symmetry. Monopole-instantons generate a mass gap, and when $m \to \infty$ the theory is confining: large Wilson loops have area-law falloff with a string tension σ [14], so the 1-form symmetry is not spontaneously broken in this limit.

If m^2 is finite and $m^2 \gg g^4$, then it is natural to integrate out ϕ to describe physics at distances large compared to $1/m$, leading to the long-distance effective action

$$S_{\mathrm{eff}} = \int_{M_3} \left(\frac{1}{2g^2} (da)^2 + \frac{c_4}{m^5} da^4 + \cdots \right), \tag{4}$$

where $c_4 \sim \mathcal{O}(1)$ and $\cdots$ represents higher-order terms containing higher powers of da and its derivative. Note that a Chern-Simons term cannot appear given parity

[1] Some $(3+1)$D systems have more exotic topological surface operators that only act on other surface operators, not line operators, see, e.g., [11]. Our goal here is to understand more conventional 1-form symmetries that act on line operators.

invariance, and thus all terms which can appear in S_{eff} are invariant under shifts of a by a closed 1-form λ. One might therefore think that there is a robust infared-emergent 1-form symmetry when m^2 is finite and $m^2 \gg g^4$. This turns out not to be true.

The key point is that the effective action in Eq. (4) is not actually "effective" for all long-distance observables. While it is effective for calculating correlation functions of widely separated local operators, it fails to correctly describe the correlation functions of large Wilson loops. In particular, consider $\langle W(C)\rangle$, $\langle U_\alpha(\Gamma)\rangle$, and $\langle U_\alpha(\Gamma)W(C)\rangle$. The path integral over ϕ produces a formal sum over all possible insertions of "dynamical" minimal-charge Wilson loops, so that schematically

$$\langle U_\alpha(\Gamma)\rangle \sim \sum_{C'} e^{-\mu P(C')}\langle U_\alpha(\Gamma)W(C')\rangle_0 + \cdots \tag{5a}$$

$$\langle W(C)\rangle \sim \sum_{C'} e^{-\mu P(C')}\langle W(C)W(C')\rangle_0 + \cdots \tag{5b}$$

$$\langle U_\alpha(\Gamma)W(C)\rangle \sim \sum_{C'} e^{-\mu P(C')}\langle U_\alpha(\Gamma)W(C)W(C')\rangle_0 + \cdots, \tag{5c}$$

where $\langle\cdot\rangle_0$ denotes a (connected) correlator evaluated in the pure gauge theory, μ is some appropriate mass scale (such as the mass of ϕ), and $\cdots$ represents terms with multiple dynamical Wilson loops. The above representation of expectation values arises naturally in the large mass (hopping) expansion on the lattice [15], or in the worldline formalism in the continuum [16–20].

We start by considering the one-point function of the Gukov-Witten operator in Eq. (5a). The fact that large dynamical Wilson loops $W(C')$ come with a suppression factor $e^{-\mu P(C')}$ implies that for large Γ the sum in Eq. (5a) is dominated by small loops $C' \sim 0$. Summing over Wilson loop insertions on small curves C' that link Γ (illustrated on the right-hand side of Fig. 1) generates perimeter-law behavior for the Gukov-Witten operators

$$\langle U_\alpha(\Gamma)\rangle \sim e^{-\mu_\alpha P(\Gamma)}, \tag{6}$$

where μ_α is a nonuniversal mass scale that depends on α as well as, e.g., the mass of the charged particle. But we can set μ_α to zero by a counterterm localized on Γ, producing operators which are topological for large Γ so long as $m^2 \gtrsim g^4$. This is consistent with a naive analysis based on the effective field theory Eq. (4).

We now consider Eqs. (5b) and (5c). Since $\langle W(C)\rangle_0$ has confining behavior, there is a competition between the perimeter suppression $e^{-\mu P(C')}$ of dynamical Wilson loops $W(C')$ and the more severe (in this case area-law) suppression of the probe Wilson loop $W(C)$ in the pure gauge theory. As a result, for large contours C, large dynamical

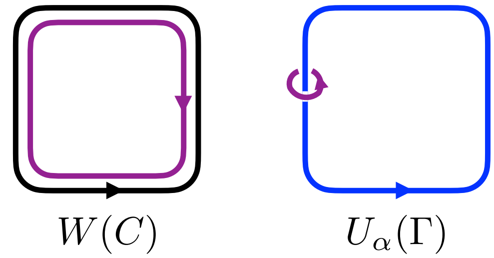

FIG. 1. Left: in a confining theory coupled to heavy matter, the expectation value of a large Wilson loop $W(C)$ (black curve) is dominated by contributions that include a large screening "matter" Wilson loop (purple curve) arising from integrating out a charged matter field. Right: the expectation value of $U_\alpha(\Gamma)$ (blue curve) is dominated by the contributions of small Wilson loops (purple curve) arising from integrating out the charged matter field.

Wilson loops are favored rather than suppressed, and $\langle W(C)\rangle$ is dominated by fluctuations around the "screening loop" $C' = \bar{C}$ running opposite to C. This is illustrated in the left-hand side of Fig. 1. Such contributions go to zero with the perimeter of C rather than its area. This is simply the standard physics of screening, which is not captured in the local effective field theory of Eq. (4). The same conclusion holds for the correlator of the Wilson loop and Gukov-Witten operator in Eq. (5c). Again, the dominant contribution involves a screening loop, and as a result the Gukov-Witten operator measures zero charge.

To summarize, the fact that large Wilson loops are screened implies that their charge cannot be detected at long distances. Instead,

$$\frac{\langle U_\alpha(\Gamma)W(C)\rangle}{\langle U_\alpha(\Gamma)\rangle\langle W(C)\rangle} = 1 \tag{7}$$

for well-separated curves Γ, C which tend to infinity. Effectively, confinement in the limit $m \to \infty$ [meaning that $\langle W(C)\rangle \to 0$ faster than a perimeter law in that limit] implies that when m is finite all Wilson loops flow to the identity line operator at long distances, leaving nothing for the wouldbe emergent 1-form symmetry to act on. As a result the correlation functions of $U_\alpha(\Gamma)$ are trivial—all charges are screened. Therefore our ES definition is not satisfied, and there is no emergent 1-form symmetry at finite positive m^2. Finally, if $-m^2 \gtrsim g^4$, screening loops due to ϕ proliferate. Then there is no approximation in which we can neglect the matter field, and so there is also no emergent 1-form symmetry for $-m^2 \gtrsim g^4$. So there is no reason to expect an emergent 1-form symmetry for any finite m^2 in this model.

We conclude that confinement is almost entirely incompatible with the emergence of 1-form symmetries. For example, there is no infrared-emergent 1-form symmetry in 4D QCD with fundamental quarks of any finite mass m_q, no matter how large m_q is compared to the strong scale.

V. DECONFINEMENT AND EMERGENCE

We now discuss what happens when an exact 1-form symmetry is spontaneously broken, and then also explicitly broken by the addition of heavy charged matter fields. To this end, we couple 3D $U(1)$ gauge theory to a charge N scalar χ,

$$S = \int_{M_3} \left(\frac{1}{2g^2}(da)^2 + |d\chi - iNa\chi|^2 + \dots \right), \qquad (8)$$

and assume χ is the only electrically charged field. This theory has an exact $\mathbb{Z}_N$ 1-form symmetry generated by Gukov-Witten operators $U_k(\Gamma)$, defined to have holonomies $e^{\frac{2\pi ik}{N}}$ on infinitesimal contours linking Γ. We condense χ, Higgsing the gauge group to $\mathbb{Z}_N$. In this phase large Wilson loops have a perimeter-law scaling, and the 1-form symmetry is spontaneously broken.

We now put in a unit-charge scalar ϕ with mass m to explicitly break the 1-form symmetry. When $m^2 \gtrsim g^4$ the 1-form symmetry emerges in the long-distance limit with m held fixed. To see this we examine the behavior of the correlation functions in Eq. (5) resulting from formally summing over ϕ configurations, with $\langle \cdot \rangle_0$ now interpreted as correlation functions of the theory without ϕ. The analysis of $\langle U_\alpha(\Gamma) \rangle$ goes through as before, but the perimeter-law scaling of $\langle W(C) \rangle_0$ now implies that small matter loops (as opposed to screening loops with $C' \sim \bar{C}$) dominate $\langle U_\alpha(\Gamma) W(C) \cdots \rangle$ when C is very large, and

$$\frac{\langle U_k(\Gamma) W(C) \rangle}{\langle U_k(\Gamma) \rangle \langle W(C) \rangle} = e^{\frac{2\pi ik}{N} \mathrm{Link}(\Gamma, C)} \qquad (9)$$

for very large and well-separated loops C, Γ. Therefore we have a set of operators $U_\alpha(\Gamma)$ which have nontrivial topological correlation functions in the long-distance limit, our definition (ES) is satisfied, and there is an infrared emergent $\mathbb{Z}_N$ 1-form symmetry so long as χ is condensed and ϕ is not.

This analysis resonates with recent discussions of emergent 1-form symmetries in Refs. [4,6–9], which assumed (explicitly or implicitly) that they were working in the spontaneously broken situation discussed in this section. In fact, there is a standard way to understand the above result without invoking the modern language of higher-form symmetries. The charge-N Abelian Higgs model flows to a $\mathbb{Z}_N$ gauge theory at long distances, described in the continuum by a BF action $S_{\mathrm{BF}} = \frac{iN}{2\pi} \int_{M_3} b \wedge da$ where b is a (emergent) $U(1)$ 1-form gauge field, see, e.g., Ref. [21]. This is a topological field theory with 2^N ground states on a large spatial torus, and it has long been known (see Refs. [22–24]) that these ground states remain degenerate (and the long-distance BF description remains valid) after the addition of unit-charge matter so long as it does not "condense." From the more modern perspective, BF theory can be thought of as an effective field theory for a spontaneously broken $\mathbb{Z}_N$ 1-form symmetry [1].

VI. 'T HOOFT ANOMALY AND CHARGED OPERATORS

We now explore the interplay between 't Hooft anomalies and emergent 1-form symmetries. We again start with $U(1)$ gauge theory in 3D, $S = \int_{M_3} \frac{1}{2g^2}(da)^2$, but this time we do not allow dynamical magnetic monopole-instantons in the UV completion. This gives rise to a 0-form "magnetic" symmetry $U(1)_m^{(0)}$ associated with the conserved current $j_m^{(1)} = \frac{1}{2\pi} \star da$. This symmetry is generated by topological surface operators $V_\beta(\Sigma)$ which act on pointlike charge k monopole operators $e^{ik\sigma}$ where σ is the 2π-periodic scalar field dual to a. As before, the $U(1)_e^{(1)}$ electric symmetry is generated by topological line operators $U_\alpha(\Gamma)$ which act on charge q Wilson lines $W_q(C)$.

The $U(1)_m^{(0)}$ and $U(1)_e^{(1)}$ symmetries have a mixed 't Hooft anomaly, see, e.g., [1]. This can be detected by turning on 1-form and 2-form $U(1)$ background gauge fields A_m and B_e, with 0-form and 1-form background gauge transformations $A_m \to A_m + d\Lambda_m$, $B_e \to B_e + d\Lambda_e$, $a \to a + \Lambda_e$, so that

$$S[A,B] = \int_{M_3} \left[\frac{1}{2g^2}(da - B_e)^2 + \frac{i}{2\pi} A_m \wedge da - \frac{i}{2\pi} b_{\mathrm{CT}} A_m \wedge B_e + \cdots \right], \qquad (10)$$

where $\cdots$ stands for other background-field counterterms in addition to the $A_m \wedge B_e$ term. While it may seem from Eq. (10) one can preserve background gauge invariance for $U(1)_m^{(0)}$ by setting $b_{\mathrm{CT}} = 0$ (or do the same for $U(1)_e^{(1)}$ by setting $b_{\mathrm{CT}} = 1$), the situation is more subtle and is detailed in the Appendix. In any case there is no choice of counterterms which can enforce background gauge invariance for both A_m and B_e simultaneously, indicating an 't Hooft anomaly.

The 't Hooft anomaly implies the following correlation functions:

$$\frac{\langle V_{2\pi q}(D) U_\beta(\Gamma) \rangle}{\langle V_{2\pi q}(D) \rangle \langle U_\beta(\Gamma) \rangle} = e^{iq\beta(1-b_{\mathrm{CT}}) \mathrm{Link}(\partial D, \Gamma)}, \qquad (11a)$$

$$\frac{\langle V_\alpha(\Sigma) U_{2\pi k}(L) \rangle}{\langle V_\alpha(\Sigma) \rangle \langle U_{2\pi k}(L) \rangle} = e^{ik\alpha b_{\mathrm{CT}} \mathrm{Link}(\Sigma, \partial L)}, \qquad (11b)$$

where $q, k \in \mathbb{Z}$, and D and Σ are (respectively) open and closed surfaces, while L and Γ are open and closed lines, which are sketched in Fig. 2. We note that an open surface operator can be modified by attaching an arbitrary line operator to its boundary, with similar remarks holding for

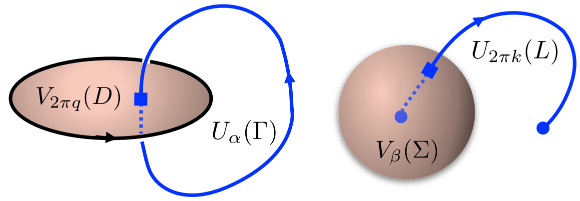

FIG. 2. The 't Hooft anomaly of 3D pure $U(1)$ gauge theory with a $U(1)^{(0)}_m \times U(1)^{(1)}_e$ symmetry implies that certain unusual correlation functions of the symmetry generators are nontrivial. These correlation functions involve operator intersections (denoted by squares in the figure) and are sensitive to the choice of counterterms. But it is not possible to fully trivialize the expectation value of the combination of operators given in Eq. (12), which is illustrated in this figure.

open line operators. Our choice in Eq. (11) is such that together, the bulk and boundary of the operators $V_{2\pi q}(D)$, $U_{2\pi k}(L)$ are topological.

Absent the anomaly, $V_{2\pi q}(D)$ would be a trivial surface operator that contributes only contact terms to correlation functions which can all be trivialized by a choice of local counterterms. But if we take $b_{\mathrm{CT}} = 0$, Eq. (11a) tells us that the boundary of $V_{2\pi q}(D)$ acts like a charge q topological Wilson line. By comparison, the genuine Wilson lines $W_q(C)$, which are not attached to surfaces, are not topological in this model—they are (logarithmically) confined. Similar remarks hold for Eq. (11b), which says that the endpoints of $U_{2\pi k}(L)$ behave like a pair of charge $\pm k$ monopole operators (when $b_{\mathrm{CT}} = 1$), with only a topological dependence on L. In summary, Eq. (11) tells us that in a given scheme the 't Hooft anomaly adds $V_{2\pi q}(D)$ or $U_{2\pi k}(L)$ to the list of distinct operators charged under $U(1)^{(0)}_m$ and $U(1)^{(1)}_e$, which would otherwise have only included $W_q(C)$ and $e^{ik\sigma}$. The scheme-independent statement is that at least one of the correlators in Eq. (11) is nontrivial. Equivalently, the following correlator is nontrivial in any scheme:

$$\frac{\langle V_{2\pi q}(D) U_\beta(\Gamma) V_\alpha(\Sigma) U_{2\pi k}(L)\rangle}{\langle V_{2\pi q}(D)\rangle\langle U_\beta(\Gamma)\rangle\langle V_\alpha(\Sigma)\rangle\langle U_{2\pi k}(L)\rangle} = e^{ik\alpha b_{\mathrm{CT}}\mathrm{Link}(\Sigma,\partial L)} e^{iq\beta(1-b_{\mathrm{CT}})\mathrm{Link}(\partial D,\Gamma)} \neq 1. \quad (12)$$

The quick way to see why Eq. (11) is true is to consider the effects of singular background gauge transformations. An insertion of $V_{2\pi q}(D)$ is equivalent to turning on the background $A_m = 2\pi q \delta^{(1)}(D)$, which can be removed by a singular gauge transformation Λ_A which winds by $-2\pi q$ around ∂D. Similarly, an insertion of $U_\beta(\Gamma)$ with Γ contractible is equivalent to taking $B_e = \beta\delta^{(2)}(\Gamma)$, which can be removed by the background gauge transformation by $\Lambda_B = -\beta\delta^{(1)}(S)$, where $\partial S = \Gamma$. Doing these transformations in $S[A, B]$, we obtain

$$\delta S[A,B] = \frac{i2\pi q\beta(1-b_{\mathrm{CT}})}{2\pi}\int_{M_3}\delta^{(1)}(D)\wedge\delta^{(2)}(\Gamma) = iq\beta(1-b_{\mathrm{CT}})\mathrm{Link}(\partial D,\Gamma), \quad (13)$$

reproducing Eq. (11a). A very similar computation gives Eq. (11b). We also derive this result in the Appendix using the language of coordinate patches, cochains, and transition functions for readers who are nervous about manipulations of singular gauge transformations. This analysis shows the precise sense in which the 't Hooft anomaly remains nontrivial even in locally flat background fields, which is not obvious from its naive form in Eq. (10).

There is a well-known argument that an 't Hooft anomaly implies that the ground state cannot be trivially gapped. If it were trivially gapped, then one could simultaneously gauge both $U(1)^{(0)}_m$ and $U(1)^{(1)}_e$ in the (empty) long-distance Effective Field Theory (EFT), which would be inconsistent with the 't Hooft anomaly. The same constraint on the ground state structure follows from Eq. (12). If the ground state on $M_3 = \mathbb{R}^3$ were trivially gapped, then all correlation functions must either go to zero or unity in the long-distance limit.[2] But Eq. (12) is nontrivial for any contractible Γ and Σ, no matter how large, so the ground state on $\mathbb{R}^3$ cannot be trivially gapped.

VII. 'T HOOFT ANOMALY AND EMERGENCE

Now we reintroduce our charge-1 scalar ϕ with mass m,

$$S = \int_{M_3}\left(\frac{1}{2g^2}(da)^2 + |d\phi - ia\phi|^2 + m^2|\phi|^2 + \dots\right), \quad (14)$$

so that $U(1)^{(1)}_e$ is explicitly broken for any finite m^2. However, in contrast to the analysis after Eq. (3), we continue to assume that there are no dynamical monopole-instantons, so that $U(1)^{(0)}_m$ is preserved. The explicit breaking of $U(1)^{(1)}_e$ means that the 't Hooft anomaly seems to be gone, as it no longer makes sense to consider (background) gauging both symmetries. However, is there a range of microscopic parameters for which this QFT has an emergent 1-form symmetry in the long-distance limit?.

This question was considered from the particle-vortex dual perspective [25–27] in Ref. [5]. The authors of Ref. [5] argued that the 't Hooft anomaly of the Nambu-Goldstone effective field theory for a $(2+1)$D superfluid, which is dual to our S_{eff} from Eq. (4), implies the existence of a robust gapless mode. Here we are posing the question with regard to our definition (ES), which requires us to analyze the correlation functions of wouldbe topological operators.

[2]This may require adjusting the coefficients of local counterterms. The more precise statement is that the long-distance limit of any correlation function in the trivially gapped theory reduces to a pure contact term which can be removed by an appropriate choice of scheme.

When m^2 is sufficiently positive, the discussion following Eq. (5) implies that we can construct approximately topological operators $U_\beta(\Gamma)$ and $U_{2\pi k}(L)$, with Γ (respectively, L) a closed (respectively, open) line. However, $\langle W_q(C)\rangle \to 0$ faster than a perimeter law when $m^2 \to \infty$, so $W_q(C)$ flows to the trivial operator for large C for any finite m. So one may worry that the wouldbe 1-form symmetry generators $U_\beta(\Gamma)$ have nothing to act on in the long-distance limit.

However, the situation is more interesting. Since $U(1)_m^{(0)}$ is not explicitly broken, $V_{2\pi q}(D)$ and $V_\beta(\Sigma)$ are topological surface operators. We can now consider Eq. (10) with B_e replaced by a 2-form background field B, which can no longer be interpreted as a background gauge field, since $U(1)_e^{(1)}$ is explicitly broken. Nevertheless, setting $B = \beta\delta^{(2)}(\Gamma)$ has the effect of inserting a nontopological Gukov-Witten operator $U_\beta(\Gamma)$. For large Γ, however, $U_\beta(\Gamma)$ is approximately topological. If $b_{\mathrm{CT}} \neq 1$, then $V_{2\pi q}(D)$ has nontrivial approximately topological correlation functions with $U_\beta(\Gamma)$ when Γ is large. Crucially, the fact that $V_{2\pi q}(D)$ is exactly topological for any m^2 means that it is not screened by charged matter loops. If $b_{\mathrm{CT}} = 1$, then the approximately topological operator $U_{2\pi k}(L)$ has nontrivial correlation functions with the exactly topological operator $V_\beta(\Sigma)$. Therefore Eq. (11) remains nontrivial in the long-distance limit for sufficiently positive fixed m^2.

If m^2 is not sufficiently positive, then matter Wilson loops proliferate, and $U_\beta(\Gamma)$, $U_\beta(L)$ flow to either the zero or identity operators at long distances depending on the choice of counterterms.

We thus see that our definition (ES) is satisfied when m^2 is sufficiently positive, so that the theory described by Eq. (14) has a $U(1)_e^{(1)}$ emergent 1-form symmetry, despite the fact that the $U(1)_e^{(1)}$ symmetry is not spontaneously broken in the limit $m \to \infty$, where test charges are (logarithmically) confined.

VIII. CONSEQUENCES

One interesting consequence of the emergent $U(1)_e^{(1)}$ symmetry of 3D massive scalar QED with a $U(1)_m^{(0)}$ magnetic symmetry involves its ground state structure. While naively the explicit breaking of the $U(1)_e^{(1)}$ kills the mixed 't Hooft anomaly, the anomaly manages to maintain its grip on the ground state structure thanks to Eq. (11) and the discussion in the preceding section. The nontriviality of Eq. (11) at long distances means that when m is finite (but sufficiently positive) the ground state must remain nontrivial. This is in accordance with Ref. [5], which argues that the gapless mode in this regime is a direct consequence of an emergent 't Hooft anomaly. On the other hand if m^2 is sufficiently negative, Eq. (11) trivializes. As a result, the ground state can be (and is) trivially gapped, as can be verified by a standard semiclassical calculation, see Fig. 3 for an illustration.

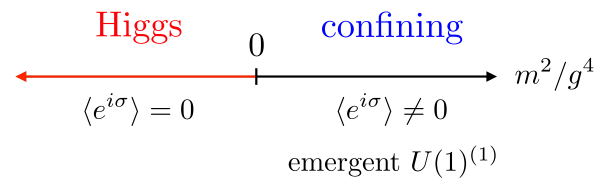

FIG. 3. A sketch of the minimal phase diagram of 3D scalar QED, which has Higgs and confining phases which are separated by at least one phase boundary where the realization of the $U(1)_m^{(0)}$ symmetry changes and the expectation values of monopole operators $e^{i\sigma}$ are nonanalytic. The confining phase is characterized by an emergent $U(1)_e^{(1)}$ symmetry.

If we had preserved the $U(1)_e^{(1)}$ symmetry but explicitly broken $U(1)_m^{(0)}$ by adding unit-charge monopole operators to the action, we would have landed on Eq. (3), a QFT with area-law-like "confinement" (in the same loose sense that QCD with finite mass quarks is confining) and a trivially gapped ground state. This is another illustration of the robustness [4,6–9,22–24,28,29] of 1-form symmetries compared to 0-form symmetries. Even when 1-form symmetries are explicitly broken, they can enforce interesting constraints on the low-energy physics, in contrast to 0-form symmetries.

Another interesting byproduct of the discussion above has to do with Higgs-confinement continuity, namely the common lore that the Higgs and confining regimes of gauge theories with fundamental-representation Higgs fields should be smoothly connected. Higgs-confinement continuity was proven in some lattice models in Refs. [30,31], where a key assumption was the absence of any global symmetries under which the Higgs field is charged. The heuristic argument is that in such a situation the Higgs and confining regimes are smoothly connected because there are no order parameters that could distinguish them. The model we have discussed above in Eq. (14) provides an immediate and nontrivial counterexample to the popular interpretation of the results of [30,31] as implying Higgs-confinement continuity even outside the context of the specific models analyzed in those references.

The model in Eq. (14) has a unit-charge Higgs field ϕ and a $U(1)_m^{(0)}$ global symmetry under which the Higgs field is neutral. The Higgs and confining phases are separated by at least one phase transition where the realization of $U(1)_m^{(0)}$ changes, and our results imply that the "confining" phase can be defined as the phase with an emergent 1-form symmetry, which is separated by at least one phase boundary from the "deconfined" Higgs phase. What makes this counterexample to the standard lore nontrivial is that the Higgs field ϕ is not charged under any global symmetry, yet its "condensation" controls the emergent mixed anomaly and subsequent breaking of $U(1)_m^{(0)}$. Other possible nontrivial counterexamples to Higgs-confinement continuity in some systems with global symmetries were recently discussed in, e.g., Refs. [32,33].

IX. CONCLUSIONS

We have put forth necessary conditions for the emergence of 1-form symmetries at long distances. The key requirement is that the long-distance correlation functions of operators be topological and nontrivial. By this criterion, a 1-form symmetry that is not spontaneously broken nor involved in ’t Hooft anomalies cannot emerge at long distances when broken explicitly by finite-mass charged matter. Our definition agrees with the usual lore that the implications of spontaneously broken higher-form symmetries are robust against explicit breaking. Moreover, the implications of a higher-form symmetry can remain robust against explicit breaking even when the symmetry is not spontaneously broken when the symmetry is involved in an ’t Hooft anomaly.

While for simplicity we focused on 1-form symmetries in 3D, our definition can in principle be applied to higher-form symmetries in any dimension. It would be particularly interesting to analyze a wider class of ’t Hooft anomalies that arise in situations where the higher-form symmetries are not spontaneously broken. For instance, 4D $SU(N)$ $\mathcal{N}=1$ super-Yang-Mills (YM) theory has a $\mathbb{Z}_N$ 1-form symmetry and a $\mathbb{Z}_N$ 0-form chiral symmetry with a mixed ’t Hooft anomaly [1,34]. If we add a massive fundamental-representation fermion, do we expect an emergent 1-form symmetry at long distances? The arguments in Secs. VI and VIII can be applied to the linelike intersections of Gukov-Witten surface operators with codimension-1 operators generating the chiral symmetry—while ordinary Wilson lines are confined in $\mathcal{N}=1$ SYM, these nongenuine lines will be approximately topological at long distances and contribute to nontrivial correlation functions.

It may also be worthwhile to explore the connection of our emergence criterion with the phenomenon of symmetry fractionalization, see, e.g., Refs. [35–42], where the charged particles breaking the 1-form symmetry transform in projective representations of a 0-form global symmetry.

Finally, throughout this paper, we focused on the interplay of the large-mass limit, in which the symmetry-breaking matter field contributions are clearly suppressed, and the large-distance limit, where it is less obvious that symmetry breaking effects are suppressed. We should mention that $SU(N)$ QCD has another limit where matter field contributions are clearly suppressed, namely the large N limit where the ’t Hooft coupling, number of flavors N_f and mass parameters are fixed as $N\to\infty$. Does a 1-form symmetry emerge in the large N limit? We recently studied this question together with M. Neuzil in Ref. [10]. It is well known that screening of large Wilson loops is $1/N$ suppressed so long as the large N limit is taken before the large-loop limit, so the specific failure mechanism for the emergence of a 1-form symmetry discussed in the present paper does not apply. However, a 1-form symmetry fails to emerge anyway, because the $U_\alpha(\Gamma)$ operators do not become topological when $N\to\infty$. Their expectation values take the form of Eq. (6) with $\mu_\alpha\sim N$, so that $\langle U_\alpha(\Gamma)\rangle=0$ at large N. It remains an interesting open question whether there is some notion of an “emergent symmetry” which would explain the Wilson loop selection rules of large N QCD.

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APPENDIX: $U(1)^{(0)}\times U(1)^{(1)}$ ’T HOOFT ANOMALY

We give a more precise characterization of the anomaly described in the main text. This allows us to identify operators with nontrivial correlation functions that would not be obvious in a naive continuum approach.

We follow the formalism of Ref. [43], see also [44], where we describe gauge fields in terms of patches, transition functions, and cocycle conditions. We choose an open cover $\{U_I\}$ of the spacetime manifold. Then we pick an associated partition of the manifold into three-dimensional closed regions $\{\sigma_I\}$ with $\sigma_I\subset U_I$, such that $\sigma_{IJ}=\sigma_I\cap\sigma_J\subset U_I\cap U_J$ are two-dimensional and contained in double overlaps, $\sigma_{IJK}=\sigma_I\cap\sigma_J\cap\sigma_K\subset U_I\cap U_J\cap U_K$ are one-dimensional and contained in triple overlaps, and $\sigma_{IJKL}=\sigma_I\cap\sigma_J\cap\sigma_K\cap\sigma_L\subset U_I\cap U_J\cap U_K\cap U_L$ are points contained in quadruple overlaps.

The dynamical $U(1)$ gauge field is described by a collection of fields $(a_I^{(1)},a_{IJ}^{(0)},a_{IJK}^{(-1)})$. The starting point is an $\mathbb{R}$-valued 1-form gauge field on each patch $a_I^{(1)}$. In the language of Ref. [43] this is a 1-form-valued 0 cochain. We use a shorthand notation δ to take differentials of fields on various overlaps: for instance $(\delta a^{(1)})_{IJ} = a_J^{(1)} - a_I^{(1)}$. On double overlaps have $(\delta a^{(1)})_{IJ} = da_{IJ}^{(0)}$ where $a_{IJ}^{(0)} \in \mathbb{R}$ is a real-valued, 0-form transition function. Since $a^{(0)}$ has two "patch" indices it is a 1 cochain (with values in 0-forms). It satisfies $(\delta a^{(0)})_{IJK} = a_{JK}^{(0)} - a_{IK}^{(0)} + a_{IJ}^{(0)} = 2\pi a_{IJK}^{(-1)}$ with $a_{IJK}^{(-1)} \in \mathbb{Z}$ is a constant integer defined on triple-overlaps (an integer 2 cochain). The δ operation (which is the coboundary operator) squares to 0 ($\delta^2 = 0$ and commutes with d, so that, e.g., under an ordinary gauge transformation $(\delta a^{(1)})_{IJ} \to (\delta a^{(1)})_{IJ} + (\delta d\lambda^{(0)})_{IJ} = (\delta a^{(1)})_{IJ} + d(\delta\lambda^{(0)})_{IJ}$. To summarize, the set of relations is

$$(\delta a^{(1)})_{IJ} = da_{IJ}^{(0)}, \tag{A1a}$$

$$(\delta a^{(0)})_{IJK} = 2\pi a_{IJK}^{(-1)}, \tag{A1b}$$

$$(\delta a^{(-1)})_{IJKL} = 0, \tag{A1c}$$

where in the last equation we are looking at quadruple overlaps, and the "cocycle condition" $\delta a^{(-1)} = 0$ means there are no magnetic monopoles. The full gauge redundancy is

$$a_I^{(1)} \to a_I^{(1)} + d\lambda_I^{(0)}, \tag{A2a}$$

$$a_{IJ}^{(0)} \to a_{IJ}^{(0)} + (\delta\lambda^{(0)})_{IJ} + 2\pi m_{IJ}^{(-1)}, \tag{A2b}$$

$$a_{IJK}^{(-1)} \to a_{IJK}^{(-1)} + (\delta m^{(-1)})_{IJK}, \tag{A2c}$$

where the function $\lambda_I^{(0)}$ and constants $m_{IJ}^{(-1)} \in \mathbb{Z}$ represent small and large gauge transformations, respectively.

We take the same set of data for the background 1-form $U(1)$ gauge field, $(A_I^{(1)}, A_{IJ}^{(0)}, A_{IJK}^{(-1)})$, where $A^{(1)}$ is a 1-form, $A^{(0)}$ is a 0-form, $A^{(-1)}$ is a constant integer, and $\Lambda_I^{(0)}$ and $M_{IJ}^{(-1)}$ will denote the small and large background gauge transformations. In this formalism, a symmetry operator supported on a surface is described by $A_I^{(1)} = 0$ everywhere and $A_{IJ}^{(0)}$ constant on a set of double overlaps, so that the corresponding set of σ_{IJ}s constitute the surfaces on which the defect is supported. The surface may have a boundary as long as $\delta A^{(0)}$ a multiple of 2π, with $A_{IJK}^{(-1)}$ activated on the boundary. The well-known statement that a generic $U(1)$ background gauge field cannot be expressed in terms of a network of symmetry defects is reflected in the fact that it is possible to activate a nonzero 1-form $A_I^{(1)}$ on each patch.

Up to signs, the precise version of $\frac{i}{2\pi}\int A \wedge da$ is

$$\begin{aligned}
&\text{“}\frac{i}{2\pi}\int A \wedge da\text{”} \\
&= \frac{i}{2\pi}\sum_I \int_{\sigma_I} A_I^{(1)} \wedge da_I^{(1)} \\
&\quad + \frac{i}{2\pi}\sum_{I<J}\int_{\sigma_{IJ}} A_{IJ}^{(0)} \wedge da_J^{(1)} - \frac{i}{2\pi}\sum_{I<J<K} A_{IJK}^{(-1)}\int_{\sigma_{IJK}} a_K^{(1)} \\
&\quad - i\sum_{I<J<K<L} A_{IJK}^{(-1)} a_{KL}^{(0)}|_{\sigma_{IJKL}}.
\end{aligned} \tag{A3}$$

It is invariant under gauge transformations of all fields and is well defined. What happens if we turn on $A_{IJK}^{(-1)} = q$ on some triple overlap? First, we see that this inserts a Wilson line, and the cocycle condition on $A^{(-1)}$ implies that this line must be closed. Second, the relations between $A_{IJK}^{(-1)}$ and neighboring $A_{IJ}^{(0)}$ implies that we must also turn on $A_{IJ}^{(0)} = 2\pi q$ on some neighboring double overlap, which inserts a field-strength surface operator involving $da^{(1)}$ on that overlap. This gives a rigorous definition of the operator $V_{2\pi q}(D)$ in Eq. (11) the main text.

The 2-form gauge field is described by $(B_I^{(2)}, B_{IJ}^{(1)}, B_{IJK}^{(0)}, B_{IJKL}^{(-1)})$. Here $B^{(2)}$ is a real 2-form on each patch, $B^{(1)}$ is a real 1-form on double overlaps, $B^{(0)}$ is a real 0-form on triple overlaps, and $B^{(-1)}$ is a constant integer on quadruple overlaps. They are related via

$$(\delta B^{(2)})_{IJ} = dB_{IJ}^{(1)}, \tag{A4a}$$

$$(\delta B^{(1)})_{IJK} = dB_{IJK}^{(0)}, \tag{A4b}$$

$$(\delta B^{(0)})_{IJKL} = 2\pi B_{IJKL}^{(-1)}. \tag{A4c}$$

Under gauge transformations we have

$$B_I^{(2)} \to B_I^{(2)} + d\Pi_I^{(1)}, \tag{A5a}$$

$$B_{IJ}^{(1)} \to B_{IJ}^{(1)} + (\delta\Pi^{(1)})_{IJ} + d\Pi_{IJ}^{(0)}, \tag{A5b}$$

$$B_{IJK}^{(0)} \to B_{IJK}^{(0)} + (\delta\Pi^{0})_{IJK} + 2\pi L_{IJK}^{(-1)}, \tag{A5c}$$

$$B_{IJKL}^{(-1)} \to B_{IJKL}^{(-1)} + (\delta L^{(-1)})_{IJKL}, \tag{A5d}$$

where $\Pi_I^{(1)}$ is a real 1-form, Π_{IJ}^{0} is a real 0-form, $L_{IJK}^{(-1)}$ is a constant integer. Again, a generic 2-form $U(1)$ gauge field cannot be associated with a network of symmetry defects, but if $B^{(2)} = 0$, $B^{(1)} = 0$ we can think of $B^{(0)}$ activated on a set of triple overlaps as inserting a codimension-2 symmetry defect on lines (corresponding to a set of σ_{IJK}s).

The dynamical gauge field $a^{(1)}$ shifts under background 1-form gauge transformations. Accordingly, in the presence of the background field for the 1-form symmetry the relations Eq. (A1) are modified to

$$(\delta a^{(1)})_{IJ} = da^{(0)}_{IJ} + B^{(1)}_{IJ}, \tag{A6a}$$

$$(\delta a^{(0)})_{IJK} = 2\pi a^{(-1)}_{IJK} - B^{(0)}_{IJK}, \tag{A6b}$$

$$(\delta a^{(-1)})_{IJKL} = B^{(-1)}_{IJKL}. \tag{A6c}$$

These relations are invariant under the background gauge transformations

$$a^{(1)}_I \to a^{(1)}_I + \Pi^{(1)}_I, \tag{A7a}$$

$$a^{(0)}_{IJ} \to a^{(0)}_{IJ} - \Pi^{(0)}_{IJ}, \tag{A7b}$$

$$a^{(-1)}_{IJK} \to a^{(-1)}_{IJK} + L^{(-1)}_{IJK}. \tag{A7c}$$

The anomalous shift of Eq. (A3) under 1-form background gauge transformations is then

$$\begin{aligned}&\frac{i}{2\pi}\sum_I \int_{\sigma_I} A^{(1)}_I \wedge d\Pi^{(1)}_I + \frac{i}{2\pi}\sum_{I<J}\int_{\sigma_{IJ}} A^{(0)}_{IJ} \wedge d\Pi^{(1)}_J \\ &\quad - \frac{i}{2\pi}\sum_{I<J<K} A^{(-1)}_{IJK}\int_{\sigma_{IJK}} \Pi^{(1)}_K + i\sum_{I<J<K<L} A^{(-1)}_{IJK}\Pi^{(0)}_{KL}\Big|_{\sigma_{IJKL}},\end{aligned} \tag{A8}$$

As we mentioned above, if we insert $V_{2\pi q}(D)$ by taking $A^{(0)}_{IJ} = 2\pi q$ on a set of double-overlaps tiling D, then we must also have $A^{(-1)}_{IJK} = \pm q$ on the set of triple overlaps containing the boundary of D. We now insert $U_\beta(\Gamma)$ by setting $B^{(0)}_{IJK}$ equal to a constant β on the triple overlaps tiling the closed loop Γ, which we assume pierces D. Removing this curve with an appropriate 1-form gauge transformation with $\Pi^{(0)} = \beta$, we pick up a phase $e^{i\beta q}$ from the last term above. This reproduces Eq. (11a) in the main text, except that we have not yet discussed the counterterm dependence.

We also have anomalous shifts of Eq. (A3) involving 0-form gauge transformations of A coming from the modified cocycle condition Eq. (A7c),

$$\begin{aligned}&-\frac{i}{2\pi}\sum_{I<J}\int_{\sigma_{IJ}} \Lambda^{(0)}_I \wedge dB^{(1)}_{IJ} - i\sum_{I<J<K} M^{(-1)}_{IJ}\int_{\sigma_{IJK}} B^{(1)}_{JK} \\ &\quad + i\sum_{I<J<K<L} M^{(-1)}_{IJ} B^{(0)}_{JKL}\Big|_{\sigma_{IJKL}}.\end{aligned} \tag{A9}$$

We can now contemplate adding various counterterms involving B. In particular if we add

$$\begin{aligned}&\text{“}\frac{-i}{2\pi}\int A\wedge B\text{”} \\ &= -\frac{i}{2\pi}\sum_I\int_{\sigma_I} A^{(1)}_I \wedge B^{(2)}_I \\ &\quad - \frac{i}{2\pi}\sum_{I<J}\int_{\sigma_{IJ}} A^{(0)}_{IJ}\wedge B^{(2)}_J - \frac{i}{2\pi}\sum_{I<J<K<L} A^{(0)}_{IJ}B^{(0)}_{JKL}\Big|_{\sigma_{IJKL}},\end{aligned} \tag{A10}$$

then we can cancel all the terms in Eq. (A9) and the first two terms in Eq. (A8). This is equivalent to setting $b_{\mathrm{CT}} = 1$ in Eq. (10). We then have

$$\begin{aligned}&\text{“}\frac{i}{2\pi}\int A\wedge(da - B)\text{”} \\ &= \frac{i}{2\pi}\sum_I\int_{\sigma_I} A^{(1)}_I\wedge(da^{(1)} - B^{(2)})_I \\ &\quad + \frac{i}{2\pi}\sum_{I<J}\int_{\sigma_{IJ}} A^{(0)}_{IJ}\wedge(da^{(1)} - B^{(2)})_J \\ &\quad - \frac{i}{2\pi}\sum_{I<J<K} A^{(-1)}_{IJK}\int_{\sigma_{IJK}} a^{(1)}_K - i\sum_{I<J<K<L} A^{(-1)}_{IJK} a^{(0)}_{KL}\Big|_{\sigma_{IJKL}} \\ &\quad - \frac{i}{2\pi}\sum_{I<J<K<L} A^{(0)}_{IJ}B^{(0)}_{JKL}\Big|_{\sigma_{IJKL}}.\end{aligned} \tag{A11}$$

The anomalous shift of Eq. (A11) is

$$\begin{aligned}&S_{\mathrm{anomaly},b_{\mathrm{CT}}=1} \\ &\quad = -i\sum_{I<J} M^{(-1)}_{IJ}\int_{\sigma_{IJ}} B^{(2)}_J - \frac{i}{2\pi}\sum_{I<J<K} A^{(-1)}_{IJK}\int_{\sigma_{IJK}} \Pi^{(1)}_K \\ &\qquad + i\sum_{I<J<K<L} A^{(-1)}_{IJK}\Pi^{(0)}_{KL}\Big|_{\sigma_{IJKL}} \\ &\qquad - \frac{i}{2\pi}\sum_{I<J<K<L} A^{(0)}_{IJ}(\delta\Pi^{(0)} + 2\pi L^{(-1)})_{JKL}\Big|_{\sigma_{IJKL}} \\ &\qquad - i\sum_{I<J<K<L} M^{(-1)}_{IJ}(\delta\Pi^{(0)})_{JKL}\Big|_{\sigma_{IJKL}} \\ &\qquad - \frac{i}{2\pi}\sum_{I<J<K<L} (\delta\Lambda^{(0)})_{IJ} B^{(0)}_{JKL}\Big|_{\sigma_{IJKL}}.\end{aligned} \tag{A12}$$

Now if we consider the same configuration from before with $A^{(0)}_{IJ} = 2\pi q$, $A^{(-1)}_{IJK} = q$, the two terms above in the second and third line cancel and $\langle V_{2\pi q}(D)U_\beta(\Gamma)\rangle$ has no nontrivial phases.

However, we can consider a configuration where $B^{(0)}_{IJK} = 2\pi k$ on a set of triple overlaps containing an open line L, with $B^{(-1)}_{IJKL} = \pm k$ at the quadruple overlaps containing the endpoints of L. This represents the insertion of an open generator of the 2π1-form transformation $U_{2\pi k}(L)$ (which would be trivial absent an anomaly). If we also insert a 0-form generator $V_\alpha(\Sigma)$ on a closed surface Σ enclosing the endpoint, with $A^{(0)}_{IJ} = \alpha$ for some constant α on an appropriate set of double-overlaps containing Σ, then the gauge transformation $A^{(0)}_{IJ} \to A^{(0)}_{IJ} + (\delta\Lambda^{(0)})_{IJ}$ used to unlink the surface Σ from the endpoint of the 1-form generator $U_{2\pi k}(L)$ will yield the phase $e^{i\alpha k}$ from the last term in Eq. (A12). This reproduces Eq. (11b) for the behavior of $\langle V_\alpha(\Sigma) U_{2\pi k}(L)\rangle$.

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