Vacuum zero point energy of self-interacting quantum fields in dS background

Hassan Firouzjahi* and Haidar Sheikhahmadi†
School of Astronomy, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5531, Tehran, Iran

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We consider self-interacting scalar fields with a conformal coupling in the de Sitter background and study the quantum corrections from bubble loop diagrams. Incorporating the perturbative in-in formalism, we calculate the quantum corrections in the vacuum zero point energy and pressure of self-interacting fields with the potential $V \propto \Phi^n$ for even values of n. We calculate the equation of state corresponding to these quantum corrections and examine the scaling of the divergent terms in the vacuum zero point energy and pressure associated to the dimensional regularization scheme. In particular, we show that for a quartic self-interacting scalar field the conformal invariance is respected at two-loop order at the conformal point.

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I. INTRODUCTION

Quantum field theory in de Sitter (dS) background is a rich topic which has important implications both observationally and theoretically [1–5]. Observationally, there are strong evidences that the early Universe experienced a period of inflation in which the background was nearly a dS spacetime. In the simplest realization, inflation is driven by a scalar field, the inflaton field, with a near-flat potential [6,7]. While the inflaton field slowly rolls on top of its potential, its quantum fluctuations are stretched on superhorizon scales, which provide the seeds of large-scale structure and the perturbations on the cosmic microwave background [8,9]. The basic predictions of the models of inflation are that these primordial perturbations are nearly scale invariant, Gaussian, and adiabatic, which are well supported by cosmological observations [10,11]. In addition, numerous observations indicate that the late Universe is undergoing a phase of accelerating expansion. The origin of dark energy as the source of the recent cosmological acceleration is not known, but a cosmological constant associated with the quantum zero point energy of fields may be a good option [12–16].

On the theoretical side, while there is no compelling theory of quantum gravity at hand, understanding quantum fields in curved backgrounds, including the dS background, may shed some light for the pursuit of a theory of quantum gravity. Understanding important issues such as regularizations and renormalization of cosmological correlations of quantum perturbations in dS background can play important roles as well. More specifically, similar to quantum field theories in flat spacetime, physical quantities such as the energy-momentum tensor, energy density, and pressure suffer from infinities in a curved spacetime. Therefore, it is an important question as how one can regularize and renormalize the infinities to extract the finite physical quantities. Furthermore, the fact that there is no unique vacuum in a curved spacetime adds more complexities for the treatment of regularization and renormalization in a curved spacetime [17–22].

In this work, we study the quantum fluctuations of a self-interacting scalar field with nonminimal coupling to gravity in a dS background. The free quantum fields with the conformal coupling in dS background [23–37] and the self-interacting scalar field with a quartic potential $V \propto \lambda\Phi^4$ [38–49] are vastly studied in the literature. In this paper, we extend those works to more general self-interacting potentials with the emphasis on vacuum zero point energy associated to the bubble diagrams. More specifically, we study potentials of the form $V \propto \lambda\Phi^n$ for even values of $n = 4, 6, \ldots$ and calculate the expectation values of the vacuum zero point energy and pressure associated to the bubble diagrams. The case $n = 4$ is renormalizable, but higher-order interactions with $n > 4$ are not renormalizable. However, it is understood that we work in an effective field theory approximation where renormalizability is not a fundamental requirement. The current setup may be viewed as a low-energy limit of some unknown UV complete theories which can be useful for phenomenological inflationary model building. We employ the dimensional regularization scheme [50–55] to regularize the quantum infinities (for earlier works concerning the dimensional regularization scheme in dS spacetime, see, for example, [38–49]). This paper extends our earlier work [56] in which the vacuum zero point energy associated to the bubble diagram for a free field with a conformal coupling in a dS background was studied. In the presence of self-interaction,

*firouz@ipm.ir
†h.sh.ahmadi@gmail.com; h.sheikhahmadi@ipm.ir

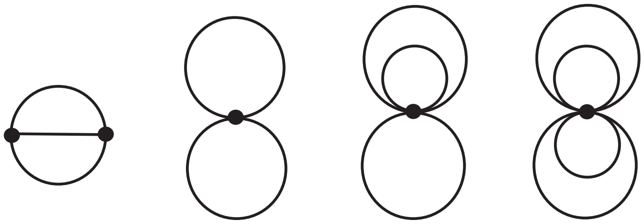

FIG. 1. The Feynman bubble diagrams for $\lambda\Phi^n$ theory at the leading order. For $n = 3$ one is dealing with a nested (double time) in-in integral, while for even values of $n = 4, 6, 8, \ldots$ one has a single time in-in integral. For each even value of n, one has to consider a diagram with $\frac{n}{2}$ loops to calculate the order λ corrections in expectation values such as $\langle\rho\rangle_\Omega$.

we have to consider multiple bubble loop diagrams as depicted in Fig. 1. For an even value of n, we have to consider a bubble diagram with $\frac{n}{2}$ loops to obtain the order λ corrections in the expectation values of physical quantities.

II. THE SETUP

We consider a real scalar field Φ in a dS spacetime which is nonminimally coupled to gravity with the conformal coupling ξ and the self-interacting potential $V(\Phi) = \lambda\Phi^n/n$, in which λ is the self-interacting constant. As will become evident soon, we consider even values of n with $n = 4, 6, \ldots$. For the special case $n = 4$, the coupling constant λ is dimensionless, while for higher values of n it has the dimension M^{4-n}. In addition, as we are interested in the conformal limit, and also, to simplify the analysis, we assume the field is massless, $m = 0$. However, as we shall see below, one can easily restore the mass in our formalism, though the equations will become more complicated. In our analysis below, we treat the contribution of the self-interaction as a perturbation to the free theory and calculate the vacuum zero point energy and pressure to first order in coupling constant λ.

With the above discussions in mind, the action involving the scalar field is given by

$$S = \int d^D x \sqrt{-g_D}\left(-\frac{1}{2}\xi\Phi^2 R - \frac{1}{2}\nabla^\mu\Phi\nabla_\mu\Phi - \frac{\lambda}{n}\Phi^n\right), \quad (1)$$

where D refers to the dimension of the spacetime and g_D stands for the determinant of the metric. Since we employ dimensional regularization to regularize the quantum infinities, we keep the spacetime dimension general and set $D = 4 - \epsilon$ with $\epsilon \to 0$ as in the conventional dimensional regularization approach. We work in the test field limit, where the background geometry is the solution of the Einstein field equations with no backreactions from the scalar fields. In order for this approximation to be consistent, the vacuum zero point energy and pressure associated with the fluctuations of Φ should be much smaller than the corresponding background quantities.

In the absence of the self-interaction, the theory is classically conformal invariant in four-dimensional spacetime when $\xi = \frac{1}{6}$. However, as is well known, this classical symmetry is anomalous under quantum corrections. Furthermore, the addition of the potential can break the conformal invariance at the classical level, since the coupling constant λ induces scale into the theory when $n \neq 4$. While the theory is still classically conformal invariant for $n = 4$, the quantum corrections break conformal invariance in this case as well. We consider the arbitrary values of even n, while the special case of $n = 4$ was studied extensively in the literature; see, for example, the works of Woodard and collaborators [38–49].

The background spacetime has the form of the Friedmann-Lemaître-Robertson-Walker metric:

$$ds^2 = a(\tau)^2(-d\tau^2 + d\mathbf{x}^2), \quad (2)$$

where τ is the conformal time related to cosmic time via $dt = a(\tau)d\tau$, in which $a(\tau)$ is the scale factor. In our limit of a fixed dS background, $aH\tau = -1$, in which H is the Hubble expansion rate associated to the dS background. Since the dS spacetime is maximally symmetric, the Ricci tensor and Ricci scalar are given as follows:

$$R_{\mu\nu} = (D-1)H^2 g_{\mu\nu}, \qquad R = D(D-1)H^2. \quad (3)$$

The scalar field equation is given by

$$\Box\Phi - \xi R\Phi - \lambda\Phi^{n-1} = 0. \quad (4)$$

Note that, to simplify the above field equation, we use the convention that the coupling constant has the form λ/n instead of $\lambda/n!$ which is usually used in quantum field theory (QFT) textbooks.

To study the quantum fluctuations, we introduce the canonically normalized field $\sigma(\tau)$:

$$\sigma(\tau) \equiv a^{\frac{D-2}{2}}\Phi(\tau), \quad (5)$$

in terms of which the action takes the following diagonal form:

$$S = \frac{1}{2}\int d\tau d^{D-1}\mathbf{x}\left[\sigma'(\tau)^2 - (\nabla\sigma)^2 + \left(\frac{(D-4)(D-2)}{4}\left(\frac{a'}{a}\right)^2 + \frac{D-2}{2}\frac{a''}{a} - \left(\xi R + \frac{\lambda}{a^D}\sigma^{n-2}\right)a^2\right)\sigma^2\right], \quad (6)$$

where a prime indicates the derivative with respect to the conformal time.

To quantize the field, we expand it in terms of the creation and annihilation operators in $D-1$-dimensional Fourier space as follows:

$$\sigma(x^\mu) = \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{\frac{(D-1)}{2}}}(\sigma_k(\tau)e^{i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}} + \sigma_k^*(\tau)e^{-i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}}^\dagger), \quad (7)$$

in which $\sigma_k(\tau)$ is the quantum mode function and $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$ satisfy the following commutation relation in $D-1$ spatial dimensions:

$$[a_{\mathbf{k}}, a_{\mathbf{k}}'^\dagger] = \delta^{D-1}(\mathbf{k}-\mathbf{k}'). \quad (8)$$

Correspondingly, the equation of motion of the free mode function (with $\lambda = 0$) from the action (6) is given by

$$\sigma_k''(\tau) + \left[k^2 + \frac{1}{\tau^2}\left(D(D-1)\xi - \frac{D(D-2)}{4}\right)\right]\sigma_k(\tau) = 0. \quad (9)$$

Note that the above equation is similar to the Mukhanov-Sasaki equation associated to the inflaton perturbations in an inflationary background [9]. Notice that for $\xi = \frac{1}{6}$ in $D = 4$ the second term in the big bracket vanishes and the mode function reduces to its simple flat form. This corresponds to the conformal limit which we consider in our analysis below. However, note that, in a general D-dimensional spacetime, the conformal limit corresponds to the special value

$$\xi = \xi_D \equiv \frac{D-2}{4(D-1)}. \quad (10)$$

Imposing the Bunch-Davies (Minkowski) initial conditions for the modes deep inside the horizon, the solution of the mode function from Eq. (9) is given in terms of the Hankel function:

$$\Phi_k(\tau) = a^{\frac{2-D}{2}}\sigma_k(\tau) = (-H\tau)^{\frac{D-1}{2}}\left(\frac{\pi}{4H}\right)^{\frac{1}{2}} e^{\frac{i\pi}{2}(\nu+\frac{1}{2})}H_\nu^{(1)}(-k\tau), \quad (11)$$

where

$$\nu \equiv \frac{1}{2}\sqrt{(D-1)^2 - 4D(D-1)\xi}. \quad (12)$$

From the above expression, we see that ν can be either real or pure imaginary, depending on the values of ξ. In our limit of interest where ξ is near its conformal value, ν is real. In particular, for $\xi = \frac{1}{6}$ with $D = 4$, we obtain $\nu = \frac{1}{2}$. As the in-in integrals become nontrivial, and in order to prevent complications associated to an imaginary ν in the mode functions, in our analysis below we assume that ν is real. This imposes the bound $0 < \xi \le \frac{D-1}{4D}$, which for $D = 4$ corresponds to $\xi \le \frac{3}{16}$.

The energy-momentum tensor in the presence of self-interaction is given by

$$T_{\mu\nu} = (1-2\xi)\partial_\mu\Phi\partial_\nu\Phi + \left(2\xi - \frac{1}{2}\right)g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi + \xi\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\Phi^2 + 2\xi(g_{\mu\nu}\Phi\Box\Phi - \Phi\nabla_\nu\nabla_\mu\Phi) - \frac{\lambda}{n}g_{\mu\nu}\Phi^n. \quad (13)$$

Employing the field equation (4), one can eliminate $\Box\Phi$, and, using Eq. (3), $T_{\mu\nu}$ is further simplified to

$$T_{\mu\nu} = \partial_\mu\Phi\partial_\nu\Phi + \frac{g_{\mu\nu}}{2}(4\xi-1)\partial^\alpha\Phi\partial_\alpha\Phi + \frac{\xi}{2}(D-1)(2+(4\xi-1)D)H^2 g_{\mu\nu}\Phi^2 - \xi\nabla_\mu\nabla_\nu\Phi^2 + g_{\mu\nu}\lambda\left(2\xi - \frac{1}{n}\right)\Phi^n. \quad (14)$$

Similarly, the trace of the energy-momentum tensor $T \equiv T^\mu_\mu$ is obtained to be

$$T = 2\left((D-1)\xi + \frac{2-D}{4}\right)(\partial^\alpha\Phi\partial_\alpha\Phi + D(D-1)\xi H^2\Phi^2) + \lambda\left(2\xi(D-1) - \frac{D}{n}\right)\Phi^n. \quad (15)$$

The energy density $\rho = T_{00}$ is

$$\rho = \frac{(1+4\xi)}{2}\dot{\Phi}^2 + \frac{(1-4\xi)}{2}\nabla^i\Phi\nabla_i\Phi + \frac{H^2}{2} \times [(1-4\xi)(D(D-1)\xi) - 2(D-1)\xi]\Phi^2 - \xi\nabla_0\nabla_0\Phi^2 - \lambda\left(2\xi - \frac{1}{n}\right)\Phi^n. \quad (16)$$

Finally, the pressure P is given by

$$P = \frac{1}{D-1}\perp^{\mu\nu}T_{\mu\nu}, \quad (17)$$

in which $\perp^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu$ represents the projection operator and $u^\mu = (1,0,0,0)$ is the comoving four-velocity. Consequently, we obtain

$$P = \frac{1}{D-1}(T+\rho). \quad (18)$$

In our analysis below, we will be mainly interested in vacuum expectation values such as $\langle\rho\rangle$, $\langle P\rangle$, and $\langle T\rangle$. This was studied for a free theory with a nonzero mass in [56], and here we extend these analyses in the presence of the self-interaction $\lambda\Phi^n$. However, it is important to note that the expectation value is with respect to the full vacuum, which we denote by $|\Omega\rangle$ so $\langle\rho\rangle \equiv \langle\Omega|\rho|\Omega\rangle$ and so on. Because of the interaction term $\lambda\Phi^n$, the vacuum $|\Omega\rangle$ is different than the vacuum associated to the free theory, which is denoted by $|0\rangle$. To prevent confusion, we define $\langle\Omega|\rho|\Omega\rangle \equiv \langle\rho\rangle_\Omega$ while $\langle 0|\rho|0\rangle \equiv \langle\rho\rangle_0$ and so on.

III. DIMENSIONAL REGULARIZATIONS AND IN-IN FORMALISM

In this section, we calculate vacuum expectation values such as $\langle\rho\rangle_\Omega$ using the dimensional regularization scheme in D dimensions. As in [56], let us define

$$\rho_1 \equiv \frac{1}{2}\dot{\Phi}^2, \qquad \rho_2 \equiv \frac{1}{2}g^{ij}\nabla_i\Phi\nabla_j\Phi, \qquad \rho_3 \equiv \frac{1}{2}H^2\Phi^2. \tag{19}$$

Then,

$$\begin{aligned}\langle\rho\rangle_\Omega &= (1+4\xi)\langle\rho_1\rangle_\Omega + (1-4\xi)\langle\rho_2\rangle_\Omega \\ &\quad + [(1-4\xi)D(D-1)\xi - 2(D-1)\xi]\langle\rho_3\rangle_\Omega \\ &\quad - \xi\langle\nabla_0\nabla_0\Phi^2\rangle_\Omega - \lambda\left(2\xi - \frac{1}{n}\right)\langle\Phi^n\rangle_\Omega.\end{aligned} \tag{20}$$

Note that the first three terms in the first line in Eq. (20) are formally the same as in [56] except that in [56] the expectation values were with respect to the vacuum of the free theory. The two terms in second line in Eq. (20) are new. First, we have a term with the specific λ coupling. Second, the contribution $\langle\nabla_0\nabla_0\Phi^2\rangle_\Omega$ is nontrivial. In the analysis of [56], it was noticed that $\langle\nabla_0\nabla_0\Phi^2\rangle_0 = 0$. This is because $\langle\Phi^2\rangle_0$ is a constant, so it is easy to understand that $\langle\nabla_0\nabla_0\Phi^2\rangle_0 = \nabla_0\nabla_0\langle\Phi^2\rangle_0 = 0$. However, in the presence of the interaction, we notice that $\nabla_0|\Omega\rangle \neq 0$, so one cannot simply take the derivative outside the expectation value, i.e., $\langle\nabla_0\nabla_0\Phi^2\rangle_\Omega \neq \nabla_0\nabla_0\langle\Phi^2\rangle_\Omega$.

Out of the five contributions into $\langle\rho\rangle_\Omega$ in Eq. (20), the last term $\lambda\langle\Phi^n\rangle_\Omega$ is the easiest term to calculate. This is because it has a factor of λ, and, since we are interested in first-order corrections in λ, we can simply assume the vacuum in this case is the free vacuum and

$$\lambda\langle\Phi^n\rangle_\Omega \simeq \lambda\langle\Phi^n\rangle_0 + \mathcal{O}(\lambda^2). \tag{21}$$

Note that the assumption that n is even was necessary to obtain the above result to leading order in λ. For odd values of n, the linear term in λ vanishes, and one has to go to higher orders of λ to calculate $\lambda\langle\Phi^n\rangle_\Omega$. This brings additional complexities involving the in-in integral as we shall see in the next section.

Since Φ is a Gaussian free field in the absence of interaction, one can easily see that, for even values of n,

$$\langle\Phi^n\rangle_0 \simeq (n-1)!!(\langle\Phi^2\rangle_0)^{\frac{n}{2}}, \tag{22}$$

in which $(n-1)!! = (n-1)(n-3)\ldots 1$. For example, for $n=4$, we have $\langle\Phi^4\rangle_0 = 3(\langle\Phi^2\rangle_0)^2$. As a result, the last term in Eq. (20) reads

$$\lambda\left(2\xi - \frac{1}{n}\right)\langle\Phi^n\rangle_\Omega \simeq \left(2\xi - \frac{1}{n}\right)\lambda(\langle\Phi^2\rangle_0)^{\frac{n}{2}} + \mathcal{O}(\lambda^2). \tag{23}$$

The quantity $\langle\Phi^2\rangle_0$ can be viewed as the coincident limit of the Feynman propagator. It plays important roles in our analysis below, in which the expectation values of the physical quantities in the presence of interaction can be expressed in terms of $\langle\Phi^2\rangle_0$.

A. Free theory

Here, we briefly review the results of [56] for the free theory, which will be used in our following analysis as well. In the free theory with $\lambda = 0$, the vacuum $|0\rangle$ is dS invariant, so from Eq. (20) we obtain

$$\begin{aligned}\langle\rho\rangle_0 &= (1+4\xi)\langle\rho_1\rangle_0 + (1-4\xi)\langle\rho_2\rangle_0 + [(1-4\xi)D(D-1)\xi \\ &\quad - 2(D-1)\xi]\langle\rho_3\rangle_0.\end{aligned} \tag{24}$$

As we shall see below, all three components of $\langle\rho_i\rangle_0$ are expressed in terms of $\langle\Phi^2\rangle_0$, so let us calculate this quantity. Performing the dimensional regularization analysis, we have

$$\langle\Phi^2\rangle_0 = \mu^{4-D}\int\frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}}|\Phi_k(\tau)|^2, \tag{25}$$

in which μ is a mass scale to keep track of the dimensionality of physical quantities. We decompose the integral into the radial and angular parts as follows:

$$\mathrm{d}^{D-1}\mathbf{k} = k^{D-2}\mathrm{d}k\mathrm{d}^{D-2}\Omega, \tag{26}$$

where $\mathrm{d}^{D-2}\Omega$ represents the $D-2$-dimensional angular part with the volume

$$\int\mathrm{d}^{D-2}\Omega = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma(\frac{D-1}{2})}. \tag{27}$$

Combining all numerical factors and defining the dimensionless variable $x \equiv -k\tau$, we obtain

$$\langle\Phi^2\rangle_0 = \frac{\pi^{\frac{3-D}{2}}\mu^{4-D}H^{D-2}}{2^D\Gamma(\frac{D-1}{2})}\int_0^\infty dx x^{D-2}\left|H_\nu^{(1)}(x)\right|^2. \tag{28}$$

Performing the integral, this yields [57]

$$\langle\Phi^2\rangle_0 = \frac{\mu^{4-D}\pi^{-\frac{D}{2}-1}}{2^D}\Gamma\left(\nu+\frac{D}{2}-\frac{1}{2}\right)\Gamma\left(-\nu+\frac{D}{2}-\frac{1}{2}\right) \times\Gamma\left(-\frac{D}{2}+1\right)\cos(\pi\nu)H^{D-2}. \tag{29}$$

In particular, note that, for the conformal limit with $\nu=\frac{1}{2}$, the above expression vanishes. With $\langle\Phi^2\rangle_0$ at hand and using Eq. (23), the last term for $\langle\rho\rangle_\Omega$ in Eq. (20) is calculated accordingly.

Following similar steps to calculate $\langle\rho_i\rangle_0$, we obtain the following relations [56]:

$$\langle\rho_1\rangle_0 = (D-1)\xi\langle\rho_3\rangle_0, \\ \langle\rho_2\rangle_0 = -(D-1)\langle\rho_1\rangle_0 = -(D-1)^2\xi\langle\rho_3\rangle_0, \tag{30}$$

where, from Eq. (19), $\langle\rho_3\rangle_0 = \frac{H^2}{2}\langle\Phi^2\rangle_0$ with $\langle\Phi^2\rangle_0$ given in Eq. (29). In the conformal limit where $\langle\Phi^2\rangle_0=0$, we see that $\langle\rho_i\rangle_0=0$ and, correspondingly, $\langle\rho\rangle_0=0$.

Similarly, calculating $\langle T\rangle_0$, one can show that $\langle T\rangle_0 = -D\langle\rho\rangle_0$ and, correspondingly [56],

$$\langle P\rangle_0 = -\langle\rho\rangle_0. \tag{31}$$

This is the expected result indicating the local Lorentz invariance for the free theory in which one expects to locally have $\langle T_{\mu\nu}\rangle = -\langle\rho\rangle g_{\mu\nu}$.

It is important to note that, in the above results, D is general, so to perform the regularization we consider $D=4-\epsilon$. One sets $\epsilon\to0$ at the end with the understanding that the singular terms in physical quantities with inverse powers of ϵ are canceled by appropriate counterterms as in standard QFT analysis.

It is useful to look at the results in some limits of interest. In the case of a massless field with no conformal limit, $\xi=0$, one obtains

$$\langle\rho\rangle_0^{\mathrm{reg}} = \frac{3H^4}{32\pi^2} = -\frac{1}{4}\langle T\rangle_0^{\mathrm{reg}} \quad (\xi=0). \tag{32}$$

On the other hand, as we noticed before, for the special case of conformal point with $\xi=\frac{1}{6}$, $\langle\rho\rangle_0^{\mathrm{reg}} = \langle T\rangle_0^{\mathrm{reg}} = 0$. However, if one restores the mass so the theory is not conformally invariant, one obtains [56]

$$\langle\rho\rangle_0^{\mathrm{reg}} = -\frac{H^4}{96\pi^2}\beta^2 + \frac{H^4}{64\pi^2}\left[\ln\left(\frac{H^2}{4\pi\mu^2}\right)-\frac{1}{2}\right]\beta^4 + \mathcal{O}(\beta^6)\left(\xi=\frac{1}{6}\right), \tag{33}$$

in which $\beta\equiv m/H$.

B. In-in formalism

To calculate the first four terms in Eq. (20) to first order in λ, we need to implement the in-in formalism, which perturbatively relates the vacuum expectation values of the interacting theory to the vacuum expectation of the free theory as follows [58]:

$$\langle\mathcal{O}(\tau)\rangle_\Omega = \left\langle 0\left|\bar{T}e^{i\int_{\tau_0}^{\tau}H_I(\tau')d\tau'}\mathcal{O}_I(\tau)Te^{-i\int_{\tau_0}^{\tau}H_I(\tau')d\tau'}\right|0\right\rangle, \tag{34}$$

where T and $\bar{T}$ stand for time ordering and antitime ordering, respectively. The subscript I in the right-hand side of the above equation indicates that all quantities are calculated in the interaction picture, i.e., with the mode function of the free theory given by Eq. (11). The initial time is $\tau_0=-\infty$, while the final slicing τ is the time when the measurement on the quantum operator $\mathcal{O}$ is made. As we work in an unperturbed dS background, we have $-\infty<\tau'\le\tau<0$. Since the time integrals in Eq. (34) are nontrivial, we shall restrict ourselves to the case where the upper limit $\tau\to0$, i.e., the measurement is being made toward the future boundary of dS. In an inflationary setup with deviations from an exact dS background, the final slicing $\tau\to0$ corresponds to the time of the end of inflation. Finally, H_I represents the interacting Hamiltonian, which in our case is

$$H_I = \frac{\lambda}{n}a^D\int d^{D-1}\mathbf{x}\Phi(x)^n. \tag{35}$$

Note that the factor a^D appears because of the volume element $\sqrt{-g}$.

To leading order in λ, the correction in $\langle\mathcal{O}(\tau)\rangle_\Omega$ is given by

$$\langle\mathcal{O}(\tau)\rangle_\Omega = \langle\mathcal{O}(\tau)\rangle_0 + 2\mathrm{Im}\int_{-\infty}^{\tau}d\tau'\langle\mathcal{O}(\tau)H_I(\tau')\rangle_0. \tag{36}$$

The first term above is the contribution in the absence of interaction which was calculated in [56] for $\mathcal{O}=\rho$, P, etc. Our goal below is to calculate the above integrals for four different operators ρ_1, ρ_2, ρ_3, and $\nabla_0\nabla_0\Phi^2$ as appearing in Eq. (20). In order to isolate the contribution of the free theory, we denote the last term in Eq. (36) by $\Delta\langle\mathcal{O}(\tau)\rangle$.

Let us start with $\langle\rho_3(\tau)\rangle_\Omega$, yielding

$$\Delta\langle\rho_3(\tau,\mathbf{x_0})\rangle = \frac{H^2}{2}\Delta\langle\Phi^2(\tau,\mathbf{x_0})\rangle = \frac{\lambda H^2}{n}\mathrm{Im}\left[\int_{-\infty}^{\tau}d\tau' a(\tau')^D \times\int d^{D-1}x\langle\Phi(\tau,\mathbf{x}_0)^2\Phi(\tau',\mathbf{x})^n\rangle_0\right]. \tag{37}$$

Note that $\mathbf{x}_0$ is an arbitrary reference point in background where the measurement is made. However, because of the spatial translation invariance, we can set $\mathbf{x}_0=0$, so we do not specify $\mathbf{x}_0$ in the rest of the analysis below. From the above expression, we see that, for odd values of n, the expectation values vanish in the light of the Wick theorem. Therefore, for odd values of n, one needs to go to second order of perturbations, leading to order $\mathcal{O}(\lambda^2)$ corrections. This, in turn, requires nested time integrals (i.e., a double time integral) which are more complicated than the single time integral over τ' which we encounter for even values of n as given in Eq. (37). For this reason, as mentioned before, we restrict our analysis to even values of $n = 4, 6, \ldots$.

There are two different types of contributions when performing the contractions in Eq. (37). The first type is in the form $\langle\Phi(\tau,\mathbf{x}_0)^2\rangle_0\langle\Phi(\tau',\mathbf{x})^n\rangle_0$. With a bit of effort, one can show that this contribution has no imaginary component, so this contribution vanishes. The second type contracts each term of $\Phi(\tau,\mathbf{x}_0)$ with a term in $\Phi(\tau',\mathbf{x})^n$. There are total $n(n-1)!!$ possibilities for these contractions. After performing the Wick contractions, we obtain (for further details of Wick contractions, see the Appendix)

$$\Delta\langle\rho_3(\tau)\rangle = (n-1)!!\lambda H^2(\langle\Phi^2\rangle_0)^{\frac{n-2}{2}}\mu^{4-D}I_3(\tau), \tag{38}$$

in which

$$I_3(\tau) \equiv \int_{\tau_0}^{\tau} d\tau' a(\tau')^D \int \frac{d^{D-1}\mathbf{q}}{(2\pi)^{(D-1)}} \mathrm{Im}[\Phi_q(\tau)^2\Phi_q^*(\tau')^2]. \tag{39}$$

Following similar steps for ρ_1 and ρ_2, we have

$$\Delta\langle\rho_1(\tau)\rangle = (n-1)!!\lambda H^2(\langle\Phi^2\rangle_0)^{\frac{n-2}{2}}\mu^{4-D}I_1(\tau), \tag{40}$$

with

$$I_1(\tau) \equiv \int_{-\infty}^{\tau} d\tau' \frac{a(\tau')^D}{a(\tau)^2} \int \frac{d^{D-1}\mathbf{q}}{(2\pi)^{(D-1)}} \mathrm{Im}[\Phi'_q(\tau)^2\Phi_q^*(\tau')^2], \tag{41}$$

and

$$\Delta\langle\rho_2(\tau)\rangle = (n-1)!!\lambda H^2(\langle\Phi^2\rangle_0)^{\frac{n-2}{2}}\mu^{4-D}I_2(\tau), \tag{42}$$

with

$$I_2(\tau) \equiv \int_{-\infty}^{\tau} d\tau' \frac{q^2 a(\tau')^D}{a(\tau)^2} \int \frac{d^{D-1}\mathbf{q}}{(2\pi)^{(D-1)}} \mathrm{Im}[\Phi_q(\tau)^2\Phi_q^*(\tau')^2]. \tag{43}$$

In addition, we have to calculate $\langle\nabla_0\nabla_0\Phi^2\rangle_\Omega$ as well, which is given by

$$\langle\nabla_0\nabla_0\Phi^2\rangle_\Omega = \frac{2}{a^2}\left[\langle\Phi'^2\rangle_\Omega + \langle\Phi\Phi''\rangle_\Omega + \frac{1}{\tau}\langle\Phi\Phi'\rangle_\Omega\right]. \tag{44}$$

Calculating each term as above, we obtain

$$\langle\nabla_0\nabla_0\Phi^2\rangle_\Omega = (n-1)!!\lambda H^2(\langle\Phi^2\rangle_0)^{\frac{n-2}{2}}\mu^{4-D}I_4(\tau), \tag{45}$$

in which

$$I_4(\tau) \equiv \int_{-\infty}^{\tau} d\tau' \frac{a(\tau')^D}{a(\tau)^2} \int \frac{d^{D-1}\mathbf{q}}{(2\pi)^{(D-1)}} \mathrm{Im}[(\Phi'_q(\tau)^2 + \Phi_q(\tau)\Phi''_q(\tau) + \frac{1}{\tau}\Phi_q(\tau)\Phi'_q(\tau))\Phi_q^*(\tau')^2]. \tag{46}$$

IV. MEASUREMENTS AT FUTURE DS BOUNDARY

So far, our analyses were general except that we assumed that n is even so we deal with a single time integral as in Eq. (39). To proceed further, we should calculate each of $I_i(\tau)$ listed above. We start with $I_3(\tau)$, which is easier. Let us denote the $(D-2)$-dimensional angular part of the momentum integral by V_{D-2} as given in Eq. (27). Defining the dimensionless variables $x \equiv -q\tau'$ and $y \equiv -q\tau$ and switching the orders of the time and momentum integrals, we obtain

$$I_3 = \frac{V_{D-2}}{(2\pi)^{D-1}}\mathrm{Im}\left[\frac{\pi^2 H^{D-4}}{16}\int_0^\infty dy y^{D-2}(H_\nu^{(1)}(y))^2 \times \int_y^\infty \frac{dx}{x}(H_\nu^{(2)}(x))^2\right]. \tag{47}$$

Looking at the integral over the x variable, we see that it is in the form of a nested integral. Furthermore, its integrand falls off quickly for large x as the integrand oscillates rapidly with a decaying amplitude. Therefore, one expects the dominant contribution for the interior integral comes from the lower bound when $x \to y$.

To proceed further and to calculate the integrals analytically, we have to impose some simplification approximations. As a reasonable approximation, we take $\tau \to 0$. This corresponds to performing the measurement at the future boundary of dS. In inflationary models, this corresponds to performing the vacuum expectation value at the time of the end of inflation. In the limit $\tau \to 0$, the mode function $\Phi_q(\tau)$ in Eq. (11) simplifies to

$$\Phi_q(\tau) \to -\frac{i\Gamma(\nu)}{\pi}\left(\frac{\pi}{4H}\right)^{\frac{1}{2}} e^{\frac{i\pi}{2}(\nu+\frac{1}{2})}(-H\tau)^{\frac{D-1}{2}}\left(\frac{-2}{k\tau}\right)^\nu. \tag{48}$$

In the context of inflation, this represents the superhorizon limit of cosmological perturbations when $q \ll aH$ so $y = -q\tau \to 0$.

Expanding the interior integrand for $x \ll 1$ and taking the lower bound of integral with $x \to y \to 0$, the rest of the integral over y can be taken analytically, yielding

$$I_3 = \frac{4^\nu \pi^{\frac{1}{2}} H^{D-4}}{32\nu} \frac{\Gamma(-\nu + \frac{d}{2} - \frac{1}{2})\Gamma(\nu - \frac{d}{2} + 1)\Gamma(\frac{d}{2} - \frac{1}{2})}{\Gamma(2\nu - \frac{d}{2} + \frac{3}{2})\Gamma(1-\nu)^2} \times \sin(\pi\nu) \sin\left(\pi\nu - \frac{\pi D}{2}\right). \quad (49)$$

Following the same strategy, we can calculate I_1, I_2, and I_4 analytically. It turns out that I_i are related to each other, so all of them can be expressed in terms of I_3 as follows:

$$I_1 = \frac{(4\nu^2 + d - 2\nu - 1)(-2\nu + d - 1)}{4(d - 2\nu)} I_3, \quad (50)$$

$$I_2 = -\frac{(d - 1 - 4\nu)(d - 1 - 2\nu)(d - 1)}{4(d - 2\nu)} I_3, \quad (51)$$

and

$$I_4 = 8\nu^2 I_3. \quad (52)$$

With the above analytical values of the in-in integrals at hand, we can calculate $\langle\rho\rangle_\Omega$, $\langle P\rangle_\Omega$, and $\langle T\rangle_\Omega$. The expressions for these quantities for a general value of ξ are too complicated to report here, so we consider two special limits for analytical presentations: first the conformal point where $\xi = \frac{1}{6}$ and second the limit of small deviation from the conformal point with $\delta\xi \equiv \xi - \frac{1}{6} \ll 1$. For general values of ξ where the analytical results are intractable, we present the numerical plots of $\langle\rho\rangle_\Omega$ and $\langle T\rangle_\Omega$.

A. Conformal point: $\xi = \frac{1}{6}$

At the conformal point with $\xi = \frac{1}{6}$, we obtain

$$\Delta\langle\rho\rangle = \frac{\lambda(3-n)}{3n}(n-1)!!\left(\frac{-H^2}{24\pi^2}\right)^{\frac{n}{2}} \quad (53)$$

and

$$\Delta\langle P\rangle = \frac{\lambda(2n-9)}{9n}(n-1)!!\left(\frac{-H^2}{24\pi^2}\right)^{\frac{n}{2}}. \quad (54)$$

From the above formulas for $\Delta\langle\rho\rangle$ and $\Delta\langle P\rangle$, and noting that when $\lambda = 0$ both $\langle\rho\rangle$ and $\langle P\rangle$ vanish, the equation of state $w = \frac{P}{\rho}$ is obtained to be

$$w = \frac{-2n + 9}{3n - 9}. \quad (55)$$

For example, for $n = 4$, corresponding to two-loop quantum corrections, we obtain $w = \frac{1}{3}$ so the quantum corrections in the energy-momentum tensor behave like radiation. On the other hand, for large values of n, the equation of state approaches $w \to -\frac{2}{3}$. It is intriguing that the quantum corrections from self-interactions are not in the form of $w = -1$, which is expected from local Lorentz invariance.

It is instructive to calculate $\Delta\langle T\rangle$ as well, yielding

$$\Delta\langle T\rangle = \frac{\lambda(n-4)}{n}(n-1)!!\left(\frac{-H^2}{24\pi^2}\right)^{\frac{n}{2}}. \quad (56)$$

Interestingly, for the case $n = 4$, we see that the quantum corrections in the trace of energy-momentum tensor vanish. This is consistent with the fact that for $n = 4$ the parameter λ is dimensionless, so the theory is classically scale invariant and $T = 0$ at the classical level. It is intriguing that the two-loop quantum correction respects this result as well. However, it is an open question whether or not higher-order loop corrections (i.e., λ^2 and higher-order corrections) respect this conclusion.

B. Small deviation from conformal point

Now suppose we slightly deviate from the conformal point with $\delta\xi = \xi - \frac{1}{6} \ll 1$. We calculate the quantum corrections to leading order in $\delta\xi$. By increasing the value of n, the analysis becomes complicated, so here we present the results for two cases $n = 4$ and $n = 6$.

Starting with $n = 4$ to linear order in $(\xi - \frac{1}{6})$, we obtain

$$\Delta\langle\rho\rangle \simeq -\frac{\lambda H^4}{2304\pi^4} + \frac{\lambda H^4}{32\pi^4}\left[-\frac{1}{\epsilon} + \ln\left(\frac{H^2}{4\pi\mu^2}\right) + \frac{5}{2} - \gamma\right]\left(\xi - \frac{1}{6}\right) + \mathcal{O}\left(\left(\xi - \frac{1}{6}\right)^2\right) \quad (n = 4) \quad (57)$$

and

$$\Delta\langle P\rangle \simeq -\frac{\lambda H^4}{6912\pi^4} + \frac{\lambda H^4}{96\pi^4}\left[-\frac{1}{\epsilon} + \ln\left(\frac{H^2}{4\pi\mu^2}\right) + \frac{23}{6} - \gamma\right] \times \left(\xi - \frac{1}{6}\right) + \mathcal{O}\left(\left(\xi - \frac{1}{6}\right)^2\right) \quad (n = 4), \quad (58)$$

in which γ is the Euler number. We have the divergent ϵ^{-1} term in both $\Delta\langle\rho\rangle$ and $\Delta\langle P\rangle$ which appears when $\xi \neq \frac{1}{6}$ which should be removed by appropriate counterterms. Interestingly, in this case, we see that the singular terms in $\Delta\langle\rho\rangle$ and $\Delta\langle P\rangle$ have the equation of state associated to radiation, $w = \frac{1}{3}$, as suggested in Eq. (55), while this does not hold for the finite terms. Indeed, we have

$$\Delta\langle\rho\rangle - 3\Delta\langle P\rangle \simeq \frac{19\lambda H^4}{96\pi^4}\left(\xi - \frac{1}{6}\right) \quad (n = 4), \quad (59)$$

so the divergent ϵ^{-1} terms are canceled in the above expression. In addition, the trace of energy-momentum tensor is not zero:

$$\Delta\langle T\rangle \simeq -\frac{3\lambda H^4}{16\pi^4}\left(\xi-\frac{1}{6}\right) \quad (n=4). \tag{60}$$

However, we notice that it has no singular part.

Now consider the case $n=6$, corresponding to three-loop bubble diagrams. In this case, the coupling constant λ has the dimension of M^{-2}, so the quantum corrections would scale like λH^6. To linear order in $(\xi-\frac{1}{6})$, one obtains

$$\Delta\langle\rho\rangle \simeq \frac{5\lambda H^6}{27648\pi^6} - \frac{15\lambda H^6}{512\pi^6}\left[-\frac{2}{3\epsilon}+\ln\left(\frac{H^2}{4\pi\mu^2}\right)+\frac{71}{54}-\gamma\right] \times\left(\xi-\frac{1}{6}\right)+\mathcal{O}\left(\left(\xi-\frac{1}{6}\right)^2\right) \quad (n=6). \tag{61}$$

As expected, we have the singular term ϵ^{-1}. In addition, there will be singular terms ϵ^{-2}, but it comes at second order $(\xi-\frac{1}{6})^2$. Similarly, for the pressure we obtain

$$\Delta\langle P\rangle \simeq \frac{-5\lambda H^6}{82944\pi^6} + \frac{5\lambda H^6}{512\pi^6}\left[-\frac{2}{3\epsilon}+\ln\left(\frac{H^2}{4\pi\mu^2}\right)+\frac{127}{54}-\gamma\right] \times\left(\xi-\frac{1}{6}\right)+\mathcal{O}\left(\left(\xi-\frac{1}{6}\right)^2\right) \quad (n=6). \tag{62}$$

From Eq. (55) for w in conformal point with $n=6$ we obtain $w=-\frac{1}{3}$, so we expect the singular terms in $\Delta\langle\rho\rangle$ and $\Delta\langle P\rangle$ to be related with this equation of state. Indeed, we have

$$\Delta\langle\rho\rangle + 3\Delta\langle P\rangle \simeq \frac{35\lambda H^6}{1152\pi^6}\left(\xi-\frac{1}{6}\right) \quad (n=6), \tag{63}$$

so the singular terms ϵ^{-1} are canceled in the above expression. We have checked that the equation of state $w=-\frac{1}{3}$ also holds for the most singular terms ϵ^{-2} which appear at second order $(\xi-\frac{1}{6})^2$.

A conclusion is that the equation of state Eq. (55), which is obtained for the conformal point, is the equation of state for the singular terms in inverse powers of ϵ in $\langle\rho\rangle$ and $\langle P\rangle$ at each order of $(\xi-\frac{1}{6})$ as well.

C. Numerical plots for general value of ξ

As the analytical expressions for $\Delta\langle\rho\rangle$ and $\Delta\langle T\rangle$ for the general values of ξ are very complicated, here we present their numerical plots for the special case of $n=4$.

In Fig. 2, we have presented the three-dimensional behavior of $\Delta\langle\rho\rangle$ and $\Delta\langle T\rangle$ as functions of two parameters (ξ,μ). We have varied μ in units of H, while ξ is varied in the interval $0<\xi<\frac{3}{16}$ in which the index ν is real. In the left panel in this figure, we have presented $\Delta\langle\rho\rangle/(\frac{-\lambda H^4}{2304\pi^4})$, while in the right panel we have presented $\Delta\langle T\rangle/(-\frac{3\lambda H^4}{16\pi^4})$. In the left panel, the horizontal surface represents the surface with the value equal to unity, while in the right panel the horizontal surface represents the surface $z=\xi-\frac{1}{6}$, independent of μ. In the left panel, we see that, near the conformal point $\xi=\frac{1}{6}$, $\Delta\langle\rho\rangle$ approaches the constant value given in Eq. (57). On the other hand, in the right panel, we see that near the conformal point $\Delta\langle T\rangle$ approaches the formula given in Eq. (60) in which the quantum corrections in trace vanish at the conformal point.

To have a better visualization, in Figs. 3 and 4 we have presented the two-dimensional sections of the above plot as

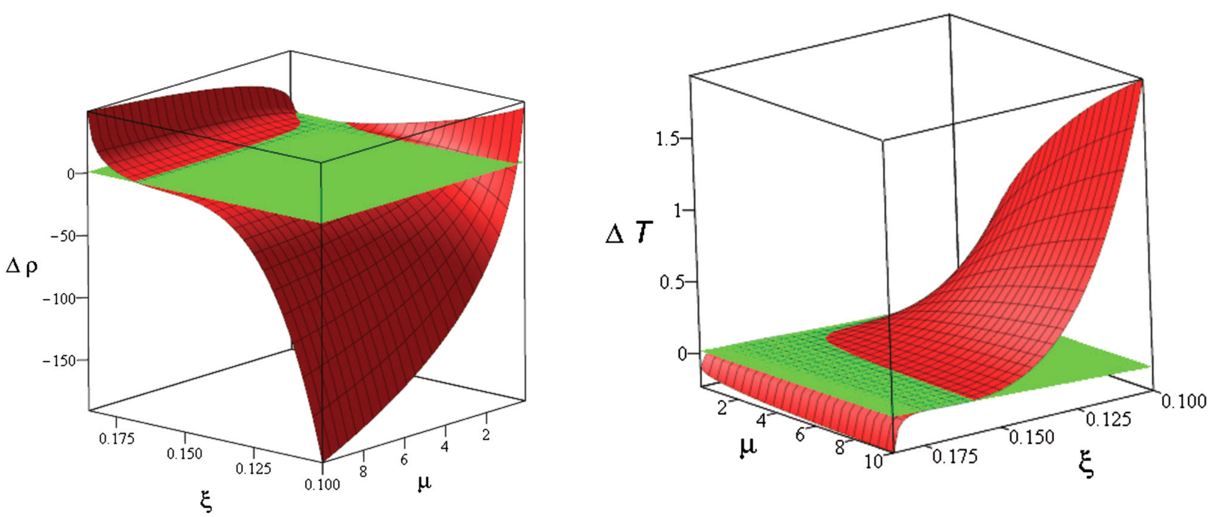

FIG. 2. The 3D diagrams of $\Delta\langle\rho\rangle$ and $\Delta\langle T\rangle$ as functions of variables (ξ,μ) for $n=4$. We have varied μ in units of H, while ξ is varied in the interval $0<\xi<\frac{3}{16}$. In the left panel $\Delta\langle\rho\rangle$ is measured in the scale of $\frac{-\lambda H^4}{2304\pi^4}$, while in the right panel $\Delta\langle T\rangle$ is measured in the scale of $-\frac{3\lambda H^4}{16\pi^4}$. The green horizontal surface in the left panel represents the surface $z=1$, while in the right panel it represents the surface $z=\xi-\frac{1}{6}$. In the left panel near the conformal point $\xi=\frac{1}{6}$, $\Delta\langle\rho\rangle$ approaches the constant value given in Eq. (57), while in the right panel $\Delta\langle T\rangle$ approaches the formula given in Eq. (60) near the conformal point.

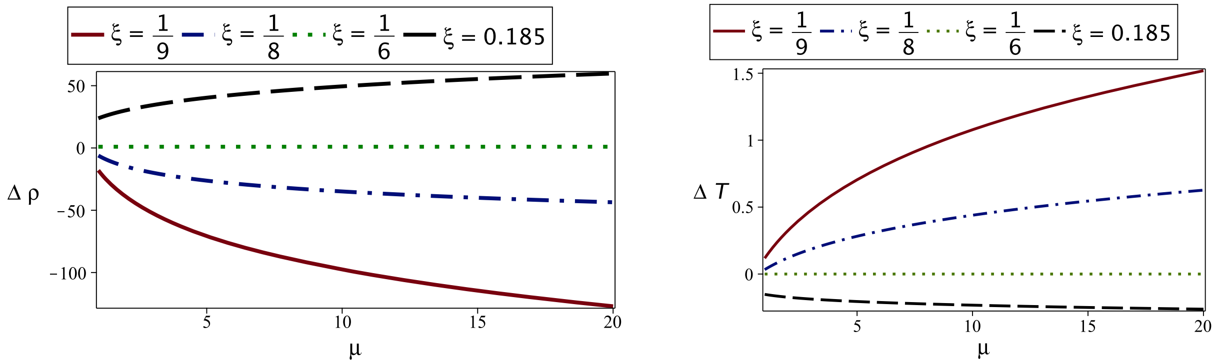

FIG. 3. The diagram shows the behavior of $\Delta\langle\rho\rangle$ and $\Delta\langle T\rangle$ as functions of μ for various fixed values of ξ. As in Fig. 2, $\Delta\langle\rho\rangle$ is measured in the scale of $\frac{-\lambda H^4}{2304\pi^4}$, while $\Delta\langle T\rangle$ is measured in the scale of $\frac{-3\lambda H^4}{16\pi^4}$. At the conformal point $\xi = \frac{1}{6}$, $\Delta\langle\rho\rangle$ and $\Delta\langle T\rangle$ are constant, while the behaviors of the curves change for values of ξ below and above this value.

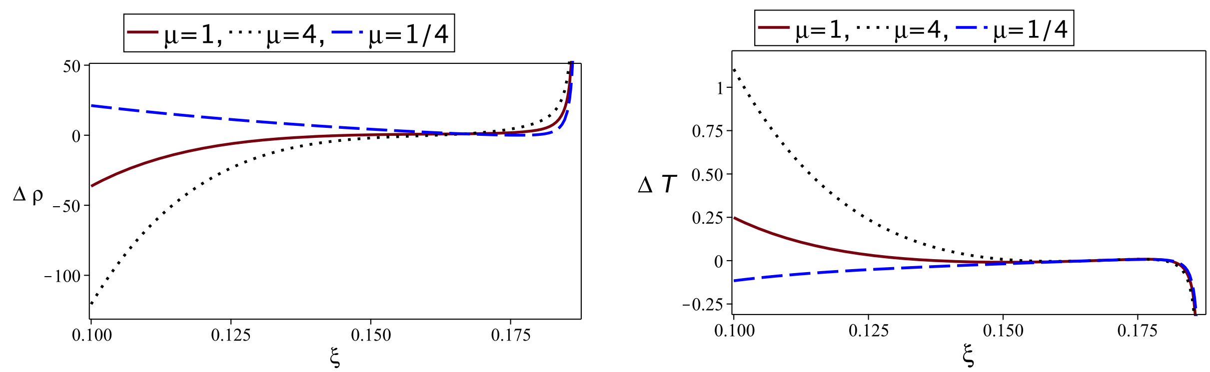

FIG. 4. The diagram shows the behavior of $\Delta\langle\rho\rangle$ and $\Delta\langle T\rangle$ as functions of μ for various fixed values of ξ. As in previous plots, $\Delta\langle\rho\rangle$ is measured in the scale of $\frac{-\lambda H^4}{2304\pi^4}$, while $\Delta\langle T\rangle$ is measured in the scale of $\frac{-3\lambda H^4}{16\pi^4}$. At the conformal point $\xi = \frac{1}{6}$, all curves merge to a fixed value in each panel.

functions of μ (ξ) for fixed values of ξ (μ). In both panels in Fig. 3, we see that $\xi = \frac{1}{6}$ plays a special role in which at this point $\Delta\langle\rho\rangle = \frac{-\lambda H^4}{2304\pi^4}$ and $\Delta\langle T\rangle = 0$ independent of the value of μ. This property is reinforced in Fig. 4, in which all curves merge to each other at the conformal point $\xi = \frac{1}{6}$.

V. SUMMARY AND DISCUSSIONS

In this work, we have studied the quantum fields with self-interaction potential $V \propto \lambda\Phi^n$ in a dS background and calculated the vacuum expectation values of the energy density, pressure, and the trace of the energy-momentum tensor. We have employed the perturbative in-in formalism to calculate the corrections in $\langle\rho\rangle_\Omega$, etc., to first order in λ. To simplify the analysis, we have considered even values of n. This is because for even values of n the nonzero corrections appear at the first order of λ, while for odd values of n the nonzero corrections appear at the order λ^2. Technically speaking, this corresponds to having a single time in-in integral for even values of n, while for odd values of n we need to consider nested (double time) in-in integrals. For the case $n = 4$ the self-coupling λ is dimensionless, so the theory is renormalizable. However, for higher-order interactions with $n > 4$, the theory is not renormalizable. In view of effective field theory, one may consider the current nonrenormalizable interactions as the low-energy limit of some unknown UV complete theory. Since our motivation is phenomenological, looking for implications of zero point energy for potentials which may be employed for inflationary model building, the condition of renormalizability may not be a fundamental requirement at this stage.

Our analysis were performed for general values of ξ, but to report the analytical results we have considered two special limits: the conformal point $\xi = \frac{1}{6}$ and the case with a small deviation from the conformal point $\delta\xi \ll 1$. At the conformal point, we obtain the equation of state Eq. (55) between $\langle\rho\rangle_\Omega$ and $\langle P\rangle_\Omega$. In particular, for $n = 4$ we obtain the intriguing result that $w = \frac{1}{3}$ with $\langle T\rangle_\Omega = 0$. This suggests that the classical conformal invariance associated to the case $n = 4$ is respected under two-loop (i.e., order λ) quantum corrections. It is an open question if this symmetry does hold to higher-order loop corrections. On the other hand, for other values of n, the coupling λ is dimensionful, so the theory is not conformally invariant even at the classical level. Therefore, the equation of state is different that that of radiation. It approaches $w \to -\frac{2}{3}$ for large values of n. In the case of $\delta\xi \neq 0$ but being small, we have calculated $\langle\rho\rangle_\Omega$ and $\langle P\rangle_\Omega$ to first order in $\delta\xi$ for cases $n = 4$ and $n = 6$. Unlike the conformal point, we have the divergent terms ϵ^{-1} at the order $\delta\xi$ and ϵ^{-2} for $n = 6$ at the order $\delta\xi^2$. We have found that these singular terms between $\langle\rho\rangle_\Omega$ and $\langle P\rangle_\Omega$ are related by the equation of state Eq. (55) at each order of $\delta\xi$.

While we presented the analytical results only for the above two special cases, we have presented the numerical plots of $\Delta\langle\rho\rangle$ and $\Delta\langle T\rangle$ for general values of ξ for $n = 4$ in Figs. 2–4. All these figures highlight the special roles played by the conformal limit $\xi = \frac{1}{6}$.

There are a number of directions in which the current study can be extended. The first which comes to mind is repeating this investigation for odd values of n. In this case, the in-in analysis become more complicated, as one has to go to second order in in-in integrals, with corrections appearing at the order of λ^2. The second question is to extend the current analysis for $n = 4$ to second order in λ and see if the classical conformal invariance with $w = \frac{1}{3}$ and $\langle T\rangle_\Omega = 0$ holds at the order of λ^2 or not. Since we were mostly interested in conformal limits, we have set the mass of the field to be zero. However, one can easily incorporate the effects of mass in the current analysis as well. In particular, as a physical example, one may consider a symmetry-breaking potential like $V = \lambda(\Phi^2 - v^2)^2$ in which the effective mass and the cubic and the quartic couplings are all related via the coupling constant λ. Employing in-in formalism to second order in λ, one can calculate the quantum corrections in vacuum zero point energy from both the cubic and quartic self-interactions. As the theory is not scale invariant even at the classical level, then one may not expect the conclusions $w = \frac{1}{3}$ and $\langle T\rangle_\Omega = 0$ to hold under loop corrections. We would like to come back to the interesting example of symmetry-breaking potential in the future.

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APPENDIX: WICK CONTRACTIONS

In this appendix, we briefly review some steps concerning Wick contractions.

Let us consider Eq. (37) in configuration space:

$$\Delta\langle\rho_3(\tau,\mathbf{x_0})\rangle = \frac{\lambda H^2}{n}\mathrm{Im}\left[\int_{-\infty}^{\tau} d\tau' a(\tau')^D \int d^{D-1}x \langle 0|\Phi(\tau,\mathbf{x_0})^2\Phi(\tau',\mathbf{x})^n|0\rangle_0\right]. \tag{A1}$$

Going to Fourier space and setting $\mathbf{x_0} = 0$ because of the translation invariance, this yields

$$\begin{aligned}\Delta\langle\rho_3(\tau,\mathbf{x_0})\rangle = \bigg\langle 0\bigg|\frac{\mu^{(n/2)(4-D)}}{n}\lambda H^2\mathrm{Im}\bigg[\int_{-\infty}^{\tau} d\tau' a(\tau')^D \int \frac{d^{D-1}\mathbf{p}_1}{(2\pi)^{D-1}}\frac{d^{D-1}\mathbf{p}_2}{(2\pi)^{D-1}}\frac{d^{D-1}\mathbf{q}_1}{(2\pi)^{D-1}}\cdots\frac{d^{D-1}\mathbf{q}_n}{(2\pi)^{D-1}}\\ \times\int d^{D-1}x[a_{\mathbf{p}_1}\phi_{\mathbf{p}_1}(\tau) + a^\dagger_{\mathbf{p}_1}\phi^*_{\mathbf{p}_1}(\tau)][a_{\mathbf{p}_2}\phi_{\mathbf{p}_2}(\tau) + a^\dagger_{\mathbf{p}_2}\phi^*_{\mathbf{p}_2}(\tau)]\\ \times[a_{\mathbf{q}_1}\phi_{\mathbf{q}_1}(\tau')e^{i\mathbf{q}_1\cdot\mathbf{x}} + a^\dagger_{\mathbf{q}_1}\phi^*_{\mathbf{q}_1}(\tau')e^{-i\mathbf{q}_1\cdot\mathbf{x}}][a_{\mathbf{q}_2}\phi_{\mathbf{q}_2}(\tau')e^{i\mathbf{q}_2\cdot\mathbf{x}} + a^\dagger_{\mathbf{q}_2}\phi^*_{\mathbf{q}_2}(\tau')e^{-i\mathbf{q}_2\cdot\mathbf{x}}]\\ \times[a_{\mathbf{q}_3}\phi_{\mathbf{q}_3}(\tau')e^{i\mathbf{q}_3\cdot\mathbf{x}} + a^\dagger_{\mathbf{q}_3}\phi^*_{\mathbf{q}_3}(\tau')e^{-i\mathbf{q}_3\cdot\mathbf{x}}][a_{\mathbf{q}_4}\phi_{\mathbf{q}_4}(\tau')e^{i\mathbf{q}_4\cdot\mathbf{x}} + a^\dagger_{\mathbf{q}_4}\phi^*_{\mathbf{q}_4}(\tau')e^{-i\mathbf{q}_4\cdot\mathbf{x}}]\\ \times\cdots\times[a_{\mathbf{q}_n}\phi_{\mathbf{q}_n}(\tau')e^{i\mathbf{q}_n\cdot\mathbf{x}} + a^\dagger_{\mathbf{q}_n}\phi^*_{\mathbf{q}_n}(\tau')e^{-i\mathbf{q}_n\cdot\mathbf{x}}]\bigg|0\bigg\rangle\bigg].\end{aligned} \tag{A2}$$

As a specific example, consider $n = 6$, where we present the Wick contractions for one case:

$$\begin{aligned}\Delta\langle\rho_3(\tau,\mathbf{x_0})\rangle \supset \bigg\langle 0\bigg|\frac{\mu^{(3)(4-D)}}{6}\lambda H^2\mathrm{Im}\bigg[\int_{-\infty}^{\tau} d\tau' a(\tau')^D \int \frac{d^{D-1}\mathbf{p}_1}{(2\pi)^{D-1}}\frac{d^{D-1}\mathbf{p}_2}{(2\pi)^{D-1}}\frac{d^{D-1}\mathbf{q}_1}{(2\pi)^{D-1}}\cdots\frac{d^{D-1}\mathbf{q}_6}{(2\pi)^{D-1}}\\ \times\int d^{D-1}x[a_{\mathbf{p}_1}a_{\mathbf{p}_2}a_{\mathbf{q}_1}a_{\mathbf{q}_2}a^\dagger_{\mathbf{q}_3}a^\dagger_{\mathbf{q}_4}a^\dagger_{\mathbf{q}_5}a^\dagger_{\mathbf{q}_6}]\times[e^{i(\mathbf{q}_1+\mathbf{q}_2-\mathbf{q}_3-\mathbf{q}_4-\mathbf{q}_5-\mathbf{q}_6)\cdot\mathbf{x}}]\\ \times[\phi_{\mathbf{p}_1}(\tau)\phi_{\mathbf{p}_2}(\tau)\phi_{\mathbf{q}_1}(\tau')\phi_{\mathbf{q}_2}(\tau')\phi^*_{\mathbf{q}_3}(\tau')\phi^*_{\mathbf{q}_4}(\tau')\phi^*_{\mathbf{q}_5}(\tau')\phi^*_{\mathbf{q}_6}(\tau')]\bigg]\bigg|0\bigg\rangle.\end{aligned} \tag{A3}$$

Using Eq. (8) and noting that

$$\int d^{D-1}x e^{i(\mathbf{q_i}\pm\mathbf{q_j})\cdot\mathbf{x}} = (2\pi)^{D-1}\delta^{(D-1)}(\mathbf{q_i}\pm\mathbf{q_j}), \tag{A4}$$

then Eq. (A3) is cast into the following form:

$$\begin{aligned}\Delta\langle\rho_3(\tau,\mathbf{x_0})\rangle \supset \Bigg\langle 0\Bigg|\frac{\mu^{3(4-D)}}{6}\lambda H^2 \mathrm{Im}\Bigg[\int_{-\infty}^{\tau} d\tau' a(\tau')^D \int \frac{d^{D-1}\mathbf{p}_1}{(2\pi)^{D-1}}\frac{d^{D-1}\mathbf{p}_2}{(2\pi)^{D-1}}\frac{d^{D-1}\mathbf{q}_1}{(2\pi)^{D-1}}\cdots\frac{d^{D-1}\mathbf{q}_6}{(2\pi)^{D-1}}\\ \times\delta(\mathbf{q}_2-\mathbf{q}_3)\delta(\mathbf{q}_1-\mathbf{q}_4)\delta(\mathbf{p}_1-\mathbf{q}_5)\delta(\mathbf{p}_2-\mathbf{q}_6)\delta(\mathbf{q}_1+\mathbf{q}_2-\mathbf{q}_3-\mathbf{q}_4-\mathbf{q}_5-\mathbf{q}_6)\\ \times[\phi_{\mathbf{p}_1}(\tau)\phi_{\mathbf{p}_2}(\tau)\phi_{\mathbf{q}_1}(\tau')\phi_{\mathbf{q}_2}(\tau')\phi^*_{\mathbf{q}_3}(\tau')\phi^*_{\mathbf{q}_4}(\tau')\phi^*_{\mathbf{q}_5}(\tau')\phi^*_{\mathbf{q}_6}(\tau')]\Bigg]\Bigg|0\Bigg\rangle.\end{aligned} \tag{A5}$$

After some manipulations, the above equation takes the following form:

$$\langle\rho_3(\tau,\mathbf{x_0})\rangle \supset \frac{\mu^{3(4-D)}}{6}\lambda H^2 \mathrm{Im}\Bigg[\int_{-\infty}^{\tau} d\tau' a(\tau')^D \int \frac{d^{D-1}\mathbf{q}_1}{(2\pi)^{D-1}}\frac{d^{D-1}\mathbf{q}_2}{(2\pi)^{D-1}}\frac{d^{D-1}\mathbf{q}_5}{(2\pi)^{D-1}}\quad [|\phi_{\mathbf{q}_1}(\tau')|^2|\phi_{\mathbf{q}_2}(\tau')|^2\phi^2_{\mathbf{q}_5}(\tau)\phi^{*2}_{\mathbf{q}_5}(\tau')]\Bigg].$$

It is easy to show that the desired equation for $\Delta\langle\rho_3(\tau,\mathbf{x_0})\rangle$, i.e., Eq. (38), for $n=4$ is obtained accordingly.

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