Angular spectrum of quantum fluctuations in causal structure

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(Received 28 November 2023; accepted 12 May 2024; published 3 June 2024)

Scaling arguments are used to constrain the angular spectrum of distortions on boundaries of macroscopic causal diamonds, produced by Planck scale vacuum fluctuations of causally coherent quantum gravity. The small-angle spectrum of displacement is derived from a form of scale invariance: The variance and fluctuation rate of distortions normal to the surface of a causal diamond of radius *R* at transverse physical separation $c\tau \ll R$ should depend only on τ , with a normalization set by the Planck time t_P , and should not depend on *R*. For measurements on scale *R*, the principle leads to universal scaling for variance on angular scale Θ , $\langle \delta \tau^2 \rangle_{\Theta} \simeq \tau t_p \sim \Theta R t_P / c$ and angular power spectrum $C_{\ell} \sim (R l_P) / \ell^3$ at $\ell \gg 1$. This spectrum is consistent with a relational model of holographic noise based on causally coherent virtual null gravitational shocks, a general picture conjectured for all ℓ . The high ℓ scaling is contrasted with that predicted in some other quantum models, which differ by one power of angular wave number ℓ and are shown to predict excessive blurring of images from distant sources.

DOI: 10.1103/PhysRevD.109.123505

I. INTRODUCTION

There is no widely accepted theory of how quantum gravity works [1,2]. It is generally agreed that standard relativity and quantum mechanics become inconsistent below the Planck scale $t_P = \sqrt{\hbar G/c^5}$, determined by Planck's constant \hbar , Newton's constant G, and the speed of light c. However, there is no consensus on large-scale phenomenology of quantum geometry, apart from the requirement that any theory must agree with classical gravity on large scales. The standard quantization of linearized gravity, based on quantum field theory, is not causally coherent and cannot form the foundation of a complete consistent theory, even on large scales [1,2].

A key breakthrough would be to identify a uniquely identifiable large-scale phenomenon associated with quantum gravitational effects. Such a research program may be possible if quantum states of geometry are nonlocally coherent on null surfaces, which leads to observable correlations even on macroscopic scales. Predictions of causally coherent macroscopic effects from Planck scale fluctuations have been derived from semiclassical models of holographic noise [3–7] and from more formal quantum theories of causal fluctuations [8–12].

In these scenarios, macroscopic null surfaces acquire large-scale distortions due to Planck scale quantum fluctuations, with an amplitude much larger than the Planck time. In principle, their correlations may be observable, either in the signals of suitable configured interferometers [3-7,10-14] or in the relic cosmic perturbations measured in the cosmic microwave background [15-17].

In this paper, we use scale invariance to make a modelindependent estimate of the small-scale angular spectrum of macroscopic corrugations of causal diamond surfaces and horizons due to coherent Planck scale quantum gravitational vacuum fluctuations.

II. ANGULAR SPECTRUM AND CORRELATION

Consider distortions of the spherical boundary of a causal diamond of radius R with displacement in time $\delta \tau$, measured for example as displacements of clocks (as considered in, e.g., Ref. [6]). In general, the displacement is relational, specific to the vantage point of the world line that defines the causal diamond.

We adopt standard notation from the CMB literature (see, e.g., Refs. [15,16]). A function on a unit sphere with directions $\vec{\Omega}$ labeled by standard polar coordinates θ, ϕ is decomposed into spherical harmonics $Y_{\ell m}$:

$$\delta\tau(\vec{\Omega}) = \sum_{\ell} \sum_{m} Y_{\ell m}(\theta, \phi) a_{\ell m}, \qquad (1)$$

where $a_{\ell m}$ are the harmonic coefficients of the discrete 2D angular spectrum. The angular power spectrum is related to the harmonic coefficients by

$$C_{\ell} = \frac{1}{2\ell' + 1} \sum_{m=-\ell}^{m=+\ell'} |a_{\ell m}|^2.$$
(2)

The angular correlation function is defined as

$$C(\Theta) = \langle \delta \tau_1 \delta \tau_2 \rangle_{\Theta},\tag{3}$$

where the average is taken over all points on the sphere with angular separation $\Theta = |\vec{\Omega}_1 - \vec{\Omega}_2|$. It is related to the angular power spectrum by

$$C(\Theta) = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_{\ell} P_{\ell}(\cos \Theta), \qquad (4)$$

where P_{ℓ} are the Legendre polynomials.

III. CAUSALLY COHERENT NOISE

Our main aim is to derive general scaling laws for C_{ℓ} and $C(\Theta)$ applicable to scaling of Planckian quantum noise in the high- ℓ limit. Extrapolation to low ℓ is affected by causal symmetries, whose main features can be captured in a geometrical model of coherent distortions on intersecting spherical null surfaces, as described in the example in the Appendix.

We will consider anisotropy generated in a broad class of models based on the principle that quantum gravitational vacuum fluctuation states are coherent in causal diamonds. This general construction allows states of space-time to have coherent causal structures consistent with particles they couple to. A theory that obeys the correspondence principle can have geometrical states that match the coherent angular pattern of distortions on a spherical null surface produced by classical gravity.

Consider quantum states of geometry that are coherent on causal diamonds, with holographic granularity at the Planck scale. Vacuum fluctuations produce directional causal distortions with nonlocal angular coherence on the surface of every causal diamond. A causally coherent model leads to vacuum fluctuations with variance and coherence similar to that generated by displacements by coherent gravitational shock waves that would be produced by a classical gas of many pointlike photons with randomly oriented Planck scale momenta [4–7,18], whose number density saturates gravitational bounds. For harmonic distortions that add in quadrature, the total variance of distortion on the surface of a causal diamond of radius R is of order [6]

$$\langle \delta \tau^2 \rangle \sim R t_P / c.$$
 (5)

This semiclassical estimate corresponds to the total holographic noise in the emergence of relational positions among world lines from a Planck scale quantum system with causal coherence and agrees with the standard quantum limit for such a system. Approximately the same overall variance has been derived more formally from a conformal field theory of near-horizon vacuum states [8–12].

IV. SMALL-ANGLE SCALING INVARIANCE

In a scale-invariant theory, holographic distortions measured locally in a region of size $\ll R$ should not depend on

the scale R. This property imposes general constraints on how the variance in Eq. (5) is distributed among different angular scales in a system of size R.

At very small angular separation, we can ignore the curvature of a causal diamond surface. A causal diamond that intersects a small patch of the surface contributes the same physical fluctuation in the normal direction to any much larger diamond, independent of its size. This property leads to the following form of scale-invariance symmetry for the spectrum: On small angular scales, correlations of physical displacement corrugations $\delta \tau$ in the normal direction to a null surface at transverse spacelike separation $c\tau$, which fluctuate coherently on timescale τ , should not depend on the total radial propagation distance R, only on the transverse separation scale $c\tau$. With this kind of invariance, measured fluctuations only depend on the causal diamonds encompassed by an actual measurement, not on much larger systems it is embedded in.

Scale invariance means that the variance of corrugation in patches of physical size τ matches the variance of causal diamonds of that size [Eq. (5)]:

$$\langle \delta \tau^2 \rangle_{\tau} \simeq \tau t_P.$$
 (6)

The contribution to total variance from patches of size $\Theta \ll 1$ is given by Eq. (3), so scaling invariance in the small angle limit

$$\ell \sim \pi/\Theta \sim \pi R/c\tau \gg 1 \tag{7}$$

leads to variance in patches of size Θ

$$C(0) - C(\Theta) \sim \langle \delta \tau^2 \rangle_{\Theta} \sim \tau t_p \sim \Theta R t_P / c, \qquad (8)$$

and an equivalent scaling of the power spectrum

$$C_{\ell} \sim C(<\Theta)/\ell^2 \sim \tau t_P/\ell^2 \sim \ell^{-3} R t_P/c.$$
(9)

With this scaling, the bulk of the total anisotropy is always dominated by the large angular scales and loworder modes. It agrees with scaling computed in concrete examples, for example, the angular spectrum of gravitational shock waves from photons emitted in directionally indeterminate *S* decay in an EPR experiment [6].

In general, the angular spectrum at low ℓ , and the correlation function at large Θ , have nontrivial structure that depart from these scalings due to the curvature in null shock fronts not accounted for in this argument. A specific illustration of a complete causally coherent candidate model spectrum is presented in the Appendix.

V. SMALL-ANGLE SPECTRUM IN THE GEONTROPIC MODEL

The scale-invariant spectrum contrasts with that of the "pixellon" in the Verlinde-Zurek geontropic model [10–12], which yields a variance that grows at small angles:

$$\langle \delta \tau^2 \rangle_{\Theta} = (Rt_P/\sqrt{8\pi^3 c}) \log(\pi/\Theta),$$
 (10)

and leads to a different high- ℓ scaling,

$$C_{\ell} \propto \ell^{-2}. \tag{11}$$

{See, for example, Eqs. (27) and (28) of Ref. [10], Eq. (30) of Ref. [11], or Eq. (57) of Ref. [12]}.

The geontropic model shows approximately the same magnitude for the macroscopic variance as the scaleinvariant, causally coherent holographic noise model just described, but with contributions spread out slightly more evenly in angular scale. This spectrum is not consistent with our conjectured scale invariance since the variation in a causal diamond increases with the size of other causal diamonds in which it is embedded.

As seen in Eq. (10), the VZ geontropic spectrum leads to a variance $\langle \delta \tau^2 \rangle$ that decreases only logarithmically on scales less than *R*. With this spectrum, wave front displacements randomly vary across the aperture of size $A \sim \pi R/\ell$ by an amount

$$\langle c^2 \delta \tau^2 \rangle \sim (R l_P) / \log(R/A).$$
 (12)

This scaling requires correlations of wave front corrugations with a small transverse separation but large R to "remember" that they have been travelling for a long time. We regard this as implausible because it violates the scale invariance described above. Unlike the spectrum derived from scale invariance, the variation in a patch depends on Ras well as A.

The difference in angular spectrum matters for experimental design. While both spectra have comparable total variance, the fluctuation power in the geontropic model is approximately evenly divided over logarithmic intervals of scale. One consequence is that the signal fluctuation power predicted with the geontropic model in interferometer experiments is typically reduced by a factor $\sim 1/\log(R/l_P)$.

VI. BLURRING OF DISTANT IMAGES

The shallower spectrum in Eq. (11) leads to an observable physical effect, a blurring of images from distant astronomical sources (as noted in Ref. [7]). A distortion of causal structure should lead to a comparable distortion in the wave fronts of light propagating from distant sources. The tilt of a wave front changes the apparent angular position of a distant point source. This effect of coherent geometrical distortions is not the same as previous models of blurring based on local interactions of quantum spacetime "foam" with propagating photons [19].

Consider the displacement of wave fronts entering a telescope aperture A from a source at distance D. The source appears in the image plane in a direction normal to the mean wave front entering the aperture. Fluctuations in mean wave front tilt lead to an effect analogous to atmospheric seeing: the apparent location of a source fluctuates on the sky, with an angular variance

$$\langle \delta \Theta^2 \rangle \sim c^2 \langle \delta \tau^2 \rangle_{\Theta = A/D} / A^2,$$
 (13)

and a timescale $\tau \sim A/c$. In the image plane, a pointlike source appears to be a point jittering very fast in an angular patch of area $\langle \delta \Theta^2 \rangle$ around its expected location. As expected from scale invariance, the jitter does not depend on the distance to a source, only on the size of the causal diamonds determined by the size of the measurement apparatus, $A \sim c\tau$.

Let us estimate the blurring produced in the VZ geontropic model. For a source with angular size distance $D \sim 1$ Gpc typical of a source at high redshift, the variance of angular position, which appears as a blurring in a time averaged image, is

$$\langle \delta \Theta^2 \rangle \sim \frac{Dct_P}{\sqrt{8\pi^3}A^2} \log(D/A) \sim \left(\frac{5\ \mu m}{A}\right)^2 \left(\frac{D}{1\ \text{Gpc}}\right).$$
 (14)

The geontropic blurring does not depend on the wavelength of photons, only the aperture used to image them. Since standard diffraction of a photon wave produces images of angular size $\sim \lambda/A$, geontropic blurring dominates diffraction at wavelengths shorter than few microns for sources at cosmological distances.

Distortions of this magnitude would have prevented high resolution imaging of cosmologically distant point sources at infrared and shorter wavelengths. Contemporary astronomers routinely obtain subarcsecond images $(\theta < 5 \times 10^{-6})$ of distant QSOs with apertures of the order of a meter, which rules out such blurring. To choose one example, the Hubble Space Telescope (A = 2.4 m) makes diffraction-limited UV images of cosmologically distant objects as small as $\sqrt{\langle \delta \Theta^2 \rangle} \sim 1.5 \times 10^{-7}$, an order of magnitude smaller than the expected geontropic blurring.

The predicted blurring for the scale-invariant ℓ^{-3} spectrum is smaller by a very large factor since the amount of variation on every scale is determined by causal diamonds of size *A*, not D_H :

$$\langle \delta \Theta^2 \rangle \sim (A l_P) / A^2 \sim l_P / A,$$
 (15)

which produces a negligible effect on images. In this case, the wave front tilt is of the same magnitude as the relational holographic displacement of mirrors in interferometers.

VII. CONCLUSION

We have derived constraints of a general character on the angular structure of large-scale causal distortions from coherent quantum gravity. Different models of holographic coherent quantum gravity lead to different predictions for angular spectra, some of which are constrained by existing astronomical data. These constraints complement inferences from primordial perturbations and from direct interferometric experiments.

APPENDIX: SPECTRAL MODELS WITH COHERENT NULL SHOCKS

In recent previous work [16], a classical noise model was developed to analyze holographic correlations of largeangle cosmic microwave background (CMB) anisotropy, based on virtual relational shock displacements at intersections of inflationary horizons. It is instructive to adapt this approach to estimate the large-angle spectrum of causal distortions on constant-time surfaces in flat space-time.

In the model, a single coherent shock produces coherent displacements on circular intersections of null spheres, described by an axially symmetric kernel $\delta \tau(\theta)$ that depends on the polar angle θ from the shock axis. A noise realization is a sum of $N \gg 1$ distortions from shocks in different directions $\vec{\Omega}_i$:

$$\delta \tau_{\text{total}}(\vec{\Omega}) = \sum_{i}^{N} \delta_{i} \, \delta \tau(\theta = |\vec{\Omega} - \vec{\Omega}_{i}|)/c, \qquad (A1)$$

where δ_i represents a random variable with zero mean and unit variance. For a model with holographic gravitational entropy, $N \simeq (R/ct_P)^2$ and $\langle \delta \tau^2 \rangle \sim t_P^2$, which leads to distortions on macroscopic causal surfaces given by Eq. (5). Since the shocks map onto a duration R/c, the total variation corresponds to a Planck variance per Planck time, like a random walk [4–7,18]. For large N, the sum of many such shocks produces a universal holographic angular power spectrum via Eq. (2), determined by the transform of the kernel:

$$a_{\ell 0} = 2\pi \int_0^{\pi} d\theta \, \sin(\theta) Y^*_{\ell 0}(\theta) \, \delta\tau(\theta). \tag{A2}$$

For the CMB :[16], a geometrical model for $\delta \tau(\theta)$ was constructed based on causal projections and axial displacements appropriate for slow-roll inflation (Fig. 1). The background curvature affects the high- ℓ spectrum since classical displacements stretch along with the comoving background outside the inflationary horizon.

Flat space-time and inflation have the same conformal causal structure, so many elements of the model construction apply to both cases. For flat space-time, we write the kernel as

$$\delta \tau(\theta) \propto P_{ABC}(\theta) d_{AB}(\theta).$$
 (A3)

The two factors represent an axial displacement P_{ABC} and a projection function d_{AB} for causal diamond surface intersections of world lines A, B, C with angular relationships as shown in Fig. 2.



FIG. 1. Angular power spectra of causal distortions on spacelike spheres. Causally coherent model spectra are shown for constant-time surfaces in slow-roll inflation (from Ref. [16]) and in flat space-time (from formulas in the text). As a reference, the dashed curve approximately follows a $C_{\ell} \propto \ell^{-3}$ power law, which is the scale invariant limit at high ℓ in the flat case. For comparison, the standard expectation for CMB temperature anisotropy is also shown, as well as CMB data for various Galaxy-subtracted maps and their average [15,16].



FIG. 2. Azimuthal angles associated with intersections of *ABC* spheres along an axis, which represent boundaries of causal diamonds entangled with shocks in the polar direction.

We adopt the same candidate form for d_{AB} as the CMB model in Ref. [16], which has the same conformal causal structure:

$$d_{AB}(\theta) = \cos(\theta)\cos(2\theta) \left[\cos(\theta) - \frac{1}{2}\sin(2\theta)\right].$$
(A4)

This expression is an approximate interpolation that accounts for the coherent projections and displacements of causal shocks as viewed from the center of a causal diamond.

The P_{ABC} term in flat space is not the same as inflation but is constrained to agree with the invariant scaling derived above. The variance of distortion for diamonds centered on the surface of the distorted sphere is proportional to duration, so the axial displacement is

$$P_{ABC}(\theta) = 1/\sqrt{\sin(\theta/2)}.$$
 (A5)

As shown in Fig. 1, the flat space-time spectrum reproduces the power law scaling expected at high ℓ . At $\ell \lesssim 7$, the model resembles the spectrum measured on the CMB sky, with significant extra power in odd modes, especially $\ell = 3, 5,$ and 7. Both features are also consistent with the spectrum posited in a previous white paper, Ref. [7]. These features could be important in the design and interpretation of laboratory experiments.

- S. Hollands and R. M. Wald, Quantum field theory is not merely quantum mechanics applied to low energy effective degrees of freedom, Gen. Relativ. Gravit. 36, 2595 (2004).
- [2] P. C. E. Stamp, Rationale for a correlated worldline theory of quantum gravity, New J. Phys. 17, 065017 (2015).
- [3] Y. J. Ng and H. van Dam, Measuring the foaminess of space-time with gravity-wave interferometers, Found. Phys. 30, 795 (2000).
- [4] C. J. Hogan, O. Kwon, and J. W. Richardson, Statistical model of exotic rotational correlations in emergent spacetime, Class. Quantum Gravity 34, 135006 (2017).
- [5] C. J. Hogan, Exotic rotational correlations in quantum geometry, Phys. Rev. D 95, 104050 (2017).
- [6] K. Mackewicz and C. J. Hogan, Gravity of two photon decay and its quantum coherence, Class. Quantum Gravity 39, 075015 (2021).
- [7] O. Kwon, Phenomenology of holography via quantum coherence on causal horizons, arXiv:2204.12080.
- [8] T. I. Banks and K. M. Zurek, Conformal description of nearhorizon vacuum states, Phys. Rev. D 104, 126026 (2021).
- [9] E. P. Verlinde and K. M. Zurek, Modular fluctuations from shockwave geometries, Phys. Rev. D 106, 106011 (2022).
- [10] E. P. Verlinde and K. M. Zurek, Observational signatures of quantum gravity in interferometers, Phys. Lett. B 822, 136663 (2021).
- [11] K. M. Zurek, On vacuum fluctuations in quantum gravity and interferometer arm fluctuations, Phys. Lett. B 826, 136910 (2022).

- [12] D. Li, V. S. H. Lee, Y. Chen, and K. M. Zurek, Interferometer response to geontropic fluctuations, Phys. Rev. D 107, 024002 (2023).
- [13] A. Chou, H. Glass, H. R. Gustafson, C. J. Hogan, B. L. Kamai, O. Kwon, R. Lanza, L. P. McCuller, S. S. Meyer, J. W. Richardson, C. Stoughton, R. E. Tomlin, and R. Weiss, Interferometric constraints on quantum geometrical shear noise correlations, Class. Quantum Gravity 34, 165005 (2017).
- [14] J. W. Richardson, O. Kwon, H. R. Gustafson, C. J. Hogan, B. L. Kamai, L. P. McCuller, S. S. Meyer, C. Stoughton, R. E. Tomlin, and R. Weiss, Interferometric constraints on spacelike coherent rotational fluctuations, Phys. Rev. Lett. **126**, 241301 (2021).
- [15] C. J. Hogan and S. S. Meyer, Angular correlations of causally coherent primordial quantum perturbations, Class. Quantum Gravity **39**, 055004 (2022).
- [16] C. J. Hogan, S. S. Meyer, N. Selub, and F. Wehlen, Angular correlations on causally coherent inflationary horizons, Class. Quantum Gravity 40, 165012 (2023).
- [17] C. Hogan, O. Kwon, S. S. Meyer, N. Selub, and F. Wehlen, Causal bounds on cosmological angular correlation, arXiv: 2312.16147.
- [18] C. J. Hogan and O. Kwon, Statistical measures of Planck scale signal correlations in interferometers, Class. Quantum Gravity 34, 075006 (2017).
- [19] E. S. Perlman, S. A. Rappaport, W. A. Christiansen, Y. J. Ng, J. DeVore, and D. Pooley, New constraints on quantum gravity from X-ray and gamma-ray observations, Astrophys. J. 805, 10 (2015).