# Analysis of $B_s^0 \to X(3872)[\psi(2S)]\pi^+\pi^-(K^+K^-)$ decays

Hao-Nan Wang,[1,2,*] Li-Sheng Geng,[3,4,5,6,†] Gang Li,[7,‡] and Ju-Jun Xie[1,2,6,§]

[1]*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*
[2]*School of Nuclear Sciences and Technology, University of Chinese Academy of Sciences, Beijing 101408, China*
[3]*School of Physics, Beihang University, Beijing 102206, China*
[4]*Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 102206, China*
[5]*Peng Huanwu Collaborative Center for Research and Education, Beihang University, Beijing 100191, China*
[6]*Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, China*
[7]*College of Physics and Engineering, Qufu Normal University, Qufu 273165, China*

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We have phenomenologically investigated the decays $B_s^0 \to X(3872)\pi^+\pi^-(K^+K^-)$ and $B_s^0 \to \psi(2S)\pi^+\pi^-(K^+K^-)$. In our analysis, the scalar meson $f_0(980)$ is formed through the final state interactions of coupled channels $\pi\pi$ and $K\bar{K}$. Our findings indicate that the $\pi^+\pi^-$ invariant mass distribution of the $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay can be accurately reproduced. Furthermore, we have explored the $\pi^+\pi^-(K^+K^-)$ invariant mass distribution of the $B_s^0 \to X(3872)\pi^+\pi^-(K^+K^-)$ decay, accounting for the different production mechanisms between $X(3872)$ and $\psi(2S)$, up to a global factor. It is found that the production rates for $X(3872)$ and $\psi(2S)$ are much different, which indicates that the structure of $X(3872)$ is more complicated than the $\psi(2S)$, which is a conventional $c\bar{c}$ state. Additionally, we have considered the contributions from $f_0(1500)$ to $\pi^+\pi^-$ and the $\phi$ meson to $K^+K^-$ in our analysis. Utilizing the model parameters, we have calculated the branching fraction of $B_s^0 \to X(3872)K^+K^-$, and anticipate that the findings of our study can be experimentally tested in the future.

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## I. INTRODUCTION

The nonleptonic weak decays of bottom hadrons are widely acknowledged as valuable means to elucidate the nature of certain enigmatic hadrons [1–6], especially these decays with charmonia in the final states [7,8]. For example, it was found that the scalar meson $f_0(500)$ has a relatively bigger signal than $f_0(980)$ in the decay of $\bar{B}^0$ into $J/\psi\pi^+\pi^-$ [9]. While the decay of $B_s^0 \to J/\psi\pi^+\pi^-$ was measured by the LHCb collaboration [10], and a pronounced peak was found for the scalar meson $f_0(980)$ in the $\pi^+\pi^-$ invariant mass distributions. However, there was no appreciable signal for the scalar meson $f_0(500)$ [10]. New measurements about the $B$ and $B_s$ decays have been performed by Belle Collaboration [11], CDF Collaboration [12], D0 Collaboration [13], and LHCb Collaboration [14,15].

The $B_s^0 \to J/\psi\pi^+\pi^-$ decay was studied in Ref. [6] based on the final-state interaction of pseudoscalar meson-pseudoscalar meson provided by the chiral unitary approach, where the scalar mesons $f_0(500)$ and $f_0(980)$ were dynamically generated. The theoretical results are in agreement with the experimental data [10]. The approach of Ref. [6] was successfully extended to study other weak decays of $B_s^0$ and $B$ mesons [16–21] (see also Ref. [1] for an extensive review). The $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay was firstly measured by the LHCb collaboration [22] and the $f_0(980)$ meson played an important role in the $\pi^+\pi^-$ invariant mass distributions. Recently, the $B_s^0 \to X(3872)\pi^+\pi^-$ decay was also firstly observed by the LHCb collaboration [23], where a large contribution from $B_s^0 \to X(3872)[f_0(980) \to \pi^+\pi^-]$ was found. Determining the $f_0(980)$ nature in the $B_s^0$ decays is possible. Indeed, it is interesting to investigate $f_0(980)$ in $B_s^0 \to X(3872)\pi^+\pi^-$, since it is the analogous decay compared with the decay of $B_s^0$ into $\psi(2S)\pi^+\pi^-$ within the

*wanghaonan@impcas.ac.cn
†lisheng.geng@buaa.edu.cn
‡gli@qfnu.edu.cn
§xiejujun@impcas.ac.cn

assumption that $X(3872)$ can be generated by the hadronization of $c\bar{c}$ which is used to produce $\psi(2S)$ in the former case. The $B_s^0 \to X(3872)\pi^+\pi^-$ decay is also a useful platform to explore the exotic feature of $X(3872)$ [24–33]. Even if it was discovered about two decades ago [34–43], its nature is still unclear. For instance, molecular perspective is one common explanation for $X(3872)$ rather than a pure chamonium. As discussed in Ref. [33], one has investigated the decays of $B$ meson into $X(3872)$ with a pseudoscalar or vector meson based on the molecular perspective of $X(3872)$ from the interaction of $D\bar{D}^* +$ c.c. (charge conjugate). Following the analysis about the $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay, we will also study the $B_s^0 \to X(3872)\pi^+\pi^-$ decay.

It is natural to study the role of $f_0(980)$ in the $K^+K^-$ invariant mass distribution of $B_s^0 \to \psi(2S)K^+K^-$ and $B_s^0 \to X(3872)K^+K^-$ decays using the chiral unitary approach, since $f_0(980)$ has strong coupling to the $K\bar{K}$ channel [44,45]. Note that, within the chiral unitary approach [46–49], the production of $f_0(980)$ and $f_0(500)$ mesons in $B^0$ and $B_s^0$ into $J/\psi$ and a $\pi^+\pi$ or $K^+K^-$ pair were investigated in Refs. [4,6,21]. To understand the new experimental data collected by the LHCb collaboration [23] and study the nature of $X(3872)$ and the scalar meson $f_0(980)$, in this work, we perform a coherent analysis of the $B_s^0 \to X(3872)\pi^+\pi^-(K^+K^-)$ and $B_s^0 \to \psi(2S)\pi^+\pi^-(K^+K^-)$ decays. In addition to the $f_0(980)$, we also consider the contribution from the scalar meson $f_0(1500)$, since its signal is clearly seen in the invariant $\pi^+\pi^-$ mass distributions [23,50,51].

This paper is organized as follows. In Sec. II, we present the theoretical formalism for the production of the scalar meson $f_0(980)$ in the $B_s^0$ decays into $\psi(2S)$ or $X(3872)$ and $\pi^+\pi^-$ or $K^+K^-$, together with a discussion about the scalar meson $f_0(1500)$ in the corresponding decays, while the contribution of the $\phi$ meson in the $B_s^0 \to X(3872)K^+K^-$ decay is also shown. In Sec. III, we show our theoretical numerical results and discussions, followed by a summary in the last section.

## II. THEORETICAL FORMALISM

### A. The $B_s^0 \to \psi(2S)[f_0(980), f_0(1500) \to \pi^+\pi^-]$ decay

The leading contributions to the decays of $B_s^0$ into $\psi(2S)$ plus a scalar meson is the Cabibbo favored $\bar{b} \to c\bar{c}\,\bar{s}$ process, therefore, the decay diagram of $B_s^0 \to \psi(2S)[f_0(980) \to \pi^+\pi^-]$, at the quark level, is shown in Fig. 1, which can be separated into two steps. The first step, namely the Cabbibo favored process, consists of the $\bar{b}$ decaying into a $\bar{c}$ quark and a $W^+$ boson followed by its decay into a $c$ quark and an $\bar{s}$ quark. Then, in addition to the hadronization of $c\bar{c}$ to produce $\psi(2S)$, we need another $q\bar{q}$ ($\equiv u\bar{u} + d\bar{d} + s\bar{s}$) pair to generate the $\pi^+\pi^-$ in the final states from $s\bar{s}$.

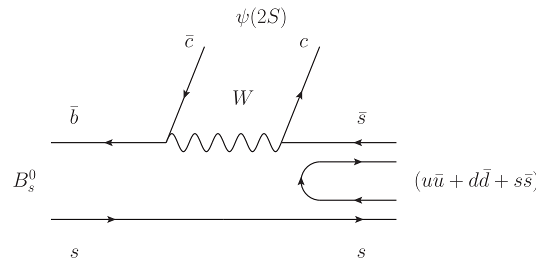


FIG. 1. Diagram for the decay of $B_s^0$ into $\psi(2S)$ (formed by the $c\bar{c}$ pair) and a primary $s\bar{s}$ pair, which hadronizes with an extra $(u\bar{u} + d\bar{d} + s\bar{s})$ pair from the vacuum.

Following Refs. [4,6], the hadronization of $s\bar{s}$, in terms of pseudoscalar mesons, can be written as

$$s\bar{s}(u\bar{u} + d\bar{d} + s\bar{s}) \to K^+K^- + K^0\bar{K}^0 + \frac{2}{3}\eta\eta. \quad (1)$$

After the pseudoscalar meson-pseudoscalar meson pair is produced, final-state interactions between the mesons occur, where the $\pi^+\pi^-$ pair can be obtained in the final states. The scalar meson $f_0(980)$ is dynamically generated from the $s$-wave interaction of the pseudoscalar meson-pseudoscalar meson in coupled channels [52–54]. Hence, the decay amplitude for $B_s^0 \to \psi(2S)[f_0(980) \to \pi^+\pi^-]$ can be written as [6],

$$\mathcal{M}^{f_0(980)}_{B_s^0\to\psi(2S)\pi^+\pi^-} = g_1\mathcal{M}_a = \frac{g_1 V_{cs}|\vec{p}_{\psi(2S)}|\cos\theta}{m_{B_s^0}} \times \Big( G_{K^+K^-}t_{K^+K^-\to\pi^+\pi^-} + G_{K^0\bar{K}^0}t_{K^0\bar{K}^0\to\pi^+\pi^-} + \frac{2}{3}\frac{1}{2}G_{\eta\eta}t_{\eta\eta\to\pi^+\pi^-}\Big), \quad (2)$$

where $\boldsymbol{p}_{\psi(2S)}$ is the three momentum of $\psi(2S)$ in the center-mass system of $B_s^0$ and $\theta$ is an integration variable of final-state phase space. Note that for the $B_s^0 \to \psi(2S)[f_0(980) \to \pi^+\pi^-]$ decay, we shall need a $p$-wave interaction to match angular momentum conservation. We introduce a parameter $g_1$ to contain all dynamical factors, which is assumed to be real and positive in this work. The $V_{cs}$ is one matrix element of the Cabbibo-Kobayashi-Maskawa matrix which is related to the Cabbibo angle [21]: $V_{cs} = \cos\theta_c = 0.97427$.

In Eq. (2), $G_i$ is the loop function of two meson propagators

$$G_i(s) = \mathrm{i}\int \frac{\mathrm{d}^4q}{(2\pi)^4}\frac{1}{(P-q)^2 - m_1^2 + \mathrm{i}\varepsilon}\frac{1}{q^2 - m_2^2 + \mathrm{i}\varepsilon}, \quad (3)$$

where "$i$" represents the $i$th channel, and $m_1$, $m_2$, and $q$ are the masses and four momentum of one meson in this channel, respectively. $P$ is the total momentum in this system, satisfying $s = P^2$. The three-momentum integral is

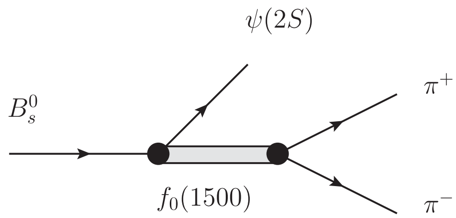


FIG. 2. Diagram for the decay of $B_s^0$ into $\psi(2S)$ and $\pi^+\pi^-$ through the resonance $f_0(1500)$.

carried out by precisely integrating the $q^0$ variable and applying a cutoff $\Lambda$ of the order of 1 GeV, which is impacted by the number of channels. The element of the scattering matrix, $t_{ij}$, for the transition of channel $i$ to $j$, is given by $t = (1 - VG)^{-1}V$. Now numbering the channels as 1 for $\pi^+\pi^-$, 2 for $\pi^0\pi^0$, 3 for $K^+K^-$, 4 for $K^0\bar{K}^0$, and 5 for $\eta\eta$, the $V$ matrix can be used in the same form as [6]. It is worth noting whether or not considering the $\eta\eta$ channel does not affec<!——>t the results much, as long as a reasonable cutoff $\Lambda$ is used. See more details in Refs. [6,44,53]. We do not consider the $\eta\eta$ channel in this work and take $\Lambda = 903$ MeV. The loop function $G$ and two-body scattering amplitude $t$ depend on the invariant mass $M_{\pi\pi}$ of the $\pi^+\pi^-$ system.

In addition to the scalar meson $f_0(980)$, we consider the scalar meson, namely $f_0(1500)$ as shown in Fig. 2. It is treated in the amplitude as a Breit-Wigner (BW) propagator.

$$\mathcal{M}^{f_0(1500)}_{B_s^0\to\psi(2S)\pi^+\pi^-} = g_2\mathcal{M}_b = \frac{\mathrm{i}g_2 m_{f_0(1500)}\Gamma_{f_0(1500)}|\vec{\boldsymbol{p}}_{\psi(2S)}|\cos\theta}{m_{B_s^0}(M_{\pi\pi}^2 - m_{f_0(1500)}^2 + \mathrm{i}m_{f_0(1500)}\Gamma_{f_0(1500)})}, \tag{4}$$

where $m_{f_0(1500)}$ and $\Gamma_{f_0(1500)}$ are the mass and width of $f_0(1500)$. Here, $g_2$ is a free parameter, and we consider it real and positive. Furthermore, ongoing debates exist about the nature of $f_0(1500)$, and its mass and width are not well determined [55]. Hence, $m_{f_0(1500)}$ and $\Gamma_{f_0(1500)}$ will be fitted to the experimental data.

Then, the total decay amplitude for $B_s^0 \to \psi(2S)\pi^+\pi^-$ is written as

$$\mathcal{M}_{B_s^0\to\psi(2S)\pi^+\pi^-} = g_1\mathcal{M}_a + g_2\mathcal{M}_b e^{\mathrm{i}\varphi}, \tag{5}$$

where $\varphi$ is the relative phase between $\mathcal{M}_a$ and $\mathcal{M}_b$, and it is a free parameter. In fact, as discussed in Ref. [23], there are indeed contributions from the interference between $f_0(980)$ and $f_0(1500)$ to the $\pi^+\pi^-$ invariant mass spectrum of the $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay.

### B. The mechanism of $B_s^0 \to X(3872)\pi^+\pi^-(K^+K^-)$

In contrast with the charmonium state $\psi(2S)$, the production of $X(3872)$ in the decay of $B_s^0 \to X(3872)\pi^+\pi^-$ may have a more involved mechanism because of the exotic nature of the $X(3872)$ state. Therefore, we should involve a different parameter $g_1'$ [see Eq. (2)] for the $B_s^0 \to X(3872)\pi^+\pi^-$ decay[1]:

$$\mathcal{M}_{B_s^0\to X(3872)\pi^+\pi^-} = \frac{g_1' V_{cs}|\vec{\boldsymbol{p}}_{X(3872)}|\cos\theta}{m_{B_s^0}} \times (G_{K^+K^-}t_{K^+K^-\to\pi^+\pi^-} + G_{K^0\bar{K}^0}t_{K^0\bar{K}^0\to\pi^+\pi^-}). \tag{6}$$

In other words, the mechanism for the production of $X(3872)$ is the same as that shown in Fig. 1 if we only consider the short-range contribution to the hadronization of $c\bar{c}$.

On the other hand, the contribution of $f_0(1500) \to \pi^+\pi^-$ in $B_s^0 \to X(3872)\pi^+\pi^-$ is different from that in the decay $B_s^0 \to \psi(2S)\pi^+\pi^-$. Referring to the masses of relevant particles in the Review of Particle Physics (RPP) [55], the phase space is tiny for the $B_s^0 \to X(3872)\pi^+\pi^-$ decay. The upper limit of the invariant mass of $M_{\pi\pi}$ is barely bigger than the mass of $f_0(1500)$, which means that the peak of $f_0(1500)$ in the $\pi^+\pi^-$ invariant mass distribution is seriously suppressed. Even if there is some contribution from $f_0(1500)$, it can be omitted in our mechanism. Thus, Eq. (6) is essentially the complete amplitude of the $B_s^0 \to X(3872)\pi^+\pi^-$ decay.

The $K^+K^-$ pair can not only be directly produced by the hadronization of $s\bar{s}$ with $u\bar{u}$ from the vacuum in Fig. 1, but also be dynamically produced by the final-state interaction of $K\bar{K}$ in $s$-wave. According to the diagrams shown in Fig. 3, the decay amplitude of $B_s^0 \to X(3872)f_0(980) \to X(3872)K^+K^-$ is given by

$$\mathcal{M}^{f_0(980)}_{B_s^0\to X(3872)K^+K^-} = \frac{g_1' V_{cs}|\vec{\boldsymbol{p}}_{X(3872)}|\cos\theta}{m_{B_s^0}} \times (1 + G_{K^+K^-}t_{K^+K^-\to K^+K^-} + G_{K^0\bar{K}^0}t_{K^0\bar{K}^0\to K^+K^-}), \tag{7}$$

where we have used the same coupling constant $g_1'$ as in Eq. (6) because of the similar mechanism and the same final state $X(3872)$.

On the other hand, we also consider the contribution of the $\phi$ meson to the $B_s^0 \to X(3872)K^+K^-$ decay. In this case, the $K^+K^-$ is produced in $p$-wave. The decay amplitude is written as

[1]Note that the $\eta\eta$ channel is also neglected.

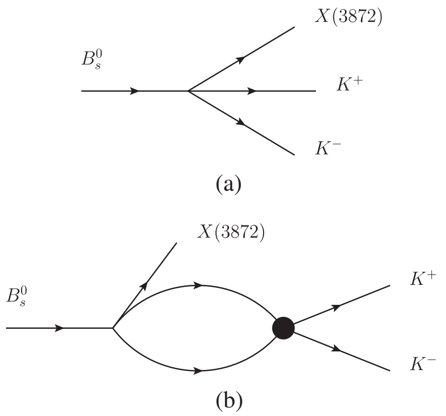


FIG. 3. Diagram for the decay of $B_s^0 \to X(3872)K^+K^-$ where $K^+K^-$ is produced in $s$-wave. (a) is the tree diagram, and (b) is the rescattering.

$$\mathcal{M}^{\phi}_{B_s^0\to X(3872)K^+K^-} = g_{BX\phi} g_{\phi K\bar{K}} \epsilon^{\mu\nu\rho\sigma} \times \epsilon^*_\mu(\boldsymbol{p}_X) p_{X\nu} q_\sigma \frac{\mathrm{i}(p_{K^+} - p_{K^-})_\rho}{q^2 - m_\phi^2 + \mathrm{i} m_\phi \Gamma_\phi}, \quad (8)$$

where $\epsilon^*_\mu(\boldsymbol{p}_X)$ and $p_X$ are the polarization and four momentum of $X(3872)$. And $\epsilon^*_\nu(\boldsymbol{q})$, $q$, $m_\phi$, and $\Gamma_\phi$ are the polarization, four momentum, mass, and width of the $\phi$ meson. Besides, $g_{BX\phi}$ and $g_{\phi K\bar{K}}$ are the coupling parameters of the vertexes of $B_s^0 X(3872)\phi$ and $\phi K\bar{K}$. With the branching fractions of $\mathcal{B}[B_s^0 \to X(3872)\phi] = (1.1 \pm 0.4) \times 10^{-4}$ and $\mathcal{B}[\phi \to K^+K^-] = (49.1 \pm 0.5)\%$ from RPP [55], one can obtain that $g^2_{BX\phi} = (7.3 \pm 2.7) \times 10^{-22}$ MeV$^{-2}$ and $g^2_{\phi KK} = (20.0 \pm 0.2)$. In general, $g_{BX\phi}$ and $g_{\phi K\bar{K}}$ are complex. However, from the partial decay width, one can only obtain the absolute value of the coupling constants, but not the phase. In this work, we assume that $g_{BX\phi}$ and $g_{\phi K\bar{K}}$ are real and positive.

### C. The mass distribution and partial decay width of $B_s^0 \to X(3872)[\psi(2S)]\pi^+\pi^-(K^+K^-)$

With these decay amplitudes obtained above, the $\pi^+\pi^-$ and $K^+K^-$ invariant mass distributions of $B_s^0 \to X(3872)[\psi(2S)]\pi^+\pi^-(K^+K^-)$ decay can be easily obtained as follows:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}M_{\mathrm{inv}}} = \frac{1}{512\pi^5 m^2_{B_s^0}} \int \mathrm{d}\Omega \mathrm{d}\Omega^* |\boldsymbol{p}||\boldsymbol{p}^*||\mathcal{M}|^2, \quad (9)$$

where $(\boldsymbol{p}, \Omega)$ is the three momentum of $X(3872)$ or $\psi(2S)$ in the rest frame of $B_s^0$, while $(\boldsymbol{p}^*, \Omega^*)$ is the three momentum of one $\pi$ ($K$) in the final $\pi^+\pi^-$ ($K^+K^-$) center-of-mass frame with invariant mass $M_{\mathrm{inv}}$. Here, $\mathcal{M}$ is taken as $\mathcal{M}^{f_0(980)}_{B_s^0\to\psi(2S)\pi^+\pi^-} + \mathcal{M}^{f_0(1500)}_{B_s^0\to\psi(2S)\pi^+\pi^-}$ or $\mathcal{M}^{f_0(980)}_{B_s^0\to X(3872)K^+K^-} + \mathcal{M}^{\phi}_{B_s^0\to X(3872)K^+K^-}$ for the case of $B_s^0 \to \psi(2S)\pi^+\pi^-$ or $B_s^0 \to X(3872)K^+K^-$, respectively.

TABLE I. The fitted parameters in this work.

| Parameters | Fitting results |
|---|---|
| $Cg_2^2$ | $(2.77 \pm 0.35) \times 10^8$ |
| $g_1/g_2$ | $0.68 \pm 0.04$ |
| $\varphi(°)$ | $-85.11 \pm 8.65$ |
| $m_{f_0(1500)}$ (MeV) | $1450.0 \pm 6.8$ |
| $\Gamma_{f_0(1500)}$ (MeV) | $164.4 \pm 22.4$ |
| $\chi^2$/d.o.f. | 1.4 |

## III. NUMERICAL RESULTS AND DISCUSSIONS

We calculate the invariant $\pi^+\pi^-$ mass distributions of the process $B_s^0 \to \psi(2S)\pi^+\pi^-$ with the above theoretical formalism. There are five free parameters to be obtained by fitting to the experimental data: $g_1$ in Eq. (2), $g_2$, $m_{f_0(1500)}$, and $\Gamma_{f_0(1500)}$ in Eq. (4), and $\varphi$ in Eq. (5). Since our numerical results are $\mathrm{d}\Gamma/\mathrm{d}M_{\mathrm{inv}}$, and the experimental data are events as a function of the $\pi^+\pi^-$ invariant mass, there is a global factor $C$ between our theoretical calculations and the experimental data. On the other hand, in the fitting to the experimental data, we use the following form:

$$\mathrm{data} = C\frac{\mathrm{d}\Gamma}{\mathrm{d}M_{\mathrm{inv}}} = \frac{Cg_2^2}{512\pi^5 m^2_{B_s^0}} \int \mathrm{d}\Omega \mathrm{d}\Omega^* |\boldsymbol{p}||\boldsymbol{p}^*| \times \left[\left(\frac{g_1}{g_2}\right)^2 |\mathcal{M}_a|^2 + |\mathcal{M}_b|^2 + \frac{2g_1}{g_2} \mathrm{Re}(\mathcal{M}_a^* \mathcal{M}_b e^{\mathrm{i}\varphi})\right]. \quad (10)$$

Thus, we take $Cg_2^2$, $g_1/g_2$, $m_{f_0(1500)}$, $\Gamma_{f_0(1500)}$, and $\varphi$ as free parameters. It should be noted that the global factor $C$ can normalize the theoretical results to match the experimental mass distribution. And more importantly, the factor $C$ is the same for the two processes $B_s^0$ into $\psi(2S)\pi^+\pi^-$ and $X(3872)\pi^+\pi^-$. In this way, the fitted parameters are listed in Table I. The obtained $\chi^2$/d.o.f is 1.4, which is reasonably small.

The fitted results of the $\pi^+\pi^-$ invariant mass distributions of $B_s^0 \to \psi(2S)\pi^+\pi^-$ are shown in Fig. 4. One can see that thanks to the contributions from $f_0(980)$ and $f_0(1500)$, the experimental data can be well reproduced. In the calculations, the scalar meson $f_0(980)$ is produced in the final-state interaction of $K\bar{K}$ and $\pi\pi$ in coupled channels. The first higher peak can be described by only the $f_0(980)$ state. In contrast, the second small peak and the long tail between the two peaks can be reproduced by the $f_0(1500)$ and the interference between $f_0(980)$ and $f_0(1500)$. It is worth mentioning that the mass and width of $f_0(1500)$ state are

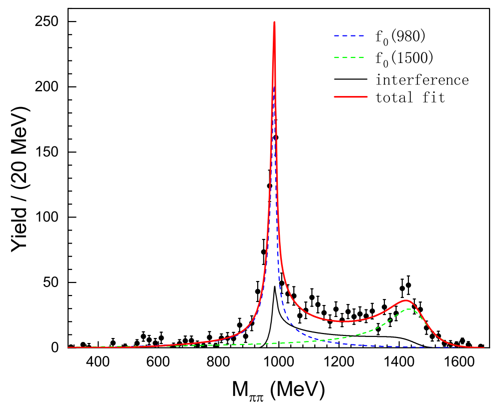


FIG. 4. Invariant mass distribution of $\pi^+\pi^-$ for the $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay, compared with the experimental data taken from Ref. [23]. The blue-dashed, green-dashed, and black-solid curves are the contributions from the $f_0(980)$, $f_0(1500)$, and their interference, respectively. The red-solid line is their total contribution.

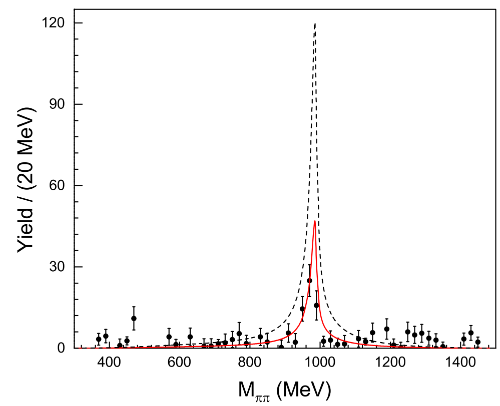


FIG. 5. Invariant mass distribution of $\pi^+\pi^-$ for the $B_s^0 \to X(3872)\pi^+\pi^-$ decay, compared with the experimental data [23]. The red-solid and black-dashed curves are obtained with different values for the production parameter $g_1'$.

mainly determined by the second peak, and the fitted results are different from the values quoted in the RPP [55].

With the fitted parameters and the branching ratio of $\mathcal{B}[B_s^0 \to \psi(2S)\pi^+\pi^-] = (6.9 \pm 1.2) \times 10^{-5}$ [55], we can extract the global factor $C$, which is $C = (8.28 \pm 1.44) \times 10^{17}$. Then, we can also get $g_2 = (1.83 \pm 0.20) \times 10^{-5}$ and $g_1 = (1.24 \pm 0.15) \times 10^{-5}$. If we take the same coupling constant $g_1$ for the $B_s^0 \to J/\psi\pi^+\pi^-$ decay, we obtain $\Gamma[B_s^0 \to J/\psi f_0(980) \to J/\psi\pi^+\pi^-] = (3.9 \pm 1.0) \times 10^{-14}$ MeV, which is in agreement with the value of $(5.4 \pm 0.6) \times 10^{-14}$ MeV quoted in the RPP [55]. This indicates that the coupling constants for producing charmonium states in the $B_s^0$ decays are universal.

Next, we turn to the $B_s^0 \to X(3872)\pi^+\pi^-$ decay. We firstly set $g_1' = g_1$. The resulting invariant mass $M_{\pi\pi}$ distribution of $B_s^0 \to X(3872)\pi^+\pi^-$ is shown as the black-dashed curve in Fig. 5. In this case, the obtained peak of $f_0(980)$ is too high compared with the available experimental data around 980 MeV. This indicates that the coupling of $g_1'$ should differ from that of $g_1$. In another words, the production mechanism of $X(3872)$ and $\psi(2S)$ in the $B_s^0 \to X(3872)[\psi(2S)]\pi^+\pi^-$ decays are different. Indeed, the contributions from the long-distance $\bar{D}D^*$ scattering to the $X(3872)$ production in the $B_s^0$ decays are important [32,33].

To get a good description of the experimental data on the $B_s^0 \to X(3872)\pi^+\pi^-$ decay, we modify the value of $g_1'$ and enable the theoretical results to pass through the highest experimental point around $M_{\pi\pi} = 980$ MeV. We get $g_1' = 0.69 g_1$, and the corresponding results are shown in Fig. 5 by the red curve. To explore more details about the difference between the production of $X(3872)$ and the charmonium states in the $B_s^0$ decays, we compare the modulus squared of their decay amplitudes, where the effect of the phase space is removed. For this purpose, we write

$$|\mathcal{M}_{B_s^0 \to R f_0(980)}|^2 = \Gamma_{B_s^0 \to R f_0(980)} / |\boldsymbol{p}_R|, \tag{11}$$

where $R$ represents the $X(3872)$, $\psi(2S)$, or $J/\psi$, respectively. For the partial decay width of $\Gamma_{B_s^0 \to R f_0(980)}$, we calculate them with the spectral function for the $\pi^+\pi^-$ distribution as follows [2,21]:

$$\Gamma_{B_s^0 \to R f_0(980)} = \frac{\int_{M_{\pi\pi}^{\rm min}}^{M_{\pi\pi}^{\rm max}} \frac{d\Gamma_{B_s^0 \to R\pi^+\pi^-}}{dM_{\pi\pi}} / S(M_{\pi\pi}^2) dM_{\pi\pi}}{\int_{M_{\pi\pi}^{\rm min}}^{M_{\pi\pi}^{\rm max}} dM_{\pi\pi}}, \tag{12}$$

with the special function $S(M_{\pi\pi}^2)$,[2]

$$S(M_{\pi\pi}^2) = -{\rm Im} \frac{2 m_{f_0(980)}/\pi}{M_{\pi\pi}^2 - m_{f_0(980)}^2 + i m_{f_0(980)} \Gamma_{f_0(980)}}, \tag{13}$$

where we take $m_{f_0(980)} = 985$ MeV and $\Gamma_{f_0(980)} = 100$ MeV as quoted in the RPP [55], while $M_{\pi\pi}^{\rm max} = m_{B_s^0} - m_R$ and $M_{\pi\pi}^{\rm min} = 2m_\pi$.

On the other hand, we can also evaluate the modulus squared of decay amplitudes for $B_s^0 \to R\phi(\eta, \eta')$ by replacing $f_0(980)$ with $\phi$, $\eta$, or $\eta'$, which contain an $s\bar{s}$ component. And we define:

$$R_1 = \frac{|\mathcal{M}_{B_s^0 \to X(3872) f_0(980)[\phi,\eta,\eta']}|^2}{|\mathcal{M}_{B_s^0 \to J/\psi f_0(980)[\phi,\eta,\eta']}|^2}, \tag{14}$$

[2]Here, we use a Breit–Wigner form for the $f_0(980)$, and it will not change our main conclusion if we worked in the dynamically generated picture.

TABLE II. Ratios of the two-body decays of $B_s^0 \to X(3872)[\psi(2S)]f_0(980)[\phi,\eta,\eta']$ to the $B_s^0 \to J/\psi f_0(980)[\phi,\eta,\eta']$.

| | $R_1$ | $R_2$ |
|---|---|---|
| $f_0(980)$ | $0.11 \pm 0.03$ | $0.34 \pm 0.08$ |
| $\phi$ | $0.18 \pm 0.07$ | $0.71 \pm 0.06$ |
| $\eta$ | $0.05 \pm 0.03$ | $1.07 \pm 0.35$ |
| $\eta'$ | $0.08 \pm 0.04$ | $0.54 \pm 0.16$ |

$$R_2 = \frac{|\mathcal{M}_{B_s^0 \to \psi(2S) f_0(980)[\phi,\eta,\eta']}|^2}{|\mathcal{M}_{B_s^0 \to J/\psi f_0(980)[\phi,\eta,\eta']}|^2}. \tag{15}$$

These obtained numerical results for $R_1$ and $R_2$ are listed in Table II. In the calculations, we take the two-body decay branching fractions from the RPP [55] except for the $\mathcal{B}[B_s^0 \to X(3872)\eta]$ and $\mathcal{B}[B_s^0 \to X(3872)\eta']$. Table II shows that the results for $R_2$ are close to one since both $\psi(2S)$ and $J/\psi$ are charmonium states. Furthermore, the obtained ratios of $R_1$ are much smaller than that of $R_2$, which indicates that the $X(3872)$ state is not pure charmonium.

For the decays of $B_s^0 \to X(3872)\eta$ and $B_s^0 \to X(3872)\eta'$, there are still no experimental measurements. Thus, we rely on the results obtained in Ref. [32] with the subtraction parameter $\alpha = -1.91$ (see more details in that reference). These values are listed in Table III. Note that Ref. [33] also gives the results of $\mathcal{B}[B_s^0 \to X(3872)\eta]$ and $\mathcal{B}[B_s^0 \to X(3872)\eta']$ based on the molecular picture of $X(3872)$.

Finally, we consider the $B_s^0 \to X(3872)[\psi(2S)]K^+K^-$ process. The theoretical results for the invariant $K^+K^-$ mass distributions are shown in Fig. 6, where the numerical results for the invariant $\pi^+\pi^-$ mass distributions of the $B_s^0 \to X(3872)[\psi(2S)]\pi^+\pi^-$ decays are also shown. The production rate of the $X(3872)$ is almost an order of magnitude smaller than that of the production of $\psi(2S)$ for both $\pi^+\pi^-$ and $K^+K^-$ final states. The final-state interactions of $\pi^+\pi^-$ and $K^+K^-$ occur in $s$-wave, where only the $f_0(980)$ contribution is considered. It is expected that future experimental measurements can test these calculations.

TABLE III. Branching fractions of $B_s^0$ decaying into $X(3872)[\psi(2S), J/\psi]$ and $\phi[\eta,\eta']$.

| Decay modes | Branching fractions ($\times 10^{-4}$) |
|---|---|
| $X(3872)\phi$ | $1.1 \pm 0.4$ |
| $X(3872)\eta$ | $0.15 \pm 0.07$ |
| $X(3872)\eta'$ | $0.17 \pm 0.08$ |
| $\psi(2S)\phi$ | $5.2 \pm 0.4$ |
| $\psi(2S)\eta$ | $3.3 \pm 0.9$ |
| $\psi(2S)\eta'$ | $1.29 \pm 0.35$ |
| $J/\psi\phi$ | $10.4 \pm 0.4$ |
| $J/\psi\eta$ | $4.0 \pm 0.7$ |
| $J/\psi\eta'$ | $3.3 \pm 0.4$ |

It is interesting to compare the branching fractions through the integral of invariant mass $M_{\pi\pi}$ and $M_{KK}$. The results are given by

$$\frac{\mathcal{B}[B_s^0 \to X(3872)(f_0(980) \to K^+K^-)]}{\mathcal{B}[B_s^0 \to X(3872)(f_0(980) \to \pi^+\pi^-)]} = 0.5, \tag{16}$$

$$\frac{\mathcal{B}[B_s^0 \to \psi(2S)(f_0(980) \to K^+K^-)]}{\mathcal{B}[B_s^0 \to \psi(2S)(f_0(980) \to \pi^+\pi^-)]} = 0.6, \tag{17}$$

which show that the branching fraction obtained from the integral over invariant mass $M_{KK}$ is of the same order of magnitude as that for $M_{\pi\pi}$ while the strength of $K^+K^-$ invariant mass distribution below the peak of $f_0(980)$ is much smaller than that for $\pi^+\pi^-$.

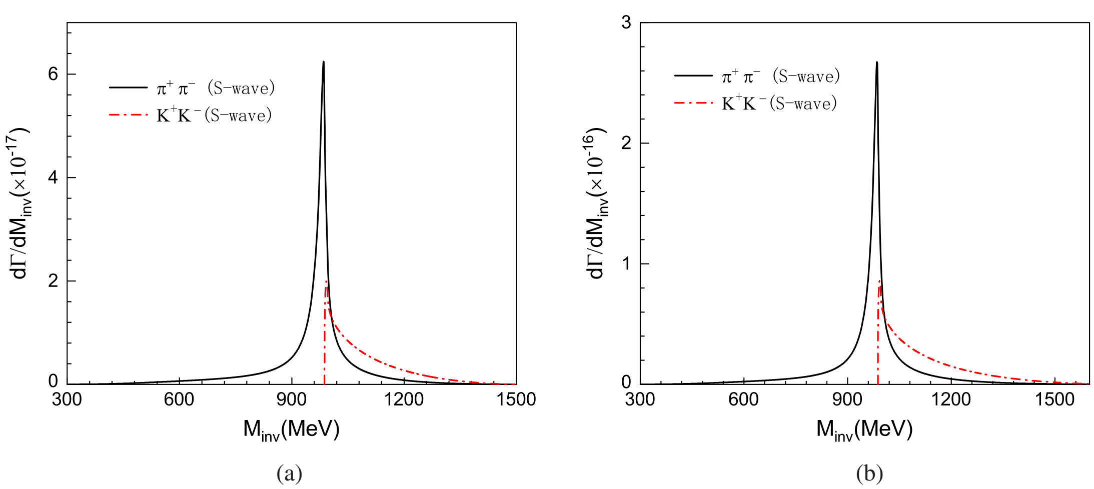


FIG. 6. The $\pi^+\pi^-$ and $K^+K^-$ invariant mass distribution of $B_s^0$ decay with the final state (a) $X(3872)$ and (b) $\psi(2S)$.

Moreover, it is easy to get the branching fraction from the measurements of Ref. [56],

$$\mathcal{B}[B_s^0 \to X(3872)(K^+K^-)_{\text{non}-\phi}] = (8.6 \pm 3.5) \times 10^{-5}. \quad (18)$$

Then, one can also get the following ratio,

$$\frac{\mathcal{B}[B_s^0 \to X(3872)(f_0(980) \to K^+K^-)]}{\mathcal{B}[B_s^0 \to X(3872)(K^+K^-)_{\text{non}-\phi}]} = 0.06 \pm 0.02, \quad (19)$$

which means that the $s$-wave $K^+K^-$ contribution from $f_0(980)$ is extremely small compared with other non-$\phi$ contributions.

For the contribution of the $\phi$ meson to the $B_s^0 \to X(3872)K^+K^-$, with the above obtained couplings of $g^2_{BX\phi}$ and $g^2_{\phi KK}$, we get the branching fraction

$$\mathcal{B}[B_s^0 \to X(3872)(\phi \to K^+K^-)] = (8.3 \pm 3.0) \times 10^{-5}, \quad (20)$$

which is consistent with the following result from the narrow width approximation within the uncertainty:

$$\mathcal{B}[B_s^0 \to X(3872)(\phi \to K^+K^-)] = \mathcal{B}[B_s^0 \to X(3872)\phi] \times \mathcal{B}[\phi \to K^+K^-] = (5.4 \pm 2.0) \times 10^{-5}, \quad (21)$$

where we have used $\mathcal{B}[\phi \to K^+K^-] = (49.1 \pm 0.5)\%$ from the RPP [55].

## IV. SUMMARY

We have investigated the decays of $B_s^0$ into $\psi(2S)\pi^+\pi^-$ and $X(3872)\pi^+\pi^-$ and performed a $\chi^2$ fit to the $\pi^+\pi^-$ invariant mass distributions based on the experimental data from the LHCb collaboration. Taking the dominant Cabibbo favored weak decay mechanism of $B_s^0$, we firstly get $\psi(2S)$ or $X(3872)$ and an $s\bar{s}$ pair. Second, after the hadronization of $s\bar{s}$, we get $\pi^+\pi^-$ and $K^+K^-$ in the final state, and this interaction is mediated by the scalar meson $f_0(980)$. In addition, the contribution from the scalar meson $f_0(1500)$ is also considered for the $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay. It is found that the recent LHCb experimental measurements on the $\pi^+\pi^-$ invariant mass distributions of $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay can be well reproduced.

Within the same picture, we also studied the $B_s^0 \to X(3872)\pi^+\pi^-$ decay. We find that, to reproduce the experimental data, one needs a different production coupling parameter for $X(3872)$, which indicates that the production of $X(3872)$ is not the same as the production of the charmonium state $\psi(2S)$. Moreover, we have compared the modulus squared of amplitudes of $B_s^0$ decays into $X(3872)$ or $\psi(2S)$ and one light meson, namely $f_0(980)$, $\phi$, $\eta$, and $\eta'$. The results indicate that the production amplitudes of $X(3872)$ in $B_s^0$ decays are different from that of one charmonium in the same $B_s^0$ decays. This may indicate that the $X(3872)$ is not a pure charmonium state.

The $\pi^+\pi^-$ and $K^+K^-$ invariant mass distributions for the processes $B_s^0 \to \psi(2S)[X(3872)]\pi^+\pi^-$ and $B_s^0 \to \psi(2S)[X(3872)]K^+K^-$ are calculated, where we have naturally considered the $K^+K^-$ final state from $f_0(980)$ for the decays of $B_s^0$ into $\psi(2S)K^+K^-$ and $X(3872)K^+K^-$ in the coupled channel approach, and compared with the $\pi^+\pi^-$ final state in the same situation. On the one hand, it is found that the peak strength of $f_0(980)$ in $m_{\pi\pi}$ is higher than that in $m_{KK}$ for the production of $X(3872)$ or $\psi(2S)$ in the $B_s^0$ decays. On the other hand, we realize that $\mathcal{B}[B_s^0 \to X(3872)\pi^+\pi^-]$ is bigger than $\mathcal{B}[B_s^0 \to X(3872)K^+K^-]$ while both are of the same order of magnitude. The above result does not change with the substitution of $\psi(2S)$ for $X(3872)$. The results here shed light on the fact that the low-lying scalar meson $f_0(980)$ is formed from the interaction of pseudoscalar meson and pseudoscalar meson and that $X(3872)$ is indeed not a pure charmonium state.

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