Binned top quark spin correlation and polarization observables for the LHC at 13.6 TeV

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We consider top-antitop quark ($t\bar{t}$) production at the Large Hadron Collider (LHC) with subsequent decays into dileptonic final states. We use and investigate a set of leptonic angular correlations and distributions with which all the independent coefficient functions of the top-spin-dependent parts of the $t\bar{t}$ production spin density matrices can be experimentally probed. We compute these observables for the LHC center-of-mass energy 13.6 TeV within the Standard Model at next-to-leading order in the QCD coupling including the mixed QCD weak corrections. We determine also the $t\bar{t}$ charge asymmetry where we take in addition also the mixed QCD-QED corrections into account. In addition we analyze and compute possible new physics (NP) effects on these observables within effective field theory in terms of a gauge-invariant effective Lagrangian that contains the operators up to mass dimension six that are relevant for hadronic $t\bar{t}$ production. First, we compute our observables inclusive in phase space. Then, in order to investigate which region in phase space has, for a specific observable, a high NP sensitivity, we determine our observables also in two-dimensional $(M_{t\bar{t}}, \cos\theta_t^*)$ bins, where $M_{t\bar{t}}$ denotes the $t\bar{t}$-invariant mass and θ_t^* is the top quark scattering angle in the $t\bar{t}$ zero-momentum frame.

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I. INTRODUCTION

The exploration of top quark spin effects in hadronic top quark pair production has become an established tool for investigating Standard Model (SM) interactions and for searches of new physics (NP). Evidence for $t\bar{t}$ spin correlations was found first by the D0 experiment [1] at the Tevatron and they were first observed by the ATLAS experiment [2] at the Large Hadron Collider (LHC). Subsequently, the ATLAS and CMS experiments at the LHC performed at c.m. energies 7 and 8 TeV, and more recently also at 13 TeV, a number of top spin correlation and polarization measurements in the dileptonic and lepton plus jet final states, using various sets of observables [3–10]. Recently, the ATLAS Collaboration used top spin correlations to claim quantum entanglement in top quark pairs [11]. This was confirmed by CMS [12]. On the theory side, SM predictions for spin correlations and polarizations were made at next-to-leading order (NLO) QCD including electroweak interactions for a number of $t\bar{t}$ spin correlation and top polarization observables in dileptonic and lepton plus jet final states [13–16]. Radiative corrections to off-shell $t\bar{t}$ production and decay were determined in [17,18]. In [19,20] several $t\bar{t}$ spin correlations were computed at next-to-next-to-leading order (NNLO) QCD for dileptonic final states.

A suggestion was made in [21] to perform a comprehensive study of spin effects in hadronic $t\bar{t}$ production by measuring all coefficients of the $t\bar{t}$ production spin density matrices. For this aim the $gg, q\bar{q} \to t\bar{t}$ spin density matrices were decomposed in a suitable orthonormal basis and a set of spin observables was proposed that project onto the different coefficients of these density matrices. The $t\bar{t}$ spin correlation and t and $\bar{t}$ polarization observables were computed at NLO QCD including weak-interaction corrections, and possible NP effects on these coefficients and observables were analyzed by using an effective NP Lagrangian. Other recent analyses of using $t\bar{t}$ spin correlations at the LHC for probing new physics include [22–25].

The suggestions of [21] were taken up first by the ATLAS experiment in [26] that measured at $\sqrt{s_{\rm had}} =$ 8 TeV a subset of the proposed spin observables and compared it with SM predictions [21]. A more comprehensive analysis was performed by the CMS experiment [10] at $\sqrt{s_{\rm had}} = 13$ TeV. Both experiments measured these spin

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observables for dileptonic final states inclusive in phase space and found agreement with the SM results. Employing the NP computations of [21] the CMS Collaboration used their data also to constrain anomalous NP top quark couplings, in particular, the anomalous chromomagnetic and chromoelectric dipole moments of the top quark [10]. Reference [24] calculated for a CP-even subset of the proposed spin observables in [21] the NP contributions to quadratic order in the anomalous couplings and determined these observables differentially in phase space.

In this paper, we extend the analysis of [21] in several ways. We consider $t\bar{t}$ production and decay into dileptonic final states at the present LHC c.m. energy $\sqrt{s_{\mathrm{had}}} = 13.6$ TeV. We compute the $t\bar{t}$ charge asymmetry A_C and the set of top spin observables [21] both in the SM at NLO QCD including weak-interaction corrections and in the framework of an effective Lagrangian $\mathcal{L}_{\mathrm{NP}}$ describing new physics effects in $t\bar{t}$ production [27–31]. We analyze two additional $t\bar{t}$ spin correlations besides those considered in [21] that are useful in pinning down two anomalous couplings of $\mathcal{L}_{\mathrm{NP}}$. First, we calculate our observables inclusive in phase space. Then, in order to explore which areas in phase space are particularly sensitive to the various anomalous couplings, we choose a set of two-dimensional $(M_{t\bar{t}}, \cos\theta_t^*)$ bins, where $M_{t\bar{t}}$ denotes the $t\bar{t}$-invariant mass and θ_t^* is the top quark scattering angle in the $t\bar{t}$ zero-momentum frame, and compute our observables bin by bin.

Our paper is organized as follows. In Sec. II we briefly recapitulate the description of $t\bar{t}$ production and decay in the spin density matrix framework. Section III contains the $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times U(1)_Y$ invariant effective NP Lagrangian that we use. Our observables are introduced in Sec. IV. In Sec. V we present our results for the charge asymmetry A_C and the spin observables, both inclusive in phase space and in two-dimensional bins of $(M_{t\bar{t}}, \cos\theta_t^*)$. The results for the bins are given in detail in the Appendix. We conclude in Sec. VI.

II. FORMALISM

We consider $t\bar{t}$ production at NLO QCD including weak-interaction corrections and subsequent semileptonic decays of t and $\bar{t}$ quarks. At LO QCD top pairs are produced by gg fusion and $q\bar{q}$ annihilation and at NLO also gq and $g\bar{q}$ fusion contribute. We analyze $t\bar{t}$ production and decay in the so-called factorizable approximation and use the narrow width approximation $\Gamma_t/m_t \to 0$. The t and $\bar{t}$ spin degrees of freedom are fully taken into account. In this approximation the squared matrix element $|\mathcal{M}_I|^2$ of the parton reactions $I \to F$ [where F denotes here the dileptonic final state from t and $\bar{t}$ decay, $F = b\bar{b}\ell^+\ell'^-\nu_\ell\bar{\nu}_{\ell'} + X(\ell = e,\mu,\tau)$] is of the form

$$|\mathcal{M}_I|^2 \propto \mathrm{Tr}\Big[\rho R^I \bar{\rho}\Big], \tag{1}$$

where R^I denotes the density matrix that describes the production by one of the above-mentioned initial parton reactions I of on-shell $t\bar{t}$ pairs in a specific spin configuration, and ρ and $\bar{\rho}$ are the density matrices that encode the semileptonic decays of polarized t and $\bar{t}$ quarks, see below. The trace extends over the t and $\bar{t}$ spin indices.

We recall the structure of the $t\bar{t}$ production density matrices for the $2 \to 2$ reactions

$$g(p_1) + g(p_2) \to t(k_1, s_1) + \bar{t}(k_2, s_2), \tag{2}$$

$$q(p_1) + \bar{q}(p_2) \to t(k_1, s_1) + \bar{t}(k_2, s_2), \tag{3}$$

where p_j, k_j and s_1, s_2 refer to the 4-momenta of the partons and to the spin 4-vectors of the t and $\bar{t}$ quarks, respectively. The production density matrix R^I of each of these reactions is defined by the squared modulus of the respective matrix element, averaged over the spins and colors of I and summed over the colors of t, $\bar{t}$. The structure of the R^I $(I = gg, q\bar{q})$ in the spin spaces of t and $\bar{t}$ is as follows:

$$R^I = f_I\Big[A^I \mathbb{1}\otimes\mathbb{1} + \tilde{B}_i^{I+}\sigma^i\otimes\mathbb{1} + \tilde{B}_i^{I-}\mathbb{1}\otimes\sigma^i + \tilde{C}_{ij}^I\sigma^i\otimes\sigma^j\Big]. \tag{4}$$

The first (second) factor in the tensor products of the 2×2 unit matrix $\mathbb{1}$ and of the Pauli matrices σ^i refers to the t $(\bar{t})$ spin space. The prefactors are given to LO QCD by

$$f_{gg} = \frac{(4\pi\alpha_s)^2}{N_c(N_c^2-1)}, \qquad f_{q\bar{q}} = \frac{(N_c^2-1)(4\pi\alpha_s)^2}{N_c^2},$$

where $N_c = 3$ denotes the number of colors.

The functions $\tilde{B}_i^{I\pm}$ and $\tilde{C}_{ij}^I$ can be further decomposed, using an orthonormal basis which we choose as in [21]. The top quark direction of flight in the $t\bar{t}$ zero-momentum frame (ZMF) is denoted by $\hat{\mathbf{k}}$, and $\hat{\mathbf{p}} = \hat{\mathbf{p}}_1$ denotes the direction of one of the incoming partons in this frame. A right-handed orthonormal basis is obtained as follows:

$$\{\hat{\mathbf{r}}, \hat{\mathbf{k}}, \hat{\mathbf{n}}\}\colon\ \hat{\mathbf{r}} = \frac{1}{r}(\hat{\mathbf{p}} - y\hat{\mathbf{k}}), \qquad \hat{\mathbf{n}} = \frac{1}{r}(\hat{\mathbf{p}}\times\hat{\mathbf{k}}),$$
$$y = \hat{\mathbf{k}}\cdot\hat{\mathbf{p}}, \qquad r = \sqrt{1-y^2}. \tag{5}$$

Using rotational invariance, we decompose the 3-vectors $\tilde{\mathbf{B}}^{I\pm}$ and the 3×3 matrices $\tilde{C}_{ij}^I$ (which have a symmetric and antisymmetric part with six and three entries, respectively) with respect to the basis (5),

$$\tilde{B}_i^{I\pm} = b_r^{I\pm}\hat{r}_i + b_k^{I\pm}\hat{k}_i + b_n^{I\pm}\hat{n}_i, \tag{6}$$

$$\begin{aligned}\tilde{C}_{ij}^I = {}& c_{rr}^I\hat{r}_i\hat{r}_j + c_{kk}^I\hat{k}_i\hat{k}_j + c_{nn}^I\hat{n}_i\hat{n}_j + c_{rk}^I(\hat{r}_i\hat{k}_j + \hat{k}_i\hat{r}_j)\\ &+ c_{kn}^I(\hat{k}_i\hat{n}_j + \hat{n}_i\hat{k}_j) + c_{rn}^I(\hat{r}_i\hat{n}_j + \hat{n}_i\hat{r}_j)\\ &+ \epsilon_{ijl}(c_r^I\hat{r}_l + c_k^I\hat{k}_l + c_n^I\hat{n}_l).\end{aligned} \tag{7}$$

The coefficients $b_v^{I\pm}$, $c_{vv'}^I$ are functions of the partonic c.m. energy squared $\hat{s}$ and of $y = \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}$, which is equal to the cosine of the top quark scattering angle in the c.m. frame of the initial partons. Notice that the terms in the antisymmetric part of (7) can be written as follows:

$$c_r^I \epsilon_{ijl} \hat{r}_l = c_r^I (\hat{k}_i \hat{n}_j - \hat{n}_i \hat{k}_j), \qquad c_k^I \epsilon_{ijl} \hat{k}_l = c_k^I (\hat{n}_i \hat{r}_j - \hat{r}_i \hat{n}_j),$$
$$c_n^I \epsilon_{ijl} \hat{n}_l = c_n^I (\hat{r}_i \hat{k}_j - \hat{k}_i \hat{r}_j). \tag{8}$$

Bose symmetry of the initial gg state implies that the matrix R^{gg} must satisfy

$$R^{gg}(-\mathbf{p}, \mathbf{k}) = R^{gg}(\mathbf{p}, \mathbf{k}). \tag{9}$$

If CP invariance holds, then

$$R^I_{\alpha_1\alpha_2,\beta_1\beta_2}(\mathbf{p}, \mathbf{k}) = R^I_{\beta_1\beta_2,\alpha_1\alpha_2}(\mathbf{p}, \mathbf{k}), \qquad I = gg, q\bar{q}. \tag{10}$$

The conditions (9) and (10) imply transformation properties of the coefficient functions $b_v^{I\pm}$, $c_{vv'}^I$ defined in (6) and (7). These properties are listed, together with the implications of parity invariance, in detail in Table 1 of [21].

We briefly recall the structure of the decay density matrices ρ, $\bar{\rho}$ that we use. In this paper, we concentrate on semileptonic top quark decays. At NLO QCD we have

$$t \to b\ell^+\nu_\ell, \qquad b\ell^+\nu_\ell g, \tag{11}$$

where $\ell = e, \mu, \tau$. Considering a fully polarized ensemble of top quarks in the top rest frame and integrating over all energy and angular variables in the decay matrix element, except over the angle θ between the polarization vector of the top quark and the direction of flight of the charged lepton ℓ^+, one obtains a decay distribution of the form $d\Gamma_\ell/d\cos\theta = \Gamma_\ell(1 + \kappa_\ell \cos\theta)/2$, where Γ_ℓ is the partial width of the respective semileptonic decay. From this decay distribution, one obtains the respective normalized one-particle inclusive t decay density matrix

$$\rho = \frac{1}{2}\left(1 + \kappa_\ell \boldsymbol{\sigma} \cdot \hat{\boldsymbol{\ell}}_+\right). \tag{12}$$

The factor κ_ℓ is the top spin analyzing power of the charged lepton. Its value is $\kappa_\ell = 0.999$ at NLO QCD [32,33]. For semileptonic $\bar{t}$ decays the respective normalized decay density matrix is

$$\bar{\rho} = \frac{1}{2}\left(1 - \kappa_\ell \boldsymbol{\sigma} \cdot \hat{\boldsymbol{\ell}}_-\right). \tag{13}$$

Equations (12) and (13) will be used in (1) for the computation of the spin observables of Sec. IV. The proportionality factor in (1) contains, in the narrow width approximation, the branching fractions of semileptonic t and $\bar{t}$ decay.

III. EFFECTIVE NP LAGRANGIAN

Assuming that NP effects in hadronic $t\bar{t}$ production and decay are characterized by a mass scale Λ that is significantly larger than the moduli of the kinematic invariants of the $t\bar{t}$ production and decay processes, one may describe these (nonresonant) effects by a local effective Lagrangian $\mathcal{L}_{\rm NP}$ that involves the SM degrees of freedom and respects the SM symmetries. Respective analyses include [27–31].

The gauge-invariant operators with $\dim \mathcal{O} \leq 6$ relevant for $t\bar{t}$ production that involve gluon fields are [27,29,31]

$$\mathcal{O}_{gt} = [\bar{t}_R \gamma^\mu T^a D^\nu t_R] G^a_{\mu\nu}, \tag{14}$$

$$\mathcal{O}_{gQ} = [\bar{Q}_L \gamma^\mu T^a D^\nu Q_L] G^a_{\mu\nu}, \tag{15}$$

$$\mathcal{O}_{\rm CDM} = [(\tilde{\Phi}\bar{Q}_L)\sigma^{\mu\nu} T^a t_R] G^a_{\mu\nu}. \tag{16}$$

Here $Q_L = (t_L, b_L)$ is the left-handed third generation doublet and $\tilde{\Phi} = i\sigma_2\Phi^\dagger = (\phi_0^*, -\phi_-)$ is the charge-conjugate Higgs doublet field. Moreover, $D_\mu = \partial_\mu + ig_s T^a G^a_\mu$, and $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu$ is the gluon field strength tensor. Furthermore, T^a are the generators of $\mathrm{SU}(3)_c$ in the fundamental representation, with $\mathrm{tr}(T^a T^b) = \delta_{ab}/2$.

The sums $\mathcal{O}_{gt} + \mathcal{O}_{gt}^\dagger$ and $\mathcal{O}_{gQ} + \mathcal{O}_{gQ}^\dagger$ are given by linear combinations of four-quark operators, as can be shown using the equation of motion for the gluons [31]. These linear combinations of four-quark operators are redundant in our case, because they are included in the set of four-quark operators given below that we use. The combinations $\mathcal{O}_{gt} - \mathcal{O}_{gt}^\dagger$ and $\mathcal{O}_{gQ} - \mathcal{O}_{gQ}^\dagger$ cannot be expressed in terms of the four-quark operators below. With these two combinations and (16) one can construct an Hermitian effective Lagrangian which reads, after spontaneous symmetry breaking, $\langle\Phi\rangle = v/\sqrt{2}$, ($v \simeq 246$ GeV), restricted to the top quark gluon sector, and using operators with definite P and CP properties,

$$\mathcal{L}_{\rm NP,g} = -\frac{g_s}{2m_t}[\hat{\mu}_t \bar{t}\sigma^{\mu\nu} T^a t G^a_{\mu\nu} + \hat{d}_t \bar{t} i\sigma^{\mu\nu}\gamma_5 T^a t G^a_{\mu\nu}]. \tag{17}$$

In the convention used in (17) we used the top quark mass m_t for setting the mass scale. The real and dimensionless coupling parameters $\hat{\mu}_t$ and $\hat{d}_t$ are, respectively, the chromomagnetic and chromoelectric dipole moment of the top quark.

In [21] we used an effective gluon Lagrangian that is $\mathrm{SU}(3)_c \times U(1)_{em}$ invariant, but not $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times U(1)_Y$ invariant, for reasons of a more agnostic, phenomenological analysis. In addition to (17) it contains two additional CP-odd operators that contribute to P- and CP-odd spin correlations and to a P-even, CP-odd polarization observable, respectively. These observables were measured in [10] and the respective coupling parameters $\hat{c}(--)$ and

$\hat{c}(-+)$ were constrained. In this paper, we will not use these operators, but stick to the $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times U(1)_Y$ invariant framework.

There are a number of gauge-invariant dim $\mathcal{O} = 6$ four-quark operators that generate nonzero tree-level interference terms with the $q\bar{q} \to t\bar{t}$ QCD amplitude. Assuming universality of the new interactions with respect to the light quarks $q \neq t$ and considering only operators with u, d quarks in the initial state that interfere with the tree-level $q\bar{q} \to t\bar{t}$ QCD matrix elements, seven gauge-invariant operators remain [29,31]. It is useful to combine these seven operators such that one obtains four isospin-zero operators with definite P and C properties and three isospin-one operators [31]. The resulting NP effective Lagrangian involving the u, d quarks reads (as above we use m_t for setting the mass scale)

$$\mathcal{L}_{\mathrm{NP,q}} = \mathcal{L}_{\mathrm{NP,0}} + \mathcal{L}_{\mathrm{NP,1}}, \tag{18}$$

where the isospin-zero part is

$$\mathcal{L}_{\mathrm{NP,0}} = \frac{g_s^2}{2m_t^2} \sum_{I,J=V,A} \hat{c}_{IJ} \mathcal{O}_{IJ}, \tag{19}$$

and

$$\begin{aligned} \mathcal{O}_{VV} &= (\bar{q}\gamma^\mu T^a q)(\bar{t}\gamma_\mu T^a t), \\ \mathcal{O}_{AA} &= (\bar{q}\gamma^\mu T^a \gamma_5 q)(\bar{t}\gamma_\mu \gamma_5 T^a t), \end{aligned} \tag{20}$$

$$\begin{aligned} \mathcal{O}_{VA} &= (\bar{q}\gamma^\mu T^a q)(\bar{t}\gamma_\mu \gamma_5 T^a t), \\ \mathcal{O}_{AV} &= (\bar{q}\gamma^\mu T^a \gamma_5 q)(\bar{t}\gamma_\mu T^a t). \end{aligned} \tag{21}$$

Here and in the following $q = (u, d)$ denotes the isospin doublet. The isospin-one contribution can be represented in the form

$$\mathcal{L}_{\mathrm{NP,1}} = \frac{g_s^2}{4m_t^2} \sum_{i=1}^{3} \hat{c}_i \mathcal{O}_i^1, \tag{22}$$

where

$$\mathcal{O}_1^1 = (\bar{q}\gamma^\mu T^a \sigma_3 q)(\bar{t}\gamma_\mu T^a t) + (\bar{q}\gamma^\mu \gamma_5 T^a \sigma_3 q)(\bar{t}\gamma_\mu T^a t), \tag{23}$$

$$\mathcal{O}_2^1 = (\bar{q}\gamma^\mu \gamma_5 T^a \sigma_3 q)(\bar{t}\gamma_\mu \gamma_5 T^a t) - (\bar{q}\gamma^\mu \gamma_5 T^a \sigma_3 q)(\bar{t}\gamma_\mu T^a t), \tag{24}$$

$$\mathcal{O}_3^1 = (\bar{q}\gamma^\mu T^a \sigma_3 q)(\bar{t}\gamma_\mu \gamma_5 T^a t) + (\bar{q}\gamma^\mu \gamma_5 T^a \sigma_3 q)(\bar{t}\gamma_\mu T^a t). \tag{25}$$

In the isospin-one case, it is not possible to combine the operators such that they have definite properties with respect to C and P.

In summary, we use the effective NP Lagrangian

$$\mathcal{L}_{\mathrm{NP}} = \mathcal{L}_{\mathrm{NP,g}} + \mathcal{L}_{\mathrm{NP,q}} \tag{26}$$

that contains the real, dimensionless coupling parameters $\hat{\mu}_t$, $\hat{d}_t$, $\hat{c}_{IJ}$ $(I, J = V, A)$, and $\hat{c}_1, \hat{c}_2, \hat{c}_3$. The NP contributions to the coefficients of the $t\bar{t}$ spin density matrices (4) induced by interference with the tree-level QCD amplitudes of $gg, q\bar{q} \to t\bar{t}$ are listed in the Appendix of [21]. In the following we will stick to these dependencies to first order in the anomalous couplings. This is justified *a posteriori* by the results [10] of the CMS experiment. The experimental constraints on the dimensionless anomalous couplings of (26) signify that they are markedly smaller than 1.

A remark on NP contributions to top quark decay $t \to b\ell\nu_\ell$ is in order. The top quark decay vertex $t \to Wb$ may be affected by new physics interactions, but the upper bounds on the respective anomalous couplings inferred from measurements of the W-boson helicity fractions [34,35] show that these effects are very small if nonzero. In this paper, we consider $t\bar{t}$ production and decay in the dileptonic channel, $pp \to t\bar{t}X \to \ell\ell' X$. We use as top spin analyzers the charged lepton from W decay and we analyze only charged-lepton angular observables that are inclusive in the lepton energies. These observables are not affected by anomalous couplings from top quark decay if a linear approximation is justified, that is, if these couplings are small [36–39]. As just mentioned, this is the case. Thus for the observables that we analyze in this paper, only contributions to $t\bar{t}$ production matter as far as NP effects are concerned.

IV. OBSERVABLES

We consider $t\bar{t}$ production at the LHC for the present center-of-mass energy $\sqrt{s_{\mathrm{had}}} = 13.6$ TeV. We compute the $t\bar{t}$ cross section and the $t\bar{t}$ charge asymmetry A_C defined in (27) below in the SM and determine, in addition, the contributions from $\mathcal{L}_{\mathrm{NP}}$. Then we focus on the dileptonic $t\bar{t}$ decay channels and investigate a number of spin correlation and polarization observables. First we perform our computations within the SM and to first order in $\mathcal{L}_{\mathrm{NP}}$ inclusive in phase space. Because future experimental investigations at the LHC aim at more detailed analyses, we then determine these observables in two-dimensional bins of the $t\bar{t}$-invariant mass and the cosine of the top quark scattering angle in the $t\bar{t}$ ZMF that will be specified below. We do not apply acceptance cuts because experiments usually unfold their data for comparison with theoretical (top spin) predictions [10,26].

We use the LHC $t\bar{t}$ charge asymmetry defined by

$$A_C = \frac{\sigma(\Delta|y| > 0) - \sigma(\Delta|y| < 0)}{\sigma(\Delta|y| > 0) + \sigma(\Delta|y| < 0)}, \tag{27}$$

where $\Delta|y| = |y_t| - |y_{\bar{t}}|$ is the difference of the moduli of the t and $\bar{t}$ rapidities in the laboratory frame.

For the dileptonic final states

$$pp \to t\bar{t} + X \to \ell^+\ell'^- + \text{jets} + X, \tag{28}$$

we consider the well-known polar angle double distributions [13,14] for a choice of reference axes $\hat{\mathbf{a}}, \hat{\mathbf{b}}$,

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_+ d\cos\theta_-} = \frac{1}{4}(1 + B_1\cos\theta_+ + B_2\cos\theta_- - C\cos\theta_+\cos\theta_-), \tag{29}$$

where

$$\cos\theta_+ = \hat{\boldsymbol{\ell}}_+ \cdot \hat{\mathbf{a}}, \qquad \cos\theta_- = \hat{\boldsymbol{\ell}}_- \cdot \hat{\mathbf{b}}, \tag{30}$$

and, as above, the unit vectors $\hat{\boldsymbol{\ell}}_+$, $\hat{\boldsymbol{\ell}}_-$ are the ℓ^+ and ℓ'^- directions of flight in the t and $\bar{t}$ rest frames, respectively. The coefficients B_1, B_2, and C signify the t, $\bar{t}$ polarizations and $t\bar{t}$ spin correlations, respectively. As we apply no acceptance cuts on the final states and consider only factorizable radiative corrections, the coefficients in (29) can be related to the expectation values of the spin observables at the level of the intermediate top quarks. We have [14]

$$C(a,b) = \kappa_\ell^2\langle 4(\mathbf{S}_t\cdot\hat{\mathbf{a}})(\mathbf{S}_{\bar{t}}\cdot\hat{\mathbf{b}})\rangle = \kappa_\ell^2\langle\boldsymbol{\sigma}\cdot\hat{\mathbf{a}}\otimes\boldsymbol{\sigma}\cdot\hat{\mathbf{b}}\rangle. \tag{31}$$

Here $\mathbf{S}_t$ and $\mathbf{S}_{\bar{t}}$ denote the t and $\bar{t}$ spin operators and σ_i are the Pauli matrices. The value of κ_ℓ is listed below Eq. (12).

The coefficients $B_{1,2}$ in (29) are given by

$$B_1(\hat{\mathbf{a}}) = P(\hat{\mathbf{a}})\kappa_\ell, \qquad B_2(\hat{\mathbf{b}}) = -\bar{P}(\hat{\mathbf{b}})\kappa_\ell, \tag{32}$$

where P, $\bar{P}$ denote the polarization degrees of the t and $\bar{t}$ ensembles in $t\bar{t}$ events with respect to the reference axes $\hat{\mathbf{a}}, \hat{\mathbf{b}}$,

$$P(\hat{\mathbf{a}}) = \langle 2\mathbf{S}_t\cdot\hat{\mathbf{a}}\rangle, \qquad \bar{P}(\hat{\mathbf{b}}) = \langle 2\mathbf{S}_{\bar{t}}\cdot\hat{\mathbf{b}}\rangle. \tag{33}$$

The relative signs in front of the coefficients $B_{1,2}$ in the distribution (29) are chosen such that in a CP-invariant theory and for the choice $\hat{\mathbf{a}} = -\hat{\mathbf{b}}$,

$$B_1 = B_2. \tag{34}$$

For the choice of reference axes $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ at the hadron level, one cannot use the orthonormal basis at the parton level introduced in Sec. II because the incoming quark will be either in the right-moving or in the left-moving proton. As in [21] we choose the following set: We use the unit vector $\hat{\mathbf{k}}$, which is the top quark direction of flight in the $t\bar{t}$ ZMF. Moreover, we use the direction of one of the proton beams in the laboratory frame $\hat{\mathbf{p}}_p$ and define unit vectors $\hat{\mathbf{r}}_p$ and $\hat{\mathbf{n}}_p$ as follows:

$$\hat{\mathbf{p}}_p = (0,0,1), \quad \hat{\mathbf{r}}_p = \frac{1}{r_p}(\hat{\mathbf{p}}_p - y_p\hat{\mathbf{k}}), \quad \hat{\mathbf{n}}_p = \frac{1}{r_p}(\hat{\mathbf{p}}_p \times \hat{\mathbf{k}}),$$
$$y_p = \hat{\mathbf{p}}_p\cdot\hat{\mathbf{k}} = \cos\theta_t^*, \quad r_p = \sqrt{1-y_p^2}. \tag{35}$$

TABLE I. Choice of reference axes at the hadron level. The unit vectors $\hat{\mathbf{n}}_p$, $\hat{\mathbf{r}}_p$ and the variable y_p are defined in (35).

	Label	$\hat{\mathbf{a}}$	$\hat{\mathbf{b}}$
Transverse	n	$\text{sign}(y_p)\hat{\mathbf{n}}_p$	$-\text{sign}(y_p)\hat{\mathbf{n}}_p$
r axis	r	$\text{sign}(y_p)\hat{\mathbf{r}}_p$	$-\text{sign}(y_p)\hat{\mathbf{r}}_p$
Helicity	k	$\hat{\mathbf{k}}$	$-\hat{\mathbf{k}}$

The angle θ_t^* is the top quark scattering angle in the $t\bar{t}$ ZMF. Only in the case of $2\to 2$ parton reactions and if the incoming parton 1 is parallel to $\hat{\mathbf{p}}_p$, the unit vectors defined in (5) are the same as those in (35). The set (35) defines our choice of reference axes $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$, which we list in Table I. The factors $\text{sign}(y_p)$ are required because of the Bose symmetry of the initial gg state. In the following, the label (a,b) refers to the choice of reference axes $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ from Table I. The correlation coefficient C associated with this choice of axes is denoted by $C(a,b)$ as in (31) and is called $t\bar{t}$ spin correlation for short. The analogous labeling applies to $B_1(a)$ and $B_2(b)$, which we refer to as t and $\bar{t}$ polarization with respect to the chosen axis.

Table II contains the set of spin correlations and polarizations that we consider. The second column shows to which coefficient of the gg and $q\bar{q}$ spin density matrices the respective observable is sensitive. The third column indicates the P- and CP-symmetry properties of the observables.[1] The label "absorptive" means that the respective observable is generated by absorptive parts in the scattering matrix.

Apart from computing the observables of Table II in the SM we determine also their sensitivity to the couplings of the effective NP Lagrangian (26). It contains the following dimensionless, real parameters: $\hat{\mu}_t$, $\hat{d}_t$, $\hat{c}_{VV}$, $\hat{c}_{AA}$, $\hat{c}_{AV}$, $\hat{c}_{VA}$, $\hat{c}_1$, $\hat{c}_2$, $\hat{c}_3$. The appendixes A.1 and A.2 of [21] show on which parameters a specific coefficient of the gg and $q\bar{q}$ spin density matrices depends.[2] This dependence

[1] We recall that, strictly speaking, a classification at the hadron level with respect to CP is not possible, because the initial pp state is not a CP eigenstate. The third column of Table II refers to the P- and CP-symmetry properties of the coefficients of the gg and $q\bar{q}$ spin density matrices [21].

[2] There are typos in the following two coefficients in Appendix A.2 of [21]. The overall sign of the first term of c_{rr} must be positive, instead of negative. The second term in the coefficient c_k should read $\beta(-1+y^2)\hat{d}_t/4$. The numerical results of [21] are not affected by these typos.

TABLE II. The spin correlations and polarizations of (29) and sums and differences for different choices of reference axes. The unit vectors associated with the labels k^* and r^* are defined in (36) and (37).

Correlation		Sensitive to
$C(n,n)$	c^I_{nn}	P even, CP even
$C(r,r)$	c^I_{rr}	P even, CP even
$C(k,k)$	c^I_{kk}	P even, CP even
$C(r,k)+C(k,r)$	c^I_{rk}	P even, CP even
$C(n,r)+C(r,n)$	c^I_{rn}	P odd, CP even, absorptive
$C(n,k)+C(k,n)$	c^I_{kn}	P odd, CP even, absorptive
$C(r,k)-C(k,r)$	c^I_{n}	P even, CP odd, absorptive
$C(n,r)-C(r,n)$	c^I_{k}	P odd, CP odd
$C(n,k)-C(k,n)$	c^I_{r}	P odd, CP odd
$C(k,k^*)$	c^I_{kk}	P even, CP even
$C(r^*,k)+C(k,r^*)$	c^I_{rk}	P even, CP even
$B_1(n)+B_2(n)$	$b^{I+}_n+b^{I-}_n$	P even, CP even, absorptive
$B_1(n)-B_2(n)$	$b^{I+}_n-b^{I-}_n$	P even, CP odd
$B_1(r)+B_2(r)$	$b^{I+}_r+b^{I-}_r$	P odd, CP even
$B_1(r)-B_2(r)$	$b^{I+}_r-b^{I-}_r$	P odd, CP odd, absorptive
$B_1(k)+B_2(k)$	$b^{I+}_k+b^{I-}_k$	P odd, CP even
$B_1(k)-B_2(k)$	$b^{I+}_k-b^{I-}_k$	P odd, CP odd, absorptive
$B_1(r^*)+B_2(r^*)$	$b^{I+}_r+b^{I-}_r$	P odd, CP even
$B_1(r^*)-B_2(r^*)$	$b^{I+}_r-b^{I-}_r$	P odd, CP odd, absorptive
$B_1(k^*)+B_2(k^*)$	$b^{I+}_k+b^{I-}_k$	P odd, CP even
$B_1(k^*)-B_2(k^*)$	$b^{I+}_k-b^{I-}_k$	P odd, CP odd, absorptive

determines the dependence on specific NP parameters of σ, A_C and the spin observables B and C. It turns out that, in order to significantly increase the sensitivity to some of these parameters, it is useful to introduce, in addition to those of Table I, another set of reference axes [21] to which we assign the labels k^* and r^*,

$$k^*:\ \hat{\mathbf{a}} = \mathrm{sign}(\Delta|y|)\hat{\mathbf{k}}, \qquad \hat{\mathbf{b}} = -\mathrm{sign}(\Delta|y|)\hat{\mathbf{k}}, \tag{36}$$

$$\begin{aligned} r^*:\ \hat{\mathbf{a}} &= \mathrm{sign}(\Delta|y|)\mathrm{sign}(y_p)\hat{\mathbf{r}}_p, \\ \hat{\mathbf{b}} &= -\mathrm{sign}(\Delta|y|)\mathrm{sign}(y_p)\hat{\mathbf{r}}_p. \end{aligned} \tag{37}$$

Here $\Delta|y|$ denotes the difference of the moduli of the t and $\bar{t}$ rapidities in the laboratory frame as defined below (27). With these vectors, one may consider the spin observables

$$\begin{aligned} &C(k,k^*), \qquad C(r^*,k), C(k,r^*), \\ &B_1(k^*), B_2(k^*), \qquad B_1(r^*), B_2(r^*), \end{aligned} \tag{38}$$

respectively, their sums and differences, see Table II. The sum of $B_{1,2}(k^*)$ and of $B_{1,2}(r^*)$ is sensitive to the NP contributions from the operator $\mathcal{O}_{AV}$ and from a P-odd combination of the operators $\mathcal{O}^1_i$, while the sum of $B_{1,2}(k)$ and $B_{1,2}(r)$ projects onto the contributions of $\mathcal{O}_{VA}$ and $\mathcal{O}^1_3$. The spin correlations $C(k,k^*)$ and $C(r^*,k)+C(k,r^*)$, which were not computed in [21], project onto different regions of phase space than $C(k,k)$ and $C(r,k)+C(k,r)$. While the latter are sensitive to $\hat{c}_{VV}$, $\hat{c}_1$, and $\hat{\mu}_t$, the former probe the couplings $\hat{c}_{AA}$ and $\hat{c}_2$. The P- and CP-odd correlations $C(n,r)-C(r,n)$ and $C(n,k)-C(k,n)$ are equivalent to CP-odd triple correlations, cf. [21], and they probe the chromoelectric dipole moment of the top quark.

Table II contains a number of P-odd observables that require absorptive parts. In the SM they result from absorptive parts of weak-interaction contributions and are very small, so we do not compute them. There are also no such contributions from our $\mathcal{L}_{\mathrm{NP}}$ at tree level. The observables $B_1(n)-B_2(n)$ and $C(r,k)-C(k,r)$ are P even, but CP odd. The latter requires in addition absorptive parts. Neither SM nor NP interactions from (26) contribute to these observables.

We close this section with a remark on the opening angle distribution [13,14] $\sigma^{-1}d\sigma/d\cos\varphi = (1-D\cos\varphi)/2$, where $\cos\varphi = \hat{\boldsymbol{\ell}}_+\cdot\hat{\boldsymbol{\ell}}_-$ is the scalar product of the two lepton directions determined in their parent t and $\bar{t}$ rest frames. Measurements by ATLAS and CMS have shown (see, for instance, [10,26]) that this distribution is highly sensitive to $t\bar{t}$ spin correlations. It can be obtained from the diagonal spin correlation coefficients. Using that the vectors defined in Table I form orthonormal sets, one gets [21]

$$D = -\frac{1}{3}[C(r,r)+C(k,k)+C(n,n)]. \tag{39}$$

The opening angle distribution can be determined with this formula from the diagonal correlations that will be computed in the next section.

V. RESULTS FOR 13.6 TEV

We compute the cross section, the charge asymmetry A_C, and the expectation values of the above spin observables for pp collisions at the c.m. energy of 13.6 TeV. As already emphasized above we do not apply acceptance cuts on the final states, because experiments compare with theory predictions by correcting their measurements to the parton level and extrapolating to the full phase space (see, e.g., [10,26]). We use the CT18 NLO parton distribution functions [40]. This set provides also the NLO QCD coupling α_s in the $\overline{\mathrm{MS}}$ scheme. We use the on-shell top quark mass $m_t = 172.5$ GeV. Moreover, we use $\Gamma_t = 1.3$, $m_Z = 91.2$, $m_W = 80.4, \Gamma_W = 2.09$, $m_H = 125$ GeV, and $\alpha(m_t) = 0.008$. We perform our computations for three values of the renormalization and factorization scale $\mu_R = \mu_F = \mu$, namely, for $\mu = m_t/2, m_t, 2m_t$.

First we compute our observables inclusively, i.e., by integrating over the complete phase space. Then we determine their values in two-dimensional bins of the $t\bar{t}$-invariant mass $M_{t\bar{t}}$ and $y_p = \cos\theta^*_t$, where θ^*_t is the top quark scattering angle in the $t\bar{t}$ ZMF.

We choose the four $M_{t\bar{t}}$ intervals

$$0 \leq M_{t\bar{t}} \leq 450\ \text{GeV}, \qquad 450 < M_{t\bar{t}} \leq 600\ \text{GeV},$$
$$600 < M_{t\bar{t}} \leq 800\ \text{GeV}, \qquad 800\ \text{GeV} < M_{t\bar{t}}. \tag{40}$$

For each of the four $M_{t\bar{t}}$ bins, we select four bins in $y_p = \cos\theta_t^*$,

$$-1 \leq y_p < -\frac{1}{2}, \qquad -\frac{1}{2} \leq y_p < 0,$$
$$0 \leq y_p < \frac{1}{2}, \qquad \frac{1}{2} \leq y_p \leq 1. \tag{41}$$

Our SM computations are performed at NLO QCD including the weak-interaction corrections. We refer to it with the acronym NLOW. In the calculation of the charge asymmetry A_C, the mixed QCD-QED corrections of order $\alpha_s^2\alpha$ [41] are taken into account in addition.[3]

The charge asymmetry and the polarization and spin correlation observables B, C used in this paper are ratios. They are, in the SM at NLOW and to first order in the anomalous couplings, schematically of the form

$$X = \frac{N_0 + N_1 + \delta N_{\text{NP}}}{\sigma_0 + \sigma_1 + \delta\sigma_{\text{NP}}}, \qquad X = A_C, B, C, \tag{42}$$

where N_0 (N_1) and σ_0 (σ_1) are the contributions at LO QCD (NLOW) and δN_{NP} and $\delta\sigma_{\text{NP}}$ denote the first-order anomalous contributions to the numerator of the respective observables and the $t\bar{t}$ cross section, respectively. We use this schematic notation both for results inclusive in phase space and for bins in $M_{t\bar{t}}$ and $\cos\theta_t^*$. *A priori*, it is not clear how to evaluate these ratios where the numerator and denominator consist of truncated perturbation series. A typical Monte Carlo analysis would determine the numerators and denominators of (42) to the attainable order and evaluate the ratio without expanding it. In the spirit of perturbation theory, one may expand the ratio. One gets at NLOW and to first order in the anomalous couplings,

$$X = \left(1 - \frac{\sigma_1}{\sigma_0} - \frac{\delta\sigma_{\text{NP}}}{\sigma_0}\right)\frac{N_0}{\sigma_0} + \frac{N_1}{\sigma_0} + \frac{\delta N_{\text{NP}}}{\sigma_0} + \mathcal{O}(\delta^2). \tag{43}$$

The difference in the two prescriptions (42) and (43) for computing X are nominally of higher order in the SM and NP couplings. It may be considered as an additional theory uncertainty.

The results of our inclusive calculations will be given in expanded form. When computing our observables for the two-dimensional bins (40) and (41), all the six quantities in the ratio (42) will be separately determined. This allows for evaluation of the ratios in either way. From the binned results listed in the Appendix, one can also obtain the inclusive results for these quantities which allows for an unexpanded evaluation of the ratios. Moreover, we display our results for the three scale choices $\mu = m_t/2, m_t, 2m_t$. This allows one to correctly account for the correlations of theory uncertainties when different observables of Table II are combined, which is advantageous for measurements (cf., e.g., [26]).

TABLE III. The SM and NP contributions to the $t\bar{t}$ cross section and the LHC charge asymmetry (27) at $\sqrt{s_{\text{had}}} = 13.6$ TeV for three renormalization and factorization scales μ.

		NLOW	$\propto \hat{c}_{VV}$	$\propto \hat{c}_1$	$\propto \hat{\mu}_t$
σ (pb)	$\mu = m_t/2$	874.71	826.06	97.00	3655.54
	$\mu = m_t$	784.87	666.66	79.30	2838.32
	$\mu = 2m_t$	693.55	546.86	65.78	2242.91

		NLO + EW	$\propto \hat{c}_{AA}$	$\propto \hat{c}_2$
A_C	$\mu = m_t/2$	7.41×10^{-3}	0.335	6.87×10^{-2}
	$\mu = m_t$	6.96×10^{-3}	0.346	7.14×10^{-2}
	$\mu = 2m_t$	6.48×10^{-3}	0.357	7.40×10^{-2}

Table III contains the $t\bar{t}$ cross section at NLOW for the three scales μ. Theory predictions for the cross section are, as is well known, available at NNLO QCD [45,46], including electroweak (EW) corrections [47]. We need the NLOW result for the normalization of our observables. There are three contributions to $\sigma_{t\bar{t}}$ from the NP Lagrangian (26). The effect of the chromomagnetic dipole operator is most significant and it is dominated by the contribution to $t\bar{t}$ production by gg fusion. The contributions from the four-quark operators $\mathcal{O}_{VV}$ and $\mathcal{O}_3^1$ are subdominant. Notice that the contributions from $\mathcal{O}_3^1$ to $u\bar{u} \to t\bar{t}$ and $d\bar{d} \to t\bar{t}$ have opposite sign and thus tend to cancel.[4]

Table III contains also the charge asymmetry A_C in expanded form (43). We recall that there is no contribution to its numerator at LO QCD. The NP contributions result from the isospin-zero operator $\mathcal{O}_{AA}$ and the P-even part of the isospin-one operator $\mathcal{O}_2^1$. These operators induce contributions to the differential cross section that are odd under interchange of the t and $\bar{t}$ momenta while those of the initial (anti)quark are kept fixed.

Tables IV and V contain our results in expanded form for the spin correlations and polarizations at NLOW in the SM and the NP contributions from the effective Lagrangian (26). The dominant NP contribution to the first four spin correlations in Table IV, which are P and CP even, is from the chromodipole operator that affects also $gg \to t\bar{t}$ besides

[3]The LHC charge asymmetry A_C was computed, for different c.m. energies, also at NLO and NNLO QCD in [42,43], respectively. In both references, the electroweak corrections were taken into account, too. The relevance of the EW corrections for the charge asymmetry was first shown by [44].

[4]This remark applies also to the contributions of $\mathcal{O}_3^2$ and $\mathcal{O}_3^3$ to the other observables.

TABLE IV. The spin correlations C at NLOW in the SM and the nonzero contributions of the NP Lagrangian (26) for the c.m. energy $\sqrt{s_{\rm had}} = 13.6$ TeV and three renormalization and factorization scales μ.

		NLOW	$\propto \hat{c}_{VV}$	$\propto \hat{c}_1$	$\propto \hat{\mu}_t$
$C(n,n)$	$\mu = m_t/2$	0.324	-6.87×10^{-2}	-6.32×10^{-3}	2.061
	$\mu = m_t$	0.325	-7.73×10^{-2}	-7.38×10^{-3}	2.037
	$\mu = 2m_t$	0.327	-8.53×10^{-2}	-8.36×10^{-3}	2.014
$C(r,r)$	$\mu = m_t/2$	7.94×10^{-2}	−0.676	-7.88×10^{-2}	2.506
	$\mu = m_t$	7.04×10^{-2}	−0.703	-8.30×10^{-2}	2.487
	$\mu = 2m_t$	6.41×10^{-2}	−0.728	-8.70×10^{-2}	2.468
$C(k,k)$	$\mu = m_t/2$	0.330	−1.195	−0.143	0.912
	$\mu = m_t$	0.331	−1.234	−0.149	0.918
	$\mu = 2m_t$	0.333	−1.272	−0.155	0.924
$C(r,k) + C(k,r)$	$\mu = m_t/2$	−0.203	−0.294	-3.25×10^{-2}	0.736
	$\mu = m_t$	−0.206	−0.308	-3.45×10^{-2}	0.739
	$\mu = 2m_t$	−0.208	−0.320	-3.64×10^{-2}	0.740

		NLOW	$\propto \hat{c}_{AA}$	$\propto \hat{c}_2$
$C(k,k^*)$	$\mu = m_t/2$	1.7×10^{-4}	−0.335	-6.87×10^{-2}
	$\mu = m_t$	2.1×10^{-4}	−0.346	-7.14×10^{-2}
	$\mu = 2m_t$	2.5×10^{-4}	−0.357	-7.40×10^{-2}
$C(r^*,k) + C(k,r^*)$	$\mu = m_t/2$	$< 10^{-4}$	−0.269	-5.45×10^{-2}
	$\mu = m_t$	1.4×10^{-4}	−0.281	-5.74×10^{-2}
	$\mu = 2m_t$	3.4×10^{-4}	−0.293	-6.01×10^{-2}

		$\propto \hat{d}_t$
$C(n,r) - C(r,n)$	$\mu = m_t/2$	−4.226
	$\mu = m_t$	−4.172
	$\mu = 2m_t$	−4.119
$C(n,k) - C(k,n)$	$\mu = m_t/2$	−0.801
	$\mu = m_t$	−0.799
	$\mu = 2m_t$	−0.794

$q\bar{q} \to t\bar{t}$. In particular, the correlation $C(r,r)$ which is small in the SM appears to have a good sensitivity to $\hat{\mu}_t$.

The observables $C(k,k^*)$ and $C(r^*,k) + C(k,r^*)$ are the spin correlation analogs of A_C. The effect of using the vectors k^* and r^* is that these correlations project onto different y_p intervals than their unstarred analogs. They are sensitive to the couplings $\hat{c}_{AA}$ and $\hat{c}_2$. Therefore, they are useful, together with A_C, to obtain information about these couplings from experimental data, once they are available.

Likewise, the use of the vectors r, k and r^*, k^* in the polarization observables B play analogous roles. The respective variables project onto different y_p intervals and are thus sensitive to different (combinations of) four-quark operators, as the results of Table V show.

The observable $B_1(n) + B_2(n)$ corresponds to the sum of the t and $\bar{t}$ polarizations normal to the scattering plane and is generated by QCD absorptive parts [48–50]. The absorptive parts of the electroweak corrections to the $t\bar{t}$ production matrix elements contribute also, but are not shown here. They are roughly half of the QCD contributions and have the same sign. There are no contributions from the Hermitian effective NP Lagrangian to LO QCD.

The charge asymmetry A_C and the spin correlation and polarization observables of Tables IV and V provide a set that is large enough to measure, respectively constrain, the couplings of the effective NP Lagrangian (26).

One may expect that the sensitivity to a specific anomalous coupling is not uniform in phase space. Therefore, we compute our observables also more differentially, namely, within the two-dimensional $(M_{t\bar{t}}, y_p)$ bins specified in (40) and (41) in order to investigate which region in phase space provides the highest sensitivity to a specific NP coupling.

One may ask whether any of these observables will depend, within a $(M_{t\bar{t}}, y_p)$ bin, on additional NP parameters besides those shown in Tables III–V. For instance, none of the four y_p bins (41) is parity symmetric; thus it could be that the P-even observables have additional NP-parameter

TABLE V. The polarizations B at NLOW in the SM and the nonzero contributions of the NP Lagrangian (26) for the c.m. energy $\sqrt{s_{\rm had}} = 13.6$ TeV and three renormalization and factorization scales μ.

		NLOW	$\propto \hat{c}_{VA}$	$\propto \hat{c}_3$
$B_1(r) + B_2(r)$	$\mu = m_t/2$	1.6×10^{-3}	0.203	2.35×10^{-2}
	$\mu = m_t$	3.3×10^{-3}	0.211	2.49×10^{-2}
	$\mu = 2m_t$	5.8×10^{-3}	0.220	2.62×10^{-2}
$B_1(k) + B_2(k)$	$\mu = m_t/2$	5.8×10^{-3}	1.580	0.191
	$\mu = m_t$	8.4×10^{-3}	1.628	0.199
	$\mu = 2m_t$	1.21×10^{-2}	1.678	0.208

		NLOW	$\propto \hat{c}_{AV}$	$\propto \hat{c}_1 - \hat{c}_2 + \hat{c}_3$
$B_1(r^*) + B_2(r^*)$	$\mu = m_t/2$	$< 10^{-4}$	0.752	0.152
	$\mu = m_t$	1.7×10^{-4}	0.790	0.161
	$\mu = 2m_t$	8.3×10^{-4}	0.828	0.169
$B_1(k^*) + B_2(k^*)$	$\mu = m_t/2$	$< 10^{-4}$	0.858	0.175
	$\mu = m_t$	4.9×10^{-4}	0.891	0.183
	$\mu = 2m_t$	1.3×10^{-3}	0.925	0.191

		NLO QCD
$B_1(n) + B_2(n)$	$\mu = m_t/2$	8.40×10^{-3}
	$\mu = m_t$	7.50×10^{-3}
	$\mu = 2m_t$	6.77×10^{-3}

dependencies within a bin that cancel in the sum over bins. We checked for all our observables that within the above $(M_{t\bar{t}}, y_p)$ bins no significant additional parameter dependencies appear; that is to say, the numerical dependence on an additional NP parameter is at least 3 orders of magnitude smaller than the significant dependencies displayed in the tables of the Appendix and are therefore discarded.

Our results for the binned $\sigma_{t\bar{t}}$, A_C, and the spin observables are given in Tables IX–XXXIII of appendix. As already mentioned above, we compute for each observable each of the six quantities (if nonzero) in the ratio (42) separately. This allows one to compute A_C and the spin observables either in unexpanded or expanded form.

TABLE VI. As in Table III, but for the two-dimensional bin $M_{t\bar{t}} > 600$ GeV and $-0.5 \le y_p \le 0.5$.

		NLOW	$\propto \hat{c}_{VV}$	$\propto \hat{c}_1$	$\propto \hat{\mu}_t$
σ(pb)	$\mu = m_t/2$	30.04	168.29	21.34	233.56
	$\mu = m_t$	31.00	132.52	17.05	176.47
	$\mu = 2m_t$	29.44	106.36	13.85	136.26

		NLO + EW	$\propto \hat{c}_{AA}$	$\propto \hat{c}_2$
A_C	$\mu = m_t/2$	7.03×10^{-3}	0.885	0.185
	$\mu = m_t$	6.74×10^{-3}	0.920	0.193
	$\mu = 2m_t$	6.55×10^{-3}	0.955	0.202

An inspection of the bins in Tables XXII–XXXIII of the NP contributions to our observables indicates that in almost all cases the two bins at large $t\bar{t}$-invariant mass $M_{t\bar{t}} > 800$ GeV in the central region, $-0.5 \le y_p \le 0.5$, seem to have the highest sensitivity to the parameters of the effective NP Lagrangian. In order to eventually obtain a reasonable large dileptonic $t\bar{t}$ data sample, we suggest here to consider the phase-space region $M_{t\bar{t}} > 600$ GeV and $-0.5 \le y_p \le 0.5$. Assuming that at the LHC at 13.6 TeV an integrated luminosity of 300 fb^{-1} will eventually be collected, and using the $t\bar{t}$ cross section given in Table VI for an estimate, one expects about 5×10^5 dileptonic $\ell\ell'$ $(\ell, \ell' = e, \mu)$ events in this region.

We use the four two-dimensional bins $M_{t\bar{t}} > 600$ GeV and $-0.5 \le y_p \le 0.5$ of Tables IX–XXXIII and compute, by summing the respective numbers of the four bins, the charge asymmetry and the normalized spin observables in this phase-space region in expanded form (43). The results are given Tables VI–VIII. Comparing the coefficients

TABLE VII. As in Table IV, but for the two-dimensional bin $M_{t\bar{t}} > 600$ GeV and $-0.5 \le y_p \le 0.5$.

		NLOW	$\propto \hat{c}_{VV}$	$\propto \hat{c}_1$	$\propto \hat{\mu}_t$
$C(n,n)$	$\mu = m_t/2$	0.414	1.176	0.547	2.101
	$\mu = m_t$	0.418	1.193	0.551	2.037
	$\mu = 2m_t$	0.421	1.211	0.555	1.975
$C(r,r)$	$\mu = m_t/2$	−0.530	−1.673	−0.745	2.949
	$\mu = m_t$	−0.539	−1.710	−0.754	2.889
	$\mu = 2m_t$	−0.547	−1.745	−0.762	2.831
$C(k,k)$	$\mu = m_t/2$	−0.254	−2.315	−0.534	2.014
	$\mu = m_t$	−0.255	−2.379	−0.548	2.017
	$\mu = 2m_t$	−0.257	−2.440	−0.563	2.021
$C(r,k) + C(k,r)$	$\mu = m_t/2$	−0.230	−0.388	−0.273	−0.945
	$\mu = m_t$	−0.234	−0.397	−0.276	−0.944
	$\mu = 2m_t$	−0.238	−0.406	−0.278	−0.946

		NLOW	$\propto \hat{c}_{AA}$	$\propto \hat{c}_2$
$C(k,k^*)$	$\mu = m_t/2$	0.5×10^{-4}	−0.885	−0.185
	$\mu = m_t$	3.0×10^{-4}	−0.920	−0.193
	$\mu = 2m_t$	3.8×10^{-4}	−0.955	−0.202
$C(r^*,k) + C(k,r^*)$	$\mu = m_t/2$	3.7×10^{-4}	−1.278	−0.265
	$\mu = m_t$	3.9×10^{-4}	−1.341	−0.280
	$\mu = 2m_t$	2.1×10^{-4}	−1.402	−0.294

		$\propto \hat{d}_t$
$C(n,r) - C(r,n)$	$\mu = m_t/2$	−4.524
	$\mu = m_t$	−4.395
	$\mu = 2m_t$	−4.270
$C(n,k) - C(k,n)$	$\mu = m_t/2$	1.261
	$\mu = m_t$	1.263
	$\mu = 2m_t$	1.269

TABLE VIII. As in Table V, but for the two-dimensional bin $M_{t\bar{t}} > 600$ GeV and $-0.5 \leq y_p \leq 0.5$.

		NLOW	$\propto \hat{c}_{VA}$	$\propto \hat{c}_3$
$B_1(r)+B_2(r)$	$\mu = m_t/2$	4.1×10^{-3}	0.547	6.80×10^{-2}
	$\mu = m_t$	4.5×10^{-3}	0.573	7.24×10^{-2}
	$\mu = 2m_t$	5.0×10^{-3}	0.599	7.66×10^{-2}
$B_1(k)+B_2(k)$	$\mu = m_t/2$	0.013	6.717	0.860
	$\mu = m_t$	0.012	6.980	0.907
	$\mu = 2m_t$	0.011	7.237	0.952

		NLOW	$\propto \hat{c}_{AV}$	$\propto \hat{c}_1 - \hat{c}_2 + \hat{c}_3$
$B_1(r^*)+B_2(r^*)$	$\mu = m_t/2$	7.7×10^{-4}	2.842	0.589
	$\mu = m_t$	1.1×10^{-3}	2.987	0.623
	$\mu = 2m_t$	1.3×10^{-3}	3.127	0.655
$B_1(k^*)+B_2(k^*)$	$\mu = m_t/2$	7.5×10^{-4}	1.935	0.403
	$\mu = m_t$	7.9×10^{-4}	2.016	0.423
	$\mu = 2m_t$	8.5×10^{-4}	2.094	0.442

		NLO QCD
$B_1(n)+B_2(n)$	$\mu = m_t/2$	1.86×10^{-2}
	$\mu = m_t$	1.67×10^{-2}
	$\mu = 2m_t$	1.51×10^{-2}

that multiply the contributions of the NP parameters $\hat{c}_{IJ}, (I, J = V, A), \hat{c}_1, \hat{c}_2, \hat{c}_3$ in these tables with the respective numbers in Tables III–V of the inclusive results, one sees that the sensitivity to the couplings of the four-quark operators increases significantly in the high $M_{t\bar{t}}$, central region. In particular, the spin correlations $C(k, k^*)$ and $C(r^*, k) + C(k, r^*)$ appear to be useful for disentangling the contributions from the operators associated with $\hat{c}_{AA}$ and $\hat{c}_2$. Not much is gained in this phase-space region for the sensitivity to the chromodipole moments $\hat{\mu}_t$ and $\hat{d}_t$ compared with the inclusive case.

VI. SUMMARY

We have elaborated on a set of spin correlation and polarization observables, proposed previously by two of the authors of this paper, that allows one to probe the hadronic $t\bar{t}$ production dynamics in detail. These observables project out all entries of the hadronic production spin density matrices. We considered $t\bar{t}$ production and decays into dileptonic final states at the LHC for the present c.m. energy 13.6 TeV. We computed these observables within the Standard Model at NLO QCD including the mixed QCD weak-interaction contributions. Possible new physics effects were incorporated by using an $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times U(1)_Y$ effective Lagrangian with operators that generate tree-level interferences with the LO QCD $gg, q\bar{q} \to t\bar{t}$ amplitudes. The effect of these NP operators on our observables was taken into account to linear order in the anomalous couplings. This can be justified by the results of the CMS experiment at 13.6 TeV [10], which show that these dimensionless couplings must be markedly smaller than 1. We considered also the LHC charge asymmetry A_C and two additional spin correlations that turn out to be very useful in disentangling contributions from NP four-quark operators. We emphasize that several our of observables allow for direct searches of nonstandard P and CP violation in $t\bar{t}$ events.

In addition to computing our observables inclusive in phase space, we determined them also in two-dimensional $(M_{t\bar{t}}, \cos\theta_t^*)$ bins, where $M_{t\bar{t}}$ denotes the $t\bar{t}$-invariant mass and θ_t^* is the top quark scattering angle in the $t\bar{t}$ zero-momentum frame. Our analysis shows that the sensitivity to a number of anomalous couplings significantly increases in the high-energy central region.

Experimental measurements of these observables were so far made only inclusively, and no deviation from the SM was found. The contributions of anomalous couplings to an observable are, however, not uniform in phase space as our results show. More differential measurements in the future, especially in the high-energy central region, promise to significantly increase our knowledge about top quark interactions beyond the Standard Model.

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APPENDIX: SM AND NP VALUES OF THE BINNED OBSERVABLES

In the tables of this appendix, we present our results for the cross section $\sigma_{t\bar{t}}$, the charge asymmetry A_C, and the spin correlations and polarization observables defined in Sec. IV and listed in Table II for the two-dimensional bins (40) and (41) at $\sqrt{s_{\mathrm{had}}} = 13.6$ TeV. For the cross section, we list the respective values at LO QCD, the contributions at NLOW, and to first order in the NP couplings. For A_C and the spin observables, we list the value N_0 of the respective numerator [cf. (42)]—if it is significantly different from zero—the contributions N_1 at NLOW, and those of the NP operators δN_{NP}. These quantities allow one to compute A_C and the spin observables either in unexpanded or expanded form, cf. Eqs. (42) and (43). Tables IX–XXI contain our SM results for the three chosen scales μ, while Tables XXII–XXXIII contain the various NP contributions to each observable. Here the values for the three scales are shown in separate tables.

TABLE IX. The $t\bar{t}$ cross section at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y_p bin. The numbers in the second, third, and fourth column are the values of $\sigma_{t\bar{t}}$ at LO QCD (σ_0) for $\mu = m_t/2, m_t$, and $2m_t$, respectively. The fifth, sixth, and seventh column contain the NLO QCD plus weak-interaction contributions to $\sigma_{t\bar{t}}$ (σ_1) for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All cross section numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV						
(−1.0, −0.5)	74.596	59.133	47.579	30.112	33.509	33.911
(−0.5, 0.0)	58.987	46.915	37.852	22.347	25.519	26.202
(0.0, 0.5)	59.071	46.971	37.876	18.071	22.316	23.847
(0.5, 1.0)	74.355	58.987	47.488	29.831	33.313	33.780
$450 < M_{t\bar{t}} \le 600$ GeV						
(−1.0, −0.5)	81.987	63.546	50.087	28.030	33.922	35.307
(−0.5, 0.0)	42.980	33.454	26.488	2.410	9.103	12.139
(0.0, 0.5)	42.855	33.371	26.433	2.273	8.823	12.025
(0.5, 1.0)	82.015	63.521	50.054	28.667	34.385	35.554
$600 < M_{t\bar{t}} \le 800$ GeV						
(−1.0, −0.5)	42.366	31.993	24.665	12.792	16.603	17.532
(−0.5, 0.0)	14.643	11.144	8.667	−2.538	0.910	2.622
(0.0, 0.5)	14.653	11.151	8.662	−2.118	1.234	2.858
(0.5, 1.0)	42.413	32.029	24.698	12.066	16.116	17.234
$M_{t\bar{t}} > 800$ GeV						
(−1.0, −0.5)	23.747	17.292	12.915	4.418	7.836	8.750
(−0.5, 0.0)	5.500	4.072	3.087	−2.769	−0.768	0.241
(0.0, 0.5)	5.499	4.069	3.084	−2.832	−0.809	0.221
(0.5, 1.0)	23.753	17.301	12.924	4.526	7.905	8.762

TABLE X. The numerator of the LHC charge asymmetry A_C in the SM defined in Eq. (27) at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y_p bin. The numbers in the second, third, and fourth column are the values of A_C at NLO QCD plus electroweak interactions for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV			
(−1.0, −0.5)	0.614	0.452	0.336
(−0.5, 0.0)	0.114	0.082	0.062
(0.0, 0.5)	0.129	0.098	0.068
(0.5, 1.0)	0.584	0.428	0.317
$450 < M_{t\bar{t}} \le 600$ GeV			
(−1.0, −0.5)	0.729	0.536	0.396
(−0.5, 0.0)	0.140	0.116	0.093
(0.0, 0.5)	0.199	0.147	0.111
(0.5, 1.0)	0.799	0.581	0.422
$600 < M_{t\bar{t}} \le 800$ GeV			
(−1.0, −0.5)	0.433	0.311	0.225
(−0.5, 0.0)	0.086	0.061	0.046
(0.0, 0.5)	0.089	0.065	0.052
(0.5, 1.0)	0.445	0.320	0.234
$M_{t\bar{t}} > 800$ GeV			
(−1.0, −0.5)	0.321	0.224	0.160
(−0.5, 0.0)	0.054	0.039	0.028
(0.0, 0.5)	0.054	0.040	0.028
(0.5, 1.0)	0.320	0.225	0.159

TABLE XI. The numerator of the spin correlation $C(n,n) = N_{nn}/\sigma_{t\bar{t}}$ in the SM at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y_p bin. The numbers in the second, third, and fourth column are the values of $N_{0,nn}$ at LO QCD for $\mu = m_t/2, m_t$, and $2m_t$, respectively. The fifth, sixth, and seventh column contain the NLO QCD plus weak-interaction contributions $N_{1,nn}$ for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV						
(−1.0, −0.5)	34.099	26.853	21.453	13.612	15.427	15.728
(−0.5, 0.0)	25.677	20.277	16.243	8.558	10.361	10.918
(0.0, 0.5)	25.708	20.301	16.262	6.724	9.037	9.928
(0.5, 1.0)	34.004	26.777	21.395	13.698	15.473	15.740
$450 < M_{t\bar{t}} \le 600$ GeV						
(−1.0, −0.5)	19.913	15.398	12.107	8.343	9.303	9.339
(−0.5, 0.0)	13.877	10.798	8.549	0.230	2.525	3.650
(0.0, 0.5)	13.833	10.765	8.527	0.208	2.491	3.588
(0.5, 1.0)	19.913	15.396	12.104	8.491	9.408	9.432
$600 < M_{t\bar{t}} \le 800$ GeV						
(−1.0, −0.5)	6.998	5.291	4.084	2.371	2.927	3.013
(−0.5, 0.0)	5.954	4.548	3.544	−1.334	0.162	0.916
(0.0, 0.5)	5.959	4.550	3.544	−1.137	0.295	1.019
(0.5, 1.0)	7.005	5.296	4.089	2.191	2.807	2.926
$M_{t\bar{t}} > 800$ GeV						
(−1.0, −0.5)	3.212	2.352	1.764	0.168	0.758	0.973
(−0.5, 0.0)	3.104	2.300	1.748	−1.784	−0.588	0.023
(0.0, 0.5)	3.105	2.302	1.749	−1.817	−0.602	0.017
(0.5, 1.0)	3.218	2.355	1.767	0.203	0.768	0.982

TABLE XII. The numerator of the spin correlation $C(r,r) = N_{rr}/\sigma_{t\bar{t}}$ in the SM at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y_p bin. The numbers in the second, third, and fourth column are the values of $N_{0,rr}$ at LO QCD for $\mu = m_t/2, m_t$, and $2m_t$, respectively. The fifth, sixth, and seventh column contain the NLO QCD plus weak-interaction contributions $N_{1,rr}$ for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV						
(−1.0, −0.5)	23.907	18.608	14.684	11.339	12.228	12.187
(−0.5, 0.0)	0.491	−0.042	−0.382	3.378	2.773	2.259
(0.0, 0.5)	0.466	−0.062	−0.400	3.059	2.502	2.043
(0.5, 1.0)	23.824	18.538	14.631	11.350	12.261	12.215
$450 < M_{t\bar{t}} \le 600$ GeV						
(−1.0, −0.5)	4.665	3.468	2.618	6.238	5.301	4.514
(−0.5, 0.0)	−16.485	−13.003	−10.423	2.112	−1.292	−3.070
(0.0, 0.5)	−16.458	−12.972	−10.407	2.108	−1.266	−3.054
(0.5, 1.0)	4.666	3.467	2.620	6.308	5.334	4.525
$600 < M_{t\bar{t}} \le 800$ GeV						
(−1.0, −0.5)	−2.359	−1.824	−1.438	1.678	0.739	0.216
(−0.5, 0.0)	−8.431	−6.459	−5.051	2.511	0.211	−0.990
(0.0, 0.5)	−8.440	−6.463	−5.054	2.287	0.046	−1.120
(0.5, 1.0)	−2.356	−1.823	−1.436	1.801	0.823	0.283
$M_{t\bar{t}} > 800$ GeV						
(−1.0, −0.5)	−2.457	−1.809	−1.364	0.997	0.171	−0.227
(−0.5, 0.0)	−3.866	−2.870	−2.183	2.345	0.810	0.016
(0.0, 0.5)	−3.863	−2.870	−2.182	2.372	0.838	0.043
(0.5, 1.0)	−2.464	−1.811	−1.366	0.990	0.154	−0.237

TABLE XIII. The numerator of the spin correlation $C(k,k) = N_{kk}/\sigma_{t\bar{t}}$ in the SM at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y_p bin. The numbers in the second, third, and fourth column are the values of $N_{0,kk}$ at LO QCD for $\mu = m_t/2, m_t$, and $2m_t$, respectively. The fifth, sixth, and seventh column contain the NLO QCD plus weak-interaction contributions $N_{1,kk}$ for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV						
(−1.0, −0.5)	40.027	31.090	24.493	11.974	15.708	17.026
(−0.5, 0.0)	29.272	22.926	18.219	8.820	11.339	12.214
(0.0, 0.5)	29.295	22.939	18.229	6.653	9.746	10.986
(0.5, 1.0)	39.892	30.981	24.399	11.936	15.683	17.043
$450 < M_{t\bar{t}} \le 600$ GeV						
(−1.0, −0.5)	29.247	22.194	17.135	9.381	12.297	13.144
(−0.5, 0.0)	6.892	5.178	3.945	−0.336	1.084	1.759
(0.0, 0.5)	6.857	5.152	3.927	−0.499	0.958	1.667
(0.5, 1.0)	29.224	22.182	17.133	9.631	12.455	13.243
$600 < M_{t\bar{t}} \le 800$ GeV						
(−1.0, −0.5)	6.546	4.768	3.544	3.885	4.135	3.986
(−0.5, 0.0)	−2.559	−2.008	−1.606	0.231	−0.254	−0.501
(0.0, 0.5)	−2.561	−2.009	−1.606	0.199	−0.277	−0.519
(0.5, 1.0)	6.575	4.787	3.554	3.972	4.198	4.043
$M_{t\bar{t}} > 800$ GeV						
(−1.0, −0.5)	−2.166	−1.628	−1.259	3.028	1.677	0.934
(−0.5, 0.0)	−2.861	−2.130	−1.625	1.458	0.432	−0.097
(0.0, 0.5)	−2.861	−2.131	−1.627	1.489	0.447	−0.084
(0.5, 1.0)	−2.173	−1.630	−1.257	2.985	1.647	0.899

TABLE XIV. The numerator of the spin correlation $C(r,k) + C(k,r) = N_{rk}/\sigma_{t\bar{t}}$ in the SM at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y_p bin. The numbers in the second, third, and fourth column are the values of $N_{0,rk}$ at LO QCD for $\mu = m_t/2, m_t$, and $2m_t$, respectively. The fifth, sixth, and seventh column contain the NLO QCD plus weak-interaction contributions $N_{1,rk}$ for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV						
(−1.0, −0.5)	−17.107	−13.884	−11.423	−6.623	−7.214	−7.258
(−0.5, 0.0)	−10.935	−8.831	−7.231	−3.927	−4.506	−4.653
(0.0, 0.5)	−10.953	−8.849	−7.245	−2.802	−3.698	−4.062
(0.5, 1.0)	−17.092	−13.866	−11.410	−6.639	−7.229	−7.277
$450 < M_{t\bar{t}} \le 600$ GeV						
(−1.0, −0.5)	−20.499	−16.052	−12.784	−5.027	−6.985	−7.670
(−0.5, 0.0)	−10.670	−8.364	−6.671	0.199	−1.670	−2.602
(0.0, 0.5)	−10.650	−8.346	−6.656	0.193	−1.661	−2.580
(0.5, 1.0)	−20.505	−16.060	−12.788	−5.131	−7.058	−7.743
$600 < M_{t\bar{t}} \le 800$ GeV						
(−1.0, −0.5)	−10.152	−7.723	−5.994	−0.561	−2.286	−3.009
(−0.5, 0.0)	−3.923	−3.000	−2.343	1.136	0.065	−0.486
(0.0, 0.5)	−3.927	−3.004	−2.343	1.046	−0.007	−0.541
(0.5, 1.0)	−10.148	−7.719	−5.996	−0.285	−2.086	−2.856
$M_{t\bar{t}} > 800$ GeV						
(−1.0, −0.5)	−4.592	−3.373	−2.539	1.376	−0.038	−0.696
(−0.5, 0.0)	−1.329	−0.988	−0.752	0.868	0.319	0.034
(0.0, 0.5)	−1.328	−0.988	−0.752	0.876	0.323	0.035
(0.5, 1.0)	−4.600	−3.377	−2.542	1.357	−0.052	−0.706

TABLE XV. The numerator of the spin correlation $C(k,k^*) = N_{kk^*}/\sigma_{t\bar{t}}$ in the SM at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y bin. The numbers in the second, third, and fourth column are the values N_{1,kk^*} at NLOW for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV			
(−1.0, −0.5)	0.001	0.040	0.070
(−0.5, 0.0)	0.279	0.221	0.183
(0.0, 0.5)	−0.304	−0.242	−0.192
(0.5, 1.0)	−0.030	−0.019	0.003
$450 < M_{t\bar{t}} \le 600$ GeV			
(−1.0, −0.5)	0.366	0.314	0.278
(−0.5, 0.0)	0.008	0.016	0.009
(0.0, 0.5)	0.195	0.153	0.120
(0.5, 1.0)	0.613	0.520	0.453
$600 < M_{t\bar{t}} \le 800$ GeV			
(−1.0, −0.5)	−0.024	0.000	0.026
(−0.5, 0.0)	−0.000	0.001	0.002
(0.0, 0.5)	0.005	0.005	0.005
(0.5, 1.0)	−0.037	−0.016	0.000
$M_{t\bar{t}} > 800$ GeV			
(−1.0, −0.5)	0.029	0.049	0.041
(−0.5, 0.0)	−0.004	−0.002	−0.001
(0.0, 0.5)	0.001	0.004	0.003
(0.5, 1.0)	0.069	0.073	0.071

TABLE XVI. The numerator of the spin correlation $C(r^*,k) + C(r,k^*) = N_{r^*k}/\sigma_{t\bar{t}}$ in the SM at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y_p bin. The numbers in the second, third, and fourth column are the values N_{1,r^*k} at NLOW for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV			
(−1.0, −0.5)	0.126	0.121	0.115
(−0.5, 0.0)	−0.161	−0.137	−0.108
(0.0, 0.5)	0.042	0.037	0.031
(0.5, 1.0)	0.041	0.052	0.066
$450 < M_{t\bar{t}} \le 600$ GeV			
(−1.0, −0.5)	0.138	0.134	0.126
(−0.5, 0.0)	0.059	0.046	0.035

(Table continued)

TABLE XVI. *(Continued)*

$450 < M_{t\bar{t}} \le 600$ GeV			
(0.0, 0.5)	−0.217	−0.168	−0.141
(0.5, 1.0)	−0.127	−0.065	−0.031
$600 < M_{t\bar{t}} \le 800$ GeV			
(−1.0, −0.5)	−0.003	0.012	0.013
(−0.5, 0.0)	0.006	0.006	0.002
(0.0, 0.5)	0.008	0.006	0.004
(0.5, 1.0)	0.042	0.027	0.029
$M_{t\bar{t}} > 800$ GeV			
(−1.0, −0.5)	0.001	−0.001	0.009
(−0.5, 0.0)	−0.002	−0.002	−0.001
(0.0, 0.5)	0.003	0.002	0.000
(0.5, 1.0)	−0.000	0.004	0.005

TABLE XVII. The numerator of the polarization observable $B_1(r) + B_2(r) = N_r/\sigma_{t\bar{t}}$ at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y_p bin. The numbers in the second, third, and fourth column are the values $N_{1,r}$ at NLOW for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV			
(−1.0, −0.5)	0.021	0.201	0.357
(−0.5, 0.0)	0.043	0.096	0.143
(0.0, 0.5)	0.043	0.096	0.143
(0.5, 1.0)	0.022	0.202	0.358
$450 < M_{t\bar{t}} \le 600$ GeV			
(−1.0, −0.5)	0.121	0.240	0.347
(−0.5, 0.0)	0.110	0.113	0.119
(0.0, 0.5)	0.111	0.114	0.120
(0.5, 1.0)	0.120	0.239	0.346
$600 < M_{t\bar{t}} \le 800$ GeV			
(−1.0, −0.5)	0.096	0.116	0.139
(−0.5, 0.0)	0.058	0.049	0.044
(0.0, 0.5)	0.058	0.049	0.044
(0.5, 1.0)	0.096	0.117	0.139
$M_{t\bar{t}} > 800$ GeV			
(−1.0, −0.5)	0.061	0.056	0.056
(−0.5, 0.0)	0.025	0.019	0.015
(0.0, 0.5)	0.025	0.019	0.015
(0.5, 1.0)	0.061	0.056	0.056

TABLE XVIII. The numerator of the polarization observable $B_1(k) + B_2(k) = N_k/\sigma_{t\bar{t}}$ in the SM at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y_p bin. The numbers in the second, third, and fourth column are the values of $N_{1,k}$ at NLOW for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV			
(−1.0, −0.5)	0.058	0.285	0.482
(−0.5, 0.0)	0.047	0.007	−0.025
(0.0, 0.5)	0.046	0.007	−0.026
(0.5, 1.0)	0.061	0.287	0.485
$450 < M_{t\bar{t}} \le 600$ GeV			
(−1.0, −0.5)	0.428	0.630	0.817
(−0.5, 0.0)	0.181	0.114	0.063
(0.0, 0.5)	0.181	0.114	0.063
(0.5, 1.0)	0.426	0.628	0.815
$600 < M_{t\bar{t}} \le 800$ GeV			
(−1.0, −0.5)	0.484	0.531	0.587
(−0.5, 0.0)	0.150	0.104	0.071
(0.0, 0.5)	0.150	0.104	0.071
(0.5, 1.0)	0.485	0.531	0.588
$M_{t\bar{t}} > 800$ GeV			
(−1.0, −0.5)	0.531	0.494	0.485
(−0.5, 0.0)	0.121	0.085	0.060
(0.0, 0.5)	0.121	0.085	0.060
(0.5, 1.0)	0.532	0.494	0.485

TABLE XIX. The numerator of the polarization observable $B_1(r^*) + B_2(r^*) = N_{r^*}/\sigma_{t\bar{t}}$ in the SM at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y_p bin. The numbers in the second, third, and fourth column are the values of N_{1,r^*} at NLOW for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV			
(−1.0, −0.5)	−0.036	0.013	0.054
(−0.5, 0.0)	−0.037	0.013	0.053
(0.0, 0.5)	−0.036	0.013	0.054
(0.5, 1.0)	−0.035	0.014	0.055
$450 < M_{t\bar{t}} \le 600$ GeV			
(−1.0, −0.5)	−0.030	0.002	0.028
(−0.5, 0.0)	−0.009	0.007	0.020

(Table continued)

TABLE XIX. *(Continued)*

$450 < M_{t\bar{t}} \le 600$ GeV			
(0.0, 0.5)	−0.009	0.007	0.020
(0.5, 1.0)	−0.031	0.001	0.027
$600 < M_{t\bar{t}} \le 800$ GeV			
(−1.0, −0.5)	−0.013	−0.003	0.005
(−0.5, 0.0)	0.007	0.008	0.009
(0.0, 0.5)	0.006	0.008	0.009
(0.5, 1.0)	−0.013	−0.003	0.005
$M_{t\bar{t}} > 800$ GeV			
(−1.0, −0.5)	−0.006	−0.003	0.000
(−0.5, 0.0)	0.009	0.008	0.006
(0.0, 0.5)	0.009	0.008	0.007
(0.5, 1.0)	−0.006	−0.003	0.000

TABLE XX. The numerator of the polarization observable $B_1(k^*) + B_2(k^*) = N_{k^*}/\sigma_{t\bar{t}}$ in the SM at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y_p bin. The numbers in the second, third, and fourth column are the values of N_{1,k^*} at NLOW for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV			
(−1.0, −0.5)	−0.040	0.028	0.083
(−0.5, 0.0)	−0.008	0.005	0.015
(0.0, 0.5)	−0.008	0.005	0.015
(0.5, 1.0)	−0.038	0.030	0.086
$450 < M_{t\bar{t}} \le 600$ GeV			
(−1.0, −0.5)	−0.023	0.037	0.086
(−0.5, 0.0)	0.001	0.005	0.008
(0.0, 0.5)	0.001	0.005	0.008
(0.5, 1.0)	−0.025	0.035	0.084
$600 < M_{t\bar{t}} \le 800$ GeV			
(−1.0, −0.5)	−0.006	0.023	0.047
(−0.5, 0.0)	0.006	0.005	0.005
(0.0, 0.5)	0.006	0.005	0.005
(0.5, 1.0)	−0.005	0.024	0.048
$M_{t\bar{t}} > 800$ GeV			
(−1.0, −0.5)	0.005	0.021	0.035
(−0.5, 0.0)	0.009	0.007	0.005
(0.0, 0.5)	0.009	0.007	0.005
(0.5, 1.0)	0.005	0.021	0.035

TABLE XXI. The numerator of the polarization observable $B_1(n) + B_2(n) = N_n/\sigma_{t\bar{t}}$ in the SM at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for three scales μ. For each invariant mass bin, the first column displays the range of the y_p bin. The numbers in the second, third, and fourth column are the values $N_{1,n}$ at NLO QCD for $\mu = m_t/2, m_t$, and $2m_t$, respectively. All numbers are in units of picobarns.

$2m_t \le M_{t\bar{t}} \le 450$ GeV			
(−1.0, −0.5)	0.260	0.187	0.138
(−0.5, 0.0)	0.184	0.132	0.097
(0.0, 0.5)	0.184	0.132	0.097
(0.5, 1.0)	0.260	0.187	0.138
$450 < M_{t\bar{t}} \le 600$ GeV			
(−1.0, −0.5)	0.761	0.533	0.384
(−0.5, 0.0)	0.457	0.320	0.230
(0.0, 0.5)	0.457	0.320	0.230
(0.5, 1.0)	0.761	0.533	0.384
$600 < M_{t\bar{t}} \le 800$ GeV			
(−1.0, −0.5)	0.546	0.373	0.263
(−0.5, 0.0)	0.261	0.179	0.126
(0.0, 0.5)	0.261	0.179	0.126
(0.5, 1.0)	0.546	0.373	0.263
$M_{t\bar{t}} > 800$ GeV			
(−1.0, −0.5)	0.310	0.204	0.140
(−0.5, 0.0)	0.113	0.075	0.052
(0.0, 0.5)	0.113	0.075	0.052
(0.5, 1.0)	0.310	0.204	0.140

TABLE XXII. The NP contributions to the $t\bar{t}$ cross section, to the numerator of A_C, and to the numerator of $C(n,n)$ at 13.6 TeV in bins of $M_{t\bar{t}}$ and $y_p = \cos\theta_t^*$ for the scale $\mu = m_t/2$. For each invariant mass bin listed in the first column, the second to fifth columns show the respective contribution in the indicated range of the y_p bin. All numbers are in units of picobarns.

	(−1.0, −0.5)	(−0.5, 0.0)	(0.0, 0.5)	(0.5, 1.0)
NP contribution $\propto \hat{c}_{VV}$ to $\sigma_{t\bar{t}}$				
$(2m_t, 450\ \mathrm{GeV})$	52.055	48.679	48.678	52.058
(450 GeV, 600 GeV)	62.353	52.787	52.793	62.353
(600 GeV, 800 GeV)	46.484	36.325	36.328	46.480
>800 GeV	66.005	47.819	47.818	66.006
NP contribution $\propto \hat{c}_1$ to $\sigma_{t\bar{t}}$				
$(2m_t, 450\ \mathrm{GeV})$	5.497	5.138	5.139	5.497
(450 GeV, 600 GeV)	6.940	5.873	5.873	6.940
(600 GeV, 800 GeV)	5.488	4.287	4.287	5.488
>800 GeV	8.831	6.385	6.385	8.832
NP contribution $\propto \hat{\mu}_t$ to $\sigma_{t\bar{t}}$				
$(2m_t, 450\ \mathrm{GeV})$	432.138	338.583	338.586	432.147
(450 GeV, 600 GeV)	406.136	226.624	226.618	406.126
(600 GeV, 800 GeV)	196.492	82.400	82.400	196.477
>800 GeV	106.903	34.376	34.377	106.903
NP contribution $\propto \hat{c}_{AA}$ to numerator of A_C				
$(2m_t, 450\ \mathrm{GeV})$	11.069	3.699	3.699	11.068
(450 GeV, 600 GeV)	21.875	7.311	7.311	21.874
(600 GeV, 800 GeV)	20.221	6.759	6.758	20.222
>800 GeV	33.083	11.057	11.057	33.083
NP contribution $\propto \hat{c}_2$ to numerator of A_C				
$(2m_t, 450\ \mathrm{GeV})$	2.190	0.732	0.732	2.190
(450 GeV, 600 GeV)	4.390	1.467	1.4672	4.390
(600 GeV, 800 GeV)	4.137	1.383	1.383	4.137
>800 GeV	6.985	2.335	2.335	6.985
NP contribution $\propto \hat{c}_{VV}$ to numerator of $C(n,n)$				
$(2m_t, 450\ \mathrm{GeV})$	2.904	6.389	6.389	2.904
(450 GeV, 600 GeV)	8.087	17.792	17.792	8.087
(600 GeV, 800 GeV)	8.555	18.823	18.823	8.556
>800 GeV	15.292	33.639	33.640	15.291
NP contribution $\propto \hat{c}_1$ to numerator of $C(n,n)$				
$(2m_t, 450\ \mathrm{GeV})$	0.309	0.679	0.679	0.309
(450 GeV, 600 GeV)	0.902	1.985	1.985	0.902
(600 GeV, 800 GeV)	1.011	2.225	2.225	1.011
>800 GeV	2.057	4.526	4.526	2.057
NP contribution $\propto \hat{\mu}_t$ to numerator of $C(n,n)$				
$(2m_t, 450\ \mathrm{GeV})$	326.065	285.418	285.405	326.077
(450 GeV, 600 GeV)	259.501	181.143	181.143	259.498
(600 GeV, 800 GeV)	112.016	62.159	62.163	112.016
>800 GeV	55.321	23.612	23.611	55.323

TABLE XXIII. Continuation of Table XXII. The NP contributions to the binned numerators of the displayed spin observables at 13.6 TeV for the scale $\mu = m_t/2$. All numbers are in units of picobarns.

	(−1.0, −0.5)	(−0.5, 0.0)	(0.0, 0.5)	(0.5, 1.0)
NP contribution $\propto \hat{c}_{VV}$ to numerator of $C(r,r)$				
$(2m_t, 450$ GeV$)$	−20.041	−44.092	−44.092	−20.042
(450 GeV, 600 GeV)	−21.322	−46.909	−46.912	−21.321
(600 GeV, 800 GeV)	−14.424	−31.733	−31.730	−14.424
>800 GeV	−18.650	−41.031	−41.028	−18.650
NP contribution $\propto \hat{c}_1$ to numerator of $C(r,r)$				
$(2m_t, 450$ GeV$)$	−2.115	−4.654	−4.654	−2.115
(450 GeV, 600 GeV)	−2.372	−5.217	−5.218	−2.372
(600 GeV, 800 GeV)	−1.702	−3.744	−3.744	−1.702
>800 GeV	−2.489	−5.475	−5.476	−2.489
NP contribution $\propto \hat{\mu}_t$ to numerator of $C(r,r)$				
$(2m_t, 450$ GeV$)$	285.346	184.670	184.671	285.348
(450 GeV, 600 GeV)	198.573	60.0668	60.0695	198.578
(600 GeV, 800 GeV)	74.609	4.613	4.612	74.611
>800 GeV	32.643	−4.231	−4.231	32.642
NP contribution $\propto \hat{c}_{VV}$ to numerator of $C(k,k)$				
$(2m_t, 450$ GeV$)$	−34.919	−10.978	−10.978	−34.918
(450 GeV, 600 GeV)	−49.117	−23.673	−23.673	−49.119
(600 GeV, 800 GeV)	−40.619	−23.417	−23.416	−40.619
>800 GeV	−62.646	−40.430	−40.430	−62.648
NP contribution $\propto \hat{c}_1$ to numerator of $C(k,k)$				
$(2m_t, 450$ GeV$)$	−3.691	−1.164	−1.164	−3.691
(450 GeV, 600 GeV)	−5.471	−2.640	−2.640	−5.470
(600 GeV, 800 GeV)	−4.797	−2.767	−2.767	−4.797
>800 GeV	−8.400	−5.435	−5.435	−8.399
NP contribution $\propto \hat{\mu}_t$ to numerator of $C(k,k)$				
$(2m_t, 450$ GeV$)$	294.511	219.665	219.669	294.507
(450 GeV, 600 GeV)	195.302	82.720	82.715	195.294
(600 GeV, 800 GeV)	53.398	13.586	13.587	53.395
>800 GeV	3.385	0.969	0.969	3.381
NP contribution $\propto \hat{c}_{VV}$ to numerator of $C(r,k) + C(k,r)$				
$(2m_t, 450$ GeV$)$	−41.107	−22.184	−22.184	−41.106
(450 GeV, 600 GeV)	−40.864	−22.052	−22.052	−40.863
(600 GeV, 800 GeV)	−24.050	−12.979	−12.979	−24.049
>800 GeV	−21.363	−11.529	−11.529	−21.363
NP contribution $\propto \hat{c}_1$ to numerator of $C(r,k) + C(k,r)$				
$(2m_t, 450$ GeV$)$	−4.338	−2.341	−2.341	−4.338
(450 GeV, 600 GeV)	−4.543	−2.452	−2.452	−4.543
(600 GeV, 800 GeV)	−2.836	−1.530	−1.530	−2.836
>800 GeV	−2.791	−1.506	−1.506	−2.791

TABLE XXIV. Continuation of Table XXIII. The NP contributions to the binned numerators of the displayed spin observables at 13.6 TeV for the scale $\mu = m_t/2$. All numbers are in units of picobarns.

	(−1.0, −0.5)	(−0.5, 0.0)	(0.0, 0.5)	(0.5, 1.0)
NP contribution $\propto \hat{\mu}_t$ to numerator of $C(r,k) + C(k,r)$				
$(2m_t, 450$ GeV$)$	−3.809	−20.567	−20.567	−3.809
(450 GeV, 600 GeV)	2.673	−30.307	−30.308	2.676
(600 GeV, 800 GeV)	−22.284	−23.897	−23.899	−22.287
>800 GeV	−48.061	−20.339	−20.339	−48.060
NP contribution $\propto \hat{d}_t$ to numerator of $C(n,r) - C(r,n)$				
$(2m_t, 450$ GeV$)$	−261.754	−297.544	−297.549	−261.750
(450 GeV, 600 GeV)	−327.141	−229.623	−229.627	−327.118
(600 GeV, 800 GeV)	−161.682	−70.331	−70.327	−161.679
>800 GeV	−83.506	−20.787	−20.785	−83.502
NP contribution $\propto \hat{d}_t$ to numerator of $C(n,k) - C(k,n)$				
$(2m_t, 450$ GeV$)$	−128.717	−48.007	−48.007	−128.716
(450 GeV, 600 GeV)	−116.332	−17.503	−17.503	−116.337
(600 GeV, 800 GeV)	−21.641	9.430	9.431	−21.642
>800 GeV	31.137	15.955	15.955	31.136
NP contribution $\propto \hat{c}_{AA}$ to numerator of $C(k,k^*)$				
$(2m_t, 450$ GeV$)$	−11.069	−3.699	−3.699	−11.069
(450 GeV, 600 GeV)	−21.874	−7.311	−7.310	−21.874
(600 GeV, 800 GeV)	−20.222	−6.759	−6.759	−20.222
>800 GeV	−33.083	−11.057	−11.057	−33.083
NP contribution $\propto \hat{c}_2$ to numerator of $C(k,k^*)$				
$(2m_t, 450$ GeV$)$	−2.190	−0.732	−0.732	−2.190
(450 GeV, 600 GeV)	−4.390	−1.467	−1.467	−4.390
(600 GeV, 800 GeV)	−4.137	−1.383	−1.383	−4.137
>800 GeV	−6.985	−2.335	−2.335	−6.985
NP contribution $\propto \hat{c}_{AA}$ to numerator of $C(r^*,k) + C(k,r^*)$				
$(2m_t, 450$ GeV$)$	−7.694	−11.985	−11.985	−7.694
(450 GeV, 600 GeV)	−11.936	−18.592	−18.591	−11.935
(600 GeV, 800 GeV)	−8.340	−12.991	−12.992	−8.339
>800 GeV	−8.177	−12.738	−12.738	−8.178
NP contribution $\propto \hat{c}_2$ to numerator of $C(r^*,k) + C(k,r^*)$				
$(2m_t, 450$ GeV$)$	−1.522	−2.370	−2.370	−1.522
(450 GeV, 600 GeV)	−2.394	−3.729	−3.729	−2.394
(600 GeV, 800 GeV)	−1.705	−2.656	−2.656	−1.705
>800 GeV	−1.718	−2.677	−2.677	−1.718

TABLE XXV. Continuation of Table XXIV. The NP contributions to the binned numerators of the displayed spin observables at 13.6 TeV for the scale $\mu = m_t/2$. All numbers are in units of picobarns.

	(−1.0, −0.5)	(−0.5, 0.0)	(0.0, 0.5)	(0.5, 1.0)
NP contribution $\propto \hat{c}_{VA}$ to numerator of $B_1(r)+B_2(r)$				
$(2m_t, 450\ \mathrm{GeV})$	9.855	5.319	5.319	9.855
(450 GeV, 600 GeV)	15.032	8.113	8.113	15.032
(600 GeV, 800 GeV)	10.349	5.585	5.585	10.349
>800 GeV	10.059	5.429	5.429	10.059
NP contribution $\propto \hat{c}_3$ to numerator of $B_1(r)+B_2(r)$				
$(2m_t, 450\ \mathrm{GeV})$	1.044	0.563	0.563	1.044
(450 GeV, 600 GeV)	1.673	0.903	0.903	1.673
(600 GeV, 800 GeV)	1.221	0.659	0.659	1.221
>800 GeV	1.316	0.710	0.710	1.316
NP contribution $\propto \hat{c}_{VA}$ to numerator of $B_1(k)+B_2(k)$				
$(2m_t, 450\ \mathrm{GeV})$	42.446	29.114	29.116	42.445
(450 GeV, 600 GeV)	82.476	56.574	56.576	82.480
(600 GeV, 800 GeV)	75.132	51.535	51.536	75.135
>800 GeV	122.071	83.737	83.740	122.076
NP contribution $\propto \hat{c}_3$ to numerator of $B_1(k)+B_2(k)$				
$(2m_t, 450\ \mathrm{GeV})$	4.500	3.087	3.086	4.500
(450 GeV, 600 GeV)	9.192	6.306	6.306	9.193
(600 GeV, 800 GeV)	8.876	6.088	6.088	8.875
>800 GeV	16.386	11.240	11.240	16.386
NP contribution $\propto \hat{c}_{AV}$ to numerator of $B_1(r^*)+B_2(r^*)$				
$(2m_t, 450\ \mathrm{GeV})$	32.047	49.919	49.917	32.0473
(450 GeV, 600 GeV)	32.432	50.517	50.518	32.4316
(600 GeV, 800 GeV)	19.379	30.184	30.187	19.379
>800 GeV	17.368	27.052	27.052	17.367
NP contribution $\propto (\hat{c}_1-\hat{c}_2+\hat{c}_3)$ to numerator of $B_1(r^*)+B_2(r^*)$				
$(2m_t, 450\ \mathrm{GeV})$	6.332	9.865	9.864	6.332
(450 GeV, 600 GeV)	6.503	10.130	10.129	6.503
(600 GeV, 800 GeV)	3.962	6.171	6.171	3.962
>800 GeV	3.648	5.683	5.683	3.648
NP contribution $\propto \hat{c}_{AV}$ to numerator of $B_1(k^*)+B_2(k^*)$				
$(2m_t, 450\ \mathrm{GeV})$	45.297	15.139	15.139	45.299
(450 GeV, 600 GeV)	59.106	19.755	19.755	59.108
(600 GeV, 800 GeV)	46.881	15.670	15.669	46.885
>800 GeV	69.737	23.308	23.308	69.739
NP contribution $\propto (\hat{c}_1-\hat{c}_2+\hat{c}_3)$ to numerator of $B_1(k^*)+B_2(k^*)$				
$(2m_t, 450\ \mathrm{GeV})$	8.953	2.992	2.992	8.953
(450 GeV, 600 GeV)	11.857	3.963	3.963	11.857
(600 GeV, 800 GeV)	9.589	3.205	3.205	9.590
>800 GeV	14.718	4.919	4.919	14.718

TABLE XXVI. As in Table XXII, but for the scale $\mu = m_t$. All numbers are in units of picobarns.

	(−1.0, −0.5)	(−0.5, 0.0)	(0.0, 0.5)	(0.5, 1.0)
NP contribution $\propto \hat{c}_{VV}$ to $\sigma_{t\bar{t}}$				
$(2m_t, 450\ \mathrm{GeV})$	43.315	40.512	40.511	43.315
(450 GeV, 600 GeV)	51.091	43.258	43.261	51.089
(600 GeV, 800 GeV)	37.408	29.235	29.238	37.405
>800 GeV	51.072	37.023	37.023	51.074
NP contribution $\propto \hat{c}_1$ to $\sigma_{t\bar{t}}$				
$(2m_t, 450\ \mathrm{GeV})$	4.643	4.340	4.341	4.643
(450 GeV, 600 GeV)	5.778	4.890	4.890	5.777
(600 GeV, 800 GeV)	4.490	3.508	3.508	4.490
>800 GeV	6.935	5.017	5.017	6.935
NP contribution $\propto \hat{\mu}_t$ to $\sigma_{t\bar{t}}$				
$(2m_t, 450\ \mathrm{GeV})$	341.976	268.633	268.640	341.975
(450 GeV, 600 GeV)	314.438	176.420	176.419	314.436
(600 GeV, 800 GeV)	148.422	62.785	62.780	148.415
>800 GeV	77.902	25.452	25.453	77.896
NP contribution $\propto \hat{c}_{AA}$ to numerator of A_C				
$(2m_t, 450\ \mathrm{GeV})$	9.227	3.084	3.084	9.227
(450 GeV, 600 GeV)	17.966	6.004	6.005	17.966
(600 GeV, 800 GeV)	16.307	5.450	5.450	16.307
>800 GeV	25.577	8.548	8.548	25.576
NP contribution $\propto \hat{c}_2$ to numerator of A_C				
$(2m_t, 450\ \mathrm{GeV})$	1.837	0.614	0.614	1.837
(450 GeV, 600 GeV)	3.630	1.213	1.213	3.630
(600 GeV, 800 GeV)	3.359	1.123	1.123	3.359
>800 GeV	5.438	1.817	1.817	5.437
NP contribution $\propto \hat{c}_{VV}$ to numerator of $C(n,n)$				
$(2m_t, 450\ \mathrm{GeV})$	2.412	5.306	5.306	2.412
(450 GeV, 600 GeV)	6.621	14.566	14.567	6.621
(600 GeV, 800 GeV)	6.882	15.142	15.141	6.882
>800 GeV	11.813	25.988	25.989	11.813
NP contribution $\propto \hat{c}_1$ to numerator of $C(n,n)$				
$(2m_t, 450\ \mathrm{GeV})$	0.260	0.573	0.573	0.260
(450 GeV, 600 GeV)	0.751	1.651	1.652	0.751
(600 GeV, 800 GeV)	0.827	1.819	1.820	0.827
>800 GeV	1.613	3.548	3.548	1.613
NP contribution $\propto \hat{\mu}_t$ to numerator of $C(n,n)$				
$(2m_t, 450\ \mathrm{GeV})$	256.332	224.457	224.464	256.335
(450 GeV, 600 GeV)	199.762	139.508	139.509	199.765
(600 GeV, 800 GeV)	84.137	46.710	46.713	84.137
>800 GeV	40.075	17.144	17.144	40.076

TABLE XXVII. As in Table XXIII, but for the scale $\mu = m_t$. All numbers are in units of picobarns.

	(−1.0, −0.5)	(−0.5, 0.0)	(0.0, 0.5)	(0.5, 1.0)
NP contribution $\propto \hat{c}_{VV}$ to numerator of $C(r,r)$				
$(2m_t, 450\ \mathrm{GeV})$	−16.679	−36.694	−36.694	−16.679
(450 GeV, 600 GeV)	−17.473	−38.441	−38.439	−17.473
(600 GeV, 800 GeV)	−11.609	−25.539	−25.536	−11.608
>800 GeV	−14.442	−31.772	−31.771	−14.442
NP contribution $\propto \hat{c}_1$ to numerator of $C(r,r)$				
$(2m_t, 450\ \mathrm{GeV})$	−1.787	−3.931	−3.931	−1.787
(450 GeV, 600 GeV)	−1.975	−4.345	−4.345	−1.975
(600 GeV, 800 GeV)	−1.393	−3.064	−3.064	−1.393
>800 GeV	−1.956	−4.304	−4.304	−1.956
NP contribution $\propto \hat{\mu}_t$ to numerator of $C(r,r)$				
$(2m_t, 450\ \mathrm{GeV})$	223.378	143.188	143.186	223.376
(450 GeV, 600 GeV)	152.112	44.606	44.605	152.120
(600 GeV, 800 GeV)	55.692	2.691	2.690	55.697
>800 GeV	23.4443	−3.512	−3.512	23.444
NP contribution $\propto \hat{c}_{VV}$ to numerator of $C(k,k)$				
$(2m_t, 450\ \mathrm{GeV})$	−29.048	−9.124	−9.124	−29.047
(450 GeV, 600 GeV)	−40.237	−19.385	−19.385	−40.238
(600 GeV, 800 GeV)	−32.682	−18.840	−18.840	−32.682
>800 GeV	−48.443	−31.241	−31.239	−48.443
NP contribution $\propto \hat{c}_1$ to numerator of $C(k,k)$				
$(2m_t, 450\ \mathrm{GeV})$	−3.116	−0.982	−0.982	−3.116
(450 GeV, 600 GeV)	−4.553	−2.197	−2.197	−4.553
(600 GeV, 800 GeV)	−3.924	−2.263	−2.263	−3.924
>800 GeV	−6.592	−4.262	−4.262	−6.592
NP contribution $\propto \hat{\mu}_t$ to numerator of $C(k,k)$				
$(2m_t, 450\ \mathrm{GeV})$	230.169	172.697	172.697	230.166
(450 GeV, 600 GeV)	149.336	63.629	63.625	149.331
(600 GeV, 800 GeV)	39.655	10.153	10.153	39.654
>800 GeV	2.318	0.678	0.678	2.317
NP contribution $\propto \hat{c}_{VV}$ to numerator of $C(r,k) + C(k,r)$				
$(2m_t, 450\ \mathrm{GeV})$	−34.212	−18.463	−18.463	−34.210
(450 GeV, 600 GeV)	−33.490	−18.075	−18.074	−33.492
(600 GeV, 800 GeV)	−19.362	−10.450	−10.449	−19.361
>800 GeV	−16.641	−8.982	−8.981	−16.642
NP contribution $\propto \hat{c}_1$ to numerator of $C(r,k) + C(k,r)$				
$(2m_t, 450\ \mathrm{GeV})$	−3.664	−1.978	−1.978	−3.664
(450 GeV, 600 GeV)	−3.783	−2.042	−2.042	−3.783
(600 GeV, 800 GeV)	−2.321	−1.253	−1.253	−2.321
>800 GeV	−2.209	−1.192	−1.192	−2.209

TABLE XXVIII. As in Table XXIV, but for the scale $\mu = m_t$. All numbers are in units of picobarns.

	(−1.0, −0.5)	(−0.5, 0.0)	(0.0, 0.5)	(0.5, 1.0)
NP contribution $\propto \hat{\mu}_t$ to numerator of $C(r,k) + C(k,r)$				
$(2m_t, 450\ \mathrm{GeV})$	−5.148	−17.288	−17.289	−5.147
(450 GeV, 600 GeV)	0.326	−24.253	−24.253	0.328
(600 GeV, 800 GeV)	−17.617	−18.435	−18.436	−17.616
>800 GeV	−35.158	−15.070	−15.070	−35.158
NP contribution $\propto \hat{d}_t$ to numerator of $C(n,r) - C(r,n)$				
$(2m_t, 450\ \mathrm{GeV})$	−204.974	−232.705	−232.704	−204.972
(450 GeV, 600 GeV)	−251.135	−175.592	−175.602	−251.131
(600 GeV, 800 GeV)	−121.113	−52.183	−52.181	−121.113
>800 GeV	−60.277	−14.686	−14.685	−60.278
NP contribution $\propto \hat{d}_t$ to numerator of $C(n,k) - C(k,n)$				
$(2m_t, 450\ \mathrm{GeV})$	−100.490	−37.421	−37.421	−100.489
(450 GeV, 600 GeV)	−88.709	−13.048	−13.047	−88.710
(600 GeV, 800 GeV)	−15.686	7.403	7.402	−15.684
>800 GeV	22.747	11.818	11.818	22.747
NP contribution $\propto \hat{c}_{AA}$ to numerator of $C(k,k^*)$				
$(2m_t, 450\ \mathrm{GeV})$	−9.227	−3.084	−3.084	−9.228
(450 GeV, 600 GeV)	−17.965	−6.005	−6.005	−17.965
(600 GeV, 800 GeV)	−16.306	−5.450	−5.450	−16.305
>800 GeV	−25.576	−8.548	−8.548	−25.576
NP contribution $\propto \hat{c}_2$ to numerator of $C(k,k^*)$				
$(2m_t, 450\ \mathrm{GeV})$	−1.837	−0.614	−0.614	−1.837
(450 GeV, 600 GeV)	−3.630	−1.213	−1.213	−3.630
(600 GeV, 800 GeV)	−3.359	−1.123	−1.123	−3.359
>800 GeV	−5.438	−1.817	−1.817	−5.437
NP contribution $\propto \hat{c}_{AA}$ to numerator of $C(r^*,k) + C(k,r^*)$				
$(2m_t, 450\ \mathrm{GeV})$	−6.415	−9.994	−9.993	−6.415
(450 GeV, 600 GeV)	−9.807	−15.275	−15.275	−9.806
(600 GeV, 800 GeV)	−6.728	−10.481	−10.481	−6.728
>800 GeV	−6.369	−9.922	−9.923	−6.370
NP contribution $\propto \hat{c}_2$ to numerator of $C(r^*,k) + C(k,r^*)$				
$(2m_t, 450\ \mathrm{GeV})$	−1.277	−1.989	−1.989	−1.277
(450 GeV, 600 GeV)	−1.981	−3.085	−3.085	−1.981
(600 GeV, 800 GeV)	−1.385	−2.158	−2.158	−1.385
>800 GeV	−1.348	−2.099	−2.099	−1.348

TABLE XXIX. As in Table XXV, but for the scale $\mu = m_t$. All numbers are in units of picobarns.

	(−1.0, −0.5)	(−0.5, 0.0)	(0.0, 0.5)	(0.5, 1.0)
NP contribution $\propto \hat{c}_{VA}$ to numerator of $B_1(r) + B_2(r)$				
$(2m_t, 450$ GeV$)$	8.193	4.422	4.422	8.193
(450 GeV, 600 GeV)	12.316	6.647	6.647	12.317
(600 GeV, 800 GeV)	8.330	4.496	4.495	8.330
>800 GeV	7.832	4.227	4.227	7.832
NP contribution $\propto \hat{c}_3$ to numerator of $B_1(r) + B_2(r)$				
$(2m_t, 450$ GeV$)$	0.881	0.475	0.475	0.881
(450 GeV, 600 GeV)	1.393	0.752	0.752	1.393
(600 GeV, 800 GeV)	0.999	0.539	0.539	0.999
>800 GeV	1.041	0.562	0.562	1.041
NP contribution $\propto \hat{c}_{VA}$ to numerator of $B_1(k) + B_2(k)$				
$(2m_t, 450$ GeV$)$	35.279	24.199	24.199	35.279
(450 GeV, 600 GeV)	67.549	46.334	46.336	67.548
(600 GeV, 800 GeV)	60.447	41.465	41.463	60.452
>800 GeV	94.368	64.731	64.731	94.368
NP contribution $\propto \hat{c}_3$ to numerator of $B_1(k) + B_2(k)$				
$(2m_t, 450$ GeV$)$	3.797	2.604	2.604	3.797
(450 GeV, 600 GeV)	7.650	5.248	5.247	7.650
(600 GeV, 800 GeV)	7.260	4.980	4.980	7.260
>800 GeV	12.855	8.818	8.818	12.855
NP contribution $\propto \hat{c}_{AV}$ to numerator of $B_1(r^*) + B_2(r^*)$				
$(2m_t, 450$ GeV$)$	26.749	41.667	41.664	26.749
(450 GeV, 600 GeV)	26.657	41.522	41.522	26.656
(600 GeV, 800 GeV)	15.637	24.356	24.356	15.636
>800 GeV	13.536	21.085	21.083	13.535
NP contribution $\propto (\hat{c}_1 - \hat{c}_2 + \hat{c}_3)$ to numerator of $B_1(r^*) + B_2(r^*)$				
$(2m_t, 450$ GeV$)$	5.320	8.286	8.286	5.320
(450 GeV, 600 GeV)	5.382	8.382	8.383	5.382
(600 GeV, 800 GeV)	3.219	5.014	5.014	3.219
>800 GeV	2.863	4.459	4.459	2.863
NP contribution $\propto \hat{c}_{AV}$ to numerator of $B_1(k^*) + B_2(k^*)$				
$(2m_t, 450$ GeV$)$	37.799	12.633	12.633	37.799
(450 GeV, 600 GeV)	48.561	16.231	16.232	48.564
(600 GeV, 800 GeV)	37.810	12.637	12.638	37.815
>800 GeV	53.951	18.032	18.032	53.952
NP contribution $\propto (\hat{c}_1 - \hat{c}_2 + \hat{c}_3)$ to numerator of $B_1(k^*) + B_2(k^*)$				
$(2m_t, 450$ GeV$)$	7.519	2.513	2.513	7.519
(450 GeV, 600 GeV)	9.808	3.278	3.278	9.808
(600 GeV, 800 GeV)	7.787	2.603	2.602	7.788
>800 GeV	11.465	3.832	3.832	11.465

TABLE XXX. As in Table XXII, but for the scale $\mu = 2m_t$. All numbers are in units of picobarns.

	(−1.0, −0.5)	(−0.5, 0.0)	(0.0, 0.5)	(0.5, 1.0)
NP contribution $\propto \hat{c}_{VV}$ to $\sigma_{t\bar{t}}$				
$(2m_t, 450$ GeV$)$	36.475	34.116	34.119	36.475
(450 GeV, 600 GeV)	42.441	35.938	35.942	42.442
(600 GeV, 800 GeV)	30.584	23.905	23.908	30.582
>800 GeV	40.357	29.271	29.270	40.357
NP contribution $\propto \hat{c}_1$ to $\sigma_{t\bar{t}}$				
$(2m_t, 450$ GeV$)$	3.961	3.703	3.703	3.960
(450 GeV, 600 GeV)	4.866	4.119	4.119	4.866
(600 GeV, 800 GeV)	3.724	2.910	2.910	3.724
>800 GeV	5.550	4.018	4.018	5.550
NP contribution $\propto \hat{\mu}_t$ to $\sigma_{t\bar{t}}$				
$(2m_t, 450$ GeV$)$	274.578	216.227	216.232	274.574
(450 GeV, 600 GeV)	247.699	139.703	139.700	247.710
(600 GeV, 800 GeV)	114.447	48.812	48.809	114.436
>800 GeV	58.237	19.318	19.317	58.239
NP contribution $\propto \hat{c}_{AA}$ to numerator of A_C				
$(2m_t, 450$ GeV$)$	7.781	2.601	2.601	7.782
(450 GeV, 600 GeV)	14.954	4.998	4.998	14.954
(600 GeV, 800 GeV)	13.355	4.464	4.463	13.355
>800 GeV	20.192	6.749	6.749	20.192
NP contribution $\propto \hat{c}_2$ to numerator of A_C				
$(2m_t, 450$ GeV$)$	1.558	0.521	0.521	1.558
(450 GeV, 600 GeV)	3.039	1.016	1.016	3.039
(600 GeV, 800 GeV)	2.767	0.925	0.925	2.767
>800 GeV	4.318	1.443	1.443	4.318
NP contribution $\propto \hat{c}_{VV}$ to numerator of $C(n, n)$				
$(2m_t, 450$ GeV$)$	2.028	4.461	4.461	2.028
(450 GeV, 600 GeV)	5.497	12.092	12.092	5.496
(600 GeV, 800 GeV)	5.625	12.377	12.377	5.625
>800 GeV	9.321	20.507	20.508	9.321
NP contribution $\propto \hat{c}_1$ to numerator of $C(n, n)$				
$(2m_t, 450$ GeV$)$	0.222	0.488	0.488	0.222
(450 GeV, 600 GeV)	0.632	1.390	1.390	0.632
(600 GeV, 800 GeV)	0.686	1.509	1.509	0.686
>800 GeV	1.289	2.835	2.835	1.289
NP contribution $\propto \hat{\mu}_t$ to numerator of $C(n, n)$				
$(2m_t, 450$ GeV$)$	204.452	179.096	179.090	204.454
(450 GeV, 600 GeV)	156.463	109.313	109.318	156.465
(600 GeV, 800 GeV)	64.508	35.828	35.832	64.511
>800 GeV	29.785	12.767	12.766	29.785

TABLE XXXI. As in Table XXIII, but for the scale $\mu = 2m_t$. All numbers are in units of picobarns.

	(−1.0, −0.5)	(−0.5, 0.0)	(0.0, 0.5)	(0.5, 1.0)
NP contribution $\propto \hat{c}_{VV}$ to numerator of $C(r,r)$				
$(2m_t, 450\ \text{GeV})$	−14.047	−30.903	−30.904	−14.047
(450 GeV, 600 GeV)	−14.517	−31.937	−31.939	−14.517
(600 GeV, 800 GeV)	−9.493	−20.884	−20.882	−9.492
>800 GeV	−11.419	−25.121	−25.122	−11.419
NP contribution $\propto \hat{c}_1$ to numerator of $C(r,r)$				
$(2m_t, 450\ \text{GeV})$	−1.524	−3.354	−3.354	−1.524
(450 GeV, 600 GeV)	−1.663	−3.660	−3.660	−1.664
(600 GeV, 800 GeV)	−1.1553	−2.542	−2.541	−1.1552
>800 GeV	−1.567	−3.447	−3.447	−1.567
NP contribution $\propto \hat{\mu}_t$ to numerator of $C(r,r)$				
$(2m_t, 450\ \text{GeV})$	177.401	112.593	112.598	177.403
(450 GeV, 600 GeV)	118.557	33.649	33.650	118.559
(600 GeV, 800 GeV)	42.437	1.470	1.470	42.438
>800 GeV	17.274	−2.941	−2.941	17.275
NP contribution $\propto \hat{c}_{VV}$ to numerator of $C(k,k)$				
$(2m_t, 450\ \text{GeV})$	−24.456	−7.676	−7.676	−24.455
(450 GeV, 600 GeV)	−33.420	−16.095	−16.094	−33.419
(600 GeV, 800 GeV)	−26.717	−15.399	−15.400	−26.718
>800 GeV	−38.257	−24.655	−24.654	−38.258
NP contribution $\propto \hat{c}_1$ to numerator of $C(k,k)$				
$(2m_t, 450\ \text{GeV})$	−2.658	−0.837	−0.837	−2.658
(450 GeV, 600 GeV)	−3.834	−1.849	−1.849	−3.834
(600 GeV, 800 GeV)	−3.254	−1.877	−1.877	−3.255
>800 GeV	−5.272	−3.406	−3.406	−5.272
NP contribution $\propto \hat{\mu}_t$ to numerator of $C(k,k)$				
$(2m_t, 450\ \text{GeV})$	182.493	137.730	137.733	182.493
(450 GeV, 600 GeV)	116.172	49.789	49.789	116.172
(600 GeV, 800 GeV)	30.056	7.745	7.745	30.056
>800 GeV	1.606	0.484	0.484	1.606
NP contribution $\propto \hat{c}_{VV}$ to numerator of $C(r,k)+C(k,r)$				
$(2m_t, 450\ \text{GeV})$	−28.814	−15.550	−15.550	−28.813
(450 GeV, 600 GeV)	−27.829	−15.019	−15.018	−27.829
(600 GeV, 800 GeV)	−15.836	−8.546	−8.546	−15.836
>800 GeV	−13.227	−7.138	−7.138	−13.226
NP contribution $\propto \hat{c}_1$ to numerator of $C(r,k)+C(k,r)$				
$(2m_t, 450\ \text{GeV})$	−3.126	−1.687	−1.687	−3.126
(450 GeV, 600 GeV)	−3.187	−1.720	−1.720	−3.187
(600 GeV, 800 GeV)	−1.926	−1.039	−1.039	−1.926
>800 GeV	−1.779	−0.960	−0.960	−1.779

TABLE XXXII. As in Table XXIV, but for the scale $\mu = 2m_t$. All numbers are in units of picobarns.

	(−1.0, −0.5)	(−0.5, 0.0)	(0.0, 0.5)	(0.5, 1.0)
NP contribution $\propto \hat{\mu}_t$ to numerator of $C(r,k)+C(k,r)$				
$(2m_t, 450\ \text{GeV})$	−5.820	−14.686	−14.687	−5.819
(450 GeV, 600 GeV)	−1.104	−19.720	−19.720	−1.104
(600 GeV, 800 GeV)	−14.182	−14.508	−14.507	−14.181
>800 GeV	−26.424	−11.453	−11.453	−26.424
NP contribution $\propto \hat{d}_t$ to numerator of $C(n,r)-C(r,n)$				
$(2m_t, 450\ \text{GeV})$	−162.885	−184.656	−184.655	−162.877
(450 GeV, 600 GeV)	−196.175	136.606	−136.612	−196.167
(600 GeV, 800 GeV)	−92.616	−39.517	−39.518	−92.620
>800 GeV	−44.645	−10.633	−10.634	−44.645
NP contribution $\propto \hat{d}_t$ to numerator of $C(n,k)-C(k,n)$				
$(2m_t, 450\ \text{GeV})$	−79.610	−29.593	−29.593	−79.609
(450 GeV, 600 GeV)	−68.817	−9.881	−9.882	−68.818
(600 GeV, 800 GeV)	−11.584	5.920	5.921	−11.584
>800 GeV	17.091	8.980	8.981	17.090
NP contribution $\propto \hat{c}_{AA}$ to numerator of $C(k,k^*)$				
$(2m_t, 450\ \text{GeV})$	−7.782	−2.601	−2.601	−7.782
(450 GeV, 600 GeV)	−14.953	−4.998	−4.998	−14.953
(600 GeV, 800 GeV)	−13.355	−4.464	−4.464	3.355
>800 GeV	−20.192	−6.749	−6.749	−20.192
NP contribution $\propto \hat{c}_2$ to numerator of $C(k,k^*)$				
$(2m_t, 450\ \text{GeV})$	−1.558	−0.521	−0.521	−1.558
(450 GeV, 600 GeV)	−3.039	−1.016	−1.016	−3.039
(600 GeV, 800 GeV)	−2.767	−0.925	−0.925	−2.767
>800 GeV	−4.318	−1.443	−1.443	−4.318
NP contribution $\propto \hat{c}_{AA}$ to numerator of $C(r^*,k)+C(k,r^*)$				
$(2m_t, 450\ \text{GeV})$	−5.412	−8.430	−8.430	−5.412
(450 GeV, 600 GeV)	−8.165	−12.720	−12.719	−8.166
(600 GeV, 800 GeV)	−5.513	−8.587	−8.587	−5.512
>800 GeV	−5.061	−7.884	−7.884	−5.062
NP contribution $\propto \hat{c}_2$ to numerator of $C(r^*,k)+C(k,r^*)$				
$(2m_t, 450\ \text{GeV})$	−1.083	−1.687	−1.687	−1.083
(450 GeV, 600 GeV)	−1.659	−2.584	−2.584	−1.659
(600 GeV, 800 GeV)	−1.142	−1.778	−1.778	−1.142
>800 GeV	−1.077	−1.678	−1.678	−1.077

TABLE XXXIII. As in Table XXV, but for the scale $\mu = 2m_t$. All numbers are in units of picobarns.

	(−1.0, −0.5)	(−0.5, 0.0)	(0.0, 0.5)	(0.5, 1.0)
NP contribution $\propto \hat{c}_{VA}$ to numerator of $B_1(r) + B_2(r)$				
$(2m_t, 450\ \mathrm{GeV})$	6.894	3.721	3.721	6.894
(450 GeV, 600 GeV)	10.231	5.522	5.522	10.231
(600 GeV, 800 GeV)	6.812	3.676	3.676	6.812
>800 GeV	6.222	3.358	3.358	6.222
NP contribution $\propto \hat{c}_3$ to numerator of $B_1(r) + B_2(r)$				
$(2m_t, 450\ \mathrm{GeV})$	0.751	0.405	0.405	0.751
(450 GeV, 600 GeV)	1.173	0.633	0.633	1.173
(600 GeV, 800 GeV)	0.829	0.447	0.447	0.829
>800 GeV	0.838	0.452	0.452	0.838
NP contribution $\propto \hat{c}_{VA}$ to numerator of $B_1(k) + B_2(k)$				
$(2m_t, 450\ \mathrm{GeV})$	29.677	20.357	20.358	29.678
(450 GeV, 600 GeV)	56.092	38.478	38.477	56.090
(600 GeV, 800 GeV)	49.418	33.895	33.897	49.416
>800 GeV	74.507	51.106	51.108	74.506
NP contribution $\propto \hat{c}_3$ to numerator of $B_1(k) + B_2(k)$				
$(2m_t, 450\ \mathrm{GeV})$	3.236	2.220	2.220	3.236
(450 GeV, 600 GeV)	6.441	4.418	4.418	6.441
(600 GeV, 800 GeV)	6.020	4.130	4.130	6.020
>800 GeV	10.278	7.050	7.050	10.278

(Table continued)

TABLE XXXIII. *(Continued)*

NP contribution $\propto \hat{c}_{AV}$ to numerator of $B_1(r^*) + B_2(r^*)$				
$(2m_t, 450\ \mathrm{GeV})$	22.586	35.181	35.180	22.586
(450 GeV, 600 GeV)	22.204	34.587	34.586	22.205
(600 GeV, 800 GeV)	12.813	19.961	19.958	12.813
>800 GeV	10.761	16.761	16.761	10.760
NP contribution $\propto (\hat{c}_1 - \hat{c}_2 + \hat{c}_3)$ to numerator of $B_1(r^*) + B_2(r^*)$				
$(2m_t, 450\ \mathrm{GeV})$	4.517	7.036	7.036	4.517
(450 GeV, 600 GeV)	4.509	7.023	7.023	4.509
(600 GeV, 800 GeV)	2.653	4.133	4.133	2.653
>800 GeV	2.289	3.566	3.566	2.289
NP contribution $\propto \hat{c}_{AV}$ to numerator of $B_1(k^*) + B_2(k^*)$				
$(2m_t, 450\ \mathrm{GeV})$	31.907	10.664	10.664	31.907
(450 GeV, 600 GeV)	40.433	13.514	13.515	40.437
(600 GeV, 800 GeV)	30.970	10.351	10.351	30.971
>800 GeV	42.618	14.244	14.244	42.620
NP contribution $\propto (\hat{c}_1 - \hat{c}_2 + \hat{c}_3)$ to numerator of $B_1(k^*) + B_2(k^*)$				
$(2m_t, 450\ \mathrm{GeV})$	6.383	2.133	2.13	6.382
(450 GeV, 600 GeV)	8.215	2.746	2.745	8.215
(600 GeV, 800 GeV)	6.416	2.144	2.144	6.416
>800 GeV	9.111	3.045	3.045	9.111

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