Right-handed neutrino as a common progenitor of baryon number asymmetry and dark matter

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Cosmological and astrophysical observations suggest that both energy densities of baryon and dark matter take the same order values in the present Universe. We propose a scenario to give an answer for this problem in a scotogenic model and its extension. The model naturally provides dark matter candidates as its crucial ingredients to explain the small neutrino mass. If a neutral component of an inert doublet scalar plays a role of dark matter and leptogenesis occurs at TeV scales, both dark matter abundance and baryon number asymmetry could be explained with a same mother particle. Coincidence between the order of their energy densities might be understood through such a background.

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I. INTRODUCTION

Existence of dark matter (DM) suggested through various astrophysical and cosmological observations is one of unsolved problems in the standard model (SM) [1]. Since there is no corresponding particle in the SM, it can be a crucial clue to study new physics beyond the SM. A promising candidate is a weakly interacting massive particle called a WIMP. If it is in the thermal equilibrium in the early Universe and is frozen out at a certain stage, it is known that the expected DM abundance could be naturally realized as its relic. However, such a particle has not yet been discovered in various kind of experiments but a lot of proposed models for it have been ruled out or constrained by them [2,3].

Another mysterious point on DM is cosmological coincidence of its abundance with baryon number asymmetry, that is, why their energy densities take the same order values in the present Universe. The WIMP scenario cannot answer this problem since its abundance is usually considered to be explained through physics irrelevant to the baryon number asymmetry. Asymmetric DM model [4] is a promising scenario which is motivated to give an answer for it. In this scenario, the DM abundance is explained as asymmetry of a certain conserved charge like the baryon number asymmetry. If we consider an alternative possibility to explain it, a scenario may be constructed by assuming that both the baryon number asymmetry and the DM are caused from a same mother particle. In that case, the same physics could be relevant to them and then it could give an answer for the problem. As such we study a class of model for the neutrino mass here.

The scotogenic model [5] has been proposed to explain the small neutrino mass and the existence of DM, simultaneously. In this model, the DM abundance is usually explained following the WIMP scenario. The model includes DM candidates as its important ingredients to generate the small neutrino mass. They are a lightest neutral component of an inert doublet scalar η and a lightest right-handed neutrino. If we choose the right-handed neutrino as the DM, a serious lepton-flavor violating problem appears [6]. On the other hand, if we adopt a neutral component of the η as the DM, any serious phenomenological problem occurs, and then it can be considered as a promising DM candidate in the model. Its relic abundance and various features have been extensively studied in a lot of papers [7,8].

In this paper, we reconsider the DM abundance in this model from a view point of its relation to the baryon number asymmetry. In the scotogenic model, leptogenesis is known to generate baryon number asymmetry successfully [9,10]. Since model parameters relevant to DM and leptogenesis could be almost independent in usually supposed cases, they have been studied separately. However, there could be an exceptional situation where they are closely related; this situation seems not to have been noticed and has not been studied. In the present study, we focus our attention on a case where the model allows successful leptogenesis at TeV scales as such a situation. There, we find that the usual estimation of the relic abundance of the DM should be modified. The DM abundance is considered to be explained as a cosmological relic which is neither a pure thermal relic nor an asymmetry

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of any conserved charge. It might give a new viewpoint for the cosmological coincidence between the DM and the baryon number asymmetry.

Following parts are organized as follows. In Sec. II we briefly review some features of the scotogenic model relevant to the present study. In Sec. III, after we overview leptogenesis and the DM physics in the model, we discuss a scenario in which the baryon number asymmetry and the DM abundance can be closely correlated. We show its realization in the model which can give the origin of the CP phases in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [11]. We summarize the paper in Sec. IV. In the Appendix, we present a rough sketch of the derivation of the CP phases in the PMNS matrix in the model.

II. SCOTOGENIC MODEL

The scotogenic model [5] is a simple extension of the SM with three right-handed neutrinos N_j and an inert doublet scalar η. These new ingredients are assumed to have an odd parity under imposed Z_2 symmetry although all the SM contents have even parity. Relevant parts to these new ingredients in Lagrangian are given as

$$\mathcal{L} \supset \sum_{j=1}^{3}\left[\sum_{i=1}^{3} h^{\nu}_{ij}\bar{\ell}_{L_i}\eta N_j + M_{N_j}\bar{N}_j N_j^c + \text{H.c.}\right] + V, \qquad (1)$$

$$V = m_\phi^2 \phi^\dagger\phi + m_\eta^2 \eta^\dagger\eta + \lambda_1(\phi^\dagger\phi)^2 + \lambda_2(\eta^\dagger\eta)^2 + \lambda_3(\phi^\dagger\phi)(\eta^\dagger\eta) + \lambda_4(\phi^\dagger\eta)(\eta^\dagger\phi) + \left[\frac{\lambda_5}{2}(\eta^\dagger\phi)^2 + \text{H.c.}\right], \qquad (2)$$

where ℓ_{L_i} and ϕ stand for SM doublet leptons and a doublet Higgs scalar, respectively. The model can have a stable vacuum only if the potential V in Eq. (2) satisfies the condition,

$$\lambda_1, \lambda_2 > 0, \qquad \lambda_+ > -2\sqrt{\lambda_1\lambda_2}, \qquad (3)$$

where λ_+ is defined as $\lambda_+ = \lambda_3 + \lambda_4 + \lambda_5$.

Since η is assumed to have no vacuum expectation value (VEV), neutrino mass is forbidden at tree level but it can be generated by a VEV of the SM Higgs scalar through a one-loop diagram whose internal lines are composed of N_j and η. Additionally, since η gets no VEV, the imposed Z_2 symmetry remains as an exact symmetry and it guarantees the stability of the lightest Z_2 odd particle, which could be a cold DM candidate as long as it is neutral. If $\lambda_4 < 0$ is satisfied, a lightest neutral component of η can be such a candidate. In the following study, $\lambda_5 < 0$ is assumed and then its real part η_R^0 is supposed to be DM. Its mass $m_{\eta_R^0}$ has to satisfy $m_{\eta_R^0} < M_{N_1}$ where M_{N_1} is mass of the lightest right-handed neutrino N_1. In that case, N_1 can decay to the lepton through the Yukawa coupling in Eq. (1). This decay could generate lepton number asymmetry, which can be transformed to baryon number asymmetry through a sphaleron process [12,13]. It should be also noted that η is also produced in this decay. Thus, the lightest right-handed neutrinos could be a common mother particle of both the baryon number asymmetry and the DM in this model.

The neutrino mass formula derived from the one-loop diagram is given as

$$\mathcal{M}_{\nu_{ij}} = \sum_{k=1}^{3} h^{\nu}_{ik}h^{\nu}_{jk}\Lambda_k,$$
$$\Lambda_k = \frac{\lambda_5\langle\phi\rangle^2}{8\pi^2 M_{N_k}}\left[\frac{M_{N_k}^2}{M_\eta^2 - M_{N_k}^2}\left(1 + \frac{M_{N_k}^2}{M_\eta^2 - M_{N_k}^2}\ln\frac{M_{N_k}^2}{M_\eta^2}\right)\right], \qquad (4)$$

where $M_\eta^2 = m_\eta^2 + (\lambda_3+\lambda_4)\langle\phi\rangle^2$. This formula can explain the small neutrino mass required by neutrino oscillation data [2] even for N_j with the mass of TeV scales as long as $|\lambda_5|$ takes a sufficiently small value.[1] Moreover, if we note that only two right-handed neutrinos are necessary to derive two mass differences required to explain the neutrino oscillation data, we find that the N_1 could be irrelevant to the small neutrino mass generation. This suggests that the neutrino Yukawa coupling h^{ν}_{i1} can take a much smaller value than others h^{ν}_{ij} $(j = 2, 3)$, which should be fixed so as to satisfy the neutrino oscillation data through Eq. (4).

Neutrino oscillation data suggest that the PMNS matrix which characterizes lepton flavor mixing could be described approximately through tribimaximal mixing [14]. Although it cannot cause nonzero mixing angle θ_{13}, it can be modified by a mixing matrix for charged leptons even if the tribimaximal mixing matrix is assumed for the neutrino sector.[2] Tribimaximal mixing in the neutrino sector can be easily realized simply by assuming the neutrino Yukawa couplings in Eq. (1) to satisfy [16],

$$h_{ek} = 0, \qquad h_{\mu k} = h_{\tau k} = h_k \quad (k = 1, 2);$$
$$h_{e3} = h_{\mu 3} = -h_{\tau 3} = h_3. \qquad (5)$$

Using this assumption, we can present examples which explain the neutrino oscillation data well. Here, we take h_1 to be sufficiently small like $O(10^{-8})$ so that N_1 is irrelevant to the neutrino mass determination as mentioned above.

[1] A lower bound on $|\lambda_5|$ can be derived through a possible inelastic scattering of η_R^0 with a nucleon in DM direct search experiments [10].

[2] A concrete example of it is presented in [15] and also in the Appendix of this article. In the following study, parameters given there are used.

Taking account of this, if we apply to Eq. (4) the parameters,[3]

$$\text{(a)}\ \lambda_5 = -10^{-5}, \qquad m_{\eta_R^0} = 2\times 10^3\ \text{GeV},$$
$$M_{N_2} = 6\times 10^4\ \text{GeV}, \qquad M_{N_3} = 7\times 10^4\ \text{GeV},$$
$$\text{(b)}\ \lambda_5 = -10^{-5}, \qquad m_{\eta_R^0} = 2\times 10^3\ \text{GeV},$$
$$M_{N_2} = 6\times 10^3\ \text{GeV}, \qquad M_{N_3} = 7\times 10^3\ \text{GeV}, \tag{6}$$

the mass differences required to explain the neutrino oscillation data are found to be realized for the couplings

$$\text{(a)}\ h_2 = 1.16\times 10^{-2}, \qquad h_3 = 4.15\times 10^{-3},$$
$$\text{(b)}\ h_2 = 6.87\times 10^{-3}, \qquad h_3 = 2.37\times 10^{-3}. \tag{7}$$

We use them in the following study.

III. LEPTOGENESIS AND DARK MATTER ABUNDANCE

A. Low-scale leptogenesis

This model makes leptogenesis [13] work well to generate the baryon number asymmetry in the same way as the type-I seesaw model [17]. Sufficient lepton number asymmetry could be produced as a seed of the baryon number asymmetry through the out-of-equilibrium decay of N_1 [10]. Moreover, it could happen at much lower scales than the type-I seesaw model if N_1 is in the thermal equilibrium [18]. However, it is difficult to make it in the thermal equilibrium only by the neutrino Yukawa coupling h^ν_{i1} in Eq. (1) in a consistent way with successful leptogenesis as suggested in [10]. To overcome this difficulty, interactions, which can produce a sufficient amount of N_1 to reach its equilibrium value, are proposed in some extended frameworks of the scotogenic model in connection with various problems of the SM, for example, inflation [19,20], right-handed neutrino mass [21,22], and CP violation [15,23]. For a while, we assume that N_1 is in the thermal equilibrium through certain interactions. Study of the coincidence problem based on a concrete interaction is given later.

The CP asymmetry ε in the decay $N_1 \to \ell_{L_i}\eta^\dagger$ is expressed as [24]

$$\varepsilon \equiv \frac{\sum_i[\Gamma(N_1\to \ell_{L_i}\eta^\dagger) - \Gamma(N_1^c \to \bar{\ell}_{L_i}\eta)]}{\sum_i \Gamma(N_1\to \ell_{L_i}\eta^\dagger)}$$
$$= \frac{1}{8\pi}\sum_{j=2,3}\text{Im}\left[\frac{(\sum_i h^\nu_{i1}h^\nu_{ij})^2}{\sum_i h^{\nu 2}_{i1}}\right] F\left(\frac{M^2_{N_j}}{M^2_{N_1}}\right), \tag{8}$$

where $F(x) = \sqrt{x}[1-(1+x)\ln\frac{1+x}{x}]$. Since the dependence on h^ν_{i1} in this asymmetry ε can be suppressed under a suitable flavor structure, h^ν_{i1} can be assigned a much smaller value compared with h^ν_{i2} and h^ν_{i3} keeping a value of ε to be a magnitude required for successful leptogenesis. On the other hand, a small value of h^ν_{i1} makes the decay of N_1 delay so that the washout of generated lepton number asymmetry could be ineffective when its substantial decay starts. These could make successful leptogenesis possible for the N_1 with the TeV scale mass in the scotogenic model [15,18,20,22,23].

The decay of N_1 is expected to start around the temperature T_L, which satisfies a condition $\Gamma^D_{N_1} = H(T_L)$, where $\Gamma^D_{N_1}$ and $H(T)$ are the decay width of N_1 and the Hubble parameter at the temperature T, respectively. They are expressed as

$$\Gamma^D_{N_1} = \sum_{i=1}^{3}\frac{h^{\nu 2}_{i1}}{8\pi}M_{N_1}\sqrt{1-\frac{M^2_\eta}{M^2_{N_1}}}, \quad H(T) = \left(\frac{\pi^2}{90}g_*\right)^{1/2}\frac{T^2}{M_{\rm pl}}, \tag{9}$$

where g_* is relativistic degrees of freedom at T and $M_{\rm pl}$ is a reduced Planck mass. If we use the tribimaximal assumption given in Eq. (5) for the neutrino Yukawa couplings, the temperature T_L can be estimated as

$$\frac{T_L}{M_{N_1}} = 5.1\times\left(\frac{h_1}{10^{-8}}\right)\left(\frac{10^4\ \text{GeV}}{M_{N_1}}\right)^{1/2}, \tag{10}$$

where $g_* = 116$ is used.

Here, we define Y_i as $Y_i \equiv \frac{n_i}{s}$ by using the number density n_i of a particle species i and the entropy density s. Lepton number asymmetry is expressed by $Y_L \equiv \frac{n_\ell - n_{\bar\ell}}{s}$, where n_ℓ and $n_{\bar\ell}$ are the number density of leptons and antileptons, respectively. If washout processes of the lepton number asymmetry Y_L decouple at a temperature T_F and $T_L < T_F$ is satisfied, the Y_L generated through the N_1 decay is not affected by the washout effect which is caused by the inverse decay of the right-handed neutrinos and 2-2 scatterings mediated by $N_{2,3}$.[4] In such a case, the lepton number asymmetry at T_L can be roughly estimated by using ε in Eq. (8) as $Y_L = \varepsilon Y^{\rm eq}_{N_1}(T_L)$ where $Y^{\rm eq}_{N_1}(T_L)$ takes the equilibrium number density of the relativistic particle as $Y^{\rm eq}_{N_1}(T_L) = O(10^{-3})$. Since $T_L < M_{N_1}$ has to be satisfied at least, Eq. (10) shows

$$h_1 < 6.1\times 10^{-7}\left(\frac{M_{N_1}}{10^4\ \text{GeV}}\right)^{1/2}. \tag{11}$$

[3] We note that $m_{\eta_R^0} \simeq M_\eta$ is satisfied for this small $|\lambda_5|$.

[4] If the lepton asymmetry is generated at the temperature lower than the η mass as a result of a small h_1, it can escape the washout by these processes since they could be sufficiently suppressed by the Boltzmann factor. It is confirmed through the numerical study shown in Fig. 3 which shows that T_L and T_F satisfy $T_L < T_F \lesssim m_{\eta_R^0}/10$ as expected.

The required baryon number asymmetry can be generated for $Y_L = O(10^{-10})$ and then ε should have a value of $O(10^{-7})$. Equation (8) shows that such an ε is consistent with the values of $h_{2,3}$ given in Eq. (7) as long as a maximal CP violation is assumed.

The generated lepton number asymmetry is converted to the baryon number asymmetry through the sphaleron process which is considered to be in the thermal equilibrium at temperature higher than 100 GeV [12]. Since the lepton number violating N_1 decay has to occur at a higher temperature than it, this imposes that $\Gamma^D_{N_1} > H(T)$ should be satisfied at $T = 100$ GeV. This condition can be expressed as

$$h_1 > 6.1 \times 10^{-9}\left(\frac{10^4\ \mathrm{GeV}}{M_{N_1}}\right)^{1/2}. \tag{12}$$

If we impose that leptogenesis occurs successfully at TeV regions, we find from Eqs. (11) and (12) that h_1 should take a value of $O(10^{-8})$ for M_{N_1} which is larger than $m_{\eta_R^0}$ assumed in Eq. (6).

To examine these qualitative arguments, we solve the Boltzmann equations for Y_L and Y_{N_1} given in [10] numerically by assuming that N_1 is initially in the thermal equilibrium and using the parameters given in Eqs. (6) and (7). The results for both cases (a) and (b) are given in the left and right panels of Fig. 1, respectively. Both panels show that the N_1 decay delays and Y_{N_1} keeps its relativistic value until the temperature T reaches the value which satisfies $T \ll M_{N_1}$. Since Y_L is found to be realized as $\varepsilon Y^{\rm eq}_{N_1}(T_L)$ from it, the above discussion can be justified quantitatively. The sufficient lepton number is found to be produced in both cases.

B. DM abundance

We consider the η_R^0 abundance as DM in this model. The relic abundance of η_R^0 is determined by solving Boltzmann equation for it. If we define a dimensionless parameter x as $x \equiv m_{\eta_R^0}/T$, Boltzmann equations for $Y_{\eta_R^0}$ and Y_{N_1} can be written as

$$\frac{dY_{\eta_R^0}}{dx} = -\frac{s(m_{\eta_R^0})}{H(m_{\eta_R^0})x^2}\langle\sigma v\rangle\left(Y^2_{\eta_R^0} - Y^{\rm eq^2}_{\eta_R^0}\right) + \frac{2x}{H(m_{\eta_R^0})}\Gamma^D_{N_1}\left(Y_{N_1} - Y^{\rm eq}_{N_1}\right), \tag{13}$$

$$\frac{dY_{N_1}}{dx} = -\frac{x}{H(m_{\eta_R^0})}\Gamma^D_{N_1}\left(Y_{N_1} - Y^{\rm eq}_{N_1}\right), \tag{14}$$

where $H(m_{\eta_R^0})$ and $s(m_{\eta_R^0})$ represent the Hubble parameter and the entropy density at $T = m_{\eta_R^0}$, respectively. $\langle\sigma v\rangle$ is a thermally averaged annihilation cross section of η_R^0 and $\Gamma^D_{N_1}$ is the decay width of N_1 given in Eq. (9). We take account of an effect of the N_1 decay to η as the second term in the right-hand side of Eq. (13). Here, we define $Y^{(n)}_{\eta_R^0}$ and $Y^{(o)}_{\eta_R^0}$ as the solution of Eq. (13) with the second term and the one without it, respectively. Since this term can be safely neglected for the case $M_{N_1} \gg m_{\eta_R^0}$, we can expect $Y^{(n)}_{\eta_R^0} = Y^{(o)}_{\eta_R^0}$ there and DM is considered as thermal relic. We consider such a case first.

In that case, the present abundance of η_R^0 is fixed through its equilibrium density at temperature T_D where its annihilation processes are frozen out [25,26]. We can estimate T_D by using its thermally averaged annihilation cross

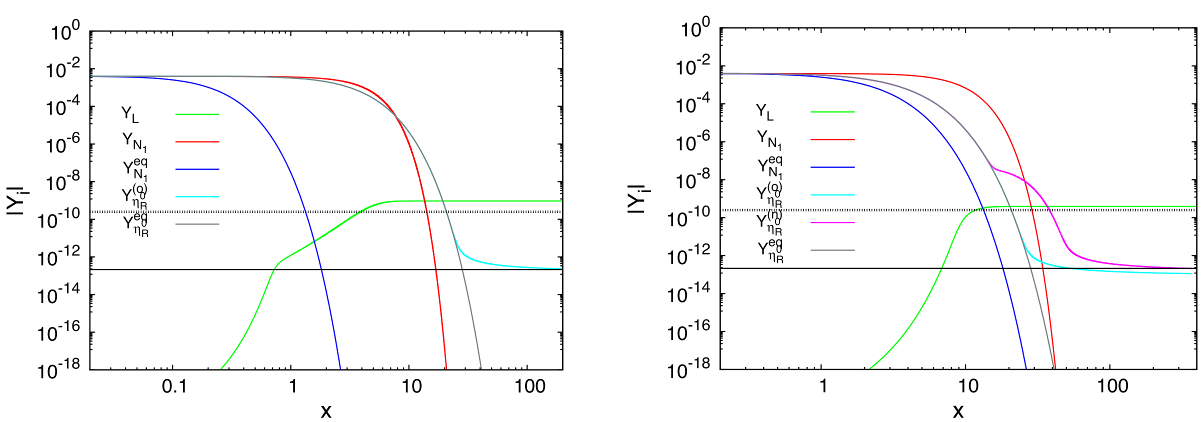

FIG. 1. Evolution of the lepton number asymmetry Y_L, the lightest right-handed neutrino number density Y_{N_1} and the η_R^0 number density $Y_{\eta_R^0}$ is displayed as a function of a dimensionless parameter $x(\equiv m_{\eta_R^0}/T)$. N_1 contribution is not taken into account in $Y^{(o)}_{\eta_R^0}$ but it is taken into account in $Y^{(n)}_{\eta_R^0}$. Y_i with superscript "eq" stands for its equilibrium value. In the left panel for the case (a), $h_1 = 2.0 \times 10^{-8}$ and $M_{N_1} = 3.1 \times 10^4$ GeV are used. In the right panel for the case (b), $h_1 = 5.4 \times 10^{-8}$ and $M_{N_1} = 3.1 \times 10^3$ GeV are used. Horizontal lines represent values of Y_L (dotted) and $Y_{\eta_R^0}$ (solid) which are required to explain the abundance of the baryon number asymmetry and the DM abundance in the present universe, respectively.

section $\langle\sigma v\rangle$ and Hubble parameter $H(T)$ through a condition $2n^{\rm eq}_{\eta_R^0}(T_D)\langle\sigma v\rangle = H(T_D)x_D$ as found from Eq. (13). Using $H(T)$ given in Eq. (9), x_D can be determined by solving the above mentioned condition as

$$x_D = \ln\frac{0.384 g M_{\rm pl}\langle\sigma v\rangle m_{\eta_R^0}}{(g_* x_D)^{1/2}}, \qquad (15)$$

where g is an internal degree of freedom of η_R^0. Since $Y_{\eta_R^0}$ converges to a constant value $Y^\infty_{\eta_R^0}$ at $x > x_D$, the present DM abundance can be expressed as $\Omega_{\eta_R^0} = m_{\eta_R^0}Y^\infty_{\eta_R^0}s_0/\rho_0$ where ρ_0 and s_0 are the energy density and the entropy density in the present Universe. They are given as $\rho_0 = 3M_{\rm pl}^2H_0^2$ and $s_0 = 2.27\times10^{-38}$ GeV^3. If we solve Eq. (13) by taking account of $Y_{\eta_R^0} > Y^{\rm eq}_{\eta_R^0}$ at $x \gg x_D$ and also $Y^\infty_{\eta_R^0} < Y_{\eta_R^0}(x_D)$, we find that $\Omega_{\eta_R^0}$ can be approximately expressed as

$$\Omega_{\eta_R^0}h^2 = \frac{Y^\infty_{\eta_R^0} m_{\eta_R^0} s_0}{3M_{\rm pl}^2H_0^2/h^2} = \frac{2.13\times10^8\ {\rm GeV}}{\sqrt{g_*}M_{\rm pl}\int_{x_D}^\infty \frac{\langle\sigma v\rangle}{x^2}dx}, \qquad (16)$$

where we use $H_0 = 2.13\times10^{-42}h$ GeV. Applying it to the present observational result $\Omega_{\rm DM}h^2 = 0.12$, we find that $Y^\infty_{\eta_R^0} = 2.13\times10^{-13}(2000\ {\rm GeV}/m_{\eta_R^0})$ should be satisfied. It constrains the parameters λ_+ and λ_3 in the potential (2) which determine the annihilation cross section $\langle\sigma v\rangle$. Since the masses of the components of η are almost degenerate, coannihilation [26] among them should be taken into account to estimate $\langle\sigma v\rangle$ [8,10].

In Fig. 2, we draw contours of $\Omega_{\eta_R^0}h^2 = 0.12$ for typical values of $m_{\eta_R^0}$ in the (λ_+, λ_3) plane. In this plane, we have to take account of both the stability condition given in Eq. (3) and $\lambda_4 = \lambda_+ - \lambda_3 - \lambda_5 < 0$ which is a necessary condition for η_R^0 to be lighter than the charged components. Combining them, we find that only points on the contours included in a triangle region of the upper-right quadrant in the (λ_+, λ_3) plane are allowed.

On the other hand, the same parameters are also constrained by the present results of DM direct search experiments. In this model, η_R^0-nucleon elastic scattering is caused by the t-channel Higgs exchange. Its cross section can be expressed as

$$\sigma_N = \frac{\lambda_+^2}{8\pi}\frac{\bar f_N^2}{m^2_{\eta_R^0}}\frac{m_N^4}{m_h^4}, \qquad (17)$$

where $\bar f_N$ is a coupling between the Higgs scalar and a nucleon. Masses of the nucleon and the Higgs boson are represented by m_N and m_h, respectively. A present most stringent bound on σ_N through the direct search experiment

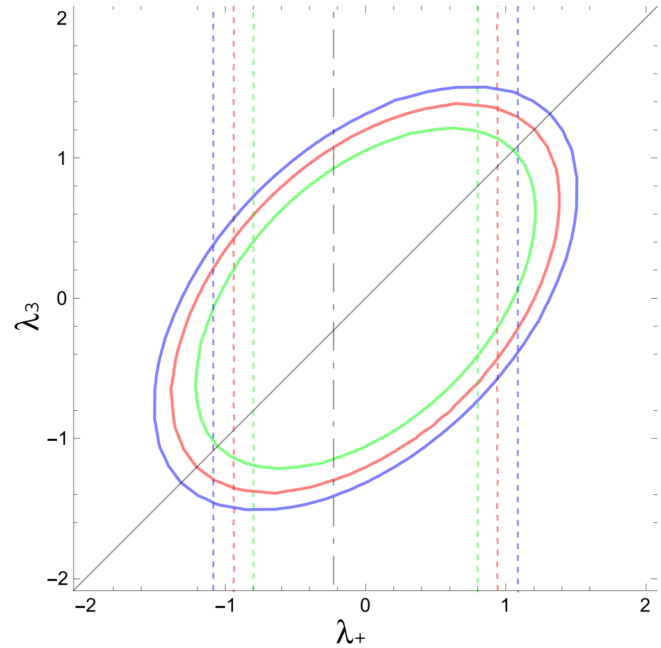

FIG. 2. Contours of the η_R^0 relic density $\Omega_{\eta_R^0}h^2 = 0.12$ are plotted by colored solid lines in the (λ_+, λ_3) plane. Each contour corresponds to the one with $m_{\eta_R^0} = 2200$ GeV (blue), 2000 GeV (red), and 1800 GeV (green), respectively. An upper bound on $|\lambda_+|$ based on the direct search of XENONnT is represented by vertical dashed lines with the same color as the one of the relic density for each mass. A diagonal black solid line and a vertical black dash-dotted line represent the condition $\lambda_4 = 0$ and $\lambda_+ = -2\sqrt{\lambda_1\lambda_2}$ with $\lambda_2 = 0.1$, respectively. The allowed region is restricted into an upper-triangle region surrounded by these lines.

is given by the XENONnT experiment [27]. Since it gives an upper bound on $|\lambda_+|$, as found from Eq. (17), a region included in a band sandwiched with dotted vertical lines fixed for each $m_{\eta_R^0}$ is allowed. These suggest that only restricted points in the (λ_+, λ_3) plane can be consistent with the present data for the DM abundance. Future direct detection experiments and collider experiments might find a η_R^0 signature in this parameter region.

C. Coincidence of DM and baryon number asymmetry

We reconsider realization of the DM abundance in relation to the baryon number asymmetry generated through leptogenesis. Since the required relic abundance is known to be realized for $x_D \sim 25$ in the WIMP scenario, $T_D < 100\ {\rm GeV} < T_L$ can be satisfied for η_R^0 whose mass is less than 3 TeV. If $M_{N_1} \gg m_{\eta_R^0}$ is satisfied, $Y^{\rm eq}_{\eta_R^0}(x_D) \gg Y^{\rm eq}_{N_1}(x_D)$ is expected and then $Y^{(n)}_{\eta_R^0} = Y^{(o)}_{\eta_R^0}$ is guaranteed. So that the estimation of the relic abundance of η_R^0 in the previous section can be justified.

The solution of the Boltzmann equation for $Y_{\eta_R^0}$ added in the left panel of Fig. 1 proves it. In this calculation, λ_+ and

λ_3 are fixed to 0.66 and 1.56, which are the consistent values with the contour for $m_{\eta_R^0} = 2$ TeV shown in Fig. 2. There is no contribution to $Y_{\eta_R^0}$ from the N_1 decay and then $Y^{(o)}_{\eta_R^0}$ is found to be equal to $Y^{(n)}_{\eta_R^0}$. The annihilation processes of η_R^0 are frozen out at $x_D \sim 26$ where Y_{N_1} becomes much smaller than $Y^{\rm eq}_{\eta_R^0}$. After that, $Y_{\eta_R^0}$ converges to a constant value which gives $\Omega_{\eta_R^0} h^2 = 0.12$. It suggests that $Y_{\eta_R^0}$ is determined irrelevantly from the determination of Y_L. It is generally expected for the case where N_1 is much heavier than η_R^0.

Next, we consider a case where $Y^{\infty}_{\eta_R^0}$ is determined through a different way from the above case. It can happen in a situation where h_1 takes a small value and also N_1 has the similar mass to η_R^0. The condition for h_1 required by the leptogenesis is given by Eqs. (11) and (12). If the Yukawa coupling h_1 takes a value such as $H(T_D) \gtrsim \Gamma^D_{N_1}$, the N_1 abundance could be larger than $Y^{\rm eq}_{\eta_R^0}(x_D)$ which saturates the required DM abundance. To escape such a disastrous situation, h_1 has to satisfy,

$$h_1 > 4.9 \times 10^{-9} \left(\frac{10^4 \text{ GeV}}{M_{N_1}} \right)^{1/2}. \tag{18}$$

This is consistent with Eq. (12). If these conditions are satisfied and $Y^{\rm eq}_{\eta_R^0}(x_D)$ is smaller than the value required by the DM abundance, the decay of N_1 could give a crucial contribution to the relic density of η_R^0 since $Y_{N_1}(x_D) \gtrsim Y^{\rm eq}_{\eta_R^0}(x_D)$ could occur.

We can estimate $Y_{N_1}(x_D)$ by solving Eq. (14) as

$$Y_{N_1}(x_D) = Y_{N_1}(x_i) \exp\left[-\frac{\Gamma^D_{N_1}}{2H(m_{\eta_R^0})} (x_D^2 - x_i^2) \right]. \tag{19}$$

Since $Y_{N_1}(x_i)$ is of $O(10^{-3})$ at $T_i = M_{N_1}$ which stands for $x_i = m_{\eta_r^0}/M_{N_1} (< 1)$, the exponential factor in Eq. (19) can be rewritten as

$$\exp\left[-\frac{\Gamma^D_{N_1}}{2H(m_{\eta_R^0})} x_D^2 \right] = \exp\left[-4.3 \times 10^{12} \left(\frac{x_D}{25} \right)^2 h_1^2 M_{N_1} \sqrt{1 - \left(\frac{m_{\eta_R^0}}{M_{N_1}} \right)^2} \right], \tag{20}$$

where we use $m_{\eta_R^0} = 2000$ GeV. The required DM abundance can be realized for $Y_{\eta_R^0}(x_D) = O(10^{-12})$ as found from the left panel of Fig. 1. If we impose $Y_{N_1}(x_D) = O(10^{-12})$, Eqs. (19) and (20) suggest that the η_R^0 produced by the N_1 decay at $x > x_D$ could supply a substantial part of the DM abundance for

$$h_1^2 M_{N_1} \sqrt{1 - \left(\frac{2000}{M_{N_1}} \right)^2} = O(10^{-12}). \tag{21}$$

This relation is consistent with conditions (11) and (12).

If h_1 and M_{N_1} take values following the condition (21), $Y_{N_1}(x_D) \gtrsim Y_{\eta_R^0}(x_D)$ can be realized although N_1 is heavier than η_R^0. The N_1 decay which causes the lepton number asymmetry could also generate a dominant part of the relic η_R^0 for parameters λ_+ and λ_3 which are ruled out in the ordinary estimation of the relic η_R^0. The present abundance of η_R^0 should be estimated based on

$$Y_{\eta_R^0}(x_D) = Y^{\rm eq}_{\eta_R^0}(x_D) + 2Y_{N_1}(x_D), \tag{22}$$

since all components of η produced through the N_1 decay finally come to η_R^0. As the Boltzmann equation for $Y_{\eta_R^0}$, we have to use Eq. (13) which takes account of the N_1 decay to η.

To examine the above observation quantitatively, we solve Boltzmann equations (13) and (14) numerically assuming that N_1 is initially in the thermal equilibrium. Although the mass difference between η_R^0 and N_1 is small, their coannihilation can be neglected in $\langle \sigma v \rangle$ because their coupling h_1 is small enough. In the right panel of Fig. 1, the evolution of both $Y^{(n)}_{\eta_R^0}$ and $Y^{(o)}_{\eta_R^0}$ obtained as the solutions of the Boltzmann equations in the case (b) is plotted. In this calculation, λ_+ and λ_3 are fixed to 0.66 and 2.14, respectively. These make $Y^{(o)}_{\eta_R^0}$ coming from the thermal η_R^0 smaller than the required value $\Omega_{\eta_R^0} h^2 = 0.12$. It occupies about 50% of the total and a remaining part is supplied through the N_1 decay. The figure shows that both the sufficient baryon number asymmetry and the DM abundance are simultaneously realized through the N_1 decay.

Comparison of both panels in Fig. 1 clarifies features of the present case. In the left panel which corresponds to the usually supposed case, the η_R^0 abundance is realized by the freeze-out of the thermal η_R^0 and the yields from the N_1 decay is irrelevant. The baryon number asymmetry and the DM abundance is explained based on irrelevant physics. On the other hand, we can find that the N_1 decay plays a crucial role to determine the η_R^0 abundance in the right panel. In this example, the contribution from the thermal η_R^0 is only 50% and the remaining one is caused by the N_1 decay. It suggests that the η_R^0 yielded through the N_1 decay could supply the substantial part of the η_R^0 abundance. The coincidence of the baryon number density and the dark matter density in the present Universe could be explained naturally there since common parameters relevant to the N_1 decay control them simultaneously.

D. A feasible model for the scenario

Finally, in order to show that this scenario works in a realistic way, we adopt a well motivated model with possible interactions which can generate N_1 in the thermal equilibrium even if the Yukawa coupling h_1 is too small to produce it effectively. The model has been proposed to give a prospect to the CP issues in the SM [15]. It can solve the strong CP problem [28] through the Nelson-Barr mechanism [29,30] and give the origin of CP phases in the PMNS and CKM matrices [11,31].

This model is an extension of the scotogenic model with a singlet scalar S and vectorlike fermions $D_{L,R}$ and $E_{L,R}$, where $D_{L,R}$ and $E_{L,R}$ are down-type singlet quarks and charged leptons, respectively. Their Yukawa couplings are given as

$$\sum_{k=1}^{3}[(y_k^d S+\tilde{y}_k^d S^\dagger)\bar{D}_L d_{R_k}+(y_k^e S+\tilde{y}_k^e S^\dagger)\bar{E}_L e_{R_k} + (y_{N_k} S+\tilde{y}_{N_k} S^\dagger)\bar{N}_k N_k^c]+(y_E S+\tilde{y}_E S^\dagger)\bar{E}_L E_R+\text{H.c.}, \tag{23}$$

where d_{R_k} and e_{R_k} are the singlet down-type quark and the singlet charged lepton in the SM, respectively. Since CP invariance is assumed in the model, all coupling constants in Eq. (23) are considered to be real. If the singlet scalar S gets a VEV as $\langle S\rangle=\frac{1}{\sqrt{2}}ue^{i\rho_0}$, spontaneous CP violation is caused. Complex phases due to this CP violation could be brought about in the PMNS and CKM matrices through the mixing between the SM fermions and vectorlike fermions which is caused by these interactions.[5] The last term in the first line of (23) induces the mass of the right-handed neutrino N_k. If we redefine N_k to make its mass real, M_{N_k} and Λ_k in Eq. (4) can be expressed as

$$M_{N_k}=(y_{N_k}^2+\tilde{y}_{N_k}^2+2y_{N_k}\tilde{y}_{N_k}\cos 2\rho_0)^{1/2}u,$$
$$\Lambda_k=|\lambda_k|e^{i\theta_k}, \qquad \tan\theta_k=\frac{y_{N_k}-\tilde{y}_{N_k}}{y_{N_k}+\tilde{y}_{N_k}}\tan\rho_0. \tag{24}$$

This θ_k determines the CP violation in the N_1 decay and then it fixes the CP asymmetry ε given in Eq. (8).

In the context of this paper, we should especially note that these interactions can cause scatterings $\bar{D}_L d_{R_k}\to\bar{N}_1 N_1^c$, $\bar{E}_L e_{R_k}\to\bar{N}_1 N_1^c$, $\bar{E}_L E_R\to\bar{N}_1 N_1^c$, and $\bar{N}_k N_k^c\to\bar{N}_1 N_1^c$ through the exchange of S. If the mass of the vectorlike fermions is smaller than the reheating temperature, they could be in the thermal equilibrium through the SM gauge interactions. Heavier right-handed neutrinos $N_{2,3}$ could also be in the thermal equilibrium effectively through their neutrino Yukawa interactions at the temperature less than $O(10^8)$ GeV if their couplings take the values given in Eq. (7) [21,22]. In such a case, these processes can produce N_1 effectively to reach its equilibrium density. Thus, if its mass M_{N_1} takes a similar value to $m_{\eta_R^0}$ as in the case (b) shown in the right panel of Fig. 1, we can expect that the N_1 decay generates the lepton number asymmetry sufficiently and it also contributes to the relic abundance of η_R^0 substantially.[6]

For its quantitative check, we solve the Boltzmann equations for Y_{N_1}, $Y_{\eta_R^0}$, and Y_L by taking account of these scattering processes. Equation (14) should be modified by introducing the right-hand side additional terms,

$$-\frac{s(m_{\eta_R^0})}{H(m_{\eta_R^0})x^2}\sum_\alpha\langle\sigma v\rangle_\alpha(Y_{N_1}^2-Y_{N_1}^{\mathrm{eq}2}), \tag{25}$$

where the suffix α describes the above processes. We take $Y_{N_1}=0$ as an initial value of Y_{N_1}. Couplings in Eq. (23) and the mass of the vectorlike fermions, which are crucial for the determination of the PMNS matrix, are fixed to the ones used in [15]. They are presented in the Appendix. We assume $u=10^6$ GeV and $\rho_0=\frac{\pi}{4}$ as a VEV of the singlet scalar S and then these couplings fix mass eigenvalues of the fourth charged lepton and $N_{2,3}$ as $M_E=3165$ GeV and $M_{N_{2,3}}$ given in Eq. (6), respectively. It is noticeable that these can realize the present experimental results for the PMNS matrix well through a framework presented in the Appendix as described in [15]. We also note that the CP violation, which fixes the CP asymmetry ε, can take a maximal value.

The results of this calculation are given in Fig. 3. It shows that N_1 reaches its equilibrium number density through the introduced interactions and its decay produces both the sufficient lepton number asymmetry and a substantial part of the required relic η_R^0 abundance as in the same way as the right panel of Fig. 1. In this example, the η_R^0 yielded through the N_1 decay occupies about 70% of the required abundance $\Omega_{\eta_R^0}h^2=0.12$. The remaining part is supplied from η_R^0 in the thermal equilibrium by setting the relevant parameters as $\lambda_+=0.66$ and $\lambda_3=2.6$. Since the N_1 decay starts at a larger x due to a smaller value of h_1 compared with the right panel of Fig. 1, its contribution to the relic η_R^0 becomes larger.

Finally, we examine how the relative share of η_R^0 produced from the N_1 decay in the total relic depends

[5]A rough sketch of this scenario is given in the Appendix.

[6]We should note that a scalar interaction $\kappa_{S\eta}S^\dagger S\eta^\dagger\eta$ is not forbidden in the model, which could give an additional source of η_R^0 through the scattering. However, since this process is effective at the temperature higher than $m_{\eta_R^0}$, it does not affect the present calculation of the DM abundance. It should be also noted that any change in the neutrino mass formula (4) is not caused by this interaction since the one-loop neutrino mass depends on the squared mass difference of η_R^0 and η_I^0 which is not changed by this interaction.

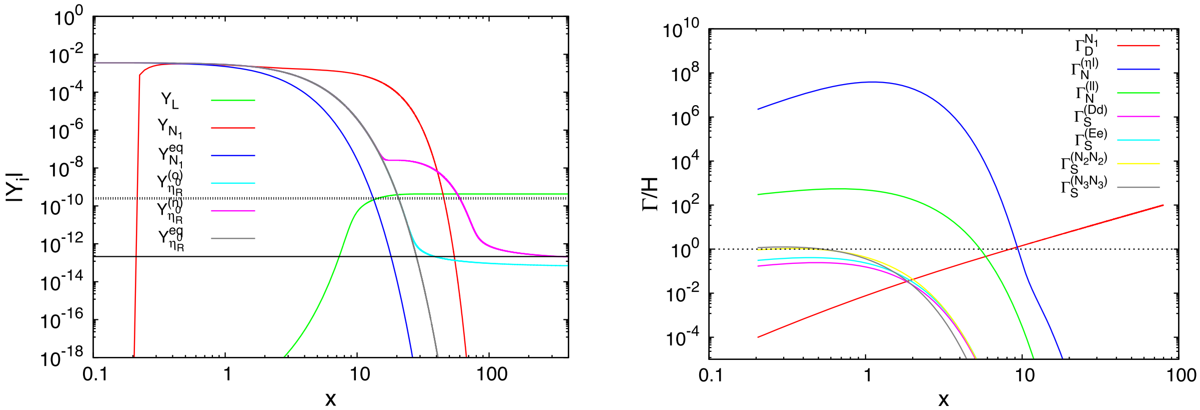

FIG. 3. Left: Evolution of the lepton number asymmetry Y_L and the η_R^0 number density $Y_{\eta_R^0}$. In the calculation, $h_1 = 3.4 \times 10^{-8}$ and $M_1 = 3.1 \times 10^3$ GeV for the case (b) are used. Horizontal lines represent the required values of Y_L (dotted) and $Y_{\eta_R^0}$ (solid) to explain the baryon number asymmetry and the DM abundance, respectively. Right: Evolution of the relevant reaction rates included in the Boltzmann equations. $\Gamma_f^{(ab)}$ stands for the reaction rate of the scattering $ab \to N_1 N_1$ mediated by f. The washout of the lepton number asymmetry Y_L is caused by $\Gamma_N^{(\eta\ell)}$ and $\Gamma_N^{(\ell\ell)}$.

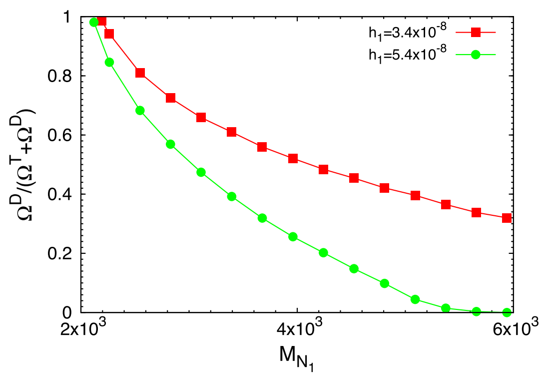

FIG. 4. The lightest right-handed neutrino mass M_{N_1} dependence of the ratio of the relic abundance of η_R^0 originated from the N_1 decay to the total relics which are composed of the thermal one Ω^T and Ω^D from the N_1 decay. The masses of $N_{2,3}$ and η_R^0 are fixed to the ones given as (b) in Eq. (6).

on the N_1 mass. Results are shown in Fig. 4 where the abundance of the thermal η_R^0 and the η_R^0 produced from the N_1 decay is respectively represented as Ω^T and Ω^D, and $(\Omega^T + \Omega^D)h^2 = 0.12$ is imposed. The scalar couplings $\lambda_{3,4}$ are fixed to realize this value. In this study, $m_{\eta_R^0}$ and $M_{N_{2,3}}$ are fixed to the ones given as (b) in Eq. (6), and then M_{N_1} is allowed in the range 2000 GeV $< M_{N_1} <$ 6000 GeV. We also choose two values of h_1 which are used in the right panel of Figs. 1 and 3. The figure shows that the relic η_R^0 can be entirely produced by the N_1 decay if N_1 takes a close value with the η_R^0 mass. In that case, λ_3 and $|\lambda_4|$ have to take large values near their perturbative bound since the η_R^0 abundance produced by the N_1 decay is large and its substantial part has to be reduced through the annihilation processes. We also find from the figure that $\Omega^D h^2$ can reach the order of the required DM abundance in a wide region of M_{N_1}. This result suggests that the coincidence between the baryon number density and the DM density in the present Universe can be recognized as a natural consequence of the model. If η_R^0 is discovered as the DM, the origin of the relic η_R^0 can be an interesting subject. In that case, the present study suggests that detailed experimental study of the scalar couplings $\lambda_{3,4}$ might tell us the ratio of Ω^D to Ω^T.

IV. SUMMARY

We study a new scenario for the DM abundance in the scotogenic model from a view point of the coincidence of the baryon number asymmetry and the DM abundance in the present Universe. In this model, the abundance of the inert doublet DM is usually considered to be explained as the thermal relic following the WIMP scenario. Since it is irrelevant to the baryon number asymmetry in that case, the model cannot give any answer for this problem. However, if we note that the decay of the lightest right-handed neutrino N_1 generates both the lepton number asymmetry and the DM candidate η_R^0, a correlation can be found between them in a certain situation such that N_1 takes the same order mass as η_R^0.

The mass of the right-handed neutrinos can take a TeV scale value consistently with the neutrino oscillation data in the model. Moreover, the decay of such N_1 is known to generate the sufficient baryon number asymmetry through leptogenesis if the N_1 is in the thermal equilibrium through certain interactions. These suggest that the relic η_R^0 as the DM could be supplied not only as the thermal relic but also as the yields of the decay of N_1 which also causes the lepton number asymmetry. If the latter gives a dominant part of the DM abundance, the baryon number asymmetry has a close relation to the DM abundance. The model could give an insight for the coincidence problem.

We examine this idea quantitatively by solving the Boltzmann equations under the assumption that N_1 is in the thermal equilibrium initially. The result shows that their coincidence can be explained well. We also propose a well-motivated model which contains the interactions to make N_1 reach the thermal equilibrium and show how the coincidence of the baryon number asymmetry and the DM abundance can happen in this extended model. An interesting point of the model is that these interactions can give an explanation for the *CP* issues in the SM.

If both the baryon number asymmetry and a substantial part of the DM are produced through the decay of a common mother particle, it could be a promising scenario to give an answer to the coincidence problem. In such a context, low-scale leptogenesis may provide an interesting possibility not only from a phenomenological viewpoint but also from a cosmological viewpoint. The extended model studied in this paper might be considered as a prototype model which can realize such a scenario naturally.

APPENDIX: DERIVATION OF THE *CP* PHASES IN THE PMNS MATRIX

In this Appendix, we briefly address how *CP* phases in the PMNS matrix can be induced through Yukawa interactions given in Eq. (23) following [15]. If the singlet scalar gets a VEV as $\langle S\rangle = \frac{1}{\sqrt{2}}ue^{i\rho_0}$, which causes spontaneous *CP* violation, the *CP* phase can appear in the PMNS matrix through the couplings of the singlet S with vectorlike charged leptons $E_{L,R}$. These Yukawa interactions extend the SM charged lepton mass matrix m^e_{ij} to a 4×4 mass matrix $\mathcal{M}_e$ such as

$$(\bar{\ell}_{L_i}, \bar{E}_L)\begin{pmatrix} m^e_{ij} & \mathcal{G}_i \\ \mathcal{F}^e_j & \mu_E \end{pmatrix}\begin{pmatrix} e_{R_j} \\ E_R \end{pmatrix}, \tag{A1}$$

where ℓ_{L_i} and e_{R_i} are the charged leptons in the SM. $\mathcal{F}^e_j$, $\mathcal{G}_i$, and μ_E are defined as $\mathcal{F}^e_j = \frac{1}{\sqrt{2}}(y^e_j e^{i\rho_0} + \tilde{y}^e_j e^{-i\rho_0})u$, $\mathcal{G}_i = x_i\langle\phi\rangle$ and $\mu_E = \frac{1}{\sqrt{2}}(y_E e^{i\rho_0} + \tilde{y}_E e^{-i\rho_0})u$.

Diagonalization of a matrix $\mathcal{M}_e\mathcal{M}_e^\dagger$ by a 4×4 unitary matrix $\tilde{V}_L$ is represented as

$$\begin{pmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{pmatrix}\begin{pmatrix} m^e m^{e\dagger} + \mathcal{G}\mathcal{G}^\dagger & m^e\mathcal{F}^{e\dagger} + \mu_E^*\mathcal{G} \\ \mathcal{F}^e m^{e\dagger} + \mathcal{G}^\dagger\mu_E & |\mu_E|^2 + \mathcal{F}^e\mathcal{F}^{e\dagger} \end{pmatrix} \times \begin{pmatrix} \tilde{A}^\dagger & \tilde{C}^\dagger \\ \tilde{B}^\dagger & \tilde{D}^\dagger \end{pmatrix} = \begin{pmatrix} \tilde{m}_e^2 & 0 \\ 0 & \tilde{M}_E^2 \end{pmatrix}, \tag{A2}$$

where a 3×3 matrix $\tilde{m}_e^2$ in the right-hand side is diagonal. Equation (A2) requires,

$$\begin{aligned} m^e m^{e\dagger} + \mathcal{G}\mathcal{G}^\dagger &= \tilde{A}^\dagger\tilde{m}_e^2\tilde{A} + \tilde{C}^\dagger\tilde{M}_E^2\tilde{C}, \\ \mathcal{F}^e m^{e\dagger} + \mathcal{G}^\dagger\mu_E &= \tilde{B}^\dagger\tilde{m}_e^2\tilde{A} + \tilde{D}^\dagger\tilde{M}_E^2\tilde{C}, \\ |\mu_E|^2 + \mathcal{F}^e\mathcal{F}^{e\dagger} &= \tilde{B}^\dagger\tilde{m}_e^2\tilde{B} + \tilde{D}^\dagger\tilde{M}_E^2\tilde{D}. \end{aligned} \tag{A3}$$

If $|\mu_E|^2 + \mathcal{F}^e\mathcal{F}^{e\dagger}$ is much larger than each component of $\mathcal{F}^e m^{e\dagger} + \mathcal{G}\mu_E^*$, we find that $\tilde{B}, \tilde{C}$, and $\tilde{D}$ can be approximately expressed as

$$\tilde{B} \simeq -\frac{\tilde{A}(m^e\mathcal{F}^{e\dagger} + \mu_E^*\mathcal{G})}{|\mu_E|^2 + \mathcal{F}^e\mathcal{F}^{e\dagger}}, \qquad \tilde{C} \simeq \frac{\mathcal{F}^e m^{e\dagger} + \mathcal{G}^\dagger\mu_E}{|\mu_E|^2 + \mathcal{F}^e\mathcal{F}^{e\dagger}}, \qquad \tilde{D} \simeq 1. \tag{A4}$$

These guarantee the approximate unitarity of the matrix $\tilde{A}$. In that case, it is also easy to find that

$$\tilde{A}^{-1}\tilde{m}^{e2}\tilde{A} = m^e m^{e\dagger} + \mathcal{G}\mathcal{G}^\dagger - \frac{1}{|\mu_E|^2 + \mathcal{F}^e\mathcal{F}^{e\dagger}}(m^e\mathcal{F}^{e\dagger} + \mu_E^*\mathcal{G})(\mathcal{F}^e m^{e\dagger} + \mu_E\mathcal{G}^\dagger). \tag{A5}$$

The charged lepton effective mass matrix $\tilde{m}_e$ is obtained as a result of the mixing between the light charged leptons and the extra heavy lepton. If $\tilde{y}^e_j$ is not equal to y^e_j and $|\mu_E|^2 < \mathcal{F}^e\mathcal{F}^{e\dagger}$ is satisfied, the matrix $\tilde{A}$ could have a large *CP* phase.

We assume that the neutrino mass matrix $\mathcal{M}_\nu$ is diagonalized by a tribimaximal matrix U_ν as $U_\nu^T\mathcal{M}_\nu U_\nu = \mathcal{M}_\nu^{\rm diag}$, where the matrix U_ν can be expressed as

$$U_\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{-i\alpha_2} \end{pmatrix}. \tag{A6}$$

Majorana phases α_1 and α_2 are written by using Eq. (24) as

$$\alpha_1 = \frac{\theta_3}{2}, \qquad \alpha_2 = \frac{1}{2}\tan^{-1}\left[\frac{h_1^2|\Lambda_1|\sin\theta_1 + h_2^2|\Lambda_2|\sin\theta_2}{h_1^2|\Lambda_1|\cos\theta_1 + h_2^2|\Lambda_2|\cos\theta_2}\right]. \tag{A7}$$

The PMNS matrix is obtained as $V_{\rm PMNS} = \tilde{A}^\dagger U_\nu$ where $\tilde{A}$ is fixed through Eq. (A5). Since the matrix $\tilde{A}$ is expected to be almost diagonal from hierarchical masses of the charged leptons, the structure of $V_{\rm PMNS}$ is considered to be mainly determined by U_ν in the neutrino sector. Although tribimaximal mixing cannot realize a nonzero mixing angle θ_{13} which is required by the neutrino oscillation data, the matrix $\tilde{A}$ could compensate this fault and a desirable $V_{\rm PMNS}$ may be derived as $V_{\rm PMNS} = \tilde{A}^\dagger U_\nu$. If we fix the relevant parameters in Eq. (23) as

$$y^e = (0, 3\times 10^{-3}, 0), \qquad \tilde{y}^e = (0, 0, 10^{-3}), \qquad y_E = \tilde{y}_E = 3.3\times 10^{-6},$$
$$y_N = (2.2\times 10^{-3}, 6\times 10^{-3}, 7\times 10^{-3}), \qquad \tilde{y}_N = (2.2\times 10^{-3}, 0, 0), \tag{A8}$$

$V_{\rm PMNS}$ obtained in this way is found to be rather good realization of the experimental results including nonzero θ_{13} as shown in [15].

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