

QCD sum rules for positive and negative parity heavy baryons at next-to-leading order in α_s -expansion

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QCD sum rules for positive and negative parity heavy baryons in the heavy quark limit are formulated. We apply the method to Λ and Σ channels. We include the next-to-leading order corrections in α_s -expansion to dimension 0 and 3 terms in the operator product expansion. The corrections lead to the considerable reduction of the predicted masses and significantly improves the stability with respect to the Borel parameter, especially for negative parity states. It is also found that, in the heavy quark limit, chiral odd condensates do not contribute to the negative parity states.

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I. INTRODUCTION

In the past decades, a remarkable experimental progress has been made in the field of heavy baryon physics. In fact, many excited states of singly charmed baryon have been observed in the 2000s. Excited singly bottom baryons have also been discovered in recent years one after another, although only two states were known until 2012 [1,2]. Those data have been investigated comprehensively from various theoretical perspective so far (see Ref. [3] and references therein). To study negative parity heavy baryons is especially important in the sense that they can be key subjects to clarify the mechanism of excitation in baryon systems.

QCD sum rule is one of the useful nonperturbative method based on QCD, which can connect the nontrivial vacuum condensates with the hadron properties in a model independent way. So far, by many authors QCD sum rule has been used to study not only ground (positive parity) [4–11] but also excited (negative parity) heavy baryons [3,12–21]. In many works, [7,11,13,16,19], the transverse γ -matrices were adopted to ensure that the interpolating fields with positive parity uniquely couple to the states with

positive parity. In Ref. [12], employing the transverse γ -matrices combined with or without a covariant derivative, the interpolating fields with negative parity are also ensured that they couple only to the states with negative parity. The authors in Refs. [3,14–21] adopted the p -wave heavy baryon interpolating fields with a covariant derivative systematically constructed from Bethe-Salpeter equation to study the masses and the decay properties of p -wave heavy baryons.

In this paper we employ a completely different approach that was originally proposed in Ref. [22]. We use the interpolating field of positive intrinsic parity without covariant derivatives. Noting the fact that the interpolating field of positive intrinsic parity couples not only to positive parity states but also to negative parity ones, we “project” the correlation function of the interpolating fields onto each parity [22], which enables us to construct the sum rules for respective parity states. Nucleons and hyperons were investigated within this method and the origin of the mass splitting between positive and negative parity was discussed in connection with chiral condensates [22–25]. Up to now, there exist no work applying properly this method to studying heavy baryons.

We consider heavy baryons containing one heavy quark and construct the QCD sum rules in the framework of the heavy quark effective theory (HQET), since the physics of hadrons containing one infinitely heavy quark is well described with HQET and the analysis is greatly simplified due to the heavy quark symmetry.

We take into account the next-to-leading order corrections in α_s to the terms in the operator product expansion.

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It is known that in the systems containing heavy quarks α_s -correction gives significantly large contribution, which amounts to 100% or more to the leading order contribution [9,10,26,27]. In Refs. [9,10], the α_s -corrections to dimension 0 and dimension 3 terms in the operator product expansion of the heavy baryon correlation functions are calculated to study positive parity Λ and Σ . However, as will be pointed out in Sec. III A, the authors excluded the interpolating fields that couple to negative parity states. In the present paper, we use the interpolating field including both of the parity components, and compute the α_s -corrections in the correlation function.

The paper is organized as follows. In the second section, we formulate the QCD sum rules for positive and negative parity heavy baryons. The method is applied to Λ and Σ heavy baryons and demonstrate the derivation of the sum rules at leading order in α_s , in the third section. In the fourth section, we calculate the next-to-leading order correction in α_s and present the sum rules including the corrections. The sum rules derived in the third and fourth sections are analyzed numerically in the fifth section. The sixth section is devoted to summary and discussion.

II. QCD SUM RULES FOR POSITIVE AND NEGATIVE PARITY HEAVY BARYONS IN THE HQET

In this section, we set up the QCD sum rule for positive and negative parity heavy baryons in the HQET. We consider the following correlation function in the HQET:

$$\Pi_B(\omega) = i \int d^4x e^{i\omega v \cdot x} \langle 0 | T[\eta_B(x) \bar{\eta}_B(0)] | 0 \rangle, \quad (1)$$

where η_B is the interpolating field of heavy baryon B and v is the four velocity of the heavy baryon. In the general procedures in the QCD sum rule approach, the correlation functions are evaluated by the operator product expansion (OPE) in unphysical region $\omega \rightarrow -\infty$ on one hand and expressed in terms of the properties of physical states (masses, coupling constants and so on) on the other hand; we relate the two descriptions exploiting the dispersion relations, which yields the QCD sum rule.

Let us first consider how Eq. (1) is expressed in terms of physical states. The interpolating field η_B couples not only to positive parity states but also to negative parity ones [22,28], in the way that

$$\langle 0 | \eta(0) | B_{j(+)}(v, \alpha) \rangle = \lambda_{j(+)} u(v, \alpha), \quad (2)$$

$$\langle 0 | \eta(0) | B_{j(-)}(v, \alpha) \rangle = \lambda_{j(-)} \gamma_5 u(v, \alpha), \quad (3)$$

where $|B_{j(\pm)}(v, \alpha)\rangle$ is the j -th positive/negative parity resonance state with velocity v and spin α and $u(v, \alpha)$ is the Dirac spinor for the baryon at heavy quark limit. Hence, inserting a complete set of physical states between the two

interpolating fields in Eq. (1) and neglecting the widths of the resonance states, Eq. (1) in the baryon rest frame $\mathbf{v} = \mathbf{0}$ can be expressed as follows,

$$\Pi(\omega) = \sum_j \left[\frac{-|\lambda_{j(+)}|^2}{\omega - \bar{\Lambda}_{j(+)} + i\epsilon} P_+ + \frac{-|\lambda_{j(-)}|^2}{\omega - \bar{\Lambda}_{j(-)} + i\epsilon} P_- \right], \quad (4)$$

where $P_{\pm} \equiv \frac{1}{2}(\gamma_0 \pm 1)$ are the parity projection operators and $\bar{\Lambda}_{j(\pm)} \equiv M_{j(\pm)} - m_Q$ with $M_{j(\pm)}$ being the masses of j -th positive/negative parity states of the heavy baryon and m_Q the heavy quark mass. It should be noticed that Eq. (4) does not have poles at negative ω because of the absence of antiheavy baryons in the heavy quark limit. In the correlation function of nucleons or hyperons, the term corresponding to the first (second) one in Eq. (4) has poles of antiparticles of negative (positive) parity at negative ω . As a result, even after performing ‘‘parity projection’’ (see below), the contributions of positive and negative parity states are not separated from each other [24,25].

We apply ‘‘parity projection’’ onto the correlation function, namely, consider

$$\Pi_{B(\pm)}(\omega) = \frac{1}{4} \text{Tr}[P_{\pm} \Pi(\omega)]. \quad (5)$$

From Eq. (4) we see $\Pi_{B(+)}(\omega)$ contain only the contribution of the positive parity states and $\Pi_{B(-)}(\omega)$ only the negative parity. Calculating Eq. (5) using the OPE and matching the results with the corresponding parity components in Eq. (4), we obtain the sum rules. The matching can be done via the dispersion relations for $\Pi_{B(\pm)}$ and utilizing Borel transformation. $\Pi_{B(\pm)}$ obeys the dispersion relations,

$$\Pi_{B(\pm)}(\omega) = \int_0^{\infty} d\omega' \frac{\rho_{B(\pm)}(\omega')}{\omega' - \omega - i\epsilon}, \quad (6)$$

(the subtraction terms, which are polynomial in ω , are not written explicitly here). In Eq. (6), $\rho_{B(\pm)}(\omega)$ is the spectral function defined by

$$\rho_{B(\pm)}(\omega) = \frac{1}{\pi} \text{Im} \Pi_{B(\pm)}(\omega), \quad (7)$$

We apply the Borel transformation operator, defined by

$$\hat{B} \equiv \lim_{\substack{n \rightarrow \infty, -\omega \rightarrow \infty \\ M = -\omega/n \text{ fixed}}} \frac{\omega^n}{\Gamma(n)} \left(-\frac{d}{d\omega} \right)^n, \quad (8)$$

on both sides of the dispersion relation, Eq. (6). This transformation introduces an exponential weight in the integral as

$$\hat{B} \Pi_{B(\pm)}(\omega) = \frac{1}{M} \int_0^{\infty} d\omega' e^{-\omega'/M} \rho_{B(\pm)}(\omega'), \quad (9)$$

and eliminate the subtraction terms. In the left-hand side (lhs) of Eq. (9), where $\Pi_{B(\pm)}(\omega)$ are calculated by OPE, the convergence of the series in the OPE is improved since the higher dimensional terms in the OPE are suppressed factorially ($\sim 1/n!$). Simultaneously, in the right-hand side (rhs), the contributions of higher resonances and continuum are suppressed exponentially compared with that of the lowest-lying state. It is therefore allowed to approximate excited-state contributions to the rhs of Eq. (9) by the imaginary part of the OPE result which starts from the “continuum threshold” ω_{th} ; namely, we use

$$\rho_{B(\pm)}(\omega) = |\lambda_{B(\pm)}|^2 \delta(\omega - \bar{\Lambda}_{B(\pm)}) + \frac{1}{\pi} \text{Im} \Pi_{B(\pm)}^{\text{OPE}}(\omega) \theta(\omega - \omega_{\text{th}}), \quad (10)$$

where $\Pi_{B(\pm)}^{\text{OPE}}(\omega)$ is the correlation function calculated by OPE. Substituting Eq. (10) into the rhs of Eq. (9), we obtain,

$$|\lambda_{B(\pm)}|^2 e^{-\bar{\Lambda}_{B(\pm)}/M} = \int_0^{\omega_{\text{th}}} d\omega e^{-\omega/M} \frac{1}{\pi} \text{Im} \Pi_{B(\pm)}^{\text{OPE}}(\omega), \quad (11)$$

Derivative of the logarithm of Eq. (11) with respect to $-1/M$ gives the expressions for $\bar{\Lambda}_{B(\pm)}$,

$$\bar{\Lambda}_{B(\pm)} = \frac{\frac{\partial}{\partial(-1/M)} \int_0^{\omega_{\text{th}}} d\omega e^{-\omega/M} \frac{1}{\pi} \text{Im} \Pi_{B(\pm)}^{\text{OPE}}(\omega)}{\int_0^{\omega_{\text{th}}} d\omega e^{-\omega/M} \frac{1}{\pi} \text{Im} \Pi_{B(\pm)}^{\text{OPE}}(\omega)}. \quad (12)$$

Calculating the correlation function by OPE and substitute the results into the rhs of the above equations, we obtain the sum rules for $\bar{\Lambda}_{B(\pm)}$.

III. APPLICATION TO Λ AND Σ CHANNEL

In this section, we apply the method described in the previous section to $I = 0$ (Λ) and $I = 1$ (Σ) heavy baryons.

A. Interpolating fields for Λ and Σ

The candidates of the interpolating field for Λ are

$$\eta_S \equiv \epsilon_{abc} (u_a C \gamma_5 d_b) h_c, \quad (13)$$

$$\eta_P \equiv \epsilon_{abc} (u_a C d_b) \gamma_5 h_c, \quad (14)$$

$$\eta_V \equiv \epsilon_{abc} (u_a C \gamma_5 \gamma_\mu d_b) \gamma^\mu h_c, \quad (15)$$

and those for Σ ,

$$\eta_A \equiv \epsilon_{abc} (u_a C \gamma_\mu d_b) \gamma^\mu \gamma_5 h_c, \quad (16)$$

$$\eta_T \equiv \frac{1}{2} \epsilon_{abc} (u_a C \sigma_{\mu\nu} d_b) \sigma^{\mu\nu} \gamma_5 h_c, \quad (17)$$

where u and d are the up and down quark fields, h the effective heavy quark field in the HQET, C the charge conjugation operator, $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$ and a, b and c the color indices. The subscripts, S, P, V, A , and T stand for the channel of the light diquark fields, scalar, pseudoscalar, vector, axialvector, and tensor, respectively. The general interpolating fields for Λ and Σ should be given by their linear combinations,

$$\eta_\Lambda = t_S \eta_S + t_P \eta_P + t_V \eta_V, \quad (18)$$

$$\eta_\Sigma = t_A \eta_A + t_T \eta_T, \quad (19)$$

with t_X ($X = S, P, V, A, T$) being arbitrary mixing parameters.

Some important remarks are in order here. $\eta_{S(P)}$ couples only to positive (negative) parity states, as in the form of Eq. (2) [Eq. (3)], since the effective heavy quark field, h , which is constrained to satisfy $\not{v}h = h$, is projected onto positive parity states at its rest frame $\mathbf{v} = \mathbf{0}$. On the other hand, η_V, η_A , and η_T can couple with both of parity states, which becomes obvious if we decompose them as

$$\eta_V = (u C \gamma_5 \gamma_0 d) \gamma^0 h + (u C \gamma_5 \gamma_i d) \gamma^i h, \quad (20)$$

$$\eta_A = (u C \gamma_i d) \gamma^i \gamma_5 h + (u C \gamma_0 d) \gamma^0 \gamma_5 h, \quad (21)$$

$$\eta_T = (u C \sigma_{0i} d) \sigma^{0i} \gamma_5 h + \frac{1}{2} (u C \sigma_{ij} d) \sigma^{ij} \gamma_5 h, \quad (22)$$

(Color indices are suppressed here for simplicity.) The first term in the right hand side of each of the above equations couples only with positive parity state, while the second term only with negative parity states. In all the previous studies on positive parity heavy baryons in QCD sum rules [4–7,9,10], the second terms of Eqs. (20)–(22) are excluded consistently. However, as was pointed out above, they couple to negative parity heavy baryons and are not allowed to be excluded for the purpose of constructing the sum rules for positive and negative parity states.

Let us define the correlators of η_X 's,

$$\Pi_{XY}(\omega) \equiv i \int d^4 x e^{i\omega v \cdot x} \langle 0 | T [\eta_X(x) \bar{\eta}_Y(0)] | 0 \rangle. \quad (23)$$

Then the correlation function, Eq. (1), for Λ and Σ can be written in terms of $\Pi_{XY}(\omega)$ as

$$\Pi_\Lambda(\omega) = \sum_{X,Y=S,P,V} t_X t_Y \Pi_{XY}(\omega), \quad (24)$$

$$\Pi_\Sigma(\omega) = \sum_{X,Y=A,T} t_X t_Y \Pi_{XY}(\omega). \quad (25)$$

In the following two subsections, we evaluate Eq. (23) by OPE to derive the sum rules at leading order (LO) in α_s -expansion and those at next-to-leading order (NLO). To this end, we write Eq. (23) as

$$\Pi_{XY}(\omega) = \Pi_{XY}^{(0)}(\omega) + \Pi_{XY}^{(1)}(\omega) + \dots, \quad (26)$$

where $\Pi_{XY}^{(0)}(\omega)$ and $\Pi_{XY}^{(1)}(\omega)$ denote LO and NLO contributions in α_s -expansion, respectively, and the ellipsis the higher order ones.

B. Sum rules at LO in α_s -expansion

We carry out the OPE of $\Pi_{XY}^{(0)}(\omega)$. The nonvanishing contribution in the OPE up to dimension 6 operators and at LO in α_s -expansion are represented by the Feynman diagrams shown in Fig. 1.

First, we show the results for the components of $\Pi_{XY}^{(0)}(\omega)$ that contribute to $\Pi_{\Lambda}(\omega)$,

$$\begin{aligned} \Pi_{SS}^{(0)}(\omega) &= \frac{-N_c!}{120\pi^4} \omega^5 \ln(-\omega) P_+ \\ &+ \frac{-1}{32\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \omega \ln(-\omega) P_+ \\ &+ \frac{-N_c!}{4N_c^2} \langle \bar{u}u \rangle \langle \bar{d}d \rangle \frac{1}{\omega} P_+ \\ &+ \dots, \end{aligned} \quad (27)$$

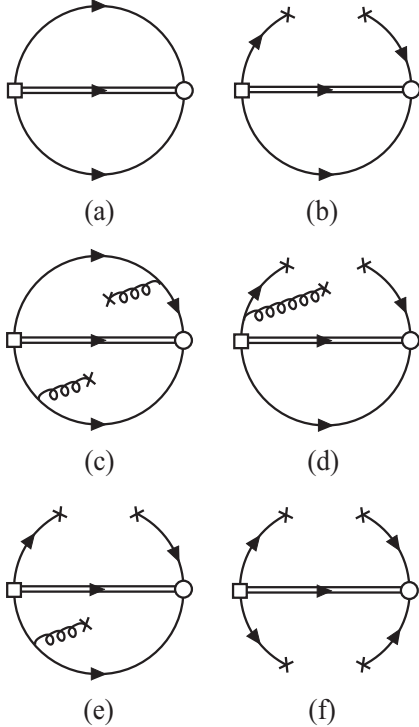


FIG. 1. Nonvanishing diagrams representing the terms in the OPE at LO in α_s -expansion. (a) leading term (dimension 0), (b) dimension 3, (c) dimension 4, (d) and (e) dimension 5, (f) dimension 6 term. The single and the double lines stand for the light and the heavy quark propagators, respectively.

$$\begin{aligned} \Pi_{PP}^{(0)}(\omega) &= \frac{-N_c!}{120\pi^4} \omega^5 \ln(-\omega) P_- \\ &+ \frac{-1}{32\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \omega \ln(-\omega) P_- \\ &+ \frac{N_c!}{4N_c^2} \langle \bar{u}u \rangle \langle \bar{d}d \rangle \frac{1}{\omega} P_- \\ &+ \dots, \end{aligned} \quad (28)$$

$$\begin{aligned} \Pi_{VV}^{(0)}(\omega) &= \frac{-N_c!}{120\pi^4} \omega^5 \ln(-\omega) (P_+ + 3P_-) \\ &+ \frac{-1}{32\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \omega \ln(-\omega) (P_+ - P_-) \\ &+ \frac{-N_c!}{4N_c^2} \langle \bar{u}u \rangle \langle \bar{d}d \rangle \frac{1}{\omega} (P_+ - 3P_-) \\ &+ \dots, \end{aligned} \quad (29)$$

$$\Pi_{SP,PS}^{(0)}(\omega) = 0, \quad (30)$$

$$\begin{aligned} \Pi_{SV,VS}^{(0)}(\omega) &= \frac{N_c!}{4\pi^2 N_c} (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \omega^2 \ln(-\omega) P_+ \\ &+ \frac{-N_c!}{32\pi^2 N_c} (g \langle \bar{u}\sigma \cdot Gu \rangle + g \langle \bar{d}\sigma \cdot Gd \rangle) \ln(-\omega) P_+ \\ &+ \frac{1}{32\pi^2} (g \langle \bar{u}\sigma \cdot Gu \rangle + g \langle \bar{d}\sigma \cdot Gd \rangle) \ln(-\omega) P_+ \\ &+ \dots, \end{aligned} \quad (31)$$

$$\Pi_{PV,VP}^{(0)}(\omega) = 0, \quad (32)$$

where the ellipsis denotes the terms that will disappear when the Borel transformation is carried out. Calculation of the dimension 6 (four-quark condensate) term is done by applying the factorization hypothesis. N_c is the number of colors, $\langle \bar{q}q \rangle \equiv \langle 0 | \bar{q}q | 0 \rangle$, $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \equiv \langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a\mu\nu} | 0 \rangle$, $g \langle \bar{q}\sigma \cdot Gq \rangle \equiv g \langle 0 | \bar{q} \sigma_{\mu\nu} \frac{\lambda^a}{2} G^{a\mu\nu} q | 0 \rangle$, where g is the strong coupling constant, λ^a the usual Gell-Mann SU(3) matrix, and $G_{\mu\nu}^a$ the gluon field strength.

Let us discuss Eqs. (27)–(32).

- (i) $\Pi_{SS(PP)}$ has only $P_{+(-)}$ component and $\Pi_{SP,PS} = 0$. This is due to the fact that $\eta_{S(P)}$ couples only to positive (negative) parity states, as was mentioned in the previous subsection. In contrast, η_V can couple with both of parity states. Therefore Π_{VV} has both of P_+ and P_- components and $\Pi_{SV,VS}$ has only P_+ .
- (ii) Diagonal correlators, Π_{SS} , Π_{PP} , Π_{VV} , have only chiral even condensates. On the other hand, non-diagonal correlator, $\Pi_{SV,VS}$, has only chiral odd condensates. This can be understood by introducing right- and left-handed quark fields, q_R and q_L [29],

$$\eta_S = (u_R C d_R - u_L C d_L) h, \quad (33)$$

$$\eta_P = (u_R C d_R + u_L C d_L) \gamma_5 h, \quad (34)$$

$$\eta_V = (u_R C \gamma_\mu d_L - u_L C \gamma_\mu d_R) \gamma^\mu h. \quad (35)$$

(Color indices are suppressed here for simplicity.) In general, in the OPE of correlation functions, $\langle \bar{q}q \rangle$ or $g\langle \bar{q}\sigma \cdot Gq \rangle$ appear only if q_R is paired with q_L , since $\bar{q}q = \bar{q}_R q_L + \bar{q}_L q_R$, $\bar{q}\sigma \cdot Gq = \bar{q}_R \sigma \cdot Gq_L + \bar{q}_L \sigma \cdot Gq_R$. In $\Pi_{SV,VS}$ this is possible if one $q_{R(L)}$ in η_S is paired with one $q_{L(R)}$ in η_V , as shown in Fig. 2(a). In Π_{SS}, Π_{PP} , and Π_{VV} , this is possible only if two pairs, i.e., u_R and u_L , d_R and d_L , are formed at once, yielding the term $\langle 0|\bar{u}u\bar{d}d|0 \rangle$ [see Figs. 2(b) and 2(c)].

(iii) $\Pi_{PV,VP}$ is vanishing, although one may think that $\Pi_{PV,VP}$ has P_- component, since both of η_P and η_V can couple with negative parity states, and have chiral odd condensates if one $q_{R(L)}$ in η_P is paired with $q_{L(R)}$ in η_V . In reality, however, the pair yields the term $\langle \bar{q}\gamma_5 q \rangle$, which is vanishing.

Next, we show the OPE results for $\Pi_{XY}^{(0)}(\omega)$ related with $\Pi_\Sigma(\omega)$,

$$\begin{aligned} \Pi_{AA}^{(0)}(\omega) &= \frac{-N_c!}{120\pi^4} \omega^5 \ln(-\omega) (3P_+ + P_-) \\ &+ \frac{1}{32\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \omega \ln(-\omega) (P_+ - P_-) \\ &+ \frac{-N_c!}{4N_c^2} \langle \bar{u}u \rangle \langle \bar{d}d \rangle \frac{1}{\omega} (3P_+ - P_-) \\ &+ \dots, \end{aligned} \quad (36)$$

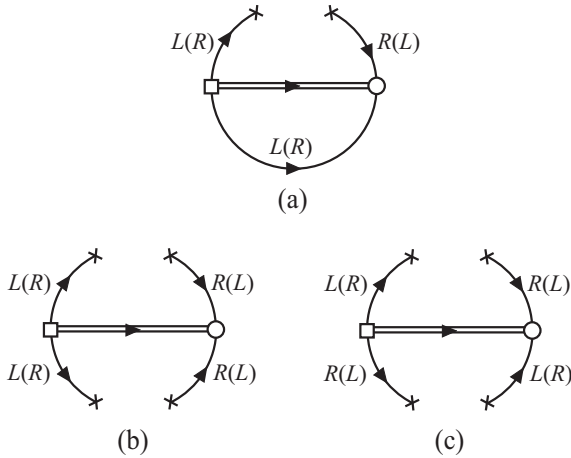


FIG. 2. Diagrams with explicit dependence on the chirality of light quarks, corresponding to (a) dimension 3 term in Π_{SV} , (b) dimension 6 term in $\Pi_{SS,PP}$, and (c) that in Π_{VV} .

$$\begin{aligned} \Pi_{TT}^{(0)}(\omega) &= \frac{-N_c!}{40\pi^4} \omega^5 \ln(-\omega) (P_+ + P_-) \\ &+ \frac{1}{32\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \omega \ln(-\omega) (P_+ + P_-) \\ &+ \frac{-3N_c!}{4N_c^2} \langle \bar{u}u \rangle \langle \bar{d}d \rangle \frac{1}{\omega} (P_+ - P_-) \\ &+ \dots, \end{aligned} \quad (37)$$

$$\begin{aligned} \Pi_{AT,TA}^{(0)}(\omega) &= \frac{3N_c!}{4\pi^2 N_c} (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \omega^2 \ln(-\omega) P_+ \\ &+ \frac{-3N_c!}{32\pi^2 N_c} (g\langle \bar{u}\sigma \cdot Gu \rangle + g\langle \bar{d}\sigma \cdot Gd \rangle) \ln(-\omega) P_+ \\ &+ \frac{-1}{32\pi^2} (g\langle \bar{u}\sigma \cdot Gu \rangle + g\langle \bar{d}\sigma \cdot Gd \rangle) \ln(-\omega) P_+ \\ &+ \dots. \end{aligned} \quad (38)$$

Again the ellipsis denotes the terms that will vanish after the Borel transformation is applied.

In order to discuss Eqs. (36)–(38), we decompose η_A as

$$\eta_A = \eta_{\bar{A}} + \eta_{\underline{A}}, \quad (39)$$

where

$$\eta_{\bar{A}} = (u C \gamma_i d) \gamma^i \gamma_5 h, \quad \eta_{\underline{A}} = (u C \gamma_0 d) \gamma^0 \gamma_5 h. \quad (40)$$

Then Π_{AA} and $\Pi_{AT,TA}$ can also be decomposed as

$$\Pi_{AA}(\omega) = \Pi_{\bar{A}\bar{A}}(\omega) + \Pi_{\underline{A}\underline{A}}(\omega) + \Pi_{\bar{A}\underline{A}}(\omega) + \Pi_{\underline{A}\bar{A}}(\omega), \quad (41)$$

$$\Pi_{AT,TA}(\omega) = \Pi_{\bar{A}T,\bar{A}}(\omega) + \Pi_{\underline{A}T,\underline{A}}(\omega). \quad (42)$$

Following Eqs. (41) and (42), one can decompose Eqs. (36) and (38) as

$$\begin{aligned} \Pi_{\bar{A}\bar{A}}^{(0)}(\omega) &= \frac{-N_c!}{120\pi^4} \omega^5 \ln(-\omega) \cdot 3P_+ \\ &+ \frac{1}{32\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \omega \ln(-\omega) P_+ \\ &+ \frac{-N_c!}{4N_c^2} \langle \bar{u}u \rangle \langle \bar{d}d \rangle \frac{1}{\omega} \cdot 3P_+ \\ &+ \dots, \end{aligned} \quad (43)$$

$$\begin{aligned} \Pi_{\underline{A}\underline{A}}^{(0)}(\omega) &= \frac{-N_c!}{120\pi^4} \omega^5 \ln(-\omega) P_- \\ &+ \frac{-1}{32\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \omega \ln(-\omega) P_- \\ &+ \frac{N_c!}{4N_c^2} \langle \bar{u}u \rangle \langle \bar{d}d \rangle \frac{1}{\omega} P_- \\ &+ \dots, \end{aligned} \quad (44)$$

$$\Pi_{\bar{A}\bar{A}\bar{A}\bar{A}}^{(0)}(\omega) = 0, \quad (45)$$

$$\begin{aligned} \Pi_{\bar{A}T\bar{T}\bar{A}}^{(0)}(\omega) = & \frac{3N_c!}{4\pi^2 N_c} (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \omega^2 \ln(-\omega) P_+ \\ & + \frac{-3N_c!}{32\pi^2 N_c} (g\langle \bar{u}\sigma \cdot Gu \rangle + g\langle \bar{d}\sigma \cdot Gd \rangle) \ln(-\omega) P_+ \\ & + \frac{-1}{32\pi^2} (g\langle \bar{u}\sigma \cdot Gu \rangle + g\langle \bar{d}\sigma \cdot Gd \rangle) \ln(-\omega) P_+ \\ & + \dots, \end{aligned} \quad (46)$$

$$\Pi_{\bar{A}T\bar{T}\bar{A}}^{(0)}(\omega) = 0. \quad (47)$$

Using Eqs. (43)–(47) instead of Eqs. (36) and (38), one can interpret Eqs. (36)–(38) completely in parallel with Eqs. (27)–(32).

(i) $\Pi_{\bar{A}\bar{A}\bar{A}\bar{A}}$ has only $P_{+(-)}$ component and $\Pi_{\bar{A}\bar{A}\bar{A}\bar{A}} = 0$. This is due to the fact that $\eta_{\bar{A}(\underline{A})}$ couples only to positive (negative) parity states. On the other hand, η_T can couple with both of parity states. Therefore Π_{TT} has both of P_+ and P_- components and $\Pi_{\bar{A}T\bar{T}\bar{A}}$ has only P_+ .

(ii) Diagonal correlators, $\Pi_{\bar{A}\bar{A}}$, $\Pi_{\bar{A}\bar{A}}$ and Π_{TT} , have only chiral even condensates, while nondiagonal correlator, $\Pi_{\bar{A}T\bar{T}\bar{A}}$, has only chiral odd condensates. To understand this, we write $\eta_{\bar{A}}$ and η_T in terms of right- and left-handed quark fields,

$$\eta_{\bar{A}} = (u_R C \gamma_i d_L + u_L C \gamma_i d_R) \gamma^i \gamma_5 h, \quad (48)$$

$$\eta_{\underline{A}} = (u_R C \gamma_0 d_L + u_L C \gamma_0 d_R) \gamma^0 \gamma_5 h, \quad (49)$$

$$\eta_T = \frac{1}{2} (u_R C \sigma_{\mu\nu} d_R + u_L C \sigma_{\mu\nu} d_L) \sigma^{\mu\nu} \gamma_5 h. \quad (50)$$

In $\Pi_{\bar{A}T\bar{T}\bar{A}}$, $\langle \bar{q}q \rangle$ or $g\langle \bar{q}\sigma \cdot Gq \rangle$ can appear if one $q_{R(L)}$ in $\eta_{\bar{A}}$ is paired with one $q_{L(R)}$ in η_T . In $\Pi_{\bar{A}\bar{A}}$, $\Pi_{\bar{A}\bar{A}}$ and Π_{TT} , this can occur only if u_R is paired with u_L and simultaneously d_R is paired with d_L , yielding the term $\langle 0 | \bar{u}u \bar{d}d | 0 \rangle$.

(iii) $\Pi_{\bar{A}T\bar{T}\bar{A}}$ is vanishing, although it may be possible that $\Pi_{\bar{A}T\bar{T}\bar{A}}$ has P_- component, since both of $\eta_{\bar{A}}$ and η_T can couple with negative parity states, and have chiral odd condensates if one $q_{R(L)}$ in $\eta_{\bar{A}}$ is paired with $q_{L(R)}$ in η_T . But in fact, the pair yields the term $\langle \bar{q}\gamma_5 q \rangle$, which is vanishing.

Now we have all the ingredients to derive the sum rules. Substituting Eq. (24) in which the OPE results Eqs. (27)–(32) are used as $\Pi_{XY}(\omega)$, and Eq. (25) in which Eqs. (36)–(38) are used, into the rhs of Eq. (12), we obtain the QCD sum rules for $\bar{\Lambda}_{\Lambda(\pm)}$ and $\bar{\Lambda}_{\Sigma(\pm)}$ at LO in α_s -expansion,

$$\bar{\Lambda}_{B(\pm)} = \frac{\frac{\partial}{\partial(-1/M)} P_{B(\pm)}^{(0)}(M)}{P_{B(\pm)}^{(0)}(M)}, \quad (B = \Lambda, \Sigma), \quad (51)$$

where $P_{B(\pm)}^{(0)}(M)$ are given by

$$\begin{aligned} P_{\Lambda(+)}^{(0)}(M) = & \frac{t_S^2 + t_V^2}{20\pi^4} E_5(M, \omega_{th}) \\ & - \frac{t_S t_V}{\pi^2} (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) E_2(M, \omega_{th}) \\ & + \frac{t_S^2 + t_V^2}{32\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle E_1(M, \omega_{th}) \\ & + \frac{t_S t_V}{16\pi^2} (g\langle \bar{u}\sigma \cdot Gu \rangle + g\langle \bar{d}\sigma \cdot Gd \rangle) E_0(M, \omega_{th}) \\ & + \frac{t_S^2 + t_V^2}{6} \langle \bar{u}u \rangle \langle \bar{d}d \rangle, \end{aligned} \quad (52)$$

$$\begin{aligned} P_{\Lambda(-)}^{(0)}(M) = & \frac{t_P^2 + 3t_V^2}{20\pi^4} E_5(M, \omega_{th}) \\ & + \frac{t_P^2 - t_V^2}{32\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle E_1(M, \omega_{th}) \\ & - \frac{t_P^2 + 3t_V^2}{6} \langle \bar{u}u \rangle \langle \bar{d}d \rangle, \end{aligned} \quad (53)$$

$$\begin{aligned} P_{\Sigma(+)}^{(0)}(M) = & \frac{3(t_A^2 + t_T^2)}{20\pi^4} E_5(M, \omega_{th}) \\ & - \frac{3t_A t_T}{\pi^2} (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) E_2(M, \omega_{th}) \\ & - \frac{t_A^2 + t_T^2}{32\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle E_1(M, \omega_{th}) \\ & + \frac{7t_A t_T}{16\pi^2} (g\langle \bar{u}\sigma \cdot Gu \rangle + g\langle \bar{d}\sigma \cdot Gd \rangle) E_0(M, \omega_{th}) \\ & + \frac{t_A^2 + t_T^2}{2} \langle \bar{u}u \rangle \langle \bar{d}d \rangle, \end{aligned} \quad (54)$$

$$\begin{aligned} P_{\Sigma(-)}^{(0)}(M) = & \frac{t_A^2 + 3t_T^2}{20\pi^4} E_5(M, \omega_{th}) \\ & + \frac{t_A^2 - t_T^2}{32\pi^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle E_1(M, \omega_{th}) \\ & - \frac{t_A^2 + 3t_T^2}{6} \langle \bar{u}u \rangle \langle \bar{d}d \rangle. \end{aligned} \quad (55)$$

In the above equations,

$$E_n(M, \omega) \equiv \int_0^\omega d\omega' \omega'^n e^{-\omega'/M}, \quad (56)$$

and the explicit value of the number of colors $N_c = 3$ is substituted. The numerator in Eq. (51) is easily obtained by using the property,

$$\frac{\partial}{\partial(-1/M)} E_n(M, \omega) = E_{n+1}(M, \omega). \quad (57)$$

A noticeable feature of the above sum rules is that the sum rules for negative parity states do not have chiral odd terms ($\langle \bar{q}q \rangle$, $g\langle \bar{q}\sigma \cdot Gq \rangle$, ...). As was pointed out in the discussion about Eqs. (27)–(32) and Eqs. (36)–(38), chiral odd condensates contribute only through the nondiagonal correlators, Π_{SV} and Π_{AT} . However, in the case of heavy baryons, Π_{SV} and Π_{AT} have only positive parity component. As a result, only the positive parity sum rules receive the contribution of chiral odd condensates but the negative parity sum rules not. This feature of the sum rules for heavy baryons is in contrast to those for nucleons and hyperons, where chiral odd condensates contribute to both of parity states but with opposite sign, which increase the mass of negative parity and make the mass difference between positive and negative parity large [22–25].

Also note that while the general interpolating field of Λ , Eq. (18), has three mixing parameters, t_P , t_S , and t_V , the sum rule for $\bar{\Lambda}_{\Lambda(+)}$ depends on two of them, t_S and t_V , and that for $\bar{\Lambda}_{\Lambda(-)}$ on t_P and t_V . This is due to the fact that $\eta_{P(S)}$ in Eq. (18), whose coefficient is $t_{P(S)}$, does not couple to positive (negative) parity states. As a result, $\bar{\Lambda}_{\Lambda(+)}$ depends on the mixing parameters via their ratio, t_V/t_S , and $\bar{\Lambda}_{\Lambda(-)}$ via t_V/t_P .

It is interesting that Eq. (53) and Eq. (55) coincide with each other, which means that the mass difference between $\Lambda(-)$ and $\Sigma(-)$ is not given within this calculation. The difference is produced by including the α_s -correction [see Eqs. (82) and (84)].

C. Sum rules at NLO in α_s -expansion

Let us next consider NLO correction in α_s -expansion. We take into account the correction to the leading dimension (dimension 0) and the next dimension (dimension 3) terms in the OPE. For convenience, we write the dimension

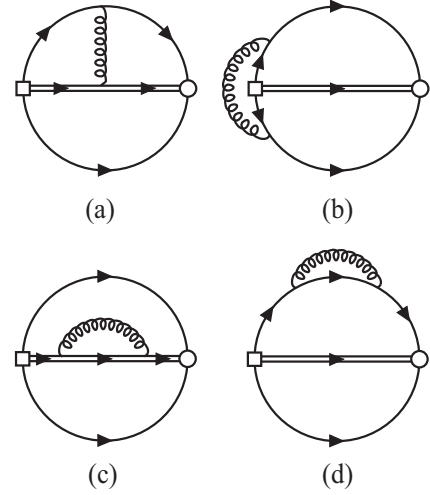


FIG. 3. Diagrams corresponding to NLO correction in α_s -expansion to dimension 0 term.

0 and 3 terms in $\Pi_{XY}(\omega)$ respectively as

$$\Pi_{XY}(\omega)_0 = \Pi_{XY}^{(0)}(\omega)_0 + \Pi_{XY}^{(1)}(\omega)_0 + \dots, \quad (58)$$

$$\Pi_{XY}(\omega)_3 = \Pi_{XY}^{(0)}(\omega)_3 + \Pi_{XY}^{(1)}(\omega)_3 + \dots, \quad (59)$$

where $\Pi_{XY}^{(0)}(\omega)_{0,3}$ denotes LO contribution in α_s -expansion, $\Pi_{XY}^{(1)}(\omega)_{0,3}$ NLO contribution, and the ellipsis the higher order.

First, let us calculate the correction to the dimension 0 term, $\Pi_{XY}^{(1)}(\omega)_0$. Diagrams corresponding to $\Pi_{XY}^{(1)}(\omega)_0$ are shown in Fig. 3. The calculation of those diagrams in $D = 4 + 2\epsilon$ space-time dimension can be performed by using the method described in [30,31]. The results are presented in Eqs. (A5)–(A7) and Eqs. (A9)–(A11). Expanding them in powers of $1/\epsilon$, we obtain for each component of $\Pi_{XY}^{(1)}(\omega)_0$,

$$\Pi_{SS,PP}^{(1)}(\omega)_0 = \frac{\alpha_s}{40\pi^5} N_c C_F \left[\omega^5 L \frac{1}{\epsilon} + 3\omega^5 L \left(L - \frac{2\pi^2}{27} - \frac{22}{5} \right) + \dots \right] P_{+,-}, \quad (60)$$

$$\begin{aligned} \Pi_{VV,AA}^{(1)}(\omega)_0 &= \frac{\alpha_s}{80\pi^5} N_c C_F \left[\omega^5 L \frac{1}{\epsilon} + 3\omega^5 L \left(L - \frac{4\pi^2}{27} - \frac{223}{45} \right) + \dots \right] P_{+,-} \\ &+ \frac{3\alpha_s}{80\pi^5} N_c C_F \left[\omega^5 L \frac{1}{\epsilon} + 3\omega^5 L \left(L - \frac{4\pi^2}{27} - \frac{71}{15} \right) + \dots \right] P_{-,+}, \end{aligned} \quad (61)$$

$$\begin{aligned} \Pi_{TT}^{(1)}(\omega)_0 &= \frac{\alpha_s}{40\pi^5} N_c C_F \left[\omega^5 L \frac{1}{\epsilon} + 3\omega^5 L \left(L - \frac{2\pi^2}{9} - \frac{248}{45} \right) + \dots \right] P_{+} \\ &+ \frac{\alpha_s}{40\pi^5} N_c C_F \left[\omega^5 L \frac{1}{\epsilon} + 3\omega^5 L \left(L - \frac{2\pi^2}{9} - \frac{263}{45} \right) + \dots \right] P_{-}, \end{aligned} \quad (62)$$

where $C_F = (N_c^2 - 1)/(2N_c)$, $L \equiv \ln(-\omega/\mu_{\text{MS}}) + \gamma_E/2 - (1/2)\ln\pi$ with γ_E the Euler constant, μ_{MS} is the renormalization scale in the MS scheme and the ellipsis denotes the terms that will vanish after the Borel transformation is carried out. L can be written in the $\overline{\text{MS}}$ scheme as $L = \ln(-2\omega/\mu_{\overline{\text{MS}}})$ with the renormalization scale in the $\overline{\text{MS}}$ scheme, $\mu_{\overline{\text{MS}}}$. Later calculation is done in the $\overline{\text{MS}}$ scheme and we denote $\mu_{\overline{\text{MS}}}$ simply by μ .

$\Pi_{XX}(\omega)_0$ is renormalized by the renormalization constant Z_{η_X} of the interpolating field η_X [9] as

$$\Pi_{XX}(\omega)_0 = Z_{\eta_X}^2 \Pi_{XX}^{\text{ren}}(\omega)_0. \quad (63)$$

Z_{η_X} are given by (see Appendix B for derivation)

$$Z_{\eta_{S,P}} = 1 - \frac{3}{4} \left(1 + \frac{1}{N_c}\right) \frac{\alpha_s}{\pi} \frac{1}{\epsilon}, \quad (64)$$

$$Z_{\eta_{V,A}} = 1 - \frac{3}{8} \left(1 + \frac{1}{N_c}\right) \frac{\alpha_s}{\pi} \frac{1}{\epsilon}, \quad (65)$$

$$Z_{\eta_T} = 1 - \frac{1}{4} \left(1 + \frac{1}{N_c}\right) \frac{\alpha_s}{\pi} \frac{1}{\epsilon}. \quad (66)$$

By rewriting Eq. (63) as

$$\begin{aligned} \Pi_{XX}^{\text{ren}}(\omega)_0 &= \Pi_{XX}^{(0)}(\omega)_0 + \Pi_{XX}^{(1)}(\omega)_0 \\ &+ \left(\frac{1}{Z_{\eta_X}^2} - 1\right) \Pi_{XX}^{(0)}(\omega)_0, \end{aligned} \quad (67)$$

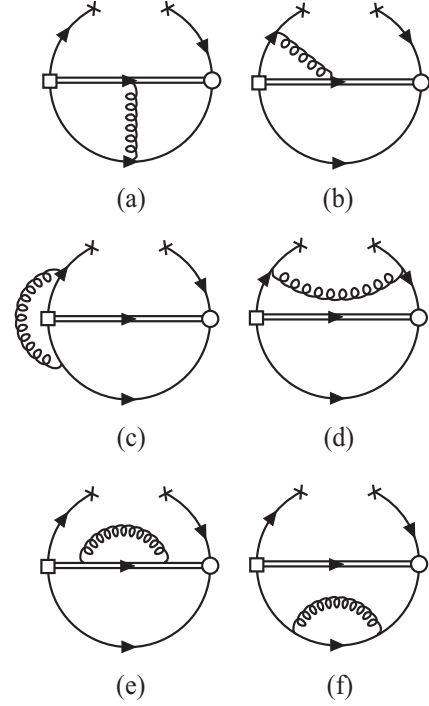


FIG. 4. Diagrams corresponding to NLO correction in α_s -expansion to dimension 3 term.

one can confirm that the pole in $\Pi_{XX}^{(1)}(\omega)_0$ [the second term in Eq. (67)] is canceled by the third term (counterterm). As a result, we obtain the renormalized dimension 0 term,

$$\Pi_{SS,PP}^{\text{ren}}(\omega)_0 = \left[-\frac{1}{20\pi^4} \omega^5 L + \frac{3\alpha_s}{10\pi^5} \omega^5 L \left(L - \frac{2}{27} \pi^2 - \frac{22}{5} \right) \right] P_{+,-}, \quad (68)$$

$$\Pi_{VV,AA}^{\text{ren}}(\omega)_0 = \left[-\frac{1}{20\pi^4} \omega^5 L + \frac{3\alpha_s}{20\pi^5} \omega^5 L \left(L - \frac{4}{27} \pi^2 - \frac{223}{45} \right) \right] P_{+,-} + \left[-\frac{3}{20\pi^4} \omega^5 L + \frac{9\alpha_s}{20\pi^5} \omega^5 L \left(L - \frac{4}{27} \pi^2 - \frac{71}{15} \right) \right] P_{-,+}, \quad (69)$$

$$\Pi_{TT}^{\text{ren}}(\omega)_0 = \left[-\frac{3}{20\pi^4} \omega^5 L + \frac{3\alpha_s}{10\pi^5} \omega^5 L \left(L - \frac{2}{9} \pi^2 - \frac{248}{45} \right) \right] P_{++} + \left[-\frac{3}{20\pi^4} \omega^5 L + \frac{3\alpha_s}{10\pi^5} \omega^5 L \left(L - \frac{2}{9} \pi^2 - \frac{263}{45} \right) \right] P_{--}, \quad (70)$$

where the number of colors is fixed to be $N_c = 3$.

Next, we calculate the correction to the dimension 3 term. Diagrams corresponding to $\Pi_{XY}^{(1)}(\omega)_3$ are shown in Fig. 4. The results of the calculation are given in Eqs. (A16)–(A21). Expansion of them in powers of $1/\epsilon$ yields

$$\Pi_{SV,VS}^{(1)}(\omega)_3 = -\frac{3\alpha_s}{16\pi^3} C_F \left[\omega^2 L \frac{1}{\epsilon} + 2\omega^2 L \left(L - \frac{2\pi^2}{9} - \frac{29}{6} \right) + \dots \right] (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) P_+, \quad (71)$$

$$\Pi_{AT,TA}^{(1)}(\omega)_3 = \frac{3\alpha_s}{16\pi^3} C_F \left[\omega^2 L \frac{1}{\epsilon} + 2\omega^2 L \left(L + \frac{2\pi^2}{3} + \frac{29}{6} \right) + \dots \right] (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) P_+, \quad (72)$$

where the ellipsis again denotes the terms that will disappear when the Borel transformation is applied. $\Pi_{XY}(\omega)_3$ is renormalized by Z_{η_X} and $Z_{\bar{q}q}$ [10],

$$\Pi_{SV,VS}(\omega)_3 = \frac{Z_{\eta_s} Z_{\eta_V}}{Z_{\bar{q}q}} \Pi_{SV,VS}^{\text{ren}}(\omega)_3, \quad (73)$$

$$\Pi_{AT,TA}(\omega)_3 = \frac{Z_{\eta_A} Z_{\eta_T}}{Z_{\bar{q}q}} \Pi_{AT,TA}^{\text{ren}}(\omega)_3. \quad (74)$$

$Z_{\bar{q}q}$ is the renormalization constant of $\bar{q}q$ (see Appendix C),

$$Z_{\bar{q}q} = 1 - 3C_F \frac{\alpha_s}{4\pi\epsilon}. \quad (75)$$

Multiplication of the renormalization constants cancels the pole in $\Pi_{XY}^{(1)}(\omega)_3$, providing the renormalized dimension 3 term,

$$\Pi_{SV,VS}^{\text{ren}}(\omega)_3 = \left[\frac{1}{2\pi^2} \omega^2 L - \frac{\alpha_s}{2\pi^3} \omega^2 L \left(L - \frac{2\pi^2}{9} - \frac{29}{6} \right) \right] \times (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) P_+, \quad (76)$$

$$\Pi_{AT,TA}^{\text{ren}}(\omega)_3 = \left[\frac{3}{2\pi^2} \omega^2 L + \frac{\alpha_s}{2\pi^3} \omega^2 L \left(L + \frac{2\pi^2}{3} + \frac{29}{6} \right) \right] \times (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) P_+, \quad (77)$$

where the number of colors is fixed to be $N_c = 3$.

Now we can obtain the sum rules for $\bar{\Lambda}_{B(\pm)}$ at NLO in α_s -expansion. Substituting Eqs. (27)–(32) and Eqs. (36)–(38) with the dimension 0 terms replaced by Eqs. (68)–(70) and dimension 3 terms by and Eqs. (76) and (77), into the rhs of Eq. (12), we obtain

$$\bar{\Lambda}_{B(\pm)} = \frac{\frac{\partial}{\partial(-1/M)} [P_{B(\pm)}^{(0)}(M) + P_{B(\pm)}^{(1)}(M)_0]}{P_{B(\pm)}^{(0)}(M) + P_{B(\pm)}^{(1)}(M)}, \quad (78)$$

where $P_{B(\pm)}^{(1)}(M)$ denote the contributions of NLO terms in α_s , and they consist of the corrections to dimension 0 and 3 terms,

$$P_{B(+)}^{(1)}(M) = P_{B(+)}^{(1)}(M)_0 + P_{B(+)}^{(1)}(M)_3, \quad (79)$$

$$P_{B(-)}^{(1)}(M) = P_{B(-)}^{(1)}(M)_0. \quad (80)$$

Here, $P_{B(\pm)}^{(1)}(M)_0$ correspond to the contributions from NLO corrections to dimension 0 terms and their explicit expressions for Λ and Σ are given by,

$$P_{\Lambda(+)}^{(1)}(M)_0 = t_S^2 \frac{-3\alpha_s}{10\pi^5} \left[2F_5(M, \omega_{th}, \mu) + E_5(M, \omega_{th}) \left(\ln 4 - \frac{2}{27} \pi^2 - \frac{22}{5} \right) \right] + t_V^2 \frac{-3\alpha_s}{20\pi^5} \left[2F_5(M, \omega_{th}, \mu) + E_5(M, \omega_{th}) \left(\ln 4 - \frac{4}{27} \pi^2 - \frac{223}{45} \right) \right], \quad (81)$$

$$P_{\Lambda(-)}^{(1)}(M)_0 = t_P^2 \frac{-3\alpha_s}{10\pi^5} \left[2F_5(M, \omega_{th}, \mu) + E_5(M, \omega_{th}) \left(\ln 4 - \frac{2}{27} \pi^2 - \frac{22}{5} \right) \right] + t_V^2 \frac{-9\alpha_s}{20\pi^5} \left[2F_5(M, \omega_{th}, \mu) + E_5(M, \omega_{th}) \left(\ln 4 - \frac{4}{27} \pi^2 - \frac{71}{15} \right) \right], \quad (82)$$

$$P_{\Sigma(+)}^{(1)}(M)_0 = t_A^2 \frac{-9\alpha_s}{20\pi^5} \left[2F_5(M, \omega_{th}, \mu) + E_5(M, \omega_{th}) \left(\ln 4 - \frac{4}{27} \pi^2 - \frac{71}{15} \right) \right] + t_T^2 \frac{-3\alpha_s}{10\pi^5} \left[2F_5(M, \omega_{th}, \mu) + E_5(M, \omega_{th}) \left(\ln 4 - \frac{2}{9} \pi^2 - \frac{248}{45} \right) \right], \quad (83)$$

$$P_{\Sigma(-)}^{(1)}(M)_0 = t_A^2 \frac{-3\alpha_s}{20\pi^5} \left[2F_5(M, \omega_{th}, \mu) + E_5(M, \omega_{th}) \left(\ln 4 - \frac{4}{27} \pi^2 - \frac{223}{45} \right) \right] + t_T^2 \frac{-3\alpha_s}{10\pi^5} \left[2F_5(M, \omega_{th}, \mu) + E_5(M, \omega_{th}) \left(\ln 4 - \frac{2}{9} \pi^2 - \frac{263}{45} \right) \right], \quad (84)$$

and $P_{B(+)}^{(1)}(M)_3$ are those from the corrections to dimension 3 terms,

$$P_{\Lambda(+)}^{(1)}(M)_3 = t_S t_V \frac{\alpha_s}{\pi^3} \left[2F_2(M, \omega_{th}, \mu) + E_2(M, \omega_{th}) \left(\ln 4 - \frac{2}{9} \pi^2 - \frac{29}{6} \right) \right] (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle), \quad (85)$$

$$P_{\Sigma(+)}^{(1)}(M)_3 = t_A t_T \frac{-\alpha_s}{\pi^3} \left[2F_2(M, \omega_{th}, \mu) + E_2(M, \omega_{th}) \left(\ln 4 + \frac{2}{3}\pi^2 + \frac{29}{6} \right) \right] (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle). \quad (86)$$

In the above equations,

$$F_n(M, \omega, \mu) \equiv \int_0^\omega d\omega' \omega'^n \ln \left(\frac{\omega'}{\mu} \right) e^{-\omega'/M}, \quad (87)$$

and the derivative in the numerator of Eq. (78) can be obtained by using the relation,

$$\frac{\partial}{\partial(-1/M)} F_n(M, \omega, \mu) = F_{n+1}(M, \omega, \mu). \quad (88)$$

IV. NUMERICAL ANALYSIS

In this section, we give a numerical analysis of the sum rules for $\bar{\Lambda}$, Eqs. (51) and (78) in detail.

We use the standard values for vacuum condensates collected in Table I. The value of the strong coupling constant is taken to be $\alpha_s(\mu = 1 \text{ GeV}) = 0.47$, which is consistent with the world average [27].

We analyze Eqs. (51) and (78) in the following procedure. First, we search the value of the mixing parameter of the interpolating field that makes the curve of $\bar{\Lambda}$ as a function of the Borel parameter M stable as much as possible. Next, for thus found optimal value of the mixing parameter, we find ω_{th} such that the region of M , so called ‘‘Borel window,’’ opens. The Borel window is the region satisfying the standard criterion applied in the QCD sum rule approach: the lowest pole contribution exceeds 50% in the spectral function

$$\frac{\int_0^{\omega_{th}} d\omega \rho(\omega)}{\int_0^\infty d\omega \rho(\omega)} \geq 50\%, \quad (89)$$

and the magnitude of the highest order (dimension 6) term in OPE is less than 10%,

$$\frac{\int_0^\infty d\omega \rho(\omega)_{\text{dim.6}}}{\int_0^\infty d\omega \rho(\omega)_{\text{full}}} \leq 10\%. \quad (90)$$

Finally, we examine whether the curve of $\bar{\Lambda}$ stabilize or not as ω_{th} is varied. If so, the value of $\bar{\Lambda}$ in the stability plateau yields a prediction of the heavy baryon mass.

TABLE I. Values of the vacuum condensates at the normalization scale $\mu = 1 \text{ GeV}$.

Condensate	Value
$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \equiv \langle \bar{q}q \rangle$	$(-0.24 \pm 0.02 \text{ GeV})^3$
$\langle \frac{\alpha_s}{\pi} G^2 \rangle$	$(0.012 \pm 0.006) \text{ GeV}^4$
$g \langle \bar{u}\sigma \cdot Gu \rangle = g \langle \bar{d}\sigma \cdot Gd \rangle$	$(0.8 \pm 0.2) \langle \bar{q}q \rangle$

A. $\Lambda(+)$

In this subsection, we consider positive parity Λ . As was mentioned in the comments on Eq. (51), $\bar{\Lambda}$ depends on the mixing parameter t_S and t_V via their ratio, so we write $t_V/t_S = t$.

Let us first analyze the sum rule at LO in α_s -expansion, Eq. (51). The curve of $\bar{\Lambda}$ as a function of M is found to be stable for $t \simeq 0.5$ – 2.0 . For negative values of t , stability is much worse. Here we fix $t = 0.5$ since the results are not changed if we vary the value of t in the range $t = 0.5$ – 2.0 . The Borel window opens for $\omega_{th} \gtrsim 1.6 \text{ GeV}$. Namely, if we increase ω_{th} , the Borel window starts to open at $\omega_{th} \simeq 1.6 \text{ GeV}$, and becomes wider. The lower bound of the window is $M \simeq 0.38 \text{ GeV}$. In Fig. 5, the curve of $\bar{\Lambda}$ is plotted for several values of ω_{th} in the range $\omega_{th} = 1.6$ – 1.9 GeV . We see that $\bar{\Lambda}$ does not stabilize as ω_{th} is varied.

Next, we consider the sum rule at NLO in α_s -expansion, Eq. (78). As in LO sum rule, good stability is obtained for $t \simeq 0.5$ – 2.0 and the stability is much worse for $t < 0$. We fix $t = 0.5$ for the same reason as in LO sum rule. Borel window opens for $\omega_{th} \gtrsim 1.2 \text{ GeV}$. The lower bound of the window is $M \simeq 0.31 \text{ GeV}$. M -dependence of $\bar{\Lambda}$ for $\omega_{th} = 1.2$ – 1.5 GeV is plotted in Fig. 6. Although $\bar{\Lambda}$ does not stabilize as ω_{th} is varied, the stability is slightly improved and $\bar{\Lambda}$ is reduced from that in LO sum rule. Since the stability plateau does not appear, we cannot make any reliable prediction of $\bar{\Lambda}_{\Lambda(+)}$, and we give conservatively only the lower bound:

$$\bar{\Lambda}_{\Lambda(+)} \gtrsim 0.6 \text{ GeV}. \quad (91)$$

The value of the lower bound is $\bar{\Lambda}$ at $M = 0.31 \text{ GeV}$ and $\omega_{th} = 1.2 \text{ GeV}$, where the Borel window starts to show up.

For comparison, we show M -dependence of $\bar{\Lambda}$ at LO, that with NLO correction only to dimension 0 term, and

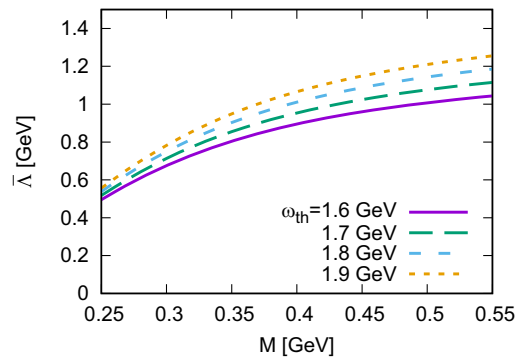


FIG. 5. Borel parameter (M) dependence of $\bar{\Lambda}$ for $\Lambda(+)$ at LO in α_s -expansion, Eq. (51).

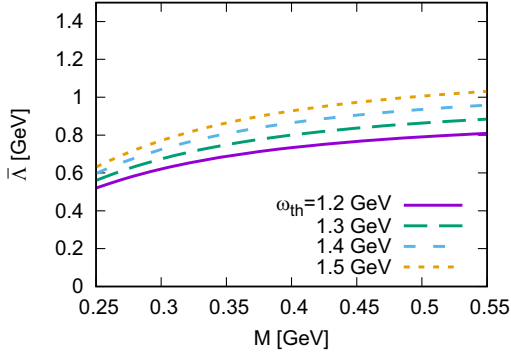


FIG. 6. M -dependence of $\bar{\Lambda}$ for $\Lambda(+)$ at NLO in α_s -expansion, Eq. (78).

that with the corrections to both of dimension 0 and 3 terms in Fig. 7. We see that the correction to dimension 3 term is more important than that to dimension 0 term.

B. $\Lambda(-)$

In this subsection, we consider negative parity Λ . Since $\bar{\Lambda}$ depends on t_p and t_V via their ratio, we write $t_V/t_p = t$.

First we consider the sum rule at LO in α_s , Eq. (51) for $\Lambda(-)$. In this case $\bar{\Lambda}$ is weakly dependent on t , since Eq. (51) for $\Lambda(-)$ depends only on t_p^2 and t_V^2 . Note that Eq. (51) for $\Lambda(+)$ has not only t_p^2 and t_V^2 but also the cross term $t_S t_V$ and therefore t -dependence is stronger than that for $\Lambda(-)$. For any value of t , Borel window opens for $\omega_{th} \gtrsim 2.5$ GeV. Therefore we fix $t = 0.5$. In Fig. 8, we plot $\bar{\Lambda}$ as a function of M for several values of ω_{th} in the range $\omega_{th} = 2.5\text{--}3.1$ GeV. We see $\bar{\Lambda}$ stabilizes for $\omega_{th} \simeq 2.9$ GeV, and the Borel window is found to be $0.42 \text{ GeV} \lesssim M \lesssim 0.5 \text{ GeV}$.

Next we turn to NLO sum rule, Eq. (78) for $\Lambda(-)$. t -dependence is very weak. The reason is the same as that for LO sum rule. For any t , Borel window opens for $\omega_{th} \gtrsim 1.9$ GeV. So we fix $t = 0.5$. We plot M -dependence of $\bar{\Lambda}$ in Fig. 9 for $\omega_{th} = 1.9\text{--}2.5$ GeV. $\bar{\Lambda}$ stabilizes for

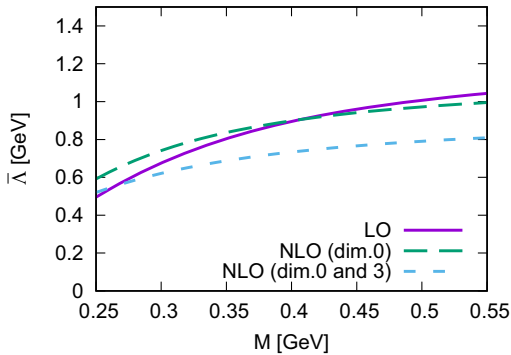


FIG. 7. M -dependence of $\bar{\Lambda}$ for $\Lambda(+)$ at LO, that with NLO correction only to dimension 0 term, and that with the corrections to dimension 0 and 3 terms. ω_{th} is taken to be 1.6 GeV, 1.4 GeV, and 1.2 GeV, respectively.

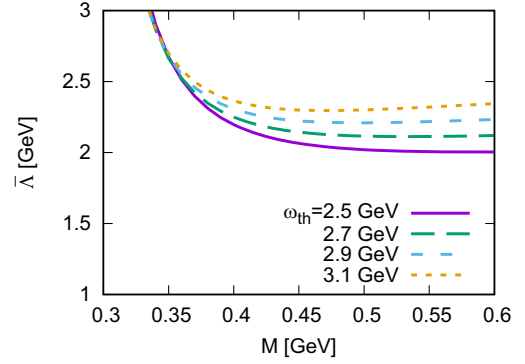


FIG. 8. M -dependence of $\bar{\Lambda}$ for $\Lambda(-)$ at LO in α_s -expansion, Eq. (51).

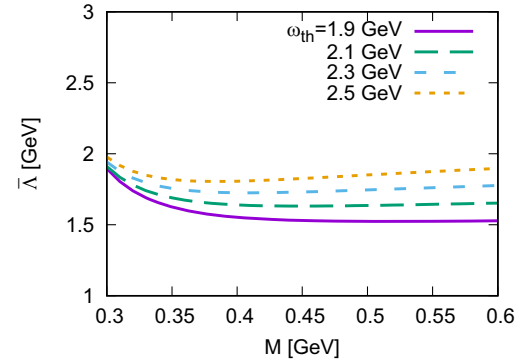


FIG. 9. M -dependence of $\bar{\Lambda}$ for $\Lambda(-)$ at NLO in α_s -expansion, Eq. (78).

$\omega_{th} \simeq 2.1$ GeV, and the Borel window is found to be $0.35 \text{ GeV} \lesssim M \lesssim 0.40 \text{ GeV}$. For this value of ω_{th} and the Borel window, $\bar{\Lambda}_{\Lambda(-)}$ is estimated to be

$$\bar{\Lambda}_{\Lambda(-)} \simeq 1.6 - 1.7 \text{ GeV}. \quad (92)$$

In Fig. 10, we show $\bar{\Lambda}$ for the above two cases for comparison, from which we see the α_s -correction significantly reduce the value of $\bar{\Lambda}$ and improve the stability.

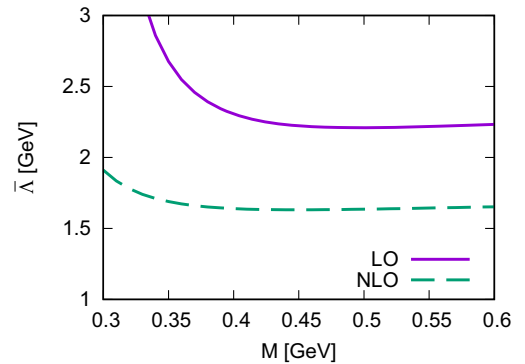


FIG. 10. M -dependence of $\bar{\Lambda}$ for $\Lambda(-)$ at LO and NLO. ω_{th} was taken to be 2.9 GeV and 2.1 GeV, respectively.

C. $\Sigma(+)$

In this subsection we analyze the sum rule of $\Sigma(+)$. We define $t = t_T/t_A$, since $\bar{\Lambda}$ depends on t_A and t_T via their ratio.

We begin with the sum rule at LO in α_s , Eq. (51) for $\Sigma(+)$. The best stability is obtained for $t \simeq 1.0$. Borel window opens for $\omega_{\text{th}} \gtrsim 1.7$ GeV. M -dependence of $\bar{\Lambda}$ for $\omega_{\text{th}} = 1.7\text{--}2.0$ GeV is plotted in Fig. 11.

$\bar{\Lambda}$ at NLO in α_s , Eq. (78) for $\Sigma(+)$, is most stable for $t \simeq 1.0$. Borel window opens for $\omega_{\text{th}} \gtrsim 1.4$ GeV. The lower bound of the window is $M \simeq 0.33$ GeV. M -dependence of $\bar{\Lambda}$ is plotted in Fig. 12 for $\omega_{\text{th}} = 1.4\text{--}1.7$ GeV. The curve does not stabilize as ω_{th} is varied. As in the case of $\Lambda(+)$, we can determine only the lower bound of $\bar{\Lambda}$:

$$\bar{\Lambda}_{\Sigma(+)} \gtrsim 0.9 \text{ GeV}. \quad (93)$$

The value of the lower bound corresponds to $\bar{\Lambda}$ at $M = 0.33$ GeV and $\omega_{\text{th}} = 1.4$ GeV, above which the Borel window opens.

In Fig. 13, we show M -dependence of $\bar{\Lambda}$ at LO, that with NLO correction only to dimension 0 term, and that with the corrections to both of dimension 0 and 3 terms. We see that the stability is slightly improved by including the α_s -correction and the correction to dimension 3 term is more important than that to dimension 0 term.

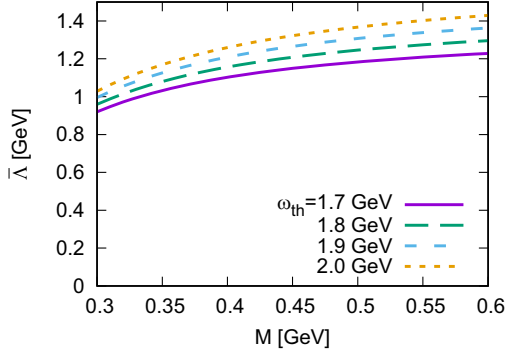


FIG. 11. M -dependence of $\bar{\Lambda}$ for $\Sigma(+)$ at LO, Eq. (51).

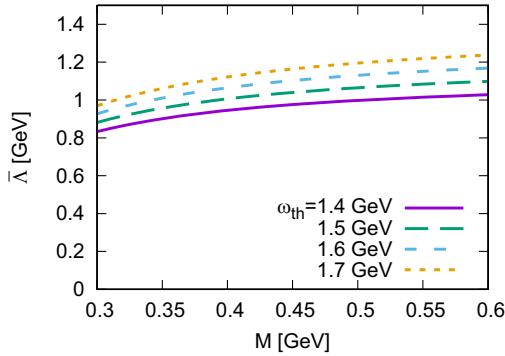


FIG. 12. M -dependence of $\bar{\Lambda}$ for $\Sigma(+)$ at NLO, Eq. (78).

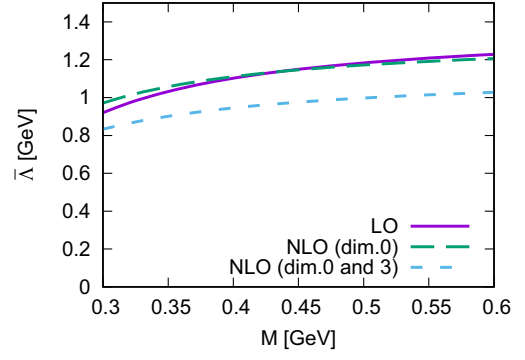


FIG. 13. M -dependence of $\bar{\Lambda}$ for $\Sigma(+)$ at LO in comparison with that including NLO correction only to dimension 0 term and that to dimension 0 and 3 terms. ω_{th} was taken to be 1.7 GeV, 1.6 GeV, and 1.4 GeV, respectively.

D. $\Sigma(-)$

In this subsection, we consider the sum rule for negative parity Σ .

We begin with LO sum rule, Eq. (51) for $\Sigma(-)$. The dependence on t is weak, because of the absence of the cross term, $t_A t_T$. For any t , Borel window opens for $\omega_{\text{th}} \gtrsim 2.5$ GeV. We fix $t = 1.0$. For $\omega_{\text{th}} = 2.5\text{--}3.1$ GeV, M -dependence of $\bar{\Lambda}$ is plotted in Fig. 14.

The NLO sum rule, Eq. (78) for $\Sigma(-)$, is also weakly dependent on t . For any t , Borel window opens for $\omega_{\text{th}} \gtrsim 2.1$ GeV. So We fix $t = 1.0$. For $\omega_{\text{th}} = 2.1\text{--}2.4$ GeV, M -dependence of $\bar{\Lambda}$ is plotted in Fig. 15. At $\omega_{\text{th}} \simeq 2.2$ GeV, $\bar{\Lambda}$ stabilizes and Borel window is $0.36 \text{ GeV} \lesssim M \lesssim 0.40 \text{ GeV}$. We can make an estimate of $\bar{\Lambda}_{\Sigma(-)}$ from the $\bar{\Lambda}$ for ω_{th} and the Borel window found above,

$$\bar{\Lambda}_{\Sigma(-)} \simeq 1.7\text{--}1.8 \text{ GeV}. \quad (94)$$

In Fig. 16, we show $\bar{\Lambda}$ for the above two cases for comparison, from which we see the α_s -correction significantly reduce the value of $\bar{\Lambda}$ and improve the stability.

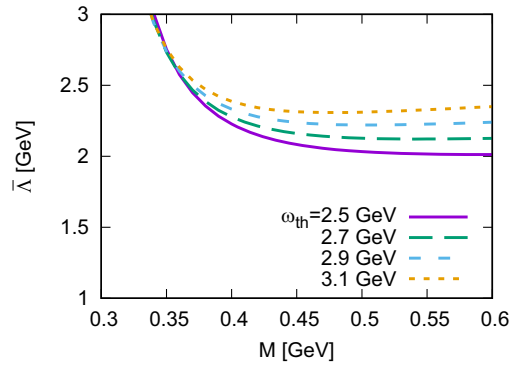
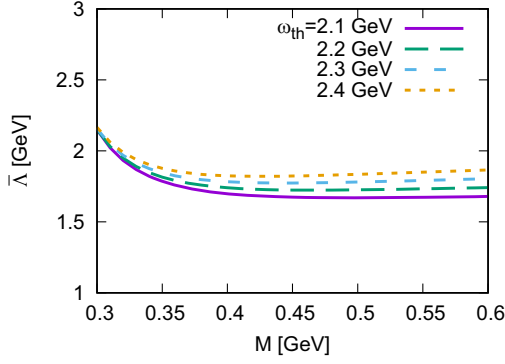
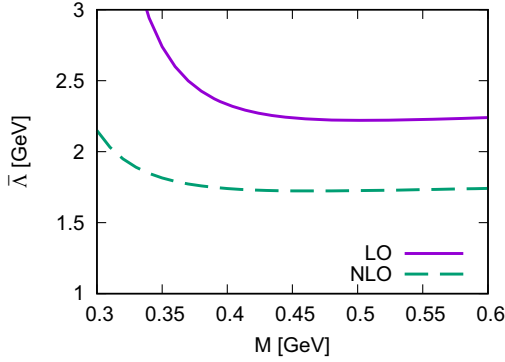


FIG. 14. M -dependence of $\bar{\Lambda}$ for $\Sigma(-)$ at LO, Eq. (51).


 FIG. 15. M -dependence of $\bar{\Lambda}$ for $\Sigma(-)$ at NLO, Eq. (78).

 FIG. 16. M -dependence of $\bar{\Lambda}$ of $\Sigma(-)$ at LO and NLO. ω_{th} was taken to be 2.9 GeV and 2.2 GeV, respectively.

V. SUMMARY AND DISCUSSION

We have formulated the QCD sum rules for positive and negative parity states of heavy baryon containing one heavy quark in the framework of HQET. Choice of the interpolating field is a crucial problem in the QCD sum rule approach. We used the general interpolating field of positive intrinsic parity without covariant derivatives for given isospin and included the components that couple with negative parity states, which were discarded in the previous studies. It has been shown that by applying parity projection onto the correlation function of the interpolating fields the sum rules for respective parity states can be constructed. The sum rule enables us to evaluate the positive or negative parity heavy baryon masses relative to the heavy quark mass, $\bar{\Lambda}$. We have applied the method to Λ and Σ channels. In the OPE we have taken into account the operators up to dimension 6 and included α_s -corrections to the terms of the leading dimension and those of the next dimension.

The effects of α_s -correction were found to be significant, especially for negative parity states. For positive parity states, by including α_s -corrections the Borel parameter (M)-stability is slightly improved, but the improvement is not enough to give any reliable predictions of $\bar{\Lambda}$. It is allowed to give only a lower bound of $\bar{\Lambda}$:

$$\bar{\Lambda}_{\Lambda(+)} \gtrsim 0.6 \text{ GeV}, \quad \bar{\Lambda}_{\Sigma(+)} \gtrsim 0.9 \text{ GeV}. \quad (95)$$

For negative parity states significant improvement of M -stability and large reduction of $\bar{\Lambda}$ were found. In fact, stability plateau appears, which allows us to give an estimation of $\bar{\Lambda}$:

$$\bar{\Lambda}_{\Lambda(-)} \simeq 1.6\text{--}1.7 \text{ GeV}, \quad \bar{\Lambda}_{\Sigma(-)} \simeq 1.7\text{--}1.8 \text{ GeV}. \quad (96)$$

The mass of the heavy baryon is given by $m_B = m_Q + \bar{\Lambda}$ at leading order of $1/m_Q$ expansion in HQET, where m_Q is the current mass of the heavy quark Q . When we use the $\overline{\text{MS}}$ masses of c and b quark [32],

$$m_c = 1.27 \text{ GeV}, \quad m_b = 4.18 \text{ GeV}. \quad (97)$$

we obtain the lower bound of the masses of positive parity states,

$$\begin{aligned} m_{\Lambda_c(+)} &\gtrsim 1.87 \text{ GeV}, \\ m_{\Sigma_c(+)} &\gtrsim 2.17 \text{ GeV}, \\ m_{\Lambda_b(+)} &\gtrsim 4.78 \text{ GeV}, \\ m_{\Sigma_b(+)} &\gtrsim 5.08 \text{ GeV}, \end{aligned} \quad (98)$$

which do not contradict the masses of the observed $J^P = 1/2^+$ states [32]: $\Lambda_c(2286)$, $\Sigma_c(2455)$, $\Lambda_b(5620)$, $\Sigma_b(5811)$. For the masses of negative parity states, we obtain,

$$\begin{aligned} m_{\Lambda_c(-)} &\simeq 2.87\text{--}2.97 \text{ GeV}, \\ m_{\Sigma_c(-)} &\simeq 2.97\text{--}3.07 \text{ GeV}, \\ m_{\Lambda_b(-)} &\simeq 5.78\text{--}5.88 \text{ GeV}, \\ m_{\Sigma_b(-)} &\simeq 5.88\text{--}5.98 \text{ GeV}. \end{aligned} \quad (99)$$

Observed $J^P = 1/2^-$ states of Λ are $\Lambda_c(2595)$ and $\Lambda_b(5912)$ [32]. Our prediction of $m_{\Lambda_b(-)}$ is close to the experimental value, while that of $m_{\Lambda_c(-)}$ is not. The deviation may be attributed to the fact that for charmed baryons HQET is inappropriate and $1/m_Q$ correction is important. Our predictions of $m_{\Sigma_c(-)}$ and $m_{\Sigma_b(-)}$ suggest that observed $\Sigma_c(2800)$ and $\Sigma_b(6097)$ [32], whose spin-parity has not yet been specified, can be the candidates of $J^P = 1/2^-$ states.

A notable feature of the heavy baryon sum rule, which is not seen in the light baryon sum rule, is that the sum rule for the negative parity states do not have chiral odd condensates. Chiral odd condensates contribute only through the nondiagonal correlators, owing to the property of the light diquark. On the other hand, since the heavy quark field couples only with positive parity states, the nondiagonal correlators have only positive parity component. Therefore,

in the heavy quark limit, it is inevitable that the negative parity states do not depend on chiral odd condensates.

If the mass of the heavy quark is large but finite and heavy quark condensation occurs, the diagonal correlators yield heavy quark condensate $\langle \bar{Q}Q \rangle$ terms while the nondiagonal correlators $\langle \bar{q}q \rangle$ terms. As a result, the chiral odd term of the correlation function reads

$$\Pi_{\text{chiral odd}} \sim \langle \bar{Q}Q \rangle (P_+ - P_-) + \langle \bar{q}q \rangle (P_+ + \alpha P_-) + \dots, \quad (100)$$

where the ellipsis denotes higher dimensional terms and α is a coefficient suppressed by powers of μ/m_Q . In the heavy quark limit, Eq. (100) is reduced to

$$\Pi_{\text{chiral odd}} \sim \langle \bar{q}q \rangle P_+ + \dots, \quad (101)$$

and accordingly the chiral odd condensates do not appear in the sum rule for negative parity.

ACKNOWLEDGMENTS

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APPENDIX A: CALCULATION OF THE DIAGRAMS WITH α_s -CORRECTION (FIGS. 3 AND 4)

In this appendix, we explain briefly how the calculation of the diagrams including α_s -correction shown in Figs. 3 and 4 is performed and present the full results.

Although a little bit lengthy, the calculation can be performed straightforwardly by exploiting the method invented by Grozin [31]. It is convenient to calculate the diagrams in momentum space. The diagrams in Figs. 3 and 4 can be expressed with the integrals of subdiagrams shown in Fig. 17(a) or 17(b). Figure 17(a) is the massless particle diagram, whose explicit expressions are defined by

$$\int \frac{d^D k d^D l}{D_1^{n_1} D_2^{n_2} D_3^{n_3} D_4^{n_4} D_5^{n_5}} = -\pi^D (-q^2)^{D-\sum_i n_i} G(n_1, n_2, n_3, n_4, n_5), \quad (A1)$$

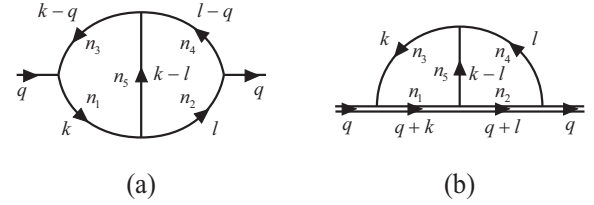


FIG. 17. Subdiagrams of Figs. 3 and 4. The single (double) line stands for the massless (heavy) particle propagator.

where

$$D_1 = -k^2, \quad D_2 = -l^2, \\ D_3 = -(k-q)^2, \quad D_4 = -(l-q)^2, \quad D_5 = -(k-l)^2. \quad (A2)$$

Figure 17(b) is the diagram of heavy particle defined by

$$\int \frac{d^D k d^D l}{D_1^{n_1} D_2^{n_2} D_3^{n_3} D_4^{n_4} D_5^{n_5}} = -\pi^D (-2q_0)^{2(D-n_3-n_4-n_5)} I(n_1, n_2, n_3, n_4, n_5), \quad (A3)$$

where

$$D_1 = \frac{k_0 + q_0}{q_0}, \quad D_2 = \frac{l_0 + q_0}{q_0}, \\ D_3 = -k^2, \quad D_4 = -l^2, \quad D_5 = -(k-l)^2. \quad (A4)$$

Therefore we first calculate those subdiagrams, namely G or I in the above equation, which can be expressed in terms of more easily calculable diagrams [31]. Next, integrating the subdiagrams, we finally obtain the expressions of Figs. 3 and 4.

The results of Figs. 3(a), 3(c), and 3(d) are summarized as follows,

$$\Pi_{XY}(\omega)_{\text{Fig. 3(a)}} = \frac{g^2 N_c C_F}{32\pi^{3D/2}} (-\omega)^5 \left(\frac{-\omega}{\mu}\right)^{3(D-4)} \frac{\Gamma(D/2)\Gamma(7-3D)}{\Gamma(6-2D)} \\ \times [2I(1, 1, 0, 1, 1) - I(1, 1, 1, 1, 0) - 2I(0, 1, 1, 1, 1) + I(1, 1, 1, 1, 1)] M_2, \quad (A5)$$

$$\Pi_{XY}(\omega)_{\text{Fig. 3(c)}} = \frac{g^2 N_c C_F}{8\pi^{3D/2}} (-1)^{3D+7} (-\omega)^5 \left(\frac{-\omega}{\mu}\right)^{3(D-4)} \frac{\Gamma(D/2)^2 \Gamma(7-3D) \Gamma(3-D) \Gamma(D/2-1)}{\Gamma(5-D)} M_2, \quad (A6)$$

$$\Pi_{XY}(\omega)_{\text{Fig. 3(d)}} = \frac{g^2 N_c C_F}{32\pi^{3D/2}} (-1)^{2D} (-\omega)^5 \left(\frac{-\omega}{\mu}\right)^{3(D-4)} \frac{(2-D)\Gamma(D/2)\Gamma(7-3D)\Gamma(2-D/2)\Gamma(D/2-1)^2}{\Gamma(3-D/2)} M_2, \quad (\text{A7})$$

where the matrix M_2 is given by

$$M_2 = \begin{cases} 4P_{\pm} & (XY = SS, PP) \\ 4[P_{\pm} + (D-1)P_{\mp}] & (XY = VV, AA) \\ 8(D-1)[2P_+ + (D-2)P_-] & (XY = TT) \\ 0 & (XY = \text{else}) \end{cases} \quad (\text{A8})$$

The result of Fig. 3(b) has more complicated form. For each component they are given by

$$\begin{aligned} \Pi_{SS,PP}(\omega)_{\text{Fig. 3(b)}} &= \frac{g^2 N_c C_F}{64\pi^{3D/2}} (-\omega)^5 \left(\frac{-\omega}{\mu}\right)^{3(D-4)} \frac{\Gamma(3D/2-3)\Gamma(7-3D)}{\Gamma(3-D)} \\ &\times \{4[-G(0,1,1,1,1) - G(1,0,1,1,1) - G(1,1,0,1,1) - G(1,1,1,0,1) + G(1,1,1,1,1)] \\ &- 2(D-4)G(1,1,1,1,0) + 2(D-2)[G(0,1,1,0,1) + G(1,0,0,1,1)]\}, \end{aligned} \quad (\text{A9})$$

$$\begin{aligned} \Pi_{VV,AA}(\omega)_{\text{Fig. 3(b)}} &= \frac{g^2 N_c C_F}{64\pi^{5D/2}} (-\omega)^5 \left(\frac{-\omega}{\mu}\right)^{3(D-4)} \frac{\Gamma(3D/2-2)\Gamma(7-3D)}{(D-1)\Gamma(3-D)} \\ &\times \left\{ \left[A_{VA} + \frac{7-3D}{2(3-D)} B_{VA} \right] P_{\pm} + (1-D) \left[A_{VA} + \frac{1}{2(3-D)} B_{VA} \right] P_{\mp} \right\}, \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} \Pi_{TT}(\omega)_{\text{Fig. 3(b)}} &= -\frac{g^2 N_c C_F}{32\pi^{5D/2}} (-\omega)^5 \left(\frac{-\omega}{\mu}\right)^{3(D-4)} \frac{\Gamma(3D/2-3)\Gamma(7-3D)}{\Gamma(3-D)} \\ &\times \left\{ (1-D) \left[2A_T + \frac{8-3D}{3-D} B_T \right] P_+ + (2-D) \left[(1-D)A_T + \frac{7-4D}{3-D} B_T \right] P_- \right\}, \end{aligned} \quad (\text{A11})$$

where A_{VA} , B_{VA} , A_T and B_T are given in terms of G ,

$$\begin{aligned} A_{VA} &= 2\pi^D (D-2) [4G(-1,1,1,0,1) + 2(D-5)G(0,1,1,0,1) - (D-8)G(1,1,1,1,0) \\ &- 8G(0,1,1,1,1) + 2G(1,1,1,1,-1) + 2G(1,1,1,1,1)], \end{aligned} \quad (\text{A12})$$

$$\begin{aligned} B_{VA} &= -2\pi^D (D-2) [4DG(-1,1,1,0,1) - 8G(0,1,1,0,1) - (D-8)G(1,1,1,1,0) \\ &- 8G(0,1,1,1,1) + 2G(1,1,1,1,-1) + 2G(1,1,1,1,1)], \end{aligned} \quad (\text{A13})$$

$$\begin{aligned} A_T &= \frac{2\pi^D}{(D-1)(D-2)} \{8(D-4)^2 G(-1,1,1,0,1) - 4(D-3)(D-4)G(1,1,1,1,-1) \\ &+ 8(D-2)(D-3)G(0,1,1,1,1) + 2[28 + 2D(D-7) - (D-2)(D-4)(D-5)]G(1,0,0,1,1) \\ &+ [2(D-8) + (D-4)(D^2 - 13D + 20)]G(1,1,1,1,0) - 2(D-2)(D-3)G(1,1,1,1,1)\}, \end{aligned} \quad (\text{A14})$$

$$\begin{aligned} B_T &= \frac{2\pi^D}{(D-1)(D-2)} \{4D(D-4)^2 G(-1,1,1,0,1) + 2(D-4)^2 G(1,1,1,1,-1) \\ &- 8(D-2)^2 G(0,1,1,1,1) - 8(D^2 - 6D + 10)G(1,0,0,1,1) \\ &- [D(D-8)^2 - 80]G(1,1,1,1,0) + 2(D-2)^2 G(1,1,1,1,1)\}. \end{aligned} \quad (\text{A15})$$

The results of Fig. 4 are summarized as follows,

$$\begin{aligned} \Pi_{XY}(\omega)_{\text{Fig. 4(a)}} &= \frac{-g^2 C_F}{128\pi^D} (-\omega)^2 \left(\frac{-\omega}{\mu}\right)^{2(D-4)} \\ &\quad \times [2I(1, 1, 0, 1, 1) - I(1, 1, 1, 1, 0) + 2I(0, 1, 1, 1, 1) + I(1, 1, 1, 1, 1)] M_{31}, \end{aligned} \quad (\text{A16})$$

$$\Pi_{XY}(\omega)_{\text{Fig. 4(b)}} = \frac{-g^2 C_F}{32\pi^D} (-\omega)^2 \left(\frac{-\omega}{\mu}\right)^{2(D-4)} \frac{\Gamma(D/2 - 1)\Gamma(D/2)\Gamma(4 - D)\Gamma(6 - 2D)}{\Gamma(5 - D)} M_{31}, \quad (\text{A17})$$

$$\Pi_{XY}(\omega)_{\text{Fig. 4(c)}} = \frac{-g^2 C_F}{256\pi^D} (-\omega)^2 \left(\frac{-\omega}{\mu}\right)^{2(D-4)} \frac{\Gamma(3 - D/2)\Gamma(D/2 - 1)^2\Gamma(6 - 2D)}{\Gamma(4 - D/2)} M_{32}, \quad (\text{A18})$$

$$\Pi_{XY}(\omega)_{\text{Fig. 4(d)}} = \frac{g^2 C_F (-1)^{7-D}}{64\pi^D} (-\omega)^2 \left(\frac{-\omega}{\mu}\right)^{2(D-4)} D\Gamma(D/2 - 2)\Gamma(D/2)\Gamma(6 - 2D) M_{31}, \quad (\text{A19})$$

$$\Pi_{XY}(\omega)_{\text{Fig. 4(e)}} = \frac{g^2 C_F}{16\pi^D} (-\omega)^2 \left(\frac{-\omega}{\mu}\right)^{2(D-4)} \frac{\Gamma(D/2 - 1)\Gamma(D/2)\Gamma(6 - 2D)\Gamma(3 - D)}{\Gamma(5 - D)} M_{31}, \quad (\text{A20})$$

$$\Pi_{XY}(\omega)_{\text{Fig. 4(f)}} = \frac{g^2 C_F (-1)^D}{64\pi^D} (-\omega)^2 \left(\frac{-\omega}{\mu}\right)^{2(D-4)} \frac{\Gamma(D/2 - 1)\Gamma(D/2)\Gamma(6 - 2D)\Gamma(2 - D/2)}{\Gamma(3 - D/2)} M_{31}, \quad (\text{A21})$$

where the matrices M_{31} and M_{32} are given by

$$M_{31} = (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \times \begin{cases} 4P_+ & (XY = SV, VS) \\ 8(D-1)P_+ & (XY = AT, TA) \\ 0 & (XY = \text{else}) \end{cases} \quad (\text{A22})$$

$$M_{32} = (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle) \times \begin{cases} \frac{4(D-2)(D-6)(D+1)}{D-4} P_+ & (XY = SV, VS) \\ \frac{8(D-1)(D-2)(D-3)(D-6)}{D-4} P_+ & (XY = AT, TA) \\ 0 & (XY = \text{else}) \end{cases} \quad (\text{A23})$$

Following the description in Ref. [31], the subdiagram integral, G and I appearing in Eq. (A5), Eqs. (A9)–(A11) and Eq. (A16), can be reduced to some combinations of the diagrams which can be easily calculated. The procedure is straightforward, but the results are so lengthy that we cannot help omitting them here.

APPENDIX B: RENORMALIZATION OF INTERPOLATING FIELD

In this appendix, we calculate α_s -corrections of the interpolating fields, Eqs. (13)–(17), in $D = 4 + 2\epsilon$ space-time dimension, and determine the corresponding renormalization constants.

The interpolating field under consideration is

$$\eta_X = (qC\Gamma_X q)\Gamma'_X h, \quad (X = S, P, V, A, T), \quad (\text{B1})$$

where $\Gamma_X^{(\prime)}$ is

$$(\Gamma_X, \Gamma'_X) = \begin{cases} (1, \gamma_5) & (X = P) \\ (\gamma_5, 1) & (X = S) \\ (\gamma_5 \gamma_\mu, \gamma^\mu) & (X = V) \\ (\gamma_\mu, \gamma^\mu \gamma_5) & (X = A) \\ (\sigma_{\mu\nu}, \sigma^{\mu\nu} \gamma_5) & (X = T) \end{cases} \quad (\text{B2})$$

Renormalization constant of the interpolating field, Z_{η_X} , is defined by

$$\eta_X^B = Z_{\eta_X} \eta_X^R, \quad (\text{B3})$$

where η_X^B and η_X^R denote the bare and renormalized interpolating fields, respectively. Then Eq. (B3) can be rewritten as

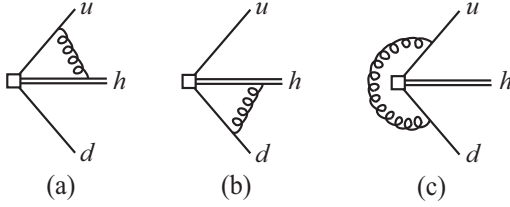


FIG. 18. Diagrams corresponding to α_s -corrections to the interpolating field, Eq. (B1).

$$\eta_X^R = (q^R C \Gamma_X q^R) \Gamma'_X h^R + \left(\frac{Z_q \sqrt{Z_h}}{Z_{\eta_X}} - 1 \right) (q^R C \Gamma_X q^R) \Gamma'_X h^R \quad (\text{B4})$$

with Z_q and Z_h respectively being the wave function renormalization of the light quark and the heavy quark field given by

$$Z_q = 1 + C_F \frac{\alpha_s}{4\pi\epsilon}, \quad Z_h = 1 - 2C_F \frac{\alpha_s}{4\pi\epsilon}. \quad (\text{B5})$$

Z_{η_X} is determined so that the counterterm [the second term in Eq. (B4)] cancels the UV-pole of the α_s -correction of η_X . The diagram corresponding to α_s -correction of η_X are depicted in Fig. 18, and its UV-pole reads

$$\text{UV-pole of Fig. 18} = -k \left(1 + \frac{1}{N_c} \right) \frac{\alpha_s}{\pi \epsilon} \epsilon_{abc} (u_a^T C \Gamma_X d_b) \Gamma'_X h_c, \quad (\text{B6})$$

where the coefficient k for each channel is given by

$$k = \begin{cases} \frac{3}{4} & (X = S, P) \\ \frac{3}{8} & (X = V, A) \\ \frac{1}{4} & (X = T) \end{cases} \quad (\text{B7})$$

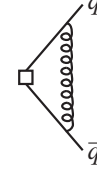


FIG. 19. Diagrams corresponding to α_s -corrections to $\bar{q}q$.

Z_{η_X} is chosen so that the counter term in Eq. (B4) is equal to the minus of Eq. (B6). Z_{η_X} obtained in this way are Eqs. (64)–(66).

APPENDIX C: RENORMALIZATION OF $\bar{q}q$

In this appendix, we determine the renormalization constant of $\bar{q}q$.

Renormalization constant of $\bar{q}q$, $Z_{\bar{q}q}$, is defined by

$$(\bar{q}q)^B = Z_{\bar{q}q} (\bar{q}q)^R, \quad (\text{C1})$$

where $(\bar{q}q)^B$ and $(\bar{q}q)^R$ denote the bare and renormalized operators, respectively. Then Eq. (C1) can be rewritten as

$$(\bar{q}q)^R = \bar{q}^R q^R + \left(\frac{Z_q}{Z_{\bar{q}q}} - 1 \right) \bar{q}^R q^R. \quad (\text{C2})$$

On the other hand, UV-pole of α_s -correction of $\bar{q}q$, shown in Fig. 19, reads

$$\text{UV-pole of Fig. 19} = -C_F \frac{\alpha_s}{\pi \epsilon} \bar{q}q. \quad (\text{C3})$$

$Z_{\bar{q}q}$ is determined so that the counter term [the second term in Eq. (C2)] cancels the UV-pole of the α_s -correction of $\bar{q}q$, Eq. (C3). Thus we obtain $Z_{\bar{q}q}$ as in Eq. (75).

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- [1] H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, *Rep. Prog. Phys.* **80**, 076201 (2017).
 [2] H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, *Rep. Prog. Phys.* **86**, 026201 (2022).
 [3] H. M. Yang, H. X. Chen, E. L. Cui, A. Hosaka, and Q. Mao, *Eur. Phys. J. C* **80**, 80 (2020).
 [4] E. V. Shuryak, *Nucl. Phys.* **B198**, 83 (1982).
 [5] A. G. Grozin and O. I. Yakovlev, *Phys. Lett. B* **285**, 254 (1992).
 [6] E. Bagan, M. Chabab, H. G. Dosch, and S. Narison, *Phys. Lett. B* **301**, 243 (1993).
 [7] Y. B. Dai, C. S. Huang, C. Liu, and C. D. Lü, *Phys. Lett. B* **371**, 99 (1996).
 [8] Y. B. Dai, C. S. Huang, M. Q. Huang, and C. Liu, *Phys. Lett. B* **387**, 379 (1996).
 [9] S. Groote, J. G. Körner, and O. I. Yakovlev, *Phys. Rev. D* **55**, 3016 (1997).
 [10] S. Groote, J. G. Körner, and O. I. Yakovlev, *Phys. Rev. D* **56**, 3943 (1997).
 [11] D. W. Wang, M. Q. Huang, and C. Z. Li, *Phys. Rev. D* **65**, 094036 (2002).
 [12] J. P. Lee, C. Liu, and H. S. Song, *Phys. Lett. B* **476**, 303 (2000).
 [13] S. L. Zhu, *Phys. Rev. D* **61**, 114019 (2000).
 [14] C. S. Huang, A. Zhang, and S. L. Zhu, *Phys. Lett. B* **492**, 288 (2000).

- [15] D. W. Wang and M. Q. Huang, *Phys. Rev. D* **68**, 034019 (2003).
- [16] X. Liu, H. X. Chen, Y. R. Liu, A. Hosaka, and S. L. Zhu, *Phys. Rev. D* **77**, 014031 (2008).
- [17] H. X. Chen, W. Chen, Q. Mao, A. Hosaka, X. Liu, and S. L. Zhu, *Phys. Rev. D* **91**, 054034 (2015).
- [18] Q. Mao, H. X. Chen, W. Chen, A. Hosaka, X. Liu, and S. L. Zhu, *Phys. Rev. D* **92**, 114007 (2015).
- [19] H. X. Chen, Q. Mao, W. Chen, A. Hosaka, X. Liu, and S. L. Zhu, *Phys. Rev. D* **95**, 094008 (2017).
- [20] H. M. Yang and H. X. Chen, *Phys. Rev. D* **101**, 114013 (2020).
- [21] H. M. Yang and H. X. Chen, *Phys. Rev. D* **104**, 034037 (2021).
- [22] D. Jido, N. Kodama, and M. Oka, *Phys. Rev. D* **54**, 4532 (1996).
- [23] D. Jido and M. Oka, [arXiv:9611322](https://arxiv.org/abs/9611322).
- [24] Y. Kondo, O. Morimatsu, and T. Nishikawa, *Nucl. Phys.* **A764**, 303 (2006).
- [25] Y. Kondo, O. Morimatsu, T. Nishikawa, and Y. Kanada-En'yo, *Phys. Rev. D* **75**, 034010 (2007).
- [26] E. Bagan, P. Ball, V. M. Braun, and H. G. Dosch, *Phys. Lett. B* **278**, 457 (1992).
- [27] T. Nishikawa and K. Tanaka, *Nucl. Phys.* **B879**, 110 (2014).
- [28] Y. Chung, H. G. Dosch, M. Kremer, and D. Schall, *Nucl. Phys.* **B197**, 55 (1982).
- [29] B. L. Ioffe, *Nucl. Phys.* **B188**, 317 (1981), **B191**, 591 (1981).
- [30] D. J. Broadhurst and A. G. Grozin, *Phys. Lett. B* **267**, 105 (1991).
- [31] A. G. Grozin, *Lectures on QED and QCD: Practical Calculation and Renormalization of One- and Multi-loop Feynman Diagrams* (World Scientific, Singapore, 2007).
- [32] R. L. Workman *et al.* (Particle Data Group), *Prog. Theor. Phys.* **2022**, 083C01 (2022).