Improved analysis of the decay width of $t \to Wb$ up to N^{3}LO QCD corrections

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In this paper, we analyze the top-quark decay $t \to Wb$ up to next-to-next-to-next-to-leading order (N^{3}LO) QCD corrections. For the purpose, we first adopt the principle of maximum conformality (PMC) to deal with the initial perturbative QCD (pQCD) series. Then we adopt the Bayesian analysis approach, which quantifies the unknown higher-order terms' contributions in terms of a probability distribution, to estimate the possible magnitude of the uncalculated N^{4}LO terms. In our calculation, an effective strong coupling constant $\alpha_s(Q_*)$ is determined by using all nonconformal $\{\beta_i\}$ terms associated with the renormalization group equation. This leads to a next-to-leading-log PMC scale $Q_*^{(\mathrm{NLL})} = 10.3048$ GeV, which can be regarded as the correct momentum flow of the process. Consequently, we obtain an improved scale-invariant pQCD prediction for the top-quark decay width, e.g., $\Gamma_t^{\mathrm{tot}} = 1.3120 \pm 0.0038$ GeV, whose error is the squared average of the uncertainties from the decay width of W-boson $\Delta\Gamma_W = \pm 0.042$ GeV, the coupling constant $\Delta\alpha_s(m_Z) = \pm 0.0009$, and the predicted N^{4}LO-terms. The magnitude of the top-quark pole mass greatly affects the total decay width. By further taking the PDG top-quark pole mass error from cross-section measurements into consideration, e.g., $\Delta m_t = \pm 0.7$ GeV, we obtain $\Gamma_t^{\mathrm{tot}} = 1.3120^{+0.0194}_{-0.0192}$ GeV.

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I. INTRODUCTION

The top quark is the heaviest elementary particle in the Standard Model (SM), and it is remarkable for its decay processes. Compared to other quarks, the top quark has a much larger mass and a significantly shorter lifetime. It does not have enough time to form any hadron before decaying itself. The top quark's substantial Yukawa coupling with the Higgs boson exerts considerable influence on the SM observables. Furthermore, it serves as an exceptional laboratory for probing fundamental interactions at the electroweak (EW) symmetry-breaking scale and beyond.

Within the SM, the top-quark decays almost exclusively into a W-boson and a b-quark. Thus the top-quark total decay width can be deduced from the partial decay width $\Gamma(t \to Wb)$ and the branching fraction $\mathcal{B}(t \to Wb)$. In 2012, using the integrated luminosity of 5.4 fb^{-1}, which is collected by the D0 Collaboration at the Tevatron $p\bar{p}$ Collider, $\Gamma_t = 2.00^{+0.47}_{-0.43}$ GeV was extracted [1]. In 2014, the CMS Collaboration provided a better determination of the total width, $\Gamma_t = 1.36 \pm 0.02(\mathrm{stat})^{+0.14}_{-0.11}(\mathrm{syst})$ GeV [2], where "stat." and "syst." are short notations for statistical and systematic errors, respectively. This measurement is based on the assumption $\mathcal{B}(t \to Wq) = 1$, which includes the sum over all down-type quarks $q = (b, s, d)$. In 2017, an initial direct measurement was conducted by an ATLAS analysis, which involves the direct fitting of reconstructed lepton + jets events by using the integrated luminosity of 20.2 fb^{-1} at a center-of-mass energy of $\sqrt{s} = 8$ TeV. This resulted in $\Gamma_t = 1.76 \pm 0.33(\mathrm{stat})^{+0.79}_{-0.68}(\mathrm{syst})$ GeV [3]. In 2019, a measurement by the ATLAS Collaboration, using the integrated luminosity of 139 fb^{-1} at the center-of-mass energy of $\sqrt{s} = 13$ TeV, employed a template fit to the invariant mass of the lepton-b-quark in dilepton final states. This yielded $\Gamma_t = 1.94^{+0.52}_{-0.49}$ GeV [4]. The Particle Data Group (PDG) reported the world average as $\Gamma(t \to Wq) = 1.42^{+0.19}_{-0.15}$ GeV and $\mathcal{B}(t \to Wb) = \Gamma(t \to Wb)/\Gamma(t \to Wq) = 0.957 \pm 0.034$ [5].

Theoretically, the next-to-leading order (NLO) quantum chromodynamics (QCD) corrections were first computed in Refs. [6–9], while the NLO EW corrections were provided

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in Refs. [10–12]. The next-to-next-to-leading order (N^2LO) QCD corrections for the $t \to Wb$ decay had been done by using the asymptotic expansion [13–17], and the complete N^2LO analytical results were available in Ref. [18]. The N^2LO polarized decay rates were calculated in Refs. [19,20]. Recently, the next-to-next-to-next-to-leading order (N^3LO) corrections in the large-N_C limit have been presented in Ref. [21], and the first complete high-precision numerical results of N^3LO QCD corrections have also been given in Ref. [22]. Those two predictions agree well with each other, indicating that the leading-color contributions are dominant and the approximation of the large-N_C limit is highly reliable at least for this particular process.

Because of the large kinematic scale $Q \sim \mathcal{O}(m_t)$ and the small strong coupling constant $\alpha_s(m_t) \sim 0.1$, the perturbative QCD (pQCD) series for the $t \to Wb$ total decay width up to the N^3LO-level exhibits good convergence. It, however, still has a sizable renormalization scale dependence due to the divergent renormalon terms [23–25]. Practically, one usually selects the renormalization scale as $\mu_R = m_t$ so as to eliminate the divergent large logarithmic terms such as $\ln(\mu_R^2/m_t^2)$, and then vary it within a certain range such as $\mu_R \in [m_t/\xi, \xi m_t]$ with ξ usually being chosen as 2, 3, 4, etc., to account for its uncertainty. This treatment is referred to as the conventional scale-setting approach. It is evident that this approach is arbitrary, and the perturbative nature of the series heavily relies on the choice of ξ, thereby diminishing the reliability of the final theoretical prediction. As is well-known, the physical observable, which corresponds to an infinite-order perturbative series, should be scale invariant [26–29]; and it is important to know whether such scale invariance can also be achieved for a fixed-order series. Simply requiring the fixed-order series to be scale invariant does not achieve this goal, as it is not true and explicitly breaks standard renormalization group invariance (RGI) [30]. This simple requirement implicitly assumes that all uncalculated higher-order terms contribute zero. Therefore, a proper method needs to be introduced to improve the perturbative series before applying the scale-setting procedures.

Many scale-setting approaches have been suggested to improve the fixed-order series so as to achieve a scale-invariant prediction. Especially, by using the n_f terms as a guide, the Brodsky-Lepage-Mackenzie (BLM) approach [31] automatically resums the corresponding gluons as well as the quark vacuum-polarization contributions, which then leads to a scheme-and-scale-invariant prediction [32]. The NLO commensurate scale relations that ensure the scheme-and-scale invariance at the NLO level have also been given there. Those relations indicate that if the expansion coefficients match well with the corresponding α_s, exactly scheme-independent predictions can be achieved. Since the running behavior of α_s is governed by the renormalization group equation (RGE) or the β function [33,34], to deal with the $\{\beta_i\}$ terms involved in the RGE is then more fundamental than to deal with the n_f terms. Lately, the BLM is developed to the principle of maximum conformality (PMC) [35–39], which perfects the idea behind BLM and offers a reliable extension of the BLM to all orders. In the Abelian limit [40], BLM and PMC reduce to the well-known Gell-Mann–Low approach [41] for QED. Subsequently, the PMC single-scale-setting approach (PMCs), as an effective alternative to the original PMC multi-scale-setting approach, has also been proposed in Refs. [42,43] from two distinct but equivalent perspectives. It has been demonstrated that the PMC prediction is independent of any choice of renormalization scheme and scale [44], being consistent with the self-consistency requirements of the renormalization group [45,46]. The PMCs approach also greatly suppresses the residual scale dependence [47] of the PMC predictions due to unknown even higher-order terms of the pQCD series. In this paper, we adopt the PMCs approach to deal with the $t \to Wb$ total decay width up to N^3LO QCD corrections.

For any perturbative series, there are some uncertainties caused by the unknown higher-order terms (UHO-terms). Since the exact pQCD result is unknown, it would be helpful to quantify the UHO-terms' contribution in terms of a probability distribution. Following the idea of Bayesian analysis (BA) [48–51], the conditional probability of the unknown perturbative coefficient is first given by a subjective prior distribution, which is then updated iteratively according to the Bayes' theorem as more and more information has been included. It has been found that the generally more convergent and scheme-and-scale-invariant PMC series provides a more reliable basis than conventional series for estimating the contributions from the UHO-terms [52–55]. In this paper, we adopt the BA approach to estimate the contributions from the unknown N^4LO-terms of the PMC series.

The remaining parts of the paper are organized as follows. Section II gives the formulas for the top-quark decay process, $t \to Wb$, up to N^3LO QCD corrections, and then shows how to apply the PMC scale-setting procedures to the present process. Section III presents numerical results and discussions for the top-quark decay. Section IV is reserved for a summary.

II. CALCULATION TECHNOLOGY

By neglecting the corrections caused by the finite b-quark mass and the off-shell W-boson effect, the total decay width of the top-quark decay $t \to Wb$ up to N^3LO QCD corrections can be expressed as

$$\Gamma_{t\to Wb}^{\rm QCD} = \frac{G_F m_t^3 |V_{\rm tb}|^2}{8\sqrt{2}\pi} \sum_{i=0}^{3} f_i(\omega;\mu_R)\alpha_s^i(\mu_R), \tag{1}$$

where μ_R is the renormalization scale, G_F is the Fermi constant, m_t is the top-quark pole mass, $V_{\rm tb}$ is the

Cabibbo-Kobayashi-Maskawa (CKM) matrix element, and $\omega = m_W^2/m_t^2$. The LO decay width that is caused by weak interaction provides a dominant contribution to the total decay width. Because the W-boson will decay promptly into leptons or quarks, we rewrite the top-quark decay width as

$$\Gamma^{\rm QCD}_{t\to W^* b} = \Gamma_{\rm LO}[1 + R_t(\mu_R)], \tag{2}$$

where W^* indicates the W-boson may be off-shell and the QCD corrections

$$R_t(\mu_R) = \sum_{i=1}^{3} r_i(\mu_R)\alpha_s^i(\mu_R) + \mathcal{O}(\alpha_s^4), \tag{3}$$

where for the coefficients r_i ($i \in [1,3]$), we have [6,18]

$$r_i(\mu_R) = \frac{1}{\Gamma_{\rm LO}} \frac{G_F m_t^3 |V_{\rm tb}|^2}{8\sqrt{2}\pi} \frac{\omega\gamma}{\pi} \int_0^{\bar{\varepsilon}} \frac{f_i(\varepsilon, x; \mu_R)}{(x-\omega)^2 + \omega^2\gamma^2} {\rm d}x. \tag{4}$$

Here $\varepsilon = m_b^2/m_t^2$, $\bar{\varepsilon} = (1 - m_b/m_t)^2$ and $\gamma = \Gamma_W/m_W$ with Γ_W being the W-boson total decay width. The LO decay width is scale invariant and equals

$$\Gamma_{\rm LO} = \frac{G_F m_t^3 |V_{\rm tb}|^2}{8\sqrt{2}\pi} \frac{\omega\gamma}{\pi} \int_0^{\bar{\varepsilon}} \frac{f_0(\varepsilon, x)}{(x-\omega)^2 + \omega^2\gamma^2} {\rm d}x. \tag{5}$$

To get the analytic expression, the integration involving the polylogarithm functions can be accomplished with the assistance of the POLYLOGTOOLS [56] and GINAC [57] packages. In Eqs. (4) and (5), the finite b-quark mass contributions have been included; e.g., the coefficients $f_i(\omega; \mu_R)$ in Eq. (2) have been replaced by $f_i(\varepsilon, \omega; \mu_R)$ and the upper limit of the integral in Eqs. (4) and (5) becomes $\bar{\varepsilon}$ instead of 1. Currently, the finite b-quark mass effect is known only up to NLO accuracy [6]. For convenience, we list the analytic coefficients f_0 and f_1 at the scale $\mu_R = m_t$, including and excluding b-quark mass effects, in Appendix A. The analytic form of the coefficients f_2 and f_3 can be found in Refs. [18,21], where f_3 contains only the most dominant leading color contribution that gives the most dominant contributions.

Following the standard PMC single-scale-setting procedures [42,43], by using the QCD degeneracy relations among different orders [58], one can distribute the original n_f series into conformal and nonconformal terms, and then use the RGE-involved non conformal $\{\beta_i\}$ terms to determine an overall effective running coupling $\alpha_s(Q_*)$ for $t \to Wb$ decay, where Q_* is referred to as the PMC scale, which can be viewed as the effective momentum flow for the process. More explicitly, the coefficients r_i in Eq. (2) can be parametrized as follows:

$$r_1 = r_{1,0}, \tag{6}$$

$$r_2 = r_{2,0} + \beta_0 r_{2,1}, \tag{7}$$

$$r_3 = r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}. \tag{8}$$

Here, $r_{i,0}$ are scale-invariant conformal coefficients and $r_{i,j(\neq 0)}$ are nonconformal coefficients. At present, the $\{\beta_i\}$ functions have been computed up to the five-loop level in the modified minimal-subtraction scheme ($\overline{\rm MS}$-scheme) [59,60], e.g., $\beta_0 = (11 - 2n_f/3)/(4\pi)$ and $\beta_1 = (102 - 38n_f/3)/(4\pi)^2$ for n_f active flavors.

After applying the PMC, the divergent renormalon terms, which grow as various powers of β_0 under the approximation $\beta_i \simeq \beta_0^{i+1}$, have been removed, we then obtain a more convergent pQCD series as

$$R_t|_{\rm PMC} = \sum_{i=1}^{3} r_{i,0}\alpha_s^i(Q_*) + \mathcal{O}(\alpha_s^4), \tag{9}$$

where Q_* is fixed by requiring all nonconformal terms to vanish. Using the present known pQCD series up to the N^{3}LO level, the PMC scale Q_* can be fixed up to next-to-leading-log (NLL) accuracy, i.e.,

$$\ln\frac{Q_*^2}{Q^2} = S_0 + S_1\alpha_s(Q_*) + \mathcal{O}(\alpha_s^2), \tag{10}$$

where the coefficients $S_{0,1}$ are

$$S_0 = -\frac{\hat{r}_{2,1}}{\hat{r}_{1,0}}, \tag{11}$$

$$S_1 = \frac{2(\hat{r}_{2,0}\hat{r}_{2,1} - \hat{r}_{1,0}\hat{r}_{3,1})}{\hat{r}_{1,0}^2} + \frac{\hat{r}_{2,1}^2 - \hat{r}_{1,0}\hat{r}_{3,2}}{\hat{r}_{1,0}^2}\beta_0, \tag{12}$$

where $\hat{r}_{i,j} \equiv r_{i,j}|_{\mu_R = Q}$. Since all the nonconformal $\{\beta_i\}$ terms of the series (3) that are associated with the RGE have been absorbed into $\alpha_s(Q_*)$, the resulting pQCD series (9) becomes a conformal series. The magnitude of α_s then matches well with the expansion coefficients of series, yielding naturally scheme-independent theoretical predictions at any fixed order [30,44,46,61,62].

III. NUMERICAL RESULTS AND DISCUSSIONS

For numerical calculations, the following values are taken as the input parameters [5]:

$$m_b = 4.78~{\rm GeV}, \qquad m_t = 172.5 \pm 0.7~{\rm GeV},$$

$$m_W = 80.377~{\rm GeV}, \qquad \Gamma_W = 2.085 \pm 0.042~{\rm GeV},$$

$$m_Z = 91.1876~{\rm GeV}, \qquad G_F = 1.1663788 \times 10^{-5}~{\rm GeV}^2,$$

and $\alpha_s(m_Z) = 0.1179 \pm 0.0009$. The NLO EW correction $\Delta_{\rm EW}$ can be calculated using the formulas given in Refs. [11,12,63] together with the recently issued *Mathematica* program TOPWIDTH [18,21]. By using the above inputs, we obtain $\Delta_{\rm EW} = 0.0249 \pm 0.0004$ GeV, where the errors are for $\Delta m_t = \pm 0.7$ GeV.

A. Basic properties

Using Eqs. (10), (11), and (12), we can obtain the wanted LL-accuracy and NLL-accuracy PMC scales that correspond to N^{2}LO and N^{3}LO pQCD series, respectively, e.g.,

$$Q_*^{\rm (LL,NLL)} = \{15.2143, 10.3048\}\ {\rm GeV}. \quad (13)$$

Using the NLL-accuracy Q_*, we then obtain the N^{3}LO-level pQCD approximant R_t under the PMC scale-setting approach

$$R_t|_{\rm PMC} = -0.1146. \quad (14)$$

We observe that the improved pQCD series after applying the PMC becomes independent to any choice of renormalization scale. On the other hand, we observe that the scale dependence of the original pQCD series does become smaller when more loop terms have been included, being consistent with conventional wisdom. As for the present N^{3}LO-level series, the net error under the conventional scale-setting approach is small, which is about 2.5% for $\mu_R \in [m_t/2, 2m_t]$, which will be extended to $\simeq$4.8% for a broader choice of scale range $\mu_R \in [m_t/4, 4m_t]$. More explicitly, we give the N^{3}LO-level conventional prediction in the following:

$$R_t|_{\rm Conv} = -0.1122^{+0.0009+0.0014}_{-0.0019-0.0013}, \quad (15)$$

whose central value corresponds to $\mu_R = m_t$; the first errors are for $\mu_R \in [m_t/2, 2m_t]$, and the second ones are additional errors by using a broader range $\mu_R \in [m_t/4, 4m_t]$. The net N^{3}LO R_t for conventional and PMC series are consistent with each other under the proper choice of scale range for the conventional scale-setting approach. Thus, we need to be careful of discussing the scale uncertainties under the conventional scale-setting approach. For convenience, if not specially stated, we will adopt the usual choice of $\mu_R \in [m_t/2, 2m_t]$ to do our discussions. It is found that the perturbative behavior of the conventional series still depends heavily on the choice of scale. To show this point more clearly, we define a κ-factor for the series (3) or (9), i.e.,

$$\kappa_{\rm Conv}^{{\rm N}^i{\rm LO}} = \frac{r_i(\mu_R)\alpha_s^i(\mu_R)}{r_1\alpha_s(\mu_R)}, \qquad \kappa_{\rm PMC}^{(i)} = \frac{r_{i,0}\alpha_s^i(Q_*)}{r_{1,0}\alpha_s(Q_*)}. \quad (16)$$

Numerically, we have

$$\kappa_{\rm Conv} = \{1, 0.1702, 0.0476\}, \qquad \mu_R = m_t/2, \quad (17)$$

$$\kappa_{\rm Conv} = \{1, 0.2450, 0.0792\}, \qquad \mu_R = m_t, \quad (18)$$

$$\kappa_{\rm Conv} = \{1, 0.3096, 0.1155\}, \qquad \mu_R = 2m_t, \quad (19)$$

$$\kappa_{\rm PMC} = \{1, 0.1210, 0.0548\}. \quad (20)$$

The scale-independent convergent behavior of the PMC series (9) could be regarded as the intrinsic perturbative nature of R_t. According to Eqs. (14) and (15), after including the known NLO EW correction $\Delta_{\rm EW}$, the total top-quark decay widths of $t \to Wb$ are

$$\Gamma_t^{\rm tot}|_{\rm Conv} = 1.3156^{+0.0014}_{-0.0027}\ {\rm GeV}, \quad (21)$$

$$\Gamma_t^{\rm tot}|_{\rm PMC} = 1.3120\ {\rm GeV}, \quad (22)$$

where the errors for the series under the conventional scale-setting approach is for $\mu_R \in [m_t/2, 2m_t]$.

We present the top-quark total decay width $\Gamma_t^{\rm tot}$ up to N^{3}LO QCD corrections versus the renormalization scale μ_R (divided by m_t) before and after applying the PMC scale-setting approach in Fig. 1. After applying the PMC, as shown in Fig. 1, the scale dependence of the top-quark decay width is eliminated at any fixed order. The difference between N^{2}LO-level and the N^{3}LO-level PMC predictions are much smaller than that of the conventional ones, indicating that the convergence of the pQCD series is significantly improved.

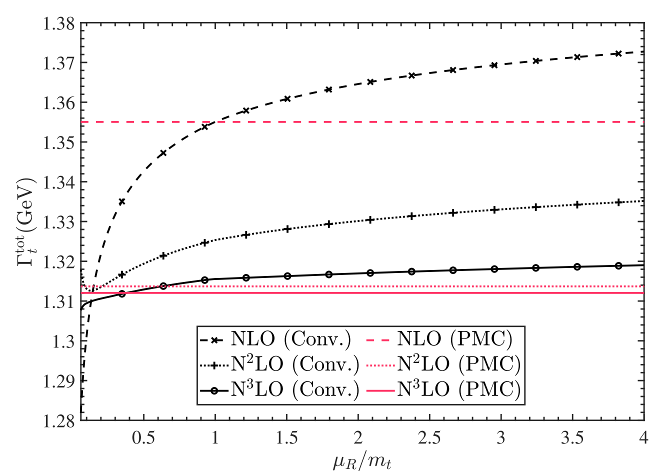

FIG. 1. The total top-quark decay width of $t \to Wb$ ($\Gamma_t^{\rm tot}$) up to N^{3}LO QCD corrections versus the renormalization scale μ_R using conventional (Conv.) (black lines) and PMC (red lines) scale-setting approaches, respectively.

B. Predictions of the uncalculated N^{4}LO contributions using the Bayesian analysis approach

It has been noted that the improved series by using the PMC scale-setting approach not only provides more precise predictions for the fixed-order pQCD series but also establishes a robust basis for estimating the potential contributions from the UHO-terms, thus significantly improving the predictive power of perturbation theory.

In the following, we adopt the BA to estimate the effects from the uncalculated N^{4}LO-terms from the known initial N^{3}LO-level series (3) and the PMC series (9), respectively. The BA quantifies the contributions of the UHO-terms in terms of a probability distribution. It becomes most effective when the series has a good convergent behavior. A detailed introduction to the BA can be found in Refs. [48–51], and its combination with the PMC can be found in Refs. [43,52–55]. Following the idea of the BA, for a fixed degree-of-belief (DoB) or equivalently the Bayesian probability, the estimated UHO-coefficient r_{p+1}, given known coefficients $\{r_1, r_2, \ldots, r_p\}$, will fall within the following specific credible interval (CI) $r_{p+1} \in [-r_{p+1}^{(\mathrm{DoB})}, r_{p+1}^{(\mathrm{DoB})}]$, where

$$r_{p+1}^{(\mathrm{DoB})} = \begin{cases} \bar{r}_{(p)} \frac{p+1}{p} \mathrm{DoB}, & \mathrm{DoB} \le \frac{p}{p+1} \\ \bar{r}_{(p)} [(p+1)(1-\mathrm{DoB})]^{-1/p}, & \mathrm{DoB} \ge \frac{p}{p+1} \end{cases}, \qquad (23)$$

with $\bar{r}_{(p)} = \max\{|r_1|, |r_2|, \ldots, |r_p|\}$. For definiteness and without loss of generality, we take $\mathrm{DoB} \equiv 95.5\%$ to estimate the contributions from the UHO-terms.

Comparison of the calculated central values using known series (simply labeled as "exact value") of the total top-quark decay width Γ_t^{tot} with the predicted credible intervals of Γ_t^{tot} up to N^{4}LO-level QCD corrections are given in Fig. 2. Because of the scale dependence of the coefficients $r_{i>1}$ in the conventional pQCD series, the BA can only be employed after specifying the renormalization scale, which introduces additional uncertainties to the total decay width. And in Fig. 2, we give two predictions for $\mu_R \in [m_t/2, 2m_t]$ and $\mu_R \in [m_t/4, 4m_t]$, respectively. In contrast, the conformal coefficients $r_{i,0}$ in the PMC series are scale independent, providing a more reliable foundation for constraining estimations from UHO contributions. Figure 2 shows that the probability distributions become more accurate, and simultaneously the resultant credible intervals become smaller for the same DoB, when more loop terms have been included. Practically, we can treat the magnitude of unknown N^{4}LO terms as one of the errors of the given N^{3}LO-level Γ_t^{tot}. If taking the scale range $\mu_R \in [m_t/2, 2m_t]$ to estimate the UHO contributions, such errors by using the BA are

$$\Delta\Gamma_t^{\mathrm{tot}}|_{\mathrm{Conv}}^{\mathrm{UHO}} = \left({}^{+0.0037}_{-0.0041}\right) \mathrm{GeV}, \qquad (24)$$

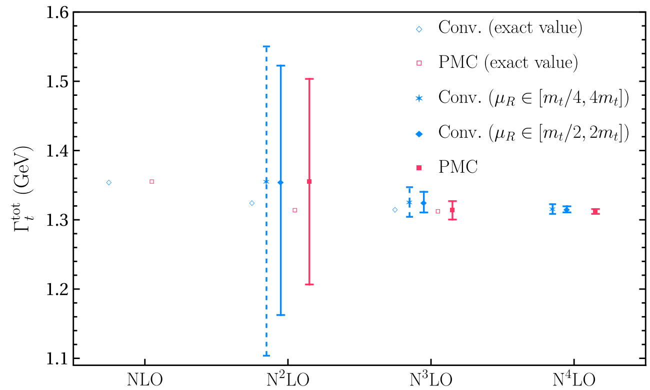

FIG. 2. Comparison of the calculated central values of the total top-quark decay width of $t \to Wb$ (Γ_t^{tot}) for the known series (labeled as "exact value") with the predicted credible intervals of Γ_t^{tot} up to N^{4}LO-level QCD corrections. The blue hollow diamonds and red hollow quadrates represent the calculated central values of the fixed-order pQCD predictions using conventional (Conv.) and PMC scale-setting approaches, respectively. The blue stars, blue solid diamonds, and red solid quadrates with error bars represent the predicted credible intervals using the Bayesian approach based on the known Conv. series and the PMC series, respectively. DoB = 95.5%.

$$\Delta\Gamma_t^{\mathrm{tot}}|_{\mathrm{PMC}}^{\mathrm{UHO}} = \pm 0.0035 \ \mathrm{GeV}. \qquad (25)$$

The predicted error range for the perturbative series under the conventional scale-setting approach will be extended to $\Delta\Gamma_t^{\mathrm{tot}}|_{\mathrm{Conv}}^{\mathrm{UHO}} = \left({}^{+0.0060}_{-0.0054}\right)$ GeV for $\mu_R \in [m_t/4, 4m_t]$. The above two equations show that after applying the PMC, the errors $\Delta\Gamma_t^{\mathrm{tot}}$ from the UHO-terms are only slightly smaller than those of the pQCD series under the conventional scale-setting approach. Both are small, e.g., $\kappa^{\mathrm{N^4LO}} \sim 0.03$ for all cases, due to the fact that the N^{3}LO-level QCD corrections already show good convergent behavior as indicated by Eqs. (17)–(20).

C. Uncertainties for the total decay width Γ_t^{tot}

In addition to the above-mentioned scale uncertainties and the uncertainties caused by the UHO-terms, there are also other error sources for the determination of $t \to Wb$ decay width. When discussing the uncertainty for one of the error sources, the other error sources are set to be their central values.

We put the uncertainties arising from $\Delta\alpha_s(m_Z) = \pm 0.0009$, $\Delta\Gamma_W = \pm 0.042$ GeV, and $\Delta m_t = \pm 0.7$ GeV under conventional and PMC scale-setting approaches in Table I. It shows that the errors are dominated by Δm_t, whose effect to the total decay width is about 5 to 10 times larger than those of other error sources. Thus, a more precise m_t will greatly improve the precision of the theoretical predictions. The squared average of the uncertainties to the total top-quark decay width arising from

TABLE I. Additional uncertainties (in unit: GeV) arising from $\Delta\alpha_s(m_Z) = \pm 0.0009$, $\Delta\Gamma_W = \pm 0.042$ GeV, and $\Delta m_t = \pm 0.7$ GeV under conventional and PMC scale-setting approaches, respectively.

	$\Delta\Gamma_t^{\rm tot}\vert^{\Delta\alpha_s}$	$\Delta\Gamma_t^{\rm tot}\vert^{\Delta\Gamma_W}$	$\Delta\Gamma_t^{\rm tot}\vert^{\Delta m_t}$
Conv.	± 0.0015	± 0.0004	$\left(^{+0.0190}_{-0.0189}\right)$
PMC	± 0.0015	± 0.0004	$\left(^{+0.0190}_{-0.0188}\right)$

$\mu_R \in [m_t/2, 2m_t]$, $\Delta\alpha_s(m_Z)$, $\Delta\Gamma_W$, and the predicted N^{4}LO terms are

$$\Gamma_t^{\rm tot}|_{\rm Conv} = 1.3156^{+0.0042}_{-0.0051}\ \text{GeV}, \quad (26)$$

$$\Gamma_t^{\rm tot}|_{\rm PMC} = 1.3120 \pm 0.0038\ \text{GeV}. \quad (27)$$

And if further including the uncertainty caused by Δm_t, we finally get

$$\Gamma_t^{\rm tot}|_{\rm Conv} = 1.3156 \pm 0.0195\ \text{GeV}, \quad (28)$$

$$\Gamma_t^{\rm tot}|_{\rm PMC} = 1.3120^{+0.0194}_{-0.0192}\ \text{GeV}. \quad (29)$$

Figure 3 depicts the relationship between the total decay width $\Gamma_t^{\rm tot}$ and the top-quark pole mass m_t, which indicates an approximately proportional relationship between the top-quark's decay widths and its pole mass. The total decay width is highly sensitive to the top-quark pole mass. Figure 4 shows the total decay width $\Gamma_t^{\rm tot}$ for the theoretical predictions given by Eqs. (28) and (29). As a comparison,

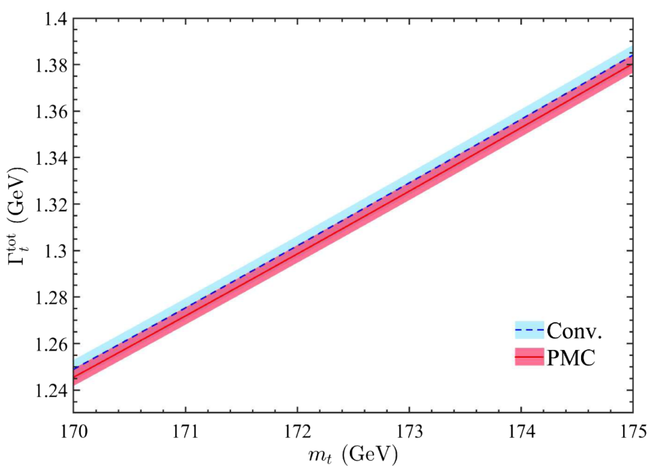

FIG. 3. Total decay width of $t \to Wb$ ($\Gamma_t^{\rm tot}$) versus the top-quark pole mass m_t, where the light-blue and dark-red bands are results for conventional (Conv.) and PMC scale-setting approaches, respectively. The dashed and solid lines are their central values. The shaded bands are for the uncertainties arising from $\mu_R \in [m_t/2, 2m_t]$, $\Delta\alpha_s(m_Z)$, $\Delta\Gamma_W$, and the predicted N^{4}LO terms.

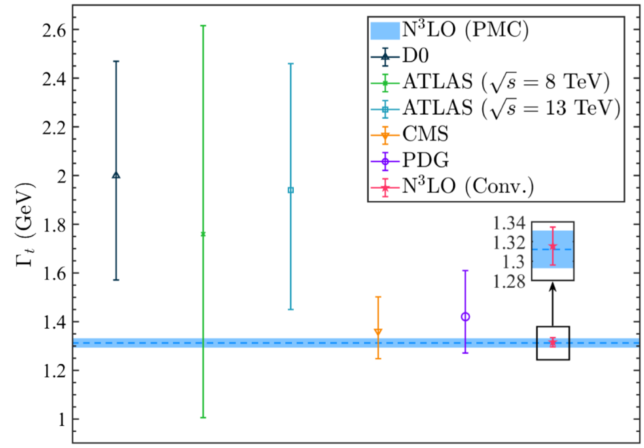

FIG. 4. Comparison between the theoretical predictions with their errors given in Eqs. (28) and (29) and the experimental measurements for the total decay width $\Gamma_t^{\rm tot}$. The errors of theoretical predictions are for the uncertainties arising from $\mu_R \in [m_t/2, 2m_t]$, $\Delta\alpha_s(m_Z)$, $\Delta\Gamma_W$, Δm_t, and the predicted N^{4}LO terms.

various experimental measurements [1–4] and the world average value reported by the PDG [5] have been included in Fig. 4. Clearly, while the top quark's decay width is highly sensitive to its pole mass, the large uncertainties in experimental measurements preclude us from deriving a reliable reference value for the top-quark pole mass from these data.

In the above analysis, we have implicitly taken the CKM matrix element $|V_{\rm tb}| = 1$ to do our numerical calculation. As a final remark, we give an inverse determination of the CKM matrix element $|V_{\rm tb}|$ by using the PDG averaged values on the top-quark total decay width and the branching fraction $\mathcal{B}(t \to Wb)$. That is, by using the PDG's averaged total decay width $\Gamma_{t,\rm PDG} = 1.42^{+0.19}_{-0.15}$ GeV [5], the branching fraction $\mathcal{B}(t \to Wb) = 0.957 \pm 0.034$, along with the theoretical predictions (28) and (29) under conventional and PMC scale-setting approaches, respectively, we inversely obtain[1]

$$|V_{\rm tb}|_{\rm Conv} = 1.033^{+0.144}_{-0.116}, \quad (30)$$

$$|V_{\rm tb}|_{\rm PMC} = 1.036^{+0.144}_{-0.116}. \quad (31)$$

Both of them are consistent with the average value of the Tevatron and LHC results, e.g., $|V_{\rm tb}| = 1.014 \pm 0.029$ [5]. By confining the prior within the SM region [0, 1], we then establish a lower limit of $|V_{\rm tb}| > 0.917$ and $|V_{\rm tb}| > 0.919$

[1]The errors are estimated by using the usual error propagation formulas. That is, the error of a quantity $Z = X/Y$ is calculated by $\Delta Z = (X_0/Y_0)\sqrt{(\Delta X/X_0)^2 + (\Delta Y/Y_0)^2}$, where $X = X_0 \pm \Delta X$, $Y = Y_0 \pm \Delta Y$, and $Z = (X_0/Y_0) \pm \Delta Z$.

for conventional and PMC scale-setting approaches, respectively.

IV. SUMMARY

In the paper, we have presented an improved analysis of the total decay width of the top-quark decay $t \to Wb$ up to N^{3}LO QCD corrections by applying the PMC scale-setting approach. In contrast to previous literature [64], our present treatment is achieved after taking both the off-shell W-boson contributions and the finite b-quark mass effects into account. Because the N^{3}LO-level QCD corrections to the total decay width $\Gamma(t \to Wb)$ already show good convergent behavior as indicated by Eqs. (17)–(20), the predictions under the conventional scale-setting approach are close to the PMC predictions. Especially because the errors are dominated by Δm_t, which dilute the great improvements on the perturbative nature of the series by applying the PMC. However, the improved pQCD series after applying the PMC is independent of any choice of the renormalization scale, which not only leads to a more precise prediction but also provides a better basis for estimating the contributions from UHO-terms. The errors of the PMC series caused by $\Delta\alpha_s(m_Z)$ and the predicted N^{4}LO terms are comparable to each other, and there are also sizable renormalization scale errors for the conventional scale-setting approach. Figure 1 indicates that the differences between the N^{2}LO-level and the N^{3}LO-level PMC predictions are much smaller than the conventional one, indicating that the convergence of the pQCD series is significantly improved and the PMC prediction shows quicker trends of approaching its physical/measured value. Thus our present results emphasize the importance of using proper scale-setting approaches to achieve precise fixed-order pQCD predictions.

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APPENDIX: ANALYTIC EXPRESSIONS FOR THE LO AND NLO COEFFICIENTS WITH OR WITHOUT b-QUARK MASS EFFECTS

The first two scale-independent coefficients $f_0(\varepsilon,\omega)$ and $f_1(\varepsilon,\omega)$ at the scale $\mu_R = m_t$ for $t \to Wb$ which contain finite b-quark mass effects are [6]

$$f_0(\varepsilon,\omega) = \lambda^{1/2}(1,\varepsilon,\omega)[(1-\varepsilon)^2 + \omega(1+\varepsilon) - 2\omega^2], \quad \text{(A1)}$$

$$\begin{aligned} f_1(\varepsilon,\omega) = -\frac{C_F}{2\pi}\Big\{&[(1-\varepsilon)^2+\omega(1+\varepsilon)-2\omega^2](1+\varepsilon-\omega)\Big[\pi^2+2\mathrm{Li}_2(u_\mathrm{W})-2\mathrm{Li}_2(1-u_\mathrm{W})-4\mathrm{Li}_2(u_q)-4\mathrm{Li}_2(u_q u_\mathrm{W}) \\ &+\ln\frac{1-u_q}{\omega}\ln(1-u_q)-\ln^2(1-u_q u_\mathrm{W})+\frac{1}{4}\ln^2\frac{\omega}{u_\mathrm{W}}-\ln u_\mathrm{W}\ln\frac{(1-u_q u_\mathrm{W})^2}{1-u_q}-2\ln u_q\ln[(1-u_q)(1-u_q u_\mathrm{W})]\Big] \\ &-2f_0(\varepsilon,\omega)\left(\ln\omega+\frac{3}{2}\ln\varepsilon-2\ln\lambda(1,\varepsilon,\omega)\right)+2(1-\varepsilon)[(1-\varepsilon)^2+\omega(1+\varepsilon)-4\omega^2]\ln u_\mathrm{W} \\ &+\frac{1}{2}[(3-\varepsilon+11\varepsilon^2-\varepsilon^3)+\omega(6-12\varepsilon+2\varepsilon^2)-\omega^2(21+5\varepsilon)+12\omega^3]\ln u_q \\ &+\frac{3}{2}\lambda^{1/2}(1,\varepsilon,\omega)(1-\varepsilon)(1+\varepsilon-\omega)\ln\varepsilon+\frac{1}{2}\lambda^{1/2}(1,\varepsilon,\omega)[-5+22\varepsilon-5\varepsilon^2-9\omega(1+\varepsilon)+6\omega^2]\Big\}, \end{aligned} \quad \text{(A2)}$$

where $\omega = m_W^2/m_t^2$, $\varepsilon = m_b^2/m_t^2$, $C_F = 4/3$ is $\mathrm{SU}_c(3)$ color factor, the Källen function $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$, the polylogarithm function $\mathrm{Li}_n(z) = \sum_{k=1}^{\infty} z^k/k^n$. The functions u_q and u_W are defined as

$$u_q = \frac{1+\varepsilon-\omega-\lambda^{1/2}(1,\varepsilon,\omega)}{1+\varepsilon-\omega+\lambda^{1/2}(1,\varepsilon,\omega)},$$
$$u_\mathrm{W} = \frac{1-\varepsilon+\omega-\lambda^{1/2}(1,\varepsilon,\omega)}{1-\varepsilon+\omega+\lambda^{1/2}(1,\varepsilon,\omega)}. \quad \text{(A3)}$$

In the case where the b-quark mass effects have been ignored, one has

$$f_0(\omega) \equiv f_0(0,\omega) = (1-\omega)^2(1+2\omega), \quad \text{(A4)}$$

$$\begin{aligned} f_1(\omega) = -\frac{C_F}{2\pi}\Big\{&f_0(\omega)[\pi^2+2\mathrm{Li}_2(\omega)-2\mathrm{Li}_2(1-\omega)] \\ &+2\omega(1+\omega)(1-2\omega)\ln\omega \\ &+(1-\omega)^2(5+4\omega)\ln(1-\omega) \\ &-\frac{1}{2}(1-\omega)(5+9\omega-6\omega^2)\Big\}. \end{aligned} \quad \text{(A5)}$$

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