Perturbative T-odd asymmetries in the Drell-Yan process revisited

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We calculate the perturbative T-odd contributions to the lepton angular distribution in the Drell-Yan process. Using collinear factorization, we work at the first order in QCD perturbation theory where these contributions appear, $\mathcal{O}(\alpha_s^2)$, and address both $W^\pm$ and γ/Z^0 boson exchange. A major focus of our calculation is on the regime where the boson's transverse momentum Q_T is much smaller than its mass Q. We carefully expand our results up to next-to-next-to-leading power in Q_T/Q. Our calculation provides a benchmark for studies of T-odd contributions that employ transverse-momentum dependent parton distribution functions. In the neutral-current case we compare our results for the T-odd structure functions to available ATLAS data.

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I. INTRODUCTION

There is a long history of the study of T-odd asymmetries in QCD hard-scattering processes, starting with the seminal papers [1–6] that showed how T-odd effects may be generated by absorptive parts of QCD loop diagrams. T-odd behavior refers to noninvariance of observables under so-called *naive* time reversal, that is, under reversal of momenta and spins without interchange of initial and final states. As shown in the early papers, such behavior can occur even in theories that are manifestly invariant under *true* time reversal. Subsequent work [7–22] explored T-odd QCD phenomena in a wide range of scattering reactions, among them especially the Drell-Yan process. In Ref. [8] it was proposed to study T-odd asymmetries appearing in the angular distributions of the charged leptons from the decay of $W^\pm$ bosons produced with high transverse momentum at hadron colliders. The asymmetries manifest themselves as terms proportional to $\sin\phi$ or $\sin 2\phi$ in the lepton distribution, where ϕ is a suitably defined azimuthal angle between the lepton plane and the hadron plane. The T-odd part of the spin-averaged differential cross section for $W^\pm$ production was expanded in [8] in terms of three structure functions which were computed to lowest order of perturbation theory. The results were later obtained independently in Ref. [10] and extended to the case of longitudinal polarization of one of the initial hadrons in [15,21].

In parallel developments, it was realized that T-odd effects in QCD may also arise in hadronic matrix elements, especially in parton distribution functions (PDFs) [23–26], where they are associated with correlations among three-momenta, transverse momenta, and polarizations of partons and hadrons and again generate azimuthal-angle dependent terms. This has given rise to an intensive experimental program aiming at the extraction of such T-odd transverse-momentum dependent parton distributions (TMDs) (for a recent review, see Ref. [27]), either in semi-inclusive lepton scattering (SIDIS) or via the Drell-Yan process.

The precise connection between T-odd effects in perturbative (collinearly factorized) hard scattering on the one hand and T-odd TMDs on the other has been an area of active research as well. This issue is important both theoretically and for phenomenology, where it is central for the "matching" of resummed calculations based on TMDs to fixed-order perturbation theory. While much progress has been made for leading-twist observables [28–39], it was also realized that for many of the azimuthal-angle dependent terms in the Drell-Yan and SIDIS cross sections—both T-odd and T-even—this matching is nontrivial and will involve TMD PDFs at next-to-leading power in the hard scale [40,41]. Correspondingly, TMD factorization theorems at next-to-leading power were developed in the literature [42–46].

In the present paper, we advance this area of research by specifically exploring the low-transverse momentum limit of the T-odd terms appearing in the Drell-Yan hard-scattering calculation. The work is carried out in the spirit of Ref. [22] that addressed T-odd effects in SIDIS, but goes well beyond it in terms of calculational techniques. For our purpose, we first perform an independent new analytical calculation of the lowest-order T-odd terms in the Drell-Yan cross section, recovering results at this order from the previous literature [8,10], but also extending them by presenting results for pure Z-boson exchange and $\gamma - Z$ interference. As one application, we will compare our results to available ATLAS data [47] for the T-odd angular terms taken around the Z resonance. Our main focus, however, is to carefully expand the results for low Q_T^2/Q^2, where Q_T and Q are the boson's transverse momentum and mass, respectively. We do this to first and second power in this ratio, identifying logarithmic behavior as well. As a byproduct we also uncover a novel simple relation between two of the T-odd structure functions valid at leading power in Q_T^2/Q^2 for both partonic channels, $q\bar{q}$ annihilation and qg Compton scattering.

We hope that our explicit results will be useful in testing TMD factorization at next-to-leading power and ultimately contribute to a better understanding of the matching between the TMD and collinearly factorized regimes. As has been shown in Refs. [48–50], at leading power γ/Z interference generates a $\sin 2\phi$ azimuthal dependence in the unpolarized Drell-Yan cross section, entering with the Boer-Mulders function [24], while a term proportional to $\sin\phi$ is not generated. Effects beyond leading power have been investigated in Ref. [51].

In more general terms, TMD factorization theorems make a prediction also for the large-Q_T "tail" of the transverse-momentum distribution they provide, which may be confronted with the terms generated by the collinearly factorized cross section expanded to low Q_T/Q. An important issue is whether there is an overlap region of Q_T where the two approaches agree. This decides whether the TMD and collinear contributions to the cross section are manifestations of the same physical origin, or should be regarded as genuinely separate pieces. While such an overlap has been demonstrated in a few important cases, notably the Sivers function [31,32,38], the situation is not clear for next-to-leading power observables [40], especially for those that arise only from loop corrections in the collinear-factorization case. In any case, knowledge of both the TMD and the large-Q_T (collinear) parts of the cross section is vital for phenomenology, in order to obtain a formalism that encompasses the full range of Q_T. We will not address the potential ramifications of our results for TMDs in this paper, but rather view our work as providing a part of a "library" of hard-scattering functions at low transverse momenta. We stress that our techniques for expanding the cross sections for low Q_T^2/Q^2 are completely general and may also be used in a variety of other settings, such as for T-even contributions, collisions of polarized hadrons, and so forth, or perhaps even at the next order in perturbation theory, as available from [18].

Our paper is organized as follows. In Sec. II we present the definition of the structure functions parametrizing the lepton-angular distribution for the Drell-Yan process and the main ingredients for the perturbative calculation of the T-odd contributions. In Sec. III we collect the analytic results, and subsequently in Sec. IV the small Q_T expansion is performed. In Sec. V we compare our results with the ATLAS data. Section VI concludes our paper. Some calculational details are collected in Appendixes A–C, and the lengthy results for the small-Q_T expansion to next-next-to-leading-power are presented in the Supplemental Material [52].

II. T-ODD STRUCTURE OF THE DRELL-YAN HADRONIC TENSOR

The hadronic tensor $W^{\mu\nu}$ for the Drell-Yan process can be written in terms of nine structure functions W_i. The most straightforward decomposition of this tensor is obtained by using the helicity formalism proposed in Ref. [53] for reactions with photon exchange and extended in Ref. [10] to the electroweak case. The results of Ref. [10] for the expansion of $W^{\mu\nu}$ can be conveniently rewritten using a basis of orthogonal unit vectors $T^\mu = q^\mu/\sqrt{Q^2} = (1,0,0,0)$, $X^\mu = (0,1,0,0)$, $Z^\mu = (0,0,0,1)$, $Y^\mu = \epsilon^{\mu\nu\alpha\beta} T_\nu Z_\alpha X_\beta = (0,0,1,0)$, proposed in Ref. [53] and constructed from the hadron and virtual-boson momenta. This expansion reads as

$$
\begin{aligned}
W^{\mu\nu} = {}& (X^\mu X^\nu + Y^\mu Y^\nu) W_T + i(X^\mu Y^\nu - Y^\mu X^\nu) W_{T_P} + Z^\mu Z^\nu W_L \\
& + (Y^\mu Y^\nu - X^\mu X^\nu) W_{\Delta\Delta} - (X^\mu Y^\nu + Y^\mu X^\nu) W_{\Delta\Delta_P} \\
& - (X^\mu Z^\nu + Z^\mu X^\nu) W_\Delta - (Y^\mu Z^\nu + Z^\mu Y^\nu) W_{\Delta_P} \\
& + i(Z^\mu X^\nu - X^\mu Z^\nu) W_\nabla + i(Y^\mu Z^\nu - Z^\mu Y^\nu) W_{\nabla_P}, \qquad (1)
\end{aligned}
$$

where q is the momentum of the gauge boson γ, $W^\pm$, or Z^0, with $q^2 = Q^2$ its Minkowski momentum squared, and $\epsilon^{\mu\nu\alpha\beta}$ is the four-dimensional Levi-Civita tensor defined via $\mathrm{tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta) = 4i\epsilon^{\mu\nu\alpha\beta}$, with $\epsilon^{0123} = -\epsilon_{0123} = -1$.

The number of structure functions, $9 = 3 \times 3$, is determined by the number of possible helicity settings of the gauge boson in the amplitude and its complex conjugate. In the case of the purely weak Drell-Yan reactions or for $\gamma - Z^0$ interference we have nine functions, while in the case of the purely electromagnetic Drell-Yan we have only four T-even structure functions. In general, the Drell-Yan hadronic structure functions may be classified as

(a) two transverse functions, the P-even W_T and the P-odd W_{T_P};
(b) one longitudinal function W_L which is P-even;

(c) two transverse-transverse interference (double-spin-flip) functions, the P-even $W_{\Delta\Delta}$ and the P-odd $W_{\Delta\Delta_P}$; and

(d) four transverse-longitudinal interference (single-spin-flip) functions, the P-even W_{Δ}, W_{∇} and the P-odd W_{Δ_P}, W_{∇_P}.

The lepton angular distribution $dN/d\Omega$ is expanded in terms of the hadronic structure functions as

$$\frac{dN}{d\Omega} = \frac{3}{8\pi(2W_T + W_L)}\big[g_T W_T + g_L W_L + g_\Delta W_\Delta + g_{\Delta\Delta} W_{\Delta\Delta} + g_{T_P} W_{T_P} + g_{\nabla_P} W_{\nabla_P} + g_\nabla W_\nabla + g_{\Delta\Delta_P} W_{\Delta\Delta_P} + g_{\Delta_P} W_{\Delta_P}\big], \quad (2)$$

where $g_i = g_i(\theta, \phi)$ denote the angular coefficients

$$\begin{aligned} g_T &= 1 + \cos^2\theta, & g_L &= 1 - \cos^2\theta, & g_{T_P} &= \cos\theta, \\ g_{\Delta\Delta} &= \sin^2\theta\cos 2\phi, & g_\Delta &= \sin 2\theta\cos\phi, & g_{\nabla_P} &= \sin\theta\cos\phi, \\ g_{\Delta\Delta_P} &= \sin^2\theta\sin 2\phi, & g_{\Delta_P} &= \sin 2\theta\sin\phi, & g_\nabla &= \sin\theta\sin\phi, \end{aligned} \quad (3)$$

with θ and ϕ being the polar and azimuthal angles of one of the decay leptons in the center-of-mass system (c.m.s.) of the lepton pair. The angle ϕ may be taken to define the orientation of the lepton plane with respect to the hadron plane. In Fig. 1 we show the polar and the azimuthal angles for the Drell-Yan process in the Collins-Soper frame.

The six angular coefficients g_i ($i = T$, L, $\Delta\Delta$, Δ, $\Delta\Delta_P$, Δ_P) in (3) are invariant under the P-parity transformation $\theta \to \pi - \theta$ and $\phi \to \pi + \phi$, while the other three coefficients g_i ($i = T_P$, ∇, ∇_P) change their sign in that case. Therefore, the six partial lepton angular distributions $dN_i/d\Omega$ ($i = T$, L, $\Delta\Delta$, Δ, T_P, ∇_P) are also P invariant, whereas the other three distributions $dN_i/d\Omega$ ($i = \Delta\Delta_P$, Δ_P, ∇) are P odd and also T odd. As can be seen from Eq. (3), the latter distributions are all proportional to either $\sin\phi$ or $\sin 2\phi$.

We note in passing that two other commonly employed, and equivalent, parametrizations of the lepton angular distribution are [10,41,53–55]

$$\frac{dN}{d\Omega} = \frac{3}{16\pi}\bigg(1 + \cos^2\theta + \frac{A_0}{2}(1 - 3\cos^2\theta) + A_1 \sin 2\theta\cos\phi + \frac{A_2}{2}\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi\bigg), \quad (4)$$

and

$$\frac{dN}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda + 3}\bigg(1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi + \tau\sin\theta\cos\phi + \eta\cos\theta + \xi\sin^2\theta\sin 2\phi + \zeta\sin 2\theta\sin\phi + \chi\sin\theta\sin\phi\bigg). \quad (5)$$

The relations between the three sets of structure functions are recalled in Appendix A.

The T-odd structure functions W_∇, $W_{\Delta\Delta_P}$, and W_{Δ_P} are generated at $\mathcal{O}(\alpha_s^2)$ in the strong coupling constant α_s by the absorptive parts of parton scattering amplitudes. The leading contributions arise from the interference of one-loop and tree-level diagrams. The relevant channels are quark-antiquark annihilation and quark-gluon Compton scattering. Their one-loop diagrams providing an absorptive part for photon exchange in Drell-Yan are shown in Figs. 2 and 3; the diagrams with Z^0 and $W^\pm$ bosons are generated analogously. The ensuing T-odd effects were first studied in Ref. [8] and later recalculated in [10]. Here we will present an independent derivation that will allow us to explore the low-Q_T limit of the results.

For our calculation of the T-odd structure functions we use a convenient orthogonal basis of vectors P, R, K [56], defined by

$$\begin{aligned} P^\mu &= (p_1 + p_2)^\mu, \\ R^\mu &= (p_1 - p_2)^\mu, \\ K^\mu &= k_1^\mu - P^\mu\frac{P\cdot k_1}{P^2} - R^\mu\frac{R\cdot k_1}{R^2} \\ &= -q^\mu + P^\mu\frac{P\cdot q}{P^2} + R^\mu\frac{R\cdot q}{R^2}, \end{aligned} \quad (6)$$

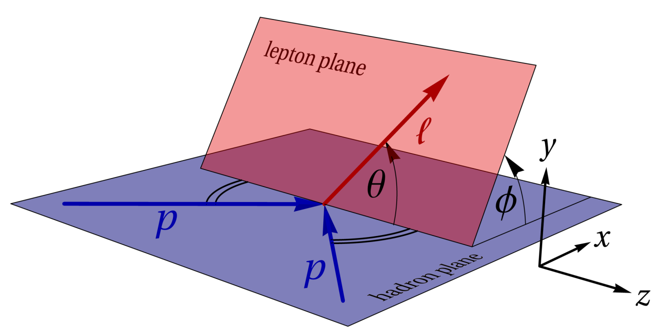

FIG. 1. Definition of the polar and the azimuthal angles for the Drell-Yan process in the Collins-Soper frame. The hadron plane is depicted in blue, the lepton plane in red.

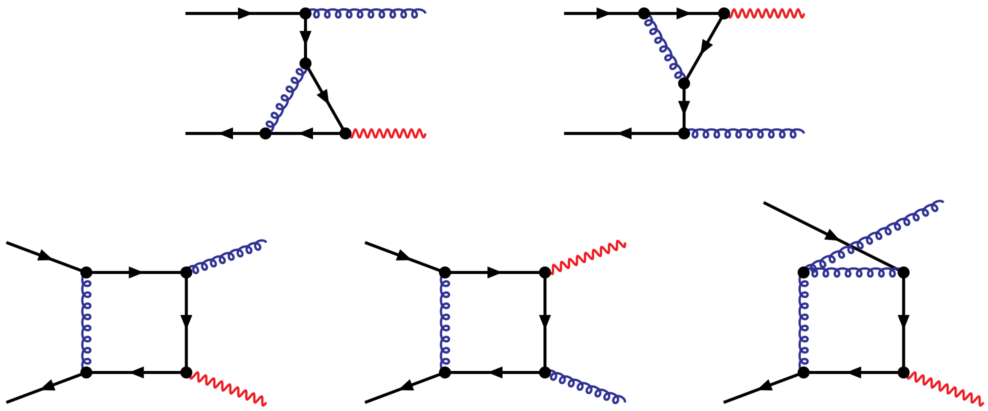

FIG. 2. One-loop diagrams for $q\bar{q} \to g\gamma$ that produce an absorptive part.

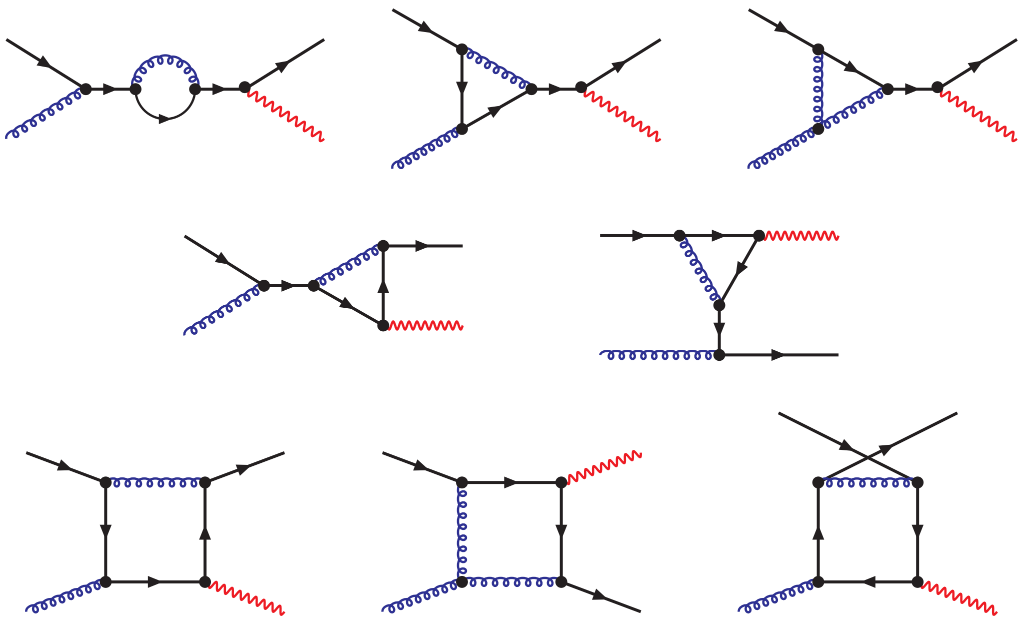

FIG. 3. One-loop diagrams for $qg \to q\gamma$ that produce an absorptive part.

which obey the conditions

$$P^2 = -R^2 = \hat{s}, \qquad K^2 = -\frac{\hat{u}\,\hat{t}}{\hat{s}}, \qquad P\cdot R = P\cdot K = R\cdot K = 0. \tag{7}$$

Here p_1, p_2, and k_1 are the momenta of the two initial partons and the final-state parton, respectively, satisfying the momentum conservation relation $p_1 + p_2 = k_1 + q$. Furthermore, $\hat{s} = (p_1 + p_2)^2$, $\hat{t} = (p_1 - q)^2$, $\hat{u} = (p_2 - q)^2$, with $\hat{s} + \hat{t} + \hat{u} = Q^2$ the parton-level Mandelstam variables.

The (P, R, K) and (T, X, Y, Z) bases are related by

$$\begin{aligned} X^\mu &= \frac{T^\mu\sqrt{1+\rho^2}}{\rho} - \frac{P^\mu z_{12}^+ + R^\mu z_{12}^-}{2Q\rho\sqrt{1+\rho^2}} \\ &= \frac{\rho(P^\mu z_{12}^+ + R^\mu z_{12}^-)}{2Q\sqrt{1+\rho^2}} - \frac{K^\mu\sqrt{1+\rho^2}}{Q\rho}, \\ Z^\mu &= \frac{P^\mu z_{12}^- + R^\mu z_{12}^+}{2Q\sqrt{1+\rho^2}}, \\ Y^\mu &= -\epsilon^{\mu PRK}\frac{z_1 z_2}{Q^3\rho(1+\rho^2)}, \end{aligned} \tag{8}$$

where $z_{12}^{\pm} = z_1 \pm z_2$, $Q = \sqrt{Q^2}$, and $\epsilon^{\mu PRK} = \epsilon^{\mu\nu\alpha\beta} P_\nu R_\alpha K_\beta$. We have $z_i = x_i/\xi_i$, with the momentum fractions ξ_i of the partons defined by $p_i = \xi_i P_i$, and with the momentum fractions of the light-cone components of the gauge boson,

$$x_{1,2} = e^{\pm y}\sqrt{\frac{Q^2+Q_T^2}{s}}, \tag{9}$$

at nonzero Q_T. We also introduce the variables

$$x_{1,2}^0 = e^{\pm y}\frac{Q}{\sqrt{s}} \tag{10}$$

relevant in the $Q_T = 0$ limit.

The hadronic structure functions $W(x_1, x_2, \rho^2)$ for the Drell-Yan process with colliding hadrons H_1 and H_2 are related to the parton-level structure functions $w^{ab}(x_1, x_2, \rho^2)$ by the QCD collinear factorization formula

$$W(x_1,x_2,\rho^2) = \frac{1}{x_1 x_2}\sum_{a,b}\int_{x_1}^1 dz_1 \int_{x_2}^1 dz_2 w^{ab}(z_1,z_2,\rho^2) \times f_{a/H_1}\left(\frac{x_1}{z_1}\right) f_{b/H_2}\left(\frac{x_2}{z_2}\right), \tag{11}$$

where $f_{i/H}(\xi)$ is the PDF describing the ξ distribution of partons of type i in hadron H. We are suppressing here the scale dependence of the PDFs.

We may project onto the parton-level T-odd structure functions in the following way (here we drop parton labels):

$$\begin{aligned} w_{\Delta\Delta_P} &= -\frac{1}{2}(X^\mu Y^\nu + X^\nu Y^\mu) w_{\mu\nu} \\ &= -\frac{z_1 z_2}{4Q^4\rho^2(1+\rho^2)^{3/2}}\left[\epsilon^{\mu PRK}(P^\nu z_{12}^+ + R^\nu z_{12}^-) + \epsilon^{\nu PRK}(P^\mu z_{12}^+ + R^\mu z_{12}^-)\right] w_{\mu\nu}, \end{aligned} \tag{12}$$

$$\begin{aligned} w_{\Delta_P} &= -\frac{1}{2}(Y^\mu Z^\nu + Y^\nu Z^\mu) w_{\mu\nu} \\ &= \frac{z_1 z_2}{4Q^4\rho(1+\rho^2)^{3/2}}\left[\epsilon^{\mu PRK}(P^\nu z_{12}^- + R^\nu z_{12}^+) + \epsilon^{\nu PRK}(P^\mu z_{12}^- + R^\mu z_{12}^+)\right] w_{\mu\nu}, \end{aligned} \tag{13}$$

$$\begin{aligned} w_\nabla &= \frac{i}{2}(X^\mu Z^\nu - X^\nu Z^\mu) w_{\mu\nu} \\ &= \frac{i\, z_1 z_2}{2Q^2\rho(1+\rho^2)}(P^\nu R^\mu - P^\mu R^\nu) w_{\mu\nu}. \end{aligned} \tag{14}$$

In the evaluation of the absorptive parts of the one loop diagrams we use the following set of imaginary parts of scalar one-loop integrals [56,57]:

$$\begin{aligned}
&\mathrm{Im}B_0(Q^2) = \mathrm{Im}B_0(\hat{s}) = \pi, \qquad \mathrm{Im}B_0(\hat{u}) = \mathrm{Im}B_0(\hat{t}) = 0, \\
&\mathrm{Im}C_0(\hat{s},0) = \frac{\pi}{\hat{s}}\left(\frac{1}{\bar{\epsilon}} - \log\frac{\hat{s}}{\mu^2}\right), \qquad \mathrm{Im}C_0(\hat{u},0) = \mathrm{Im}C_0(\hat{t},0) = 0, \\
&\mathrm{Im}C_0(Q^2,0) = \frac{\pi}{Q^2}\left(\frac{1}{\bar{\epsilon}} - \log\frac{Q^2}{\mu^2}\right), \qquad \mathrm{Im}C_0(Q^2,\hat{s}) = -\frac{\pi}{Q^2-\hat{s}}\log\frac{Q^2}{\hat{s}}, \\
&\mathrm{Im}C_0(Q^2,\hat{u}) = \frac{\pi}{Q^2-\hat{u}}\left(\frac{1}{\bar{\epsilon}} - \log\frac{Q^2}{\mu^2}\right), \qquad \mathrm{Im}C_0(Q^2,\hat{t}) = \frac{\pi}{Q^2-\hat{t}}\left(\frac{1}{\bar{\epsilon}} - \log\frac{Q^2}{\mu^2}\right), \\
&\mathrm{Im}C_0(\hat{s},\hat{u}) = \frac{\pi}{\hat{s}-\hat{u}}\left(\frac{1}{\bar{\epsilon}} - \log\frac{\hat{s}}{\mu^2}\right), \qquad \mathrm{Im}C_0(\hat{s},\hat{t}) = \frac{\pi}{\hat{s}-\hat{t}}\left(\frac{1}{\bar{\epsilon}} - \log\frac{\hat{s}}{\mu^2}\right), \\
&\mathrm{Im}D_0(Q^2,\hat{s},\hat{u}) = -\frac{2\pi}{\hat{s}\,\hat{u}}\log\frac{Q^2-\hat{u}}{Q^2}, \qquad \mathrm{Im}D_0(Q^2,\hat{s},\hat{t}) = -\frac{2\pi}{\hat{s}\,\hat{t}}\log\frac{Q^2-\hat{t}}{Q^2}, \\
&\mathrm{Im}D_0(Q^2,\hat{t},\hat{u}) = \frac{2\pi}{\hat{u}\,\hat{t}}\left(-\frac{1}{\bar{\epsilon}} + \log\frac{Q^2}{\mu^2} - \log\frac{(Q^2-\hat{u})(Q^2-\hat{t})}{\hat{u}\,\hat{t}}\right),
\end{aligned} \tag{15}$$

where B_0 denotes the two-propagator (bubble) diagram, C_0 the three-propagator (triangle) diagram, and D_0 the four-propagator (box) diagram. We have used dimensional regularization with $D = 4 - 2\epsilon$ space-time dimensions; as usual $1/\bar{\epsilon} = 1/\epsilon + \log(4\pi) + \gamma_\mathrm{E}$ with the Euler-Mascheroni constant γ_E.

We note that in the cases of the electroweak contributions to $w_{\Delta\Delta_P}$ and w_{Δ_P} we have to deal with an odd number of γ^5 matrices in the relevant Dirac traces. We adopt the Larin scheme for treating γ_5; details are discussed in Appendix B.

III. ANALYTICAL RESULTS FOR THE PARTONIC T-ODD STRUCTURE FUNCTIONS

In this section we present our analytical results for the parton-level T-odd structure functions $w_{\Delta\Delta_P}$, w_{Δ_P}, and w_∇.

We first introduce some notation. We use the QCD color factors $C_F = (N_c^2 - 1)/(2N_c) = 4/3$, $C_A = N_c = 3$, $T_F = 1/2$, $C_1 = C_F - N_c/2 = -1/(2N_c) = -1/6$, which at large N_c scale as $\mathcal{O}(N_c)$, $\mathcal{O}(N_c)$, $\mathcal{O}(1)$, $\mathcal{O}(1/N_c)$, respectively. Specifically, the color factors for $q\bar{q}$ annihilation and qg scattering are $C_{q\bar{q}} = C_F/N_c = (N_c^2 - 1)/(2N_c^2) = 4/9$ and $C_{qg} = T_F/N_c = 1/(2N_c) = 1/6$. Furthermore, it is convenient to introduce the coupling factors $g_{q\bar{q};i} = C_{q\bar{q}} g^{Z\gamma/W}_{\mathrm{EW};i} e_q^2 \alpha_s^2/(4\pi)$ and $g_{qg;i} = C_{qg} g^{Z\gamma/W}_{\mathrm{EW};i} e_q^2 \alpha_s^2/(4\pi)$, where $i = 1, 2$. Here, e_q is the electric charge of a quark of flavor q. The electroweak couplings $g_{\mathrm{EW};1}$ and $g_{\mathrm{EW};2}$, which incorporate the products of couplings of the gauge bosons ($W^\pm$, Z^0, γ) with quarks and leptons, are given by

$$
\begin{aligned}
g^{Z\gamma}_{\mathrm{EW};1} &= 2 g^A_{Zq} \big[g^V_{Zq} ((g^V_{Z\ell})^2 + (g^A_{Z\ell})^2) |D_Z(Q^2)|^2 \\
&\quad + g^V_{Z\ell} \mathrm{Re}[D_Z(Q^2)] \big], \\
g^{Z\gamma}_{\mathrm{EW};2} &= 2 g^A_{Z\ell} \big[g^V_{Z\ell} ((g^V_{Zq})^2 + (g^A_{Zq})^2) |D_Z(Q^2)|^2 \\
&\quad + g^V_{Zq} \mathrm{Re}[D_Z(Q^2)] \big],
\end{aligned}
\tag{16}
$$

in the case of electrically neutral gauge bosons (Z^0, γ) and

$$
\begin{aligned}
g^W_{\mathrm{EW};1} &= 2 g^V_{Wqq'} g^A_{Wqq'} \big((g^V_{W\ell})^2 + (g^A_{W\ell})^2 \big) |V_{qq'}|^2 |D_W(Q^2)|^2, \\
g^W_{\mathrm{EW};2} &= 2 g^V_{W\ell} g^A_{W\ell} \big((g^V_{Wqq'})^2 + (g^A_{Wqq'})^2 \big) |V_{qq'}|^2 |D_W(Q^2)|^2,
\end{aligned}
\tag{17}
$$

in the case of the $W^\pm$ gauge bosons, where

$$
\begin{aligned}
g^V_{W\ell} &= g^A_{W\ell} = g^V_{Wqq'} = g^A_{Wqq'} = \frac{1}{2\sin\theta_W \sqrt{2}}, \\
g^V_{Z\ell} &= -\frac{1 - 4\sin^2\theta_W}{2\sin 2\theta_W}, \qquad g^A_{Z\ell} = -\frac{1}{2\sin 2\theta_W}, \\
g^V_{Zu} &= \frac{1 - 8/3\sin^2\theta_W}{2e_q \sin 2\theta_W}, \qquad g^V_{Zd} = -\frac{1 - 4/3\sin^2\theta_W}{2e_q \sin 2\theta_W}, \\
g^A_{Zu} &= \frac{1}{2e_q \sin 2\theta_W}, \qquad g^A_{Zd} = -\frac{1}{2e_q \sin 2\theta_W}.
\end{aligned}
\tag{18}
$$

In the above expressions, $V_{qq'}$ is the relevant element of the Cabibbo-Kabayashi-Maskawa (CKM) matrix, and θ_W is the Weinberg angle measured to be $\sin^2\theta_W = 0.23121$ [58]. Furthermore, $D_G(Q^2)$ ($G = W^\pm, Z^0$) denotes the product of the Breit-Wigner propagator of a weak gauge boson and Q^2. Its real and imaginary part are given by

$$
\begin{aligned}
\mathrm{Re}[D_G(Q^2)] &= \frac{(Q^2 - M_G^2) Q^2}{(Q^2 - M_G^2)^2 + M_G^2 \Gamma_G^2}, \\
\mathrm{Im}[D_G(Q^2)] &= \frac{M_G \Gamma_G Q^2}{(Q^2 - M_G^2)^2 + M_G^2 \Gamma_G^2}.
\end{aligned}
\tag{19}
$$

The masses M_G and total widths Γ_G of the bosons, taken from the Particle Data Group [58], are $M_{W^\pm} = 80.377 \pm 0.012$ GeV, $M_{Z^0} = 91.1876 \pm 0.0021$ GeV, $\Gamma_{W^\pm} = 2.085 \pm 0.042$ GeV, and $\Gamma_{Z^0} = 2.4955 \pm 0.0023$ GeV. Note that in Eqs. (16) and (17) the terms proportional to the squares of the Breit-Wigner propagators ($|D_Z(Q^2)|^2$ and $|D_W(Q^2)|^2$) correspond to the purely weak ZZ and WW contributions to the couplings, while the terms proportional to the real part of the Breit-Wigner Z boson propagator $\mathrm{Re}[D_Z(Q^2)]$ correspond to γ-Z interference.

As a final ingredient, we note that all one-loop partonic structure functions contain a factor $\delta((\hat{s} + \hat{t} + \hat{u} - Q^2)/\hat{s})$ arising from phase space and corresponding to the fact that the recoil in the final state consists of a single massless parton. It is convenient to write

$$
w^{ab}(z_1, z_2, \rho^2) = \tilde{w}^{ab}(z_1, z_2, \rho^2) \delta\big((\hat{s} + \hat{t} + \hat{u} - Q^2)/\hat{s}\big).
\tag{20}
$$

With this notation in place, we obtain the following partonic T-odd structure functions. For the $q\bar{q}$ annihilation subprocess, we find

$$
\begin{aligned}
\tilde{w}^{q\bar{q}}_{\Delta\Delta_P} &= \frac{g_{q\bar{q};1}}{2} \sqrt{\frac{Q^2 \hat{s}}{(Q^2 - \hat{u})(Q^2 - \hat{t})}} \Bigg[-\frac{C_F}{2} \left(\frac{Q^2 - \hat{t}}{Q^2 - \hat{u}} + \frac{Q^2 - \hat{u}}{Q^2 - \hat{t}} \right) \\
&\quad + C_1 \left(\frac{Q^2 - \hat{t}}{\hat{t}} \left(1 - \frac{\hat{s}}{\hat{t}} \log \frac{Q^2 - \hat{u}}{\hat{s}} \right) + \frac{Q^2 - \hat{u}}{\hat{u}} \left(1 - \frac{\hat{s}}{\hat{u}} \log \frac{Q^2 - \hat{t}}{\hat{s}} \right) \right) \Bigg],
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
\tilde{w}^{q\bar{q}}_{\Delta_P} &= \frac{g_{q\bar{q};1}}{2} \frac{Q^2 \hat{s}}{\sqrt{(Q^2 - \hat{u})(Q^2 - \hat{t}) \hat{u} \hat{t}}} \Bigg[C_F \left(\frac{Q^2 - \hat{t}}{Q^2 - \hat{u}} - \frac{Q^2 - \hat{u}}{Q^2 - \hat{t}} \right) \\
&\quad + C_1 \left(\frac{Q^2 - \hat{u}}{\hat{u}} \log \frac{Q^2 - \hat{t}}{\hat{s}} - \frac{Q^2 - \hat{t}}{\hat{t}} \log \frac{Q^2 - \hat{u}}{\hat{s}} \right) \Bigg],
\end{aligned}
\tag{22}
$$

$$\tilde{w}_{\nabla}^{q\bar{q}} = g_{q\bar{q};2}\sqrt{\frac{Q^2\hat{s}}{\hat{u}\,\hat{t}}}\left[\frac{C_F}{2}\frac{(2Q^2\hat{s}+\hat{u}\,\hat{t})(Q^2+\hat{s})(\hat{u}-\hat{t})}{(Q^2-\hat{u})^2(Q^2-\hat{t})^2}\right. \\ \left. + C_1\left(-\frac{Q^2(\hat{u}-\hat{t})}{(Q^2-\hat{u})(Q^2-\hat{t})} + \frac{\hat{s}}{\hat{u}}\log\frac{Q^2-\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}}\log\frac{Q^2-\hat{u}}{\hat{s}}\right)\right]. \tag{23}$$

For qg scattering we have

$$\tilde{w}_{\Delta\Delta_P}^{qg} = \frac{g_{qg;1}}{2}\frac{\hat{u}}{\hat{s}}\sqrt{\frac{Q^2\hat{s}}{(Q^2-\hat{u})(Q^2-\hat{t})}}\left[\frac{C_F}{2}\frac{2Q^2+\hat{s}}{Q^2-\hat{t}}\right. \\ \left. - C_1\left(\frac{Q^2}{Q^2-\hat{u}} + \frac{Q^2-\hat{u}}{\hat{s}}\log\frac{(Q^2-\hat{u})(Q^2-\hat{t})}{\hat{u}\,\hat{t}} + \frac{Q^2-\hat{t}}{\hat{t}}\left(1-\frac{Q^2-\hat{u}}{\hat{t}}\log\frac{Q^2-\hat{u}}{\hat{s}}\right)\right)\right], \tag{24}$$

$$\tilde{w}_{\Delta_P}^{qg} = \frac{g_{qg;1}}{2}\frac{Q^2\hat{u}}{\sqrt{(Q^2-\hat{u})(Q^2-\hat{t})\hat{u}\,\hat{t}}}\left[\frac{C_F}{2}\frac{\hat{t}-2\hat{u}}{Q^2-\hat{t}}\right. \\ \left. - C_1\left(\frac{\hat{u}}{Q^2-\hat{u}} + \frac{Q^2}{\hat{s}} - \frac{(Q^2-\hat{u})\hat{u}}{\hat{s}^2}\log\frac{(Q^2-\hat{u})(Q^2-\hat{t})}{\hat{u}\,\hat{t}}\right)\right], \tag{25}$$

$$\tilde{w}_{\nabla}^{qg} = g_{qg;2}\frac{\hat{u}}{\hat{s}}\sqrt{\frac{Q^2\hat{s}}{\hat{u}\,\hat{t}}}\left[-\frac{C_F}{2}\frac{2Q^2\hat{u}+\hat{s}\,\hat{t}}{(Q^2-\hat{t})^2}\right. \\ \left. - C_1\left(\frac{2Q^2\hat{s}}{(Q^2-\hat{u})^2} - \frac{Q^4-\hat{u}\,\hat{t}}{(Q^2-\hat{u})(Q^2-\hat{t})} + \frac{\hat{u}}{\hat{t}}\log\frac{Q^2-\hat{u}}{\hat{s}} - \frac{\hat{u}}{\hat{s}}\log\frac{(Q^2-\hat{u})(Q^2-\hat{t})}{\hat{u}\,\hat{t}}\right)\right]. \tag{26}$$

We note that in the large N_c limit the terms proportional to C_1 in the structure functions are suppressed by a factor $1/N_c^2$ relative to those proportional to C_F and C_A. The results for $g\bar{q}$ scattering are obtained from the ones for the qg process by interchange of momenta, $p_1 \leftrightarrow p_2$, which corresponds to an interchange of Mandelstam variables $u \leftrightarrow t$.

For calculating the hadronic structure functions via the factorization formula (11) and subsequently investigating their small Q_T behavior, it is convenient to express our results in terms of the variables $z_1 = x_1/\xi_1$, $z_2 = x_2/\xi_2$, and $\rho^2 = Q^2/Q_T^2$. Using

$$\hat{s} = \frac{Q^2+Q_T^2}{z_1 z_2}, \qquad Q^2-\hat{t} = \frac{Q^2+Q_T^2}{z_1}, \qquad Q^2-\hat{u} = \frac{Q^2+Q_T^2}{z_2}, \\ \frac{\hat{u}\,\hat{t}}{Q^2\hat{s}} = \rho^2, \qquad \frac{(Q^2-\hat{u})(Q^2-\hat{t})}{Q^2\hat{s}} = 1+\rho^2, \tag{27}$$

one gets for $q\bar{q}$ annihilation

$$\tilde{w}_{\Delta\Delta_P}^{q\bar{q}} = -\frac{g_{q\bar{q};1}}{4z_1z_2}\frac{1}{\sqrt{1+\rho^2}}\left[C_A\frac{z_1^2+z_2^2}{2} + C_1(z_1^2F_1(z_2)+z_2^2F_1(z_1))\right], \tag{28}$$

$$\tilde{w}_{\Delta_P}^{q\bar{q}} = -\frac{g_{q\bar{q};1}}{2z_1z_2}\frac{1}{\rho\sqrt{1+\rho^2}}\left[C_A\frac{z_1^2-z_2^2}{2} + C_1(z_1^2F_2(z_2)-z_2^2F_2(z_1))\right], \tag{29}$$

$$\tilde{w}_{\nabla}^{q\bar{q}} = -\frac{g_{q\bar{q};2}}{z_1z_2}\frac{1}{\rho}\left[\left(C_A - \frac{\rho^2}{1+\rho^2}C_F\right)\frac{z_1^2-z_2^2}{2} + C_1(z_1F_2(z_2)-z_2F_2(z_1))\right], \tag{30}$$

and for qg scattering

$$\tilde{w}^{qg}_{\Delta\Delta_P} = -\frac{g_{qg;1}}{2}\frac{1-z_2}{z_1 z_2}\frac{1}{\sqrt{1+\rho^2}}\left[\frac{C_F}{2}z_1\left(1+\frac{2z_1z_2}{1+\rho^2}\right) + C_1 z_1 z_2\left(\left(F_1(z_1) - \frac{1-\rho^2}{1+\rho^2}\right)\frac{z_2}{2} + z_1\log\frac{\rho^2}{1+\rho^2}\right)\right], \tag{31}$$

$$\tilde{w}^{qg}_{\Delta_P} = -\frac{g_{qg;1}}{2}\frac{1-z_2}{z_1 z_2}\frac{1}{\rho\sqrt{1+\rho^2}}\left[\frac{C_F}{2}z_1(1+z_1-2z_2) + C_1 z_2\left(1-z_2-\frac{z_1^2 z_2}{1+\rho^2} + z_1^2(1-z_2)\log\frac{\rho^2}{1+\rho^2}\right)\right], \tag{32}$$

$$\tilde{w}^{qg}_{\nabla} = -g_{qg;2}\frac{1-z_2}{z_1 z_2}\frac{1}{\rho}\left[C_F z_1\left(\frac{z_1(1-z_2)}{1+\rho^2} + \frac{1-z_1}{2z_2}\right) + C_1 z_2\left(\frac{z_1^2-2z_2}{1+\rho^2} + z_1(1-z_2)\log\frac{\rho^2}{1+\rho^2} - (1-z_2)F_2(z_1)\right)\right]. \tag{33}$$

Here, the functions F_1 and F_2 are defined as

$$F_1(z) \equiv \frac{1+z}{1-z} + \frac{2z\log(z)}{(1-z)^2} = 2\sum_{N=1}^{\infty}\frac{(1-z)^N}{(N+1)(N+2)} = \mathcal{O}(1-z),$$
$$F_2(z) \equiv 1 + \frac{z\log(z)}{1-z} = \frac{1-z}{2}(1+F_1(z)) = \sum_{N=1}^{\infty}\frac{(1-z)^N}{N(N+1)} = \mathcal{O}(1-z). \tag{34}$$

They obey $F_1(1) = F_2(1) = 0$ and $F_1(0) = F_2(0) = 1$. For later reference, we have also given their expansions around $z = 1$.

We now have

$$W(x_1, x_2, \rho^2) = \frac{1}{x_1 x_2}\sum_{a,b}\int_{x_1}^{1} dz_1\int_{x_2}^{1} dz_2 \tilde{w}^{ab}(z_1, z_2, \rho^2)\delta\left((1-z_1)(1-z_2) - \frac{\rho^2}{1+\rho^2}z_1 z_2\right) f_{a/H_1}\left(\frac{x_1}{z_1}\right) f_{b/H_2}\left(\frac{x_2}{z_2}\right). \tag{35}$$

This factorization is formally valid when Q_T is of order Q, that is, for $\Lambda_{\mathrm{QCD}} \ll Q \sim Q_T$. For $Q_T \ll Q$ the appropriate factorization formalism is TMD factorization. Here, we will take the collinear factorization and extrapolate to small values of Q_T by formally expanding the result about $Q_T = 0$. This will result in an expansion in powers of ρ^2 which can be matched to TMD results in the region of intermediate Q_T for a smooth transition from the TMD to the collinear regime. We will perform the expansion in ρ^2 beyond the leading power, which has been the main focus in the existing literature, to provide information about which higher-power corrections are accounted for in the collinear formalism.

IV. SMALL-Q_T EXPANSION

In the expansion of hadronic structure functions of the form in Eq. (35) we have three contributions. First, there is the direct dependence of the partonic structure function on Q_T. Second, the phase space delta function has nontrivial Q_T dependence. Third, the variables x_1, x_2 have implicit Q_T dependence. The first type of contribution may be straightforwardly taken into account by simple expansion of the partonic structure functions. The second and third contributions require more discussion.

The phase space delta function in (35) is well known in the literature, see, e.g., Refs. [41,54,59]. Its expansion to leading power in $\rho^2 = Q_T^2/Q^2$ was also given in that reference and reads as

$$\delta\left((1-z_1)(1-z_2) - \frac{\rho^2}{1+\rho^2}z_1 z_2\right)$$
$$= \frac{\delta(1-z_1)}{(1-z_1)_+} + \frac{\delta(1-z_2)}{(1-z_2)_+} - \delta(1-z_1)\delta(1-z_2)$$
$$\times \log\rho^2 + \mathcal{O}(\rho^2). \tag{36}$$

Here the "plus" distribution is defined by

$$\int_0^1 dz\frac{f(z)}{(1-z)_+} \equiv \int_0^1 dz\frac{f(z)-f(1)}{1-z}, \tag{37}$$

for a function f that is regular at $z = 1$. We note in passing that in Ref. [60] a general method for the expansion of distributions was developed, based on Mellin integral techniques. Building on these ideas, we recently formulated [61] an algorithm for the small-Q_T expansion of singular functions valid to arbitrary order of ρ^2 and arbitrary number of radiated partons. This will be presented in a separate publication. Here we are only concerned with the expansion of *integrals* containing the phase space delta function in (36). For a general regular function $\varphi(z_1, z_2)$ such an integral can be expanded for small Q_T including $\mathcal{O}(\rho^4, \rho^4 \log\rho^2)$ terms in the following way:

$$\begin{aligned} I_0 &\equiv \int_{x_1}^{1} dz_1 \int_{x_2}^{1} dz_2 \delta\left((1-z_1)(1-z_2) - \frac{\rho^2}{1+\rho^2} z_1 z_2\right)\varphi(z_1, z_2) \\ &= \int_{x_1}^{1} dz_1 \int_{x_2}^{1} dz_2 (\delta(1-z_2) G_1(z_1, z_2) + \delta(1-z_1) G_1(z_2, z_1) \\ &\quad + \delta(1-z_1)\delta(1-z_2) G_2(z_1, z_2))\varphi(z_1, z_2) + \mathcal{O}(\rho^6, \rho^6 \log\rho^2), \end{aligned} \tag{38}$$

with

$$\begin{aligned} G_1(z_1, z_2) &= \frac{(1+\rho^2)(1+\rho^2\partial_{z_2}) + \rho^4 \partial_{z_2}^2/2}{(1-z_1)_+} - \frac{\rho^2(1+\rho^2+(1+3\rho^2)\partial_{z_2} + \rho^2\partial_{z_2}^2)}{(1-z_1)_{+,1}^2} + \frac{\rho^4(1+2\partial_{z_2} + \partial_{z_2}^2/2)}{(1-z_1)_{+,2}^3}, \\ G_2(z_1, z_2) &= \rho^2\left(1+\rho^2\left(\frac{1}{2} + \partial_{z_1} + \partial_{z_2} + \partial_{z_1 z_2}^2\right)\right) - \log\rho^2\left((1+\rho^2)\big(1+\rho^2(\partial_{z_1} + \partial_{z_2} + \partial_{z_1 z_2}^2)\big)\right. \\ &\quad \left. + \frac{\rho^4}{2}(\partial_{z_1} + \partial_{z_2})^2 + \rho^4 \partial_{z_1 z_2}^2 \left(1 + \partial_{z_1} + \partial_{z_2} + \frac{\partial_{z_1 z_2}^2}{4}\right)\right). \end{aligned} \tag{39}$$

Here $1/(1-z)_{+,m-1}^m$ is a generalized plus distribution of power m, defined by

$$\int_0^1 dz \frac{f(z)}{(1-z)_{+,m-1}^m} \equiv \int_0^1 dz \frac{f(z) - \mathcal{T}_{z=1}^{m-1} f(z)}{(1-z)^m}, \tag{40}$$

where $f(z)$ is again a sufficiently regular test function and $\mathcal{T}_{z=1}^{m-1} f(z)$ denotes the Taylor polynomial of $f(z)$ about $z=1$ to order $m-1$,

$$\mathcal{T}_{z=1}^{m-1} f(z) = \sum_{k=0}^{m-1} \frac{(-1)^k f^{(k)}(1)}{k!}(1-z)^k. \tag{41}$$

A lower integration bound of x instead of zero introduces additional boundary terms of the form

$$\begin{aligned} \int_x^1 dz \frac{f(z)}{(1-z)_{+,m-1}^m} = \int_x^1 dz \Bigg[& \frac{1}{(1-z)_{+x,m-1}^m} + \delta(1-z) \\ & \times \log(1-x_1)\frac{(-1)^{m-1}}{(m-1)!}\partial_z^{m-1} \\ & - \delta(1-z)\sum_{j=2}^{m} \frac{(-1)^{m-j}}{(j-1)(m-j)!} \\ & \times \left(\frac{1}{(1-x_1)^{j-1}} - 1\right)\partial_z^{m-j}\Bigg] f(z_1), \end{aligned} \tag{42}$$

where $f(z)/(1-z)_{+x,m-1}^m$ is the generalized plus distribution defined for an integral starting at a finite lower limit x, i.e.,

$$\int_x^1 dz \frac{f(z)}{(1-z)_{+x,m-1}^m} \equiv \int_x^1 dz \frac{f(z) - \mathcal{T}_{z=1}^{m-1} f(z)}{(1-z)^m}. \tag{43}$$

Comparing Eq. (38) with (36) one can see that we reproduce the known leading terms, while the terms of order ρ^2, $\rho^2 \log\rho^2$, ρ^4, and $\rho^4 \log\rho^2$ are new.

Substituting the small-Q_T expansion of the parton-level structure functions $w^{ab}(z_1, z_2, \rho^2)$ for the various partonic channels into Eq. (11) we get for the contributions to the small-Q_T expansion of the hadronic structure function $W(x_1, x_2, \rho^2)$:

$$\begin{aligned} W_{\delta-\mathrm{fct.}}(x_1, x_2, \rho^2) &= W_0(x_1, x_2, L_\rho) + \rho^2 W_1(x_1, x_2, L_\rho) \\ &\quad + \rho^4 W_2(x_1, x_2, L_\rho) + \mathcal{O}(\rho^6), \end{aligned} \tag{44}$$

were we have abbreviated

$$L_\rho \equiv \log\rho^2. \tag{45}$$

The expansion coefficients with $i = 1,\ 2,\ 3$ have the structure

$$W_i(x_1,x_2,L_\rho) = \frac{1}{x_1x_2}\sum_{a,b}\big[R_{ab,i}(x_1,x_2,L_\rho)f_{a/H_1}(x_1)f_{b/H_2}(x_2) + (P_{ba,i}\otimes f_{b/H_2})(x_2,x_1,L_\rho)f_{a/H_1}(x_1) + (P_{ab,i}\otimes f_{a/H_1})(x_1,x_2,L_\rho)f_{b/H_2}(x_2)\big], \tag{46}$$

where

$$(\mathcal{P}\otimes f)(x,y,L_\rho) = \int_x^1 \frac{dz}{z}\mathcal{P}(z,y,L_\rho)f\left(\frac{x}{z}\right) \tag{47}$$

denotes a generalized convolution, $R_i(x_1,x_2,L_\rho)$, $P_{ba,i}(z_2,x_1,L_\rho)$, and $P_{ab,i}(z_1,x_2,L_\rho)$ are perturbative coefficient functions containing differential operators acting on the PDFs $f_{a/H_1}(x_1)$ and $f_{b/H_2}(x_2)$. We note that the generalized convolution (47) reverts to the ordinary one,

$$(\mathcal{P}\otimes f)(x) = \int_x^1 \frac{dz}{z}\mathcal{P}(z)f\left(\frac{x}{z}\right), \tag{48}$$

when $\mathcal{P}(z,y,L_\rho)$ does not depend on y and L_ρ. Details are given in Appendix C. We stress that, as indicated in Eq. (44), the functions W_i may carry dependence on $\log\rho^2$, on top of the overall power of ρ that they multiply.

However, Eq. (44) is not yet the complete expansion. As mentioned above, we need to take into account that x_1 and x_2 are defined at finite Q_T [see Eq. (9)] and hence must also be expanded about their respective values at $Q_T = 0$, x_1^0 and x_2^0 in (10). Therefore, we substitute $x_i = x_i^0\sqrt{1+\rho^2}$ as arguments of the structure functions W_i and perform the ρ^2 expansions of the latter. We now present our final result for the full small-Q_T expansion of the hadronic structure functions, including the leading-power (LP) term $W^{\rm LP}(x_1^0,x_2^0,L_\rho)$, the next-to-leading-power (NLP) term $W^{\rm NLP}(x_1^0,x_2^0,L_\rho)$, and the next-next-to-leading-power (NNLP)term $W^{\rm NNLP}(x_1^0,x_2^0,L_\rho)$:

$$W(x_1,x_2,\rho^2) = W^{\rm LP}(x_1^0,x_2^0,L_\rho) + \rho^2 W^{\rm NLP}(x_1^0,x_2^0,L_\rho) + \rho^4 W^{\rm NNLP}(x_1^0,x_2^0,L_\rho) + \mathcal{O}(\rho^6), \tag{49}$$

where

$$W^{\rm LP}(x_1^0,x_2^0,L_\rho) = W_0(x_1^0,x_2^0,L_\rho), \tag{50}$$

$$W^{\rm NLP}(x_1^0,x_2^0,L_\rho) = W_1(x_1^0,x_2^0,L_\rho) + \frac{1}{2}\big(x_1^0\partial_{x_1^0}W_0(x_1^0,x_2^0,L_\rho) + x_2^0\partial_{x_2^0}W_0(x_1^0,x_2^0,L_\rho)\big), \tag{51}$$

$$\begin{aligned}W^{\rm NNLP}(x_1^0,x_2^0,L_\rho) = {}& W_2(x_1^0,x_2^0,L_\rho) + \frac{1}{4}x_1^0x_2^0\partial_{x_1^0}\partial_{x_2^0}W_0(x_1^0,x_2^0,L_\rho) - \frac{1}{8}\big(x_1^0\partial_{x_1^0}W_0(x_1^0,x_2^0,L_\rho) - 4x_1^0\partial_{x_1^0}W_1(x_1^0,x_2^0,L_\rho) \\ & - (x_1^0)^2\partial^2_{x_1^0}W_0(x_1^0,x_2^0,L_\rho)\big) - \frac{1}{8}\big(x_2^0\partial_{x_2^0}W_0(x_1^0,x_2^0,L_\rho) - 4x_2^0\partial_{x_2^0}W_1(x_1^0,x_2^0,L_\rho) \\ & - (x_2^0)^2\partial^2_{x_2^0}W_0(x_1^0,x_2^0,L_\rho)\big).\end{aligned} \tag{52}$$

Here $\partial^m_{x_1}\partial^n_{x_2}W_i(x_1,x_2,L_\rho)$ denotes the mth partial derivative with respect to x_1 and the nth partial derivative with respect to x_2. The calculational techniques for taking these derivatives are discussed in Appendix C.

Explicitly we obtain the following analytical results for the LP contributions $W_J^{{\rm LP};ab}(x_1^0,x_2^0,L_\rho)$ to the T-odd hadronic structure functions (here $ab = q\bar q, qg$ and $J = \Delta\Delta_P, \Delta_P, \nabla$):

$$\begin{aligned}W^{{\rm LP};q\bar q}_{\Delta\Delta_P}(x_1^0,x_2^0,L_\rho) = {}& \frac{g_{q\bar q;1}}{4x_1^0x_2^0}C_A\left(L_\rho+\frac{3}{2}\right)q_1(x_1^0)\bar q_2(x_2^0) - \frac{g_{q\bar q;1}}{4x_1^0x_2^0}\frac{C_A}{2C_F}\big[q_1(x_1^0)(P_{qq}\otimes\bar q_2)(x_2^0) + (P_{qq}\otimes q_1)(x_1^0)\bar q_2(x_2^0)\big] \\ & - \frac{g_{q\bar q;1}}{4x_1^0x_2^0}C_1\big[q_1(x_1^0)(f_1\otimes\bar q_2)(x_2^0) + (f_1\otimes q_1)(x_1^0)\bar q_2(x_2^0)\big],\end{aligned} \tag{53}$$

$$\begin{aligned}W^{{\rm LP};q\bar q}_{\nabla}(x_1^0,x_2^0,L_\rho) &= 2\beta W^{{\rm LP};q\bar q}_{\Delta_P}(x_1^0,x_2^0) \\ &= -\frac{g_{q\bar q;1}}{\rho x_1^0x_2^0}\frac{C_A}{2C_F}\big[q_1(x_1^0)(\tilde P_{qq}\otimes\bar q_2)(x_2^0) - (\tilde P_{qq}\otimes q_1)(x_1^0)\bar q_2(x_2^0)\big] \\ &\quad - \frac{g_{q\bar q;1}}{\rho x_1^0x_2^0}C_1\big[q_1(x_1^0)(f_2\otimes\bar q_2)(x_2^0) - (f_2\otimes q_1)(x_1^0)\bar q_2(x_2^0)\big],\end{aligned} \tag{54}$$

$$W^{\mathrm{LP};qg}_{\Delta\Delta_P}(x_1^0, x_2^0, L_\rho) = -\frac{g_{qg;1}}{4x_1^0 x_2^0} q(x_1^0)(P'_{qg} \otimes g)(x_2^0), \tag{55}$$

$$\begin{aligned} W^{\mathrm{LP};qg}_{\nabla}(x_1^0, x_2^0, L_\rho) &= 2\beta W^{\mathrm{LP};qg}_{\Delta_P}(x_1^0, x_2^0, L_\rho) \\ &= -\frac{g_{qg;1}}{\rho x_1^0 x_2^0} q(x_1^0)(P''_{qg} \otimes g)(x_2^0), \end{aligned} \tag{56}$$

where

$$\begin{aligned} \beta &= \frac{g_{\mathrm{EW};1}}{g_{\mathrm{EW};2}}, \\ P_{qq}(z) &= C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right], \\ \tilde{P}_{qq}(z) &= C_F(1+z), \\ P'_{qg}(z, L_\rho) &= C_F(1+2z) + C_1 z(2L_\rho - z), \\ P''_{qg}(z, L_\rho) &= C_F(1-z) + C_1 z(L_\rho(1-z) + 1 - 2z), \end{aligned} \tag{57}$$

and

$$f_i(z) = \frac{F_i(z)}{1-z}, \tag{58}$$

with F_i as defined in Eq. (34). Note that the f_i are regular functions with $f_1(1) = 1/3$, $f_2(1) = 1/2$.

As shown in (56), there is an interesting relation between the structure functions $W^{\mathrm{LP};ab}_{\nabla}(x_1^0, x_2^0, L_\rho)$ and $W^{\mathrm{LP};ab}_{\Delta_P}(x_1^0, x_2^0, L_\rho)$:

$$W^{\mathrm{LP};ab}_{\nabla}(x_1^0, x_2^0, L_\rho) = 2\beta W^{\mathrm{LP};ab}_{\Delta_P}(x_1^0, x_2^0, L_\rho), \tag{59}$$

valid both for the $q\bar{q}$ and the qg subprocess at leading power. The NLP and NNLP contributions to the T-odd hadronic structure function are listed in the Supplemental Material [52].

To illustrate the numerical behavior of these expansions, we consider the $q\bar{q}$ contribution to the hadronic double-flip structure function, $W^{q\bar{q}}_{\Delta\Delta_P}(x_1, x_2)$, as an example. In Fig. 4 we compare the full expression without Q_T expansion with the LP, NLP, and NNLP results. Here we use the CTEQ 6.1M PDFs of Ref. [62], taken from LHAPDF [63], along with their MANEPARSE [64] *Mathematica* implementation. We choose $\sqrt{s} = 8$ TeV, $Q = 100$ GeV, as representative of the kinematics in the ATLAS measurements [47], and the renormalization and factorization scales in the calculations are set to $\mu = \sqrt{Q^2 + Q_T^2}$. As one can see, the LP piece describes the full result only at low Q_T and rapidly departs from it for $Q_T > 10$ GeV or $\rho^2 > 0.01$. By contrast, already inclusion of the NLP term leads to excellent agreement with the full result out to $Q_T = 40$ GeV ($\rho^2 = 0.16$), only marginally further improved by the NNLP contribution. In particular, for $Q_T = 20$ GeV, the LP result deviates from the full one by about 20%, whereas at NNLP the relative deviation is only ~0.4%.

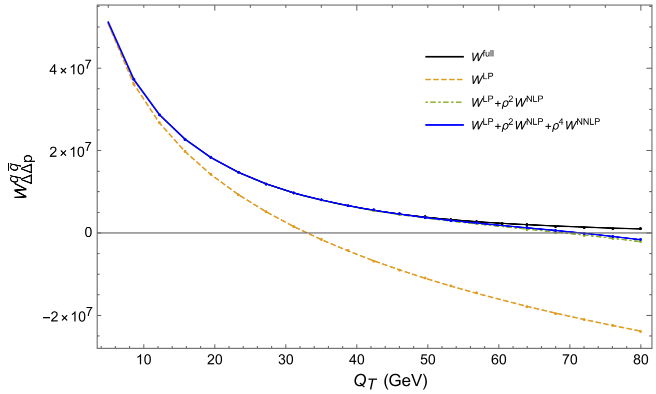

FIG. 4. Comparison of the full analytical result for the quark-channel contribution to $W_{\Delta\Delta_P}$ [black solid line, taken from Eq. (30)] with expansions to LP (dashed), NLP (dot-dashed), NNLP (blue solid) as given in the Supplemental Material [52].

V. COMPARISON TO ATLAS DATA

As shown in Eq. (4), the Drell-Yan cross section can be expressed in terms of eight angular coefficients $A_{i=0,\ldots,7}$. The relations of these coefficients to the hadronic helicity structure functions are recalled in Appendix A. Previous experimental and phenomenological studies mostly focused on the first five A_i coefficients [18,47,65–71] which are related to the T-even structure functions. The T-odd structure functions have received less attention. Experimentally, it has not yet been possible to measure the T-odd angular coefficients in W boson production, but results in neutral-current scattering in the vicinity of the Z-boson mass peak are available from the ATLAS Collaboration [47]. Specifically, the measurement was performed in the Z-boson invariant mass window of 80–100 GeV, as a function of Q_T, and also in three bins of rapidity y: (a) $|y| < 1$, (b) $1 < |y| < 2$, and (c) $2 < |y| < 3.5$. We note that near $Q = m_Z$ the contribution by $\gamma - Z^0$ interference is suppressed relative to that for pure Z^0 exchange. In the following, we compare our results for the angular coefficients A_5, A_6, and A_7 to the ATLAS data. Here we use the full expressions at $\mathcal{O}(\alpha_s^2)$ for the helicity structure functions $W_{\Delta\Delta_P}$, W_{Δ_P}, and W_∇. In the denominator of the coefficients, we use the $\mathcal{O}(\alpha_s)$ expressions for the transverse and longitudinal structure functions W_T and W_L (see details in Refs. [41,54,72]). This approach thus consistently gives the leading contribution to A_5, A_6, and A_7, which is of order α_s.

To begin with, we investigate the rapidity and Q_T dependences of the angular coefficients A_5, A_6, A_7 near the Z^0 pole. As in the previous section the calculation is done using $\sqrt{s} = 8$ TeV and $\mu = \sqrt{Q^2 + Q_T^2}$. Figure 5 shows the rapidity distribution for fixed Q_T, while Fig. 6

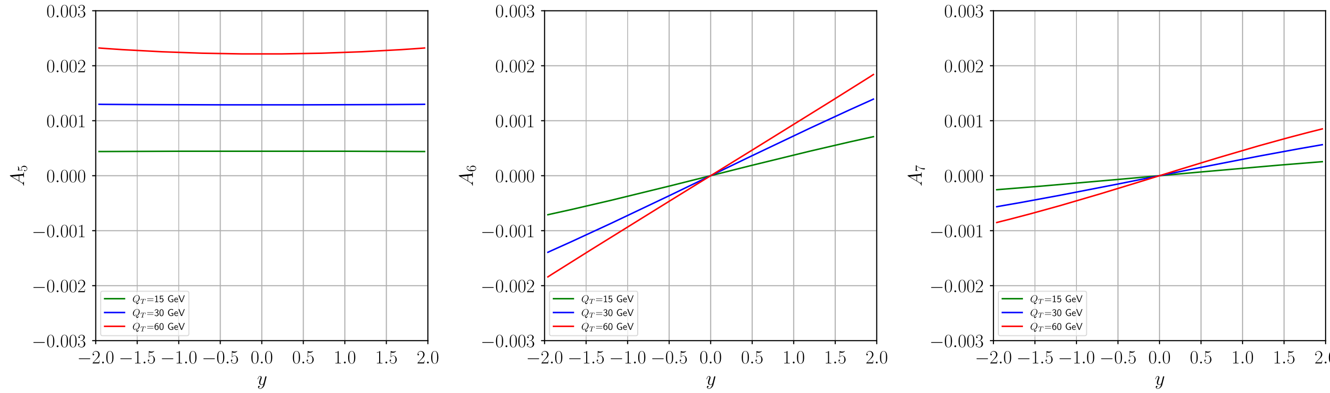

FIG. 5. T-odd angular coefficients A_5, A_6, and A_7 for various transverse momenta Q_T of the lepton pair, as functions of pair rapidity at $Q \sim m_Z$ and $\sqrt{s} = 8$ TeV.

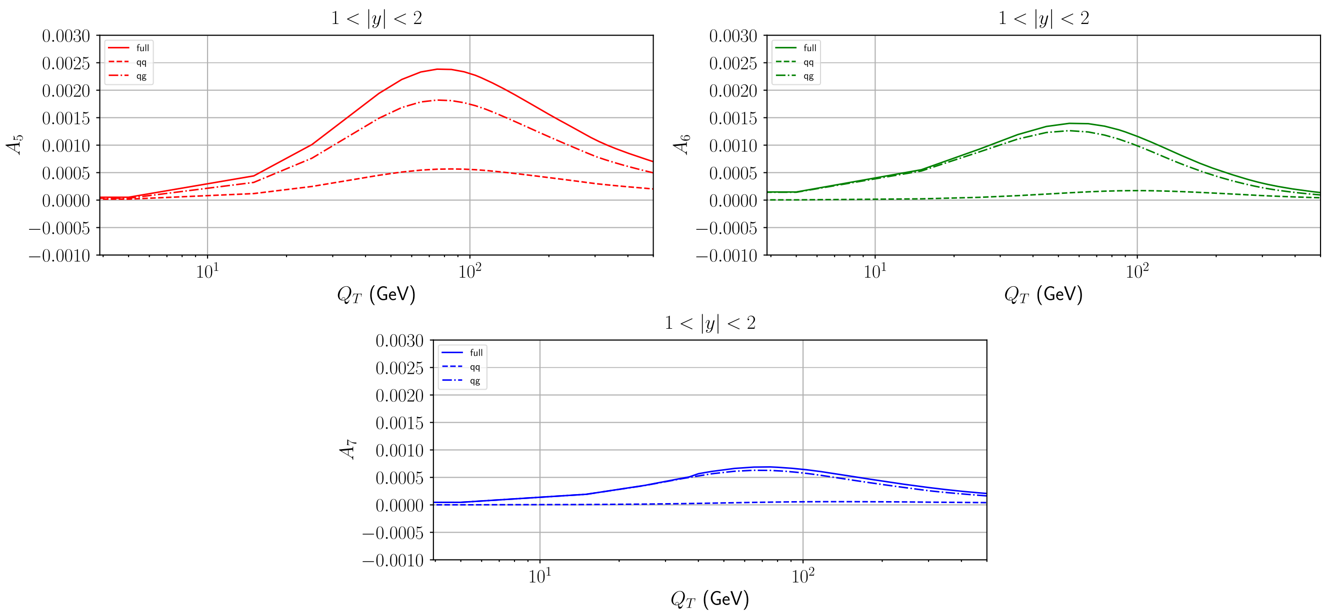

FIG. 6. Q_T dependence of the angular coefficients A_5, A_6, and A_7 for the rapidity interval $1 < |y| < 2$ at $Q \sim m_Z$ and $\sqrt{s} = 8$ TeV. We also show the individual $q\bar{q}$ and qg contributions.

presents the results as functions of Q_T in one of the three rapidity bins accessed by ATLAS. As the plots show, the coefficients are overall small, reaching at most 0.1–0.2% near $Q_T \sim m_Z$ or toward larger $|y|$. This finding does not really come as a surprise: Small values of the T-odd $A_{5,6,7}$ coefficients have been predicted in Refs. [8,10] also for W boson production. In the case of Z boson production $A_{5,6,7}$ are further suppressed because of the smallness of the corresponding weak couplings, relative to the couplings appearing in W_T and W_L.

The increase of $A_{5,6,7}$ at larger values of rapidity in Fig. 5—which is consistent with the leading-power relation between W_∇ and W_{Δ_P} we found in Eq. (59)—follows the trend observed in Ref. [8] for W boson production in $p\bar{p}$ collisions at $\sqrt{s} = 540$ GeV. Likewise, a similar dependence on rapidity was found for the angular coefficients in Refs. [47,66]. At fixed rapidity, the T-odd coefficients are small for small Q_T and then increase, peaking when Q_T is near the Z mass (see Fig. 6). We also show in the figure the individual contributions by $q\bar{q}$ and qg scattering, the latter dominating for all kinematics. Among the T-odd structure functions, $W_{\Delta\Delta_P}$, being symmetric under interchange $z_1 \leftrightarrow z_2$, has the largest contribution from quark-antiquark annihilation.

Figure 7 explores the role played by the pair mass Q for the Q_T distribution. We show results for $Q = 80$ GeV, $Q = m_Z$, and $Q = 100$ GeV, which span the range of Q used for the ATLAS measurements. As one can see, A_5 and A_6 are rather insensitive to Q, whereas A_7 exhibits a strong dependence, even turning negative at high Q. This suggests that A_7 will be quite sensitive to smearing effects if data are sampled over a sizable range in Q.

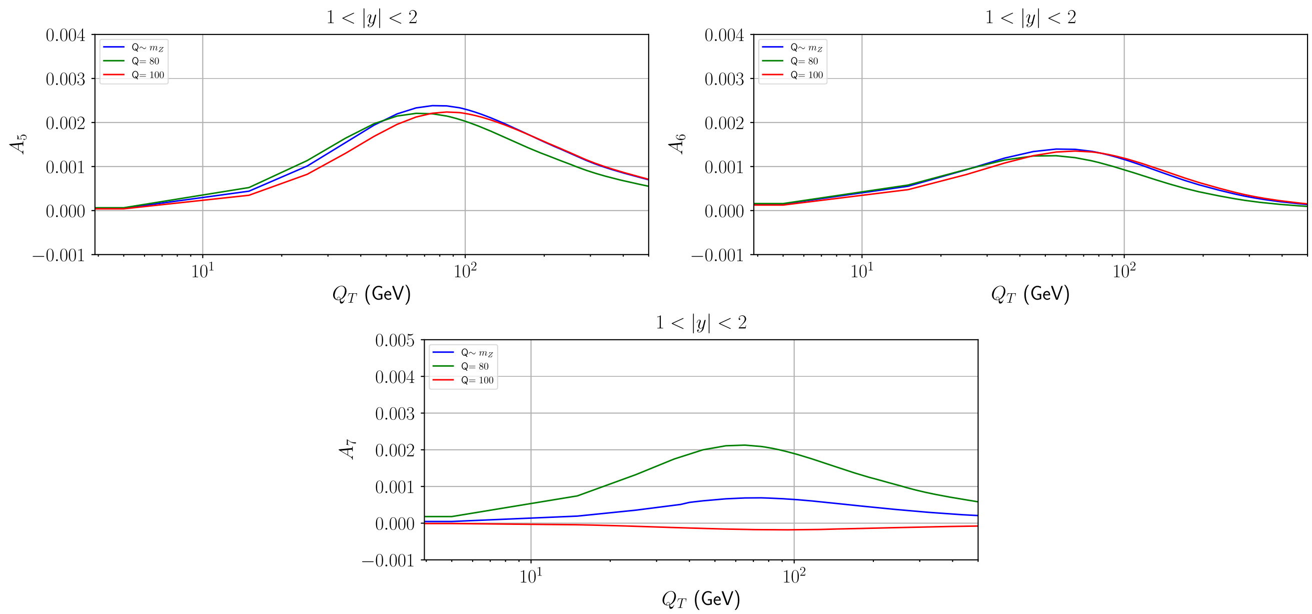

FIG. 7. Q_T dependence of the angular coefficients A_5, A_6, and A_7 for rapidity $1 < |y| < 2$ at $\sqrt{s} = 8$ TeV and for different Q.

We now turn to the actual comparison with the ATLAS data [47]. Of their three rapidity bins ($|y| < 1$, $1 < |y| < 2$, and $2 < |y| < 3.5$) we only use the two with higher $|y|$ since, as we saw above, the angular coefficients are very small for $|y| < 1$. We note that ATLAS presents the data in two ways, as an "unregularized" and a "regularized" set. The regularization smoothes the data by correcting for bin migration. This procedure involves the use of Monte-Carlo pseudo data which are at lowest-order accuracy. Details about the data regularization method are presented in Appendixes C and E of Ref. [47].

Figures 8–10 show the comparison. Neither the regularized nor the unregularized data are in particularly good agreement with our theoretical predictions. At best, there is qualitative agreement in that the theoretical results show positive values for all three T-odd angular coefficients, with a similar trend in the data. In particular, for the bin $1 < |y| < 2$ one observes a rise of $A_{5,6,7}$ with Q_T up to about $Q_T \sim m_Z$, exactly as predicted theoretically. Quantitatively, however, the regularized data—which is the set primarily to be used for comparisons—shows overall much higher coefficients than obtained in our calculation. We note that ATLAS used the DYNNLO package [73] to obtain theoretical results for $A_{5,6,7}$. DYNNLO predicts values of up to 0.005 for the coefficients $A_{5,6,7}$. However, as stated in [47], the prediction of nonzero values is at the limit of sensitivity of

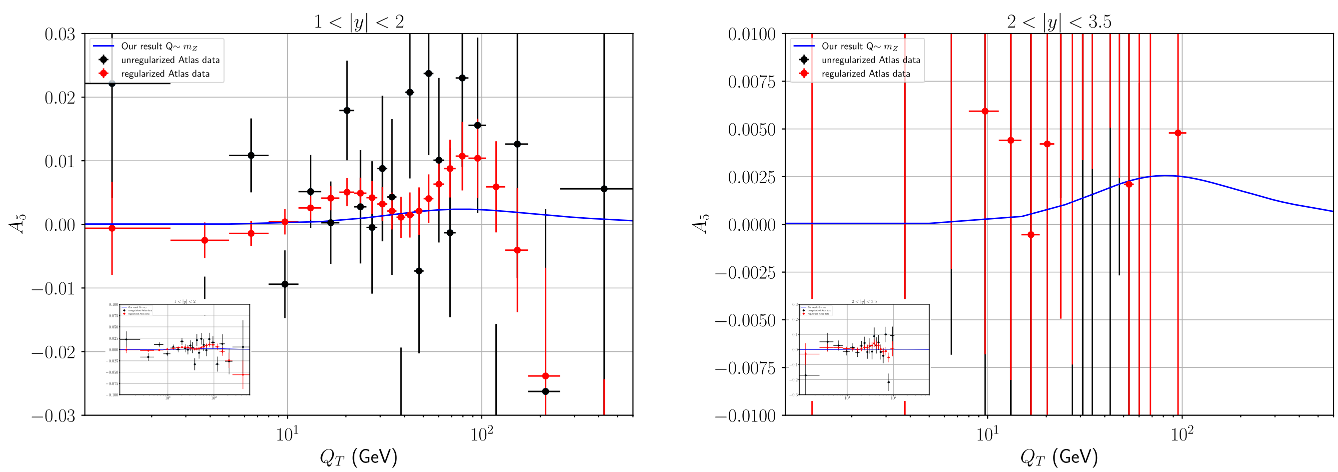

FIG. 8. Comparison of our result for A_5 to ATLAS data [47] at $Q \sim m_Z$. The black and red experimental points denote the unregularized and regularized data, respectively, and show their statistical error. The left panel shows the results for $1 < |y| < 2$, while the right is for $2 < |y| < 3.5$. (The scale on the y axis has been chosen for better visibility of the small values of the coefficient; as a result, some data points fall outside the plot range. The full dataset is shown in the left lower inset in each plot).

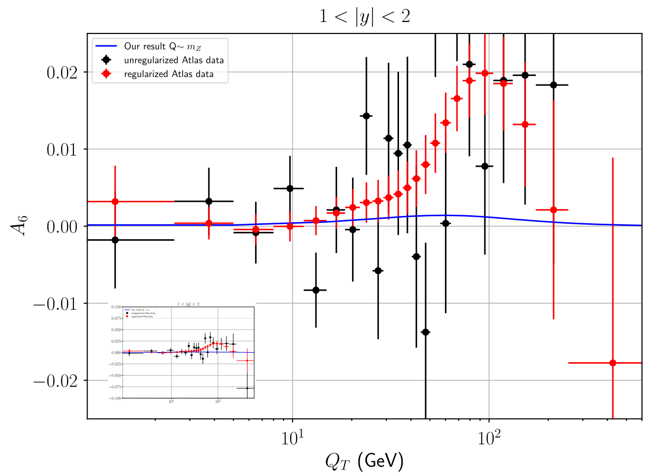

FIG. 9. Same as Fig. 8, but for the coefficient A_6 for rapidity $1 < |y| < 2$.

both the theoretical calculation and the data. Hence it seems preliminary to delve into a detailed analysis of the visible discrepancies between our result and the existing data.

More robust conclusions regarding the agreement of data and theory for the T-odd structure functions will only become possible after significant improvements of both the experimental results and the theoretical description. A natural question to ask is whether higher-order (say, NLO) QCD corrections to the angular coefficients could lead to a better agreement with the data. In this context it is important to keep in mind that the T-odd coefficients effectively carry an overall factor α_s. This immediately means that they will be more susceptible to QCD corrections than ratios of cross sections would normally be. Just to give a simple estimate: For the values of Q_T relevant here, varying the scale in α_s from $Q_T/2$ to $2Q_T$ easily generates differences of $\pm 15\%$ or more in the calculated coefficients. On top of this, there will be smaller uncertainties associated with the scale dependence and uncertainties of the PDFs. One would thus expect the NLO corrections to $A_{5,6,7}$ to be overall non-negligible, even though judging by the sizeable discrepancy in magnitude and shape between the existing data and our result it would come as a surprise if higher -order corrections were to account for the entire difference. We note that NLO corrections could in fact be obtained from Ref. [18]. Clearly, a phenomenological study of $A_{5,6,7}$ at NLO will be an interesting project for the future. Along with hopefully improved future data it would open the door to careful assessments of the validity of fixed-order perturbation theory for the T-odd angular coefficients.

VI. CONCLUSION

We have performed a detailed analysis of perturbative T-odd effects in charged- and neutral-current Drell-Yan processes, taking into account $W^{\pm}$ and Z^0 exchange, as well as $\gamma - Z^0$ interference. To this end, we have computed the relevant T-odd structure functions for the $q\bar{q}$ annihilation and qg Compton channels at order $\mathcal{O}(\alpha_s^2)$, where they become nonvanishing thanks to absorptive contributions to loop amplitudes. While the corresponding results are not new, we have used them in novel ways. Foremost, we have presented a new formalism to expand the results for low transverse momentum, or low $\rho = Q_T/Q$, with the goal of facilitating comparisons to frameworks that analyze T-odd effects in terms of TMDs, especially at nonleading power. Our new formalism is completely general and can in principle be used to obtain expansions to arbitrary order in Q_T/Q. As a proof of concept, we have applied it to the T-odd structure functions and expanded them to order $\mathcal{O}(\rho^4)$. In doing so, we uncovered a new relation between two of the T-odd structure functions, $W_{\nabla}^{\mathrm{LP}}(x_1, x_2)$ and $W_{\Delta_p}^{\mathrm{LP}}(x_1, x_2)$, valid at leading power in the small-Q_T expansion. Although in the present paper we have not attempted to connect our results to calculations based on

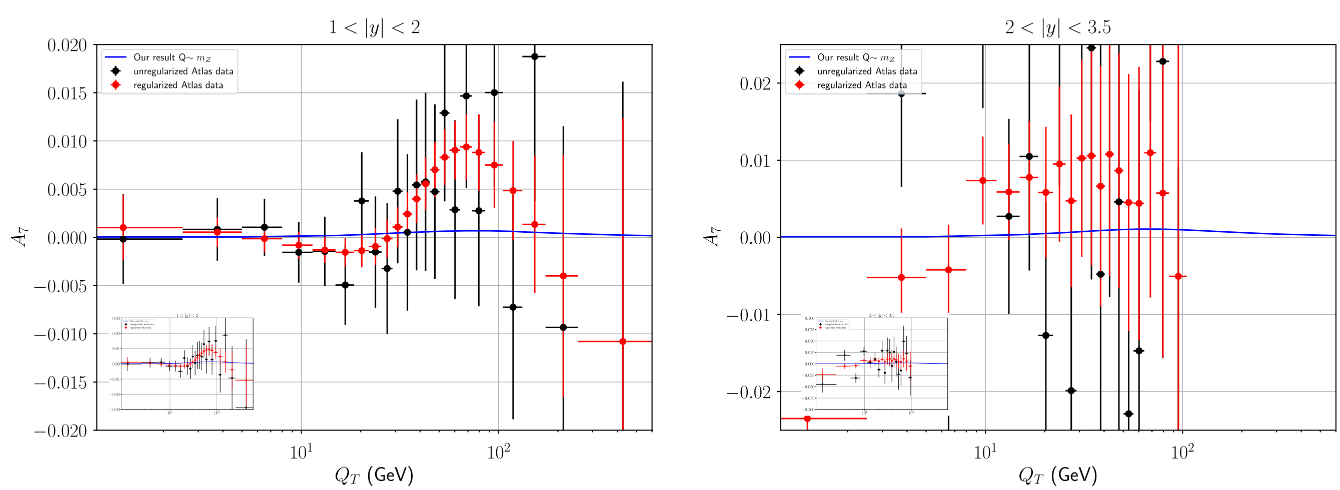

FIG. 10. Same as Fig. 8, but for the coefficient A_7.

TMD factorization, we think that our paper has much to offer for such comparisons in the future.

We have also presented numerical results for the validity of the expansion in ρ^2, and we have compared our full results for the T-odd structure functions to available data from the ATLAS experiment. We found that in the present situation it is impossible to draw any quantitative conclusions from this comparison.

In the present paper we have restricted our analysis to the case of the T-odd effects in the Drell-Yan process with unpolarized beams. Extensions to the T-even sector and to polarized scattering will be natural extensions of our work.

ACKNOWLEDGMENTS

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APPENDIX A: RELATIONS AMONG DIFFERENT SETS OF THE STRUCTURE FUNCTIONS

The three sets of structure functions $\{A_i\}$, $\{W_i\}$, and $\{\lambda, \mu, \nu, \ldots\}$ are related as [10,41,53–55]

$$
\begin{aligned}
&\lambda = \frac{W_T - W_L}{W_T + W_L} = \frac{2 - 3A_0}{2 + A_0}, \qquad \mu = \frac{W_\Delta}{W_T + W_L} = \frac{2A_1}{2 + A_0}, \qquad \nu = \frac{2W_{\Delta\Delta}}{W_T + W_L} = \frac{2A_2}{2 + A_0},\\
&\tau = \frac{W_{\nabla_P}}{W_T + W_L} = \frac{2A_3}{2 + A_0}, \qquad \eta = \frac{W_{T_P}}{W_T + W_L} = \frac{2A_4}{2 + A_0}, \qquad \xi = \frac{W_{\Delta\Delta_P}}{W_T + W_L} = \frac{2A_5}{2 + A_0},\\
&\zeta = \frac{W_{\Delta_P}}{W_T + W_L} = \frac{2A_6}{2 + A_0}, \qquad \chi = \frac{W_\nabla}{W_T + W_L} = \frac{2A_7}{2 + A_0},
\end{aligned}
\tag{A1}
$$

and

$$
\begin{aligned}
&A_0 = \frac{2W_L}{2W_T + W_L} = \frac{2(1-\lambda)}{3 + \lambda}, \qquad A_1 = \frac{2W_\Delta}{2W_T + W_L} = \frac{4\mu}{3 + \lambda}, \qquad A_2 = \frac{4W_{\Delta\Delta}}{2W_T + W_L} = \frac{4\nu}{3 + \lambda},\\
&A_3 = \frac{2W_{\nabla_P}}{2W_T + W_L} = \frac{4\tau}{3 + \lambda}, \qquad A_4 = \frac{2W_{T_P}}{2W_T + W_L} = \frac{4\eta}{3 + \lambda}, \qquad A_5 = \frac{2W_{\Delta\Delta_P}}{2W_T + W_L} = \frac{4\xi}{3 + \lambda},\\
&A_6 = \frac{2W_{\Delta_P}}{2W_T + W_L} = \frac{4\zeta}{3 + \lambda}, \qquad A_7 = \frac{2W_\nabla}{2W_T + W_L} = \frac{4\chi}{3 + \lambda}.
\end{aligned}
\tag{A2}
$$

APPENDIX B: TREATMENT OF γ^5 MATRIX IN CALCULATION OF STRUCTURE FUNCTIONS

In the calculation of the T-odd structure functions we have to deal with the γ^5 matrix in dimensional regularization. We encounter two different cases:

(a) Dirac traces with an even number (in practice, two) of γ^5 matrices in case of w_∇; and

(b) Dirac traces with an odd number (in practice, one) of γ^5 matrices in case of $w_{\Delta\Delta_P}$ and w_{Δ_P}.

For case (a), it is permissible even in dimensional regularization to anticommute the two γ^5 matrices toward each other and to use $(\gamma^5)^2 = 1$. As a result, the contributions of [vector ⊗ vector] and [axial-vector ⊗ axial-vector] couplings to w_∇ are identical.

In case (b) we use techniques established in the literature for the treatment of γ^5 in dimensional regularization. In particular, for the axial-vector spin matrix we use the Larin prescription [74–77], expressing γ^5 as the product of the four-dimensional Levi-Civita tensor $\epsilon^{\mu\nu\alpha\beta}$ and three gamma matrices:

$$
\gamma^\mu\gamma^5 = \frac{i}{6}\epsilon^{\mu\nu\alpha\beta}\gamma_\nu\gamma_\alpha\gamma_\beta. \tag{B1}
$$

Next, in order to evaluate the structure functions $w_{\Delta\Delta_P}$ and w_{Δ_P}, we apply the method proposed in Ref. [77] for the contraction of two Levi-Civita tensors. One Levi-Civita tensor occurs in the definition of the structure functions $w_{\Delta\Delta_P}$ and w_{Δ_P} [see Eqs. (12) and (13)], and the other appears because of the Larin substitution (B1). In general, the product of two Levi-Civita tensors must be evaluated in terms of the D-dimensional Kronecker tensor δ^μ_ν in order to preserve Lorentz invariance [77]:

$$\epsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\nu_1\nu_2\nu_3\nu_4} = -\det\begin{vmatrix} \delta^{\mu_1}_{\nu_1} & \delta^{\mu_1}_{\nu_2} & \delta^{\mu_1}_{\nu_3} & \delta^{\mu_1}_{\nu_4} \\ \delta^{\mu_2}_{\nu_1} & \delta^{\mu_2}_{\nu_2} & \delta^{\mu_2}_{\nu_3} & \delta^{\mu_2}_{\nu_4} \\ \delta^{\mu_3}_{\nu_1} & \delta^{\mu_3}_{\nu_2} & \delta^{\mu_3}_{\nu_3} & \delta^{\mu_3}_{\nu_4} \\ \delta^{\mu_4}_{\nu_1} & \delta^{\mu_4}_{\nu_2} & \delta^{\mu_4}_{\nu_3} & \delta^{\mu_4}_{\nu_4} \end{vmatrix}. \tag{B2}$$

For our purposes we need to evaluate two types of contractions [see Eqs. (12) and (13)],

$$i\epsilon^{\mu PRK}\gamma_\mu\gamma_5 = -\frac{1}{6}\epsilon^{\mu PRK}\epsilon_{\mu\rho\alpha\beta}\gamma^\rho\gamma^\alpha\gamma^\beta, \tag{B3}$$

and

$$i\epsilon^{\mu PRK}\gamma_\nu\gamma_5 = -\frac{1}{6}\epsilon^{\mu PRK}\epsilon_{\nu\rho\alpha\beta}\gamma^\rho\gamma^\alpha\gamma^\beta. \tag{B4}$$

Using Eq. (B2) we get

$$i\epsilon^{\mu PRK}\gamma_\mu\gamma_5 = (D-3)\not{P}\not{R}\not{K}, \tag{B5}$$

and

$$i\epsilon^{\mu PRK}\gamma_\nu\gamma_5 = \delta^\mu_{\nu;\perp}\not{P}\not{R}\not{K} + \gamma^\mu_\perp\big(P_\nu\not{K}\not{R} + R_\nu\not{P}\not{K} + K_\nu\not{R}\not{P}\big), \tag{B6}$$

where $D = 4 - 2\epsilon$ and $\delta^\mu_{\nu;\perp}$ and $\gamma^\mu_\perp$ are the perpendicular D-dimensional Kronecker tensor and gamma matrices, respectively, defined as

$$\begin{aligned} \delta^\mu_{\nu;\perp} &\equiv \delta^\mu_\nu - \frac{P^\mu P_\nu}{P^2} - \frac{R^\mu R_\nu}{R^2} - \frac{K^\mu K_\nu}{K^2}, \\ \gamma^\mu_\perp &\equiv \delta^\mu_{\nu;\perp}\gamma^\nu = \gamma^\mu - \frac{P^\mu \not{P}}{P^2} - \frac{R^\mu \not{R}}{R^2} - \frac{K^\mu \not{K}}{K^2}, \end{aligned} \tag{B7}$$

which are manifestly orthogonal to all momenta of the basis (P, R, K). The obey the following conditions:

$$\begin{aligned} \delta^\mu_{\nu;\perp}\delta^\nu_{\mu;\perp} &= \delta^\mu_{\nu;\perp}\delta^\nu_\mu = \delta^\mu_{\mu;\perp} = D-3, \\ \delta^\mu_{\nu;\perp}P_\mu &= \delta^\mu_{\nu;\perp}R_\mu = \delta^\mu_{\nu;\perp}K_\mu = 0. \end{aligned} \tag{B8}$$

Similarly one can define the perpendicular D-dimensional metric tensor $g^{\mu\nu}_\perp$ introduced in Ref. [56],

$$g^{\mu\nu}_\perp = g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} - \frac{R^\mu R^\nu}{R^2} - \frac{K^\mu K^\nu}{K^2}. \tag{B9}$$

One should mention that the identities (B4) and (B6) are generalizations of (B3) and (B5), respectively. In particular, Eq. (B5) follows from (B6) for $\nu \to \mu$. In this limit, $\delta^\mu_{\mu;\perp} = D - 3$, the second term on the rhs of Eq. (B6) vanishes, and we arrive at the identity (B5).

We also stress that using the orthogonal basis (P, R, K) along with the orthogonal metric tensors and gamma matrices turns out to be very useful and economical in our analytical calculations. In addition, we note that the use of identity (B6) is further simplified in the case of the evaluation of $w_{\Delta\Delta_P}$ and w_{Δ_P}. The reason is that the Levi-Civita tensor $\epsilon^{\mu PRK}$ on the lhs of Eq. (B6) is accompanied by the basis vector P^ν or R^ν [see Eqs. (12) and (13)]. Therefore, the first Lorentz structure $\delta^\mu_{\nu;\perp}\not{P}\not{R}\not{K}$ on the rhs of Eq. (B6) vanishes thanks to $P^\nu\delta^\mu_{\nu;\perp} = R^\nu\delta^\mu_{\nu;\perp} = 0$. The second Lorentz structure $\gamma^\mu_\perp(P_\nu\not{K}\not{R} + R_\nu\not{P}\not{K} + K_\nu\not{R}\not{P})$ on the rhs of the identity (B6) is also simplified because of the orthogonality of the (P, R, K) basis. In particular, depending on the accompanying momentum P^ν or R^ν, we deduce from the identity (B6) two simplified identities useful for the calculation of $w_{\Delta\Delta_P}$ and w_{Δ_P}:

$$\begin{aligned} i\epsilon^{\mu PRK}\not{P}\gamma_5 &= \gamma^\mu_\perp\not{K}\not{R}P^2, \\ i\epsilon^{\mu PRK}\not{R}\gamma_5 &= \gamma^\mu_\perp\not{P}\not{K}R^2. \end{aligned} \tag{B10}$$

APPENDIX C: CALCULATIONAL TECHNIQUE FOR THE PARTIAL DERIVATIVES OF THE STRUCTURE FUNCTIONS $W_i(x_1, x_2)$

In this appendix we discuss the calculational technique for the partial derivatives of the structure functions $W_i(x_1, x_2)$ defined in Eq. (46), which have the following form:

$$\begin{aligned} &W_i(x_1, x_2, L_\rho) \\ &= \frac{1}{x_1 x_2}\sum_{a,b}\big[R_{ab,i}(x_1, x_2, L_\rho)f_{a/H_1}(x_1)f_{b/H_2}(x_2) \\ &\quad + (P_{ba,i}\otimes f_{b/H_2})(x_2, x_1, L_\rho)f_{a/H_1}(x_1) \\ &\quad + (P_{ab,i}\otimes f_{a/H_1})(x_1, x_2, L_\rho)f_{b/H_2}(x_2)\big], \end{aligned} \tag{C1}$$

where $R_{ab,i}(x_1, x_2, L_\rho)$, $P_{ab,i}(z_1, x_2, L_\rho)$, and $P_{ba,i}(z_2, x_1, L_\rho)$ are perturbative coefficient functions containing differential operators acting on the PDFs $f_{a/H_1}(x_1)$ and $f_{b/H_2}(x_2)$. In particular, the functions $R_{ab,i}(x_1, x_2, L_\rho)$, $P_{ab,i}(z_1, x_2, L_\rho)$, and $P_{ba,i}(z_2, x_1, L_\rho)$ can be expanded using the set of the differential operators to factorize the dependence on the variables x_1 and x_2, z_1 and x_2, z_2 and x_1, respectively (see detailed discussion in the Supplemental Material [52]). One can see that $W_i(x_1, x_2, L_\rho)$ is composed of three main terms, one term proportional to the perturbative function $R_i(x_1, x_2, L_\rho)$ times the PDFs, and two terms each containing a single integral representation of the convolution of the perturbative functions $P_{a,i}(z_1, L_\rho)$, and $P_{b,i}(z_2, L_\rho)$ with the PDFs. Also the latter terms depend on the second variable $x_i = x_1$ or x_2 via the simple form of a product of $f(x_i)$ with some polynomial in x_i.

The first term can be trivially differentiated with respect to x_1 and x_2 to the desired order. In the case of the other two terms one can straightforwardly differentiate the non-integral terms. It remains to specify how to take the partial derivative of the terms with integrals. Let us discuss the treatment of such terms by considering the following integral:

$$J(x) = \frac{I(x)}{x}, \qquad I(x) = (P \otimes f)(x) = \int_x^1 \frac{dz}{z} P(z) f\left(\frac{x}{z}\right). \tag{C2}$$

The nth derivative of the integral $J(x)$ can be taken using the binomial formula

$$\frac{\partial^n J(x)}{\partial x^n} = \sum_{m=0}^{n} \frac{n!}{m!} \frac{(-1)^{n-m}}{x^{n-m+1}} \frac{\partial^m I(x)}{\partial x^m}. \tag{C3}$$

Therefore, we only need to derive an analytical formula for the nth derivative of the integral $I(x)$, where n is an arbitrary natural number. In our derivation we will consider two possible choices for the function $P(z)$:

(a) $P(z) = R(z)$ is a regular function of the variable z, and
(b) $P(z) = [1/(1-z)^m]_{+,m-1}$ is the generalized plus distribution of power m, defined in Eq. (40) in the main text.

We first consider the simpler case (a). Here, the first-order derivative of $I(x)$ reads as

$$\begin{aligned}\frac{\partial I(x)}{\partial x} &= \frac{\partial}{\partial x}\int_x^1 \frac{dz}{z} R(z) f\left(\frac{x}{z}\right) = \frac{\partial}{\partial x}\int_x^1 \frac{d\xi}{\xi} R(x/\xi) f(\xi) \\ &= -\int_x^1 \frac{d\xi}{\xi}\delta(\xi - x) R(x/\xi) f(\xi) + \int_x^1 \frac{d\xi}{\xi}\frac{\partial R(x/\xi)}{\partial x} f(\xi) \\ &= -\int_x^1 \frac{d\xi}{\xi^2}\delta(1 - x/\xi) R(x/\xi) f(\xi) + \int_x^1 \frac{d\xi}{\xi^2}\frac{\partial R(x/\xi)}{\partial(x/\xi)} f(\xi) \\ &= -\frac{1}{x}\int_x^1 dz\,\delta(1-z) R(z) f\left(\frac{x}{z}\right) + \frac{1}{x}\int_x^1 dz \frac{\partial R(z)}{\partial z} f\left(\frac{x}{z}\right) \\ &= \frac{1}{x}\left[-R(1) f(x) + \int_x^1 dz R'(z) f\left(\frac{x}{z}\right)\right],\end{aligned} \tag{C4}$$

where $R'(z) = \partial R(z)/\partial z$.

One can prove by induction that the nth derivative of $I(x)$ is given by

$$\frac{\partial^n I(x)}{\partial x^n} = -\sum_{k=1}^{n}\left(\frac{f(x)}{x^k}\right)_x^{(n-k)} R^{(k-1)}(1) + \frac{1}{x^n}\int_x^1 dz z^{n-1} R^{(n)}(z) f\left(\frac{x}{z}\right), \tag{C5}$$

where

$$R^{(k)}(z) = \partial^k R(z)/\partial z^k, \qquad (\cdots)_x^{(k)} = \partial^k(\cdots)/\partial x^k. \tag{C6}$$

For case (b) we recall the definition for the generalized plus distribution when applied to a PDF $f(x/z)$:

$$\begin{aligned}\int_x^1 dz \frac{f(\frac{x}{z})}{(1-z)^m_{+,m-1}} = \int_x^1 dz \Bigg[&\frac{1}{(1-z)^m_{+x,m-1}} + \delta(1-z) \\ &\times \log(1-x)\frac{(-1)^{m-1}}{(m-1)!}\partial_z^{m-1} \\ &- \delta(1-z)\sum_{j=2}^{m}\frac{(-1)^{m-j}}{(j-1)(m-j)!} \\ &\times\left(\frac{1}{(1-x)^{j-1}} - 1\right)\partial_z^{m-j}\Bigg] f\left(\frac{x}{z}\right).\end{aligned} \tag{C7}$$

The x dependence of $f(x/z)/(1-z)^m_{+x,m-1}$ induces a subtraction of the $(m-1)$-th order Taylor polynomial $T[f(x/z)]_{z=1}^{m-1}$ evaluated at $z=1$,

$$\begin{aligned}\frac{f(\frac{x}{z})}{(1-z)^m_{+x,m-1}} &= \frac{f(\frac{x}{z}) - T\left[f\left(\frac{x}{z}\right)\right]_{z=1}^{m-1}}{(1-z)^m}, \\ T\left[f\left(\frac{x}{z}\right)\right]_{z=1}^{m-1} &= \sum_{k=0}^{m-1}\frac{(z-1)^k}{k!}\partial_z^k f\left(\frac{x}{z}\right)\Big|_{z=1} \\ &= f(x) + \sum_{k=1}^{m-1}\sum_{\ell=1}^{k} C_{k-1}^{\ell-1}(1-z)^k \frac{x^\ell}{\ell!} f^{(\ell)}(x),\end{aligned} \tag{C8}$$

where $C_m^\ell = m!/(\ell!(m-\ell)!)$ is the binomial coefficient and $f^{(\ell)}(x) = \partial^\ell f(x)/\partial x^\ell$. Note that the partial derivatives $\delta(1-z)\partial_z^{m-j} f(x/z)$ with respect to z in Eq. (C7) can be simplified and reduced to derivatives with respect to x as

$$\delta(1-z)\partial_z^m f\left(\frac{x}{z}\right) = \delta(1-z)\sum_{\ell=1}^{m}(-1)^m C_m^\ell \times \frac{(m-1)!}{(\ell-1)!} x^\ell f^{(\ell)}(x). \tag{C9}$$

In all cases the derivatives $\partial^n I(x)/\partial x^n$ can be easily taken by changing the integration variable $z \to x/\xi$ and, after some simplifications, returning back to the integration over z. We stress that the choice $P(z) = R(z)[1/(1-z)^m]_{+,m-1}$, where $R(z)$ is a regular function, can be reduced to case (b) by carrying out a Taylor expansion of $R(z)$ around $z = 1$:

$$R(z) = \sum_{k=0}^{\infty} \frac{(z-1)^k}{k!} R^{(k)}(1), \tag{C10}$$

where $R^{(k)}(1) = (\partial^k R(z)/\partial z^k)_{z=1}$, which results in the cancellation of the respective powers of $(1-z)$ between the numerator and the denominator of the integrand of $I(x)$. In particular, the distribution $P_{qq}(z)$ defined in Eq. (57) contains the term $(1+z^2)/(1-z)_+$, which can be represented as the sum of a regular term corresponding to case (a) and a single distribution $2/(1-z)_+$ corresponding to case (b),

$$\frac{1+z^2}{(1-z)_+} = -(1+z) + \frac{2}{(1-z)_+}. \tag{C11}$$

In case (b) the nth derivative of the integral $I(x)$ reads as

$$\begin{aligned}
\frac{\partial^n I(x)}{\partial x^n} &= -\lim_{z\to 1} \sum_{k=1}^{n} \left(\frac{1}{x^k} \frac{\partial^{k-1}}{\partial z^{k-1}} \left[\frac{1}{(1-z)^m_{+x,m-1}} \right] z^{k-2} f\left(\frac{x}{z}\right) \right)^{(n-k)}_x + \frac{1}{x^n} \int_x^1 dz \frac{\partial^n}{\partial z^n} \left[\frac{1}{(1-z)^m_{+,m-1}} \right] z^{n-1} f\left(\frac{x}{z}\right) \\
&= -\lim_{z\to 1} \sum_{k=1}^{n} \left(\frac{(m+k-2)!}{x^k (m-1)!} \frac{z^{k-2} f\left(\frac{x}{z}\right)}{(1-z)^{m+k-1}_{+x,m+k-2}} \right)^{(n-k)}_x + \frac{(m+n-1)!}{x^n (m-1)!} \int_x^1 dz \frac{z^{n-1} f\left(\frac{x}{z}\right)}{(1-z)^{m+n}_{+,m+n-1}} \\
&= \lim_{z\to 1} \sum_{k=1}^{n} \left(\frac{(-1)^{m+k}}{x^k (m-1)!(m+k-1)} \frac{\partial^{m+k-1}}{\partial z^{m+k-1}} \left[z^{k-2} f\left(\frac{x}{z}\right) \right] \right)^{(n-k)}_x + \frac{(m+n-1)!}{x^n (m-1)!} \int_x^1 dz \frac{z^{n-1} f\left(\frac{x}{z}\right)}{(1-z)^{m+n}_{+,m+n-1}}.
\end{aligned} \tag{C12}$$

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