Electromagnetic properties of vector doubly charmed tetraquark states

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We conduct a systematic study of the electromagnetic properties of multiquark systems with undetermined internal structures. Motivated by the recent observation of the T_{cc}^+ state, we apply the light-cone version of the QCD sum rule method to extract the magnetic dipole moments of several possible doubly charmed vector tetraquark states. When analyzing the magnetic dipole moment of these states, they are modeled to have the diquark-antidiquark configurations. The magnetic dipole moments for the members are extracted as $\mu_{T_{cc\bar{u}\bar{d}}} = 1.17^{+0.44}_{-0.32}\mu_N$, $\mu_{T_{cc\bar{u}\bar{s}}} = 1.35^{+0.50}_{-0.37}\mu_N$, $\mu_{T_{cc\bar{d}\bar{s}}} = -2.69^{+1.02}_{-0.75}\mu_N$, $\mu_{T_{cc\bar{u}\bar{u}}} = 1.33^{+0.56}_{-0.40}\mu_N$, $\mu_{T_{cc\bar{d}\bar{d}}} = 1.41^{+0.57}_{-0.43}\mu_N$, and $\mu_{T_{cc\bar{s}\bar{s}}} = 1.44^{+0.53}_{-0.41}\mu_N$. Comparing the results obtained for the magnetic dipole moments of the $T_{cc\bar{u}\bar{d}}$ state with the $T_{cc\bar{u}\bar{s}}$ state, the U symmetry is seen to be broken at about 15%, while, for the $T_{cc\bar{d}\bar{d}}$ and $T_{cc\bar{s}\bar{s}}$ states, this symmetry is minimally broken. The obtained results may be useful to determine the true nature of these new interesting states.

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I. INTRODUCTION

Investigation of the various physical properties of multiquark systems constitutes one of the main directions of research in hadron physics. Among these systems, tetraquarks occupy a special place, since the two main revolutionary discoveries on multiquark systems belong to these states: the discovery of $X(3872)$ by Belle collaboration [1] in 2003 and the discovery of $T_{cc}^+(3875)$ by LHCb collaboration [2,3] in 2021. The first discovery was the observation of the first multiquark system and the second one was the observation of the first doubly charmed exotic meson: the previously discovered states were hidden heavy flavored states. At present, we have plenty number of exotic states discovered by different experiments or have been introduced as possible candidates. Despite the good experimental and theoretical progress on the exotic states, their internal structure and quark-gluon configuration are not clear yet [4–19] and more investigations are needed.

The discovered $T_{cc}^+(3875)$ by the LHCb collaboration was an axial-vector tetraquark with the quark content $cc\bar{u}\bar{d}$ and spin-parity $J^P = 1^+$ in the $D^0D^0\pi^+$ invariant mass distribution as a narrow peak. Its mass lies just a little bit below the $D^0D^*(2010)^+$ threshold. The two-meson D^0D^{*+} threshold has a mass of 3875.1 MeV, whereas T_{cc}^+ has the mass $m_{\text{exp}} = 3875.1\text{ MeV} + \delta m_{\text{exp}}$ with $\delta m_{\text{exp}} = -273 \pm 61 \pm 5^{+11}_{-14}$ KeV. It has also a very narrow width $\Gamma = 410 \pm 165 \pm 43^{+18}_{-38}$ KeV, making this particle the longest-living exotic meson discovered so far. Because of its unique feature, the T_{cc}^+ and doubly heavy tetraquark states at all have been under intense investigations via various models and approaches [20–82].

It is very natural to search for the possible T_{cc} states of different spin parities other than $J^P = 1^+$ now both in the experiment and theory. Among them, the vector state with $J^P = 1^-$ can be very interesting both from the spectroscopic and electromagnetic properties points of view. It is of a great importance to exactly determine the properties of vector T_{cc} and compare the results with those of its axial vector partner. From spectroscopic analyses, it is expected that the vector state lies above the related two-meson thresholds [20,78]. This means that the vector state is unstable and strongly decays to the two-meson states. This situation makes the vector T_{cc} be very different than the axial one. The electromagnetic properties of the vector state is also expected to be different than the axial state. Theoretical investigations of different properties of the vector state and the obtained results can help experimental groups in their search for the vector T_{cc} in the experiment. There are a few studies in the literature where the spectroscopic parameters of the vector doubly

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charmed tetraquark states have been investigated in [20,45,78,83,84]. In [20], the authors developed a systematic approach to investigating potential doubly heavy states with possible spin parities by constructing possible tetraquark interpolating operators without derivatives. Their predictions indicate that there are no bound states in the doubly charmed sectors. In [78], the mass and coupling values of the vector states were obtained at the next-to-leading order using the two possible interpolating currents constructed in [20]. It is shown that the mass values obtained from this study are approximately 200–300 MeV lower than the mass values in [20]. In Ref. [83], the author constructed axialvector-diquark-axialvector-antidiquark type currents to interpolate vector and other spin-parities doubly charmed tetraquark states. These currents were then used to study these states with QCD sum rules, with the operator product expansion carried out up to vacuum condensates of dimension 10 in a consistent manner. In [45], the mass of the vector and other possible double-heavy states are studied in the chiral-diquark picture. Based on these findings, it was concluded that there are no bound doubly charmed tetraquark states. In [84], the masses of the vector and other possible doubly heavy states were calculated within the framework of the one-boson exchange model, considering their molecular structure. The results demonstrated that only doubly charmed axial states can be stable. It should be noted, however, that their existence and stability are contingent upon the specific model in question, at least for the time being. Further theoretical investigation would be useful in clarifying this issue.

The masses and residues of doubly charmed tetraquark states are available [20,78], allowing us to use them as inputs to discover the electromagnetic parameters of the vector states of different configurations. The physical quantities related to the electromagnetic properties of hadrons are useful parameters to reveal the nature and inner structures of specially the exotic states. The electromagnetic form factors (FFs) and the resultant multipole moments help us to investigate the charge and magnetism distributions inside the hadrons. This also allows us to know where and how the quark antiquark (the valence and sea) and gluons are distributed inside the volume of the hadrons and to gain useful information about the geometric shapes and electric and magnetic radii of the hadrons. In the case of the proton, now, we see that there is a puzzle regarding the electromagnetic, mass, and mechanical radii of this relatively well-known particle. Hence, investigation of the interaction of exotic states with nonzero spin and charge with the photon can provide useful information on many aspects of the exotic hadrons. Calculation of such kinds of nonperturbative objects requires some reliable nonperturbative approaches. Among them is QCD sum rule formalism [85,86], which is one of the powerful and predictive nonperturbative approaches in hadron physics. Note that the magnetic dipole moment of the axial-vector T_{cc} states was investigated in [79] using the light-cone QCD sum rules [87–89]. However, as we mentioned above, there are important differences between the nature, internal structure, and decay properties of the vector and axial-vector states that are resulted from different currents interpolating these states. The axial state was discovered in the experiment, so we hope theoretical calculations of different related parameters will help and motivate experimental investigations of the vector T_{cc} state.

The rest of this article is arranged as follows. In Sec. II, we apply the QCD light-cone sum rules to calculate the magnetic dipole moments (MDMs) of doubly charmed vector tetraquark states. Section III is dedicated to the numeric analyses of the attained sum rules in the previous section. The final section contains a brief discussion and concluding remarks.

II. FORMALISM

As is well known, the QCD light-cone sum rule is a widely used method to probe the hadronic properties and has proven to be a robust nonperturbative technique to extract different physical quantities in the nonperturbative regime of QCD. For deriving the magnetic dipole moments, we take into account an appropriate correlator in the presence of a weak external electromagnetic field:

$$\Pi_{\mu\nu}(p,q) = i \int d^4x e^{ip\cdot x} \langle 0|\mathcal{T}\{J_\mu(x) J_\nu^\dagger(0)\}|0\rangle_\gamma, \quad (1)$$

where the subindex γ indicates the weak background electromagnetic field and $J_\mu(x)$ represents the current interpolating the vector T_{cc} states with the total angular momentum-parity $J^P = 1^-$. To determine the magnetic dipole moments of doubly charmed vector tetraquark states, we apply diquark-antidiquark interpolating currents as shown below

$$J^1_\mu(x) = [c^{aT}(x) C c_b(x)][\bar{q}^a_1(x)\gamma_\mu C \bar{q}^b_2(x)], \quad (2)$$

$$J^2_\mu(x) = [c^{aT}(x) C\gamma_\mu\gamma_5 c_b(x)][\bar{q}^a(x)\gamma_5 C \bar{q}^b(x)], \quad (3)$$

where q denotes the u, d and s quarks; the q_1 is the u or d quark, and the q_2 is the d or s quark. While q_1 is the u quark, q_2 can be the d or s quark. If q_1 is the d quark, then q_2 can be only the s quark. In principle, various interpolating currents of the molecular and compact tetraquark forms couple to these states [20]. Calculations up to next-to-leading order on the spectroscopic parameters show that the above currents lead to more reliable results [78]. Hence, using these currents, the values of the mass and current coupling as the main input parameters in the calculations of the electromagnetic properties of the states under study are available with higher accuracy, allowing us to use them in extraction of the magnetic dipole moments of the vector T_{cc} states. These parameters are not available for other possible interpolating currents.

The calculation of magnetic dipole moments starts with the analysis of the correlation function with the help of the hadronic parameters. To achieve this, we inject a full set of hadronic states, with the same quantum numbers as the currents under study, into the correlation function. By following this procedure and performing the resultant four integrals, we obtain

$$\Pi^{\rm Had}_{\mu\nu}(p,q) = \frac{\langle 0|J_\mu(x)|T_{cc}(p,\varepsilon^i)\rangle}{p^2 - m^2_{T_{cc}}} \langle T_{cc}(p,\varepsilon^i)|T_{cc}(p+q,\varepsilon^f)\rangle_\gamma \frac{\langle T_{cc}(p+q,\varepsilon^f)|J^\dagger{}_\nu(0)|0\rangle}{(p+q)^2 - m^2_{T_{cc}}} + \text{higher states}. \tag{4}$$

To continue the calculations, we parametrize the matrix elements appearing in the above equation in terms of different quantities:

$$\langle 0|J_\mu(x)|T_{cc}(p,\varepsilon^i)\rangle = \lambda_{T_{cc}}\varepsilon^i_\mu, \tag{5}$$

$$\langle T_{cc}(p,\varepsilon^i)|T_{cc}(p+q,\varepsilon^f)\rangle_\gamma = -\varepsilon^\gamma(\varepsilon^i)^\alpha(\varepsilon^f)^\beta\Big\{G_1(Q^2)(2p+q)_\gamma g_{\alpha\beta} + G_2(Q^2)(g_{\gamma\beta}q_\alpha - g_{\gamma\alpha}q_\beta) - \frac{1}{2m^2_{T_{cc}}}G_3(Q^2)(2p+q)_\gamma q_\alpha q_\beta\Big\}, \tag{6}$$

where ε^γ is the polarization vector of the photon, ε^i and ε^f indicate the polarization vectors of the initial and final doubly charmed vector states, and $\lambda_{T_{cc}}$ is residue or current coupling of the doubly charmed tetraquark state T_{cc}. Here, $G_i(Q^2)$ with $i = 1$, 2, and 3 are the electromagnetic form factors at $Q^2 = -q^2$.

The physical or phenomenological side of the correlator is acquired by making use of the Eqs. (4)–(6) as follows:

$$\Pi^{\rm Had}_{\mu\nu}(p,q) = \frac{\varepsilon_\rho\lambda^2_{T_{cc}}}{[m^2_{T_{cc}} - (p+q)^2][m^2_{T_{cc}} - p^2]}\Big\{G_2(Q^2)\Big(q_\mu g_{\rho\nu} - q_\nu g_{\rho\mu} - \frac{p_\nu}{m^2_{T_{cc}}}\Big(q_\mu p_\rho - \frac{1}{2}Q^2 g_{\mu\rho}\Big) + \frac{(p+q)_\mu}{m^2_{T_{cc}}}\Big(q_\nu(p+q)_\rho + \frac{1}{2}Q^2 g_{\nu\rho}\Big) - \frac{(p+q)_\mu p_\nu p_\rho}{m^4_{T_{cc}}}Q^2\Big) + \cdots\Big\}, \tag{7}$$

where $\cdots$ stands for the contributions of the higher states and continuum as well as other Lorentz structures that are not used to extract the required FFs. As is seen, we kept only the form factor $G_2(Q^2)$ and the corresponding Lorentz structure. We need only this form factor which is called magnetic FF,

$$F_M(Q^2) = G_2(Q^2), \tag{8}$$

the static limit of which is used to extract the MDM, $\mu_{T_{cc}}$:

$$\mu_{T_{cc}} = \frac{e}{2m_{T_{cc}}}F_M(Q^2 = 0). \tag{9}$$

The physical side led us to define the MDM of the state under study. Now, we need to evaluate the correlator in high energies and short distances in terms of QCD parameters called the QCD representation. To this end, we insert the explicit forms of the currents in terms of the quark fields into the correlator and contract the corresponding fields of the heavy and light quarks by means of Wick's theorem. When this is done, we obtain the following representations,

$$\begin{aligned}\Pi^{\rm QCD}_{\mu\nu}(p,q) = i\int d^4x e^{ip\cdot x}\langle 0|\{&\mathrm{Tr}[S_c^{bb'}(x)\tilde{S}_c^{aa'}(-x)]\mathrm{Tr}[\gamma_\mu\tilde{S}_{q_2}^{b'b}(-x)\gamma_\nu S_{q_1}^{a'a}(-x)] \\ &- \mathrm{Tr}[S_c^{ba'}(x)\tilde{S}_c^{ab'}(-x)]\mathrm{Tr}[\gamma_\mu\tilde{S}_{q_2}^{b'b}(-x)\gamma_\nu S_{q_1}^{a'a}(-x)] \\ &- \mathrm{Tr}[S_c^{bb'}(x)\tilde{S}_c^{aa'}(-x)]\mathrm{Tr}[\gamma_\mu\tilde{S}_{q_1q_2}^{a'b}(-x)\gamma_\nu S_{q_2q_1}^{b'a}(-x)] \\ &+ \mathrm{Tr}[S_c^{ba'}(x)\tilde{S}_c^{ab'}(-x)]\mathrm{Tr}[\gamma_\mu\tilde{S}_{q_1q_2}^{a'b}(-x)\gamma_\nu S_{q_2q_1}^{b'a}(-x)]\}|0\rangle_\gamma,\end{aligned} \tag{10}$$

for the J^1_μ and

$$\begin{aligned}\Pi^{\rm QCD}_{\mu\nu}(p,q) = i\int d^4x e^{ip\cdot x}\langle 0|\{&\mathrm{Tr}[\gamma_\mu\gamma_5 S_c^{bb'}(x)\gamma_5\gamma_\nu \tilde{S}_c^{aa'}(-x)]\mathrm{Tr}[\gamma_5 \tilde{S}_q^{b'b}(-x)\gamma_5 S_q^{a'a}(-x)]\\ &-\mathrm{Tr}[\gamma_\mu\gamma_5 S_c^{ba'}(x)\gamma_5\gamma_\nu \tilde{S}_c^{ab'}(-x)]\mathrm{Tr}[\gamma_5 \tilde{S}_q^{b'b}(-x)\gamma_5 S_q^{a'a}(-x)]\\ &-\mathrm{Tr}[\gamma_\mu\gamma_5 S_c^{bb'}(x)\gamma_5\gamma_\nu \tilde{S}_c^{aa'}(-x)]\mathrm{Tr}[\gamma_5 \tilde{S}_q^{a'b}(-x)\gamma_5 S_q^{b'a}(-x)]\\ &+\mathrm{Tr}[\gamma_\mu\gamma_5 S_c^{ba'}(x)\gamma_5\gamma_\nu \tilde{S}_c^{ab'}(-x)]\mathrm{Tr}[\gamma_5 \tilde{S}_q^{a'b}(-x)\gamma_5 S_q^{b'a}(-x)]\}|0\rangle_\gamma,\end{aligned} \tag{11}$$

for the J^2_μ current considered in the present study. Here, $S_q(x)$ and $S_c(x)$ denote the light and charm-quark propagators, which are written as

$$S_q(x) = i\frac{\not{x}}{2\pi^2 x^4} - \frac{\langle\bar{q}q\rangle}{12}\left(1 - i\frac{m_q\not{x}}{4}\right) - \frac{\langle\bar{q}q\rangle}{192}m_0^2x^2\left(1 - i\frac{m_q\not{x}}{6}\right) - \frac{ig_s}{32\pi^2x^2}G^{\mu\nu}(x)[\not{x}\sigma_{\mu\nu} + \sigma_{\mu\nu}\not{x}] + \cdots, \tag{12}$$

$$\begin{aligned}S_c(x) = &\frac{m_c^2}{4\pi^2}\left[\frac{K_1(m_c\sqrt{-x^2})}{\sqrt{-x^2}} + i\frac{\not{x}K_2(m_c\sqrt{-x^2})}{(\sqrt{-x^2})^2}\right]\\ &-\frac{g_s m_c}{16\pi^2}\int_0^1 dv\, G^{\mu\nu}(vx)\left[(\sigma_{\mu\nu}\not{x} + \not{x}\sigma_{\mu\nu})\frac{K_1(m_c\sqrt{-x^2})}{\sqrt{-x^2}} + 2\sigma_{\mu\nu}K_0(m_c\sqrt{-x^2})\right] + \cdots,\end{aligned} \tag{13}$$

where $\langle\bar{q}q\rangle$ is the light-quark condensate, m_q is the light-quark mass, $m_0^2 = \langle 0|\bar{q}g_s\,\sigma_{\mu\nu}\,G^{\mu\nu}q|0\rangle/\langle\bar{q}q\rangle$, $G^{\mu\nu}$ is the gluon field strength tensor, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu,\gamma_\nu]$, m_c is the charm-quark mass, the K_i are modified Bessel functions of the second kind and v is the line variable.

The QCD side of the correlator contains both the perturbative and nonperturbative pieces giving contributions to the problem under study. The perturbative contributions are related to the perturbative soft interaction of the photon with the light and heavy quark lines. In order to calculate such contributions, one of the heavy/light quark propagators, having interaction with the photon, is replaced by

$$S^{\rm pert}(x) \to \int d^4z S^{\rm pert}(x-z)\not{A}(z)S^{\rm pert}(z), \tag{14}$$

where $S^{\rm pert}(x)$ stands for the first term of the light or heavy quark propagator, and the rest propagators that have no interaction with the photon are taken as their perturbative parts only. The nonperturbative part should contain nonperturbative large distance interactions of the photon with the quark lines. Such contributions are written in terms of some matrix elements like $\langle\gamma(q)|\bar{q}(x)\Gamma_i q(0)|0\rangle$ and $\langle\gamma(q)|\bar{q}(x)\Gamma_i G_{\mu\nu}q(0)|0\rangle$ with $\Gamma_i = \mathbf{1},\gamma_5,\gamma_\mu,i\gamma_5\gamma_\mu,\sigma_{\mu\nu}/\sqrt{2}$, being the Dirac set. These matrix elements can be decomposed in terms of distribution amplitudes (DAs) of the photon having different twists. The relevant matrix elements and DAs of the photon together with the corresponding wave functions and entered constants all are represented/calculated in Ref. [90]. To achieve the nonperturbative contributions that represent the interaction of the photon with light quark lines at a large scale, we make use of the following replacement for one of the light quark propagators interacting with the photon

$$S^{ij}_{\mu\nu}(x) \to -\frac{1}{4}[\bar{q}^i(x)\Gamma_i q^j(0)](\Gamma_i)_{\mu\nu}, \tag{15}$$

where the other propagators are considered as their full expressions. These procedures produce the matrix elements discussed above. Adding both the resultant perturbative and nonperturbative contributions gives the QCD side of the correlation function in terms of QCD fundamental parameters as well as the quark-gluon degrees of freedom.

Finally, we choose the gauge invariant $(q_\mu\varepsilon_\nu - q_\nu\varepsilon_\mu)$ structure from both the hadronic and QCD sides of the correlation function and match its coefficients from both representations. The contributions from the higher states and the continuum are suppressed by the Borel transformation and the continuum subtraction. As a result of the procedures we have described above, we obtain the following sum rules

$$\mu^1_{T_{cc}} = \Delta_1(\mathrm{M}^2,\mathrm{s}_0), \tag{16}$$

$$\mu^2_{T_{cc}} = \Delta_2(\mathrm{M}^2,\mathrm{s}_0), \tag{17}$$

where the $\mu^1_{T_{cc}}$ and $\mu^2_{T_{cc}}$ denote the magnetic dipole moments obtained for the $T_{cc\bar{q}_1\bar{q}_2}$ and $T_{cc\bar{q}\bar{q}}$ states, respectively. The results for the $\Delta_1(\mathrm{M}^2,\mathrm{s}_0)$ and $\Delta_2(\mathrm{M}^2,\mathrm{s}_0)$ functions are listed in the Appendix.

III. NUMERICAL ANALYSIS OF THE SUM RULES

In this section, we numerically analyze the MDMs of the vector doubly charmed tetraquark states obtained via the

QCD light-cone sum rules in the previous section. The values of the input parameters required for the numerical calculations are listed as $m_u = m_d = 0$, $m_s = 93.4^{+8.6}_{-3.4}$ MeV, $m_c = 1.27 \pm 0.02$ GeV, $f_{3\gamma} = -0.0039$ GeV^2 [90], $\langle \bar{s}s \rangle = 0.8\langle \bar{u}u \rangle$ with $\langle \bar{u}u \rangle = (-0.24 \pm 0.01)^3$ GeV^3 [91], $m_0^2 = 0.8 \pm 0.1$ GeV^2 [91] and $\langle g_s^2 G^2 \rangle = 0.88$ GeV^4 [92]. The numerical values of the hadronic parameters such as the masses and residues of the doubly charmed vector tetraquark states are borrowed from Refs. [20,78]. The DAs of the photon are among the main input parameters for the calculations of the magnetic dipole moments, we use them from the Ref. [90].

In establishing the QCD light-cone sum rules, two helping parameters, M^2 and s_0, have been entered as previously mentioned. These parameters should be fixed according to the standard prescriptions that follow two criteria: Maximum possible pole contribution (PC) and convergence of the operator product expansion (COPE). To fulfill these criteria, it is convenient to impose the following conditions, typical for the exotic states,

$$\mathrm{PC} = \frac{\Delta_i(\mathrm{M}^2, \mathrm{s}_0)}{\Delta_i(\mathrm{M}^2, \infty)} \geq 30\%, \tag{18}$$

$$\mathrm{COPE}(\mathrm{M}^2, \mathrm{s}_0) = \frac{\Delta_i^{\mathrm{Dim}\,7}(\mathrm{M}^2, \mathrm{s}_0)}{\Delta_i(\mathrm{M}^2, \mathrm{s}_0)} \leq 5\%, \tag{19}$$

where $\Delta_i^{\mathrm{Dim}\,7}(\mathrm{M}^2, \mathrm{s}_0)$ denotes the last term in the OPE of $\Delta_i(\mathrm{M}^2, \mathrm{s}_0)$. By these requirements, we fix the working widows of the auxiliary parameters M^2 and s_0. We present these intervals for the states under study in Table I. In this table, we also present the values of PC and COPE obtained from the analyses for each state.

In Fig. 1, we depict the dependencies of the MDMs of the doubly charmed vector tetraquark states on M^2 at different fixed values of the continuum threshold s_0. From this figure, it follows that the MDMs exhibit mild dependence on these helping parameters in their working intervals.

Having determined all the input parameters, we are ready to calculate the corresponding MDMs of the vector doubly charmed tetraquark states. Our final results are given as

$$\begin{aligned}
\mu_{T_{cc\bar{u}\bar{d}}} &= 1.17^{+0.44}_{-0.32}\mu_N, & \mu_{T_{cc\bar{u}\bar{u}}} &= 1.33^{+0.56}_{-0.40}\mu_N, \\
\mu_{T_{cc\bar{u}\bar{s}}} &= 1.35^{+0.50}_{-0.37}\mu_N, & \mu_{T_{cc\bar{d}\bar{d}}} &= 1.41^{+0.57}_{-0.43}\mu_N, \\
\mu_{T_{cc\bar{d}\bar{s}}} &= -2.69^{+1.02}_{-0.75}\mu_N, & \mu_{T_{cc\bar{s}\bar{s}}} &= 1.44^{+0.53}_{-0.41}\mu_N,
\end{aligned} \tag{20}$$

where, the variations with respect to the errors of input parameters and the working intervals of the helping quantities M^2 and s_0 together with the errors in the numerical values of the photon DAs, are the main sources of the uncertainties in the presented results. The orders of the MDMs indicate that they are accessible in the experiment. Our analyses also depict that the magnetic dipole moments of $T_{cc\bar{q}_1\bar{q}_2}$ states are governed by the light diquarks, while those of $T_{cc\bar{q}\bar{q}}$ states are governed by the heavy diquarks. Comparing the results of MDMs for different flavors leads to a conclusion on the order of breaking of the U-symmetry among the states. Although the U-symmetry breaking effects have been taken into account through a nonzero s-quark mass and different s-quark condensate, we observe that the U-symmetry violation in the magnetic dipole moments is negligible between the $T_{cc\bar{d}\bar{d}}$ and $T_{cc\bar{s}\bar{s}}$ states and is approximately 15% between the $T_{cc\bar{u}\bar{d}}$ and $T_{cc\bar{u}\bar{s}}$ states.

We should mention that the MDMs of the doubly bottom vector tetraquarks have also been investigated together with the charmed channels; however, the results are not included in the text as reliable sum rules could not be constructed. In the analysis of the doubly bottom vector tetraquarks, the fundamental requirements of the method regarding the PC and COPE have not been sufficiently satisfied, and reliable results could not be obtained. It should be noted that, as we previously mentioned, the mass values for the vector doubly bottom tetraquark were obtained using the chosen interpolating currents in [20,78]. However, given that the physical quantities studied in the present work have different dynamics, this situation can be the case.

TABLE I. Working intervals of the s_0, M^2, the PC and COPE for the MDMs of the doubly charmed vector tetraquark states.

Current	State	s_0 (GeV^2)	M^2 (GeV^2)	PC (%)	COPE ≤ (%)
	$cc\bar{u}\bar{d}$	[20.5, 22.5]	[4.2, 6.2]	[58.3, 31.2]	2.52
J_μ^1	$cc\bar{u}\bar{s}$	[21.0, 23.0]	[4.3, 6.3]	[58.0, 32.3]	2.58
	$cc\bar{d}\bar{s}$	[21.0, 23.0]	[4.3, 6.3]	[58.1, 32.4]	2.61
	$cc\bar{u}\bar{u}$	[20.5, 22.5]	[4.2, 6.2]	[58.4, 31.4]	2.71
J_μ^2	$cc\bar{d}\bar{d}$	[20.5, 22.5]	[4.2, 6.2]	[58.1, 32.3]	2.73
	$cc\bar{s}\bar{s}$	[21.5, 23.5]	[4.5, 6.5]	[58.4, 32.4]	2.72

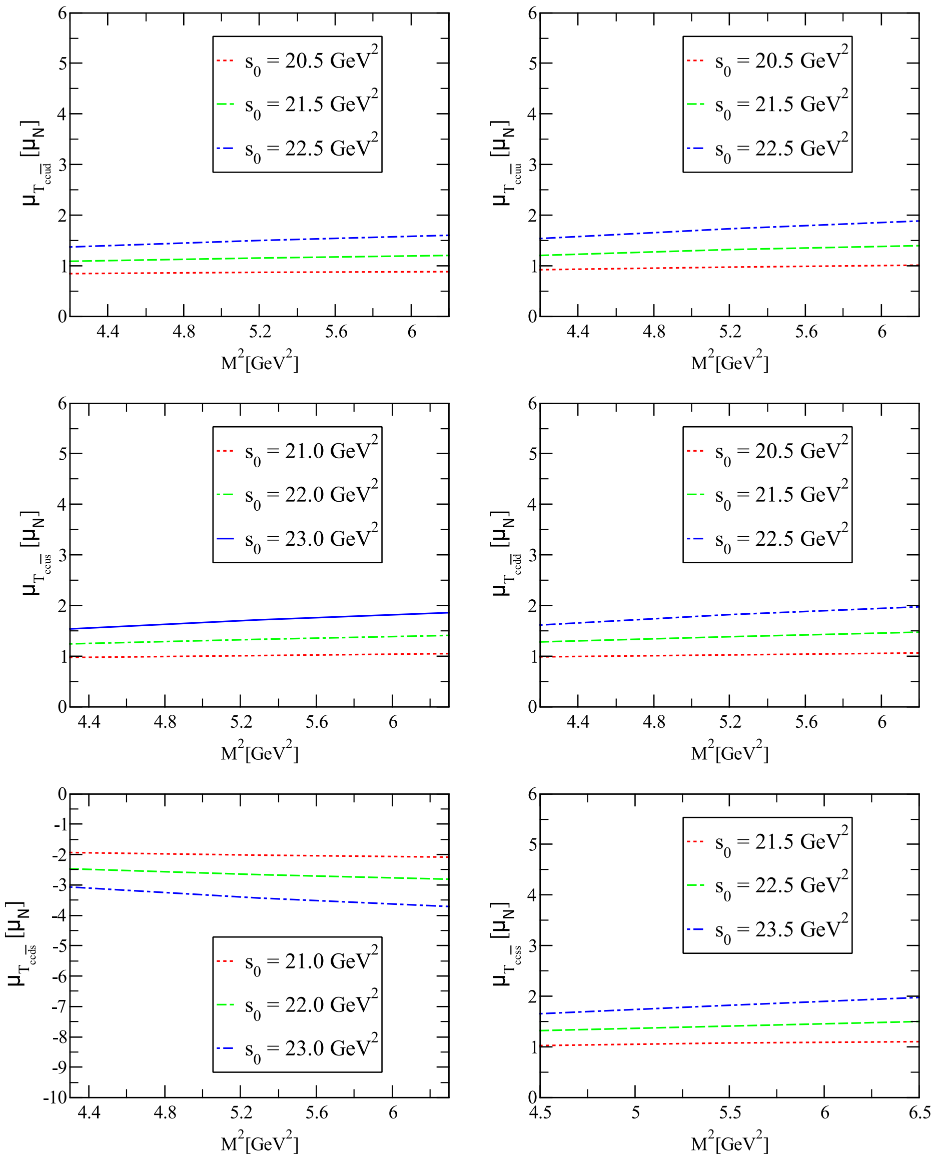

FIG. 1. Dependencies of the MDMs of the doubly charmed vector tetraquark states on M^2 at three different values of s$_0$.

As a byproduct, we also calculate the quadrupole moments of the vector doubly charmed tetraquark states. The results obtained are

$$\mathcal{D}_{T_{cc\bar{u}\bar{d}}} = (-2.91^{+0.56}_{-0.79}) \times 10^{-3}\ \mathrm{fm}^2,$$
$$\mathcal{D}_{T_{cc\bar{u}\bar{s}}} = (-3.17^{+0.62}_{-0.55}) \times 10^{-3}\ \mathrm{fm}^2,$$
$$\mathcal{D}_{T_{cc\bar{d}\bar{s}}} = (3.19^{+0.62}_{-0.56}) \times 10^{-3}\ \mathrm{fm}^2. \quad (21)$$

The quadrupole moments of the $T_{cc\bar{u}\bar{u}}$, $T_{cc\bar{d}\bar{d}}$, and $T_{cc\bar{s}\bar{s}}$ states have also been calculated. Their values are very close to zero.

Before ending this section, we would like to make some comments on the importance and indirect ways of the measurements of the electromagnetic parameters of the T_{cc} states. As we previously mentioned, the $T_{cc}^+(3875)$ with $J^P = 1^+$ discovered by the LHCb collaboration lies just below the $D^0 D^*(2010)^+$ threshold. This situation, prevents the fast strong dissociation of this particle and make it the longest-living exotic meson discovered so far. The long lifetime of this particle would allow experimental groups to measure the electromagnetic properties of this particle like its magnetic dipole moment to shed light on its yet exactly undetermined nature and inner structure. The theoretical results obtained for its electromagnetic properties together with those of its possible single and double strange partners available from the theory [79] can help the experimental groups in this respect. It is expected that the T_{cc} with

$J^P = 1^+$ is the lightest state in the $cc\bar{u}\,\bar{d}$ channel. Hence, the resonances with other quantum numbers like $J^P = 1^-$ are expected to lie above the related two-meson thresholds [20,78] making their lifetime shorter than the axial-vector state considering their possible strong decays. Although, this situation can make the measurements of its parameters difficult compared to the T_{cc} with $J^P = 1^+$, it is possible to identify the vector T_{cc} state and indirectly measure its electromagnetic properties by considering the recent progresses in the experiments. Any measurement on the properties of vector state will help us gain useful information on the nature and quark-gluon organization of the interesting doubly charmed states. There are alternative ways to indirectly determine the electromagnetic properties of the vector mesons. The first one is based on the soft photon emission of hadrons proposed in Ref. [93], where a procedure for the determination of the electromagnetic multipole moments is given. The basic idea of this procedure is that the photon carries information about the magnetic dipole and other higher multipole moments of the particle from which it is emitted. The matrix element for the radiative process can be expressed by the energy of the photon, E_γ, as

$$M \sim \frac{A}{E_\gamma} + B(E_\gamma)^0 + \cdots, \tag{22}$$

where $\frac{1}{E_\gamma}$, $(E_\gamma)^0$, and $\cdots$ represent the contributions coming from the electric charge, the magnetic dipole moment, and the higher multipole moments, respectively. Therefore, the magnetic dipole moment of the particles under consideration can be identified through measuring the cross section or decay width of the radiative process and ignoring the small contributions of the linear/higher order terms in E_γ. The $\Delta^+(1232)$ resonance also has a very short lifetime, whereas the magnetic dipole moment of this resonance was obtained by utilizing the experimental data obtained in the $\gamma N \to \Delta \to \Delta\gamma \to \pi N\gamma$ process with the help of this technique [94–96]. The second one is the possibility of measuring the electromagnetic multipole moments of the vector mesons in the radiative production and decays of such mesons, and it has been argued that the energy and the combined angular distributions of the radiated photons is an effective method to measure the electromagnetic multipole moments of the vector mesons [97]. This approach has also been used for the determination of the magnetic dipole moment of the ρ meson with preliminary data from the *BABAR* collaboration for the $e^+e^- \to \pi^+\pi^- 2\pi^0$ process [98]. Therefore, although it is currently not possible to experimentally measure the electromagnetic properties of resonances directly, they can be obtained using the experimental data of corresponding radiative processes indirectly.

IV. SUMMARY AND CONCLUDING REMARKS

The experimental observation of the axial-vector T_{cc}^+ state, and the measured values of the corresponding mass and width that give this particle a unique place and make it a long-lived exotic state discovered ever, has opened a new area to study the double-charmed tetraquark states: Scientists in this field have conducted extensive research on the nature of these states after this discovery. The studying of the electromagnetic properties of these states can help us better understand the nature, internal structure and quark-gluon configurations of these states.

Motivated by this, we determined the MDMs of the possible vector doubly charmed states with spin-parity $J^P = 1^-$. We assigned a diquark-antidiquark structure to these states and considered the possible light quark contents. We used the QCD light-cone sum rule formalism to extract the values of the MDMs. The values of the presented MDMs and quadrupole moments together with the spectroscopic/decay parameters of these states increase our understanding of the nature and inner structures of these states. They may help experimental groups in the course of their search for doubly heavy tetraquarks of different spin parities.

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APPENDIX: EXPLICIT EXPRESSION FOR $\Delta_1(\mathrm{M}^2,\mathrm{s}_0)$ AND $\Delta_2(\mathrm{M}^2,\mathrm{s}_0)$ FUNCTIONS

We present the explicit expressions of the functions $\Delta_1(\mathrm{M}^2,\mathrm{s}_0)$ and $\Delta_2(\mathrm{M}^2,\mathrm{s}_0)$ for the MDMs of vector doubly charmed tetraquark states entering the obtained sum rules:

$$\begin{aligned}\Delta_1(\mathrm{M}^2,\mathrm{s}_0) = \frac{e^{\frac{m_{T_{cc}}^2}{\mathrm{M}^2}}}{\lambda_{T_{cc}}^2}\bigg\{&\frac{(e_{q_1}+e_{q_2})m_c^2}{327680\pi^5}\Big[2I[0,5,2,1] - 5I[0,5,2,2] + 4I[0,5,2,3] - I[0,5,2,4] - 4I[0,5,3,1]\\ &+ 6I[0,5,3,2] - 2I[0,5,3,3] + 2I[0,5,4,1] - I[0,5,4,2] + 5I[1,4,2,2] - 10I[1,4,2,3] + 5I[1,4,2,4]\\ &- 10I[1,4,3,2] + 10I[1,4,3,3] + 5I[1,4,4,2]\Big]\end{aligned}$$

$$
\begin{aligned}
&-\frac{3(e_{q_1}+e_{q_2})}{1310720\pi^5}\Big[I[0,6,3,0]-4I[0,6,3,1]+6I[0,6,3,2]-4I[0,6,3,3]+I[0,6,3,4]-3I[0,6,4,0]\\
&+9I[0,6,4,1]-9I[0,6,4,2]+3I[0,6,4,3]+3I[0,6,5,0]-6I[0,6,5,1]+3I[0,6,5,2]-I[0,6,6,0]\\
&+I[0,6,6,1]+6I[1,5,3,1]-18I[1,5,3,2]+18I[1,5,3,3]-6I[1,5,3,4]-18I[1,5,4,1]+36I[1,5,4,2]\\
&-18I[1,5,4,3]+18I[1,5,5,1]-18I[1,5,5,2]-6I[1,5,6,1]\Big]\\
&-\frac{e_{q_1}m_{q_2}m_c^2\langle g_s^2G^2\rangle\langle\bar{q}_1q_1\rangle}{36864\pi^3}(4I[0,1,2,0]-3I[0,1,4,0])I_6[h_\gamma]\\
&-\frac{e_{q_2}m_{q_1}m_c^2\langle g_s^2G^2\rangle\langle\bar{q}_2q_2\rangle}{36864\pi^3}(4I[0,1,2,0]-3I[0,1,4,0])I_6[h_\gamma]\\
&-\frac{35(e_{q_1}+e_{q_2})m_c^2\langle g_s^2G^2\rangle f_{3\gamma}}{3538944\pi^3}I[0,2,2,0]I_2[\mathcal{V}]+\frac{(e_{q_1}+e_{q_2})m_c^2f_{3\gamma}}{393216\pi^3}I[0,4,4,0]I_2[\mathcal{V}]\Big\},
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
\Delta_2(\mathrm{M}^2,\mathrm{s}_0)=\frac{e^{\frac{m_{T_{cc}}^2}{\mathrm{M}^2}}}{\lambda_{T_{cc}}^2}\Big\{&\frac{e_cm_c^2}{163840\pi^5}[I[0,5,1,2]-2I[0,5,1,3]+I[0,5,1,4]-2I[0,5,2,2]+2I[0,5,2,3]+I[0,5,3,2]]\\
&+\frac{e_c}{327680\pi^5}[2I[0,6,2,1]-7I[0,6,2,2]+9I[0,6,2,3]-5I[0,6,2,4]+I[0,6,2,5]-6I[0,6,3,1]\\
&+15I[0,6,3,2]-12I[0,6,3,3]+3I[0,6,3,4]+6I[0,6,4,1]-9I[0,6,4,2]+3I[0,6,4,3]-2I[0,6,5,1]\\
&+I[0,6,5,2]+6I[1,5,2,2]-18I[1,5,2,3]+18I[1,5,2,4]-6I[1,5,2,5]-18I[1,5,3,2]+36I[1,5,3,3]\\
&-18I[1,5,3,4]+18I[1,5,4,2]-18I[1,5,4,3]-6I[1,5,5,2]]\\
&+\frac{e_qm_qm_c^2\langle g_s^2G^2\rangle\langle\bar{q}q\rangle}{147456\pi^3}I[0,1,2,0]I_6[h_\gamma]+\frac{e_qm_c^2\langle g_s^2G^2\rangle}{3538944\pi^3}I[0,3,2,0]I_2[\mathcal{V}]+\frac{e_qm_qm_c^2\langle\bar{q}q\rangle}{3072\pi^3}I[0,3,3,0]I_4[\mathcal{S}]\\
&+\frac{e_qm_q\langle\bar{q}q\rangle}{32768\pi^3}I[0,4,5,0]I_2[\mathcal{S}]-\frac{e_qm_c^2f_{3\gamma}}{24576\pi^3}I[0,4,3,0]I_2[\mathcal{V}]+\frac{e_qf_{3\gamma}}{327680\pi^3}I[0,5,5,0]I_2[\mathcal{V}]\Big\},
\end{aligned}
\tag{A2}
$$

where the functions $I[n,m,l,k]$ and $I_i[\mathcal{F}]$ are defined as

$$
\begin{aligned}
I[n,m,l,k]&=\int_{4m_c^2}^{\mathrm{s}_0}ds\int_0^1dt\int_0^1dw\,e^{-s/\mathrm{M}^2}\,s^n\,(s-4m_c^2)^m t^l w^k,\\
I_1[\mathcal{F}]&=\int D_{\alpha_i}\int_0^1dv\;\mathcal{F}(\alpha_{\bar{q}},\alpha_q,\alpha_g)\delta'(\alpha_q+\bar{v}\alpha_g-u_0),\\
I_2[\mathcal{F}]&=\int D_{\alpha_i}\int_0^1dv\;\mathcal{F}(\alpha_{\bar{q}},\alpha_q,\alpha_g)\delta'(\alpha_{\bar{q}}+v\alpha_g-u_0),\\
I_3[\mathcal{F}]&=\int D_{\alpha_i}\int_0^1dv\,\mathcal{F}(\alpha_{\bar{q}},\alpha_q,\alpha_g)\delta(\alpha_q+\bar{v}\alpha_g-u_0),\\
I_4[\mathcal{F}]&=\int D_{\alpha_i}\int_0^1dv\,\mathcal{F}(\alpha_{\bar{q}},\alpha_q,\alpha_g)\delta(\alpha_{\bar{q}}+v\alpha_g-u_0),\\
I_5[\mathcal{F}]&=\int_0^1du\,\mathcal{F}(u)\delta'(u-u_0),\\
I_6[\mathcal{F}]&=\int_0^1du\,\mathcal{F}(u),
\end{aligned}
\tag{A3}
$$

with $\mathcal{F}$ being the corresponding photon DAs.

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