On Higgs + jet production at next-to-leading power accuracy

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We present computation of the next-to-leading power corrections for Higgs plus one jet production in a hadron collider via gluon fusion channel. Shifting of spinors in the helicity amplitudes without additional radiation captures the leading next-to-soft radiative behavior and makes the calculation tractable. We establish the connection between the shifted dipole spinors and the color ordered radiative amplitudes. We find that next-to-maximal helicity violating amplitudes do not play a role in this correction. Compact analytic expressions of next-to-leading power leading logarithms coming from different helicity configurations are shown.

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I. INTRODUCTION

Precise experimental data from the Large Hadron Collider (LHC) and the lack of any persuasive new physics signature demand improvement in the understanding of the Standard Model. Typically in collider environments, the strong force dominates over other interactions, and that makes the study of theory of quantum chromodynamics (QCD) most important. Fixed order corrections by taking into account higher order perturbative terms in the strong coupling constant and resummation including certain enhanced logarithms to all orders in the perturbation series are the two ways to ameliorate the theoretical accuracy. For all collider processes, one can define a threshold variable that vanishes in the threshold limit. In terms of a generic threshold variable (ξ), the differential cross section takes the following form:

$$\frac{d\sigma}{d\xi} \approx \sum_{n=0}^{\infty} \alpha_s^n \left\{ \sum_{m=0}^{2n-1} C_{nm} \left(\frac{\log^m \xi}{\xi} \right)_+ + d_n \delta(\xi) + \sum_{m=0}^{2n-1} D_{nm} \log^m \xi \right\}. \tag{1}$$

The first set of logarithms and the delta function are associated with the leading power (LP) approximations, whereas the second set of logarithms appear due to the next-to-leading power (NLP) approximation. The LP terms are well known to originate from the emission of soft and collinear radiation. The seminal works of Refs. [1–8] based on diagrammatics helped in devising methods of LP resummation. Later, several alternative methods of LP resummation based on Wilson lines [9,10], renormalization group [11], and soft collinear effective theory [12–15] were developed. A comparative study of different approaches can be found in Refs. [16–18].

Despite substantial progress made toward understanding the infrared behavior of the NLP logarithms during the past decade, the universality of such terms is yet to be established. The numerical impacts of NLP logarithms are shown in Refs. [19–26]. Realizing the importance of these numerical impacts, several methods to resum NLP logarithms have been formulated over the years [26–79]. It is essential to investigate NLP logarithms of several processes to better understand the universal nature of the next-to-soft radiation and to come up with a global resummation formula. The universality of NLP logarithms is already established in case of color singlet production [42]; however, for colored particles in the final state, there exists no unique resummation formula.

A prescription has been developed in [42] for colorless particles and then further extended to final state colored particles in [43], in which appropriate shifting of pairs of momenta in the squared nonradiative (i.e., without additional radiation) amplitude captures the next-to-soft radiation effects. The expression of the squared nonradiative amplitude does not always have a compact analytical form, and therefore, shifting the momenta may not always give a simple result. For example, the calculation of NLP terms using the squared amplitudes appears to be very intricate for colored particles in the final state, and due to this reason, only two processes with a single colored particle in the final state are studied so far at NLP accuracy—(i) prompt

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photon plus jet production [43] and (ii) W plus jet production [48,80,81]. The scarcity of results for colored final state particles due to the complexity in such calculations clearly demands an improvement on the existing technique.

In this endeavor, we study the effect of next-to-soft gluon radiation on Higgs production via gluon fusion in association with a final state hard jet by crafting spinor helicity amplitudes. We consider the heavy top mass limit throughout. Instead of shifting momenta in the squared amplitude, we shift the spinors of the nonradiative helicity amplitudes to capture next-to-soft radiation effects, and that essentially makes the calculation lucid and tractable. We start with the soft and next-to-soft theorems developed in [82,83] and show that pairwise shifting of spinors at the nonradiative amplitudes can be realized as next-to-soft emissions from those amplitudes. We find that color dipoles with shifted spinors directly correspond to the color ordered radiative amplitudes in the next-to-soft limit. The next-to-soft amplitudes thus obtained are compact in nature. In addition, it reveals that the next-to-maximal helicity violating (NMHV) amplitudes never contribute at NLP accuracy for the case at hand. In order to obtain NLP logarithms, we integrate squared helicity amplitudes over the unresolved parton phase space and present the analytic results for different helicity configurations. Singularities that arise at the LP and NLP stages get exactly canceled, while contributions from the virtual emission and mass factorization are included.

The structure of our paper is as follows. In Sec. II, we review the soft and next-to-soft theorems in terms of spinor shifts. After detailing the shifts, in Sec. III, we apply them to calculate different color ordered helicity amplitudes. Squaring these amplitudes, we perform the phase space integration over the unresolved phase space in Sec. IV to obtain the NLP logarithms. Finally, we summarize our findings with an outlook in Sec. V. Throughout this study, we have used a combination of in-house routines based on QGRAF [84], FORM [85], and *Mathematica* [86] to calculate all helicity amplitudes and to perform the phase space integration.

II. SOFT AND NEXT-TO-SOFT CORRECTIONS

In this section, we briefly review the soft and next-to-soft theorems in terms of color ordered scattering amplitudes. Any color ordered scattering amplitude involving n particles (quarks and gluons) with specific helicities can be represented as

$$\mathcal{A} = \mathcal{A}_n(\{|1\rangle, |1]\}, \ldots, \{|n\rangle, |n]\}), \tag{2}$$

where $|i\rangle$ and $|i]$ denote the holomorphic and antiholomorphic spinors associated with the particle i carrying momentum p_i. Let us now consider that a gluon s with momenta p_s and helicity "+" is being emitted from this scattering process. Scaling the momentum of the radiated gluon $p_s \to \lambda p_s$, the scattering amplitude for $n+1$ particle can be expressed in powers of λ as written here under [82,83],

$$\mathcal{A}_{n+1}(\{\lambda|s\rangle, |s]\}, \{|1\rangle, |1]\}, \ldots, \{|n\rangle, |n]\}) = (S^{(0)} + S^{(1)})\mathcal{A}_n(\{|1\rangle, |1]\}, \ldots, \{|n\rangle, |n]\}). \tag{3}$$

Here, $S^{(0)}$ and $S^{(1)}$ denote LP and NLP terms that are of $\mathcal{O}(1/\lambda^2)$ and $\mathcal{O}(1/\lambda)$, respectively, and are given by

$$S^{(0)} = \frac{\langle n1\rangle}{\langle s1\rangle\langle ns\rangle},$$
$$S^{(1)} = \frac{1}{\langle s1\rangle}|s]\frac{\partial}{\partial|1]} - \frac{1}{\langle sn\rangle}|s]\frac{\partial}{\partial|n]}. \tag{4}$$

In order to obtain the above formulas, a holomorphic soft limit [82,83] is being used, i.e.,

$$|s\rangle \to \lambda|s\rangle, \quad |s] \to |s], \tag{5}$$

under the BCFW [87,88] deformation of the s and n pair, while particles 1 and s always form a three particle amplitude involving the on-shell cut propagator that carries complex momentum. With the help of Eq. (4), the color ordered amplitude of Eq. (3) can be rewritten as

$$\mathcal{A}^{\mathrm{LP+NLP}}_{n+1}(\{\lambda|s\rangle, |s]\}, \{|1\rangle, |1]\}, \ldots, \{|n\rangle, |n]\}) = \frac{1}{\lambda^2}\frac{\langle 1n\rangle}{\langle 1s\rangle\langle ns\rangle}\mathcal{A}_n(\{|1\rangle, |1']\}, \ldots, \{|n\rangle, |n']\}), \tag{6}$$

where

$$|1'] = |1] + \Delta_s^{(1,n)}|s],$$
$$|n'] = |n] + \Delta_s^{(n,1)}|s], \tag{7}$$

and

$$\Delta_s^{(i,j)} = \lambda\frac{\langle js\rangle}{\langle ji\rangle}. \tag{8}$$

This form of Eq. (6) signifies that the leading and subleading behavior of the amplitude can be obtained in terms of simple shifts in the spinors of tree amplitudes. Note that the emitted soft gluon is placed in between the 1 and n particles in the color ordered amplitudes and forms a color dipole $\mathcal{D}_{1n}$. Such color dipole structures play an important role in understanding the IR singularities of scattering amplitudes [89–91].

Emission of soft gluon with "−" helicity can be treated analogously by taking antiholomorphic soft gluon limit and interchanging angle and square spinors. Equipped with these formulas, we now move on to calculate the LP and

NLP amplitudes for Higgs plus one jet production in the gluon fusion channel.

III. LP AND NLP AMPLITUDES FOR $gg \to Hg$

The most dominant mechanism for Higgs boson production at the LHC is via the gluon fusion channel. In this section, we first reproduce all independent helicity amplitudes for Higgs plus one jet production via gluon fusion with(out) one extra gluon emission. Then, we obtain NLP amplitudes by (i) taking a soft gluon limit on $gg \to Hgg$ amplitudes and (ii) shifting spinors in $gg \to Hg$ amplitudes. Both ways lead to the exactly same results. Finally, we discuss that for Higgs plus one jet production NMHV amplitudes do not contribute to the NLP threshold corrections.

A. Higgs-gluon amplitudes

The Standard Model of particle physics forbids gluons to interact with Higgs at the tree level; however, they can interact via a massive quark loop. As the top quark is the heaviest among massive quarks, the coupling of Higgs with gluons is dominated via a top quark loop. In the large top mass limit $m_t \to \infty$, we can integrate out the heavy top quark effect to obtain an effective Lagrangian as follows [92,93]:

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{4} G H \mathrm{Tr}(F^a_{\mu\nu} F^{\mu\nu,a}), \tag{9}$$

where $F^a_{\mu\nu}$ is the QCD field strength tensor. The effective coupling is given at lowest order by $G = \alpha_s/3\pi v$, where v is the vacuum expectation value of the Higgs field, and α_s is the strong coupling constant. The general form of an amplitude consisting of one Higgs boson and n gluons can be represented as

$$\mathcal{A}_n(p_i, h_i, c_i) = i\left(\frac{\alpha_s}{6\pi v}\right) g_s^{n-2} \sum_{\sigma \in \mathcal{S}_{n'}} \mathrm{Tr}(\mathbf{T}^{c_1}\mathbf{T}^{c_2}\ldots\mathbf{T}^{c_n}) \times \mathcal{A}_n^{\{c_i\}}(h_1 h_2 h_3 \ldots h_n; H). \tag{10}$$

Here, $\mathcal{S}_{n'}$ represents the set of all $(n-1)!$ noncycling permutations of $1, 2, \ldots, n$. $\mathbf{T}^{c_i}$ denote the SU(3) color matrix in the fundamental representation, and they are normalized as $\mathrm{Tr}(\mathbf{T}^{c_1}, \mathbf{T}^{c_2}) = \delta^{c_1 c_2}$. For brevity, we avoid writing H explicitly in $\mathcal{A}_n^{\{c_i\}}$ in the rest of this paper.

The leading order process for Higgs plus one gluon production can be written as

$$g(p_1) + g(p_2) \to H(-p_3) + g(-p_4). \tag{11}$$

There are two independent color ordered helicity amplitudes for this process as given below,

$$\mathcal{A}^{124}_{+++} = \frac{m_H^4}{\langle 12\rangle\langle 24\rangle\langle 41\rangle}, \qquad \mathcal{A}^{124}_{-++} = \frac{[24]^3}{[12][14]}, \tag{12}$$

and amplitudes for all other helicity configurations can be constructed using these two.

Now, we consider that a gluon with momenta p_5 is being emitted from the leading order process, i.e.,

$$g(p_1) + g(p_2) \to H(-p_3) + g(-p_4) + g(-p_5). \tag{13}$$

For this process, there are only three independent helicity amplitudes, and remaining helicity configurations can be obtained by switching external momenta and spinors. These three independent helicity amplitudes containing Higgs plus four gluons are given by

$$\begin{aligned}
\mathcal{A}^{1245}_{++++} &= \frac{m_H^4}{\langle 12\rangle\langle 24\rangle\langle 45\rangle\langle 51\rangle}, \\
\mathcal{A}^{1245}_{-+++} &= \frac{\langle 1|4+5|2]^3}{\langle 4|1|2]\langle 15\rangle\langle 45\rangle s_{145}} + \frac{[25][45]\langle 1|4+5|2]^2}{\langle 4|1|2] s_{15} s_{145}} \\
&\quad + \frac{[24]\langle 1|2+4|5]^2}{\langle 24\rangle s_{12} s_{124}} \\
&\quad + \frac{[25]\langle 1|2+4|5]^2}{\langle 14\rangle\langle 24\rangle[15] s_{12}} - \frac{[25]^2\langle 1|2+5|4]^2}{s_{12} s_{15} s_{125}}, \\
\mathcal{A}^{1245}_{--++} &= -\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 24\rangle\langle 45\rangle\langle 51\rangle} - \frac{[45]^4}{[12][24][45][51]}.
\end{aligned} \tag{14}$$

Here, $s_{ij} = (p_i + p_j)^2$ and $s_{ijk} = (p_i + p_j + p_k)^2$. These amplitudes were calculated for the first time in [94]. Following Eq. (10), we can write the full amplitude for a given helicity configuration as

$$\begin{aligned}
&\mathcal{A}(\{p_i, h_i, c_i\}) \\
&= i\left(\frac{\alpha_s}{6\pi v}\right) g_s^2 \Big[\{\mathrm{Tr}(\mathbf{T}^{c_1}\mathbf{T}^{c_2}\mathbf{T}^{c_4}\mathbf{T}^{c_5}) \\
&\quad + \mathrm{Tr}(\mathbf{T}^{c_1}\mathbf{T}^{c_5}\mathbf{T}^{c_4}\mathbf{T}^{c_2})\} \mathcal{A}^{1245}_{h_1 h_2 h_4 h_5} \\
&\quad + \{\mathrm{Tr}(\mathbf{T}^{c_1}\mathbf{T}^{c_4}\mathbf{T}^{c_5}\mathbf{T}^{c_2}) + \mathrm{Tr}(\mathbf{T}^{c_1}\mathbf{T}^{c_2}\mathbf{T}^{c_5}\mathbf{T}^{c_4})\} \mathcal{A}^{1452}_{h_1 h_2 h_4 h_5} \\
&\quad + \{\mathrm{Tr}(\mathbf{T}^{c_1}\mathbf{T}^{c_5}\mathbf{T}^{c_2}\mathbf{T}^{c_4}) + \mathrm{Tr}(\mathbf{T}^{c_1}\mathbf{T}^{c_4}\mathbf{T}^{c_2}\mathbf{T}^{c_5})\} \mathcal{A}^{1524}_{h_1 h_2 h_4 h_5} \Big].
\end{aligned} \tag{15}$$

Squaring the above equation and summing over colors, we obtain the expression of squared amplitude as

$$\begin{aligned}
&\sum_{\mathrm{colors}} |\mathcal{A}(\{p_i, h_i, c_i\})|^2 \\
&= \left[\left(\frac{\alpha_s}{6\pi v}\right) g_s^2\right]^2 (N^2-1) \Big\{ 2N^2 (|\mathcal{A}^{1245}|^2 + |\mathcal{A}^{1452}|^2 \\
&\quad + |\mathcal{A}^{1524}|^2) - 4\frac{(N^2-3)}{N^2} |\mathcal{A}^{1245} + \mathcal{A}^{1452} + \mathcal{A}^{1524}|^2 \Big\}.
\end{aligned} \tag{16}$$

Here, for simplicity, we have suppressed the labels that represent helicity configurations. Due to the dual Ward identity [94,95], the term in the second line of the above equation vanishes, and we are left with only the first term.

B. Spinor shifts and color dipoles

In order to obtain NLP amplitudes for the Higgs plus two gluon production process, one needs to expand the $gg \to Hgg$ helicity amplitudes in the powers of the soft momentum keeping the subleading contributions. In parallel, following the arguments presented in Sec. II, we can get NLP amplitudes using the shifts in the spinors of $gg \to Hg$ amplitudes. We start our calculation by noting the fact that the gluon with momentum p_5 is emitted from any of the three gluons present at the leading order, and as discussed in the previous section, the emission of a soft gluon always engenders shifts in two adjacent spinors present in the color ordered nonradiative Born amplitudes.

In case of emission of a next-to-soft gluon from Higgs plus n gluon amplitudes, a total ${}^nC_2 = n(n-1)/2$ number of color dipoles can be formed. Therefore, for amplitudes consisting of Higgs plus three gluons, three dipoles are generated due to the emission of a next-to-soft gluon, and NLP amplitudes can be realized by shifting appropriate spinors depending on the helicity of the emitted gluon. For a "+" gluon emisison from the dipole $\mathcal{D}_{14}$ made up of momenta p_1 and p_4, the LP + NLP amplitude can be expressed as

$$\mathcal{A}^{1245}_{h_1 h_2 h_4 +} = \frac{\langle 14\rangle}{\langle 15\rangle\langle 45\rangle}\mathcal{A}^{1'24'}_{h_1 h_2 h_4}, \tag{17}$$

where $\mathcal{A}^{1'24'}_{h_1 h_2 h_4}$ denotes that the $|1]$ and $|4]$ spinors are shifted in the color ordered leading amplitude obeying Eq. (7). Similar contributions coming from the dipoles $\mathcal{D}_{24}$ and $\mathcal{D}_{12}$ can be written as

$$\mathcal{A}^{1452}_{h_1 h_2 h_4 +} = \frac{\langle 24\rangle}{\langle 25\rangle\langle 54\rangle}\mathcal{A}^{12'4'}_{h_1 h_2 h_4}, \tag{18}$$

and

$$\mathcal{A}^{1524}_{h_1 h_2 h_4 +} = \frac{\langle 12\rangle}{\langle 15\rangle\langle 52\rangle}\mathcal{A}^{1'2'4}_{h_1 h_2 h_4}. \tag{19}$$

So the full amplitude of Eq. (15) can now be rewritten using Eqs. (17)–(19) as

$$\begin{aligned}\mathcal{A}_{h_1 h_2 h_4 +}\big|_{\mathrm{LP+NLP}} = i\left(\frac{\alpha_s}{6\pi v}\right)g^2\Bigg[&\{\mathrm{Tr}(\mathbf{T}^{C_1}\mathbf{T}^{C_2}\mathbf{T}^{C_4}\mathbf{T}^{C_5}) + \mathrm{Tr}(\mathbf{T}^{C_1}\mathbf{T}^{C_5}\mathbf{T}^{C_4}\mathbf{T}^{C_2})\}\frac{\langle 14\rangle}{\langle 15\rangle\langle 45\rangle}\mathcal{A}^{1'24'}_{h_1 h_2 h_4} \\ &- \{\mathrm{Tr}(\mathbf{T}^{C_1}\mathbf{T}^{C_4}\mathbf{T}^{C_5}\mathbf{T}^{C_2}) + \mathrm{Tr}(\mathbf{T}^{C_1}\mathbf{T}^{C_2}\mathbf{T}^{C_5}\mathbf{T}^{C_4})\}\frac{\langle 24\rangle}{\langle 25\rangle\langle 45\rangle}\mathcal{A}^{12'4'}_{h_1 h_2 h_4} \\ &- \{\mathrm{Tr}(\mathbf{T}^{C_1}\mathbf{T}^{C_5}\mathbf{T}^{C_2}\mathbf{T}^{C_4}) + \mathrm{Tr}(\mathbf{T}^{C_1}\mathbf{T}^{C_4}\mathbf{T}^{C_2}\mathbf{T}^{C_5})\}\frac{\langle 12\rangle}{\langle 15\rangle\langle 25\rangle}\mathcal{A}^{1'2'4}_{h_1 h_2 h_4}\Bigg].\end{aligned} \tag{20}$$

To derive the above equation, we have used the reflection identity [95] that applies for Higgs plus n gluon amplitudes. This equation is one of the central results of this paper which identifies the direct correspondence of color ordered amplitudes in the next-to-soft limit to the nonradiative color ordered Born amplitudes with shifted spinors. Shift in each nonradiative spinor pair represents one color ordered radiative amplitude. The validity of this formula relies only on the cyclic and antisymmetric properties of Higgs plus gluon amplitudes. Thus, this formula is applicable to any process that satisfies such properties, namely, pure gluon amplitudes in Yang-Mills theories or gluons with a quark-antiquark pair in QCD.

C. NLP amplitudes: Absence of NMHV contribution

As evident from the discussion in the previous section, color ordered LP amplitudes always appear as a product of Born amplitudes and the corresponding Eikonal factors such as

$$\begin{aligned}\mathcal{A}^{1245}_{h_1 h_2 h_4 +}\big|_{\mathrm{LP}} &= \frac{\langle 14\rangle}{\langle 15\rangle\langle 45\rangle}\mathcal{A}^{124}_{h_1 h_2 h_4}, \\ \mathcal{A}^{1245}_{h_1 h_2 h_4 -}\big|_{\mathrm{LP}} &= \frac{[14]}{[15][45]}\mathcal{A}^{124}_{h_1 h_2 h_4}.\end{aligned} \tag{21}$$

In this section, we provide the details of NLP amplitudes for different helicity configurations. For Higgs plus four gluon amplitudes, there are altogether $2^4 = 16$ helicity configurations possible. Out of these 16 helicity amplitudes, one needs to calculate only eight, as the remaining conjugate configurations can easily be obtained by flipping the helicity of all the external gluons. As discussed earlier, NLP amplitudes can be calculated considering emission of both "+" and "−" helicity gluons from all possible Born amplitudes. In doing so, we find that the NMHV amplitudes do not add to the NLP contribution. We illustrate this by a simple example. Let us consider emission of a "+" helicity gluon out of the $\mathcal{A}^{124}_{+--}$ amplitude, which following Eq. (12) can be presented as

$$\mathcal{A}^{124}_{+--} = -\frac{\langle 24\rangle^3}{\langle 12\rangle\langle 14\rangle}. \tag{22}$$

We have already seen in the previous subsection that emission of a "+" helicity gluon always demands antiholomorphic spinors to be shifted. However, there are no square spinors present in the above equation, and therefore, $\mathcal{A}^{1245}_{+--+}|_{\rm NLP}$ vanishes. It is also straight forward to check that applying $S^{(1)}$ of Eq. (4) on $\mathcal{A}^{124}_{+--}$ gives zero. The reason behind this vanishing of NLP amplitude for NHMV amplitudes can furthermore be argued by invoking the soft Higgs limit. Due to the momentum conservation, one can choose not to bring Higgs momentum explicitly in the expressions of NLP amplitudes, and in the soft Higgs limit, these amplitudes essentially behave as pure gluon NMHV amplitudes which were shown to be noncontributing to NLP in [83]. Among 16 Higgs plus four gluon helicity amplitudes, six NMHV amplitudes vanish, and we are left with ten nonzero helicity amplitudes at NLP. Out of these ten, we need to calculate only five, as the remaining five helicity configurations can readily be obtained by flipping helicities of all external gluons. Table I shows emissions from the Born amplitudes and lists down those five different nonzero amplitudes. Their expressions including three different color orderings for each of them are given below:

(1) $++++$

$$\begin{aligned}
\mathcal{A}^{1245}_{++++}|_{\rm NLP} &= \frac{\langle 14\rangle}{\langle 15\rangle\langle 45\rangle}\frac{2(s_{15}+s_{25}+s_{45})}{(s_{12}+s_{14}+s_{24})}\mathcal{A}^{124}_{+++},\\
\mathcal{A}^{1524}_{++++}|_{\rm NLP} &= -\frac{\langle 12\rangle}{\langle 15\rangle\langle 25\rangle}\frac{2(s_{15}+s_{25}+s_{45})}{(s_{12}+s_{14}+s_{24})}\mathcal{A}^{124}_{+++},\\
\mathcal{A}^{1452}_{++++}|_{\rm NLP} &= -\frac{\langle 24\rangle}{\langle 45\rangle\langle 25\rangle}\frac{2(s_{15}+s_{25}+s_{45})}{(s_{12}+s_{14}+s_{24})}\mathcal{A}^{124}_{+++}.
\end{aligned} \tag{23}$$

TABLE I. A set of eight NLP amplitudes constructed from the Born amplitudes is given. Flipping helicities of all the particles provide the remaining eight amplitudes among which three NMHV configurations become zero again. We do not mention any explicit color ordering here as this feature stands true irrespective of that choice.

Born	Helicity of extra emission	NLP
$\mathcal{A}_{+++}$	+	$\mathcal{A}_{++++}\|_{\rm NLP}$
	−	$\mathcal{A}_{+++-}\|_{\rm NLP}$
$\mathcal{A}_{-++}$	+	$\mathcal{A}_{-+++}\|_{\rm NLP}$
	−	0
$\mathcal{A}_{+-+}$	+	$\mathcal{A}_{+-++}\|_{\rm NLP}$
	−	0
$\mathcal{A}_{++-}$	+	$\mathcal{A}_{++-+}\|_{\rm NLP}$
	−	0

(2) $-+++$

$$\begin{aligned}
\mathcal{A}^{1245}_{-+++}|_{\rm NLP} &= \frac{\langle 14\rangle}{\langle 15\rangle\langle 45\rangle}\left(\frac{3\langle 15\rangle[25]}{\langle 14\rangle[24]}-\frac{\langle 45\rangle[25]}{\langle 14\rangle[12]}-\frac{s_{15}}{s_{14}}-\frac{s_{45}}{s_{14}}\right)\mathcal{A}^{124}_{-++},\\
\mathcal{A}^{1524}_{-+++}|_{\rm NLP} &= -\frac{\langle 12\rangle}{\langle 15\rangle\langle 25\rangle}\left(-\frac{\langle 25\rangle[45]}{\langle 12\rangle[14]}-\frac{3\langle 15\rangle[45]}{\langle 12\rangle[24]}-\frac{s_{15}}{s_{12}}-\frac{s_{25}}{s_{12}}\right)\mathcal{A}^{124}_{-++},\\
\mathcal{A}^{1452}_{-+++}|_{\rm NLP} &= -\frac{\langle 24\rangle}{\langle 45\rangle\langle 25\rangle}\left(\frac{\langle 45\rangle[15]}{\langle 24\rangle[12]}-\frac{\langle 25\rangle[15]}{\langle 24\rangle[14]}+\frac{3s_{25}}{s_{24}}+\frac{3s_{45}}{s_{24}}\right)\mathcal{A}^{124}_{-++}.
\end{aligned} \tag{24}$$

(3) $++-+$

$$\begin{aligned}
\mathcal{A}^{1245}_{++-+}|_{\rm NLP} &= \mathcal{A}^{1245}_{-+++}|_{\rm NLP}\{1\leftrightarrow 4\},\\
\mathcal{A}^{1524}_{++-+}|_{\rm NLP} &= \mathcal{A}^{1452}_{-+++}|_{\rm NLP}\{1\leftrightarrow 4\},\\
\mathcal{A}^{1452}_{++-+}|_{\rm NLP} &= \mathcal{A}^{1524}_{-+++}|_{\rm NLP}\{1\leftrightarrow 4\}.
\end{aligned} \tag{25}$$

(4) $+-++$

$$\begin{aligned}
\mathcal{A}^{1245}_{+-++}|_{\rm NLP} &= \mathcal{A}^{1452}_{-+++}|_{\rm NLP}\{1\leftrightarrow 2\},\\
\mathcal{A}^{1524}_{+-++}|_{\rm NLP} &= \mathcal{A}^{1524}_{-+++}|_{\rm NLP}\{1\leftrightarrow 2\},\\
\mathcal{A}^{1452}_{+-++}|_{\rm NLP} &= \mathcal{A}^{1245}_{-+++}|_{\rm NLP}\{1\leftrightarrow 2\}.
\end{aligned} \tag{26}$$

(5) $+++-$

$$\begin{aligned}
\mathcal{A}^{1245}_{+++-}|_{\rm NLP} &= \frac{[14]}{[15][45]}\left(-\frac{\langle 25\rangle[45]}{\langle 12\rangle[14]}-\frac{\langle 25\rangle[15]}{\langle 24\rangle[14]}-\frac{s_{15}}{s_{14}}-\frac{s_{45}}{s_{14}}+\frac{2(s_{15}+s_{25}+s_{45})}{(s_{12}+s_{14}+s_{24})}\right)\mathcal{A}^{124}_{+++},\\
\mathcal{A}^{1524}_{+++-}|_{\rm NLP} &= -\frac{[12]}{[15][25]}\left(\frac{\langle 45\rangle[15]}{\langle 24\rangle[12]}-\frac{\langle 45\rangle[25]}{\langle 14\rangle[12]}-\frac{s_{15}}{s_{12}}-\frac{s_{25}}{s_{12}}+\frac{2(s_{15}+s_{25}+s_{45})}{(s_{12}+s_{14}+s_{24})}\right)\mathcal{A}^{124}_{+++},\\
\mathcal{A}^{1452}_{+++-}|_{\rm NLP} &= -\frac{[24]}{[25][45]}\left(\frac{\langle 15\rangle[45]}{\langle 12\rangle[24]}-\frac{\langle 15\rangle[25]}{\langle 14\rangle[24]}-\frac{s_{25}}{s_{24}}-\frac{s_{45}}{s_{24}}+\frac{2(s_{15}+s_{25}+s_{45})}{(s_{12}+s_{14}+s_{24})}\right)\mathcal{A}^{124}_{+++}.
\end{aligned} \tag{27}$$

IV. NLP LOGARITHMS

The obvious next step to obtain the NLP threshold logarithms is to perform phase-space integrations over the squared amplitudes at NLP, and we discuss that in the following two subsections.

A. Squared amplitudes at NLP

The amplitude for a process carrying a soft gluon radiation can be written as a sum of LP and NLP amplitudes such as

$$\mathcal{A} = \mathcal{A}_{\rm LP} + \mathcal{A}_{\rm NLP}. \tag{28}$$

Squaring the amplitude gives

$$\mathcal{A}^2 = \mathcal{A}^2_{\rm LP} + 2{\rm Re}(\mathcal{A}_{\rm NLP}\mathcal{A}^\dagger_{\rm LP}), \tag{29}$$

where the term $\mathcal{A}^2_{\rm NLP}$ is being neglected as it starts contributing at the next-to-next-to leading power. The first and the second terms represent LP and NLP contributions, respectively, and we denote the NLP contribution as $[\mathcal{A}^2]|_{\rm NLP}$ hereafter. Using Eq. (16), we obtain the squared NLP amplitude for a fixed helicity as

$$\sum_{\rm colors}[\mathcal{A}^2]|_{\rm NLP} = \left[\left(\frac{\alpha_s}{6\pi v}\right)g^2\right]^2 2N^2(N^2-1) \\ \times\{[\mathcal{A}^2]^{1245}|_{\rm NLP} + [\mathcal{A}^2]^{1452}|_{\rm NLP} \\ + [\mathcal{A}^2]^{1524}|_{\rm NLP}\}, \tag{30}$$

where N is the dimensionality of the SU(N) color, and it takes the value $N=3$ for QCD.

Using Eqs. (23)–(27), we get the squared NLP amplitudes of the following five helicity configurations:

(1) $++++$

$$[\mathcal{A}^2]^{1245}_{++++}|_{\rm NLP} = 4\left(\frac{s_{14}s_{25}}{s_{15}s_{45}} + \frac{s_{14}}{s_{15}} + \frac{s_{14}}{s_{45}}\right) \\ \times\frac{1}{(s_{12}+s_{14}+s_{24})}\mathcal{A}^2_{+++},$$
$$[\mathcal{A}^2]^{1524}_{++++}|_{\rm NLP} = [\mathcal{A}^2]^{1245}_{++++}|_{\rm NLP}\{2\leftrightarrow 4\},$$
$$[\mathcal{A}^2]^{1452}_{++++}|_{\rm NLP} = [\mathcal{A}^2]^{1245}_{++++}|_{\rm NLP}\{1\leftrightarrow 2\}. \tag{31}$$

(2) $-+++$

$$[\mathcal{A}^2]^{1245}_{-+++}|_{\rm NLP} = \left(-\frac{3s_{12}}{s_{15}s_{24}} - \frac{3}{s_{15}} + \frac{1}{s_{45}} + \frac{s_{24}}{s_{12}s_{45}} \right. \\ \left. - \frac{s_{14}s_{25}}{s_{12}s_{15}s_{45}} + \frac{3s_{14}s_{25}}{s_{15}s_{24}s_{45}}\right)\mathcal{A}^2_{-++},$$
$$[\mathcal{A}^2]^{1524}_{-+++}|_{\rm NLP} = [\mathcal{A}^2]^{1245}_{-+++}|_{\rm NLP}\{2\leftrightarrow 4\},$$
$$[\mathcal{A}^2]^{1452}_{-+++}|_{\rm NLP} = \left(\frac{s_{12}}{s_{14}s_{25}} + \frac{5}{s_{25}} + \frac{5}{s_{45}} + \frac{s_{14}}{s_{12}s_{45}} \right. \\ \left. - \frac{s_{15}s_{24}}{s_{12}s_{25}s_{45}} - \frac{s_{15}s_{24}}{s_{14}s_{25}s_{45}}\right)\mathcal{A}^2_{-++}. \tag{32}$$

(3) $++-+$

$$[\mathcal{A}^2]^{1245}_{++-+}|_{\rm NLP} = [\mathcal{A}^2]^{1245}_{-+++}|_{\rm NLP}\{1\leftrightarrow 4\},$$
$$[\mathcal{A}^2]^{1524}_{++-+}|_{\rm NLP} = [\mathcal{A}^2]^{1452}_{-+++}|_{\rm NLP}\{1\leftrightarrow 4\},$$
$$[\mathcal{A}^2]^{1452}_{++-+}|_{\rm NLP} = [\mathcal{A}^2]^{1524}_{-+++}|_{\rm NLP}\{1\leftrightarrow 4\}. \tag{33}$$

(4) $+-++$

$$[\mathcal{A}^2]^{1245}_{+-++}|_{\rm NLP} = [\mathcal{A}^2]^{1452}_{-+++}|_{\rm NLP}\{1\leftrightarrow 2\},$$
$$[\mathcal{A}^2]^{1524}_{+-++}|_{\rm NLP} = [\mathcal{A}^2]^{1524}_{-+++}|_{\rm NLP}\{1\leftrightarrow 2\},$$
$$[\mathcal{A}^2]^{1452}_{+-++}|_{\rm NLP} = [\mathcal{A}^2]^{1245}_{-+++}|_{\rm NLP}\{1\leftrightarrow 2\}. \tag{34}$$

(5) $+++-$

$$[\mathcal{A}^2]^{1245}_{+++-}|_{\rm NLP} = \left(\frac{s_{12}}{s_{15}s_{24}} - \frac{3}{s_{15}} - \frac{3}{s_{45}} + \frac{s_{24}}{s_{12}s_{45}} \right. \\ \left. - \frac{s_{14}s_{25}}{s_{12}s_{15}s_{45}} - \frac{s_{14}s_{25}}{s_{15}s_{24}s_{45}}\right)\mathcal{A}^2_{+++} \\ + [\mathcal{A}^2]^{1245}_{++++}|_{\rm NLP},$$
$$[\mathcal{A}^2]^{1524}_{+++-}|_{\rm NLP} = [\mathcal{A}^2]^{1245}_{+++-}|_{\rm NLP}\{2\leftrightarrow 4\},$$
$$[\mathcal{A}^2]^{1452}_{+++-}|_{\rm NLP} = [\mathcal{A}^2]^{1245}_{+++-}|_{\rm NLP}\{1\leftrightarrow 2\}. \tag{35}$$

Note that, the color ordering of the nonradiative squared amplitude, suppressed here and in the rest of the paper, is to be considered as $\{124\}$, i.e., $\mathcal{A}^2_{h_1h_2h_4} = [\mathcal{A}^2]^{124}_{h_1h_2h_4}$. Each of the remaining five non-NMHV squared amplitudes resemble one of the above results as their helicity amplitudes are obtained by flipping helicities of all the external particles.

B. Phase space integration

We are now ready to integrate the squared amplitudes over the unobserved parton phase space in the rest frame of p_4 and p_5 momenta to obtain the differential cross section. Following the usual method, we factorize the three-body phase space into two two-body phase spaces: (i) one containing two gluons with momenta p_4 and p_5 and (ii) the other one containing the Higgs and the collective contribution of the two gluons mentioned in (i). We choose the phase space parametrization in $d=(4-2\epsilon)$ dimension [96,97] as

$$p_1 = (E_1, 0, \ldots, 0, E_1),$$
$$p_2 = (E_2, 0, \ldots, 0, p_3\sin\psi, p_3\cos\psi - E_1),$$
$$p_3 = -(E_3, 0, \ldots, 0, p_3\sin\psi, p_3\cos\psi),$$
$$p_4 = -\frac{\sqrt{s_{45}}}{2}(1, 0, \ldots, 0, \sin\theta_1\sin\theta_2, \sin\theta_1\cos\theta_2, \cos\theta_1),$$
$$p_5 = -\frac{\sqrt{s_{45}}}{2}(1, 0, \ldots, 0, -\sin\theta_1\sin\theta_2, \\ -\sin\theta_1\cos\theta_2, -\cos\theta_1). \tag{36}$$

The differential cross section at NLP is then given by

$$s_{12}^2 \frac{d^2\sigma}{ds_{13}ds_{23}}\bigg|_{\mathrm{NLP}} = \mathcal{F}\left(\frac{s_{45}}{\bar{\mu}^2}\right)^{-\epsilon} \overline{\mathcal{A}^2_{\mathrm{NLP}}}, \tag{37}$$

where

$$\mathcal{F} = \frac{1}{2}K_{gg}G^2\left(\frac{\alpha_s(\bar{\mu}^2)}{4\pi}\right)^2\left(\frac{s_{13}s_{23}-m_H^2 s_{45}}{\bar{\mu}^2 s_{12}}\right)^{-\epsilon},$$
$$K_{gg} = \frac{N^2}{2(N^2-1)}, \qquad \bar{\mu}^2 = 4\pi e^{-\gamma_E\epsilon}\mu_r^2, \tag{38}$$

and

$$\overline{\mathcal{A}^2_{\mathrm{NLP}}} = \int_0^\pi d\theta_1(\sin\theta_1)^{1-2\epsilon}\int_0^\pi d\theta_2(\sin\theta_2)^{-2\epsilon}[\mathcal{A}^2]|_{\mathrm{NLP}}. \tag{39}$$

We can now use Eqs. (31)–(35) and formulas given in [98] to perform the angular integrations which give us the NLP threshold logarithms. We have checked that the singular terms produced after these integrations due to the hard collinear emissions get canceled, once the effects of mass factorization using helicity dependent Altareli-Parisi splitting functions [99,100] are taken into account. The helicity driven NLP leading logarithms that contribute to the differential cross sections are given by

(1) $++++$

$$s_{12}^2 \frac{d^2\sigma_{++++}}{ds_{13}ds_{23}}\bigg|_{\mathrm{NLP-LL}} = \mathcal{F}\left\{16\pi\left(s_{12}\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)+2\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right) + 16\pi\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\times\frac{1}{m_H^2}\mathcal{A}^2_{+++}. \tag{40}$$

(2) $-+++$

$$s_{12}^2 \frac{d^2\sigma_{-+++}}{ds_{13}ds_{23}}\bigg|_{\mathrm{NLP-LL}} = \mathcal{F}\left\{16\pi\left(\frac{1}{s_{13}}-\frac{1}{s_{23}}\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right) + 4\pi\left(\frac{3}{s_{13}}-\frac{1}{s_{23}}\right)\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathcal{A}^2_{-++}. \tag{41}$$

(3) $++-+$

$$s_{12}^2 \frac{d^2\sigma_{++-+}}{ds_{13}ds_{23}}\bigg|_{\mathrm{NLP-LL}} = \mathcal{F}\left\{16\pi\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right) - 4\pi\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathcal{A}^2_{++-}. \tag{42}$$

(4) $+-++$

$$s_{12}^2 s_{12}^2 \frac{d^2\sigma_{+-++}}{ds_{13}ds_{23}}\bigg|_{\mathrm{NLP-LL}} = \mathcal{F}\left\{16\pi\left(\frac{1}{s_{23}}-\frac{1}{s_{13}}\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right) + 4\pi\left(\frac{3}{s_{23}}-\frac{1}{s_{13}}\right)\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathcal{A}^2_{+-+}. \tag{43}$$

(5) $+++-$

$$s_{12}^2 \frac{d^2\sigma_{+++-}}{ds_{13}ds_{23}}\bigg|_{\mathrm{NLP-LL}} = \mathcal{F}\left\{-16\pi\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)\log\left(\frac{s_{45}}{\bar{\mu}^2}\right) - 4\pi\left(\frac{1}{s_{13}}+\frac{1}{s_{23}}\right)\log\left(\frac{s_{12}s_{45}}{s_{13}s_{23}}\right)\right\}\mathcal{A}^2_{+++} + s_{12}^2\frac{d^2\sigma_{++++}}{ds_{13}ds_{23}}\bigg|_{\mathrm{NLP-LL}}. \tag{44}$$

From the above equations, it is evident that the threshold variable is $\xi = (\frac{s_{45}}{\bar{\mu}^2})$. Flipping of all helicities together in each one of the above equations does not alter the result. Therefore, the complete result can be achieved by adding Eqs. (40)–(44) and then by multiplying by a factor of 2.

V. SUMMARY AND OUTLOOK

The avalanche of high accuracy data in the LHC demands perturbative QCD predictions to be extremely precise. From a theoretical point of view, all order resummation and fixed order calculations both are important to reach the desired precision. NLP corrections can leave numerically sizeable impacts on the differential distribution of cross sections in the threshold limit. Although there exists a method to calculate NLP corrections using momentum shifts at the squared amplitude level, the rarity of results clearly demands improvement on the method of such calculations.

We have considered the effect of next-to-soft radiation on the Higgs plus one jet production through gluon fusion. We have shifted the spinors in the nonradiative helicity amplitudes which essentially generate the helicity amplitudes in the case of an extra gluon emission in the next-to-soft limit. The squared amplitudes thus obtained are compact in nature, and it comes out that the NMHV amplitudes do not play a role in the calculation of threshold logarithms. We have performed the phase space integration over the unobserved parton phase space to obtain the NLP threshold logarithms and listed the results for each helicity configurations. A systematic method to calculate NLP leading logarithms is presented in this paper exploiting the connection between the shifted dipole spinors and color ordered radiative amplitudes, for the first time, at the helicity amplitude level. We believe that the simplicity and easy applicability of the approach presented here would facilitate bringing out more such results for several other processes.

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