Euclidean flows, solitons, and wormholes in AdS space from M-theory

Andrés Anabalón,[1,2,*] Álvaro Arboleya,[3,4,†] and Adolfo Guarino[3,4,‡]

[1]*Departamento de Física, Universidad de Concepción, Casilla 160-C, Concepción, Chile*
[2]*Instituto de Física Teórica, UNESP-Universidade Estadual Paulista, R. Dr. Bento T. Ferraz 271, Bl. II, Sao Paulo 01140-070, SP, Brazil*
[3]*Departamento de Física, Universidad de Oviedo, Avda. Federico García Lorca 18, 33007 Oviedo, Spain*
[4]*Instituto Universitario de Ciencias y Tecnologías Espaciales de Asturias (ICTEA), Calle de la Independencia 13, 33004 Oviedo, Spain*

(Received 15 January 2024; accepted 10 April 2024; published 7 May 2024)

Multiparametric families of nonsupersymmetric EAdS_4 flows as well as asymptotically EAdS_4 solitons and wormholes are constructed within the four-dimensional SO(8) gauged supergravity that describes the compactification of M-theory on S^7. More concretely, the solutions are found within the so-called STU model that describes the $\mathrm{U}(1)^4$ invariant sector of the theory. The on-shell action and gravitational free energy are computed for the regular solutions, the latter being zero for the wormholes irrespective of supersymmetry. There are special loci in parameter space yielding solutions with enhanced (super) symmetry. Examples include a supersymmetric EAdS_4 flow with $\mathrm{SO}(4)\times\mathrm{SO}(4)$ symmetry dual to a specific real mass deformation of ABJM on S^3 as well as a nonsupersymmetric wormhole with $\mathrm{SU}(3)\times\mathrm{U}(1)^2$ symmetry. Uplift formulas for these and other examples to Euclidean solutions of 11-dimensional supergravity are presented and their complex nature discussed.

DOI: 10.1103/PhysRevD.109.106007

I. INTRODUCTION

General relativity in Euclidean signature has played a central role in understanding thermodynamical aspects of gravitational systems, the most prominent example being the black hole thermodynamics [1]. In the more modern context of the gauge/gravity correspondence [2], Euclidean solutions of supergravity theories have been utilized to explore thermodynamical aspects of strongly coupled field theories in the planar limit. An example is the holographic evaluation of (the finite part of) the free energy of a three-dimensional conformal field theory (CFT) placed on S^3, which has been proposed as a measure of the number of its degrees of freedom. In analogy with the c theorem [3], this free energy is conjectured to obey a monotonicity property under the renormalisation group (RG) flow, namely, it obeys an F theorem [4,5].

This paper is framed within the specific context of the $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence. On the gravity side, our playground is the four-dimensional (Euclideanized) $\mathcal{N}=2$ STU model of [6], which describes the $\mathrm{U}(1)^4$ invariant sector of the maximal SO(8) gauged supergravity [7]. The latter describes the consistent truncation of 11-dimensional (11D) supergravity on S^7 to its zero mass sector [8]. On the field theory side, the CFT_3 of relevance is the ABJM theory describing the world volume dynamics of a stack of N planar M2-branes in flat space [9]. Placing the theory on S^3 and turning on $\mathcal{N}=2$ real mass parameters, which modify the assignment of $\mathrm{U}(1)_R$ R charges and break conformality, turns out to induce RG flows. Within the STU model, such RG flows were holographically constructed in [10] as a three-parameter family of $\mathcal{N}=2$ Euclidean solutions that preserve the isometries of the S^3 in the bulk and have an S^3 boundary. Building upon these results, we will construct various new classes of Euclidean solutions within the STU model: nonbackreacted EAdS_4 flows with nontrivial scalar fields in the bulk, as well as backreacted solutions describing singular solitons and regular wormholes.

Examples of Euclidean four-dimensional solutions with a nonbackreacted EAdS_4 geometry and nontrivial matter fields have previously been constructed in [11] and uplifted to M-theory in [12]. These examples involve nontrivial vector fields in the bulk and describe a nontrivial end point of the RG flow induced by topologically twisted (scalar) deformations, which preserve the scale invariance but not the conformal invariance of the CFT_3 in the UV. Here, we will present two different three-parameter families—one

*andres.anabalon@uai.cl
†alvaroarbo@gmail.com
‡adolfo.guarino@uniovi.es

singular and one regular—of nonbackreacted $EAdS_4$ flows with running scalars (instead of vectors), which are generically nonsupersymmetric. For a specific choice of parameters, both the singular and the regular flows become supersymmetric, and we present their uplift to 11D supergravity. The supersymmetric and regular $EAdS_4$ flow lies within the class constructed in [10]. We will review its holographic realization as a specific real mass deformation of ABJM on S^3.

In addition to the nonbackreacted flows, we also present multiparametric families of soliton and wormhole solutions of the (Euclideanized) STU model with running scalars in the bulk. While the solitons are always singular, fully regular wormholes are shown to exist upon judicious choice of the free parameters. The construction of regular Euclidean wormholes in string/M-theoretic scenarios has proven a challenging task. Some constructions for flat-space wormholes have been put forward in the context of (super)gravity coupled to proper scalars and pseudoscalars (also known as axions) [13–16], as well as for AdS wormholes in the presence of a cosmological constant or a scalar potential [17–21]. In all these constructions, the analytic continuation from Lorentzian to Euclidean signature becomes subtle as far as the axions are concerned. Fortunately, a full-fledged construction of the Euclidean STU model was done in [10] and shown to require, among other modifications, a doubling of the scalar degrees of freedom: The three complex scalars z_i of the STU model and their would-be conjugates $\tilde{z}_i$ should be treated as independent fields. This additional freedom in the Euclidean theory is precisely what allows us to construct regular wormholes for which $\tilde{z}_i \neq z_i^*$. When uplifted to 11-dimensional supergravity, both the 11D metric and the four-form flux become complex valued with nontrivial real and imaginary parts. Although this is in principle allowed in a classical theory of supergravity, it remains to be seen what the ultimate fate of these Euclidean wormholes will be in a path integral formulation of quantum gravity.

II. EUCLIDEAN SUPERGRAVITY

The STU model of [6] describes the $U(1)^4$-invariant sector of the SO(8)-gauged maximal supergravity [7]. Its Euclidean version, which is the relevant one for this work, has been discussed in full detail in [10]. In the absence of vector fields, this supergravity describes an Einstein-scalar model involving a set of complex scalars z^i and $\tilde{z}^i$ with $i = 1, 2, 3$. The bulk Euclidean (bosonic) action is given by

$$S_{\text{bulk}} = \frac{1}{2\kappa^2} \int d^4x e \left[-\frac{R}{2} + \sum_{i=1}^{3} \frac{\partial_\rho z_i \partial^\rho \tilde{z}_i}{(1 - z_i \tilde{z}_i)^2} + V \right], \quad (1)$$

with $2\kappa^2 = 8\pi G_4$ and G_4 being the four-dimensional Newton constant. The bulk action (1) includes a nontrivial scalar potential

$$V(z_i, \tilde{z}_i) = g^2 \left(3 - \sum_i \frac{2}{1 - z_i \tilde{z}_i} \right), \quad (2)$$

where g is the four-dimensional gauge coupling that is related to the (inverse) radius of the seven-sphere. Importantly, as emphasized in [10], the scalars z_i and $\tilde{z}_i$ are complex and independent in the Euclidean theory and must separately parametrize the Poincaré unit disk, namely, $|z_i|, |\tilde{z}_i| < 1 \ \forall\ i$. The spacetime metric should also be allowed to be complex in the Euclidean theory, although we are only considering real metrics in this work.

The bulk action, as specified by (1) and (2), possesses three global symmetries $\mathbb{C}_i$ acting as constant scalings of the form $z_i \to \lambda_i z_i$ and $\tilde{z}_i \to \lambda_i^{-1} \tilde{z}_i$ with $\lambda_i \in \mathbb{C}$. However, different four-dimensional solutions related by these symmetries give rise to physically inequivalent backgrounds when uplifted to 11 dimensions (see the uplift formulas collected in Appendix and also [22,23] for a discussion of this issue in the Lorentzian realm). A proper understanding of global symmetries in the bulk would also require the inclusion of the vector fields in the STU model as they couple to the scalars through kinetic and topological terms. The $\mathbb{C}_i$ symmetries would act linearly on the vector fields changing their boundary values and, therefore, also changing the dual field theory on the boundary. However, we are not considering vector fields in this work and set them to zero.

We will construct nonconformal flow solutions in the Euclidean STU model that involve nonconstant scalars. Moreover, we will demand that the spacetime geometry has an S^3 boundary and that the flows preserve the $SO(4) \sim SU(2)_l \times SU(2)_r$ isometry group of S^3 in the bulk. The Euclidean metric (in the Fefferman-Graham gauge) is then of the form

$$ds_4^2 = d\mu^2 + g^{-2} e^{2A(\mu)} d\Sigma_{S^3}^2, \quad (3)$$

where $\mu \in \mathbb{R}$ is the radial (holographic) coordinate, and $d\Sigma_{S^3}^2$ is the line element of a three-sphere S^3. As a result, the geometry is specified by the metric function $A(\mu)$, which we take to be real valued. Moreover, in order to preserve the S^3 isometries, the scalars can only depend on the radial coordinate, namely, $z_i = z_i(\mu)$ and $\tilde{z}_i = \tilde{z}_i(\mu)$. With this ansatz, the second-order equations of motion that follow from the bulk action (1) are given by

$$A'' + g^2 e^{-2A} + \sum_{i=1}^{3} \frac{z_i' \tilde{z}_i'}{(1 - z_i \tilde{z}_i)^2} = 0,$$

$$z_i'' + 3A' z_i' + 2 \frac{\tilde{z}_i (z_i')^2}{1 - z_i \tilde{z}_i} + 2g^2 z_i = 0,$$

$$\tilde{z}_i'' + 3A' \tilde{z}_i' + 2 \frac{z_i (\tilde{z}_i')^2}{1 - z_i \tilde{z}_i} + 2g^2 \tilde{z}_i = 0, \quad (4)$$

and are obviously invariant under the $\mathbb{C}_i$ scaling symmetries of the bulk action.

The Euclidean STU model admits an $\mathcal{N}=1$ rewriting in terms of a Kähler potential

$$\mathcal{K} = -\sum_{i=1}^{3} \log\left[1 - z_i \tilde{z}_i\right], \tag{5}$$

and holomorphic superpotentials

$$W(z_i) = g(1 + z_1 z_2 z_3), \qquad \tilde{W}(\tilde{z}_i) = g(1 + \tilde{z}_1 \tilde{z}_2 \tilde{z}_3). \tag{6}$$

In this language, the bulk action (1) is constructed from $\mathcal{K}$, W, and $\tilde{W}$ using standard $\mathcal{N}=1$ formulas adapted to Euclidean signature (see Refs. [10,24]), and a set of first-order BPS equations can be derived by requiring the vanishing of the supersymmetry variations of the fermions in the model. The BPS equations can be written as first-order flow equations for the scalars $(z_i, \tilde{z}_i)$ and the metric function A, with the $\mathcal{N}=1$ gravitino mass $m_{3/2} \propto e^{\frac{\mathcal{K}}{2}}(W\tilde{W})^{\frac{1}{2}}$ playing the role of a scalar superpotential. Using $\mathcal{K}$, W, and $\tilde{W}$ in (5) and (6), the set of BPS equations reads

$$\begin{aligned}
-1 + \frac{e^{2A}}{g^2}(A')^2 &= e^{2A} \frac{(1 + z_1 z_2 z_3)(1 + \tilde{z}_1 \tilde{z}_2 \tilde{z}_3)}{\prod_{i=1}^{3}(1 - z_i \tilde{z}_i)}, \\
\frac{e^A}{g} \frac{(1 + \tilde{z}_1 \tilde{z}_2 \tilde{z}_3) z_i'}{1 - z_i \tilde{z}_i} &= \left(\pm 1 - \frac{e^A}{g} A'\right)\left(z_i + \frac{\tilde{z}_1 \tilde{z}_2 \tilde{z}_3}{\tilde{z}_i}\right), \\
\frac{e^A}{g} \frac{(1 + z_1 z_2 z_3) \tilde{z}_i'}{1 - z_i \tilde{z}_i} &= \left(\mp 1 - \frac{e^A}{g} A'\right)\left(\tilde{z}_i + \frac{z_1 z_2 z_3}{z_i}\right),
\end{aligned} \tag{7}$$

where the upper (lower) choices of sign correspond to using Killing spinors that are proportional to the left-invariant (right-invariant) Killing spinors on the S^3 [10]. Note that both sign choices are related by the exchange $z_i \leftrightarrow \tilde{z}_i$. The BPS equations (7) are generically not invariant under the $\mathbb{C}_i$ global scaling symmetries of the bulk action unless two out of the three scalars z_i (equivalently for $\tilde{z}_i$) are set to zero. While the general flow solutions we will present in this note are nonsupersymmetric, and therefore satisfy (4) without satisfying (7), we will identify a particular case for which the BPS equations (7) are additionally satisfied. This supersymmetric case precisely turns off two out of the three complex scalars, i.e., $z_2 = z_3 = \tilde{z}_2 = \tilde{z}_3 = 0$, so that the remaining $\mathbb{C}_1$ scaling symmetry is also a symmetry of the BPS equations in the bulk.

All the flow solutions we will construct asymptote to the maximally (super) symmetric $EAdS_4$ vacuum of the SO(8) supergravity. This vacuum solution uplifts to the $EAdS_4 \times S^7$ Freund–Rubin background of (Euclidean) 11-dimensional supergravity with a round metric on the S^7 [25] and is dual to the superconformal ABJM theory [9] placed on S^3. This vacuum sits at the origin of field space, namely, $z_i = \tilde{z}_i = 0$, and all our solutions will reach it at $|\mu| \to \infty$ with a falloff for the scalars of the form

$$\begin{aligned}
z_i &= a_i e^{-g|\mu|} + b_i e^{-2g|\mu|} + \ldots, \\
\tilde{z}_i &= \tilde{a}_i e^{-g|\mu|} + \tilde{b}_i e^{-2g|\mu|} + \ldots.
\end{aligned} \tag{8}$$

According to the AdS/CFT correspondence, the falloff coefficients (a_i, b_i) and $(\tilde{a}_i, \tilde{b}_i)$ in the expansions (8) are related to sources and VEVs in the field theory dual to the supergravity solution. However, the precise identification turns out to be very subtle as the combination $z_i - \tilde{z}_i$ should be quantized using regular boundary conditions, whereas alternate boundary conditions should be used for $z_i + \tilde{z}_i$. A careful analysis of the boundary limit of the bulk supersymmetry performed in [10] yielded the following dictionary. Sources in the dual field theory are identified with the combinations

$$a_i - \tilde{a}_i \quad \text{and} \quad \left(b_i - \frac{\tilde{a}_1 \tilde{a}_2 \tilde{a}_3}{\tilde{a}_i}\right) + \left(\tilde{b}_i - \frac{a_1 a_2 a_3}{a_i}\right), \tag{9}$$

whereas VEVs are given by

$$a_i + \tilde{a}_i \quad \text{and} \quad \left(b_i - \frac{\tilde{a}_1 \tilde{a}_2 \tilde{a}_3}{\tilde{a}_i}\right) - \left(\tilde{b}_i - \frac{a_1 a_2 a_3}{a_i}\right). \tag{10}$$

This identification of conjugate variables (sources and VEVs) is achieved by selecting a holographic renormalization scheme compatible with supersymmetry. As stated in [10], the counterterms of holographic renormalization have a universal structure and must be valid for all solutions of the classical field equations of a given bulk theory. We will then follow the holographic renormalization prescription in [10] to compute the on-shell action and gravitational free energy of the supergravity solutions constructed in this work.

III. EUCLIDEAN SOLUTIONS

The simplest solution to the system of second order equations in (4) is given by a pure $EAdS_4$ vacuum of the form

$$EAdS_4: \; e^{2A} = \sinh^2(g\mu) \quad \text{and} \quad z_i = \tilde{z}_i = 0. \tag{11}$$

This solution is maximally supersymmetric within the full $\mathcal{N}=8$ supergravity of [7], so it satisfies the BPS equations in (7). As mentioned before, it uplifts to the $EAdS_4 \times S^7$ vacuum solution of (Euclidean) 11-dimensional supergravity with a round metric on the S^7 [25] and is dual to the superconformal ABJM theory [9] on S^3.

A. Solutions with a nonbackreacted geometry

Let us start by presenting a class of solutions with a nonbackreacted $EAdS_4$ geometry and nontrivial profiles for the scalar fields. This is possible in a spacetime with Euclidean signature because z_i and $\tilde{z}_i$ are independent fields, and only the products of z_i and $\tilde{z}_i$ (and also of their derivatives) couple to the metric A function in the Einstein equation [first equation in (4)]. Therefore, nonbackreacted solutions exist provided $z_i \tilde{z}_i = 0 \; \forall \; i$.

The simplest case involves just one nonzero scalar, z_i or $\tilde{z}_i$, and the rest vanishing. For concreteness, let us consider z_1 or $\tilde{z}_1$ being nonzero and $z_2 = z_3 = \tilde{z}_2 = \tilde{z}_3 = 0$. Within this setup, there is a singular solution given by

$$e^{2A} = \sinh^2(g\mu), \qquad z_1 = 0, \qquad \tilde{z}_1 = \frac{\tilde{c}_1}{\sinh^2\left(\frac{1}{2} g\mu\right)}, \tag{12}$$

and a regular solution given by

$$e^{2A} = \sinh^2(g\mu), \qquad z_1 = \frac{c_1}{\cosh^2\left(\frac{1}{2} g\mu\right)}, \qquad \tilde{z}_1 = 0. \tag{13}$$

These solutions satisfy the second order equations in (4) as well as the BPS equations (7) for the upper choice of sign therein. Therefore, they are supersymmetric solutions. Moreover, if we demand a nonbackreacted EAdS_4 geometry, these two solutions are the only BPS solutions of the system with the two fields z_1 and $\tilde{z}_1$ and the upper sign choice in (7).

Some comments about the solutions (12) and (13) are in order. Firstly, the exchange $z_1 \leftrightarrow \tilde{z}_1$ in (12) and (13) is a symmetry of the second order equations of motion (4) but not of the BPS equations (7): The exchange $z_1 \leftrightarrow \tilde{z}_1$ amounts to a different sign choice in the BPS equations (7) so that the preserved supersymmetries are constructed using either left-invariant or right-invariant Killing spinors on the S^3. Secondly, solutions involving just one pair of fields $(z_i, \tilde{z}_i)$ like, for example, the pair $(z_1, \tilde{z}_1)$ in the solutions above, have an enhancement of symmetry from the $\mathrm{U}(1)^4$ generic symmetry of the STU model to an $\mathrm{SU}(2)^4 \sim \mathrm{SO}(4) \times \mathrm{SO}(4) \subset \mathrm{SO}(8)$ symmetry. This $\mathrm{SO}(4) \times \mathrm{SO}(4)$ symmetry becomes manifest when the solutions are uplifted to backgrounds of 11D supergravity using the formulas in Appendix A 1. The internal geometry is of the form $S^7 = \mathcal{I} \times S^3_1 \times S^3_2$ with the two three-spheres $S^3_{1,2}$ being responsible for the residual $\mathrm{SO}(4) \times \mathrm{SO}(4)$ symmetry of the solutions. Thirdly, from (A3)–(A6) and (A7)–(A11), it becomes clear that $c_1 \in \mathbb{R}$ renders the 11D metric real and the four-form flux purely imaginary. Also, the unit-disk normalization condition along the flows requires $|z_1(0)| = |c_1| < 1$.

Since the bulk Lagrangian in (1) and (2) does not mix the different scalars, the above solutions can be generalized to the STU model. However, once generalized to the STU model, they become solutions of the second order equations (4) but no longer of the BPS equations (7). Therefore, they turn into nonsupersymmetric solutions. The singular solution in (12) is straightforwardly generalized to the STU model as

$$e^{2A} = \sinh^2(g\mu), \qquad z_i = 0, \qquad \tilde{z}_i = \frac{\tilde{c}_i}{\sinh^2\left(\frac{1}{2} g\mu\right)}, \tag{14}$$

whereas the regular solution in (13) has a generalization of the form

$$e^{2A} = \sinh^2(g\mu), \qquad z_i = \frac{c_i}{\cosh^2\left(\frac{1}{2} g\mu\right)}, \qquad \tilde{z}_i = 0. \tag{15}$$

We will further study the regular solution (15) and compute its gravitational on-shell action and free energy as a function of the parameters $c_i \in \mathbb{R}$. Note that the unit-disk normalization condition along the scalars flow implies $|z_i(0)| = |c_i| < 1 \; \forall \; i$. The uplift of the regular solution (15) to 11-dimensional supergravity requires one to generalize the Appendix A 1 to the full STU model as done in [26]. This goes beyond the scope of this paper.

B. Solutions with a backreacted geometry

Inspired by the Janus solutions of [23], let us minimally modify the EAdS_4 geometry in (11) and present two new classes of analytic and multiparametric Euclidean solutions of the STU model. These two classes of solutions are generically nonsupersymmetric, although, as we will see, a supersymmetric limit can still be considered.

1. Soliton solutions

In the first class of solutions, the spacetime metric (3) is specified by a function

$$e^{2A} = \frac{\sinh^2(g\mu)}{k^2}, \tag{16}$$

with $k^2 > 0$ so that the metric is real and Euclidean. Whenever $k^2 \neq 1$, the spacetime metric possesses a singularity at $\mu = 0$, as can be seen from the scalar curvature $R = 6g^2(k^2 - \cosh(2g\mu))/\sinh^2(g\mu)$. This restricts the range of the solution to $\mu \in \mathbb{R}^+$, so we will refer to these solutions as *solitons*.

The metric function in (16) requires both z_i and $\tilde{z}_i$ scalars to flow simultaneously. The profiles for these fields are of the form

$$z_i = \frac{\lambda_i \sqrt{k_i^2 - 1}}{k_i + \cosh(g\mu)}, \qquad \tilde{z}_i = \frac{\lambda_i^{-1} \sqrt{k_i^2 - 1}}{k_i - \cosh(g\mu)}, \tag{17}$$

with arbitrary parameters λ_i and k_i subject to the condition

$$\sum_{i=1}^{3} k_i^2 = k^2 + 2. \tag{18}$$

This class of solutions has a metric singularity at $\mu = 0$ together with a divergence of the scalars whenever $k_i^2 > 1$. However, such a scalar divergence can be eliminated by taking $k_i^2 \leq 1$ with the constraint $\sum k_i^2 > 2$ so that $k^2 > 0$ in (18). The unit-disk normalization condition then fixes $|\lambda_i|^{-2} = \frac{1-k_i}{1+k_i}$ for the scalar flows to reach the boundary of the unit disk at $\mu = 0$, i.e., $|z_i(0)| = |\tilde{z}_i(0)| = 1$. These singular solutions are very much like the Lorentzian flows to Hades in [23].

Interestingly, when two out of the three k_i^2 in (17) are set to unity, one is left with a singular but supersymmetric solution with $\mathrm{SO}(4) \times \mathrm{SO}(4) \subset \mathrm{SO}(8)$ residual symmetry. For example, setting $k_2^2 = k_3^2 = 1$ reduces (18) to $k_1^2 = k^2$ and yields

$$z_1 = e^{i\beta} \frac{k+1}{k + \cosh(g\mu)}, \qquad \tilde{z}_1 = e^{-i\beta} \frac{k-1}{k - \cosh(g\mu)}, \tag{19}$$

with arbitrary $\beta \in \mathbb{R}$ and $0 < k^2 < 1$. The solution (19) satisfies the BPS equations (7). Unfortunately, when uplifted to a background of 11D supergravity using the formulas in Appendix A 1, the singularity at $\mu = 0$ persists as it happened for the dual of the Coulomb branch flows investigated in [23]. More concretely, the 11D Ricci scalar goes as $\hat{R} \propto \mu^{-4/3}$ around $\mu = 0$. It would be interesting to further investigate the nature of this supersymmetric singularity and its potential implications.

2. *Wormhole solutions*

The second class of solutions has a regular spacetime metric (3) specified by the metric function

$$e^{2A} = \frac{\cosh^2(g\mu)}{k^2}, \tag{20}$$

with $k^2 > 0$ so that the metric is real and Euclidean, and $\mu \in \mathbb{R}$. The radius of the S^3 in the spacetime geometry (3) reaches a minimum size $(gk)^{-1}$ at $\mu = 0$, and so we will refer to this second class of solutions as *wormholes*.

The metric function in (20) requires both z_i and $\tilde{z}_i$ to flow with profiles

$$z_i = \frac{\lambda_i \sqrt{k_i^2 + 1}}{k_i - \sinh(g\mu)}, \qquad \tilde{z}_i = \frac{\lambda_i^{-1} \sqrt{k_i^2 + 1}}{k_i + \sinh(g\mu)}, \tag{21}$$

with arbitrary parameters λ_i and k_i subject to

$$\sum_{i=1}^{3} k_i^2 = k^2 - 2. \tag{22}$$

Importantly, when $0 < k^2 < 2$, the condition (22) has solutions of the form $k_i = i\mathbf{k}_i$ with $\mathbf{k}_i \in \mathbb{R}$. The scalars are nonsingular and *necessarily* complex in this case and read

$$z_i = \frac{\lambda_i \sqrt{1 - \mathbf{k}_i^2}}{i\mathbf{k}_i - \sinh(g\mu)}, \qquad \tilde{z}_i = \frac{\lambda_i^{-1} \sqrt{1 - \mathbf{k}_i^2}}{i\mathbf{k}_i + \sinh(g\mu)}. \tag{23}$$

As a result, whenever the real vector $\vec{\mathbf{k}} \equiv (\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ (with $\mathbf{k}_i \neq 0 \;\forall\; i$) specifies a point on a two-sphere of radius $(2 - k^2) > 0$, the corresponding solution is fully regular. However, the unit-disk parametrization of the scalars still requires $|z_i|, |\tilde{z}_i| < 1$ all along the scalar flow. Provided this normalization condition holds, the solution describes an Euclidean wormhole within the STU model in the SO(8) gauged supergravity. Lastly, the supersymmetric limit of setting two out of the three $\mathbf{k}_i^2$ to unity in (23) is no longer compatible with the condition (22), which reduces in this case to $-\mathbf{k}_1^2 = k^2$.

Example: A simple class of regular Euclidean wormholes follows from the identification $\mathbf{k}_i^2 = (2 - k^2)/3 \;\forall\; i$. In this case, one has

$$|\lambda_i|^{-2} |z_i(\mu)|^2 = |\lambda_i|^2 |\tilde{z}_i(\mu)|^2 = \frac{2(1 + k^2)}{(1 - 2k^2) + 3\cosh(2g\mu)}. \tag{24}$$

Since $0 < k^2 < 2$, the right-hand side of (24) is regular and has a maximum at $\mu = 0$. Then the unit-disk normalization condition requires

$$|z_i(0)|^2 = |\lambda_i|^2 \left(\frac{3}{2 - k^2} - 1 \right) < 1,$$
$$|\tilde{z}_i(0)|^2 = |\lambda_i|^{-2} \left(\frac{3}{2 - k^2} - 1 \right) < 1. \tag{25}$$

Setting for simplicity $\lambda_i = e^{i\beta} \;\forall\; i$, with arbitrary $\beta \in \mathbb{R}$, the scalars take the final form

$$z_i = -\tilde{z}_i^* = e^{i\beta} \frac{\sqrt{1 + k^2}}{i\sqrt{2 - k^2} - \sqrt{3}\sinh(g\mu)}, \tag{26}$$

are regular, and satisfy $\tilde{z}_i = -z_i^* \in \mathbb{C}$. Moreover, they are compatible with the unit-disk parametrization condition whenever $0 < k^2 < \frac{1}{2}$. Note also that both the *dilatonic* $z_i + \tilde{z}_i$ and *axionic* $z_i - \tilde{z}_i$ combinations run nontrivially in this example.

Finally, it is also worth mentioning that solutions with $z_1 = z_2 = z_3$ (same for $\tilde{z}_i$), like the one in (26), feature an enhancement of symmetry from the generic $\mathrm{U}(1)^4$ symmetry of the STU model to an $\mathrm{SU}(3) \times \mathrm{U}(1)^2 \subset \mathrm{SO}(8)$ symmetry. However, supersymmetry is not preserved in the bulk as the BPS equations (7) are not satisfied. These solutions are straightforwardly uplifted to Euclidean solutions of 11-dimensional supergravity using the uplift formulas in Appendix A 2: The result is a complex metric and a complex (not purely imaginary) four-form flux in 11D.

The complex nature of Euclidean solutions should not be surprising. Metrics with real and imaginary parts are already encountered in the Wick rotation of spinning black holes. Also, the Wick rotation of the electrically charged four-dimensional Reissner-Nordström-AdS black hole has a purely imaginary gauge field that yields a complex metric when uplifted to 11 dimensions. Lastly, black rings do not even have everywhere regular real Euclidean sections [27].

IV. ON-SHELL ACTION AND GRAVITATIONAL FREE ENERGY

The evaluation of the gravitational on-shell action gives rise to divergences due to the infinite volume in the integral of the bulk action (1). Such divergences are cured by applying the by now standard procedure of holographic renormalization [28]. However, the holographic renormalization procedure comes along with an ambiguity: Local finite counterterms can be added to the renormalized action. This reflects the scheme dependence of the renormalization procedure in quantum field theory (see Ref. [29] for the identification of the supersymmetric scheme in a five-dimensional gravitational context). As concluded in [10] after the study of the projection of the bulk supersymmetry into the boundary, the identification of sources and VEVs in (9)–(10) requires one to include specific cubic counterterms in the holographic renormalization procedure. The inclusion of such cubic counterterms was shown to be crucial to have a matching between the gravitational free energy of the supergravity flows constructed in [10] and the free energy of the dual ABJM theory on S^3 deformed with supersymmetric real mass terms computed using localization methods.

The multiparametric families of Euclidean solutions we have constructed in the previous section belong to the same bulk theory as the solutions constructed in [10]. Moreover, the solutions here possess a supersymmetric limit whenever two of the scalars z_i (same for $\tilde{z}_i$) can be smoothly set to vanish upon tuning of the free parameters. This makes us follow the same renormalization prescription of [10] and include the same (finite) cubic counterterms when renormalizing the on-shell action. The result is an on-shell action of the form

$$2\kappa^2 S_{\text{on-shell}} = 2\kappa^2 S_{\text{bulk}} + S_{\text{GH}} + S_a + S_z, \tag{27}$$

with S_{bulk} given in (1), that contains a set of boundary terms

$$\begin{aligned} S_{\text{GH}} &= -\int_{\partial M} d^3x\sqrt{h}K, \\ S_a &= \frac{1}{2g}\int_{\partial M} d^3x\sqrt{h}R(h), \\ S_z &= 2\int_{\partial M} d^3x\sqrt{h}[e^{\mathcal{K}}W\tilde{W}]^{\frac{1}{2}} \\ &= g\int_{\partial M} d^3x\sqrt{h}\left(2+\sum_i z_i\tilde{z}_i+\prod_i z_i+\prod_i \tilde{z}_i+\cdots\right). \end{aligned} \tag{28}$$

The above boundary terms include the standard counterterms that follow from the near-boundary analysis of a generic supergravity solution [see e.g., [28]), the specific cubic counterterm discussed in [10] [i.e., last line of (28)], and additional terms—denoted by the ellipsis—which vanish at the boundary when taking the limit $|\mu| \to \infty$. The quantity K entering S_{GH} is the trace of the extrinsic curvature defined as $K_{ab} = \nabla_{(a}N_{b)} = \frac{1}{2}\mathcal{L}_N h_{ab} = \pm\frac{1}{2}h'_{ab}$. Here, $N_a = \pm\delta_a^\mu$ is the vector normal to the boundary ∂M, and $h_{ab} = g_{ab} - N_a N_b$ is the induced metric [30]. The quantity $R(h)$ entering S_a is the Ricci scalar of this metric. Finally, the functions $\mathcal{K}$, W, and $\tilde{W}$ are the Kähler potential and superpotentials given in (5) and (6). We note in passing that the cubic counterterms $z_1z_2z_3$ and $\tilde{z}_1\tilde{z}_2\tilde{z}_3$ in the last line of (28) break the $\mathbb{C}_i$ scaling symmetries of the bulk action (1).

Starting from the on-shell action (27), the computation of the gravitational free energy, $J_{\text{on-shell}}$, requires a precise identification of sources and VEVs from the scalar expansions (8) around the boundary. The reason why is that a Legendre transformation of $S_{\text{on-shell}}$ might be needed for $J_{\text{on-shell}}$ to depend on the particular combinations of leading and subleading coefficients in (8) specifying the sources in the dual field theory. With the identification of sources given in (9), the gravitational free energy takes the form

$$J_{\text{on-shell}} = S_{\text{on-shell}} + \frac{1}{2\kappa^2}\Delta S, \tag{29}$$

with

$$\Delta S = -\frac{1}{2}\sum_i \int_{S^3} dx^3 (a_i+\tilde{a}_i)\left(\frac{\delta S_{\text{on-shell}}}{\delta a_i}+\frac{\delta S_{\text{on-shell}}}{\delta \tilde{a}_i}\right). \tag{30}$$

It is precisely at this point where the finite cubic counterterms entering S_z in (28) play a crucial role as they guarantee that $\delta_{a_i}S_{\text{on-shell}}+\delta_{\tilde{a}_i}S_{\text{on-shell}} \propto (\tilde{b}_i - a_1a_2a_3/a_i) + (b_i - \tilde{a}_1\tilde{a}_2\times \tilde{a}_3/\tilde{a}_i)$, and, therefore, the gravitational free energy $J_{\text{on-shell}}$ is a function of the sources in (9).

A. Nonbackreacted EAdS$_4$ solutions

Let us recall the nonbackreacted regular solution in (15), namely,

$$e^{2A} = \sinh^2(g\mu), \qquad z_i = \frac{c_i}{\cosh^2\left(\frac{1}{2}g\mu\right)}, \qquad \tilde{z}_i = 0. \tag{31}$$

When reaching the boundary at $\mu = \infty$, the scalar fields feature the asymptotic expansions

$$z_i = -4c_i\sum_{n=1}^{\infty}(-1)^n n e^{-ng\mu} \quad \text{and} \quad \tilde{z}_i = 0. \tag{32}$$

A direct comparison between (8)–(10) and (32) shows that

$$a_i \mp \tilde{a}_i = 4c_i,$$
$$\left(b_i - \frac{\tilde{a}_1\tilde{a}_2\tilde{a}_3}{\tilde{a}_i}\right) \pm \left(\tilde{b}_i - \frac{a_1a_2a_3}{a_i}\right) = -16\left(\frac{c_i}{2} \pm \frac{c_1c_2c_3}{c_i}\right), \tag{33}$$

and thus, the sources in (9) and VEVs in (10) are generically activated if $c_i \neq 0$. The evaluation of the on-shell action (27) on the solution (31) gives

$$S_{\text{on-shell}}|_{\text{non-back}} = S^{\text{EAdS}_4}_{\text{on-shell}}\left(1 + 4\prod_i c_i\right), \tag{34}$$

where

$$S^{\text{EAdS}_4}_{\text{on-shell}} = \frac{2\pi^2}{\kappa^2 g^2}, \tag{35}$$

is the on-shell action of the EAdS_4 solution in (11). The first contribution to the rhs of (34) comes from the bulk term in (27), whereas the second c_i-dependent contribution originates from the nonzero cubic counterterms in S_z of (28). Without the cubic counterterms in the holographic renormalization procedure, the on-shell action of the solution (31) would be the same as that of EAdS_4. Also, from (34), it follows that the on-shell action of the non-backreacted solution (31) is the same as that of EAdS_4 whenever (at least) one of the scalars z_i vanishes.

The gravitational free energy of the solution (31) is given by (29) and (30) with

$$\frac{\delta S_{\text{on-shell}}}{\delta a_i} = -\frac{\sqrt{g_3}}{16g^2\kappa^2}\left(\tilde{b}_i - \frac{a_1a_2a_3}{a_i}\right),$$
$$\frac{\delta S_{\text{on-shell}}}{\delta \tilde{a}_i} = -\frac{\sqrt{g_3}}{16g^2\kappa^2}\left(b_i - \frac{\tilde{a}_1\tilde{a}_2\tilde{a}_3}{\tilde{a}_i}\right), \tag{36}$$

where g_3 is the determinant of the round metric on the three-sphere S^3 of unit radius. Substituting the a_i, $\tilde{a}_i$, b_i, and $\tilde{b}_i$ coefficients that follow from direct comparison between the general expansions in (8)–(10) and the ones in (32), the resulting gravitational free energy is given by

$$J_{\text{on-shell}}|_{\text{non-back}} = J^{\text{EAdS}_4}_{\text{on-shell}}\left(1 - \sum_i \mathcal{C}_i\right), \tag{37}$$

where

$$\mathcal{C}_i = c_i^2 + \frac{2}{3}c_1c_2c_3, \tag{38}$$

and

$$J^{\text{EAdS}_4}_{\text{on-shell}} = \frac{2\pi^2}{\kappa^2 g^2} \tag{39}$$

is the universal gravitational free energy of the EAdS_4 vacuum in (11). As a consequence of F-extremisation, the gravitational free energy in (37) is maximized at the conformal case, i.e. $c_i = 0\ \forall\ i$, so that

$$J_{\text{on-shell}}|_{\text{non-back}} < J^{\text{EAdS}_4}_{\text{on-shell}}, \tag{40}$$

provided the unit-disk condition $|c_i| < 1$ holds.

Let us comment on the gravitational free energy we have obtained. For generic values of the parameters c_i, the solution (31) is nonsupersymmetric and, in principle, one could consider a renormalization scheme different from the supersymmetric one. For example, if applying the minimal subtraction scheme that follows from the near-boundary analysis–so no finite cubic counterterms are included—the gravitational free energy changes to $J^{\text{EAdS}_4}_{\text{on-shell}}(1 - \sum_i c_i^2)$ with no mixing between the z_i's. Positivity of this free energy further restricts the range of the parameters c_i to lie inside the unit radius c_i ball defined by the condition $\sum_i c_i^2 < 1$. In this paper, and in order to make contact with [10] in the supersymmetric case, we have applied the supersymmetric scheme, and, therefore, the gravitational free energy in (37) depends on the finite cubic counterterms in S_z of (28): The cubic term in the rhs of (38) stems from such finite counterterms. Positivity of (37) then requires the parameters c_i to lie inside a deformed c_i ball with four lumps defined by the cubic condition $\sum_i c_i^2 + 2c_1c_2c_3 < 1$.

1. Supersymmetric limit and holography

Setting two c_i to zero in the solution (31), i.e., $c_2 = c_3 = 0$, reduces it to the supersymmetric and $\text{SO}(4) \times \text{SO}(4) \sim \text{SU}(2)^4$ invariant solution in (13), namely,

$$e^{2A} = \sinh^2(g\mu), \qquad z_1 = \frac{c_1}{\cosh^2\left(\frac{1}{2}g\mu\right)}, \qquad \tilde{z}_1 = 0. \tag{41}$$

The general gravitational free energy in (37) reduces in this case to

$$J_{\text{on-shell}}|_{\text{SO}(4)\times\text{SO}(4)} = J^{\text{EAdS}_4}_{\text{on-shell}}(1 - c_1^2). \tag{42}$$

The solution in (41) lies precisely at the intersection between the general class of nonsupersymmetric solutions in (31) and the class of $\mathcal{N} = 2$ flows put forward in [10] describing the holography of F maximization [31]. The uplift of (41) to 11D supergravity was performed in [32] and agrees with the general uplift formulas for the $\text{SO}(4) \times \text{SO}(4)$ invariant sector collected in Appendix A 1.

Following [10], let us look at the field theory dual of the solution (41). This is the ABJM superconformal field theory (SCFT) of [9] placed on (a unit radius) S^3 and deformed with a specific supersymmetric real mass parameter δ_1. In general, there are three such real mass parameters δ_i ($i = 1, 2, 3$) compatible with $\mathcal{N} = 2$ supersymmetry that modify the assignment of $\mathrm{U}(1)_R$ R charges. The real mass parameters δ_i break conformality and also the $\mathrm{U}(1)_R \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_b \subset \mathrm{SO}(8)_R$ symmetry manifest in the $\mathcal{N} = 2$ superfield formulation of the (undeformed) ABJM SCFT down to its Cartan subgroup $\mathrm{U}(1)_R \times \mathrm{U}(1) \times \mathrm{U}(1) \times \mathrm{U}(1)_b \subset \mathrm{SO}(8)_R$.

In the $\mathcal{N} = 2$ formulation of [9], ABJM theory is a SCFT with gauge group $\mathrm{U}(N) \times \mathrm{U}(N)$ and Chern–Simons (CS) levels $(k, -k)$—in our case, $k = 1$, and the internal space in the gravity side is the round S^7. The theory consists of two vectors multiplets (A, σ, λ, D) and $(\tilde{A}, \tilde{\sigma}, \tilde{\lambda}, \tilde{D})$ for the $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge group together with four chiral matter multiplets (Z^a, χ^a, F^a) and (W_a, η_a, G_a), with $a = 1, 2$, transforming in the $(\bar{\mathbf{N}}, \mathbf{N})$ and $(\mathbf{N}, \bar{\mathbf{N}})$ representations of the gauge group. There is also a quartic superpotential for the matter multiplets which has R charge 2 and is invariant under the $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_b$ flavor symmetry. It is given by

$$W \propto \mathrm{Tr}(\epsilon_{ab}\epsilon^{cd} Z^a W_c Z^b W_d). \tag{43}$$

Particularizing the Lagrangian in [10] to describe, holographically, the solution in (41), one finds

$$\mathcal{L} = \mathcal{L}_{\mathrm{SCFT}} + \delta_1 [\mathcal{O}_B^1 - \delta_1 \mathcal{O}_S] + \delta_1 \mathcal{O}_F^1, \tag{44}$$

where $\mathcal{L}_{\mathrm{SCFT}}$ is the ABJM SCFT Lagrangian, and δ_1 is a specific supersymmetric real mass parameter that turns on bosonic and fermionic operators. In particular, it turns on the bosonic $\mathcal{O}_B^1$ and fermionic $\mathcal{O}_F^1$ operators

$$\begin{aligned} \mathcal{O}_B^1 &= \mathrm{tr}[Z_a^\dagger Z^a - W^{\dagger a} W_a], \\ \mathcal{O}_F^1 &= \mathrm{tr}[\chi^{\dagger a}\chi_a - \eta_a^\dagger \eta^a] + 2i(\sigma - \tilde{\sigma})\mathcal{O}_S, \end{aligned} \tag{45}$$

dual to the proper scalar $z_1 + \tilde{z}_1$ and the pseudoscalar $z_1 - \tilde{z}_1$ in the SO(8) supergravity, together with an additional Konishi-like bosonic operator

$$\mathcal{O}_S = \mathrm{tr}[Z_a^\dagger Z^a + W^{\dagger a} W_a], \tag{46}$$

which does not have an associated scalar in the SO(8) supergravity [33]. Remarkably, among the operators with a dual spinless field in the SO(8) supergravity, those in (45) are the only two singlets under the $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_b$ flavor symmetry of the ABJM SCFT in the $\mathcal{N} = 2$ formulation: Under this symmetry, the chiral multiplets (Z^a, χ^a, F^a) and (W_a, η_a, G_a) transform in the $(\mathbf{2}, \mathbf{1})_1$ and $(\mathbf{1}, \bar{\mathbf{2}})_{-1}$ representations, respectively [34].

The parameter δ_1 in (44) determines the assignment of $\mathrm{U}(1)_R$ R charges for the scalar component of the chiral superfields to be

$$R[Z^1] = R[Z^2] = \frac{1}{2} + \delta_1, \qquad R[W_1] = R[W_2] = \frac{1}{2} - \delta_1. \tag{47}$$

On the other hand, the free energy of the ABJM theory on S^3 deformed with $\mathcal{N} = 2$ supersymmetric real mass terms can be obtained as a function of a set of trial R charges using localization methods [31,35]. To leading order in N and for CS level $k = 1$, the result is [4]

$$\mathcal{F} = \mathcal{F}_0 \sqrt{16 R[Z^1] R[Z^2] R[W_1] R[W_2]}, \tag{48}$$

with $\mathcal{F}_0 = \sqrt{2}\pi N^{3/2}/3$ and where the Rs are the trial charges of the four chiral matter fields in the theory. The superpotential of the undeformed ABJM SCFT theory (43) is quartic on the matter fields and has R charge 2. This implies that $R[Z^1] + R[Z^2] + R[W_1] + R[W_2] = 2$, a rule that is obeyed by the R charges in (47). A direct substitution of the R charges (47) into (48) yields

$$\mathcal{F}_{\mathrm{SO}(4)\times\mathrm{SO}(4)} = \mathcal{F}_0 (1 - (2\delta_1)^2), \tag{49}$$

which matches the gravitational free energy in (42) for the supersymmetric $\mathrm{SO}(4) \times \mathrm{SO}(4)$ invariant solution (41) provided

$$\mathcal{F}_0 = J^{\mathrm{EAdS}_4}_{\text{on-shell}}, \tag{50}$$

and the identification $c_1 = 2\delta_1$. The extremization of the free energy in (49) fixes $\delta_1 = 0$ and assigns canonical R charges of $\frac{1}{2}$ to the four chiral fields in (47). The free energy in (49) then reduces to $\mathcal{F}_0$, which, using the holographic mapping between κ^2 and N in the ABJM SCFT at CS level $k = 1$, matches the gravitational free energy of the EAdS_4 vacuum in (39).

B. Backreacted wormhole solutions

Let us now investigate the backreacted solution in (20) and (23). As we demonstrated with the example of Sec. III B 2, the various parameters in the solution can be chosen such that the solution is regular in the domain $\mu \in (-\infty, \infty)$, and the scalars stay within the unit disk all along the flow. Having one of these regular and classically well-defined solutions in mind, we will proceed an compute its on-shell action.

The boundary of spacetime now consists of two pieces located at $\mu = \pm\infty$. The asymptotic expansions of the scalars in (23) around $\mu = \pm\infty$ yield

$$\begin{aligned} z_i &= 2\lambda_i \sqrt{1 - \mathbf{k}_i^2}(\pm e^{-g|\mu|} - 2i\mathbf{k}_i e^{-2g|\mu|} + \cdots), \\ \tilde{z}_i &= 2\lambda_i^{-1} \sqrt{1 - \mathbf{k}_i^2}(\mp e^{-g|\mu|} - 2i\mathbf{k}_i e^{-2g|\mu|} + \cdots), \end{aligned} \tag{51}$$

where the upper (lower) sign corresponds to the piece of the boundary at $\mu = +\infty$ ($\mu = -\infty$). Using the asymptotic expansions in (51) to evaluate the boundary terms, the on-shell action (27) for the wormholes takes the form

$$S_{\text{on-shell}}|_{\text{WH}} = S_{\text{cubic}}|_{\partial M_+} + S_{\text{cubic}}|_{\partial M_-} = 0. \tag{52}$$

The finite contributions coming from the cubic counterterms in S_z of (28) at the two pieces $\partial M_\pm$ (respectively, at $\mu = \pm\infty$) of the boundary read

$$S_{\text{cubic}}|_{\partial M_\pm} = \mp \frac{2\pi^2}{g^2 k^3}\left(\Lambda - \frac{1}{\Lambda}\right)\prod_i \sqrt{1 - \mathbf{k}_i^2}, \tag{53}$$

with $\Lambda \equiv \prod_j \lambda_j$. Except for the finite contributions in (53), all the other contributions to the on-shell action (27) cancel separately on each piece of the boundary of the wormhole.

Using the boundary expansions for the scalar fields in (51), one finds that

$$\frac{\delta S_{\text{on-shell}}}{\delta a_i} = -\frac{\sqrt{g_3}}{16 g^2 \kappa^2 k^3}\left(\tilde{b}_i - \frac{a_1 a_2 a_3}{a_i}\right),$$
$$\frac{\delta S_{\text{on-shell}}}{\delta \tilde{a}_i} = -\frac{\sqrt{g_3}}{16 g^2 \kappa^2 k^3}\left(b_i - \frac{\tilde{a}_1 \tilde{a}_2 \tilde{a}_3}{\tilde{a}_i}\right), \tag{54}$$

and an explicit computation of (30) for the wormholes in (20) and (23) yields $\Delta S|_{\partial M_+} = -\Delta S|_{\partial M_-}$. As a result, the total gravitational free energy vanishes again due to an exact cancellation between the two pieces of the wormhole boundary. Namely,

$$J_{\text{on-shell}}|_{\text{WH}} = J_{\text{on-shell}}|_{\partial M_+} + J_{\text{on-shell}}|_{\partial M_-} = 0. \tag{55}$$

Importantly, this result is independent of the holographic renormalization scheme, as any other choice of cubic counterterms would have also resulted in a null contribution to the gravitational free energy—and also to the on-shell action—due to the antisymmetry of the scalar boundary conditions (51) at $\mu = \pm\infty$.

V. CONCLUSIONS

In this paper, we have constructed new classes of $\text{U}(1)^4$-invariant Euclidean solutions in the four-dimensional STU model (with vectors turned off) that arises upon compactification of 11D supergravity on S^7. Together with some potentially pathological examples featuring spacetime singularities—like the solitons in Sec. III B 1—we presented two classes of fully regular multiparametric Euclidean solutions.

The first class describes flows with running scalars z_i ($i = 1, 2, 3$) but a nonbackreacted EAdS_4 spacetime. This is possible in Euclidean signature, where the scalars z_i and their would-be conjugates $\tilde{z}_i$ are independent fields so that a nontrivial solution with $z_i \tilde{z}_i = 0 \; \forall \; i$ has a vanishing energy-momentum tensor. We constructed the three-parameter family of nonsupersymmetric solutions in (15) and computed its on-shell action and gravitational free energy employing holographic renormalization methods. For the particular choice of parameters $c_i \neq 0$ and $c_j = c_k = 0$ in (15), with $i \neq j \neq k$, the solution features an $\text{SO}(4) \times \text{SO}(4)$ symmetry enhancement, becomes supersymmetric, and belongs to the class of flows put forward in [10]. In this supersymmetric limit, the gravitational free energy was shown to match that of ABJM placed on S^3 and deformed with a specific real mass term, which, in the $\mathcal{N} = 2$ formulation of the theory, preserves the full $\text{SU}(2) \times \text{SU}(2) \times \text{U}(1)_b$ flavor symmetry group. The embedding of this supersymmetric solution into 11D supergravity showed that, when the profile for the running scalar is taken to be real, i.e., $c_i \in \mathbb{R}$, the uplift to 11D produces a real metric and a purely imaginary four-form flux. This agrees with the standard statement that "axions" flip the sign of their kinetic term in the Euclidean theory.

The second class describes regular wormholes with scalar profiles satisfying $z_i \tilde{z}_i \neq 0$ and supporting the wormhole geometry. We constructed the multiparametric family of wormholes in (20) and (23) and showed that a supersymmetric limit is not possible for these solutions. We also computed the on-shell action and gravitational free energy of the wormholes and found that the latter always vanishes irrespectively of supersymmetry and the holographic renormalization scheme. It would be interesting to understand this result from a field-theoretic perspective. However, the absence of supersymmetry makes a holographic test of the zero gravitational free energy difficult.

Unlike for the nonbackreacted flow solutions, the scalars z_i and $\tilde{z}_i$ are necessarily complex in the wormholes. In the particular example with $\text{SU}(3) \times \text{U}(1)^2$ symmetry presented in Sec. III B 2, it happens that $\tilde{z}_i = -z_i^* \in \mathbb{C}$ (*not* $\tilde{z}_i = z_i^*$), but each pair $(z_i, \tilde{z}_i)$ still comprises two real degrees of freedom. In addition, the *a priori axionic* combination $z_i - \tilde{z}_i$ has a nontrivial profile providing an *axionic* charge to the wormhole. Upon uplift of this example to 11 dimensions, both the 11D metric and the four-form flux turn out to have nontrivial real and imaginary parts.

It is also interesting to contrast the class of wormholes presented in this note with the Maldacena–Maoz construction of Euclidean wormholes [36]. In the latter, the foliation of EAdS space is additionally compactified in order to generate the Euclidean wormhole. This compactification procedure imposes identifications that break supersymmetry even asymptotically. In our case, and despite being nonsupersymmetric, the wormholes asymptote supersymmetric EAdS vacua on both sides of the solution, thus ensuring a theory with a stable ground state. It would be interesting to further investigate the stability of the wormholes presented here as part of the swampland program and to assess their allowability in light of the recent discussion in [37]. Also, their fate in semiclassical (super)gravity—as an approximation to quantum gravity—deserves further investigations.

Finally, the class of wormholes presented in this note was inspired by (the Euclidean continuation of) the Lorentzian Janus solutions put forward in [23]. Supersymmetric Lorentzian Janus solutions with identifications that make them wormholelike (e.g., with nontrivial first homotopy group) have been constructed in [38,39] using vector fields instead of scalars. Their generalization to include nontrivial running scalars should be possible and could serve as a starting point to construct analytic families of supersymmetric Euclidean wormholes in M-theory. We hope to come back to this and related issues in the future.

ACKNOWLEDGMENTS

We are grateful to C. Hoyos, D. Rodríguez-Gómez, and P. Soler for interesting conversations. The work of A. A. is supported in part by the FAPESP visiting researcher Award 2022/11765-7 and the FONDECYT Grants No. 1200986, No. 1210635, No. 1221504, and No. 1230853. The work of A. G. is supported by the Spanish government Grant PGC2018-096894-B-100 and by the Principado de Asturias through Grant No. FC-GRUPIN-IDI/2018/000174.

APPENDIX: UPLIFTS TO 11D SUPERGRAVITY

The bosonic sector of 11-dimensional supergravity consists of a metric field $\hat{G}_{MN}$ and a three-form potential $\hat{A}_{(3)}$ with field strength $\hat{F}_{(4)} = d\hat{A}_{(3)}$. The Euclidean equations of motion read [12]

$$\begin{aligned} d(*_{11}\hat{F}_{(4)}) + \frac{i}{2}\hat{F}_{(4)} \wedge \hat{F}_{(4)} &= 0, \\ \hat{R}_{MN} - \frac{1}{12}\left((\hat{F}\hat{F})_{MN} - \frac{1}{12}(\hat{F}\hat{F})\hat{G}_{MN}\right) &= 0, \end{aligned} \tag{A1}$$

where we have denoted $(\hat{F}\hat{F})_{MN} \equiv \hat{F}_{MPQR}\hat{F}_N{}^{PQR}$ and $(\hat{F}\hat{F}) \equiv \hat{F}_{PQRS}\hat{F}^{PQRS}$. In addition, the field strength $\hat{F}_{(4)}$ satisfies the sourceless Bianchi identity

$$d\hat{F}_{(4)} = 0. \tag{A2}$$

In this appendix, we present the necessary formulas to uplift the simplest four-dimensional solutions discussed in the main text to Euclidean solutions of 11-dimensional supergravity. More concretely, we present the embedding of the $SO(4) \times SO(4)$ and $SU(3) \times U(1)^2$ invariant sectors of the maximal SO(8) gauged supergravity into 11D supergravity.

1. $SO(4) \times SO(4)$ invariant sector

When only one pair of scalars $(z_i, \tilde{z}_i)$ is nonzero—we will choose $(z, \tilde{z}) \equiv (z_1, \tilde{z}_1)$ without loss of generality—the corresponding solutions belong to the $SO(4) \times SO(4)$ invariant sector of the maximal SO(8) supergravity. The uplift of this sector to Lorentzian supergravity in 11 dimensions can be found in [40]. Here, we extend the uplift formulas therein to Euclidean signature and adapt it to the unit-disk parametrization of the scalar fields z and $\tilde{z}$ we are using in this work.

The 11D (warped) geometry is of the form $\mathcal{M}_4 \times S^7$ with $S^7 = \mathcal{I} \times S^3_1 \times S^3_2$. The two three-spheres $S^3_{1,2}$ in the internal geometry are responsible for the residual $SO(4) \times SO(4)$ symmetry of the solutions. Denoting $\alpha \in [0, \frac{\pi}{2}]$ the coordinate along the interval $\mathcal{I}$, the 11D metric is given by

$$ds^2_{11} = \Delta^2\left[ds^2_4 + \frac{4}{g^2}\left(d\alpha^2 + \frac{\cos^2\alpha}{f_1}ds^2_{S^3_1} + \frac{\sin^2\alpha}{f_2}ds^2_{S^3_2}\right)\right], \tag{A3}$$

where the external spacetime metric ds^2_4 is given in (3) and

$$ds^2_{S^3_1} = \frac{1}{4}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2), \qquad ds^2_{S^3_2} = \frac{1}{4}(\hat{\sigma}_1^2 + \hat{\sigma}_2^2 + \hat{\sigma}_3^2) \tag{A4}$$

denote the line elements on the two internal three-spheres. The warping factor in (A3) takes the form

$$\Delta^2 = f_1^{\frac{1}{3}} f_2^{\frac{1}{3}}, \tag{A5}$$

in terms of two functions

$$\begin{aligned} f_1 &= \cos^2\alpha\frac{(1+z)(1+\tilde{z})}{1-z\tilde{z}} + \sin^2\alpha, \\ f_2 &= \sin^2\alpha\frac{(1-z)(1-\tilde{z})}{1-z\tilde{z}} + \cos^2\alpha. \end{aligned} \tag{A6}$$

The four-form field strength of 11D supergravity has both a spacetime and an internal piece, namely,

$$\hat{F}_{(4)} = \hat{F}^{\text{st}}_{(4)} + \hat{F}^{\text{tr}}_{(4)}. \tag{A7}$$

The spacetime part takes the form

$$\hat{F}^{\text{st}}_{(4)} = gh\,\text{vol}_4 + g^{-1}\sin(2\alpha)\tilde{h}^{(3)} \wedge d\alpha, \tag{A8}$$

in terms of a function h and a three-form $\tilde{h}^{(3)}$ given by

$$\begin{aligned} h &= i\frac{3 - z\tilde{z} + (z+\tilde{z})\cos(2\alpha)}{(1-z\tilde{z})}, \\ \tilde{h}^{(3)} &= i\frac{(1-z^2) *_4 d\tilde{z} + (1-\tilde{z}^2) *_4 dz}{(1-z\tilde{z})^2}. \end{aligned} \tag{A9}$$

In the above expression, $*_4$ denotes the Hodge dual with respect to the external spacetime metric (3). The internal part is given by

$$\hat{F}^{\rm tr}_{(4)} = d\hat{A}^{\rm tr}_{(3)} \quad \text{with} \quad \hat{A}^{\rm tr}_{(3)} = g^{-3} h_1 \, \text{vol}_1 + g^{-3} h_2 \, \text{vol}_2, \tag{A10}$$

in terms of the functions

$$h_1 = i\,8 \frac{\cos^4\alpha}{f_1} \frac{z - \tilde{z}}{1 - z\tilde{z}}, \qquad h_2 = i\,8 \frac{\sin^4\alpha}{f_2} \frac{z - \tilde{z}}{1 - z\tilde{z}}, \tag{A11}$$

and the volume forms on the two three-spheres $\text{vol}_{1,2}$. Note that $\hat{F}^{\rm tr}_{(4)}$ is nonzero whenever the *axionic* combination $z - \tilde{z}$ is nonzero.

Due to the consistency of the truncation, any solution in the $SO(4) \times SO(4)$ invariant sector of the SO(8) gauged supergravity is also a solution of the Euclidean equations of motion in (A1) and satisfies the Bianchi identity (A2). Note also that, under the exchange $z \to \tilde{z}$, the 11D geometry in (A3) remains invariant, whereas the four-form flux in (A7) changes as $\hat{F}^{\rm st}_{(4)} \to \hat{F}^{\rm st}_{(4)}$ and $\hat{F}^{\rm tr}_{(4)} \to -\hat{F}^{\rm tr}_{(4)}$.

2. $SU(3) \times U(1)^2$ invariant sector

When the three pairs $(z_i, \tilde{z}_i)$ of scalar fields in the STU model are identified, i.e., $(z, \tilde{z}) \equiv (z_i, \tilde{z}_i) \ \forall \ i$, the corresponding solutions belong to the $SU(3) \times U(1)^2$ invariant sector of the maximal SO(8) supergravity. The uplift of this sector to 11-dimensional supergravity has been worked out in [41] (see also [42] whose conventions we closely follow) in the case of Lorentzian signature. As before, we extend the uplift formulas therein to Euclidean signature and adapt it to the unit-disk parametrization of the scalar fields z and $\tilde{z}$ we have adopted.

The 11-dimensional (warped) geometry is of the form $\mathcal{M}_4 \times S^7$ with $S^7 = \mathcal{I} \times \mathbb{CP}_2 \times S^1_\tau \times S^1_\psi$. The $\mathbb{CP}_2$ and $S^1_\tau \times S^1_\psi$ factors in the internal geometry are responsible for the $SU(3) \times U(1)^2$ residual symmetry of the four-dimensional solutions. Denoting $\alpha \in [0, \frac{\pi}{2}]$ the coordinate along the interval $\mathcal{I}$, the 11D metric takes the form

$$\begin{aligned} d\hat{s}^2_{11} = \frac{1}{2} f_1 ds^2_4 + 2g^{-2}[&f_2 d\alpha^2 \\ &+ \cos^2\alpha (f_3 ds^2_{\mathbb{CP}_2} + \sin^2\alpha f_4 (d\tau + \sigma)^2) \\ &+ f_5 (d\psi + \cos^2\alpha f_6 (d\tau + \sigma))^2], \end{aligned} \tag{A12}$$

with $\tau, \psi \in [0, 2\pi]$. We have denoted σ the one-form on $\mathbb{CP}_2$ such that $d\sigma = 2\boldsymbol{J}$ with $\boldsymbol{J}$ being the Kähler form on $\mathbb{CP}_2$. The metric in (A12) is fully specified in terms of six functions $f_{1\ldots6}$ given by

$$\begin{aligned} f_1 &= (1 - z\tilde{z})^{-1} L^{\frac{1}{3}} H^{\frac{2}{3}}, \qquad f_2 = f_3^{-2} = H^{\frac{2}{3}} L^{-\frac{2}{3}}, \\ f_4 &= (1 - z\tilde{z})^2 L^{-\frac{2}{3}} H^{\frac{2}{3}} K^{-1}, \qquad f_5 = H^{-\frac{4}{3}} K L^{-\frac{2}{3}}, \\ f_6 &= [LH + (z - \tilde{z})^2 \cos(2\alpha)] K^{-1}, \end{aligned} \tag{A13}$$

with

$$\begin{aligned} H &= 1 + z\tilde{z} - (z + \tilde{z}) \cos(2\alpha), \\ K &= 1 + (z\tilde{z})^2 - 2z\tilde{z} \cos(4\alpha), \\ L &= (1 + z)(1 + \tilde{z}). \end{aligned} \tag{A14}$$

The four-form field strength of 11D supergravity consists of a spacetime and an internal piece

$$\hat{F}_{(4)} = \hat{F}^{\rm st}_{(4)} + \hat{F}^{\rm tr}_{(4)}. \tag{A15}$$

The spacetime part reads

$$\hat{F}^{\rm st}_{(4)} = g\, h_1 \, \text{vol}_4 + g^{-1} \sin(2\alpha)\, h_2^{(3)} \wedge d\alpha, \tag{A16}$$

in terms of a zero-form h_1 and a three-form $h_2^{(3)}$ given by

$$\begin{aligned} h_1 &= i \frac{3(1 + z\tilde{z}) + (z + \tilde{z})(1 - 2\cos(2\alpha))}{2\sqrt{2}(1 - z\tilde{z})}, \\ h_2^{(3)} &= i \frac{(1 - z^2) *_4 d\tilde{z} + (1 - \tilde{z}^2) *_4 dz}{\sqrt{2}(1 - z\tilde{z})^2}. \end{aligned} \tag{A17}$$

Again, $*_4$ denotes the Hodge dual with respect to the external spacetime metric (3). Lastly, the internal part is given by

$$\begin{aligned} \hat{F}^{\rm tr}_{(4)} = 2\sqrt{2} g^{-3} [&\sin(2\alpha) h_3^{(1)} \wedge d\alpha \wedge d\psi \wedge (d\tau + \sigma) \\ &+ \cos^4\alpha h_4^{(1)} \wedge (d\tau + \sigma) \wedge \boldsymbol{J} \\ &+ \cos^2\alpha \cos(2\alpha) h_5^{(1)} \wedge d\psi \wedge \boldsymbol{J} \\ &+ \cos^2\alpha \sin(2\alpha) h_6 d\alpha \wedge d\psi \wedge \boldsymbol{J} + \cos^4\alpha h_7 \boldsymbol{J} \wedge \boldsymbol{J} \\ &+ \cos^2\alpha \sin(2\alpha) h_8 d\alpha \wedge (d\tau + \sigma) \wedge \boldsymbol{J}], \end{aligned} \tag{A18}$$

in terms of the one-forms

$$\begin{aligned} h_3^{(1)} &= \frac{i}{2} \left(\frac{d\tilde{z}}{(1 + \tilde{z})^2} - \frac{dz}{(1 + z)^2} \right), \\ h_4^{(1)} &= h_5^{(1)} = \frac{i}{2H^2} [(1 - 2\cos(2\alpha)\tilde{z} + \tilde{z}^2) dz \\ &\quad - (1 - 2\cos(2\alpha) z + z^2) d\tilde{z}], \end{aligned} \tag{A19}$$

and the zero-forms

$$\begin{aligned} h_6 &= i2(1 + z\tilde{z}) \frac{(\tilde{z} - z)}{LH^2} (1 + z\tilde{z} + (z + \tilde{z}) \sin^2\alpha), \\ h_7 &= -i \frac{(\tilde{z} - z)}{2H}, \\ h_8 &= i \frac{(\tilde{z} - z)}{H^2} (1 + z\tilde{z} + (z + \tilde{z}) \sin^2\alpha). \end{aligned} \tag{A20}$$

Note that the purely internal part of $\hat{F}^{\mathrm{tr}}_{(4)}$ is nonzero whenever the *axionic* combination $z - \tilde{z}$ is nonzero.

By virtue of the consistency of the truncation, any solution in the $\mathrm{SU}(3) \times \mathrm{U}(1)^2$ invariant sector of the SO (8) gauged supergravity is also a solution of the Euclidean equations of motion in (A1) and satisfies the Bianchi identity (A2).

[1] G. W. Gibbons and S. W. Hawking, Action integrals and partition functions in quantum gravity, Phys. Rev. D **15**, 2752 (1977).

[2] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. **2**, 231 (1998).

[3] A. B. Zamolodchikov, Irreversibility of the flux of the renormalization group in a 2D field theory, JETP Lett. **43**, 730 (1986).

[4] D. L. Jafferis, I. R. Klebanov, S. S. Pufu, and B. R. Safdi, Towards the F-theorem: $N = 2$ field theories on the three-sphere, J. High Energy Phys. 06 (2011) 102.

[5] I. R. Klebanov, S. S. Pufu, and B. R. Safdi, F-theorem without supersymmetry, J. High Energy Phys. 10 (2011) 038.

[6] M. Cvetic, M. J. Duff, P. Hoxha, J. T. Liu, H. Lu, J. X. Lu, R. Martinez-Acosta, C. N. Pope, H. Sati, and T. A. Tran, Embedding AdS black holes in ten-dimensions and eleven-dimensions, Nucl. Phys. **B558**, 96 (1999).

[7] B. de Wit and H. Nicolai, $N = 8$ supergravity, Nucl. Phys. **B208**, 323 (1982).

[8] B. de Wit and H. Nicolai, The consistency of the S^7 truncation in $D = 11$ supergravity, Nucl. Phys. **B281**, 211 (1987).

[9] O. Aharony, O. Bergman, D. L. Jafferis, and J. Maldacena, $N = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals, J. High Energy Phys. 10 (2008) 091.

[10] D. Z. Freedman and S. S. Pufu, The holography of F-maximization, J. High Energy Phys. 03 (2014) 135.

[11] Y. Nakayama, Topologically twisted renormalization group flow and its holographic dual, Phys. Rev. D **95**, 066010 (2017).

[12] Y. Nakayama, Euclidean M-theory background dual to a three-dimensional scale-invariant field theory without conformal invariance, Phys. Rev. D **95**, 046006 (2017).

[13] S. B. Giddings and A. Strominger, Axion induced topology change in quantum gravity and string theory, Nucl. Phys. **B306**, 890 (1988).

[14] E. Bergshoeff, A. Collinucci, U. Gran, D. Roest, and S. Vandoren, Non-extremal D-instantons, J. High Energy Phys. 10 (2004) 031.

[15] E. Bergshoeff, A. Collinucci, U. Gran, D. Roest, and S. Vandoren, Non-extremal instantons and wormholes in string theory, Fortschr. Phys. **53**, 990 (2005).

[16] N. Arkani-Hamed, J. Orgera, and J. Polchinski, Euclidean wormholes in string theory, J. High Energy Phys. 12 (2007) 018.

[17] M. Gutperle and W. Sabra, Instantons and wormholes in Minkowski and (A)dS spaces, Nucl. Phys. **B647**, 344 (2002).

[18] T. Hertog, M. Trigiante, and T. Van Riet, Axion wormholes in AdS compactifications, J. High Energy Phys. 06 (2017) 067.

[19] D. Marolf and J. E. Santos, AdS Euclidean wormholes, Classical Quantum Gravity **38**, 224002 (2021).

[20] G. J. Loges, G. Shiu, and T. Van Riet, A 10d construction of Euclidean axion wormholes in flat and AdS space, J. High Energy Phys. 06 (2023) 079.

[21] D. Astesiano and F. F. Gautason, Supersymmetric wormholes in string theory, arXiv:2309.02481.

[22] C. N. Pope and N. P. Warner, A dielectric flow solution with maximal supersymmetry, J. High Energy Phys. 04 (2004) 011.

[23] A. Anabalón, M. Chamorro-Burgos, and A. Guarino, Janus and Hades in M-theory, J. High Energy Phys. 11 (2022) 150.

[24] N. Bobev, V. S. Min, K. Pilch, and F. Rosso, Mass deformations of the ABJM theory: The holographic free energy, J. High Energy Phys. 03 (2019) 130.

[25] P. G. Freund and M. A. Rubin, Dynamics of dimensional reduction, Phys. Lett. **97B**, 233 (1980).

[26] A. Azizi, H. Godazgar, M. Godazgar, and C. N. Pope, Embedding of gauged STU supergravity in eleven dimensions, Phys. Rev. D **94**, 066003 (2016).

[27] H. Elvang, R. Emparan, and A. Virmani, Dynamics and stability of black rings, J. High Energy Phys. 12 (2006) 074.

[28] M. Bianchi, D. Z. Freedman, and K. Skenderis, Holographic renormalization, Nucl. Phys. **B631**, 159 (2002).

[29] M. Bianchi, D. Z. Freedman, and K. Skenderis, How to go with an RG flow, J. High Energy Phys. 08 (2001) 041.

[30] The ambiguity of the sign in N_a is because, in the wormhole solution, the boundary consists of two disconnected pieces located at $\mu = \pm\infty$.

[31] D. L. Jafferis, The exact superconformal R-symmetry extremizes Z, J. High Energy Phys. 05 (2012) 159.

[32] F. F. Gautason, V. G. M. Puletti, and J. van Muiden, Quantized strings and instantons in holography, J. High Energy Phys. 08 (2023) 218.

[33] Spinless fields in the SO(8) supergravity transform in the $\mathbf{35_v}$ (proper-scalars) and $\mathbf{35_c}$ (pseudo-scalars) symmetric and *traceless* representations of SO(8). The bosonic operator $\mathcal{O}_S$ in (46) corresponds to the trace of the boson bilinears and, therefore, has no associated scalar in the SO (8) supergravity.

[34] See Table 1 of [10] for a summary of the $\mathrm{U}(1)_R \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_b$ charges of the various fields in the ABJM SCFT.

[35] A. Kapustin, B. Willett, and I. Yaakov, Exact results for Wilson loops in superconformal Chern-Simons theories with matter, J. High Energy Phys. 03 (2010) 089.

[36] J. M. Maldacena and L. Maoz, Wormholes in AdS, J. High Energy Phys. 02 (2004) 053.

[37] E. Witten, A note on complex spacetime metrics, arXiv: 2111.06514.

[38] A. Anabalón and J. Oliva, Four-dimensional traversable wormholes and bouncing cosmologies in vacuum, J. High Energy Phys. 04 (2019) 106.

[39] A. Anabalón, B. de Wit, and J. Oliva, Supersymmetric traversable wormholes, J. High Energy Phys. 09 (2020) 109.

[40] N. Bobev, K. Pilch, and N. P. Warner, Supersymmetric Janus solutions in four dimensions, J. High Energy Phys. 06 (2014) 058.

[41] K. Pilch, A. Tyukov, and N. P. Warner, $\mathcal{N} = 2$ supersymmetric Janus solutions and flows: From gauged supergravity to M theory, J. High Energy Phys. 05 (2016) 005.

[42] G. Larios, P. Ntokos, and O. Varela, Embedding the SU(3) sector of SO(8) supergravity in $D = 11$, Phys. Rev. D **100**, 086021 (2019).