Sachdev-Ye-Kitaev model on a noisy quantum computer

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We study the Sachdev-Ye-Kitaev (SYK) model—an important toy model for quantum gravity on IBM's superconducting qubit quantum computers. By using a graph-coloring algorithm to minimize the number of commuting clusters of terms in the qubitized Hamiltonian, we find the gate complexity of the time evolution using the first-order product formula for N Majorana fermions is $\mathcal{O}(N^5 J^2 t^2/\epsilon)$ where J is the dimensionful coupling parameter, t is the evolution time, and ϵ is the desired precision. With this improved resource requirement, we perform the time evolution for $N = 6$, 8 with maximum two-qubit circuit depth and gate count of 343. We perform different error mitigation schemes on the noisy hardware results and find good agreement with the exact diagonalization results on classical computers and noiseless simulators. In particular, we compute vacuum return probability after time t and out-of-time order correlators which is a standard observable of quantifying the chaotic nature of quantum systems.

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I. INTRODUCTION

The holographic duality [1] relates a special class of quantum field theories in d dimensions and quantum gravity in $d+1$ dimensions. This strong/weak duality enables one to study the properties of strongly coupled field theory using classical supergravity and vice versa. However, there are no cases where both sides of the duality can be studied analytically at the same time. Several attempts have been made on the lattice using Monte Carlo [2,3] to study these theories but they have their limitations. Therefore, it is often of interest to find simpler models that have holographic properties and can be studied in the strong coupling limit. One such model is the Sachdev-Ye-Kitaev (SYK) model [4–7] consisting of N Majorana fermions in $0+1$ dimensions with random couplings between q fermions at a time chosen from a Gaussian distribution with zero mean and variance proportional to J^2/N^{q-1}.

An interesting feature of the SYK model is that it develops an approximate conformal symmetry in the large N, low-temperature limit i.e., $N \gg \beta J \gg 1$ (β is the inverse temperature), where it is related to near extremal black holes that develop the nAdS_2 (near AdS_2) geometry. It was shown to saturate the chaos bound [8], a feature that is associated with holographic behavior. Since this is a $0+1$-dimensional model, it is computationally tractable and has been studied up to 60 Majorana fermions [9,10].

As a toy model for quantum gravity, it is, therefore, crucial to study the real-time dynamics of this model beyond methods accessible by classical computing. This direction has already been explored starting with Ref. [11]. In another work [12], the authors studied a generalized SYK model using a four-qubit nuclear magnetic resonance quantum simulator and computed bosonic correlation functions.

We put forth an improved circuit complexity[1] and study the SYK model on noisy superconducting quantum computers for the first time to our knowledge. Specifically, we find an improved complexity from earlier proposals of $\mathcal{O}(N^{10} J^2 t^2/\epsilon)$ [11] and $\mathcal{O}(N^8 J^2 t^2/\epsilon)$ [13] to $\mathcal{O}(N^5 J^2 t^2/\epsilon)$ for the Lie-Trotter-based algorithm [14]. Using this improvement, we study the time evolution up to *eight* Trotter steps on quantum hardware available through IBM and compute the return probability and four-point out-of-time-ordered correlators.

II. SYK HAMILTONIAN

The Hamiltonian for the SYK model with N Majorana fermions and q-fermion interaction terms is

$$H = \frac{(i)^{q/2}}{q!} \sum_{i,j,k,\ldots,q=1}^{N} J_{ijk\cdots q} \chi_i \chi_j \chi_k \cdots \chi_q, \tag{1}$$

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[1] The complexity is defined as the least number of two-qubit gates in the circuit that implements the time evolution of the Hamiltonian H.

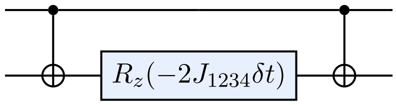

FIG. 1. Circuit implementing single Trotter step for $N = 4$.

where χ are the Majorana fermions satisfying $\{\chi_i, \chi_j\} = \chi_i\chi_j + \chi_j\chi_i = \delta_{ij}$. We consider $q = 4$ with random all-to-all quartic interactions averaged over disorder. The random (real) couplings J_{ijkl} are sampled from a Gaussian distribution with the mean $\overline{J_{ijkl}} = 0$ and variance equal to $\overline{J^2_{ijkl}} = \frac{3!J^2}{N^3}$. We set $J = 1$ in this work. The dimension of the Hilbert space is $\dim(\mathcal{H}) = 2^{N/2}$ where $n = N/2$ is the number of qubits.[2] The model can also be considered for $q > 4$ and is solvable in the large q limit [7].

A. Qubitization and Trotterization

To perform the time evolution, we first have to map the fermionic Hamiltonian to qubits. We will use the standard Jordan-Wigner transformation. The N fermions in (1) can be written in terms of tensor product of $N/2$ Pauli matrices X, Y, Z and the identity matrix $\mathbb{1}$ [11,12] as

$$\chi_{2k-1} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{k-1} Z_j\right) X_k \mathbb{1}^{\otimes(N-2k)/2},$$
$$\chi_{2k} = \frac{1}{\sqrt{2}}\left(\prod_{j=1}^{k-1} Z_j\right) Y_k \mathbb{1}^{\otimes(N-2k)/2}, \qquad (2)$$

where the square root is to ensure the normalization following Ref. [5]. In order to simulate the dynamics on quantum hardware, we first decompose the Hamiltonian into Pauli strings as $H = \sum_{j=1}^{m} H_j$ and then use the standard Lie-Trotter formula [15]:

$$e^{-iHt} = \left(\prod_{j=1}^{m} e^{-iH_j t/r}\right)^r + \mathcal{O}\left(\sum_{j<k} \|[H_j, H_k]\| \frac{t^2}{r}\right), \qquad (3)$$

where we denote the spectral norm by $\|\cdot\|$. If the terms in the decomposition of H are near to commuting, then the Trotter error is reduced and vanishes if they commute. Though the number of terms into which H is split is $m = \binom{N}{4}$, one can reduce it by only summing over a small number of clusters of Pauli strings $\mathcal{N} \ll m$. This helps in reducing the Trotter error since the number of terms summed in (3) is reduced. We find that for $N = 6$, the reduction factor i.e., $m/\mathcal{N} = 3$ while for $N = 8$, $m/\mathcal{N} = 35/3$ and this enables us to reliably evolve to larger times by controlling the Trotter error (see Supplemental Material [16] for details). If we impose total error in simulating the time evolution is ϵ, then we need $r = \mathcal{O}(t^2/\epsilon)$ Trotter steps assuming the spectral norm of commutators to be upper bounded by unity. Using these arguments, Ref. [11] estimated the circuit complexity of $\mathcal{O}(N^{10}t^2/\epsilon)$. The method based on Lie-Trotter expansion is not the only way of simulating Hamiltonians. Another way is to use a controlled version of oracles to embed the Hamiltonian in an invariant $SU(2)$ subspace [17] and a variant of this was used in Ref. [18] to bring complexity to $\mathcal{O}(N^{7/2}t + N^{5/2}t\,\mathrm{polylog}(N/\epsilon))$. However, this is not amenable to current hardware implementation.

TABLE I. The gate cost (assuming all-to-all connectivity) for the time evolution of the SYK Hamiltonian per Trotter step, number of Pauli strings, and the number of clusters $\mathcal{N}$ of commuting Pauli strings for different N.

N	Pauli strings	Clusters	Two-qubit gates
4	1	1	2
6	15	5	30
8	70	6	110
10	210	23	498
12	495	57	1504
14	1001	92	3560
16	1820	116	6812
18	3060	175	11962
20	4845	246	19984

Let us consider the simplest case of $N = 4$ where we have just one Pauli string with $H = -J_{1234}ZZ$. The time evolution circuit [19] is given by Fig. 1. The two-qubit cost of simulating various N can be found in Table I for $N \leq 20$. The circuit complexity estimate based on commuting Pauli strings exploiting the graph-coloring algorithm grows[3] like $\mathcal{O}(N^5)$, a substantial improvement over $\mathcal{O}(N^{10})$ proposed in Ref. [11].

The number of Pauli strings in the decomposition of H grows like $\mathcal{O}(N^4/4!)$ for large N, and the cost to simulate the simplest nontrivial Pauli string is $\mathcal{O}(N)$. Hence, the $\sim N^5$ circuit complexity is close to *optimal* for this approach to Hamiltonian simulation. The complexity can be improved by using Bravyi-Kitaev mapping; however, such optimizations are not required for the small N which are currently hardware accessible. We provide details of gate resource estimation in the Supplemental Material (SM) [16].

III. RETURN PROBABILITY

One of the main motivations for using quantum computers is to understand the time evolution of quantum

[2] We have $N = 2n$ since two Majorana fermions can be represented by one complex spinless fermion which can be represented by a single qubit.

[3] For $N \leq 10$, the cost based on quantum Shannon decomposition (QSD) is lower [20]. However, large N scaling is exponential.

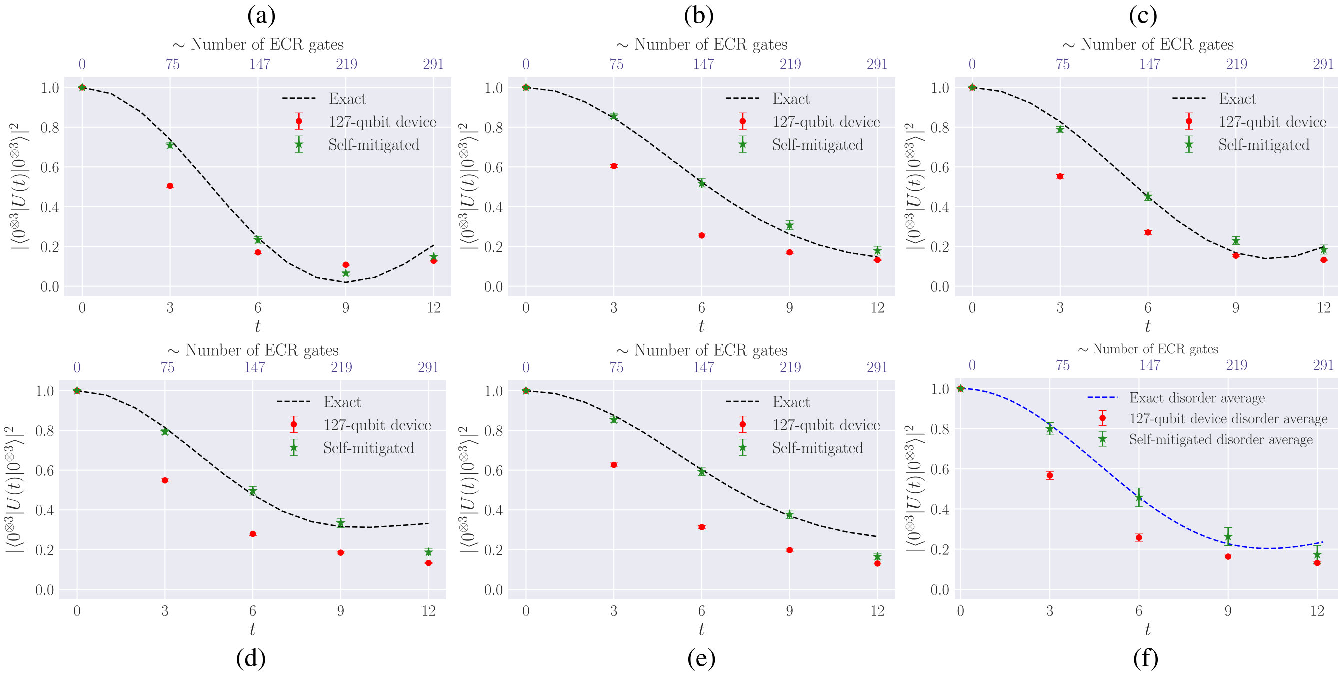

FIG. 2. The return probability for five realizations (a)–(e) of the SYK model with $N = 6$. The disordered average (f) shows the slope region and the starting of the ramp behavior. The total two-qubit circuit depth is the same as the number of ECR gates, given the topology of the qubits.

systems. An observable we compute is the return probability by evolving an initial state[4] given by $|\psi_0\rangle = |0\rangle^{\otimes n}$ for N_t Trotter steps and computing the overlap as

$$\mathcal{P}_0 = |\langle\psi_0|e^{-iHt}|\psi_0\rangle|^2. \tag{4}$$

The return probability is closely related to the spectral form factor [22] and shows similar behavior of slope, dip, ramp, and plateau upon disorder average [21,23] which are features of the SYK model.

The error-mitigated hardware results of the return probabilities for five realizations of the model with $N = 6$ and disorder average are shown in Fig. 2. The dashed black lines are the exact time evolution, while the dashed blue curve is the ensemble average of the exact evolution over the five realizations. At this stage, we do not compare to the state-of-the-art results obtained for the SYK model with classical computing methods since it is still far from the current best classical result with $N \sim 60$.

As N increases, the resource requirements quickly increase as shown in Table I. Therefore, for $N = 8$ we consider the time evolution of just one instance of the model in Fig. 3. The markers are results obtained from 127-qubit IBM machines `ibm_cusco`, `ibm_nazca`, and `ibm_kyoto` with various degrees of error mitigation applied.

[4]The initial state chosen here belongs to the set of common eigenstates of the SYK spin operators defined in [21] and forms a complete basis.

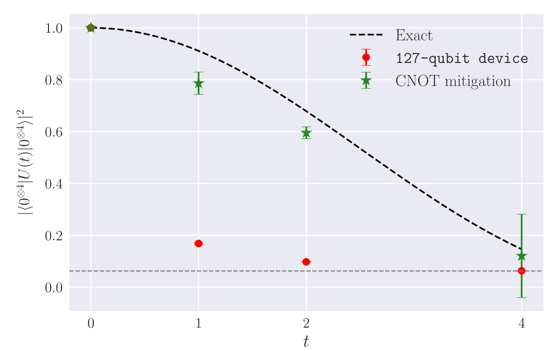

FIG. 3. The return probability for a single instance of the SYK model with $N = 8$. The results before mitigation [24] are slightly above the threshold of the fully depolarized channel (gray dashed line). For $t = 1, 2$ we use $dt = 1$ while for $t = 4$ we use $dt = 2$. The maximum 2-qubit gate depth we use is 343.

All these devices use the `eagle r3` processor, where the native two-qubit gate is not the standard controlled X (CX) but the echoed cross-resonance (ECR) gate (see SM [16] for the definition of the gate). The leading source of gate noise in current devices is the two-qubit gates. To deal with this, the error mitigation strategy we employ is a combination of Pauli twirling/randomized compiling [25] for the ECR gates and self-mitigation [26]. For the discussion on the implementation of these error-mitigation techniques and to get an estimate on the computation overhead to materialize in the quantum processing units of IBM, we refer the readers to the SM [16]. We also use the standard

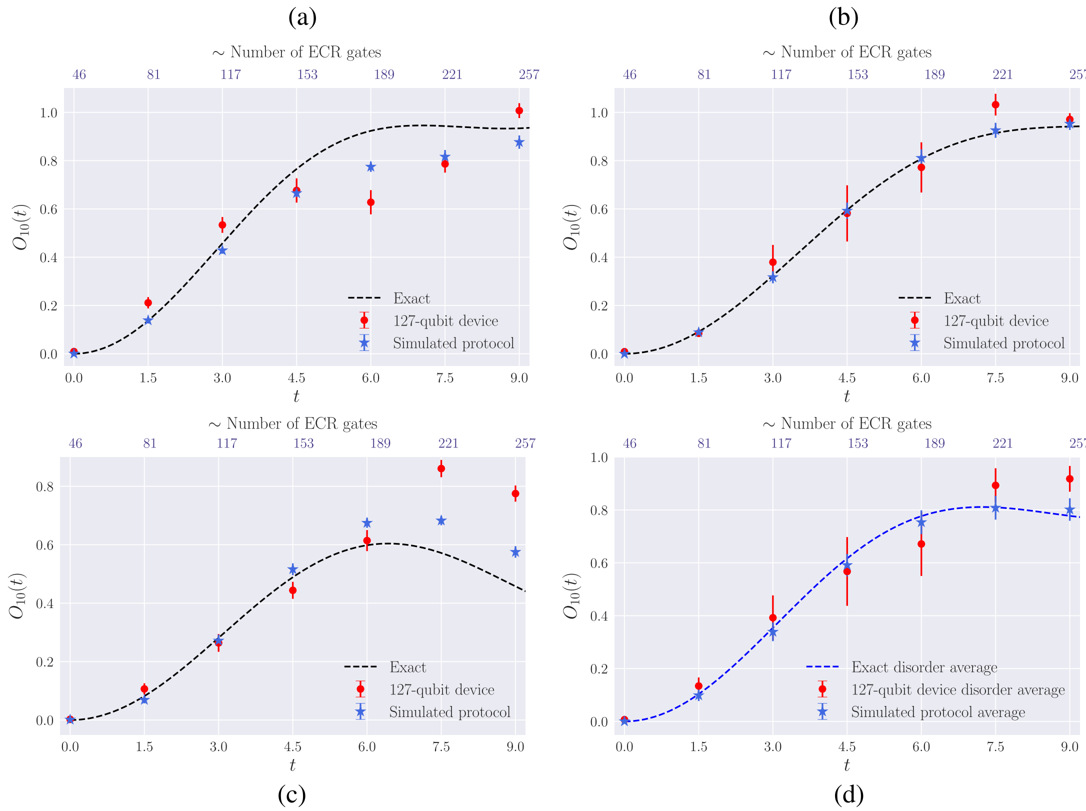

FIG. 4. The OTOC for three realizations (a)–(c) of the SYK model with $N = 6$ obtained on `ibm_kyoto` and `ibm_cusco`, (d) shows disorder average over (a)–(c).

M3 protocol [27] for qubit measurement errors and dynamical decoupling [28–31] to suppress decoherence noise. The results for the return probability[5] with different types of error mitigation applied and exact evolution are shown in Figs. 2(a)–2(e) for five different instances of the SYK Hamiltonian with $N = 6$ and the disorder average over these realizations is shown in Fig. 2(f).

For $N = 6$, the self-mitigated return probability agrees with exact results even for large evolution time and ECR gates in the circuit. To simulate the SYK model with $N = 6$ up to $t = 12$ (eight Trotter steps), we require about 300 ECR gates. For $N = 8$, we need circuit depth of 343 ECR gates for two Trotter steps ($t = 2$ with $dt = 1$, $t = 4$ with $dt = 2$) and 170 (for $t = 1$ with $dt = 1$). The return probability for $N = 8$ is shown in Fig. 3 for one realization of the model. Going beyond $t = 4$ appears to be past current hardware capability even with advanced error mitigation methods.

IV. OTOC COMPUTATION

An important feature of the SYK model is that for large N and in the low-temperature limit, it is maximally chaotic and is a quintessential example of a fast scrambler. A defining feature of such systems is that quantum information shared between a small number of elementary degrees of freedom is rapidly distributed into exponentially many degrees of freedom. This is known as "scrambling" and black holes are known to be the fastest scramblers in nature. To quantify the chaos, one considers the out-of-time ordered (OTO) commutator between two operators W and V given by [32–35]

$$C(t) = -\langle [W(t), V(0)]^{\dagger} [W(t), V(0)] \rangle, \tag{5}$$

In general $C(t)$ starts from zero and becomes significant at some later time t, which one refers to as scrambling time. By considering the commutator expansion, we can define the out-of-time order correlators (OTOC)[6]:

$$\text{OTOC} \coloneqq F(t) = \langle W(t) V(0) W(t) V(0) \rangle_{\beta}, \tag{6}$$

where W and V are generic Hermitian operators and do not have the same symmetry as the Hamiltonian. The growth of the $C(t)$ is related to the decay of the OTOCs i.e., $F(t)$ through the simple relation, $C(t) = 2(1 - F(t))$. The time

[5]We only show even Trotter steps because for self-mitigation, one requires forward and backward Trotter evolution by an equal amount.

[6]These were first introduced in the study of disordered superconductors [36]. OTOC is also closely related to the thermal average of signals from Loschmidt echo [37].

evolution of the operator W in the Heisenberg representation is $W(t) = e^{iHt}W(0)e^{-iHt}$ and $\langle\cdot\rangle_\beta = \mathrm{Tr}\{\rho\cdot\}$ denotes the thermal average at inverse temperature β. A common choice for ρ is to just use the $T \to \infty$ limit given by the normalized identity matrix, $\mathbb{1}/\dim(\mathcal{H})$ [38]. For the W and V we can either take Pauli matrices or Majorana fermions such that $W = \chi_i$ and $V = \chi_j$ with $i \neq j$ and average over the different pairs (i, j). We used the simplest case of a single Pauli matrix, i.e., $W = V = Z$. We denote $1 - F(t)$ by $1 - F(t)_{ij} = O_{ij}(t)$ where i and j denote the qubit location of the single-qubit operator W and V respectively and compute this on the hardware. The choice of these operators does not change the basic features of exponential growth and saturation. The SYK model saturates the chaos bound [8] at low temperatures and they have been extensively studied using classical computing methods [39,40]. The current resources do not allow us to access very large values of N, but we take a first step at computing in the simplest setting on the quantum computer.

A. Results for OTOC

Even though the OTOC seems simple to compute, the experimental/quantum computer measurement of the OTOC is very challenging because of the unusual time ordering. In a preliminary investigation, Ref. [41] studied OTOC of local operators on a nuclear magnetic resonance quantum simulator followed soon on trapped-ion [42]. In order to compute OTOC on quantum hardware, we need to define a protocol for the measurement. Several proposals have been put forth [43–46] and we use the protocol proposed in Ref. [45] which computes OTOC only by considering forward time evolution and exploiting the correlations between randomized measurements [47]. We discuss the details of the global protocol based on randomized measurements in the SM [16].

The results for OTOC from `ibm_cusco` and `ibm_kyoto` for three instances of $N = 6$ and the disorder average over them is shown in Fig. 4. For this computation, we used the M3 readout error mitigation protocol and dynamical decoupling. Even without self-mitigation, we find good agreement with the exact results.

V. SUMMARY AND DISCUSSION

We have proposed circuit complexity of $\mathcal{O}(N^5 t^2/\epsilon)$ for Hamiltonian simulation of the SYK model with N Majorana fermions, a substantial improvement over existing results and performed quantum simulations on noisy 127-qubit quantum computers. We studied the return probability for $N = 6$, 8 Majorana fermions and computed the out-of-time order correlators for $N = 6$, a diagnostic of the chaotic behavior of quantum many-body systems. Due to the noisy devices currently available, we applied advanced mitigation methods to the hardware result and showed that it agrees well with the exact time evolution.

It might appear that the superconducting platform is not the best method to study the quantum simulation of this model as we could have applied the ion-based approaches to quantum simulation. The advantage of superconducting platforms is the low gate times, but the limitation is the qubit connectivity. With ion-based platforms, this is the opposite—there is more freedom with connectivity, but the gate times are much longer. For the dense SYK model and other dense random Hamiltonians, both of these things are important. In this work, we take a step toward identifying which of these is more important by pushing the superconducting platform to push the limits with limited connectivity. We hope to extend this work with hardware admitting all-to-all connectivity.

Though we cannot study the strict holographic limit and see signs of saturation of chaos bound on current devices, we believe that our work will be useful in future explorations of this model. In this regard, it might also be useful to consider simplified models similar to SYK [13,48–50] that are conjectured to have the same holographic behavior as the pure SYK model considered here. Another direction is to consider $q > 4$ and explore the resource requirements and time evolution. It would be useful to study the dynamics of the model over different timescales for the return probability at finite β. These interesting problems would require resources that are beyond the contemporary hardware era. We leave these questions for future work. The use of quantum computers for models such as the SYK model in coming decades will not only provide new insights into the holographic principle but also into the interesting world of strange metals and quantum many-body systems [51].

The $N = 6$, 8 SYK Hamiltonian realizations, the Pauli decomposition, and the time evolution circuit as OPEN QASM 2.0 files for the single Trotter step can be obtained from Ref. [52].

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