Dark photon–dark energy stationary axisymmetric black holes

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Using Ernst formalism, the stationary axisymmetric black hole solution in Einstein dark matter–dark energy gravity has been elaborated. The dark sector was chosen as a dark photon concept, where an auxiliary $U(1)$-gauge field coupled to ordinary Maxwell one was introduced, while dark energy was modeled by the existence of a positive cosmological constant. Refining our studies to the case of a vanishing cosmological constant, the uniqueness theorem for the black hole in question has been proved.

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I. INTRODUCTION

The nature of the illusive ingredient of our Universe mass, the dark sector, is the most tantalizing question in contemporary physics and astrophysics. The influence of dark matter on galaxy rotation curves, motion of galaxy clusters, measurements of cosmic microwave background radiation, and baryonic oscillations [1,2] as well as pulsar timing array experiments [3] has also been revealed. The unknown dark matter sector constitutes almost 27% of the mass of the observable Universe [4], while most of the additional part of the mass comprises dark energy, responsible for the Universe expansion. The visible sector constitutes only 4% of the Universe mass.

One of the simplest conceptual ideas is to consider the existence of a hidden sector composed of particles weakly interacting with the ordinary matter. The notion of the dark photon being a hypothetical Abelian gauge boson coupled to the ordinary Maxwell gauge field [5] is very a plausible candidate for physics beyond the Standard Model. The dark photon idea has been introduced in [5] several years ago; however, it acquires a contemporary justification in the unification scheme [6], where the mixing portals coupling Maxwell and auxiliary gauge fields are under intensive exploit and the hidden sector states are charged under their own groups. On the other hand, it has been claimed that dark photons might be produced, e.g., during the inflationary phase of the Universe evolution from inflationary fluctuations [7,8], during reheating [9], from resonant creation during axion oscillations [10], and from dark Higgs [11] as well as from cosmic strings [12].

Furthermore, several anomalous astrophysical effects like 511 keV gamma rays [13], excess of the positron cosmic ray flux in galaxies [14], and the observations of an anomalous monochromatic 3.56 keV x-ray line in the spectrum of some galaxy clusters [15] may advocate the dark photon idea. Other astrophysical observations and laboratory experiments [16], like, for instance, studies of gamma ray emissions from dwarf galaxies [17], examination of dilatonlike coupling to photons caused by ultralight dark matter [18], inspections of the fine structure constant oscillations [19], dark photon emission taking place during the 1987A supernova event [20], electron excitation measurements in a CCD-like detector [21], the search for a dark photon in e^+e^- collisions at the *BABAR* experiment [22], and measurements of the muon anomalous effect [23], propose the possible range of values for a dark photon–Maxwell field coupling constant and the mass of the hidden photon [24].

It turns out that a dark photon acting as a portal to the hidden sector, which introduces dark matter self-interactions, may constitute a solution of the small-scale structure problems [25]. Moreover, it can explain the XENON1T anomaly [26]. Dark photons can affect the primordial nucleon synthesis, altering the effective number of thermally excited neutrino degrees of freedom [27], and potentially influence on transport properties and exert on stellar energy transport mechanism being the key factor during cooling neutron star processes [28].

It has been also reported that the new exclusion limit for the α-coupling constant $\alpha = 1.6 \times 10^{-9}$ and the mass range of the dark photon 2.1×10^{-7}–5.7×10^{-6} eV. These data were achieved by using the two state-of-art high-quality factor superconducting radio frequency cavities [29]. Improved limits on the coupling of ultralight bosonic dark matter to Maxwell photons, based on long-term measurements of two optical frequencies, were proposed in [30].

On the other hand, using quantum limited amplification, the first probing of the kinetic mixing coupling constant to 10^{-12} level for the majority of dark photon masses was given in [31], being the first stringent constraints on new dark matter parameter space.

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Recently, it is to be noted that the use of a cryogenic optical path and a fast spectrometer to study dark photon conversion into an ordinary one at the metal surface plate enables one to establish an upper bound on coupling constant $\alpha < 0.3\text{–}2 \times 10^{-10}$ (at 95% confidence level) [32].

On the other hand, in light of the first LIGO gravitational wave detection and Event Horizon Telescope images of a black hole shadow, investigations of black holes, and the influence of the dark sector on their physics constitute a very interesting problem on its own.

The main aim of our paper is to find the stationary axially symmetric solution to Einstein-Maxwell dark photon gravity with a cosmological constant, sometimes being identified with dark energy. Additionally, we treat the problem of the uniqueness of a rotating black hole influenced by the dark matter sector, where we restrict our consideration to the case without dark energy.

Our paper is organized as follows. In Sec. II, we consider the basic features of Einstein-Maxwell gravity replenished by the auxiliary $U(1)$-gauge field (dark photon) coupled to the Maxwell one by the so-called kinetic mixing term. Next, we derive the Ernst-like equations for stationary axisymmetric solution in the theory in question, adding to our inspection the cosmological constant which will be identified with dark energy. In Sec. III, after introducing adequate charges bounded with the gauge fields, one achieves the line element we are looking for. In Sec. IV, we pay attention to the boundary conditions of the studied solution. Using the matrix type of Ernst equations, one conducts the uniqueness proof for a stationary axisymmetric black hole in the considered theory, when the cosmological constant is absent. Namely, we reveal that all stationary axisymmetric solutions to Einstein-Maxwell dark photon gravity, being subject to the same boundary and regularity conditions, comprise the only black hole solution having a regular event horizon with nonvanishing mass and t and ϕ components of $U(1)$-gauge fields. In the last section, we conclude our investigations.

II. GRAVITY WITH DARK MATTER–DARK ENERGY SECTORS

This section will be devoted to the basic features of Einstein-Maxwell gravity influenced by the dark matter sector, which constitutes another $U(1)$-gauge field coupled to the ordinary Maxwell one, by the so-called kinetic mixing term describing interactions of both gauge fields. Moreover, we add the positive cosmological constant authorizing dark energy in the spacetime under inspection. The action related to Einstein-Maxwell dark photon–dark energy gravity is provided by

$$S_{\text{EM-dark photon-}\Lambda} = \int \sqrt{-g}d^4x(R - 2\Lambda - F_{\mu\nu}F^{\mu\nu} - B_{\mu\nu}B^{\mu\nu} - \alpha F_{\mu\nu}B^{\mu\nu}), \tag{1}$$

where α is taken as a coupling constant between Maxwell and dark matter field strength tensors.

Introducing the redefined gauge fields $\tilde{A}_\mu$ and $\tilde{B}_\mu$, in the forms as follows:

$$\tilde{A}_\mu = \frac{\sqrt{2-\alpha}}{2}(A_\mu - B_\mu), \tag{2}$$

$$\tilde{B}_\mu = \frac{\sqrt{2+\alpha}}{2}(A_\mu + B_\mu), \tag{3}$$

one can get rid of the kinetic mixing term, obtaining the following:

$$F_{\mu\nu}F^{\mu\nu} + B_{\mu\nu}B^{\mu\nu} + \alpha F_{\mu\nu}B^{\mu\nu} \Rightarrow \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu}, \tag{4}$$

where we have denoted $\tilde{F}_{\mu\nu} = 2\partial_{[\mu}\tilde{A}_{\nu]}$ and, respectively, $\tilde{B}_{\mu\nu} = 2\partial_{[\mu}\tilde{B}_{\nu]}$. Having it in mind, the rewritten action (1) implies

$$S_{\text{EM-dark photon-}\Lambda} = \int \sqrt{-g}d^4x(R - 2\Lambda - \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu}). \tag{5}$$

Variation of the action (5) with respect to $g_{\mu\nu}$, $\tilde{A}_\mu$, and $\tilde{B}_\mu$ reveals the following equations of motion for Einstein-Maxwell dark matter gravity with the positive cosmological constant (dark energy):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 2\tilde{F}_{\mu\rho}\tilde{F}_\nu{}^\rho - \frac{1}{2}g_{\mu\nu}\tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta} + 2\tilde{B}_{\mu\rho}\tilde{B}_\nu{}^\rho - \frac{1}{2}g_{\mu\nu}\tilde{B}_{\alpha\beta}\tilde{B}^{\alpha\beta}, \tag{6}$$

$$\nabla_\mu\tilde{F}^{\mu\nu} = 0, \qquad \nabla_\mu\tilde{B}^{\mu\nu} = 0. \tag{7}$$

In what follows, one will focus on stationary axisymmetric solution in the considered theory of gravity. The corresponding line element is given by

$$ds^2 = -ae^{\frac{b}{2}}(dt + \omega d\phi)^2 + ae^{-\frac{b}{2}}d\phi^2 + \frac{e^{2u}}{\sqrt{a}}(dr^2 + dz^2), \tag{8}$$

where the functions appearing in the line element (8) depend on r and z coordinates.

Furthermore, we choose the following *Ansätze* for $U(1)$-gauge fields, which will be also r and z dependent:

$$\tilde{A} = \tilde{A}_0 dt + \tilde{A}_\phi d\phi, \qquad \tilde{B} = \tilde{B}_0 dt + \tilde{B}_\phi d\phi. \tag{9}$$

For brevity of notation, we denote $\vec{\nabla}k = (\partial_r k, \partial_z k)$.

The Einstein-Maxwell dark photon equations of motion with cosmological constant Λ modeled dark energy are provided by

$$\nabla^2 a + 2\Lambda e^{2u}\sqrt{a} = 0, \qquad (10)$$

$$4a\nabla^2 u + \nabla^2 a - e^b a(\vec{\nabla}\omega)^2 + \frac{1}{4}a(\vec{\nabla}b)^2 = 0, \qquad (11)$$

$$-\frac{\vec{\nabla}\cdot(a\vec{\nabla}b)}{2} - ae^b(\vec{\nabla}\omega)^2 - \omega\cdot\vec{\nabla}(e^b a\vec{\nabla}\omega) = 2[e^{\frac{b}{2}}((\vec{\nabla}\tilde{A}_0)^2\omega^2 - (\vec{\nabla}\tilde{A}_\phi)^2) - e^{-\frac{b}{2}}(\vec{\nabla}\tilde{A}_0)^2] + 2[e^{\frac{b}{2}}((\vec{\nabla}\tilde{B}_0)^2\omega^2 - (\vec{\nabla}\tilde{B}_\phi)^2) - e^{-\frac{b}{2}}(\vec{\nabla}\tilde{B}_0)^2], \qquad (12)$$

$$\vec{\nabla}\cdot(e^b a\vec{\nabla}\omega) = -4\omega e^{\frac{b}{2}}[(\vec{\nabla}\tilde{A}_0)^2 + (\vec{\nabla}\tilde{B}_0)^2] + 4e^{\frac{b}{2}}(\vec{\nabla}\tilde{A}_\phi\cdot\vec{\nabla}\tilde{A}_0 + \vec{\nabla}\tilde{B}_\phi\cdot\vec{\nabla}\tilde{B}_0). \qquad (13)$$

The differential operators $\vec{\nabla}$ and ∇^2 appearing in the above set of equations are flat gradient and Laplacian operators written in (r, z) coordinates, while the dots mean the scalar product of ∇ operators.

On the other hand, the t and ϕ components of Maxwell dark photon relations imply, respectively, for the $\tilde{A}_\mu$ gauge field

$$\vec{\nabla}\cdot[e^{-\frac{b}{2}}\vec{\nabla}\tilde{A}_0 + \omega e^{\frac{b}{2}}(\vec{\nabla}\tilde{A}_\phi - \omega\vec{\nabla}\tilde{A}_0)] = 0, \qquad (14)$$

$$\vec{\nabla}\cdot[e^{\frac{b}{2}}(\vec{\nabla}\tilde{A}_\phi - \omega\vec{\nabla}\tilde{A}_0)] = 0 \qquad (15)$$

and for the $\tilde{B}_\mu$ one

$$\vec{\nabla}\cdot[e^{-\frac{b}{2}}\vec{\nabla}\tilde{B}_0 + \omega e^{\frac{b}{2}}(\vec{\nabla}\tilde{B}_\phi - \omega\vec{\nabla}\tilde{B}_0)] = 0, \qquad (16)$$

$$\vec{\nabla}\cdot[e^{\frac{b}{2}}(\vec{\nabla}\tilde{B}_\phi - \omega\vec{\nabla}\tilde{B}_0)] = 0. \qquad (17)$$

Equations (15) and (17) comprise the integrability conditions for a scalar potentials, say, A_3 and B_3. Namely, they imply

$$\vec{e}_\phi\times\vec{\nabla}A_3 = e^{\frac{b}{2}}(\vec{\nabla}\tilde{A}_\phi - \omega\vec{\nabla}\tilde{A}_0) \qquad (18)$$

and for B_3

$$\vec{e}_\phi\times\vec{\nabla}B_3 = e^{\frac{b}{2}}(\vec{\nabla}\tilde{B}_\phi - \omega\vec{\nabla}\tilde{B}_0). \qquad (19)$$

From the above relation, we get the following relations for A_3 and B_3:

$$\nabla_i(\epsilon^{i\phi m}e_\phi\nabla_m A_3) = 0, \qquad \nabla_i(\epsilon^{i\phi m}e_\phi\nabla_m B_3) = 0, \qquad (20)$$

where we set $m = r,\ z$ and the ϵ^{abc} in (20) denotes the Levi-Civita symbol in the orthonormal frame defined by the ordered triad $(\vec{e}_r, \vec{e}_\phi, \vec{e}_\theta)$. It, in turn, leads to the following conditions:

$$\partial_r\partial_z A_3 = \partial_z\partial_r A_3, \qquad \partial_r\partial_z B_3 = \partial_z\partial_r B_3.$$

Extracting from Eq. (18) the term $\vec{e}_\phi\times\overrightarrow{\nabla}\tilde{A}_\phi$ and calculating its diverges, we obtain the following:

$$\nabla_i(e^{-\frac{b}{2}}\nabla^i A_3 - \omega\epsilon^{i\phi m}e_\phi\nabla_m\tilde{A}_0) = 0. \qquad (21)$$

The same procedure applied to the B_3 potential reveals

$$\nabla_i(e^{-\frac{b}{2}}\nabla^i B_3 - \omega\epsilon^{i\phi m}e_\phi\nabla_m\tilde{B}_0) = 0. \qquad (22)$$

The above equations together with the relations (14) and (16) will give us the set of Maxwell dark photon–dark energy equations. Let us define the complex potential $\Phi_{(k)}$, where $k = \tilde{F}, \tilde{B}$, in the form

$$\Phi_{(\tilde{F})} = \tilde{A}_0 + iA_3, \qquad \Phi_{(\tilde{B})} = \tilde{B}_0 + iB_3. \qquad (23)$$

It can be easily seen that the Maxwell dark photon equations may be rewritten by means of $\Phi_{(k)}$ as a single set of two complex equations:

$$\vec{\nabla}\cdot(e^{-\frac{b}{2}}\vec{\nabla}\Phi_{(k)} - i\omega\vec{e}_\phi\times\vec{\nabla}\Phi_{(k)}) = 0. \qquad (24)$$

On the other hand, we apply the same procedure in respect to Eq. (13) and cast it in the form

$$\vec{\nabla}\cdot\left[e^b a\vec{\nabla}\omega - 2\vec{e}_\phi\times\mathrm{Im}\left(\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(k)}\vec{\nabla}\Phi_{(k)}\right)\right] = 0. \qquad (25)$$

Moreover, relation (25) constitutes the integrability condition for the existence of the other potential, say, h. Consequently, one achieves

$$\vec{e}_\phi\times\vec{\nabla}h = e^b a\vec{\nabla}\omega - 2\vec{e}_\phi\times\mathrm{Im}\left(\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(k)}\vec{\nabla}\Phi_{(k)}\right), \qquad (26)$$

while using relation (25) we obtain the relation given by

$$\vec{e}_\phi\times\vec{\nabla}\omega = -\frac{1}{ae^b}\left[\vec{\nabla}h + 2\mathrm{Im}\left(\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(k)}\vec{\nabla}\Phi_{(k)}\right)\right] = 0, \qquad (27)$$

which, in turn, enables us to find that the (ϕ, t) component of Einstein dark photon equations of motion may be rewritten in terms of the potential h and the complex potentials $\Phi_{(k)}$. Namely, it implies

$$\vec{\nabla}\cdot\left[\frac{1}{ae^b}\left(\vec{\nabla}h + 2\mathrm{Im}\left(\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(k)}\vec{\nabla}\Phi_{(k)}\right)\right)\right] = 0. \qquad (28)$$

Having in mind the above definitions, the relation (12) will be provided by

$$\frac{f}{a}\vec{\nabla}\cdot(a\vec{\nabla}f)-\vec{\nabla}f\cdot\vec{\nabla}f-f^2\frac{\nabla^2 a}{a} = 2f\sum_{k=\tilde{F},\tilde{B}}\vec{\nabla}\Phi_{(k)}\cdot\vec{\nabla}\Phi^*_{(k)}-\left[\vec{\nabla}h+2\mathrm{Im}\left(\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(a)}\vec{\nabla}\Phi_{(k)}\right)\right]\cdot\left[\vec{\nabla}h+2\mathrm{Im}\left(\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(k)}\vec{\nabla}\Phi_{(k)}\right)\right], \tag{29}$$

where one denotes $f = ae^{b/2}$.

To proceed further, let us define the complex function given by the relation

$$\mathcal{E} = f-\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(k)}\Phi_{(k)}+ih. \tag{30}$$

It happens that both Einstein-Maxwell dark matter–dark energy equations can be arranged in a system of the complex equations provided by

$$\left(\mathrm{Re}\mathcal{E}+\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(k)}\Phi_{(k)}\right)\frac{\vec{\nabla}\cdot(a\vec{\nabla}\mathcal{E})}{a} = \left(\vec{\nabla}\mathcal{E}+2\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(k)}\vec{\nabla}\Phi_{(k)}\right)\cdot\vec{\nabla}\mathcal{E}+\mathrm{Re}^2\left(\mathcal{E}+\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(k)}\Phi_{(k)}\right)\frac{\nabla^2 a}{a}, \tag{31}$$

$$\sum_{m=\tilde{F},\tilde{B}}\left(\mathrm{Re}\mathcal{E}+\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(k)}\Phi_{(k)}\right)\frac{\vec{\nabla}\cdot(a\vec{\nabla}\Phi_{(m)})}{a} = \sum_{m=\tilde{F},\tilde{B}}\left(\vec{\nabla}\mathcal{E}+2\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(k)}\vec{\nabla}\Phi_{(k)}\right)\cdot\vec{\nabla}\Phi_{(m)}. \tag{32}$$

The above relations authorize the generalization of Ernst's equations describing the Einstein-Maxwell system. The real and imaginary parts of the first one envisage the Einstein-Maxwell dark matter equations with a cosmological constant (sometimes interpreted as dark energy). The real and imaginary parts of (32) describe the Maxwell dark photon equations of motion. The aforementioned equations reduce to the ordinary complex Ernst differential relations for Einstein-Maxwell gravity, when one sets the auxiliary gauge field equal to zero as well as assumes that the last term in (31) vanishes. It yields that a should be a harmonic function $\nabla^2_{(r,z)}a = 0$.

On the other hand, they can be achieved by varying the effective action $S[\mathcal{E},\mathcal{E}^*,\Phi_{(k)},\Phi^*_{(k)}]$, where one denotes $k = \tilde{F},\tilde{B}$. The aforementioned action yields

$$S=\int drdz\sum_{k=\tilde{F},\tilde{B}}a\left[\frac{(\nabla^i\mathcal{E}+\Phi^*_{(k)}\nabla^i\Phi_{(k)})(\nabla_i\mathcal{E}+\Phi^*_{(k)}\nabla_i\Phi_{(k)})}{(\mathcal{E}+\mathcal{E}^*+\Phi_{(k)}\Phi^*_{(k)})^2}-\frac{\nabla_m\Phi_{(k)}\nabla^m\Phi^*_{(k)}}{(\mathcal{E}+\mathcal{E}^*+\Phi_{(k)}\Phi^*_{(k)})}-\frac{\nabla^j a}{2a}\frac{\nabla_j(\mathcal{E}+\mathcal{E}^*+\Phi_{(k)}\Phi^*_{(k)})}{(\mathcal{E}+\mathcal{E}^*+\Phi_{(k)}\Phi^*_{(k)})}\right]. \tag{33}$$

III. CHARGING SOLUTION

In Ref. [33], it was shown how to achieve the solution of the complex system of equations of the type given by relations (31) and (32). Namely, in order to find the form of the potentials $\Phi_{(k)}$, one should additionally assume that they are analytic functions and also analyze their asymptotic behavior. Having all these in mind, by using the chain rule, we arrive at

$$\frac{d^2\mathcal{E}}{d\Phi^2_{(\tilde{F})}}\vec{\nabla}\Phi_{(\tilde{F})}(\vec{\nabla}\Phi_{(\tilde{F})})^2 = \nabla^2\mathcal{E}\vec{\nabla}\Phi_{(\tilde{F})}-\nabla^2\Phi_{(\tilde{F})}\vec{\nabla}\mathcal{E} \tag{34}$$

and

$$\frac{d^2\mathcal{E}}{d\Phi^2_{(\tilde{B})}}\vec{\nabla}\Phi_{(\tilde{B})}(\vec{\nabla}\Phi_{(\tilde{B})})^2 = \nabla^2\mathcal{E}\vec{\nabla}\Phi_{(\tilde{B})}-\nabla^2\Phi_{(\tilde{B})}\vec{\nabla}\mathcal{E}. \tag{35}$$

When one implements Eq. (31) multiplied and summed by $\sum_{m=\tilde{F},\tilde{B}}\vec{\nabla}\Phi_{(m)}$ and Eq. (32) multiplied by $\vec{\nabla}\mathcal{E}$, one obtains the relations for the potentials in the forms

$$\forall\,\vec{\nabla}\Phi_{(\tilde{F})}\neq 0,$$
$$\frac{d^2\mathcal{E}}{d\Phi^2_{(\tilde{F})}}(\vec{\nabla}\Phi_{(\tilde{F})})^2-\mathrm{Re}\left(\mathcal{E}+\sum_{k=\tilde{F},\tilde{B}}\Phi^*_{(k)}\Phi_{(k)}\right)\frac{\nabla^2 a}{a}=0 \tag{36}$$

and for the $\Phi_{(\tilde{B})}$ potential

$$\forall \vec{\nabla}\Phi_{(\tilde{B})} \neq 0,$$
$$\frac{d^2\mathcal{E}}{d\Phi^2_{(\tilde{B})}}(\vec{\nabla}\Phi_{(\tilde{B})})^2 - \mathrm{Re}\left(\mathcal{E} + \sum_{k=\tilde{F},\tilde{B}} \Phi^*_{(k)}\Phi_{(k)}\right)\frac{\nabla^2 a}{a} = 0. \quad (37)$$

In the next step, we decompose the complex Ernst potential $\mathcal{E}$ in the form of a sum, where the first term does not contain Λ, while the second in Λ dependent [34]:

$$\mathcal{E} = \mathcal{E}_0 + \mathcal{E}_\Lambda. \quad (38)$$

The forms of the equations reveal that there are no Λ terms at zero order in the cosmological constant; thus, we get

$$\frac{d^2\mathcal{E}_0}{d\Phi^2_{(m)}} = 0, \quad (39)$$

which implies that to zero order in cosmological constant $\mathcal{E}_0$ is a linear function of the potential in question, i.e.,

$$\mathcal{E}_0(\Phi_{(\tilde{F})}) = f_0 + f_1\Phi_{(\tilde{F})}, \quad \mathcal{E}_0(\Phi_{(\tilde{B})}) = b_0 + b_1\Phi_{(\tilde{F})}, \quad (40)$$

where f_i and b_i are arbitrary constants. From the boundary conditions at infinity, i.e., $\Phi_{(m)} \to 0$ and $\mathcal{E}_0 \to 1$, we can fix f_0 and b_0 to be 1, while $b_1 = -\frac{2}{Q_{(\tilde{B})}}$ and $f_1 = -\frac{2}{Q_{(\tilde{F})}}$. It implies the following forms of the potentials bounded with the visible and dark sectors:

$$\Phi_{(\tilde{F})} = \frac{Q_{(\tilde{F})}}{\xi + 1}, \qquad \Phi_{(\tilde{B})} = \frac{Q_{(\tilde{B})}}{\xi + 1}, \quad (41)$$

where we have set

$$Q_{(\tilde{F})} = \frac{\sqrt{2-\alpha}}{2}(Q_{(F)} - Q_{(B)}),$$
$$Q_{(\tilde{B})} = \frac{\sqrt{2+\alpha}}{2}(Q_{(F)} + Q_{(B)}). \quad (42)$$

The quantities $Q_{(F)}$ and $Q_{(B)}$ are expressed in the standard way (as in charged Kerr metric derivation [33]) provided by

$$Q_{(F)} = \frac{e_{(F)} + ig_{(F)}}{M}, \qquad Q_{(B)} = \frac{e_{(B)} + ig_{(B)}}{M}, \quad (43)$$

where $e_{(i)}$ and $g_{(i)}$ are bounded, respectively, with electric and magnetic charges of Maxwell and dark matter sectors.

As was revealed in [33–36], the form of $\mathcal{E}$ for Kerr AdS/dS spacetime yields

$$\mathcal{E} = \frac{\xi - 1}{\xi + 1} + \frac{1}{\beta^2}((\xi+1)^2 + q^2), \quad (44)$$

where $\xi = px - iqy$, $1/\beta^2 = \pm\Lambda M^2/3$, and, respectively, the other quantities are defined as [35]

$$p = \frac{k}{M}, \qquad q = \frac{\hat{a}}{M}, \qquad x = \frac{r - M}{k},$$
$$y = \cos\theta, \qquad k = \sqrt{M^2 - \hat{a}^2}. \quad (45)$$

By M we have denoted the total mass of the black hole, while $\hat{a} = J/M$ stands for its angular momentum per unit mass.

On the other hand, one arrives at the following expressions for the metric ingredients:

$$f = \mathrm{Re}\mathcal{E} + \sum_{k=\tilde{F},\tilde{B}} \Phi^*_{(k)}\Phi_{(k)}$$
$$= \frac{\xi^*\xi - 1 + |Q_{(\tilde{F})}|^2 + |Q_{(\tilde{B})}|^2}{|\xi+1|^2} + \frac{1}{\beta^2}\mathrm{Re}((\xi+1)^2 + q^2)$$
$$= \frac{\Delta_{(\tilde{F},\tilde{B})} - \Delta_\theta \hat{a}^2 \sin^2\theta}{(r^2 + \hat{a}^2\cos^2\theta)}, \quad (46)$$

$$h = \mathrm{Im}\mathcal{E} = 2\frac{\mathrm{Im}\xi}{|\xi+1|^2} + \frac{1}{\beta^2}\mathrm{Im}(\xi^2 + 2\xi) \quad (47)$$

$$= -2\hat{a}\cos\theta\left(\frac{r}{\beta^2 M^2} + \frac{M}{r^2 + \hat{a}^2}\right), \quad (48)$$

while the other components of the rotating black hole with visible and hidden sector field line elements are provided by the following expressions:

$$\omega = \frac{\hat{a}\sin^2\theta(\Delta_{(\tilde{F},\tilde{B})} - \Delta_\theta(r^2 + \hat{a}^2))}{\hat{a}^2\Delta_\theta\sin^2\theta - \Delta_{(\tilde{F},\tilde{B})}}, \quad (49)$$

$$a = \sin\theta\sqrt{\Delta_\theta\Delta_{(\tilde{F},\tilde{B})}}, \quad (50)$$

$$e^{2u} = \sqrt{a}(r^2 + \hat{a}^2\cos^2\theta), \quad (51)$$

$$e^{\frac{b}{2}} = \frac{\Delta_{(\tilde{F},\tilde{B})} - \hat{a}^2\Delta_\theta\sin^2\theta}{\sqrt{\Delta_\theta\Delta_{(\tilde{F},\tilde{B})}}(r^2 + \hat{a}^2\cos^2\theta)\sin\theta}, \quad (52)$$

where we have denoted, in the standard way, the quantities appearing in the above relations, i.e.,

$$\Delta_r = (r^2 + \hat{a}^2)\left(1 - \frac{\Lambda}{3}r^2\right) - 2Mr, \quad (53)$$

$$\Delta_\theta = 1 + \frac{\Lambda}{3}\hat{a}^2\cos^2\theta, \quad (54)$$

$$\Delta_{(\tilde{F},\tilde{B})} = \Delta_r + e^2_{(F)} + g^2_{(F)} + e^2_{(B)} + g^2_{(B)}$$
$$+ \alpha(e_{(F)}g_{(B)} + e_{(B)}g_{(F)}). \quad (55)$$

It can be seen that dark matter influences not only the line element by the squares of electric and magnetic charges, likewise the adequate Maxwell field ingredients, but also a mixture of terms appears. They are connected with the sum of electric Maxwell magnetic dark photon and electric dark photon magnetic Maxwell charges, with the proportionality constant α being the coupling constant between visible and dark sectors [see the action (1)].

Moreover, inspection of Eq. (55) reveals that if we neglect dark charge $e_{(B)} = 0$, then the main influence is exerted by dark magnetic charge $g_{(B)}$, and the dark magnetic charge couples to the Maxwell electric one, i.e., $\alpha e_{(F)} g_{(B)}$. The same feature of the dark sector was also spotted in the analysis of the influence of dark matter on transport coefficients in chiral solids [37,38].

IV. UNIQUENESS THEOREM FOR DARK MATTER STATIONARY AXISYMMETRIC BLACK HOLE

In this section, we pay attention to the problem of the uniqueness [39] of stationary axisymmetric black holes with the dark sector. On the other hand, the uniqueness theorem for a static axially symmetric black hole in a magnetic universe (say, dark Melvin universe) was proved in Ref. [40].

In what follows, we shall restrict our considerations to the case when the cosmological constant is equal to zero. Namely, in relations (31) and (32), we set $\nabla^2_{(r,z)} a = 0$ and $a = r$.

It can be checked, by direct calculations, that by defining homographic change of the variables, for the previously defined quantities connected with both gauge fields, provided by

$$\mathcal{E} = \frac{\xi - 1}{\xi + 1}, \tag{56}$$

and taking into account the relations (41)–(43), in the case of absence of the cosmological constant, the equations of motion (31) and (32) reduce to a single complex one of the following form:

$$(\xi^* \xi - 1 + |Q_{(\tilde{F})}|^2 + |Q_{(\tilde{B})}|^2) \nabla^2 \xi = 2 \xi^* \vec{\nabla} \xi \cdot \vec{\nabla} \xi. \tag{57}$$

A. Boundary conditions

In order to study the relevant boundary conditions for a stationary axisymmetric dark photon black hole, one introduces the two-dimensional manifold $\mathcal{M}$ [41], with the spheroidal coordinates $r^2 = (\lambda^2 - c^2)(1 - \mu^2)$ and $z = \lambda\mu$, where we set $\mu = \cos\theta$. In the coordinates in question, the black hole event horizon boundary is situated at $\lambda = c$. On the other hand, two rotation axis segments distinguishing the south and the north parts of the event horizon are located at the limits $\mu = \pm 1$. The obtained line element on the two-dimensional manifold in question can be written in the form as follows:

$$dr^2 + dz^2 = (\lambda^2 - \mu^2 c^2) \left(\frac{d\lambda^2}{\lambda^2 - c^2} + \frac{d\mu^2}{1 - \mu^2} \right). \tag{58}$$

Next, let us choose the domain of outer communication $\langle\langle \mathcal{D} \rangle\rangle$ being a rectangle, which implies

$$\begin{aligned} \partial\mathcal{D}^{(1)} &= \{\mu = 1, \lambda = c, \ldots, R\}, \\ \partial\mathcal{D}^{(2)} &= \{\lambda = c, \mu = 1, \ldots, -1\}, \\ \partial\mathcal{D}^{(3)} &= \{\mu = -1, \lambda = c, \ldots, R\}, \\ \partial\mathcal{D}^{(4)} &= \{\lambda = R, \mu = -1, \ldots, 1\}. \end{aligned} \tag{59}$$

As far as the boundary conditions [41,42] are concerned, at infinity, due to the asymptotic flatness of the solution we require that f/λ^2, h, $\tilde{A}_0$, A_3, $\tilde{B}_0$, and B_3 constitute well-behaved functions of $1/\lambda$ and μ, in the limit where $1/\lambda \to 0$, and the values of $\tilde{A}_0$ and $\tilde{B}_0$ tend, respectively, to $e_{(\tilde{F})}$ and $e_{(\tilde{B})}$, while the values of A_3 and B_3 coincide with $g_{(\tilde{F})}$ and $g_{(\tilde{B})}$, respectively. In terms of the above requirements, they are given by

$$\tilde{A}_0 = -e_{(\tilde{F})}\mu + \mathcal{O}(\lambda^{-1}), \quad \tilde{B}_0 = -e_{(\tilde{B})}\mu + \mathcal{O}(\lambda^{-1}), \tag{60}$$

$$A_3 = -g_{(\tilde{F})}\mu + \mathcal{O}(\lambda^{-1}), \quad B_3 = -g_{(\tilde{B})}\mu + \mathcal{O}(\lambda^{-1}), \tag{61}$$

$$h = J\mu(3 - \mu^2) + \mathcal{O}(\lambda^{-1}), \tag{62}$$

$$\frac{f}{\lambda^2} = (1 - \mu^2)(1 + \mathcal{O}(\lambda^{-1})). \tag{63}$$

On the black hole event horizon, where $\lambda \to c$, the quantities in question should behave regularly; i.e., they yield the following relations:

$$f = \mathcal{O}(1), \qquad \frac{1}{f} = \mathcal{O}(1), \tag{64}$$

$$\partial_\mu A_3 = \mathcal{O}(1), \qquad \partial_\lambda A_3 = \mathcal{O}(1), \tag{65}$$

$$\partial_\mu \tilde{A}_0 = \mathcal{O}(1), \qquad \partial_\lambda \tilde{A}_0 = \mathcal{O}(1), \tag{66}$$

$$\partial_\mu B_3 = \mathcal{O}(1), \qquad \partial_\lambda B_3 = \mathcal{O}(1), \tag{67}$$

$$\partial_\mu \tilde{B}_0 = \mathcal{O}(1), \qquad \partial_\lambda \tilde{B}_0 = \mathcal{O}(1), \tag{68}$$

$$\partial_\mu h = \mathcal{O}(1), \qquad \partial_\lambda h = \mathcal{O}(1). \tag{69}$$

On the other hand, in the vicinity of the symmetry axis, where $\mu \to 1$ (north polar segment) and $\mu \to -1$ (south polar segment), one requires that A_3, $\tilde{A}_0$, B_3, $\tilde{B}_0$, f, and h

ought to be regular functions of λ and μ. Consequently, they are provided by

$$f = \mathcal{O}(1-\mu^2), \qquad \frac{1}{f}\partial_\mu f = 1 + \mathcal{O}(1-\mu^2), \tag{70}$$

$$\partial_\lambda A_3 = \mathcal{O}(1-\mu^2), \qquad \partial_\mu A_3 = \mathcal{O}(1), \tag{71}$$

$$\partial_\lambda \tilde{A}_0 = \mathcal{O}(1-\mu^2), \qquad \partial_\mu \tilde{A}_0 = \mathcal{O}(1), \tag{72}$$

$$\partial_\lambda B_3 = \mathcal{O}(1-\mu^2), \qquad \partial_\mu B_3 = \mathcal{O}(1), \tag{73}$$

$$\partial_\lambda \tilde{B}_0 = \mathcal{O}(1-\mu^2), \qquad \partial_\mu \tilde{B}_0 = \mathcal{O}(1), \tag{74}$$

$$\partial_\mu h + 2(\tilde{A}_0 \partial_\mu A_3 - A_3 \partial_\mu \tilde{A}_0) + 2(\tilde{B}_0 \partial_\mu B_3 - B_3 \partial_\mu \tilde{B}_0) = \mathcal{O}(1-\mu^2), \tag{75}$$

$$\partial_\lambda h = \mathcal{O}((1-\mu^2)^2). \tag{76}$$

B. Uniqueness of solutions

It was revealed in Ref. [43] that various combinations of the Ernst equations of the type given by (31) and (32) can be comprised in a matrix equation of the form

$$\partial_r[P^{-1}\partial_r P] + \partial_z[P^{-1}\partial_z P] = 0, \tag{77}$$

where by P we have denoted 3×3 Hermitian matrices with unit determinants. Additionally, it happens that, for any constant, invertible matrix A, the matrix APA^{-1} is the solution of the relation (77), enabling one to create all the transformations referred to Ernst's system of partial differential equations.

Let us assume that the matrix P components are enough differentiable in the domain of outer communication $\langle\langle \mathcal{D} \rangle\rangle$ of the two-dimensional manifold $\mathcal{M}$, with boundary $\partial\mathcal{D}$. Suppose further that we have two different matrix solutions of Eq. (77), P_1 and P_2, subject to the same boundary and differentiability conditions, and consider the difference between the aforementioned relations satisfies the equation of the form

$$\nabla(P_1^{-1}(\nabla Q)P_2) = 0, \tag{78}$$

where one sets $Q = P_1 P_2^{-1}$. Multiplying Eq. (78) by $Q^\dagger$ and taking the trace of the result, we achieve the following outcome:

$$\nabla^2 q = \mathrm{Tr}[(\nabla Q^\dagger) P_1^{-1} (\nabla Q) P_2], \tag{79}$$

where we set $q = \mathrm{Tr} Q$. Hermiticity and positive definiteness of the matrix P allow us to postulate the matrix in the form as $P = MM^\dagger$, which, in turn, yields

$$\nabla^2 q = \mathrm{Tr}(\mathcal{J}^\dagger \mathcal{J}), \tag{80}$$

where $\mathcal{J} = M_1^{-1}(\nabla Q) M_2$.

Defining homographic change of the variables, for the previously defined quantities connected with both gauge fields, provided by

$$\epsilon = \frac{\xi - 1}{\xi + 1}, \qquad \Psi_{(\tilde{F})} = \frac{\eta_{(\tilde{F})}}{\xi + 1}, \qquad \Psi_{(\tilde{B})} = \frac{\eta_{(\tilde{B})}}{\xi + 1}, \tag{81}$$

enables us to find that the P matrix implies

$$P_{\alpha\beta} = \eta_{\alpha\beta} - \frac{2\xi_\alpha \bar{\xi}_\beta}{\langle \xi_\delta \bar{\xi}^\delta \rangle}, \tag{82}$$

where we define the scalar product in the form

$$\langle \xi_\delta \bar{\xi}^\delta \rangle = -1 + \sum_\gamma \xi_\gamma \bar{\xi}^\gamma, \qquad \gamma = 1, \ldots, q. \tag{83}$$

In the case under consideration, $\xi^1 = \xi$, $\xi^2 = \eta_{(\tilde{F})}$, $\xi^3 = \eta_{(\tilde{B})}$, and $q = 3$.

Moreover, for brevity of the final notion, we change the notation in the relation (23) for the following:

$$\Phi_{(\tilde{F})} = E_{(\tilde{F})} + iB_{(\tilde{F})}, \qquad \Phi_{(\tilde{B})} = E_{(\tilde{B})} + iB_{(\tilde{B})}. \tag{84}$$

The further step in the uniqueness proof of the dark matter stationary axisymmetric black hole solution is to find the trace $q = \mathrm{Tr}(P_1 P_2^{-1})$. Consequently, after some algebra, one arrives at

$$\begin{aligned} q = P_{\alpha\beta(1)} P^{\alpha\beta}_{(2)} = 3 + \frac{1}{f_1 f_2} \Bigg\{ (f_1 - f_2)^2 + \Bigg[\sum_{i=\tilde{F},\tilde{B}} ((E_{(i)1} - E_{(i)2})^2 + (B_{(i)1} - B_{(i)2})^2) \Bigg]^2 \\ - 2(f_1 + f_2) \sum_{i=\tilde{F},\tilde{B}} [(E_{(i)1} - E_{(i)2})^2 + (B_{(i)1} - B_{(i)2})^2] \\ + \Bigg[2 \sum_{i=\tilde{F},\tilde{B}} (B_{(i)1} E_2^{(i)} - B_{(i)2} E_1^{(i)}) + (h_1 - h_2) \Bigg]^2 \Bigg\}. \end{aligned} \tag{85}$$

Let us turn our attention to the relation (80) and integrate it over the domain of outer communication $\langle\langle \mathcal{D} \rangle\rangle$ (59), using Stoke's theorem. In accordance with the choice of $\langle\langle \mathcal{D} \rangle\rangle$, one gets

$$\begin{aligned}\int_{\partial\langle\langle \mathcal{D} \rangle\rangle} \nabla_k q dS^k &= \int_{\partial\langle\langle \mathcal{D} \rangle\rangle} d\lambda \sqrt{\frac{h_{\lambda\lambda}}{h_{\mu\mu}}} \partial_\mu q|_{\mu=\mathrm{const}} + \int_{\partial\langle\langle \mathcal{D} \rangle\rangle} d\mu \sqrt{\frac{h_{\mu\mu}}{h_{\lambda\lambda}}} \partial_\lambda q|_{\lambda=\mathrm{const}} \\ &= \int_\infty^c d\lambda \sqrt{\frac{h_{\lambda\lambda}}{h_{\mu\mu}}} \partial_\mu q|_{\mu=-1} + \int_c^\infty d\lambda \sqrt{\frac{h_{\lambda\lambda}}{h_{\mu\mu}}} \partial_\mu q|_{\mu=1} + \int_1^{-1} d\mu \sqrt{\frac{h_{\mu\mu}}{h_{\lambda\lambda}}} \partial_\lambda q|_{\lambda=c} + \int_{-1}^1 d\mu \sqrt{\frac{h_{\mu\mu}}{h_{\lambda\lambda}}} \partial_\lambda q|_{\lambda\to\infty} \\ &= \int_{\langle\langle \mathcal{D} \rangle\rangle} \mathrm{Tr}(\mathcal{J}^\dagger \mathcal{J}) dV. \end{aligned} \quad (86)$$

The behavior of the left-hand side of the above equation (86) will be elaborated by considering the integrals over each part of the domain of outer communication $\langle\langle \mathcal{D}^{(i)} \rangle\rangle$, where $i = 1, \ldots, 4$, chosen as a rectangle in the two-dimensional manifold with coordinates (μ, λ).

Namely, on the black hole event horizon $\partial\mathcal{D}^{(2)}$, all the examined functions are well behaved, having asymptotic behavior given by $\mathcal{O}(1)$. As $\lambda \to c$, the r coordinate tends to $r \simeq \mathcal{O}(\sqrt{\lambda - c})$, and the square root has the form of $\sqrt{\frac{h_{\mu\mu}}{h_{\lambda\lambda}}} \simeq \mathcal{O}(\sqrt{\lambda - c})$. Then, one can conclude that $\nabla_k q$ vanishes on the dark matter stationary axisymmetric black hole event horizon.

On the symmetry axis $\partial\mathcal{D}^{(1)}$ and $\partial\mathcal{D}^{(3)}$, when $\mu \pm 1$, all the quantities under inspection are of the order of $\mathcal{O}(1)$. As $\mu \to 1$, the r coordinate tends to $\mathcal{O}(\sqrt{1 - \mu})$, and for the case when $\mu \to -1$, one has that $r \simeq \mathcal{O}(\sqrt{1 + \mu})$. The behaviors of square roots are given by $\sqrt{\frac{h_{\lambda\lambda}}{h_{\mu\mu}}} \simeq \mathcal{O}(\sqrt{1 + \mu})$ when $\mu \to -1$ and $\sqrt{\frac{h_{\lambda\lambda}}{h_{\mu\mu}}} \simeq \mathcal{O}(\sqrt{1 - \mu})$ for $\mu \to 1$. Thus, the relation (86) reveals that $\nabla_m q = 0$ for $\mu \pm 1$.

For the case when $\lambda = R \to \infty$, all functions in question are well behaved and have asymptotic behaviors given by Eqs. (60)–(63). On the other hand, the square root in the considered limit tends to $\sqrt{\frac{h_{\mu\mu}}{h_{\lambda\lambda}}} \simeq \mathcal{O}(\lambda)$. Inspection of the boundary conditions given by relations (60)–(63) and (86), where we have differentiation with respect to λ, reveal the fact that the studied integral tends to zero.

All the aforementioned arguments lead to the conclusion that

$$\int_{\langle\langle \mathcal{D} \rangle\rangle} \mathrm{Tr}(\mathcal{J}^\dagger_{(i)} \mathcal{J}_{(i)}) = 0, \quad (87)$$

which, in turn, implies that $P_{(i)1} = P_{(i)2}$, at all points belonging to the domain of outer communication, comprising a two-dimensional manifold $\mathcal{M}$ with coordinates (r, z).

It means that if one considers two stationary axisymmetric black hole solutions of Einstein-Maxwell dark photon gravity characterized, respectively, by $(f_1, h_1, \tilde{A}_{0(1)}, A_{3(1)}, \tilde{B}_{0(1)}, B_{3(1)})$ and $(f_2, h_2, \tilde{A}_{0(2)}, A_{3(2)}, \tilde{B}_{0(2)}, B_{3(2)})$, being subject to the same boundary and regularity conditions, they are identical.

In summary, the consequences of our research can be summarized as follows.

Theorem. Consider a domain of outer communication $\langle\langle \mathcal{D} \rangle\rangle$ constituting a region of two-dimensional manifold with a boundary $\langle\langle \partial\mathcal{D} \rangle\rangle$, equipped with the coordinate system (r, z) defined by $r^2 = (\lambda^2 - c^2)(1 - \mu^2)$ and $z = \lambda\mu$.

Assume further that $P_{(i)}$ are Hermitian positive, three-dimensional matrices, with unit determinants. On the boundary of the domain $\langle\langle \partial\mathcal{D} \rangle\rangle$, matrices $P_{(1)}$ and $P_{(2)}$ authorize the solution of the equation

$$\partial_r[P^{-1}\partial_r P] + \partial_z[P^{-1}\partial_z P] = 0$$

and satisfy the relation $\nabla_m q = 0$, where $q = \mathrm{Tr}(P_1 P_2^{-1})$.

Then, if $P_{(1)} = P_{(2)}$ in all domains of outer communication $\langle\langle \mathcal{D} \rangle\rangle$, implying that for at least one point $d \in \langle\langle \mathcal{D} \rangle\rangle$, one arrives at the relation $P_{(1)}(d) = P_{(2)}(d)$.

Thus, all the stationary axisymmetric solutions of Einstein-Maxwell dark photon gravity subject to the same boundary and regularity conditions, say, a Kerr-like dark matter black hole, comprise the only stationary axisymmetric symmetric black hole solution, endowed with a regular event horizon, having nonvanishing $\tilde{A}_0$, A_3, $\tilde{B}_0$, and B_3 components of Maxwell visible and hidden sector gauge fields. Having in mind equations introduced in Sec. II, the above components can be rewritten by means of A_0, A_ϕ, B_0, and B_ϕ ones, i.e.,

$$\tilde{A}_0 = \frac{\sqrt{2-\alpha}}{2}(A_0 - B_0), \quad \tilde{B}_0 = \frac{\sqrt{2+\alpha}}{2}(A_0 + B_0), \quad (88)$$

$$\tilde{A}_\phi = \frac{\sqrt{2-\alpha}}{2}(A_\phi - B_\phi), \quad \tilde{B}_\phi = \frac{\sqrt{2+\alpha}}{2}(A_\phi + B_\phi). \quad (89)$$

V. CONCLUSIONS

In our paper, we have elaborated the stationary axisymmetric solution to Einstein-Maxwell dark matter–dark energy black hole solution. The dark sector was modeled by dark photon theory, i.e., a new Abelian gauge field coupled to the ordinary Maxwell one, by means of the so-called kinetic mixing term. On the other hand, the positive cosmological constant mandated phenomenologically the features of the influence of dark energy. The equations of motion for the considered system were arranged into the form of Ernst-like system of complex relations.

The obtained metric components of the rotating Kerr-like dark matter–dark energy solution have envisaged the ordinary cosmological constant dependence (like in Kerr dS spacetime [34,36]), while the dark sector imprints its presence by square of electric Maxwell and dark photon charges, square of magnetic charges of both sectors, and mixing electric-magnetic charges pertaining to visible-dark and dark-visible sectors.

Then, we restrict our attention to the case of the asymptotically flat solution and rearrange the adequate Ernst equations into the form of a matrix equation. Choosing the domain of outer communication $\langle\langle\mathcal{D}\rangle\rangle$ as a rectangle in a two-dimensional manifold with coordinates (r, z), one shows that the two matrix solutions of the underlying equations being subject to the same boundary and regularity conditions are equal in $\langle\langle\mathcal{D}\rangle\rangle$. Thus, one can draw the conclusion that Kerr-like dark matter stationary axisymmetric black hole solution to Einstein-Maxwell dark photon gravity authorizes the only stationary axisymmetric solution in the theory under inspection, having nonzero A_0, A_ϕ, B_0, and B_ϕ gauge field components.

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