

Prospects of detection of subsolar mass primordial black hole and white dwarf binary mergers

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The subsolar mass primordial black hole (PBH) attracts attention as robust evidence of its primordial origin against the astrophysical black hole. Not only with themselves, PBHs can also form binaries with ordinary astrophysical objects, catching them by gravitational wave (GW) bremsstrahlung. We discuss the detectability of the inspiral GWs from binaries consisting of a PBH and a white dwarf (WD) by using space-borne gravitational wave interferometers like DECIGO. The conservative assessment shows the expected event number in three years by DECIGO is $\mathcal{O}(10^{-6})$ for $M_{\text{PBH}} \sim 0.1M_{\odot}$. Possible enhancement mechanisms of WD-PBH binary formation may amplify this event rate. We discuss how large enhancement associated with WDs is required to detect WD-PBH merger events without violating the existing constraints on the PBH-PBH merger by the ground-based detector.

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I. INTRODUCTION

Primordial black holes (PBHs) are hypothetical black holes that could have formed in the early Universe before star formation [1–4]. They have attracted attention more and more as a candidate for dark matter [5], seeds of supermassive black holes [6,7] (see, e.g., Refs. [8,9] for implications of supermassive PBH by high-redshift luminous galaxies found by the James Webb Space Telescope), planetary-mass microlensing objects towards the Galactic bulge detected by the Optical Gravitational Lensing Experiment [10], hypothetical Planet 9 in our solar system [11,12], triggers of faint supernovae called calcium-rich gap transients [13], etc., (see, e.g., Refs. [14,15] for recent reviews on astrophysical implications of PBHs). A significant property of PBHs is that they can have a wide range of mass even smaller than the solar mass, unlike the astrophysical black holes that are formed through the gravitational collapse of massive stars. Thus, the detection of subsolar mass black holes is a smoking gun for the existence of PBHs.

While subsolar mass PBHs have been searched mainly by the gravitational microlensing¹ so far (see Ref. [17] for a

summary of the existing observational constraints on the PBH abundance), gravitational waves (GWs) have been attracting much attention as a new tool to probe the subsolar mass PBHs. In the early Universe, PBHs can form binaries by the tidal torque exerted by a neighboring PBH [18–20]. After forming the binaries, their orbits gradually shrink as they emit the GWs. Depending on the separation and the component masses of binaries, their GWs can have the frequency to which the GW detectors are sensitive. LIGO-Virgo-KAGRA (LVK) Collaboration has already obtained the constraints on the PBH abundance from the search of the subsolar mass black hole binaries [21–24]. Several groups independently report candidates of GWs from subsolar mass black hole binaries [25–27]. Furthermore, Pujolas *et al.* [28] estimated the detectability of the PBH binaries in various future observations, such as Einstein Telescope [29], LISA [30], and DECIGO [31,32].

While the dominant formation channel of the PBH binaries is the three-body interaction in the early Universe, the GW bremsstrahlung can lead to PBH binaries in the late Universe [20,33,34]. In this channel, an unbounded PBH is captured by another PBH through the close encounter due to the energy loss by the GW emission. This channel can form not only PBH-PBH binaries but also the binaries of a PBH and another astrophysical compact object. For example, Tsai *et al.* [35] and Sasaki *et al.* [36] estimated the merger rate of binaries consisting of a PBH and a neutron star (NS). Tsai *et al.* discussed the scenario in which GW170817 is an NS-PBH merger event. Sasaki *et al.* showed that the NS-PBH binaries are subdominant, and

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¹So far any microlensing search for PBHs has not detected a PBH, but several candidate events are reported, represented by the single event by the Subaru Hyper Suprime-Camera [16].

their merger rate is not enough to explain LVK's NS-BH merger candidates.

In this work, we newly consider the binaries consisting of a subsolar mass PBH and a white dwarf (WD). The merger frequencies of the WD-PBH binaries are $\mathcal{O}(10^{-2} - 10^{-1})$ Hz, which are interesting targets of DECIGO. However, numerical simulations of the WD-PBH merger are yet to be done. We then focus on the inspiral GW and investigate the detectability of the WD-PBH merger event via the maximum detectable distance of inspiral events with a sufficiently large signal-to-noise ratio (SNR) in the DECIGO observation. As WD-PBH inspiral GWs can be mimicked by WD-WD ones, we also discuss whether they can be distinguished only by the differences in the merger frequencies. Finally, we estimate the expected event number of the WD-PBH binaries in the three-year DECIGO mission.

The paper is organized as follows. In Sec. II, we discuss the detectability and the distinguishability of WD-PBH binaries by assuming three years of observation of DECIGO. In Sec. III, we present the merger rate estimation, and Sec. IV is devoted to the conclusions and future perspective. The details of the merger rate calculation are summarized in the Appendix. Throughout the paper, we suppose that the PBH mass function is monochromatic.

II. DETECTABILITY BY DECIGO

A. Signal-to-noise ratio for the inspiraling WD-PBH binary

In this subsection, we evaluate the detectability of WD-PBH binaries only through the inspiral GWs. The most optimistic scenario, which we hereafter assume, is that DECIGO keeps observing the binary inspiral for three years from the beginning of the observation run, and the binary merges just after the end of the observation run. The detectability is quantified by the SNR of the inspiral waveform from the initial frequency f_{\min} to the final frequency f_{\max} in the detector frame. Note that we basically suppose quasicircular orbits for WD-PBH binaries for brevity though Cholis *et al.* [37] suggested that the binaries formed in the GW bremsstrahlung have high eccentricities and merge within a few years, typically keeping their eccentricities even at the merger.

The measured signal in the frequency domain is obtained by

$$\tilde{h}(f) = F_+ \tilde{h}_+(f) + F_\times \tilde{h}_\times(f), \quad (2.1)$$

where F_+ and F_\times are the antenna pattern functions, and $\tilde{h}_{+/\times}$ are plus/cross modes expressed by

$$\tilde{h}_+(f) = (1 + \cos^2 \iota) \tilde{h}_0(f), \quad \tilde{h}_\times(f) = -2i \cos \iota \tilde{h}_0(f), \quad (2.2)$$

with the inclination angle ι . \tilde{h}_0 is given by

$$\tilde{h}_0(f) = \sqrt{\frac{5}{96}} \pi^{-2/3} \left(\frac{(1+z)G\mathcal{M}_c}{c^3} \right)^{5/6} \frac{c}{d_L} f^{-7/6} e^{i\Psi(f)}, \quad (2.3)$$

where $\Psi(f)$ is the GW phase, G is the Newtonian constant of gravity, and c is the speed of light. \mathcal{M}_c is the chirp mass defined by

$$\mathcal{M}_c := \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}, \quad (2.4)$$

with the component masses m_1 and m_2 in the source frame. z and d_L are the source redshift and the luminosity distance, respectively. Their relation is given by

$$d_L = (1+z) \int_0^z \frac{cdz}{H(z)}. \quad (2.5)$$

We employ *astropy* [38] to calculate the luminosity distance and use the cosmological parameters obtained by Planck18 [39]. Given the detector noise spectrum $S_n(f)$, we define the SNR by

$$\rho_0 = \left[4 \int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)} df \right]^{1/2}. \quad (2.6)$$

Following Ref. [40], we estimate the SNR averaged over the extrinsic parameters (e.g., the source position in the sky) by the formula,

$$\rho = \frac{8}{5} \frac{A}{d_L} \sqrt{I_7}, \quad (2.7)$$

where I_7 and A are given by

$$I_7 = \int_{f_{\min}}^{f_{\max}} df \frac{f^{-7/3}}{S_n(f)}, \quad (2.8)$$

$$A = \sqrt{\frac{5}{96}} \pi^{-2/3} c \left(\frac{(1+z)G\mathcal{M}_c}{c^3} \right)^{5/6}. \quad (2.9)$$

The integration range $[f_{\min}, f_{\max}]$ in Eq. (2.8) is obtained below as a frequency evolution during the three-year observation of DECIGO.

In the present work, we use the DECIGO's design power spectral density (PSD) [41],

$$S_n^{\text{DECIGO}}(f) = \left[7.05 \times 10^{-48} \left(1 + \left(\frac{f}{f_p} \right)^2 \right) + 4.8 \times 10^{-51} \left(\frac{f}{1 \text{ Hz}} \right)^{-4} \frac{1}{1 + \left(\frac{f}{f_p} \right)^2} + 5.33 \times 10^{-52} \left(\frac{f}{1 \text{ Hz}} \right)^{-4} \right] \text{Hz}^{-1}, \quad (2.10)$$

with $f_p = 7.36 \text{ Hz}$.

Here, we focus only on GWs from the inspiral phase. We assume that f_{max} is given by

$$f_{\text{max}} = \frac{f_{\text{merge}}^{\text{WD-PBH}}}{1+z}, \quad (2.11)$$

where $f_{\text{merge}}^{\text{WD-PBH}}$ is a typical merger frequency in the source frame. In the case of WD-PBH binaries, WDs would be tidally disrupted in the final stage of the inspiral phase, and then $f_{\text{merge}}^{\text{WD-PBH}}$ is determined by the binary separation equal to the tidal radius, R_{tidal} , which is estimated by balancing the tidal force and the WD's self-gravity as

$$\begin{aligned} F_{\text{tidal}} &= F_{\text{grav}} \quad \text{at } r = R_{\text{tidal}} \\ \Rightarrow \frac{2GM_{\text{PBH}}}{r^2} \left(\frac{R_{\text{WD}}}{r} \right) &= \frac{GM_{\text{WD}}}{R_{\text{WD}}^2} \quad \text{at } r = R_{\text{tidal}} \\ \Rightarrow R_{\text{tidal}} &= \left(\frac{2M_{\text{PBH}}}{M_{\text{WD}}} \right)^{1/3} R_{\text{WD}}, \end{aligned} \quad (2.12)$$

where M_{WD} and M_{PBH} are respectively masses of the WD and the PBH, and R_{WD} is the radius of the WD. We use the following mass-radius relation:

$$R_{\text{WD}} = 0.0126 R_{\odot} \left(\frac{M_{\text{WD}}}{M_{\odot}} \right)^{-1/3}, \quad (2.13)$$

which is proposed in Ref. [42]. The corresponding frequency is given by

$$f_{\text{tidal}} = \frac{1}{\pi} \sqrt{\frac{G(M_{\text{WD}} + M_{\text{PBH}})}{R_{\text{tidal}}^3}}. \quad (2.14)$$

Equation (2.12) implies that R_{tidal} is smaller than the radius of the WD, R_{WD} , if $2M_{\text{PBH}} < M_{\text{WD}}$. In such cases, as the merger frequency, $f_{\text{merge}}^{\text{WD-PBH}}$, we should use the Kepler frequency at the time when the orbital radius becomes the sum of the radii of the WD and the PBH (R_{PBH}),

$$f_{\text{Kepler}} = \frac{1}{\pi} \sqrt{\frac{G(M_{\text{WD}} + M_{\text{PBH}})}{(R_{\text{WD}} + R_{\text{PBH}})^3}}. \quad (2.15)$$

The merger frequency is determined by the one realized earlier,

$$f_{\text{merge}}^{\text{WD-PBH}} = \min[f_{\text{Kepler}}, f_{\text{tidal}}]. \quad (2.16)$$

As for f_{min} , we introduce f_{start} as the corresponding frequency in the source frame,

$$f_{\text{min}} = \frac{f_{\text{start}}}{1+z}. \quad (2.17)$$

The duration that the binary requires to evolve from the frequency f_1 to f_2 ($> f_1$) is given by (see Refs. [43–45])

$$\begin{aligned} t(f_2) - t(f_1) &= \frac{5}{256} \left(\frac{G\mathcal{M}_c}{c^3} \right)^{-5/3} \\ &\times \pi^{-8/3} (f_1^{-8/3} - f_2^{-8/3}). \end{aligned} \quad (2.18)$$

From this expression, by assuming that the binary merges at the end of the observation period in our optimistic scenario, we can evaluate the frequency at the onset of the observation run [i.e., three years ($= T_{\text{obs}}$) before the merger], which is corresponding to f_{start} , as

$$T_{\text{obs}} = t(f_{\text{merge}}^{\text{WD-PBH}}) - t(f_{\text{start}}), \quad (2.19)$$

and it gives

$$f_{\text{start}} = \left\{ (f_{\text{merge}}^{\text{WD-PBH}})^{-8/3} + T_{\text{obs}} \frac{256\pi^{8/3}}{5} \left(\frac{G\mathcal{M}_c}{c^3} \right)^{5/3} \right\}^{-3/8}. \quad (2.20)$$

We set the detection threshold on the SNR at 8 [44] and refer to the maximum distance of the detectable binaries as the horizon distance. Figure 1 shows the horizon distance

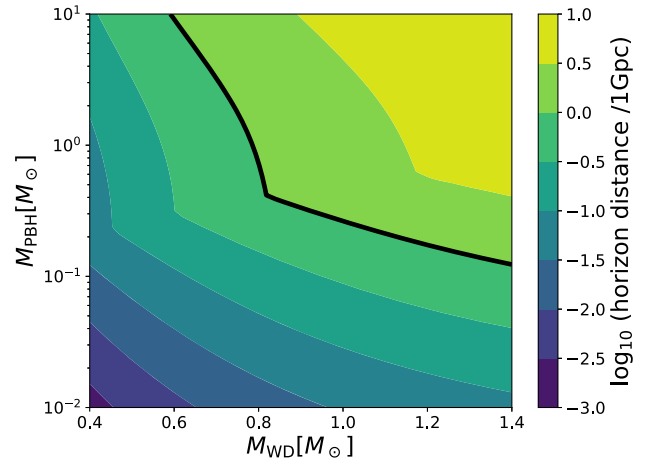


FIG. 1. Horizon distance as a function of M_{PBH} and M_{WD} . The black solid line indicates 1 Gpc. There are breaks around $M_{\text{PBH}} \sim (2-5) \times 10^{-1} M_{\odot}$. Around there, the merger frequency, defined by Eq. (2.16), changes drastically because the tidal frequency f_{tidal} and the Kepler frequency f_{Kepler} have different mass dependencies.

depending on the masses of a PBH and a WD. From this figure, we can find that we would detect the GWs from a WD-PBH binary at ~ 1 Gpc for $M_{\text{WD}} \gtrsim 0.8M_{\odot}$ and $M_{\text{PBH}} \gtrsim 0.5M_{\odot}$.

B. Distinguishability between WD-PBH and WD-WD

In the previous subsection, we evaluated the detectability of WD-PBH binaries. The GW waveform from a compact binary is characterized by the chirp mass \mathcal{M}_c . Therefore, if a WD-WD binary and a WD-PBH binary have similar component masses, they can not be distinguished only from the chirp mass. However, the merger frequency can be different because of the difference in the radii of WDs and PBHs. We discuss, here, how we can distinguish WD-WD binaries and WD-PBH binaries by the difference in the merger frequencies.

The merger frequency of WD-WD binaries, which is denoted by $f_{\text{merge}}^{\text{BWD}}$, is estimated as well as the merger frequency of WD-PBH binaries. We consider the WD-WD binary with the masses of $M_{\text{WD},1}$ and $M_{\text{WD},2}$. We use $R_{\text{tidal},1}$ to denote the tidal radius in which the first WD ($M_{\text{WD},1}$ and $R_{\text{WD},1}$) is disrupted by the companion WD ($M_{\text{WD},2}$). It is estimated by

$$R_{\text{tidal},1} = \left(\frac{2M_{\text{WD},2}}{M_{\text{WD},1}} \right)^{1/3} R_{\text{WD},1}. \quad (2.21)$$

By changing the labels $1 \leftrightarrow 2$, the tidal radius $R_{\text{tidal},2}$ can be defined. When the two WDs approach as close as the radius,

$$R_{\text{tidal}}^{\text{BWD}} = \max[R_{\text{tidal},1}, R_{\text{tidal},2}], \quad (2.22)$$

the lighter one is tidally disrupted. We define the frequency corresponding to $R_{\text{tidal}}^{\text{BWD}}$ by

$$f_{\text{tidal}}^{\text{BWD}} = \frac{1}{\pi} \sqrt{\frac{G(M_{\text{WD},1} + M_{\text{WD},2})}{(R_{\text{tidal}}^{\text{BWD}})^3}}. \quad (2.23)$$

When $M_{\text{WD},1} \simeq M_{\text{WD},2}$, $R_{\text{tidal}}^{\text{BWD}}$ could be smaller than the sum of the WD's radii. In such cases, as the merger frequency, we should employ the Kepler frequency at the time when the binary separation is equal to the sum of the WD's radii,

$$f_{\text{Kepler}}^{\text{BWD}} = \frac{1}{\pi} \sqrt{\frac{G(M_{\text{WD},1} + M_{\text{WD},2})}{(R_{\text{WD},1} + R_{\text{WD},2})^3}}. \quad (2.24)$$

Thus, as in the case of the WD-PBH binary, the merger frequency of the WD-WD binary is given by the lower one,

$$f_{\text{merge}}^{\text{BWD}} = \min[f_{\text{Kepler}}^{\text{BWD}}, f_{\text{tidal}}^{\text{BWD}}]. \quad (2.25)$$

Figure 2 shows the comparisons between the merger frequencies, $f_{\text{merge}}^{\text{WD-PBH}}$ and $f_{\text{merge}}^{\text{BWD}}$, and the initial frequency $f_{\text{start}}^{\text{WD-PBH}}$ of the WD-PBH binary as functions of M_{PBH} . If the frequency satisfies $f_{\text{merge}}^{\text{BWD}} \leq f_{\text{start}}^{\text{WD-PBH}}$, the WD-PBH binary merges at the frequency that WD-WD binaries with the same masses never reach. So, we can distinguish the WD-PBH binary and the WD-WD binary by the difference in the merger frequencies.

Figure 3 zooms in on the right panel of Fig. 2, focusing on the regions where $0.5M_{\odot} \lesssim M_{\text{WD},1} \lesssim 0.6M_{\odot}$. For some mass ranges, the merger frequency of the WD-WD binary is located between $f_{\text{start}}^{\text{WD-PBH}}$ and $f_{\text{merge}}^{\text{WD-PBH}}$. This means that the waveforms of the WD-PBH binary and the WD-WD binary overlap in the frequency domain. If this overlap is significant, it might be difficult to distinguish between the WD-PBH and WD-WD binaries. We define the quantity named ‘‘distinguishability’’ by

$$x_{\text{dis}} := \frac{q^2 - q'^2}{q^2}, \quad (2.26)$$

where q' is defined by

$$q' = \frac{8}{5} \frac{A}{d_L} \sqrt{l_7}, \quad (2.27)$$

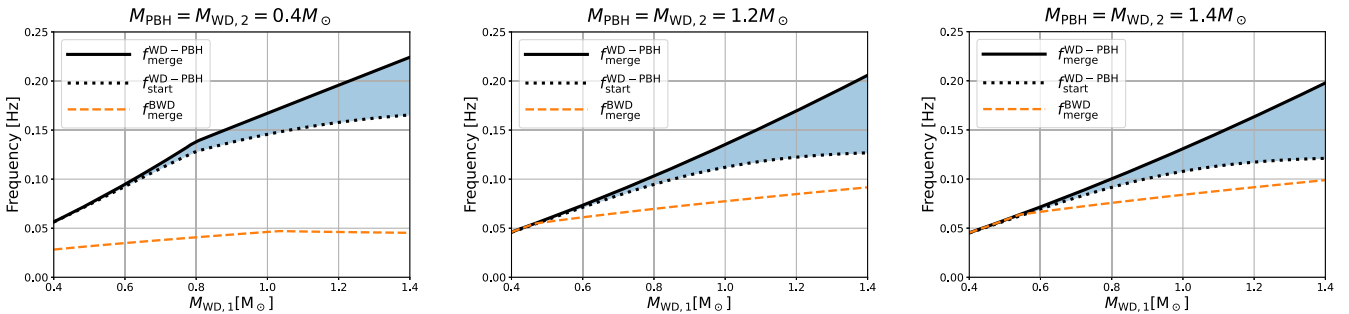


FIG. 2. Comparison of the relevant frequencies for WD-PBH and WD-WD binaries. Black solid and dotted lines are maximum and minimum frequencies realized by WD-PBH inspirals, respectively. Orange dashed lines show the merger frequency of the WD-WD binary whose component masses are $M_{\text{WD},1}$ and $M_{\text{WD},2}$. If the orange dashed line is below the black dotted line, WD-PBH and WD-WD binaries can be distinguished only by the inspiral waveform. We assume three years of observation.

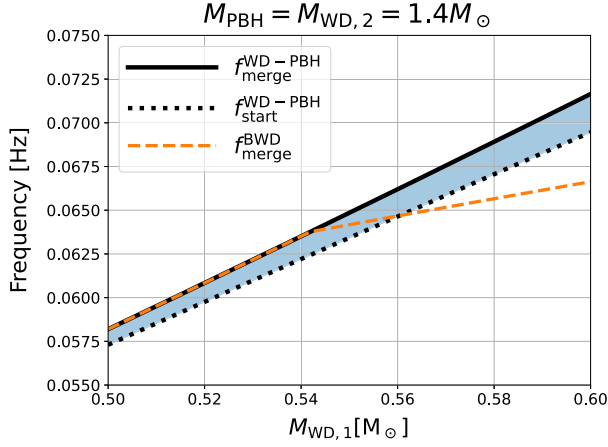


FIG. 3. Enlarged figure of the right panel of Fig. 2. While binaries are distinguishable for $M_{\text{WD},1} \gtrsim 0.56 M_{\odot}$, it is hard for $M_{\text{WD},1} \lesssim 0.56 M_{\odot}$ and impossible for $M_{\text{WD},1} \lesssim 0.54 M_{\odot}$ to distinguish them only by the inspiral waveform and hence the merger waveform is required which is beyond the scope of the paper.

with “wrongly” replacing the WD-PBH merger frequency with that of the WD-WD one,

$$I_7' = \begin{cases} \int_{f_{\text{start}}^{\text{WD-PBH}}/(1+z)}^{f_{\text{merge}}^{\text{WD}}/(1+z)} df \frac{f^{-7/3}}{S_n(f)} & \text{for } f_{\text{start}}^{\text{WD-PBH}} < f_{\text{merge}}^{\text{WD}}, \\ 0 & \text{for } f_{\text{start}}^{\text{WD-PBH}} \geq f_{\text{merge}}^{\text{WD}}. \end{cases} \quad (2.28)$$

If $f_{\text{merge}}^{\text{WD}}$ is equal or very close to $f_{\text{merge}}^{\text{WD-PBH}}$, we could not judge whether the binary is a WD-PBH binary or a WD-WD binary only from their inspiral parts because they have the same frequency evolution during the observational period. In such case, q' equals q . In this sense, the “distinguishability” x_{dis} is a quantity representing how likely we can distinguish a WD-PBH binary from a WD-WD binary. That is, for $x_{\text{dis}} \simeq 1$, we can distinguish a WD-PBH binary and a WD-WD binary, but for $x_{\text{dis}} \simeq 0$, we can not distinguish a WD-PBH binary and a WD-WD binary. Figure 4 shows the distinguishability x_{dis} as a function of the WD’s mass with $M_{\text{PBH}} = 1.2 M_{\odot}$ and $1.4 M_{\odot}$. Let us focus on the case of $M_{\text{PBH}} = 1.4 M_{\odot}$ (thick black line in Fig. 4). For $M_{\text{WD}} \gtrsim 0.55 M_{\odot}$, the distinguishability x_{dis} of a WD-PBH binary and a WD-WD binary is almost unity. This is because the merger frequency of a WD-WD binary is lower than the initial frequency of a WD-PBH binary, as Fig. 3 shows. On the other hand, for $M_{\text{WD}} \lesssim 0.55 M_{\odot}$, the merger frequencies of a WD-WD binary and a WD-PBH binary are the same. This leads to no distinguishability in SNR. Even in this case, WD-PBH binaries and WD-WD binaries could be distinguished once we go beyond the conservative approach. We can consider various effects related to the coalescence (e.g., tidal deformability). We will discuss it in more detail in the concluding section.

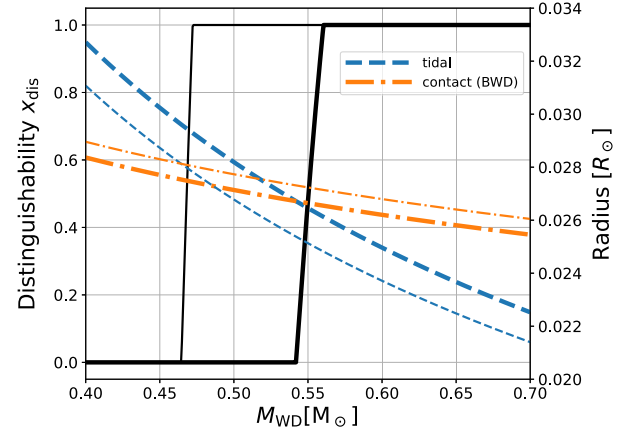


FIG. 4. Distinguishability [Eq. (2.26)] in black lines. The thick (thin) line shows the PBH mass of $1.4 M_{\odot}$ ($1.2 M_{\odot}$). The distance to the source is chosen such that the reference SNR $\rho = 30$. Although the integrals I_7 and I_7' depend on the redshift, the result is not much affected by the value of the reference SNR. The blue dashed lines represent the tidal radius of the WD-PBH binary. The orange dash-dotted lines represent the contact radius of the WD-WD binary whose component masses are M_{WD} and $1.4 M_{\odot}$ ($1.2 M_{\odot}$).

III. EXPECTED NUMBER OF EVENTS

In this section, we estimate the merger rate of the WD-PBH binaries. The expected number of events is given by

$$N_{\text{event}}^{\text{WD-PBH}} = T_{\text{obs}} \int_{M_{\text{WD,min}}}^{M_{\text{WD,max}}} \frac{dM_{\text{WD}}}{M_{\text{WD}}} \int_0^{z_{\text{up}}} dz \frac{dV}{dz} \times \int_{M_{h,\text{min}}}^{M_{h,\text{max}}} dM_h \frac{dn_{\text{halo}}}{dM_h} \frac{dR_{\text{WD-PBH}}}{d \ln M_{\text{WD}}}, \quad (3.1)$$

with the comoving volume $V(z)$ within the redshift z , the halo mass function $\frac{dn_{\text{halo}}}{dM_h}$, and the merger rate density per halo per logarithmic WD mass $\frac{dR_{\text{WD-PBH}}}{d \ln M_{\text{WD}}}$. We below neglect the redshift dependence of the halo mass function and the merger rate and use the current values for simplicity. We set the minimum and maximum masses of halos at $M_{h,\text{min}} = 400 M_{\odot}$ and $M_{h,\text{max}} = 10^{16} M_{\odot}$, respectively. We have confirmed that the event rate of WD-PBH mergers is not very sensitive to the choice of $M_{h,\text{min}}$ and $M_{h,\text{max}}$. The detailed description of each term in the integrand is given in the Appendix. Figure 5 shows the resultant event number in three years as a function of the PBH mass. Note that we assume the fraction of PBH to the dark matter, f_{PBH} , is unity though astrophysical observations have already put the constraint $f_{\text{PBH}} \lesssim \mathcal{O}(10^{-1})$ for $M_{\text{PBH}} = 0.1 M_{\odot}$. Figure 5 implies that the merger rate of WD-PBH binaries is too low to detect them by three years of observation of DECIGO.

The small event number is because of our conservative estimation. The expected event number would be increased

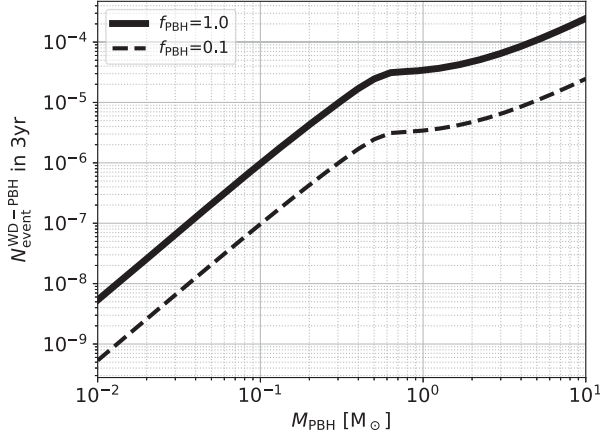


FIG. 5. Expected number of WD-PBH mergers detected by three-year observation of DECIGO. Thick solid line shows the case of $f_{\text{PBH}} = 1.0$, and thin dashed line shows the case of $f_{\text{PBH}} = 0.1$. The break at $M_{\text{PBH}} \sim 0.6M_{\odot}$ originates from the crossover between the tidal frequency and the Kepler frequency, which can be seen in Fig. 1.

by several orders of magnitude, depending on the assumptions on the number densities, the velocity dispersions, etc., of the target objects. In the context of the NS-PBH merger, several possibilities (e.g., formation in ultrafaint dwarf galaxies or globular clusters and PBH clustering at their formation) provoking such enhancement are presented [35]. Before concluding this section, we briefly see how much enhancement is required (and allowed) to detect the WD-PBH binaries. Let us parametrize the enhancement of the event number by [35]

$$\tilde{N}_{\text{event}}^{\text{WD-PBH}} = \alpha_{\text{WD}} \times \alpha_{\text{PBH}} \times f_{\text{PBH}} \times N_{\text{event}}^{\text{WD-PBH}}, \quad (3.2)$$

where α_{WD} and α_{PBH} are the enhancement factors induced by some mechanisms related to the WDs and the PBHs, respectively.

Even forgetting about the realistic astrophysical processes, α_{PBH} cannot be enlarged arbitrarily because it would conflict with the current constraint on the PBH-PBH merger rate achieved by the ground-based GW detectors. As we explained in Sec. I, PBH-PBH binaries are formed in two channels, and the early-time formation basically dominates the late-time formation. However, if the enhancement factor α_{PBH} for WD-PBH is too large, it would also enhance the late-time PBH-PBH formation, making it dominant and leading to the violation of the current constraint on the event rate of subsolar mass PBH binary coalescences. To discuss it quantitatively, we calculate the event number of PBH-PBH mergers in the late-time formation scenario. Because α_{PBH} phenomenologically parametrizes the enhancement due to the PBH properties, the PBH-PBH merger rate would also be enhanced in a similar way as

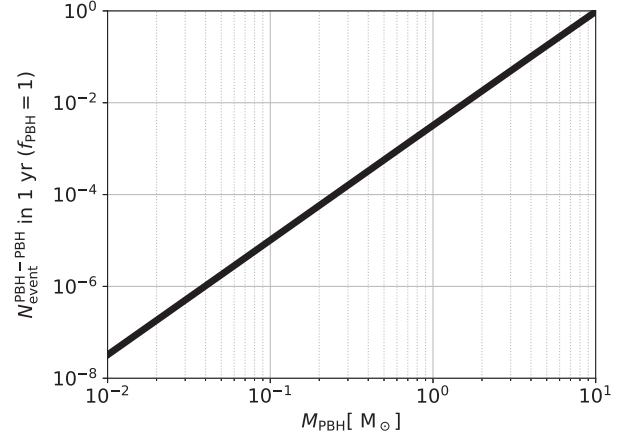


FIG. 6. Expected number of PBH-PBH mergers detected by one-year observation by LIGO.

$$\tilde{N}_{\text{event}}^{\text{PBH-PBH}} \sim (\alpha_{\text{PBH}} f_{\text{PBH}})^2 N_{\text{event}}^{\text{PBH-PBH}}, \quad (3.3)$$

where $N_{\text{event}}^{\text{PBH-PBH}}$ is the expected number of PBH-PBH mergers estimated with $f_{\text{PBH}} = 1$ and $\alpha_{\text{PBH}} = 1$. So far, binary mergers with masses of less than $1M_{\odot}$ have not been detected. Therefore, $\tilde{N}_{\text{event}}^{\text{PBH-PBH}}$ should be less than unity. For $M_{\text{PBH}} = 10^{-1}M_{\odot}$ as our interest, we find $N_{\text{event}}^{\text{PBH-PBH}} \sim 1 \times 10^{-5}$ (see Fig. 6 in the Appendix). Therefore, we get the upper bound,

$$\tilde{N}_{\text{event}}^{\text{PBH-PBH}} \lesssim 1 \Rightarrow \alpha_{\text{PBH}} f_{\text{PBH}} \lesssim 3 \times 10^2. \quad (3.4)$$

Substituting this into $\tilde{N}_{\text{event}}^{\text{WD-PBH}}$ and taking the value $N_{\text{event}}^{\text{WD-PBH}} = 2 \times 10^{-6}$, the required enhancement factor α_{WD} for the detectable event rate of WD-PBH binaries would be

$$\tilde{N}_{\text{event}}^{\text{WD-PBH}} \gtrsim 1 \Rightarrow \alpha_{\text{WD}} \gtrsim 2 \times 10^3. \quad (3.5)$$

If such an enhancement would be realized, one or more WD-PBH merger events could be detected by DECIGO without contradicting the current constraint on the PBH-PBH merger rate.

IV. CONCLUSIONS AND FUTURE PERSPECTIVE

We have investigated the detectability of WD and sub-solar mass PBH binaries formed by the GW bremsstrahlung. To this end, in Sec. II, we first investigated the maximum distance of the detectable binaries as the horizon distance assuming the simple waveform model given by Eqs. (2.7)–(2.9) and the three-year observation of DECIGO with its design sensitivity. We have shown that the GWs from WD-PBH binaries at ~ 1 Gpc are detectable when their mass satisfies $M_{\text{WD}} \gtrsim 0.8M_{\odot}$ and $M_{\text{PBH}} \gtrsim 0.5M_{\odot}$. Second, we discussed the distinguishability between

WD-PBH and WD-WD mergers by estimating the merger frequency from the comparison of their tidal and contact radii. We found that WD-PBH mergers can be distinguished from other binaries just by looking at the merger frequency unless the WD mass is less than $\sim 0.55M_\odot$. Finally, We evaluated the merger rate and the expected number of WD-PBH merger events in Sec. III. We assumed the standard halo properties and simplified WD distributions. The conservative expectation of the number of WD-PBH merger events is too small [$\sim \mathcal{O}(10^{-6})$] to be detected for $M_{\text{PBH}} \sim 10^{-1}M_\odot$.

In a specific situation such as clustering distribution or low velocity-dispersion of target objects, the expected number of WD-PBH merger events would be much enhanced. We should note that an arbitrarily large enhancement due to some mechanisms related to PBHs would contradict the current constraint on the PBH-PBH merger rate in the ground-based detectors because the same mechanisms would also promote the PBH-PBH merger. We conclude that $\mathcal{O}(10^3)$ enhancement due to WD-related mechanisms could enable the detection of $\mathcal{O}(1)$ WD-PBH merger events by three-year DECIGO observation without violating the current PBH-PBH constraint.

We assume the monochromatic spectrum for the PBH mass distribution throughout this work. In many works, it is pointed out that the mass distribution can be broader, such as log-normal distribution. If the distribution is sharp and the f_{PBH} summed over all PBH masses is the same level, the merger rate will not be changed. If the spectrum is much broader, the merger rate will be affected significantly.

One may include additional effects to improve the detectability and distinguishability of PBH-WD binaries. For example, the waveform modeling can be improved by, e.g., the tidal deformability of WDs. As a binary becomes closer, the WD is gradually deformed, resulting in the modification in the gravitational waveform. This modification is significant around the merger. Another example is the effect of mass transfer. The WD-WD binary under the stable mass transfer is discussed in the literature [46,47]. It is known that there is the possibility that the binary separation becomes wider as the mass transfers from the smaller star to the heavier star (called *outspiral*). The GWs from such a binary are much different from that of the case without the mass transfer. The synergy between GWs and electromagnetic emissions can also be beneficial in identifying the source. WD-WD binary mergers are candidates of type Ia supernovae [48,49]. WD-PBH binary mergers could emit electromagnetic emissions that are different from those of WD-WD binary mergers. On the analogy of NS-BH binaries, tidal disruption events can also be expected from the WD-PBH binary mergers. Numerical simulations for these merger events would be interesting (see [50] for a numerical simulation of a binary coalescence

of a NS and a subsolar mass PBH). As well as concrete mechanisms for the merger rate enhancement discussed in Sec. III, these points should be worth studying in future work.

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APPENDIX: DETAILS OF MERGER RATE CALCULATION

1. Halo density profile

We employ the definition of the halo mass by the mass contained in a sphere where the averaged density is 200 times larger than the critical density ρ_{crit} . The virial radius r_{vir} is defined by the radius of the sphere which is used in the definition of the halo mass. In other words, r_{vir} satisfies,

$$M_h = \frac{4\pi}{3} r_{\text{vir}}^3 \times 200\rho_{\text{crit}} = 4\pi \int_0^{r_{\text{vir}}} dr r^2 \rho_{\text{DM}}(r), \quad (\text{A1})$$

where $\rho_{\text{DM}}(r)$ is the dark matter density profile. We use the Navarro-Frenk-White (NFW) profile [51,52] for dark matter mass density profile,

$$\rho_{\text{DM}}(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}, \quad (\text{A2})$$

where ρ_s and r_s characterize the density profile and are determined from the halo mass M_h and the concentration factor $C := r_{\text{vir}}/r_s$. The virial radius is given by the halo mass as shown in Eq. (A1). So, we get the r_s from M_h and C . The density parameter ρ_s is given by

$$\frac{\rho_s}{\rho_{\text{crit}}} = \frac{200}{3} \frac{C^3}{\ln(1+C) - C/(1+C)}. \quad (\text{A3})$$

Ludlow *et al.* [53] provides us with the fitting formula of the concentration parameter as a function of the halo mass and the redshift,

$$C(\nu) = c_0 \left(\frac{\nu}{\nu_0} \right)^{-\gamma_1} \left[1 + \left(\frac{\nu}{\nu_0} \right)^{1/\beta} \right]^{-\beta(\gamma_2 - \gamma_1)}, \quad (\text{A4})$$

where $\nu(z) := \delta_{\text{sc}}/\sigma_m(M, z)$ is the dimensionless peak height, $\delta_{\text{sc}} = 1.686$ is the spherical top-hat collapse threshold, and the parameters $\{c_0, \beta, \gamma_1, \gamma_2, \nu_0\}$ are

$$c_0 = 3.395 \times x^{-0.215}, \quad (\text{A5})$$

$$\beta = 0.307 \times x^{0.540}, \quad (\text{A6})$$

$$\gamma_1 = 0.628 \times x^{-0.047}, \quad (\text{A7})$$

$$\gamma_2 = 0.317 \times x^{-0.893}, \quad (\text{A8})$$

$$\nu_0 = (4.135 - 0.564x - 0.210x^2 + 0.0557x^3 - 0.00348x^4) \times D(z)^{-1}, \quad (\text{A9})$$

with $x = 1 + z$ and the linear growth factor $D(z)^{-1}$. These relation is valid for $0 \leq \log(1 + z) \leq 1$ and $-8 \leq \log_{10}[M/(h^{-1}M_\odot)] \leq 16.5$ Table. Here, h is the Hubble parameter in the unit of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The function $\sigma_m(M, z)$ is the root mean square of the linear density field smoothed on the scale $R(M) = [3M/(4\pi\bar{\rho}_m)]^{1/3}$ and defined by

$$\sigma_m^2(M_h, z) = \frac{1}{2\pi^2} \int_0^\infty dk k^2 P(k, z) |\hat{W}(kR(M_h))|^2, \quad (\text{A10})$$

where $P(k)$ stands for the linear matter power spectrum and $\hat{W}(kR)$ is the Fourier transformation of the top-hat window function with radius R defined in the real space. σ_m is approximated by

$$\sigma_m(M, z) = D(z) \frac{22.26\xi^{0.292}}{1 + 1.53\xi^{0.275} + 3.36\xi^{0.198}}, \quad (\text{A11})$$

where

$$\xi := \left(\frac{M_h}{10^{10} h^{-1} M_\odot} \right)^{-1}. \quad (\text{A12})$$

The linear growth factor is approximated by

$$D(z) = \frac{\Omega_m(z) \Psi(0)}{\Omega_{m,0} \Psi(z)} (1 + z)^{-1}, \quad (\text{A13})$$

with

$$\Psi(z) = \Omega_m(z)^{4/7} - \Omega_\Lambda(z) + \left(1 + \frac{\Omega_m(z)}{2} \right) \left(1 + \frac{\Omega_\Lambda(z)}{70} \right),$$

$$\Omega_\Lambda(z) = \frac{\Omega_{\Lambda,0}}{\Omega_{\Lambda,0} + \Omega_{m,0}(1 + z)^3}, \quad (\text{A14})$$

and $\Omega_m(z) = 1 - \Omega_\Lambda(z)$.

2. Average of capture rate σv_{rel}

The cross-section σ [not to confuse it with the perturbation amplitude σ_m [Eq. (A10)]] of a PBH and a WD with the relative velocity v_{rel} is

$$\sigma = 2\pi \left(\frac{85\pi}{6\sqrt{2}} \right)^{2/7} \frac{G^2 (M_{\text{PBH}} + M_{\text{WD}})^{10/7} M_{\text{WD}}^{2/7} M_{\text{PBH}}^{2/7}}{c^{10/7} v_{\text{rel}}^{18/7}}. \quad (\text{A15})$$

The probability distribution of v_{rel} would be approximated by the truncated Maxwell-Boltzmann distribution [54],²

$$p(v_{\text{rel}}; v_{\text{vir}}, v_{\text{DM}}) = F_0 [e^{-v_{\text{rel}}^2/v_{\text{DM}}^2} - e^{-v_{\text{vir}}^2/v_{\text{DM}}^2}] \Theta(v_{\text{vir}} - v_{\text{rel}}), \quad (\text{A16})$$

where v_{vir} is the virial velocity, v_{DM} is the velocity dispersion of the dark matter (PBHs), and F_0 is the constant determined by the normalization,

$$1 = 4\pi \int_0^{v_{\text{vir}}} dv v^2 p(v; v_{\text{vir}}, v_{\text{DM}}). \quad (\text{A17})$$

v_{DM} and v_{vir} can be related with halo parameters via Eq. (6) of Ref. [54],

$$v_{\text{DM}} = \sqrt{\frac{GM(r < r_{\text{max}})}{r_{\text{max}}}} = \frac{v_{\text{vir}}}{\sqrt{2}} \sqrt{\frac{C}{x_{\text{max}}} \frac{g(x_{\text{max}})}{g(C)}}, \quad (\text{A18})$$

where $g(C) = \ln(1 + C) - C/(1 + C)$, and $x_{\text{max}} = r_{\text{max}}/r_s = 2.1626$ (equal to C_m of Ref. [54]). $M(r < r_{\text{max}})$ is the mass contained in the sphere of the radius r_{max} . Using the NFW profile [Eq. (A2)], we get

$$M(r < r_{\text{max}}) = 4\pi \int_0^{r_{\text{max}}} dr r^2 \rho_{\text{DM}}(r)$$

$$= 4\pi r_s^3 \rho_s \int_0^{x_{\text{max}}} dx \frac{x}{(1 + x)^2}$$

$$= 4\pi r_s^3 \rho_s \left[\ln(1 + x_{\text{max}}) - \frac{x_{\text{max}}}{1 + x_{\text{max}}} \right]. \quad (\text{A19})$$

Once we get the parameters $\{r_s, \rho_s, C\}$, we obtain the velocity dispersion v_{DM} from the first equality of Eq. (A18) and the virial velocity v_{vir} from the second equality. The average of σv_{rel} is given by

$$\langle \sigma v_{\text{rel}} \rangle = 4\pi \int_0^{v_{\text{vir}}} dv v^2 p(v; v_{\text{vir}}, v_{\text{DM}}) \sigma v. \quad (\text{A20})$$

Using Eqs. (A16) and (A18), we get

$$\langle \sigma v_{\text{rel}} \rangle \simeq N \times 2\pi \left(\frac{85\pi}{6\sqrt{2}} \right)^{2/7} \times \frac{G^2 (M_{\text{PBH}} + M_{\text{WD}})^{10/7} M_{\text{WD}}^{2/7} M_{\text{PBH}}^{2/7}}{c^{10/7} v_{\text{DM}}^{11/7}}, \quad (\text{A21})$$

²Strictly speaking, Bird *et al.* [54] employed this distribution for the PBH-PBH relative velocity.

where

$$N = \hat{F}_0 \int_0^{\hat{v}_{\text{vir}}} d\hat{v} \hat{v}^{3/7} (e^{-\hat{v}^2} - e^{-\hat{v}_{\text{vir}}^2}), \quad (\text{A22})$$

with $\hat{v} = v/v_{\text{DM}}$, $\hat{v}_{\text{vir}} = v_{\text{vir}}/v_{\text{DM}}$, and $\hat{F}_0 = 4\pi v_{\text{DM}}^3 F_0$.

3. Merger rate in a particular halo

The merger rate of a WD with the mass of M_{WD} and a PBH in a particular halo with the mass of M_{h} is

$$\frac{dR_{\text{WD-PBH}}}{d \ln M_{\text{WD}}} = 4\pi \int_0^{r_{\text{vir}}} dr r^2 \frac{dn_{\text{WD}}}{d \ln M_{\text{WD}}} n_{\text{PBH}} \langle \sigma v_{\text{rel}} \rangle, \quad (\text{A23})$$

where n_{WD} and n_{PBH} are the number density profiles of WDs and PBHs, respectively.

For PBHs, we simply assume that the density profile is proportional to the halo mass density,

$$\rho_{\text{PBH}}(r) = f_{\text{PBH}} \rho_{\text{DM}}(r), \quad (\text{A24})$$

and the mass spectrum of PBHs is monochromatic,

$$n_{\text{PBH}}(r) = \frac{\rho_{\text{PBH}}(r)}{M_{\text{PBH}}} = f_{\text{PBH}} \frac{\rho_{\text{DM}}(r)}{M_{\text{PBH}}}. \quad (\text{A25})$$

Sasaki *et al.* [36] uses the exponential density profile for NSs. We assume that the number density profile of the WDs also is an exponential form, i.e.,

$$\frac{dn_{\text{WD}}}{d \ln M_{\text{WD}}}(r) = \mathcal{N}_{\text{WD}} e^{-r/r_{\text{WD}}}, \quad (\text{A26})$$

where \mathcal{N}_{WD} is the normalization constant depending on the mass M_{WD} , and r_{WD} is the characteristic radius that is assumed to be independent of the mass. We assume [36]

$$r_{\text{WD}} = 0.1 r_{\text{s}}. \quad (\text{A27})$$

The normalization \mathcal{N}_{WD} is calculated as follows. First, we start with the derivation of the galaxy's stellar mass. We use the relation between the halo mass M_{h} and the stellar mass M_* of the galaxy. Behroozi *et al.* [55] gives the fitting function,

$$\log_{10}[M_{\text{h}}/M_{\odot}] = \log_{10} M_1 + \beta \log_{10} \left(\frac{M_*}{M_{*,0}} \right) + \frac{(M_*/M_{*,0})^{\delta}}{1 + (M_*/M_{*,0})^{-\gamma}} - \frac{1}{2}, \quad (\text{A28})$$

with

$$\log_{10} M_1 = M_{1,0} + M_{1,a}(a-1), \quad (\text{A29})$$

$$\log_{10} M_{*,0} = M_{*,0,0} + M_{*,0,a}(a-1), \quad (\text{A30})$$

$$\beta = \beta_0 + \beta_a(a-1), \quad (\text{A31})$$

$$\gamma = \gamma_0 + \gamma_a(a-1), \quad (\text{A32})$$

$$\delta = \delta_0 + \delta_a(a-1), \quad (\text{A33})$$

where a is the scale factor. The parameters are taken from Table 2 of Ref. [55], $M_{1,0} = 12.35$, $M_{1,a} = 0.28$, $M_{*,0,0} = 10.72$, $M_{*,0,a} = 0.55$, $\beta_0 = 0.44$, $\beta_a = 0.18$, $\gamma_0 = 1.56$, $\gamma_a = 2.51$, $\delta_0 = 0.57$, and $\delta_a = 0.17$. The galaxy's stellar mass determines the normalization constant of the mass spectrum of the main sequence stars through the condition,

$$\begin{aligned} M_* &= V \int_{M_{\text{MS,min}}}^{M_{\text{MS,max}}} dM_{\text{MS}} M_{\text{MS}} \frac{dn}{dM_{\text{MS}}} \\ &= V \int_{M_{\text{MS,min}}}^{M_{\text{MS,max}}} dM_{\text{MS}} \frac{dn}{d \ln M_{\text{MS}}}, \end{aligned} \quad (\text{A34})$$

where V is defined by

$$V = \frac{4\pi}{3} r_{\text{max}}^3, \quad (\text{A35})$$

and we set $M_{\text{MS,min}} = 0.01 M_{\odot}$ and $M_{\text{MS,max}} = 100 M_{\odot}$. We use the following mass spectrum [56]:

$$\frac{dn}{d \ln M_{\text{MS}}} = \begin{cases} A_{\text{BD}} \left(\frac{M_{\text{MS}}}{0.08 M_{\odot}} \right)^{1-\alpha_{\text{BD}}} & (0.01 \leq M_{\text{MS}}/M_{\odot} \leq 0.08) \\ A_{\text{MS}} \left(\frac{M_{\text{MS}}}{0.5 M_{\odot}} \right)^{1-\alpha_{\text{MS1}}} & (0.08 \leq M_{\text{MS}}/M_{\odot} \leq 0.5) \\ A_{\text{MS}} \left(\frac{M_{\text{MS}}}{0.5 M_{\odot}} \right)^{1-\alpha_{\text{MS2}}} & (M_{\text{MS}}/M_{\odot} \geq 0.5), \end{cases} \quad (\text{A36})$$

where $\alpha_{\text{BD}} = 0.8$, $\alpha_{\text{MS1}} = 1.3$, and $\alpha_{\text{MS2}} = 2.0$ [57]. The constant A_{MS} is determined by the continuity at $M_{\text{MS}} = 0.08 M_{\odot}$, and A_{BD} is chosen so that Eq. (A34) holds. We assume that the main sequence stars with masses of $1 M_{\odot} \leq M_{\text{MS}} \leq 8 M_{\odot}$ collapse to WDs. Taking into account the mass loss of the stars, the mass of a WD is estimated by the empirical relation [58],

$$\frac{M_{\text{WD}}}{M_{\odot}} = 0.339 + 0.129 \frac{M_{\text{MS}}}{M_{\odot}}, \quad (\text{A37})$$

where M_{MS} is a mass of the progenitor of the WD. Using this relation, we can convert the mass spectrum of the main sequence stars to that of the WDs which is denoted by

$$\frac{d\bar{n}_{\text{WD}}}{d \ln M_{\text{WD}}},$$

that has no radial dependence. This number density is understood as the averaged one defined by

$$\frac{d\bar{n}_{\text{WD}}}{d \ln M_{\text{WD}}} = \frac{1}{V} \int_0^{r_{\text{max}}} \mathcal{N}_{\text{WD}} e^{-r/r_{\text{WD}}} \times 4\pi r^2 dr. \quad (\text{A38})$$

The normalization constant is hence determined by

$$\begin{aligned} \mathcal{N}_{\text{WD}} &= \frac{V}{4\pi \int_0^{r_{\text{max}}} dr r^2 e^{-r/r_{\text{WD}}} d \ln M_{\text{WD}}} \frac{d\bar{n}_{\text{WD}}}{d \ln M_{\text{WD}}} \\ &\simeq \frac{0.5V}{4\pi r_{\text{WD}}^3} \frac{d\bar{n}_{\text{WD}}}{d \ln M_{\text{WD}}}. \end{aligned} \quad (\text{A39})$$

4. Merger rate per comoving volume and halo mass function

Integrating the merger rate in a particular halo with a weight due to the halo mass function (i.e., the number of halos with the mass of M_h per comoving volume), we get the merger rate of a particular mass of WD per unit time per comoving volume by

$$\int_{M_{h,\text{min}}}^{M_{h,\text{max}}} dM_h \frac{dn_{\text{halo}}}{dM_h} \frac{dR_{\text{WD-PBH}}(M_h)}{d \ln M_{\text{WD}}}, \quad (\text{A40})$$

where dn_{halo}/dM_h is the halo mass function. The fitting function of the halo mass function is given by Tinker *et al.* [59],

$$\frac{dn_{\text{halo}}}{dM_h} = f(\sigma_m) \frac{\bar{\rho}_m}{M_h} \frac{d \ln[\sigma_m^{-1}]}{dM_h}, \quad (\text{A41})$$

with the matter density $\bar{\rho}_m(z) = \Omega_m(z)\rho_{\text{crit}}(z)$ and

$$f(\sigma_m) = A \left[\left(\frac{\sigma_m}{b} \right)^{-a} + 1 \right] e^{-c/\sigma_m^2}. \quad (\text{A42})$$

For $\{A, a, b, c\}$, we use the values for $\Delta = 200$ [corresponding to the definition of the virial radius [Eq. (A1)]] in Table 2 in Ref. [59], i.e.,

$$A = 0.186, \quad a = 1.47, \quad b = 2.57, \quad c = 1.19. \quad (\text{A43})$$

5. Rate of detectable mergers of PBH and WD with a particular mass M_{WD}

The unit comoving volume is defined by

$$\frac{dV}{dz} = \frac{c}{H(z)} 4\pi \left[\int_0^z \frac{cdz'}{H(z')} \right]^2. \quad (\text{A44})$$

Using dV/dz , we get the rate of the detectable mergers of PBH and WD with a particular mass M_{WD} by

$$\int_0^{z_{\text{up}}} dz \frac{dV}{dz} \int_{M_{h,\text{min}}}^{M_{h,\text{max}}} dM_h \frac{dn_{\text{halo}}}{dM_h} \frac{dR_{\text{WD-PBH}}(M_h)}{d \ln M_{\text{WD}}}. \quad (\text{A45})$$

Here, z_{up} is determined by the horizon distance,

$$D_{\text{horizon}} = (1 + z_{\text{up}}) \int_0^{z_{\text{up}}} \frac{cdz'}{H(z')}. \quad (\text{A46})$$

The horizon distance depends on the WD and PBH masses and the detector's sensitivity, and so does z_{up} .

6. Merger rate of PBH-PBH binaries formed in the late universe

To calculate the merger rate of PBH-PBH binaries, we follow the same process as we did for PBH-WD binaries. The expected event number of PBH-PBH mergers is given by

$$N_{\text{event}}^{\text{PBH-PBH}} = T_{\text{obs}} \int_0^{z_{\text{up}}} dz \frac{dV}{dz} \int_{M_{h,\text{min}}}^{M_{h,\text{max}}} dM_h \frac{dn_{\text{halo}}}{dM_h} R_{\text{PBH-PBH}}. \quad (\text{A47})$$

The difference is the merger rate density per one halo; because we assume the monochromatic mass distribution for PBHs, we do not carry out the integration over M_{PBH} . The merger rate density per one halo $R_{\text{PBH-PBH}}$ is given by

$$R_{\text{PBH-PBH}} = 4\pi \int_0^{r_{\text{vir}}} dr r^2 f_{\text{PBH}}^2 n_{\text{DM}}^2(r) \langle \sigma v_{\text{rel}} \rangle. \quad (\text{A48})$$

When we estimate z_{up} , we use the PSD achieved in the first observation run of the LIGO-Virgo Collaboration,

$$\begin{aligned} S_n^{\text{LIGO}}(f) &= \left[\left(\frac{18 \text{ Hz}}{0.1 \text{ Hz} + f} \right)^4 \times 10^{-44} + 4.9 \times 10^{-47} \right. \\ &\quad \left. + 1.6 \times 10^{-45} \left(\frac{f}{2000 \text{ Hz}} \right)^2 \right] \text{ Hz}^{-1}, \end{aligned} \quad (\text{A49})$$

which is taken from Ref. [60]. We assume the one-year observation. Figure 6 shows the event rate of PBH-PBH binary mergers detected by one year observation of the LIGO detector. The result is consistent with preceding studies such as Bird *et al.* [54].

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