

Large extra dimensions from higher-dimensional inflation

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We propose the possibility that compact extra dimensions can obtain large size by higher-dimensional inflation, relating the weakness of the actual gravitational force to the size of the observable Universe. Solution to the horizon problem implies that the fundamental scale of gravity is smaller than 10^{13} GeV, which can be realized in a braneworld framework for any number of extra dimensions. However, requirement of an (approximate) flat power spectrum of primordial density fluctuations consistent with present observations makes this simple proposal possible only for one extra dimension at around the micron scale. After the end of five-dimensional inflation, the radion modulus can be stabilized at a vacuum with positive energy of the order of the present dark energy scale. An attractive possibility is based on the contribution to the Casimir energy of right-handed neutrinos with a mass at a similar scale.

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I. INTRODUCTION

In this work, we study the possibility of connecting different large hierarchies appearing in particle physics and cosmology, which are expected to be described by the same fundamental theory of nature. On the one hand, cosmological observations suggest that the observable Universe must have undergone a period of rapid expansion in order to make compatible its size with causal connection and explain the horizon and homogeneity problems. On the other hand, in particle physics, a major hierarchy is related to the actual weakness of gravitational relative to gauge interactions. Another hierarchy is related to the smallness of neutrino masses which appear to be of the same order of magnitude as the dark energy scale.

A way to connect these hierarchies between particle physics and cosmology is via the size of extra dimensions which are necessary ingredients for consistency of string theory. Indeed, if their size is large compared to the fundamental (string) length, the strength of gravitational interactions becomes strong at distances larger than the

actual four-dimensional (4D) Planck length [1,2]. As a result, the string scale is detached from the Planck mass consistently with all experimental bounds if the observable Universe is localized in the large compact space [2].

The possibility that large extra dimensions may be related to the smallness (in Planck units) of some physical scales [3] is also supported in the context of the swampland program [4] by the distance-duality conjecture, stating that large distances in the string landscape of vacua imply a tower of exponentially light states, such as the Kaluza-Klein (KK) tower associated to the decompactification of extra dimensions [5]. The underlying assumption is that these physical scales are determined by vacuum expectation values of moduli fields with an exponential dependence in the proper distance. The tower of states provides a 4D effective field theory description up to the so-called species scale M_* , where gravity becomes strong and an ultraviolet completion of the theory is needed [6] (such as string theory).

Our proposal is that compact dimensions may have undergone a uniform rapid expansion, together with the three-dimensional noncompact space, by regular exponential inflation driven by an (approximate) higher-dimensional cosmological constant [7]. Their size has grown from the fundamental length to a much larger value, so that at the end of inflation the emergent 4D strength of gravity became much weaker, as it is measured today. We show that this idea implies that the higher-dimensional gravity scale M_* is less than 10^{13} GeV but above 10 TeV for any number of

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extra dimensions $1 \leq d \leq 6$, consistently with accelerator constraints on new physics, laboratory tests of Newton's law, and astrophysical constraints (which impose higher lower bounds on M_* for $d = 1$ and 2) [8].

A period of higher-dimensional inflation can be easily realized by introducing a bulk d -dimensional inflaton with an appropriate scalar potential having a sufficiently flat region, as one usually considers in four dimensions. Besides solving the horizon problem, 4D slow-roll inflation predicts an approximate scale-invariant power spectrum of primordial density perturbations consistently with observations of the cosmic microwave background (CMB). This is due to the fact that the two-point function of a massless minimally coupled scalar field in de Sitter (dS) space behaves logarithmically at distances larger than the cosmological horizon, a property which is though valid for any spacetime dimensionality [9]. When some dimensions are, however, compact, this behavior is expected to hold for distances smaller than the compactification length, while deviating from scale invariance at larger distances, potentially conflicting with observations at large angles. One would, therefore, expect that a period of 4D inflation is necessary for generating the required scale-invariant power spectrum of primordial perturbations.

Here, we show that consistency with CMB observations is maintained if the size of extra dimensions is larger than about a micron, implying a change of behavior in the power spectrum at angles larger than 10° , corresponding to multiple moments $l \lesssim 30$, where experimental errors are getting large [10]. This amazing numerical coincidence singles out the case of $d = 1$ extra dimension of micron size, with a corresponding species scale $M_* \sim 10^9$ GeV, as the only possibility for the simplest realization of our proposal consistent with observations.

An interesting question is to understand the above result from the 4D perspective. It turns out that uniform 5D exponential expansion corresponds in 4D Planck units to a power-law inflation where the scale factor of the non-compact 3D space expands as t^3 in terms of the 4D Friedmann-Robertson-Walker (FRW) metric proper time t , while the radius of the extra dimension expands as t^2 . This implies that, within the 5D theory, one needs less number of e -folds of rapid cosmological expansion in order to solve the apparent 4D horizon problem. Moreover, scale invariance of the power spectrum [11,12] is obtained upon summation over the contribution of the inflaton KK-mode fluctuations that correspond to a tower of scalars from the 4D point of view.

This scenario can be nicely combined with the dark dimension proposal for the cosmological constant using the distance-duality conjecture within the swampland program [13]. It can also realize an old idea for explaining the smallness of neutrino masses by introducing the right-handed neutrinos as 5D bulk states with Yukawa couplings to the left-handed lepton and Higgs doublets that are localized

states on the Standard Model brane stack [14–16]; the neutrino masses are then suppressed due to the wave function of the bulk states.

After the end of 5D inflation, the radion modulus has a runaway exponential quintessencelike potential with an exponent near the upper allowed value for accounting the present dark energy, assuming a local minimum of the 5D scalar potential along the inflaton direction with positive energy of the order of 10^{-2} eV. Alternatively, the radion can be stabilized in a local (metastable) dS vacuum, using the contributions of bulk field gradients [17] or of the Casimir energy, assuming a mass for the bulk R-handed neutrinos of the same order of magnitude [18]. Such a bulk mass was considered in the past in order to weaken the bounds on the size of the extra dimension from an analysis of the neutrino oscillation data [19–22].

II. HIGHER-DIMENSIONAL INFLATION AND ITS 4D INTERPRETATION

We start with $(4 + d)$ -dimensional gravity having d compact extra dimensions of size R :

$$S_{4+d} = \int [d^4 x d^d y] \left(\frac{1}{2} M_*^{2+d} \mathcal{R}^{(4+d)} - \Lambda_{4+d} \right), \quad (1)$$

where $\mathcal{R}^{(4+d)}$ is the higher-dimensional curvature scalar with M_* the corresponding reduced Planck mass and Λ_{4+d} a positive higher-dimensional cosmological constant. For notational simplicity, we use brackets in the measure transforming as a density under 5D diffeomorphisms. Upon dimensional reduction to four dimensions, the effective 4D metric in the Einstein frame is obtained by the line element decomposition

$$ds_{4+d}^2 = \left(\frac{r}{R} \right)^d ds_4^2 + (R/R_0)^2 ds_d^2, \quad (2)$$

where the internal volume associated to ds_d^2 is normalized to $(2\pi R_0)^d$, with R_0 the initial value of the radius at some time instance $\tau = \tau_0$, $r \equiv \langle R \rangle$ after the end of inflation, while we neglected the graviphotons and kept only the overall internal volume $(2\pi R)^d$.¹ The resulting 4D action is

$$S_4 = \int [d^4 x] \left(\frac{1}{2} M_p^2 \mathcal{R}^{(4)} - \frac{d(d+2)}{4} M_p^2 \left(\frac{\partial R}{R} \right)^2 - (2\pi r)^{2d} \frac{\Lambda_{4+d}}{(2\pi R)^d} \right), \quad (3)$$

with the 4D reduced Planck mass M_p and the scalar potential V defined, respectively, by

¹Here, we also consider the internal space to be locally flat, which is automatic for $d = 1$.

$$M_p^2 = M_*^{2+d}(2\pi r)^d; \quad V = \frac{M_p^2}{M_*^{2+d}} \frac{\Lambda_{4+d}}{(R/r)^d}. \quad (4)$$

The vacuum solution of the higher-dimensional theory (1) is a $(4+d)$ -dimensional de Sitter spacetime, describing in flat slicing a uniform exponentially expanding universe of external and internal dimensions. The corresponding line element in conformal coordinates is conformally flat:

$$ds_{4+d}^2 = \hat{a}^2(\tau) d\hat{s}_{4+d}^2; \quad \hat{a}(\tau) = \frac{1}{H\tau}, \quad (5)$$

where $d\hat{s}_{4+d}$ stands for the flat $(4+d)$ -dimensional line element and $\hat{a}(\tau)$ is the conformal factor depending on the conformal time τ and the Hubble parameter H given by $\frac{(3+d)(2+d)}{2} H^2 = \Lambda_{4+d}/M_*^{2+d}$. Using the metric decomposition (2), one can express the uniform $(4+d)$ -dimensional inflation into a time-dependent 4D metric in the Einstein frame $ds_4^2 = a^2(\tau) d\hat{s}_4^2$ and a corresponding time-dependent background for the radius of the internal dimensions $R(\tau)$:

$$ds_{4+d}^2 = \frac{a^2(\tau)}{(R(\tau)/R_0)^d} d\hat{s}_4^2 + \left(\frac{R(\tau)}{R_0}\right)^2 ds_d^2, \quad (6)$$

implying

$$a(\tau) = (R(\tau)/R_0)^{1+d/2} = (\hat{a}(\tau))^{1+d/2}; \quad R(\tau) = \hat{a}(\tau)R_0 = R_0(a(\tau))^{2/(2+d)}, \quad (7)$$

where the normalization of the conformal factor was fixed by the initial condition $a(\tau_0) = \hat{a}(\tau_0) = 1$ at $\tau_0 = H^{-1}$. Comparing with (2), this implies that at the beginning of inflation the Universe was of the size of the higher-dimensional fundamental length M_*^{-1} , which is $(r/R_0)^{d/2}$ larger than the actual 4D Planck length.

In FRW coordinates, where the higher-dimensional cosmic time is $\hat{t} = -H^{-1} \ln(H\tau)$, both the 3D scale factor of the 4D universe and the radius of the internal space expand exponentially:

$$\hat{a}(\hat{t}) = e^{H\hat{t}} \Rightarrow a(\hat{t}) = e^{(1+d/2)H\hat{t}}; \quad R(\hat{t}) = e^{H\hat{t}}R_0. \quad (8)$$

As a result, N e -folds of 4D expansion emerge from $2N/(d+2)$ e -folds of the $(4+d)$ -dimensional metric leading to $2N/(d+2)$ e -folds of the internal radius expansion.

Note, however, that the 4D cosmic time t is different from \hat{t} , since $a(\tau) = (H\tau)^{-(1+d/2)}$. It follows that

$$Ht = \frac{2}{d}(H\tau)^{-d/2} \Rightarrow H\hat{t} = \frac{2}{d} \ln\left(\frac{d}{2}Ht\right), \quad a(t) = \left(\frac{d}{2}Ht\right)^{1+2/d}; \quad R(t) = R_0\left(\frac{d}{2}Ht\right)^{2/d}, \quad (9)$$

which gives, for example,

$$d=1: a(t) = \left(\frac{1}{2}Ht\right)^3; \quad R(t) = R_0\left(\frac{1}{2}Ht\right)^2; \\ d=2: a(t) = (Ht)^2; \quad R(t) = R_0Ht. \quad (10)$$

It follows that uniform exponential expansion in higher dimensions corresponds to a power-law inflation from the 4D perspective, together with an expanding size of the compact dimensions parametrized by a time-dependent radion modulus.

Combining Eqs. (4) and (7), one has

$$M_* = M_p(2\pi r M_*)^{-d/2}; \quad r = R_0 a^{2/(2+d)} = R_0 e^{2N/(2+d)}, \quad (11)$$

where N is the number of e -folds of expansion of the 3D space at the end of the higher-dimensional inflation. It follows that

$$M_* = M_p e^{-dN/(2+d)}. \quad (12)$$

Solution to the horizon problem requires that $N \gtrsim 30$ – 60 when inflation occurs above the TeV and below M_p , implying $M_* \lesssim 10^{13}$ GeV.² On the other hand, imposing $M_* \geq 10$ TeV, one obtains

$$14(\ln 10) \geq \frac{d}{d+2}N \Rightarrow N \lesssim 32\left(1 + \frac{2}{d}\right), \quad (13)$$

which can be easily satisfied for any d . However, in the special cases of $d=1, 2$, one obtains stronger bounds on M_* from nonaccelerator experiments. Indeed, for $d=1$ laboratory tests of Newton's law imply $r \lesssim 30$ μm corresponding to $M_* \gtrsim 10^8$ GeV [23], while for $d=2$ astrophysics constraints from supernovae imply $M_* \gtrsim 10^6$ GeV corresponding to a radius $r \lesssim 0.1$ nm ($r^{-1} \gtrsim 10$ keV) [24].

From Eq. (7), it follows that a comoving distance $d(x, x')$ between two points x and x' in 4D space at equal times corresponds to a $(4+d)$ -dimensional physical distance $d(x, x')\hat{a}(\tau) = d(x, x')a(\tau)^{2/(d+2)} = d(x, x')a(\tau)(R_0/R)^{d/2}$, leading to the relation

$$d_{\text{phys}}^r(x, x') = d(x, x')a(\tau) = d(x, x')\hat{a}(\tau)\left(\frac{R}{R_0}\right)^{d/2} \\ = \hat{d}_{\text{phys}}^r(x, x')\frac{M_p}{M_*}, \quad (14)$$

where the extra factor in the relation between the physical distances d_{phys}^r and \hat{d}_{phys}^r , in four and $d+4$ dimensions, can

²An approximate formula is $N \gtrsim \ln \frac{M_t}{M_*}$, where M_t is the inflation scale.

be understood as the change of mass units from the 4D to the $(4+d)$ -dimensional Planck length, $M_p^2(\tau) = (2\pi R(\tau))^d M_*^{2+d}$.

We now turn to the discussion of the CMB power spectrum. Precision of observational data applies to angles of less than about 10° , corresponding to the distance Mpc scale ($\sim 10^{22}$ m), equivalent to Gpc distance today. On the other hand, the number of e -folds during radiation dominance is about 29 when the inflation scale M_I is at TeV [25], corresponding to an expansion of about 4×10^{12} . The Mpc distance was then around 10^9 m. Assuming that the reheating and radiation evolutions have the same scaling [26], the above distance is scaled down by a factor TeV/M_I at a higher inflation scale, while converting it to higher-dimensional units using (14) it is scaled down by an additional factor M_*/M_p . Assuming $M_I \sim M_*$, one finds a scaling factor of $\text{TeV}/M_p \sim 10^{-15}$ leading to the distance scale of a micron which is miraculously of the order of the size of the fifth dimension for $d = 1$ at the end of inflation. Thus, compatibility with the CMB power spectrum is expected to work only for $R \gtrsim 1 \mu\text{m}$, which selects the case of one extra dimension $d = 1$ as the only possibility that does not require a period of 4D inflation. In the following, we specialize our analysis to this case.³

III. DENSITY PERTURBATIONS FROM INFLATION IN FIVE DIMENSIONS

During 5D inflation, the line element for spatially flat metric is

$$\begin{aligned} ds_5^2 &= \hat{a}^2(\tau)[-d\tau^2 + d\vec{x}^2 + dy^2] \\ &= R_0 \frac{a^2(\tau)}{R(\tau)} (-d\tau^2 + d\vec{x}^2) + (R(\tau)/R_0)^2 dy^2, \end{aligned} \quad (15)$$

with

$$R(\tau) = R_0 \hat{a}(\tau); \quad a(\tau) = \hat{a}^{3/2}(\tau). \quad (16)$$

Density fluctuations can be computed from the equal-time two-point function of a massless minimally coupled scalar field Φ in de Sitter spacetime which is given by (in Fourier space) [9]

$$\langle \Phi^2(\hat{k}, \tau) \rangle = \frac{\pi\tau}{4\hat{a}^3} [J_\nu^2(\hat{k}\tau) + Y_\nu^2(\hat{k}\tau)], \quad (17)$$

where \hat{k} is the comoving space momentum given by $\hat{k}^2 = k^2 + n^2/R_0^2$ with k the 3-space momentum (wave number) and n the KK number, while J_ν and Y_ν are Bessel functions of order $\nu = (D-1)/2 = 2$ for $D = 5$. For small argument

³Note that for $d = 1$ the metric decomposition (2) is exact when the extra dimension is compactified on an interval S^1/Z_2 , since the graviphoton is projected out of the spectrum.

(large wavelengths compared to the dS Hubble radius H^{-1}), the Bessel functions behave as

$$J_\nu(z) \simeq \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu \rightarrow 0; \quad \nu > 0, \quad (18)$$

$$\begin{aligned} Y_\nu(z) &\simeq -\frac{1}{\pi} \left(\frac{2}{z}\right)^\nu - \frac{1}{\pi} \left(\frac{2}{z}\right)^\nu \sum_{l=1}^{\nu-1} \frac{(\nu-l-1)!}{l!} \left(\frac{z}{2}\right)^{2l} \\ &\quad + \frac{2}{\pi} J_\nu(z) \ln \frac{z}{2} + \mathcal{O}(z^\nu); \quad \nu \in \mathbb{Z} \\ &\rightarrow -\frac{1}{\pi} \left(\frac{2}{z}\right)^2 \left(1 + \frac{z^2}{4}\right); \quad \nu = 2. \end{aligned} \quad (19)$$

As a result,

$$\langle \Phi^2(\hat{k}, \tau) \rangle \simeq \frac{\tau}{4\pi\hat{a}^3} \left(1 + \frac{4}{\hat{k}^2\tau^2}\right)^2 \sim \frac{4}{\pi} \frac{H^3}{(\hat{k}^2)^2}. \quad (20)$$

The two-point function at the Standard Model brane position, located, for instance, at the origin of the fifth dimension, is obtained by performing the sum over the KK excitations in \hat{k}^2 :

$$\begin{aligned} \sum_n \frac{1}{\left(\frac{n^2}{R_0^2} + k^2\right)^2} &= -\frac{\partial}{\partial k^2} \sum_n \frac{1}{\frac{n^2}{R_0^2} + k^2} \\ &= -\frac{\pi R_0}{2k} \frac{\partial}{\partial k} \left[\frac{1}{k} \coth(\pi k R_0) \right] \\ &= \frac{\pi R_0}{2k^2} \left(\frac{1}{k} \coth(\pi k R_0) + \frac{\pi R_0}{\sinh^2(\pi k R_0)} \right), \end{aligned} \quad (21)$$

which leads to

$$\langle \Phi^2(k, \tau) \rangle_{y=0} \simeq \frac{2R_0 H^3}{k^2} \left(\frac{1}{k} \coth(\pi k R_0) + \frac{\pi R_0}{\sinh^2(\pi k R_0)} \right). \quad (22)$$

In order to get an estimate of the argument $\pi k R_0$, one can express it in terms of the corresponding 5D physical wavelength $\hat{\lambda} = 2\pi\hat{a}/k$ and use the fact that $R/\hat{\lambda}$ remains the same (time independent) in comoving and physical 5D frames. Note, however, that the 4D physical wavelength (in the 4D Einstein frame) is $\lambda = 2\pi a/k = (R/R_0)^{1/2} \hat{\lambda}$, which is larger than $\hat{\lambda}$ by a factor M_p/M_* and is time dependent. One can, therefore, evaluate $\pi k R_0$ at the end of inflation, in the 5D Einstein frame. It turns out that $\pi k R_0 = 2\pi^2 R/\hat{\lambda} > 1$ for any $\hat{\lambda}$ less than a micron when the size of the extra dimension is in the range $R \sim 1-10 \mu\text{m}$. In this region, the 4D physical wavelengths λ are less than about a kilometre, as argued above.

One can now see that the two-point function changes behavior for momenta k around the compactification scale, as we expected. At large momenta (“small” wavelengths), it scales as $1/k^3$ since $\coth(\pi k R_0) \rightarrow 1$ and the resulting amplitude \mathcal{A} of the power spectrum \mathcal{P} is scale invariant:

$$\pi k R_0 > 1 \Rightarrow \mathcal{A} = \frac{k^3}{2\pi^2} \langle \Phi^2(k, \tau) \rangle_{y=0} \simeq \frac{R_0 H^3}{\pi^2} \sim \frac{H^2}{\pi^2}, \quad (23)$$

where for the last estimate R_0 was chosen of the order of H^{-1} for simplicity. The result is, therefore, the same as the one from the standard cosmology but obtained from 5D inflation. Small violation of scale invariance can be introduced as usual, for instance, by a small tachyonic mass (as in hilltop inflation), leading to a spectral index slightly below unity.

On the other hand, for very large wavelengths corresponding to physical distances in CMB much bigger than Mpc, the argument $(\pi k R_0) < 1$ and the behavior of the two-point function changes to

$$\pi k R_0 < 1 \Rightarrow \langle \Phi^2(k, \tau) \rangle_{y=0} \simeq \frac{4H^3}{\pi k^4}; \quad \mathcal{A} \simeq \frac{2H^3}{\pi^3 k}, \quad (24)$$

leading to a vanishing spectral index (or to almost vanishing by deviating slightly from de Sitter space).

IV. INTERPRETATION OF THE RESULTS FROM A 4D PERSPECTIVE

A higher-dimensional inflaton corresponds, during inflation around a flat region of the scalar potential, to an approximately massless, minimally coupled scalar field in dS space, leading to the $(4 + d)$ -dimensional action:

$$S_{4+d}^{\text{inflaton}} = \int [d^4x] d^d y \left[-\frac{1}{2} (\partial_\mu \Phi)^2 + \frac{1}{2} \left(\frac{R}{R_0} \right)^{-d-2} (\vec{\partial}_y \Phi)^2 \right], \quad (25)$$

where Eq. (2) was used and we computed explicitly the radius dependence, while we kept the 4D metric associated to the line element ds_4^2 as arbitrary background in the measure and for contracting the 4D indices. This action should be considered in the 4D background (7) with the normalization change (6) corresponding to a higher-dimensional dS spacetime, for instance, (10) in the 5D case. Thus, in FRW conformal coordinate frame, it reads in Fourier space

$$\begin{aligned} S_{4+d}^{\text{inflaton}} &= \int d\tau a^2(\tau) d^3 k \sum_{\{n\}} \left\{ \frac{1}{2} \dot{\Phi}_n^2(k) - \frac{1}{2} \left[k^2 + a^2(\tau) \left(\frac{R_0}{R} \right)^d \frac{n^2}{R^2} \right] \tilde{\Phi}_n^2(k) \right\} \\ &= \int d\tau a^2(\tau) d^3 k \sum_{\{n\}} \left\{ \frac{1}{2} \dot{\Phi}_n^2(k) - \frac{1}{2} \left(k^2 + \frac{n^2}{R_0^2} \right) \tilde{\Phi}_n^2(k) \right\}, \end{aligned} \quad (26)$$

where dot stands for derivative with respect to the conformal time τ , $\{n\}$ indicates summation over all components of the vector, and we dropped the vector symbol to simplify notation. Note that, expressing the comoving product kR_0 in terms of physical quantities, one has, for $d = 1$,

$$kR_0 = \frac{k}{a} (\hat{a}R_0) \frac{a}{\hat{a}} = 2\pi \frac{r}{\lambda} \left(\frac{r}{R_0} \right)^{1/2}, \quad (27)$$

where the last equality holds at the end of inflation with r the size of the extra dimension at the micron scale. The factor $(r/R_0)^{1/2}$ scales the physical distances at the end of 5D inflation to 4D units, as pointed out in (14).

We now define $\Phi = \chi/a$ and, thus, $\dot{\Phi} = (\dot{\chi} - (\dot{a}/a)\chi)/a = (\dot{\chi} + 3\chi/(2\tau))/a$, since $a \sim \tau^{-3/2}$ from (7). It follows that

$$\begin{aligned} S_5^{\text{inflaton}} &= \int d\tau d^3 k \sum_n \left\{ \frac{1}{2} \dot{\chi}_n^2(k) \right. \\ &\quad \left. + \frac{1}{2} \left[\frac{15}{4} \frac{1}{\tau^2} - \left(k^2 + \frac{n^2}{R_0^2} \right) \tilde{\chi}_n^2(k) \right] \right\}, \end{aligned} \quad (28)$$

which is precisely the action of a massless minimally coupled scalar in 5D de Sitter spacetime [9], leading to the solution (17). We stress again that the product $kR_0 \neq pR$ for physical 4D momenta $p = k/a$, since $a \neq \hat{a}$, as explained above.

Note that reducing (28) to the 0-mode contribution $n = 0$ does not lead to a scale-invariant power spectrum. Indeed, the coefficient 15/4 differs from the 4D value of 2 for a massless field around dS₄ and corresponds to a 4D metric background of power-law inflation $a(t) \sim t^3$, as in (10), or, equivalently, by an inflaton around dS₄ near a maximum of its potential with tachyonic mass $\sqrt{7}H/2$; for details, see the Appendix. The resulting spectral index vanishes and is already excluded by CMB observations [27]. Scale invariance is restored only upon summation over all KK modes corresponding to a tower of 4D “inflaton.” This summation is implied for fluctuations of fixed 3D wavelength observed at the position of the brane.

V. LARGE-ANGLE CMB POWER SPECTRUM

We now turn to estimate the CMB power spectrum at large angular scales. We begin by noting that a general

operator \mathcal{O}_w with conformal weight w has an equal time correlation function

$$\langle \mathcal{O}_w(\vec{x}) \mathcal{O}_w(\vec{x}') \rangle \sim |\vec{x} - \vec{x}'|^{-2w} \quad (29)$$

and a power spectrum in Fourier space given by

$$\tilde{G}_2(|\vec{k}|; w) = \langle \tilde{\mathcal{O}}_w(\vec{k}) \tilde{\mathcal{O}}_w(-\vec{k}) \rangle \sim \int d^3 \vec{x} e^{i\vec{k} \cdot \vec{x}} |\vec{x}|^{-2w} \sim |\vec{k}|^{2w-3}. \quad (30)$$

The spectral index of the energy density fluctuations in Fourier space is $n_s = 2w - 3$, and so writing as usual $\mathcal{P} \propto \mathcal{A}_s (k/k_*)^{n_s-1}$, from (23) and (24) it follows that $n_s = 1$ for the scale-invariant Harrison-Zel'dovich spectrum [11,12] and $n_s = 0$ for the 5D de Sitter spectrum at large angular scales, yielding $w = 2$ and $w = 3/2$, respectively. Here, \mathcal{A}_s is the variance of curvature perturbations in a logarithmic wave number interval centered around the pivot scale k_* . For these large angular scales, the intrinsic temperature fluctuations of the photons at decoupling are subdominant with respect to the gravitational potential perturbation they encounter *en route* to Earth, and so the scaling weight of the perturbations in the gravitational potential grows into $s = w - 2$ [28].

Assuming that the statistical distributions of matter and metric fluctuations around the background metric are isotropic, the two-point correlation function (for directions \hat{n} and \hat{n}') of CMB temperature fluctuations $\delta T/T$ is driven by the scaling dimension s :

$$\begin{aligned} G_2^{\text{CMB}}(\hat{n} \cdot \hat{n}'; s) &\equiv \left\langle \frac{\delta T}{T}(\hat{n}) \frac{\delta T}{T}(\hat{n}') \right\rangle \\ &\sim \int d^3 \vec{k} \left(\frac{1}{|\vec{k}|^2} \right)^2 \tilde{G}_2(|\vec{k}|; s+2) e^{i\vec{k} \cdot (\vec{x} - \vec{x}')} \\ &= c_s \Gamma(-s) (1 - \hat{n} \cdot \hat{n}')^{-s}, \end{aligned} \quad (31)$$

where c_s is an s -dependent constant [29]. The multipole expansion for general s gives

$$G_2^{\text{CMB}}(\hat{n} \cdot \hat{n}'; s) = \frac{1}{4\pi} \sum_{\ell=1}^{\infty} (2\ell+1) C_s(\ell) P_\ell(\hat{n} \cdot \hat{n}'), \quad (32)$$

where

$$C_s(\ell) \sim \Gamma(-s) \sin(\pi s) \frac{\Gamma(\ell+s)}{\Gamma(\ell+2-s)}, \quad (33)$$

with a pole singularity at $s = 0$ appearing in the $\ell = 0$ monopole moment. However, CMB anisotropy maps are constructed by removing the monopole and the dipole contributions, and so the $\ell = 0$ term does not appear in the multipole series expansion. The higher moments of the

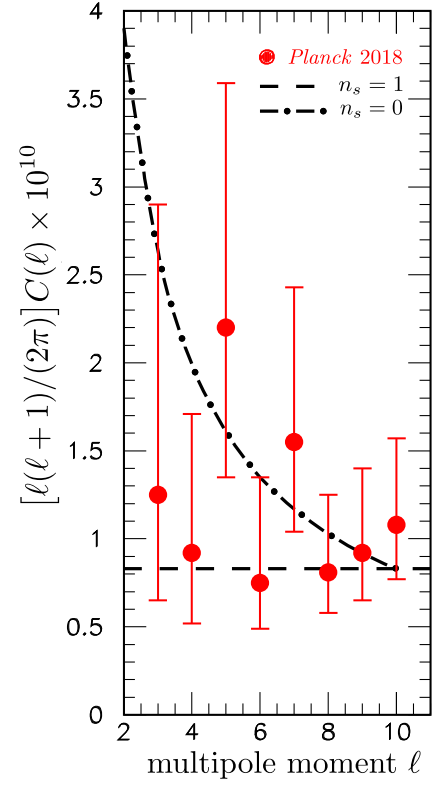


FIG. 1. Large-scale CMB power spectrum as reported by the Planck Collaboration [10], superimposed over the predictions for $n_s = 1$ and $n_s = 0$. The $n_s = 1$ curve has been normalized to the plik best fit to Planck 2018 data, i.e., $\ln(10^{10} \mathcal{A}_s) = 3.0448$ [10], whereas the $n_s = 0$ curve has been normalized to match the $n_s = 1$ curve at $\ell = 10$.

anisotropic two-point correlation are well defined for $s \rightarrow 0$. Normalizing to the quadrupole moment $C_s(2)$, we find

$$C_s(\ell) = C_s(2) \frac{\Gamma(4-s)}{\Gamma(2+s)} \frac{\Gamma(\ell+s)}{\Gamma(\ell+2-s)}. \quad (34)$$

Equation (34) provides a reasonable approximation of the anisotropy power spectrum for $\ell \lesssim 30$, and, for $n_s = 1$, it reduces to the textbook example [30]

$$C(\ell) = 6C(2) \frac{1}{\ell(\ell+1)} = \frac{2\pi}{25} \mathcal{A}_s \frac{1}{\ell(\ell+1)}. \quad (35)$$

For larger ℓ multipoles, an exhaustive transfer-function analysis (numerically solving the linearized Einstein-Boltzmann equations) would be required. However, herein we are interested in distances \gg Mpc, which correspond to $\ell \lesssim 10$. In Fig. 1, we show a comparison of the predictions for $n_s = 1$ and $n_s = 0$, which are confronted to the latest data from the *Planck* mission [10].

VI. END OF INFLATION

In order to stop the exponential expansion in higher dimensions, one may introduce an inflaton $\Phi(x, y)$ as a bulk field with a potential having a typical slow-roll region [7]. Assuming some mechanism of radius stabilization, at the end of inflation the Universe should be populated by appropriate decays of the inflaton into brane (Standard Model) fields that should dominate its decay to gravitons. For this, one may introduce a direct coupling of Φ to brane fields localized at the brane position, such as Yukawa couplings to fermions, etc. By dimensional analysis, the corresponding decay rate is

$$\Gamma_{\text{SM}}^{\Phi} \sim \frac{y^2}{(RM_*)^d} m_{\Phi}, \quad (36)$$

where m_{Φ} is the inflaton mass and y denotes the (dimensionless) coupling of the inflaton to the brane, while the denominator corresponds to the volume suppression. On the other hand, the decay rate of Φ to gravitons occurs gravitationally in $(4+d)$ dimensions with corresponding decay rate

$$\Gamma_{\text{grav}}^{\Phi} \sim \frac{m_{\Phi}^{3+d}}{M_*^{2+d}}. \quad (37)$$

Requiring the gravitational decay to be less than $\Gamma_{\text{SM}}^{\Phi}$, one finds

$$m_{\Phi} < (M_*^2/R^d)^{1/(2+d)} = M_* (M_*/M_p)^{2/(2+d)} \lesssim \mathcal{O}(1) \text{ TeV}, \quad \text{for } d=1 \text{ and } M_* \sim 10^9 \text{ GeV}. \quad (38)$$

This model represents a specific realization of the dynamical dark matter framework [31] in which the cosmic evolution of the hidden sector is primarily dominated by “dark-to-dark” decays with a small violation of the KK momentum conservation [32,33]. Alternatively, we can advocate a large violation of the KK momentum as in the fuzzy dark matter scenario discussed in [34].

VII. RADION STABILIZATION AT THE END OF INFLATION ($d=1$)

At the end of inflation, the radion acquires a runaway potential emerging from the 5D cosmological constant Λ_5^{min} at the minimum of the inflaton potential:

$$V_0 = 2\pi r^2 \frac{\Lambda_5^{\text{min}}}{R}, \quad (39)$$

where the line element decomposition (2) was taken into account; see (3). Actually, Λ_5^{min} is constrained by the Higuchi bound [35] on the mass of the lightest spin-2 KK mode at the minimum of the potential:

$$\frac{1}{r^2} \geq 2H_{\text{min}}^2 = \frac{2}{3} \frac{\Lambda_4}{M_p^2} = \frac{4}{3} \pi r \frac{\Lambda_5^{\text{min}}}{M_p^2} \Rightarrow \Lambda_5^{\text{min}} \leq \frac{3}{4\pi} \frac{M_p^2}{r^3};$$

$$(\Lambda_5^{\text{min}})^{1/5} \lesssim 100 \text{ GeV}. \quad (40)$$

V_0 has an exponential quintessencelike form, expressed in terms of the canonically normalized field in Planck units $\phi = \sqrt{3/2} \ln(R/r)$ [see (3)]; it is proportional to $e^{-\alpha\phi}$ with an exponent $\alpha \simeq 0.82$, which is curiously just at the upper limit of the experimentally allowed value [36].

On the other hand, the radion can be stabilized by taking into account some generic additional contributions to its potential:

$$V = \left(\frac{r}{R}\right)^2 \hat{V} + V_C; \quad \hat{V} = 2\pi R \Lambda_5^{\text{min}} + T_4 + 2\pi \frac{K}{R}, \quad (41)$$

where the second contribution arises from localized 3-branes or orientifolds with total tension T_4 and the third contribution arises from kinetic gradients of bulk 5D fields [17],⁴ such as a scalar background linear in the fifth (y) coordinate, while the last contribution V_C corresponds to the Casimir energy from all 5D states of mass m [18] (with periodic boundary conditions):

$$V_C = 2\pi R \left(\frac{r}{R}\right)^2 \text{Tr}(-)^F \rho(R, m);$$

$$\rho(R, m) = - \sum_{n=1}^{\infty} \frac{2m^5}{(2\pi)^{5/2}} \frac{K_{5/2}(2\pi Rmn)}{(2\pi Rmn)^{5/2}}, \quad (42)$$

where F denotes the fermion number and trace stands for the sum over all bosonic and fermionic 5D degrees of freedom.

Since the Casimir potential falls off exponentially at large R compared to the particle wavelength while at small R behaves as $1/R^4$, for simplicity, below we discuss two extreme cases of stabilization: (i) neglecting V_C and (ii) taking it into account for $K=0$.

A. Stabilization neglecting V_C

By inspection of the expression (41) neglecting V_C , one finds that a minimum of V with (approximately) vanishing energy requires $\hat{V}' = \hat{V} = 0$ at the minimum, leading to

$$r = \left(\frac{K}{\Lambda_5^{\text{min}}}\right)^{1/2}; \quad T_4 = -4\pi (K \Lambda_5^{\text{min}})^{1/2};$$

$$V'' = \hat{V}''|_{R=r} = 4\pi \frac{K}{r^3}. \quad (43)$$

It follows that the tension must be negative and the radion mass is

⁴Actually, $K = \tilde{K} R_0^2 \sim \tilde{K}/M_*^2$ with \tilde{K} of dimension five, so that K has dimension three.

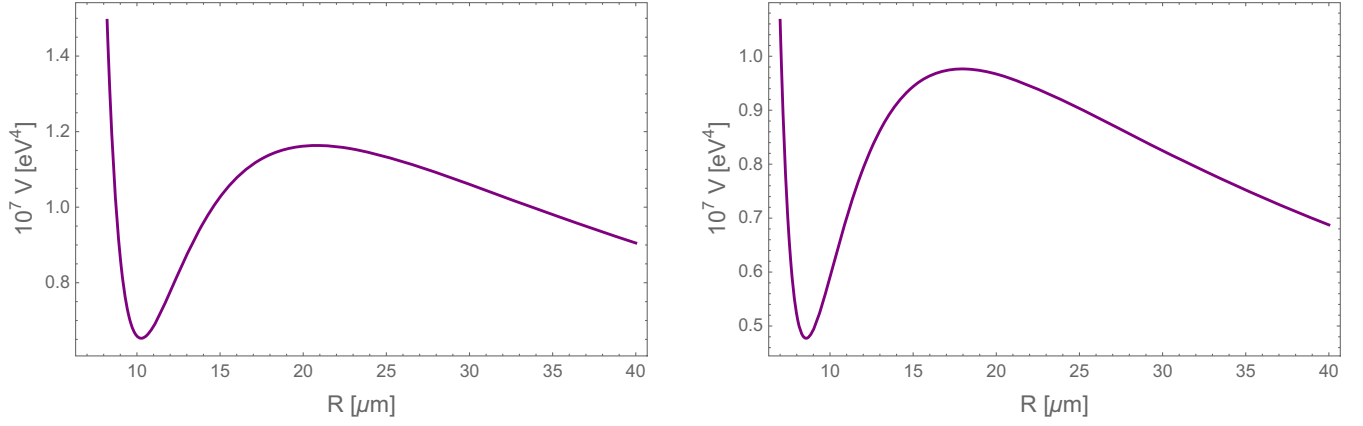


FIG. 2. The potential $V(R)$ for $(\Lambda_5^{\min})^{1/5} = 25$ meV, $|T_4|^{1/4} = 27$ meV, and $N_F - N_B = 7$ (left) and $(\Lambda_5^{\min})^{1/5} = 25$ meV, $|T_4|^{1/4} = 26$ meV, and $N_F - N_B = 3$ (right).

$$m_\phi^2 = \frac{4}{9} \frac{r^2}{M_p^2} \hat{V}'' \Big|_{R=r} = \frac{4}{9} \frac{|T_4|}{M_p^2} = \frac{8}{9} \frac{1}{r^2} \frac{K}{M_*^3}. \quad (44)$$

It is straightforward to compute also the maximum corresponding to

$$R_{\max} = 3r; \quad V_{\max} = \frac{2}{27} |T_4|. \quad (45)$$

Requiring the existence of a minimum with any vacuum energy less than V_{\max} satisfying the Higuchi bound $m_{\text{spin-2}}^2 \geq 2H^2 = 2V_{\min}/(3M_p^2)$, it follows from (44) and (45) that $m_\phi \leq 3/r$. Thus, consistency with experimental bounds on extra forces implying a radion mass heavier than 0.1 eV requires $K \sim M_*^3$ and $(\Lambda_5^{\min})^{1/5} \sim (K/r^2)^{1/5} \sim 100$ GeV, near its upper bound (40). Moreover, from (43) one obtains $|T_4|^{1/4} \sim 1$ TeV so that all three terms of the potential (41) are of the same order with tunable vacuum energy at the minimum.

Note that the presence of V_C , which is nonvanishing at least for the 5D graviton, does not affect this minimum, since its contribution is negligible compared to the others. However, for a different choice of parameters, such as in the case of vanishing K that we discuss below, its contribution will be important for the minimization of the potential. In fact, for values of K much lower than M_*^3 , the radion mass drops below the compactification scale $1/r$ and experimental limits on extra forces require a suppression of its coupling to matter that we also discuss in the next subsection.

B. Stabilization with $K = 0$

Neglecting V_C at large R , the potential (41) for K vanishing has a maximum at

$$R_{\max} = -\frac{T_4}{\pi\Lambda_5^{\min}}, \quad (46)$$

requiring again a negative tension T_4 . A minimum near the maximum can be then generated from the Casimir energy contribution along the lines of [18]. Indeed, $\rho(R, m)$ vanishes exponentially for $mR > 1$ while for mR small behaves as

$$\begin{aligned} \rho(R) &\simeq -\frac{1}{(2\pi R)^5 \pi^2} \left[\zeta(5) - \frac{2}{3} \pi^2 \zeta(3) (mR)^2 + \mathcal{O}((mR)^4) \right] \\ &\simeq -\frac{1}{(2\pi R)^5 \pi^2}, \end{aligned} \quad (47)$$

where $\zeta(5) \simeq 1.037$, leading to

$$V_C \simeq \frac{2\pi r^2}{32\pi^7 R^6} (N_F - N_B), \quad (48)$$

and where $N_F - N_B$ is the difference between the number of light fermionic and bosonic degrees of freedom which must be positive for a minimum to exist. Note from (10) that during inflation the canonically normalized field $\log R$ varies logarithmically with the 4D proper time and its speed decreases as $\sim 1/t$. It is, therefore, expected to oscillate around the minimum.

In order to obtain a minimum with positive energy and a radius around the micron, the three terms of the potential should be comparable in magnitude. In Fig. 2, we show two illustrative examples. The shape of the potential in the left panel derives from $(\Lambda_5^{\min})^{1/5} = 25$ meV, $|T_4|^{1/4} = 27$ meV, and $N_F - N_B = 7$, where we have considered contributions to the Casimir energy of the massless 5D graviton ($N_B = 5$) and three neutrino states ($N_F = 12$). We have adopted Dirac bulk mass terms $\mu_i \lesssim R_{\max}^{-1}$ satisfying $\Delta m_{21} < \Delta m_{32} \lesssim \mu_i$, where Δm_{21} and Δm_{32} are the mass differences required to explain the various neutrino oscillation phenomena and $i = 1, 2, 3$ [19–21]. As shown in [22], this particular region of the parameter space allows us to avoid restrictive bounds from neutrino oscillation experiments, which for $\mu_i \sim 0$

constrain the size of the compact dimension $R \lesssim 0.2 \mu\text{m}$ [37,38]. The shape of the potential in the right panel corresponds to $(\Lambda_5^{\min})^{1/5} = 25 \text{ meV}$, $|T_4|^{1/4} = 26 \text{ meV}$, and $N_F - N_B = 3$, where we have considered the lightest neutrino to be a massless Majorana brane state and so only two neutrinos propagate in the bulk.

In terms of the normalized field ϕ , the potential (41) can be recast as

$$V(\phi) = 2\pi\Lambda_5^{\min} r e^{-\sqrt{2/3}\phi} + T_4 e^{-2\sqrt{2/3}\phi} + \frac{1}{16\pi^6} \frac{1}{r^4} (N_F - N_B) e^{-6\sqrt{2/3}\phi}. \quad (49)$$

It follows that the radion mass, $m_\phi \sim \sqrt{V''(0)}/M_p$, is $m_\phi \sim 10^{-30} \text{ eV}$ for the illustrative example in which $R \sim 10 \mu\text{m}$ and $m_\phi \sim 10^{-31} \text{ eV}$ for $R \sim 8 \mu\text{m}$. In addition, by comparing the maximum and minimum of the potential in Fig. 2, we see that the oscillations of the radion around the minimum would have a negligible contribution to the 4D cosmological constant Λ_4 .

On the other hand, such a light scalar should have suppressed interactions to ordinary matter in order to avoid experimental limits on extra forces. In fact, in the absence of scalar potential, the radion from 5D is equivalent to a Brans-Dicke scalar with a parameter $\omega = -4/3$ that couples to the trace of the matter energy momentum tensor with a strength comparable to gravity. A way to suppress its coupling is by adding a 4D localized correction to the radion kinetic term,⁵ in analogy with the brane contribution to the potential which is constant in the 5D frame given by the tension T_4 :

$$\delta S_{\text{radion}}^{\text{localized}} = \int [d^4x] \left\{ -\zeta \left(\frac{\partial R}{R} \right)^2 - T_4 \right\}, \quad (50)$$

with ζ a positive dimensionful parameter. A possible origin of ζ is through an expectation value of a brane field coupled to the radion kinetic terms via, for instance, the internal components of the 5D Ricci tensor. When added to the 5D action (1) (for $d=1$) dimensionally reduced to 4D, it modifies the Brans-Dicke parameter by a $1/R$ correction to $\omega = -4/3 + \zeta/R$ (in M_* units) that should be large enough to suppress the physical coupling of the radion to matter (upon appropriate rescaling that normalizes its kinetic term). Note that the modification of the radion kinetic term does not change the minimization of the potential but lowers its mass by the same factor $\sim \sqrt{\omega}$ that suppresses its coupling; for this, one needs $\omega \gtrsim 10^3 - 10^4$.

In the absence of the T_4 contribution, the potential can still develop a minimum based on the negative contributions to the Casimir energy of the bosonic degrees of freedom, that produces a maximum at $R_{\text{max}}^{-1} \sim (\Lambda_5^{\min})^{1/5}$,

⁵More general corrections to the radion effective action with the same goal were discussed in [39].

and the positive contribution of 5D fermions [18] (such as the bulk R-handed neutrinos) with appropriate bulk mass around the same scale; $(\Lambda_5^{\min})^{1/5}$ should then be around the eV scale. For example, if we consider only the massless 5D graviton ($N_B = 5$) and take $(\Lambda_5^{\min})^{1/5} \sim 25 \text{ meV}$, the maximum appears at $R \sim 10 \mu\text{m}$.

VIII. CONCLUSIONS

We have investigated the possibility that compact extra dimensions can obtain large size by higher-dimensional inflation, relating the weakness of the actual gravitational force to the size of the observable Universe. The proposed 5D inflationary setup corresponds to a de Sitter (or approximate) solution of 5D Einstein equations, with cosmological constant and a 5D Planck scale $M_* \sim 10^9 \text{ GeV}$ [7]. All dimensions (compact and noncompact) expand exponentially in terms of the 5D proper time. The proposed setup requires about 40 e -folds to expand the fifth dimension from the fundamental length to the micron size. At the end of 5D inflation, or at any given moment, one can interpret the solution in terms of 4D fields using 4D Planck units from the relation $M_p^2 = 2\pi R M_*^3$, which amounts going to the 4D Einstein frame. This implies that, if R expands N e -folds, then the 3D space would expand $3N/2$ e -folds as a result of a uniform 5D inflation. Altogether, the 3D space has expanded by about 60 e -folds to solve the horizon problem while connecting this particular solution to the generation of large size extra dimension.

From the 4D perspective, the modulus potential is not constant, and there are two fields: the 5D inflaton and the radion modulus. When inflation starts, the compactification radius is small and the 4D Planck mass is of the order of the 5D Planck scale (which one can consider to be the string scale), but when inflation ends, the radius is large and the 4D Planck scale arrives at the value observed today, $M_p \sim 10^{18} \text{ GeV}$. Actually, from the 4D point of view, the 4D metric and the radion modulus evolve differently. In terms of the 4D proper time, the scale factor of the 3D space expands as t^3 which corresponds to power-law inflation, whereas the radion evolves logarithmically so that the size of the extra dimension expands as t^2 .

We have shown that this solution gives a viable 4D cosmology consistent with CMB observations. This is because, for any spacetime dimensionality, the two-point function of a massless minimally coupled scalar field (such as a slow-rolling inflaton) in de Sitter space behaves logarithmically at distances larger than the cosmological horizon [9], yielding a scale-invariant Harrison-Zel'dovich spectrum for distances smaller than the compactification length. Correspondingly, precision of CMB power spectrum applies to angles of less than about 10° in the sky or, equivalently, to a Mpc-scale distance. Now, the number of e -folds during radiation dominance is about 41 when the inflation scale is near (but below) the higher-dimensional

Planck scale, say, $M_I/\text{GeV} \sim 10^7\text{--}10^8$. This corresponds to an expansion of the 3D space by a factor of $10^{17}\text{--}10^{18}$, and thereby the critical CMB Mpc distance is shortened to 100–10 km near the end of inflation. When this distance is converted to 5D units, it is scaled down by a factor $(R_0/R)^{1/2} \sim 3 \times 10^{-10}$ to 30–3 μm , which is stunningly of the order of the size of the fifth dimension at the end of inflation. We have shown that the predicted small-angle ($<18^\circ$) CMB power spectrum is compatible with observations. Such an angle corresponds to a distance ~ 4.5 Mpc and multipole moment $\ell \simeq 10$; see Fig. 1 for details. For smaller ℓ multipoles (larger angles), one obtains more power spectrum than standard 4D inflation, corresponding to a nearly vanishing spectral index, that the present data cannot distinguish due to large errors. One caveat here is that the power spectrum is cosmic variance limited [40]. However, even though cosmic variance prevents identification at low ℓ , the transition region near $\pi k R_0$ could provide a signal for experiments in the near future. To determine ℓ multipoles in the transition region, an exhaustive transfer-function analysis (numerically solving the linearized Einstein-Boltzmann equations) would be required.

One may wonder if the existence of the extra scalar from the 4D point of view can generate additional effects, such as isocurvature perturbations and non-Gaussianities. Since the radion modulus is, however, part of the 5D graviton, we expect these effects to be of the same order as the primordial gravity waves and, thus, to be suppressed. Although the extra scalar has an amplitude that is suppressed by the slow-roll parameter, it contributes with its own power spectrum, destroying the simple power law of scalar perturbations. In other words, the scalar perturbations become a sum of two contributions, one coming from the inflaton (that is determined by the slow-roll parameters of the potential) and another that comes from the radion (that, in principle, has a different spectrum). Note that these two contributions have different momentum dependence, so they cannot be parametrized as a simple power law; i.e., there will be a correction to the slow-roll parameter that destroys the simple spectral index. Indeed, the spectral index will be a sum of two powers, with the second power being suppressed by the slow-roll parameter ϵ . An expansion in ϵ can be parametrized as a k variation of the spectral index, which is ϵ dependent. The power spectrum is then given by (23) and (24), up to a spectral tilt correction. Therefore, even if the gravitational waves are small and difficult to detect, precision measurements of the scalar perturbations could indicate that there is a correction. A dedicated analysis is under investigation for confirmation and computation of the power spectrum of the tensor modes. After the end of 5D inflation, the radion has a runaway exponential potential quintessencelike with an exponent near the upper limit of the allowed value.

Alternatively, we have proposed a mechanism to stop the time evolution of the radion at the end of inflation, which is based on some general additional contributions to the scalar

potential. The resulting minimization is controlled by one parameter, which is the magnitude of the contribution due to bulk field gradients, denoted K . For pedagogical reasons, we studied two extreme cases.

When K is of the order of the 5D fundamental scale, the radion mass reaches an upper bound proportional to the compactification scale around the eV region, while the vacuum energy can be tuned to an infinitesimal value as a result of cancellation among large contributions at the (sub)TeV scale.

When K vanishes, we have demonstrated that the radion can be stabilized in a local (metastable) dS vacuum by taking into account the contribution to the scalar potential of the Casimir energy, in the spirit of [18]. In general, one has three possible contributions. The first stems from the 5D cosmological constant Λ_5^{min} at the end of inflation, the second contribution comes from localized 3-branes or orientifolds with total tension T_4 , and the last contribution corresponds to the Casimir energy from all 5D states. For large R , the contribution from the Casimir energy can be neglected, and in the presence of a negative tension the potential develops a maximum at $R_{\text{max}} = -T_4/(\pi\Lambda_5^{\text{min}})$.⁶ We have shown that a minimum near the maximum can be accommodated using the Casimir energy contribution of light fermionic degrees of freedom driven by R-handed neutrino states with Dirac bulk masses $\lesssim 1/R_{\text{max}}$. Thus, in this case a tuning of the vacuum energy is avoided, but Λ_5^{min} and T_4 should be of the order of the eV scale.

Within the second setup, however, the radion mass is estimated to be $\mathcal{O}(10^{-30}$ eV). We have shown that the oscillations of the radion around the minimum would have a negligible contribution to the present dark energy Λ_4 ; for details, see Fig. 2. On the other hand, consistency with the experimental limits on extra forces requires suppressing its coupling to matter. We have shown that such a suppression can arise from a 4D localized correction to the radion kinetic term with a large coefficient of the order of several thousands. Alternatively, it has been argued that an appropriate modification of such theories due to bulk quantum corrections can lead to a logarithmic scale (time) dependence of the Brans-Dicke parameter ω that suppresses the radion coupling to matter [39]. For K nonvanishing but less than the 4D fundamental scale, the radion mass varies between the two extreme cases, and suppression of its coupling becomes less severe to not be necessary.

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⁶In the absence of tension, a maximum is still present due to the 5D graviton contribution to the Casimir potential.

Center for Geometry and Physics, Stony Brook University at which a last part of the research for this paper was performed.

$$S_D = \int [d^D x] \left[-\frac{1}{2} (\partial_M \Phi)^2 - m^2 \Phi^2 \right], \quad (\text{A1})$$

APPENDIX

The action of a massive scalar field Φ in a D -dimensional flat space is given by

where m is the field's mass and M runs from 0 to $(D-1)$. Introducing the dimensionless field $\chi = a^{D/2-1} \Phi$, it follows that

$$\Phi = \left(1 - \frac{D}{2}\right) a^{-D/2} \dot{a} \chi + a^{1-D/2} \dot{\chi} = a^{1-D/2} \left[\dot{\chi} + \left(1 - \frac{D}{2}\right) \frac{\dot{a}}{a} \chi \right] \quad (\text{A2})$$

and

$$\frac{\dot{a}}{a} \chi \dot{\chi} = \frac{1}{2} \chi^2 \frac{\partial}{\partial \tau} \left(\frac{\dot{a}}{a} \right). \quad (\text{A3})$$

Using (A2) and (A3), we can rewrite (A1) in Fourier space as

$$S = \int d\tau d^{D-1} k \frac{1}{2} \left\{ \dot{\tilde{\chi}}^2 + \left[\left(1 - \frac{D}{2}\right)^2 \left(\frac{\dot{a}}{a}\right)^2 + \left(1 - \frac{D}{2}\right) \frac{\partial}{\partial \tau} \left(\frac{\dot{a}}{a}\right) + k^2 - m^2 a^2 \right] \tilde{\chi}^2 \right\}. \quad (\text{A4})$$

For a power-law dependence $a(\tau) \sim \tau^{-\alpha}$ with $\alpha > 1$, corresponding to $a(t) \sim t^{\frac{\alpha}{\alpha-1}}$, it follows that $\dot{a}/a \sim -\alpha/\tau$ and $\partial/\partial \tau(\dot{a}/a) \sim \alpha/\tau^2$, and then (A4) can be rewritten as

$$S = \int d\tau d^{D-1} k \frac{1}{2} \left\{ \dot{\tilde{\chi}}^2 + \left[\alpha \left(1 - \frac{D}{2}\right) \left[\alpha \left(1 - \frac{D}{2}\right) + 1 \right] \frac{1}{\tau^2} + k^2 - m^2 a^2 \right] \tilde{\chi}^2 \right\}. \quad (\text{A5})$$

Using (A5), it is straightforward to check that for a massless scalar field the coefficient 15/4 is replaced by $L = \alpha(D/2 - 1)[\alpha(D/2 - 1) + 1] = \alpha(\alpha + 1)$ for $D = 4$.

Duplicating the procedure for a D -dimensional dS space, the action is found to be

$$S_{\text{dS}_D} = \int d\tau d^{D-1} k \frac{1}{2} \left\{ \dot{\tilde{\chi}}^2 + \left[\left(\frac{D(D-2)}{4} - \frac{m^2}{H^2} \right) \frac{1}{\tau^2} - k^2 \right] \tilde{\chi}^2 \right\}, \quad (\text{A6})$$

where $H = \dot{a}/a$ [9]. In the case of a massive field in dS_D , the coefficient 15/4 becomes $L = D(D-2)/4 - m^2/H^2$.

Following [9], one finds that the order of the Bessel functions determining the correlation function (17) is given by $\nu = \sqrt{L + 1/4}$, leading to a spectral index of the power spectrum $n_s = 4 - 2\nu$.

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