Erratum: High-energy $\pi\pi$ scattering without and with photon radiation [Phys. Rev. D 105, 014022 (2022)]

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In the following we correct some erroneous statements in this article which we have made due to a misinterpretation of the soft-photon theorem due to F. E. Low [1]. We had not appreciated the distinction between the soft-photon theorems in the versions of F. E. Low [1] and of S. Weinberg [2]. We were led astray by the fact that in the literature these two versions frequently are not clearly distinguished and Weinberg's version of the soft-photon theorem is referred to as Low's theorem. We hope to have clarified the different meaning of these two versions of soft-photon theorems in Refs. [3,4]. In the following we list the modifications which have to be made in our article Phys. Rev. D **105**, 014022 (2022) in the light of the results of [3,4].

(1) The abstract should read:

We discuss the processes $\pi\pi \to \pi\pi$ and $\pi\pi \to \pi\pi\gamma$ from a general quantum field theory (QFT) point of view. We study the soft-photon limit where the photon energy $\omega \to 0$ and where we have the theorems due to F. E. Low and S. Weinberg. We consider for the radiative amplitude the Laurent expansion in ω and calculate the terms of order ω^{-1} and ω^0. The pole term $\propto \omega^{-1}$ is given by Weinberg's soft-photon theorem. Then we calculate the amplitudes for the above reactions for high center-of-mass energies and small momentum transfers, that is, in the soft-diffraction regime using the tensor-Pomeron model. We identify places where "anomalous" soft photons could come from. Three soft-photon approximations (SPAs) are introduced. The corresponding SPA results are compared to those obtained from the full tensor-Pomeron model for center-of-mass energies $\sqrt{s} = 10$ GeV and 100 GeV. The kinematic regions where the SPAs are a good representation of the full amplitude are determined. Finally we make some remarks on the type of fundamental information one could obtain from high-energy exclusive hadronic reactions without and with soft photon radiation.

(2) The fourth paragraph of Sec. I should be replaced by the following.

With the present paper we want to start the theoretical study of soft-photon emission in hadronic exclusive diffractive high-energy reactions in the TeV energy region in the framework of the tensor-Pomeron model. Our first example will be, for simplicity, pion-pion elastic scattering. This is, of course, not easy to study in experiments but, as we shall see, we can in this example compare our "exact" model results for photon emission to approximations based on the soft-photon theorems of [1,2].

(3) In Sec. III everything after Eq. (3.28) should be deleted and replaced by the following.

Now we set

$$(k^\mu) = \omega \begin{pmatrix} 1 \\ \tilde{\boldsymbol{k}} \end{pmatrix}, \qquad \omega \geq 0, \qquad \tilde{\boldsymbol{k}}^2 \leq 1, \qquad \boldsymbol{l}_{1\perp} = \omega \tilde{\boldsymbol{l}}_{1\perp}, \qquad |\tilde{\boldsymbol{l}}_{1\perp}| = \mathcal{O}(1). \tag{3.29}$$

We consider the limit $\omega \to 0$ keeping $\tilde{\boldsymbol{k}}$ and $\tilde{\boldsymbol{l}}_{1\perp}$ fixed. Note that this implies from (3.21) that also l_1 and l_2 are proportional to ω. Inserting (3.28) in (3.27) we then find,

$$\begin{aligned}
\mathcal{M}_\lambda(p_1', p_2', k, p_a, p_b) = {} & e\mathcal{M}^{(0)}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2)\left[\frac{p_{a\lambda}}{p_a \cdot k} - \frac{p_{1\lambda}}{p_1 \cdot k}\right] + e\left\{\frac{1}{2(p_a \cdot k)^2}\left[p_{a\lambda}k^2 - k_\lambda(p_a \cdot k)\right]\right. \\
& - \frac{1}{2(p_1 \cdot k)^2}\left[p_{1\lambda}(2l_1 \cdot k - k^2) - (2l_{1\lambda} - k_\lambda)p_1 \cdot k\right] - 2\left[(p_b \cdot k)\frac{p_{a\lambda}}{p_a \cdot k} - p_{b\lambda}\right]\frac{\partial}{\partial s_L} \\
& \left. - 2\left[(p_a - p_1, k) - p_a \cdot l_1\right]\left[\frac{p_{a\lambda}}{p_a \cdot k} - \frac{p_{1\lambda}}{p_1 \cdot k}\right]\frac{\partial}{\partial t}\right\}\mathcal{M}^{(0)}\left(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2\right) + \mathcal{O}(\omega),
\end{aligned} \tag{3.30}$$

$s_L = p_a \cdot p_b + p_1 \cdot p_2$, $t = (p_a - p_1)^2 = (p_b - p_2)^2$.

With (3.30) we have given the first two terms of the Laurent expansion in ω of $\mathcal{M}_\lambda$ around $\omega = 0$. The first term on the rhs of (3.30) is the pole term $\propto \omega^{-1}$ which, for $k^2 = 0$, is exactly the soft-photon term as given by S. Weinberg; see Sec. II 1 of [2]. The term on the rhs of (3.30) with curly brackets is the term of order ω^0. Note that in (3.30) we give the expansion of the radiative amplitude $\mathcal{M}_\lambda \equiv \mathcal{M}_\lambda(p_1', p_2', k, p_a, p_b)$ [see (2.13)] around the phase-space point of zero radiation $(p_1, p_2, k = 0)$.

Now we discuss the relation of (3.30) to Low's theorem, which gives an approximate expression for $\mathcal{M}_\lambda(p_1', p_2', k, p_a, p_b)$ for this phase-space point (p_1', p_2', k). Low's formula, (1.7) of [1], which is for real photon emission, reads, using our metric convention and notation, as follows:

$$\begin{aligned}\mathcal{M}_\lambda(p_1', p_2', k, p_a, p_b) = {}& e\mathcal{M}^{(0)}\left(s_L', t_2, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2\right)\left[\frac{p_{a\lambda}}{p_a\cdot k} - \frac{p_{1\lambda}'}{p_1'\cdot k}\right] \\ & - e\left[(p_b\cdot k)\frac{p_{a\lambda}}{p_a\cdot k} + (p_2'\cdot k)\frac{p_{1\lambda}'}{p_1'\cdot k} - p_{b\lambda} - p_{2\lambda}'\right] \\ & \times \frac{\partial}{\partial s_L'}\mathcal{M}^{(0)}\left(s_L', t_2, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2\right) + \mathcal{O}(\omega), \end{aligned} \tag{3.31}$$

$s_L' = p_a\cdot p_b + p_1'\cdot p_2'$, $t_2 = (p_b - p_2')^2$.

This looks quite different from our Eqs. (3.27) and (3.30), setting there $k^2 = 0$, but this is alright since the meaning of (3.27) and (3.30) versus (3.31) is *different*. As emphasized above, (3.30) gives the first two terms of the Laurent expansion of the radiative amplitude around the phase-space point of no radiation $(p_1,\ p_2,\ k = 0)$. Low's formula (3.31) is valid only at the given phase-space point $(p_1',\ p_2',\ k)$. That is, we can use (3.31) *only* for the value of k dictated by energy-momentum conservation; $k = p_a + p_b - p_1' - p_2'$. If (3.31) is used for a different k we go outside of the physical region of the radiative amplitude $\mathcal{M}_\lambda$. Therefore, we prefer to call (3.31) Low's approximate expression for $\mathcal{M}_\lambda$ in order to distinguish it from the Laurent expansion (3.27) and (3.30). We can then construct the Laurent expansion of Low's expression (3.31) around a phase-space point of no radiation $(p_1, p_2, k = 0)$, where, of course, $(p_1,\ p_2)$ must be near to $(p_1',\ p_2')$. In this way we get the relation between Low's formula (3.31) and our Laurent expansion (3.27) and (3.30). The details of this are given in Refs. [3,4].

(4) In Eq. (4.54) the comma at the end of the first line should be deleted.

(5) The first paragraph of Sec. VI should be replaced by the following.

In this paper we have studied elastic pion-pion scattering without and with photon radiation. In Sec. II we have given a detailed analysis, from a QFT point of view, of the reactions $\pi^-\pi^0 \to \pi^-\pi^0$ and $\pi^-\pi^0 \to \pi^-\pi^0\gamma$. We have used this analysis in Sec. III to derive the expansion of the amplitude for $\pi^-\pi^0 \to \pi^-\pi^0\gamma$ in powers of ω, the photon energy in the overall center-of-mass system, for $\omega \to 0$. The term of order ω^{-1} in the Laurent expansion (3.30), specialized for $k^2 = 0$, agrees with the version of the soft-photon theorem due to S. Weinberg [2]. We have given the term of order ω^0 of this Laurent expansion. This expansion (3.30) should not be confused with the expressions for the radiative amplitude given in F. E. Low's version of the soft-photon theorem [1]. The latter gives an approximate expression for the radiative amplitude at a given phase-space point and not an expansion of the amplitude around the phase-space point of zero radiation as given in (3.30). All this is discussed in detail in [3,4] where also the relation of Low's formula, (1.7) of [1], and our formula (3.30) is given. We emphasize that our result (3.30) is a strict consequence of QFT. Therefore, absolutely no model dependence is contained there. As a nontrivial check of this general result we have considered the Laurent expansion of our tensor-Pomeron-model amplitude for $\mathcal{M}_\lambda$; see (4.19), (4.22)–(4.24). The terms of order ω^{-1} and ω^0 are found exactly as expected from (3.27).

(6) The first sentence of Appendix A should be replaced by

Here we compare our findings concerning the Laurent series for the soft-photon expansion from Sec. III to results from a number of papers from the literature.

(7) In Eq. (A2) should be added on the rhs $+\mathcal{O}(\omega)$.

(8) In Eq. (B12) it should read $\mathcal{O}(\omega)$.

Now we add some clarifying remarks concerning our paper.

(9) It should be noted that the soft-photon theorems in the versions of both Low and Weinberg hold also for spin-flip amplitudes in hadronic high-energy scattering. An example is the photon emission in pion-proton scattering in the soft-photon limit discussed from a general QFT point of view in [4]. There is no model dependence for the terms of orders ω^{-1} and ω^0 in the corresponding radiative amplitude.

(10) In the first paragraph of Sec. IV, the reader may find some historical remarks on Regge theory and the Pomeron. There we also write that the tensor-Pomeron model which we use in this article is for *soft* hadronic high-energy

reactions. For extensive discussions of this model see Refs. [5,6]. This model is convenient for us to use, but there is no claim that this model is in any sense unique. In our present paper there is also no intention to apply *this* tensor-Pomeron model to *hard* diffractive reactions. For these latter reactions other concepts and models should be and are used. In particular, methods of perturbative QCD can be applied there; see for instance [7,8]. But let us note that in [9] an extension of the above tensor-Pomeron model for soft reactions [5], including in addition a hard tensor Pomeron, gave a very satisfactory description of the low-x deep inelastic structure functions for $0\ \mathrm{GeV}^2 \leq Q^2 \leq 50\ \mathrm{GeV}^2$. This latter model was also successfully applied for a description of the HERA data on deeply virtual Compton scattering in [10].

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