Transversity generalized parton distributions in spin-3/2 particles

Dongyan Fu,[1,2,*] Yubing Dong,[1,2,†] and S. Kumano[3,4,5,‡]

[1]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*
[2]*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 101408, China*
[3]*Department of Mathematics, Physics, and Computer Science, Faculty of Science, Japan Women's University, Mejirodai 2-8-1, Tokyo 112-8681, Japan*
[4]*Theory Center, Institute of Particle and Nuclear Studies, KEK Oho 1-1, Tsukuba, Ibaraki 305-0801, Japan*
[5]*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*

(Received 21 February 2024; accepted 8 April 2024; published 8 May 2024)

The definitions of the quark and gluon transversity generalized parton distributions (GPDs) in spin-3/2 particles are obtained in the light-cone gauge. It is found that they contain 16 independent components for each parton. Their even or odd property is found in terms of a skewness variable, and the odd transversity GPDs vanish in the forward limit. There are 16 amplitudes with helicity flip of quark or gluon. We conclude that all the amplitudes, unpolarized, polarized, and transversity amplitudes, have a common factor $F(\zeta)$ carrying all the complex parts. Here, the variable ζ is the helicity difference $\zeta = (\lambda' - \lambda) - (\mu' - \mu)$ for the amplitude $\mathcal{A}_{\lambda'\mu',\lambda\mu}$. This kinematical factor is associated with the transfer of the orbital angular momentum in the quark- and gluon-spin-3/2 particle scattering amplitudes. We also derive another main physical quantity, transversity distribution, from the transversity GPDs in the forward limit for the spin-3/2 particles.

DOI: 10.1103/PhysRevD.109.096006

I. INTRODUCTION

To investigate the construction of hadrons from quarks and gluons degrees of freedom, which are the elementary particles described by quantum chromodynamics (QCD), the notion of generalized parton distributions (GPDs) was introduced [1–7]. The GPDs can provide a more detailed account of the internal structure of hadrons in terms of their elementary constituents than the parton distributions and form factors (FFs). Similar to the theoretical constructs, one of the related experiments utilized to investigate hadron structures is the deep inelastic scattering process, which indicates the parton distributions, including the unpolarized, polarized, and transversity distribution functions at the leading twist. We know that the GPDs depend on three variables: the longitudinal momentum fraction x, the square of the transferred momentum t, and the skewness variable ξ which is defined by the transferred longitudinal momentum, rather than just one variable x in the parton distribution functions (PDFs). The reaction that involves GPDs is the off-forward Compton scattering, which consists of at least one off-shell photon [7,8], including the deeply virtual Compton scattering (DVCS) process [3,5]. Furthermore, the GPDs reduce to the PDFs in the forward limit. The spacelike GPDs are measured by the DVCS and meson-production processes at the Thomas Jefferson National Accelerator Facility (JLab) and in the CERN-AMBER project. The timelike GPDs are investigated by the two-photon process or the timelike Compton scattering at KEKB [9,10] and will be studied possibly at BESIII. In addition, there are future GPD projects at EICs [11,12], by hadron reactions at the Japan Proton Accelerator Research Complex (J-PARC) [13–16], and by neutrino reactions at Fermilab [17–19].

Corresponding to transverse momentum independent parton distributions [20], there are three types of GPDs, (i) unpolarized GPD corresponding to the unpolarized parton distribution function, the probability of finding a parton with the longitudinal momentum fraction x in the unpolarized hadron; (ii) polarized GPD corresponding to the polarized distribution function, the number density of a parton with the longitudinal momentum fraction x and spin parallel minus spin antiparallel to that of the polarized hadron; (iii) transversity GPD corresponding to the transversity distribution function $h_1(x)$ [21,22] (called $\Delta_T(x)$ in Ref. [20], $\Delta(x)$ in Ref. [23] and $\delta q(x)$ in Ref. [24]), which denotes the number density of a parton with the longitudinal momentum fraction x and polarization parallel to that of the hadron with transverse polarization minus the

*fudongyan@ihep.ac.cn
†dongyb@ihep.ac.cn
‡shunzo.kumano@kek.jp

number density with antiparallel polarization. The transversity distribution function $h_1(x)$, which can be obtained from transversity GPDs in the forward limit as described above, has been studied fundamentally by Ralston *et al.* [25], Artru *et al.* [26], Jaffe *et al.* [27,28], Cortes *et al.* [29], Ji [30], and Barone *et al.* [31]. Additionally, there is no logical reason to assume that the transversity parton distribution is significantly smaller than the other two distributions [31–37]. For nearly two decades, the spin-1/2 and -1 quark transversity distributions have been measured by some experiments [38–41] and the gluon transversity is possibly measured by JLab [42], EIC (Electron-Ion Collider) [11,43,44], and EicC (Electron-Ion Collider in China) [12], and Fermilab [37,45,46].

Based on the parity and time reversal invariances, there are $2(2J+1)^2$ independent GPDs for each parton (either quark flavor or gluon) in a spin-J hadron at leading twist [8]. These independent GPDs are discussed in Sec. 3.5.4 of Ref. [8] and Ref. [47] for spin-3/2 hadrons, and it is also found by using the parity invariance in Eq. (7) of this paper.

Half of the total GPDs consist of the unpolarized and polarized GPDs that do not involve the helicity flip, while the remaining half is distributed to the transversity GPDs involving the helicity flip, i.e., there are 16 independent transversity GPDs for the spin-3/2 hadrons. The quark transversity GPDs are chiral-odd since the corresponding operator flips the quark chirality, in contrast to the chiral-even unpolarized and polarized GPDs. Moreover, the helicity flip distributions of the quark and gluon will mix in the evolution of the transversity GPDs. Many studies on the definition of GPDs [7,8,22,24,48] and the model calculations [49–51] have been conducted for the low spin ($J \leq 1$) hadrons.

It should be stressed that the unpolarized and polarized GPDs of spin-3/2 particles have been decomposed, defined, and calculated in our recent papers [47,52]. The priority of this work is to decompose and derive the definitions of the transversity GPDs for the spin-3/2 system. We expect that this series studies of GPDs for spin-3/2 particles give valuable references for possible future experiments at EIC, EicC, BESIII, KEK-B factory, and J-PARC. Moreover, the study of the gluon transversity distribution on Δ isobar is also necessary to understand the non-nucleonic degrees of the hadrons in nuclei [23,53–55]. Experimentally, the timelike GPDs could be measured in principle at BESIII and KEK-B [10] even for spin-3/2 particles like $e^+e^- \to \Delta\bar{\Delta}$ or $\Omega\bar{\Omega}$. Furthermore, there are recent interests on the transition GPDs for $N \to \Delta$ at JLab and J-PARC [13,56], so that spin-3/2 GPD experimental project could become a realistic one in future.

The main content is concentrated in Sec. II. First, the definitions of transversity GPDs and their corresponding amplitudes, along with their symmetry properties, are explained and discussed. In the next two subsections, the specific components of the parton transversity GPDs respectively for the quark and gluon, and the interpretations of their symmetry properties are shown. The corresponding tensor form factors and tensor charges are obtained by the derived sum rules. Moreover, the amplitudes and the relations connecting transversity GPDs with transversity parton distributions are derived and elucidated. To summarize our work, we provide a short discussion in Sec. III.

II. TRANSVERSITY GPDS OF SPIN-3/2 PARTICLES

A. Conventions, correlators and symmetry properties

The GPDs are defined through matrix elements of the nonlocal parton operators. Following the conventions of Ji [7,8], the leading twist GPDs can be defined by the matrix element

$$\int \frac{dz^-}{2\pi} e^{ix(P\cdot z)} \langle p', \lambda' | \mathcal{O} | p, \lambda \rangle \Big|_{z^+=0, \boldsymbol{z}_\perp=0}, \tag{1}$$

where $\mathcal{O}$ is the nonlocal operator at a lightlike separation of the corresponding GPDs. The p (p') and λ (λ') respectively denote the momentum and helicity of the initial (final) state. In this work, the light-cone coordinate is employed and any four-vector v can be rewritten as $v = (v^+, v^-, \boldsymbol{v}_\perp)$, where $v^\pm = v^0 \pm v^3$ and $\boldsymbol{v}_\perp = (v^1, v^2)$. The scalar product of any two four-vectors is $u\cdot v = \frac{1}{2}u^+v^- + \frac{1}{2}u^-v^+ - \boldsymbol{u}_\perp \cdot \boldsymbol{v}_\perp$. Moreover, the light-cone vector $n = (0, 2, \boldsymbol{0}_\perp)$ is needed and $n^2 = 0$. In addition, we use the same kinematical variables with our previous work [47],

$$P = \frac{p' + p}{2}, \qquad \Delta = p' - p, \qquad t = \Delta^2,$$
$$\xi = -\frac{\Delta^+}{2P^+} (|\xi| \leq 1), \qquad x = \frac{k^+}{P^+} (-1 \leq x \leq 1), \tag{2}$$

where $k - \Delta/2$ (or $k + \Delta/2$) is the initial (or final) parton momentum as represented in Fig. 1. Note that here we use Δ instead of q in Ref. [47] to stand for the relative momentum. There are some other conventions in the following, like $a^{[\mu}b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$, $a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$, $\sigma^{ni} = \sigma^{\rho i} n_\rho$, $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$, $\epsilon^{ni\rho\delta} = \epsilon^{\mu i\rho\delta} n_\mu$, and $\epsilon_{0123} = 1$.

In Ref. [47], the unpolarized (polarized) quark or gluon GPDs are defined and decomposed for a spin-3/2 system when we choose the nonlocal operator being $\mathcal{O}^q = \bar{\psi}\not{n}\psi$ ($\bar{\psi}\not{n}\gamma^5\psi$) or $\mathcal{O}^g = G^{n\mu}G_\mu{}^n$ ($G^{n\mu}\tilde{G}_\mu{}^n$) where $G^{n\mu} = G^{\rho\mu}n_\rho$ in Eq. (1). In the present work, $\psi(x)$ and $G^{\mu\nu}(x)$ are employed

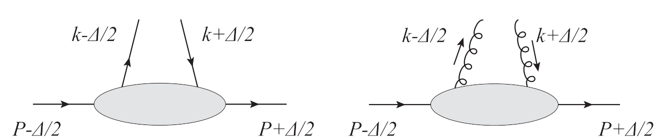

FIG. 1. Diagrams describing the quark (left) and gluon (right) GPDs.

to respectively denote the quark field and gluon field strength tensor, and the corresponding dual field strength tensor is defined as $\tilde{G}^{\rho\sigma}(x) = \frac{1}{2}\epsilon^{\rho\sigma\mu\nu}G_{\mu\nu}(x)$. There is a Wilson line $W[-\frac{1}{2}z^-, \frac{1}{2}z^-]$ along a light-like path between the two fields at positions of $-\frac{1}{2}z^-$ and $\frac{1}{2}z^-$ in the nonlocal operator $\mathcal{O}$. The Wilson line is defined as [8]

$$W[a,b] = \mathcal{P}\exp\left[ig\int_b^a dx^- A^+(x^- n_-)\right], \qquad (3)$$

where $\mathcal{P}$ stands for the path-ordering from b to a and $A(x)$ represents the gluon field. In this work, the light-cone gauge $A^+ = 0$ for the gluon field is considered and then $W[-\frac{1}{2}z^-, \frac{1}{2}z^-] = 1$, so that the Wilson line does not appear in the operators.

Equation (1) with nonlocal operators of $\mathcal{O}^q = \bar{\psi}(-\frac{1}{2}z)i\sigma^{ni}\psi(\frac{1}{2}z)$ and $\mathcal{O}^g = G^{ni}(-\frac{1}{2}z)G^{jn}(\frac{1}{2}z)$ defines the quark and gluon transversity GPDs as

$$T^{qi}_{\lambda'\lambda} = \frac{1}{2}\int\frac{dz^-}{2\pi}e^{ix(P\cdot z)}\left\langle p',\lambda'\left|\bar{\psi}\left(-\frac{1}{2}z\right)i\sigma^{ni}\psi\left(\frac{1}{2}z\right)\right|p,\lambda\right\rangle\Big|_{z^+=0,\mathbf{z}_\perp=0}, \qquad (4)$$

and

$$T^{gij}_{\lambda'\lambda} = \frac{1}{2P^+}\int\frac{dz^-}{2\pi}e^{ix(P\cdot z)}\left\langle p',\lambda'\left|\mathrm{Tr}\hat{\mathbf{S}}\left[G^{ni}\left(-\frac{1}{2}z\right)G^{jn}\left(\frac{1}{2}z\right)\right]\right|p,\lambda\right\rangle\Big|_{z^+=0,\mathbf{z}_\perp=0}, \qquad (5)$$

where i and j are the transverse indices, and the operator $\hat{\mathbf{S}}$ in Eq. (5) stands for the removal of the trace and the symmetrization between i and j. As the transversity GPDs have different definitions from the helicity nonflip GPDs, the quark and gluon transversity GPDs have the different decompositions since gluon has the more index j.

One can also define the corresponding quark and gluon amplitudes [8] as

$$\mathcal{A}^q_{\lambda'\mu',\lambda\mu} = \int\frac{dz^-}{2\pi}e^{ix(P\cdot z)}\langle p',\lambda'|\mathcal{O}^q_{\mu'\mu}(z)|p,\lambda\rangle\Big|_{z^+=0,\mathbf{z}_\perp=0},$$
$$\mathcal{A}^g_{\lambda'\mu',\lambda\mu} = \frac{1}{P^+}\int\frac{dz^-}{2\pi}e^{ix(P\cdot z)}\langle p',\lambda'|\mathcal{O}^g_{\mu'\mu}(z)|p,\lambda\rangle\Big|_{z^+=0,\mathbf{z}_\perp=0}, \qquad (6)$$

where the label μ (μ') represents the emitted (re-absorbed) parton helicity. The amplitudes satisfy the constraints of [8,22]

$$\mathcal{A}^{q/g}_{-\lambda'-\mu',-\lambda-\mu}(P,\Delta,n) = (-1)^{(\lambda'-\lambda)-(\mu'-\mu)}\mathcal{A}^{q/g*}_{\lambda'\mu',\lambda\mu}(P,\Delta,n)$$
$$\mathcal{A}^{q/g}_{-\lambda'-\mu',-\lambda,-\mu}(P,\Delta,n) = (-1)^{(\lambda'-\lambda)-(\mu'-\mu)}\mathcal{A}^{q/g}_{\lambda\mu,\lambda'\mu'}(P,-\Delta,n) \qquad (7)$$

from Hermiticity, parity and time reversal transformations. Table I lists all the operators carrying the parton helicity at leading twist. Notice that the helicity flip operators on the quark have another forms,

$$\mathcal{O}^q_{-+} = -\frac{1}{4}\bar{\psi}(i\sigma^{+1} - ii\sigma^{+2})\psi = -\frac{1}{4}\bar{\psi}i\sigma^{+1}(1+\gamma^5)\psi,$$
$$\mathcal{O}^q_{+-} = \frac{1}{4}\bar{\psi}(i\sigma^{+1} + ii\sigma^{+2})\psi = \frac{1}{4}\bar{\psi}i\sigma^{+1}(1-\gamma^5)\psi, \qquad (8)$$

because of $i\sigma^{+1}\gamma^5 = -ii\sigma^{+2}$. Amplitudes (6) with $\mu' = \mu$ contain 16 nonflip amplitudes in the spin-3/2 system, which have been explicitly given in Ref. [47]. Analogously, the parity invariance restricts 16 independent quark or gluon transversity GPDs for the spin-3/2 particles when $\mu' \neq \mu$. Note that the helicity label $\pm$ on the partons of the quark stands for $\pm\frac{1}{2}$ and those of the gluon for ± 1 according to the corresponding spin. More symmetry properties, such as Hermiticity, light-front parity, and light-front time reversal, are also satisfied [22], and they do not provide any further constraints on the number of GPDs.

TABLE I. Leading twist operators $\mathcal{O}^q_{\mu'\mu}$ and $\mathcal{O}^g_{\mu'\mu}$.

$\mathcal{O}^q_{\mu'\mu}$	$\mathcal{O}^g_{\mu'\mu}$	μ'	μ
$\frac{1}{4}\bar{\psi}\gamma^+(1+\gamma^5)\psi$	$\frac{1}{2}[G^{+\rho}G^+_\rho - iG^{+\rho}\tilde{G}^+_\rho]$	+	+
$\frac{1}{4}\bar{\psi}\gamma^+(1-\gamma^5)\psi$	$\frac{1}{2}[G^{+\rho}G^+_\rho + iG^{+\rho}\tilde{G}^+_\rho]$	−	−
$-\frac{1}{4}\bar{\psi}(i\sigma^{+1} - ii\sigma^{+2})\psi$	$\frac{1}{2}[G^{+1}G^{1+} - G^{+2}G^{2+} - i(G^{+1}G^{2+} + G^{+2}G^{1+})]$	−	+
$\frac{1}{4}\bar{\psi}(i\sigma^{+1} + ii\sigma^{+2})\psi$	$\frac{1}{2}[G^{+1}G^{1+} - G^{+2}G^{2+} + i(G^{+1}G^{2+} + G^{+2}G^{1+})]$	+	−

B. Quark transversity GPDs for a spin-3/2 particle

The quark transversity GPDs are defined by the matrix elements of transverse nonlocal quark-quark correlator in Eq. (4) as

$$T^{qi}_{\lambda'\lambda} = -\bar{u}_{\alpha'}(p',\lambda')\mathcal{H}^{qT,i,\alpha'\alpha}(x,\xi,t)u_\alpha(p,\lambda), \qquad (9)$$

where $u_\alpha(p,\lambda)$ is the spin-3/2 field Rarita-Schwinger spinor, shown in Appendix A, normalized to $\bar{u}_\alpha(p,\lambda')u^\alpha(p,\lambda) = -2M\delta_{\lambda'\lambda}$. The tensor function $\mathcal{H}^{qT,i,\alpha'\alpha}(x,\xi,t)$ can be decomposed to 16 terms corresponding to only 16 independent tensor structures. Similar to our previous analysis [47], there are two ways to obtain the sufficiently tensor structures: (1) write down all the possible structures; (2) utilize the direct product between the transvesity spin-1/2 and unpolarized spin-1 structures or between the transversity spin-1 and unpolarized spin-1/2. We can then reduce the redundant tensor structures using some on-shell identities (seen also Appendix B). Consequently, the tensor function of $\mathcal{H}^{qT,i,\alpha'\alpha}$ is decomposed at twist 2 by imposing Hermiticity, parity invariance, and time-reversal invariance as

$$\begin{aligned}
\mathcal{H}^{qT,i,\alpha'\alpha} ={}& H_1^{qT}\frac{i\sigma^{ni}}{(P\cdot n)}g^{\alpha'\alpha} + H_2^{qT}\frac{n^{[\alpha'}g^{\alpha]i}}{(P\cdot n)} + H_3^{qT}\frac{(\not{n}P^i - P\cdot n\gamma^i)}{M(P\cdot n)}g^{\alpha'\alpha} + H_4^{qT}\frac{(\not{n}P^i - P\cdot n\gamma^i)}{M^3(P\cdot n)}P^{\alpha'}P^\alpha \\
&+ H_5^{qT}\frac{(\not{n}\Delta^i - \Delta\cdot n\gamma^i)}{M(P\cdot n)}g^{\alpha'\alpha} + H_6^{qT}\frac{(\not{n}\Delta^i - \Delta\cdot n\gamma^i)}{M^3(P\cdot n)}P^{\alpha'}P^\alpha \\
&+ H_7^{qT}\frac{(\Delta^i + 2\xi P^i)}{M^2}g^{\alpha'\alpha} + H_8^{qT}\frac{(\Delta^i + 2\xi P^i)}{M^4}P^{\alpha'}P^\alpha \\
&+ H_9^{qT}\frac{(\Delta\cdot n n^{\{\alpha'}g^{\alpha\}i} - 2n^{\alpha'}n^\alpha\Delta^i)}{(P\cdot n)^2} + H_{10}^{qT}\frac{(P\cdot n n^{\{\alpha'}g^{\alpha\}i} - 2n^{\alpha'}n^\alpha P^i)}{(P\cdot n)^2} \\
&+ H_{11}^{qT}\frac{(\Delta\cdot n P^{[\alpha'}g^{\alpha]i} - P^{[\alpha'}n^{\alpha]}\Delta^i)}{M^2(P\cdot n)} + H_{12}^{qT}\frac{(P\cdot n P^{[\alpha'}g^{\alpha]i} - P^{[\alpha'}n^{\alpha]}P^i)}{M^2(P\cdot n)} \\
&+ H_{13}^{qT}\frac{M\not{n}(\Delta\cdot n n^{\{\alpha'}g^{\alpha\}i} - 2n^{\alpha'}n^\alpha\Delta^i)}{(P\cdot n)^3} + H_{14}^{qT}\frac{M\not{n}(P\cdot n n^{\{\alpha'}g^{\alpha\}i} - 2n^{\alpha'}n^\alpha P^i)}{(P\cdot n)^3} \\
&+ H_{15}^{qT}\frac{\not{n}(\Delta\cdot n P^{[\alpha'}g^{\alpha]i} - P^{[\alpha'}n^{\alpha]}\Delta^i)}{M(P\cdot n)^2} + H_{16}^{qT}\frac{\not{n}(P\cdot n P^{[\alpha'}g^{\alpha]i} - P^{[\alpha'}n^{\alpha]}P^i)}{M(P\cdot n)^2},
\end{aligned} \qquad (10)$$

where the variables x, ξ, t in the quark transversity GPDs H_i^{qT} are omitted, and M is the mass of the spin-3/2 particle.

There is another equivalent definition of quark transversity GPDs using the matrix element of $i\sigma^{ni}\gamma^5$ instead of $i\sigma^{ni}$ in Eq. (4). Due to the relation [24]

$$i\sigma^{\alpha\beta}\gamma^5 = -\frac{i}{2}\epsilon^{\alpha\beta\rho\delta}i\sigma_{\rho\delta}, \qquad (11)$$

thus the equivalent definition is

$$\begin{aligned}
&\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ix(P\cdot z)}\left\langle p',\lambda'\middle|\bar{\psi}\left(-\frac{1}{2}z\right)i\sigma^{ni}\gamma^5\psi\left(\frac{1}{2}z\right)\middle|p,\lambda\right\rangle\Big|_{z^+=0,z_\perp=0} \\
&= -\bar{u}_{\alpha'}(p',\lambda')\tilde{\mathcal{H}}^{qT,i,\alpha'\alpha}(x,\xi,t)u_\alpha(p,\lambda),
\end{aligned} \qquad (12)$$

where

$$\begin{aligned}
\tilde{\mathcal{H}}^{qT,i,\alpha'\alpha} ={}& H_1^{qT}\frac{i\sigma^{ni}\gamma^5}{(P\cdot n)}g^{\alpha'\alpha} + H_2^{qT}\frac{i\epsilon^{ni\alpha\alpha'}}{(P\cdot n)} + H_3^{qT}\frac{i\epsilon^{niP\delta}\gamma_\delta g^{\alpha'\alpha}}{M(P\cdot n)} + H_4^{qT}\frac{i\epsilon^{niP\delta}\gamma_\delta P^{\alpha'}P^\alpha}{M^3(P\cdot n)} \\
&+ H_5^{qT}\frac{i\epsilon^{ni\Delta\delta}\gamma_\delta g^{\alpha'\alpha}}{M(P\cdot n)} + H_6^{qT}\frac{i\epsilon^{ni\Delta\delta}\gamma_\delta P^{\alpha'}P^\alpha}{M^3(P\cdot n)} + H_7^{qT}\frac{i\epsilon^{ni\Delta P}g^{\alpha'\alpha}}{M^2(P\cdot n)} + H_8^{qT}\frac{i\epsilon^{ni\Delta P}P^{\alpha'}P^\alpha}{M^4(P\cdot n)} \\
&+ H_9^{qT}\frac{i\epsilon^{ni\{\alpha'\Delta}n^{\alpha\}}}{(P\cdot n)^2} + H_{10}^{qT}\frac{i\epsilon^{ni\{\alpha'P}n^{\alpha\}}}{(P\cdot n)^2} + H_{11}^{qT}\frac{i\epsilon^{ni\Delta[\alpha'}P^{\alpha]}}{M^2(P\cdot n)} + H_{12}^{qT}\frac{i\epsilon^{niP[\alpha'}P^{\alpha]}}{M^2(P\cdot n)} \\
&+ H_{13}^{qT}\frac{M\not{n}i\epsilon^{ni\{\alpha'\Delta}n^{\alpha\}}}{(P\cdot n)^3} + H_{14}^{qT}\frac{M\not{n}i\epsilon^{ni\{\alpha'P}n^{\alpha\}}}{(P\cdot n)^3} + H_{15}^{qT}\frac{\not{n}i\epsilon^{ni\Delta[\alpha'}P^{\alpha]}}{M(P\cdot n)^2} + H_{16}^{qT}\frac{\not{n}i\epsilon^{niP[\alpha'}P^{\alpha]}}{M(P\cdot n)^2},
\end{aligned} \qquad (13)$$

which is given by $\tilde{\mathcal{H}}^{qT,i',\alpha'\alpha} = i\epsilon^{0i'i3}\mathcal{H}^{qT,i,\alpha'\alpha}$. The quark transversity GPDs H_i^{qT} in Eq. (13) is the same as the corresponding ones in Eq. (10).

The relations of the amplitudes in Eq. (7) determines the even or odd property with respect to the variable of skewness ξ as

$$H_i^{qT}(x,\xi,t) = H_i^{qT}(x,-\xi,t) \quad \text{with} \quad i = 1,2,5\text{–}9,12,13,16,$$
$$H_j^{qT}(x,\xi,t) = -H_j^{qT}(x,-\xi,t) \quad \text{with} \quad j = 3,4,10,11,14,15, \tag{14}$$

where $H^{qT}_{3,4,10,11,14,15}$ are odd and others are even in ξ. Clearly, all the odd GPDs go to zero when $\xi = 0$.

It might be possibly insignificant but necessary to show the corresponding tensor FFs. In comparison with the unpolarized and polarized GPDs, one can consider the local quark-quark operator corresponding to quark transversity GPDs (4) as

$$T^{\mu\nu} = \langle p',\lambda'|\bar{\psi}(0)i\sigma^{\mu\nu}\psi(0)|p,\lambda\rangle = -2\bar{u}_{\alpha'}(p',\lambda')\mathcal{F}_q^{\mu\nu,\alpha'\alpha}u_\alpha(p,\lambda), \tag{15}$$

which give the ξ-independant tensor FFs. Since the first moments of GPDs in Eq. (10) give the tensor FFs in Eq. (15) by sum rules, the tensor structures corresponding to tensor FFs are contained in Eq. (10). In addition, the multi-n terms, like $\Delta\cdot nn^{\{\alpha'}g^{\alpha\}i}$, $n^{\alpha'}n^{\alpha}$, $\Delta\cdot n\not{n}n^{\{\alpha'}g^{\alpha\}i}$ and so on, should not exist in the tensor structures corresponding to tensor FFs because there is only a position, μ or ν, permitted to place n.

Considering the transversity GPDs (10) and the terms with nonvanishing integration, we get the decomposition of Eq. (15) in terms of the tensor FFs as

$$\begin{aligned}\mathcal{F}_q^{\mu\nu,\alpha'\alpha} &= g^{\alpha'\alpha}\left(G_1^{qT}(t)i\sigma^{\mu\nu} + G_5^{qT}(t)\frac{\gamma^{[\mu}\Delta^{\nu]}}{M} + G_7^{qT}(t)\frac{P^{[\mu}\Delta^{\nu]}}{M^2}\right) \\ &+ \frac{P^{\alpha'}P^{\alpha}}{M^2}\left(G_6^{qT}(t)\frac{\gamma^{[\mu}\Delta^{\nu]}}{M} + G_8^{qT}(t)\frac{P^{[\mu}\Delta^{\nu]}}{M^2}\right) \\ &+ G_2^{qT}(t)g^{\mu[\alpha'}g^{\alpha]\nu} + G_{12}^{qT}(t)\frac{P^{[\alpha'}g^{\alpha][\nu}P^{\mu]}}{M^2},\end{aligned} \tag{16}$$

and the tensor FFs $G_i^{qT}(t)$ are related with the transversity GPDs by the sum rules

$$\int_{-1}^{1} dx\, H_i^{qT}(x,\xi,t) = G_i^{qT}(t) \quad \text{with} \quad i = 1,2,5\text{–}8,12,$$
$$\int_{-1}^{1} dx\, H_j^{qT}(x,\xi,t) = 0 \quad \text{with} \quad j = 3,4,9,10,11,13\text{–}16. \tag{17}$$

There are three [24], five [22] and seven tensor FFs for the spin-$1/2$, -1 and $-3/2$ system, respectively. Since there is the relation, $\bar{u}_{\alpha'}(p,\lambda')(g^{\alpha'\alpha}i\sigma^{\mu\nu} + g^{\mu[\alpha'}g^{\alpha]\nu})u_\alpha(p,\lambda) = 0$, in the forward limit, which is proved in B, there is only a nonzero form factor $G_1^{qT}(0) - G_2^{qT}(0)$ describing the quark tensor charge. Beyond the Standard Model, the tensor charge is explained as the number of the electric dipole moment carried by the corresponding quark [57,58].

In addition, the quark amplitudes with helicity flip can be derived according to their definitions in Eq. (6). We use the same notations as in our previous work [47]

$$\begin{aligned}|\boldsymbol{p}_\perp|e^{\pm i\phi} &\equiv p^x \pm ip^y, \qquad |\boldsymbol{p}'_\perp|e^{\pm i\phi'} \equiv p'^x \pm ip'^y, \\ C &= \sqrt{\frac{1-\xi}{1+\xi}}\frac{|\boldsymbol{p}_\perp|}{M}e^{-i\phi} - \sqrt{\frac{1+\xi}{1-\xi}}\frac{|\boldsymbol{p}'_\perp|}{M}e^{-i\phi'} \\ &= -\frac{(\Delta+2\xi P)^x - i(\Delta+2\xi P)^y}{M\sqrt{1-\xi^2}}, \\ D &= -\frac{t}{4M^2} - \frac{\xi^2}{1-\xi^2}.\end{aligned} \tag{18}$$

The notation C represents the transfer of the transverse momentum between the initial and final hadron states, which induces that the hadron obtains the transfer of the orbital angular momentum that violates the helicity conservation, i.e. $\lambda' - \lambda = \mu' - \mu$ for $\mathcal{A}_{\lambda'\mu',\lambda\mu}$. In Eq. (18), the transversity indices x and y instead of 1 and 2 are used to avoid ambiguity. Moreover, C will be real when the azimuthal angle of the four-vector $\Delta + 2\xi P$ [8,22] is zero, i.e. $(1-\xi)p^y = (1+\xi)p'^y$, and be zero in the forward limit.

Incorporating the amplitudes with helicity nonflip in Ref. [47], we find that any amplitude $\mathcal{A}_{\lambda'\mu',\lambda\mu}$ has a common factor

$$F(\zeta) = C^\zeta\theta(\zeta) + C^{*-\zeta}\theta(-\zeta) \quad \text{with} \quad \zeta = (\lambda'-\lambda) - (\mu'-\mu), \tag{19}$$

where C^ζ represents the ζ powers of C, $\theta(\zeta)$ is the Heaviside step function with $\theta(0) = \frac{1}{2}$ and $F(0) = 1$. We conclude that $F(\zeta)$ has some properties, such as $F(\zeta \neq 0) = 0$ in the forward limit and $F^*(\zeta) = F(-\zeta)$. Therefore, the amplitudes are real if the factor $F(\zeta)$ is excluded and we can define the real amplitude as

$$\mathcal{A}'_{\lambda'\mu',\lambda\mu} = \frac{\mathcal{A}_{\lambda'\mu',\lambda\mu}}{F(\lambda'-\lambda-\mu'+\mu)}, \tag{20}$$

and the relations in Eq. (7) becomes the real form,

$$\mathcal{A}'_{-\lambda'-\mu',-\lambda-\mu} = (-1)^{(\lambda'-\lambda)-(\mu'-\mu)}\mathcal{A}'_{\lambda'\mu',\lambda\mu}. \tag{21}$$

For the quark, the factors μ' and μ are $\mu'(\mu) = \pm\frac{1}{2}$ due to the quark spin. We believe that the low spin particles [22,24] have the analogous character.[1] The specific forms of the 16 quark amplitudes are

[1]There might be a typo in (C15) of Ref. [22] according to this property.

$$\mathcal{A}'^{q}_{(3/2)-,(3/2)+} = (1+\xi)\left[\left(\frac{1}{2}H_3^{qT} - H_5^{qT}\right) + \frac{|C|^2}{8}\left(\frac{1}{2}H_4^{qT} - H_6^{qT}\right) - \left(\xi H_{15}^{qT} - \frac{1}{2}H_{16}^{qT}\right)\right] - \left(H_7^{qT} + \frac{|C|^2}{8}H_8^{qT}\right) - \frac{1}{1-\xi}\left(\xi H_{11}^{qT} - \frac{1}{2}H_{12}^{qT}\right), \tag{22}$$

$$\begin{aligned}\mathcal{A}'^{q}_{(3/2)-,(1/2)+} = &-\frac{(1+\xi)}{2\sqrt{3(1-\xi^2)}}[(1+\xi)(H_3^{qT} - 2H_5^{qT}) - (3-\xi)H_7^{qT} + (1-\xi)H_{11}^{qT} + H_{12}^{qT}] \\ &+ \frac{\sqrt{1-\xi^2}}{2\sqrt{3}}\left[\frac{|C|^2}{8}H_8^{qT} - (1+\xi)\left(H_{15}^{qT} + \frac{1}{2}H_{16}^{qT}\right)\right] \\ &- \frac{[D(1-\xi^2)+\xi]}{2\sqrt{3(1-\xi^2)}}\left[\frac{1+\xi}{1-\xi}\left(\frac{1}{2}H_4^{qT} - H_6^{qT}\right) - \frac{1}{1-\xi}H_8^{qT}\right],\end{aligned} \tag{23}$$

$$\mathcal{A}'^{q}_{(3/2)-,(-1/2)+} = -\frac{(1+\xi)}{8\sqrt{3}}\left(\frac{1}{2}H_4^{qT} - H_6^{qT} + 4H_7^{qT} - 2H_{11}^{qT} - H_{12}^{qT}\right) + \frac{1}{8\sqrt{3}}H_8^{qT} - \frac{[D(1-\xi^2)+\xi]}{4\sqrt{3}(1-\xi)}H_8^{qT}, \tag{24}$$

$$\mathcal{A}'^{q}_{(3/2)-,(-3/2)+} = -\frac{\sqrt{1-\xi^2}}{16}H_8^{qT}, \tag{25}$$

$$\begin{aligned}\mathcal{A}'^{q}_{(1/2)-,(3/2)+} = &-\frac{2\sqrt{1-\xi^2}}{\sqrt{3}}\left[H_1^{qT} - \frac{|C|^2}{2}\left(\frac{1}{2}H_3^{qT} - H_5^{qT}\right) + \frac{|C|^4}{32}H_8^{qT}\right] \\ &- \frac{4\sqrt{1-\xi^2}(1-\xi)}{\sqrt{3}}\left[\left(\xi H_{13}^{qT} - \frac{1}{2}H_{14}^{qT}\right) + \frac{|C|^2}{8}(H_{15}^{qT} - \frac{1}{2}H_{16}^{qT})\right] \\ &+ \frac{2(1-\xi)}{\sqrt{3(1-\xi^2)}}\left[H_2^{qT} - 2\left(\xi H_9^{qT} - \frac{1}{2}H_{10}^{qT}\right) - \frac{|C|^2}{4}(3+\xi)H_7^{qT}\right] \\ &- \frac{2}{\sqrt{3(1-\xi^2)}}\left[2\xi\left(\frac{1}{2}H_3^{qT} - \xi H_5^{qT}\right) + \frac{|C|^2}{4}\left(\xi\left(\frac{1}{2}H_4^{qT} - \xi H_6^{qT}\right) + (1+\xi^2)H_{11}^{qT} - H_{12}^{qT}\right)\right] \\ &- \frac{4[D(1-\xi^2)-\xi]}{\sqrt{3(1-\xi^2)}}\left[\frac{1}{1-\xi^2}\left(\xi H_{11}^{qT} - \frac{1}{2}H_{12}^{qT}\right) + \left(\xi H_{15}^{qT} - \frac{1}{2}H_{16}^{qT}\right) - \frac{|C|^2}{8}\left(\frac{1}{2}H_4^{qT} - H_6^{qT} - \frac{1}{1+\xi}H_8^{qT}\right)\right],\end{aligned} \tag{26}$$

$$\begin{aligned}\mathcal{A}'^{q}_{(1/2)-,(1/2)+} = &\frac{2(1+\xi)}{3}\left[H_1^{qT} + H_{16}^{qT} + \frac{(1+\xi)}{4(1-\xi)}H_3^{qT}\right] \\ &- \frac{(1-\xi)}{3}\left[H_2^{qT} + 2H_9^{qT} + H_{10}^{qT} - \left(\xi H_{15}^{qT} - \frac{1}{2}H_{16}^{qT}\right) + \frac{|C|^2}{8}\left(\frac{1}{2}H_4^{qT} + H_6^{qT}\right)\right] \\ &- \frac{4(1-\xi^2)}{3}\left(\frac{1}{2}H_9^{qT} + H_{13}^{qT} - \frac{1}{2}\xi H_{14}^{qT}\right) - \frac{2}{3}\left[H_2^{qT} - \left(\xi H_{15}^{qT} - \frac{1}{2}H_{16}^{qT}\right)\right] \\ &+ \frac{7}{3}\xi H_5^{qT} + \frac{1}{3}(H_5^{qT} + H_7^{qT}) - \frac{4\xi}{3}\left(\xi H_9^{qT} - \frac{1}{2}H_{10}^{qT}\right) + \frac{|C|^2}{3}\left[H_7^{qT} - \frac{5}{8}H_8^{qT} - \frac{1}{2}\left(\xi H_{11}^{qT} + \frac{1}{2}H_{12}^{qT}\right)\right] \\ &+ \frac{1}{3(1+\xi)}\left(\xi H_{11}^{qT} + \frac{1}{2}H_{12}^{qT} + \frac{|C|^2}{2}H_8^{qT}\right) - \frac{4}{3(1-\xi)}(H_5^{qT} + H_7^{qT}) + \frac{2\xi}{3(1-\xi^2)}\left(H_{11}^{qT} - \frac{1}{2}H_{12}^{qT}\right) \\ &- \frac{4[D(1-\xi^2)-\xi]}{3(1-\xi)}\left[\frac{1}{2}H_3^{qT} - H_5^{qT} - \frac{1}{1+\xi}\left(\frac{1}{2}\xi H_{11}^{qT} - \frac{3}{4}H_{12}^{qT}\right)\right] \\ &+ \frac{2[D(1-\xi^2)+\xi]}{3(1-\xi^2)}\left[\frac{\xi}{(1-\xi)}\left(\frac{1}{2}H_4^{qT} - \xi H_6^{qT}\right) + 2H_7^{qT} - (1-\xi^2)H_{16}^{qT} + \frac{|C|^2}{4}H_8^{qT}\right] \\ &- \frac{2[D(1-\xi^2)-\xi][D(1-\xi^2)+\xi]}{3(1-\xi^2)(1-\xi)}\left(\frac{1}{2}H_4^{qT} - H_6^{qT} - \frac{1}{1+\xi}H_8^{qT}\right),\end{aligned} \tag{27}$$

$$\begin{aligned}\mathcal{A}'^{q}_{(1/2)-,(-1/2)+} =& \frac{2\sqrt{1-\xi^2}}{3}\left[\frac{1}{2}H_2^{qT} + H_5^{qT} + H_9^{qT} - \frac{1}{2}\xi H_{10}^{qT} + \frac{1}{2}H_{12}^{qT} - \frac{1}{2}\left(\xi H_{15}^{qT} + \frac{1}{2}H_{16}^{qT}\right) + \frac{|C|^2}{32}H_8^{qT}\right] \\ &+ \frac{(1+\xi^2)}{\sqrt{1-\xi^2}}H_7^{qT} + \frac{1}{\sqrt{1-\xi^2}}\left[\frac{\xi}{2}\left(\frac{1}{2}H_4^{qT} - \xi H_6^{qT}\right) - \left(\xi H_{11}^{qT} + \frac{1}{2}H_{12}^{qT}\right)\right] \\ &- \frac{\xi(1-\xi)}{3\sqrt{1-\xi^2}}(H_6^{qT} - 2H_7^{qT} + H_{12}^{qT}) - \frac{\xi(1-\xi)}{3\sqrt{(1-\xi^2)^3}}H_8^{qT} + \frac{[D(1-\xi^2)+\xi]}{3\sqrt{1-\xi^2}}(H_6^{qT} - 2H_7^{qT} + H_{12}^{qT}) \\ &+ \frac{[D(1-\xi^2)+\xi]}{3\sqrt{(1-\xi^2)^3}}H_8^{qT} - \frac{[D(1-\xi^2)-\xi][D(1-\xi^2)+\xi]}{3\sqrt{(1-\xi^2)^3}}H_8^{qT}, \end{aligned} \tag{28}$$

$$\mathcal{A}'^{q}_{(1/2)-,(-3/2)+} = \frac{(1-\xi)}{2\sqrt{3}}\left[\frac{1}{4}\left(\frac{1}{2}H_4^{qT} + H_6^{qT}\right) - H_7^{qT} - \frac{1}{2}\left(H_{11}^{qT} - \frac{1}{2}H_{12}^{qT}\right)\right] + \frac{1}{8\sqrt{3}}H_8^{qT} - \frac{[D(1-\xi^2)-\xi]}{4\sqrt{3}(1+\xi)}H_8^{qT}, \tag{29}$$

$$\begin{aligned}\mathcal{A}'^{q}_{(-1/2)-,(3/2)+} =& -\frac{2(1-\xi)}{\sqrt{3}}\left[\left(H_1^{qT} - \frac{1}{2}H_2^{qT}\right) + \frac{|C|^2}{4}H_7^{qT} + \left(\xi H_9^{qT} - \frac{1}{2}H_{10}^{qT}\right)\right] \\ &- \frac{|C|^2(1+\xi)}{4\sqrt{3}}\left[\frac{1}{2}\left(\frac{1}{2}H_4^{qT} - H_6^{qT}\right) + \frac{1-\xi}{1+\xi}\left(H_{11}^{qT} - \frac{1}{2}H_{12}^{qT}\right)\right] + \frac{|C|^2}{8\sqrt{3}}H_8^{qT} + \frac{2}{\sqrt{3}}\left(\xi H_{15}^{qT} - \frac{1}{2}H_{16}^{qT}\right) \\ &- \frac{4}{\sqrt{3}(1+\xi)}\left[\xi\left(\frac{1}{2}H_3^{qT} - \xi H_5^{qT}\right) - \frac{1}{2(1-\xi)}\left(\xi H_{11}^{qT} - \frac{1}{2}H_{12}^{qT}\right)\right] \\ &- \frac{2[D(1-\xi^2)-\xi]}{\sqrt{3}(1-\xi^2)}\left[\left(\xi H_{11}^{qT} - \frac{1}{2}H_{12}^{qT}\right) + \frac{\xi}{(1+\xi)}\left(\frac{1}{2}H_4^{qT} - \xi H_6^{qT}\right) + (1-\xi)\frac{|C|^2}{8}H_8^{qT}\right], \end{aligned} \tag{30}$$

$$\begin{aligned}\mathcal{A}'^{q}_{(-1/2)-,(1/2)+} =& -\frac{4(1-\xi)^2}{3\sqrt{(1-\xi^2)}}H_1^{qT} - \frac{4(1-\xi)}{3\sqrt{1-\xi^2}}\left[\frac{|C|^2}{4}\left(3H_7^{qT} - H_{12}^{qT} + \frac{\xi(1+\xi)}{1-\xi}H_7^{qT}\right)\right] + \frac{4}{3\sqrt{1-\xi^2}}H_2^{qT} \\ &+ \frac{8\xi}{3\sqrt{1-\xi^2}}\left[-\left(\xi H_{15}^{qT} - \frac{1}{2}H_{16}^{qT}\right) - \frac{1-\xi}{1+\xi}\left(\frac{1}{2}H_3^{qT} - \xi H_5^{qT}\right) + \left(\xi H_9^{qT} - \frac{1}{2}H_{10}^{qT}\right)\right] \\ &+ \frac{2|C|^2\xi}{3\sqrt{1-\xi^2}}\left[H_3^{qT} - \frac{1}{2}\frac{1+\xi}{1-\xi}\left(\frac{1}{2}H_4^{qT} - H_6^{qT}\right) + H_{11}^{qT}\right] \\ &+ \frac{8\sqrt{1-\xi^2}}{3}\left[\xi\left(\xi H_{13}^{qT} - \frac{1}{2}H_{14}^{qT}\right) - \frac{|C|^2}{4}\frac{1+\xi^2}{1-\xi^2}H_5^{qT}\right] \\ &+ \frac{8\sqrt{1-\xi^2}}{3}\left[-\frac{|C|^2}{8}\left(H_2^{qT} + 2H_5^{qT} + 2\left(H_9^{qT} - \frac{1}{2}\xi H_{10}^{qT}\right) - \left(\xi H_{15}^{qT} + \frac{1}{2}H_{16}^{qT}\right)\right) - \frac{|C|^4}{128}H_8^{qT}\right] \\ &- \frac{8(1-\xi)}{3\sqrt{(1-\xi^2)^3}}\left[\frac{\xi}{1-\xi}\left(\xi H_{11}^{qT} - \frac{1}{2}H_{12}^{qT}\right) + \frac{\xi(1+\xi)}{(1-\xi)}\frac{|C|^2}{8}H_8^{qT}\right] \\ &+ \frac{8[D(1-\xi^2)-\xi]}{3\sqrt{1-\xi^2}}\left[H_1^{qT} - \left(\xi H_{15}^{qT} - \frac{1}{2}H_{16}^{qT}\right) - \frac{|C|^2}{8}\left(\frac{1+\xi^2}{1-\xi^2}H_6^{qT} - 2H_7^{qT} + H_{12}^{qT}\right)\right] \\ &+ \frac{8[D(1-\xi^2)-\xi]}{3\sqrt{(1-\xi^2)^3}}\left[\xi(H_3^{qT} - 2\xi H_5^{qT}) - \left(\xi H_{11}^{qT} - \frac{1}{2}H_{12}^{qT}\right) + \frac{|C|^2}{8}(\xi H_4^{qT} - H_8^{qT})\right] \\ &+ \frac{8[D(1-\xi^2)+\xi][D(1-\xi^2)-\xi]}{3\sqrt{(1-\xi^2)^5}}\left[\xi\left(\frac{1}{2}H_4^{qT} - \xi H_6^{qT}\right) + \frac{|C|^2(1-\xi^2)}{8}H_8^{qT}\right], \end{aligned} \tag{31}$$

$$\mathcal{A}'^{q}_{(-1/2)-,(-1/2)+} = -\frac{4(1-\xi^2)}{3}\left(H_{13}^{qT} - \frac{1}{2}\xi H_{14}^{qT}\right) + \frac{2(1-\xi)}{3}\left[H_1^{qT} - \frac{1-\xi}{4(1+\xi)}H_3^{qT}\right] - \frac{(1-\xi)}{3(1+\xi)}H_5^{qT}$$
$$- \frac{(1+\xi)}{3}\left[H_2^{qT} + 2\xi H_9^{qT} - H_{10}^{qT} - \left(\xi H_{15}^{qT} + \frac{1}{2}H_{16}^{qT}\right) - \frac{|C|^2}{8}\left(\frac{1}{2}H_4^{qT} - H_6^{qT}\right)\right]$$
$$- \frac{2}{3}\left[H_2^{qT} - \frac{1}{2}H_7^{qT} + 2H_9^{qT} + \xi\left(\frac{7}{2}H_5^{qT} - H_{10}^{qT} - H_{15}^{qT} - \frac{1}{2}H_{16}^{qT}\right)\right]$$
$$+ \frac{|C|^2}{3}\left[H_7^{qT} - \frac{5}{8}H_8^{qT} - \frac{1}{2}\left(\xi H_{11}^{qT} + \frac{1}{2}H_{12}^{qT}\right)\right] - \frac{2}{3(1+\xi)}\left(H_5^{qT} + 2H_7^{qT} - \xi H_{11}^{qT} - \frac{|C|^2}{4}H_8^{qT}\right)$$
$$+ \frac{1}{3(1-\xi)}\left(\xi H_{11}^{qT} + \frac{1}{2}H_{12}^{qT}\right) + \frac{2\xi}{3(1-\xi^2)}\left(\xi H_{11}^{qT} + \frac{1}{2}H_{12}^{qT}\right)$$
$$+ \frac{2[D(1-\xi^2)-\xi]}{3(1-\xi^2)}\left[\frac{\xi}{(1+\xi)}\left(\frac{1}{2}H_4^{qT} - \xi H_6^{qT}\right) + 2H_7^{qT}\right]$$
$$+ \frac{2[D(1-\xi^2)+\xi]}{3(1-\xi^2)}\left[2(1-\xi)\left(\frac{1}{2}H_3^{qT} + H_5^{qT}\right) + \xi H_{11}^{qT} - \frac{3}{2}H_{12}^{qT} - (1-\xi^2)H_{16}^{qT} + \frac{|C|^2}{4}H_8^{qT}\right]$$
$$+ \frac{2[D(1-\xi^2)+\xi][D(1-\xi^2)-\xi]}{3(1-\xi^2)(1+\xi)}\left(\frac{1}{2}H_4^{qT} + H_6^{qT} + \frac{H_8^{qT}}{1-\xi}\right), \tag{32}$$

$$\mathcal{A}'^{q}_{(-1/2)-,(-3/2)+} = \frac{\sqrt{1-\xi^2}}{\sqrt{3}}\left[\frac{1-\xi}{1+\xi}\left(\frac{1}{2}H_3^{qT} + H_5^{qT}\right) + \frac{1}{2}H_7^{qT} + \frac{1}{2}H_{11}^{qT} + \frac{|C|^2}{16}H_8^{qT}\right] + \frac{(1-\xi)}{\sqrt{3}\sqrt{1-\xi^2}}\left(H_7^{qT} - \frac{1}{2}H_{12}^{qT}\right)$$
$$+ \frac{\sqrt{1-\xi^2}(1-\xi)}{2\sqrt{3}}\left(H_{15}^{qT} - \frac{1}{2}H_{16}^{qT}\right) + \frac{[D(1-\xi^2)-\xi]}{2\sqrt{3}\sqrt{1-\xi^2}}\left[\frac{1-\xi}{1+\xi}\left(\frac{1}{2}H_4^{qT} + H_6^{qT}\right) + \frac{1}{1+\xi}H_8^{qT}\right], \tag{33}$$

$$\mathcal{A}'^{q}_{(-3/2)-,(3/2)+} = \frac{1}{\sqrt{1-\xi^2}}\left[\frac{\xi}{2}\left(\frac{1}{2}H_4^{qT} - \xi H_6^{qT}\right) + \left(\xi H_{11}^{qT} - \frac{1}{2}H_{12}^{qT}\right)\right] + \frac{|C|^2\sqrt{1-\xi^2}}{16}H_8^{qT}, \tag{34}$$

$$\mathcal{A}'^{q}_{(-3/2)-,(1/2)+} = -\frac{2(1+\xi)}{\sqrt{3}}\left[H_1^{qT} - \frac{1}{2}H_2^{qT} - \xi H_9^{qT} + \frac{1}{2}H_{10}^{qT} + \frac{|C|^2}{4}\left(H_7^{qT} - \frac{1}{2}H_{11}^{qT} - \frac{1}{4}H_{12}^{qT}\right)\right]$$
$$+ \frac{|C|^2(1-\xi)}{8\sqrt{3}}\left(\frac{1}{2}H_4^{qT} + H_6^{qT}\right) + \frac{2}{\sqrt{3}}\left(\xi H_{15}^{qT} - \frac{1}{2}H_{16}^{qT} + \frac{|C|^2}{16}H_8^{qT}\right)$$
$$- \frac{4\xi}{\sqrt{3}(1-\xi)}\left(\frac{1}{2}H_3^{qT} - \xi H_5^{qT}\right) + \frac{2}{\sqrt{3}(1-\xi^2)}\left(\xi H_{11}^{qT} - \frac{1}{2}H_{12}^{qT}\right)$$
$$- \frac{2[D(1-\xi^2)+\xi]}{\sqrt{3}(1-\xi^2)}\left[\frac{\xi}{1-\xi}\left(\frac{1}{2}H_4^{qT} - \xi H_6^{qT}\right) + \left(\xi H_{11}^{qT} - \frac{1}{2}H_{12}^{qT}\right) + \frac{|C|^2}{8(1+\xi)}H_8^{qT}\right], \tag{35}$$

$$\begin{aligned}\mathcal{A}'^{q}_{(-3/2)-,(-1/2)+} = &+\frac{4(1+\xi)\sqrt{1-\xi^2}}{\sqrt{3}}\left[\left(\xi H^{qT}_{13}-\frac{1}{2}H^{qT}_{14}\right)+\frac{|C|^2}{8}\left(H^{qT}_{15}+\frac{1}{2}H^{qT}_{16}\right)\right]\\&-\frac{2\sqrt{1-\xi^2}}{\sqrt{3}}\left(H^{qT}_{1}-\frac{1}{1-\xi}H^{qT}_{2}+\frac{|C|^2}{2}\left(\frac{1}{2}H^{qT}_{3}+H^{qT}_{5}+\frac{1}{2}H^{qT}_{7}\right)+\frac{|C|^4}{32}H^{qT}_{8}\right)\\&-\frac{2\xi}{\sqrt{3(1-\xi^2)}}\left[(H^{qT}_{3}-2\xi H^{qT}_{5})+\frac{|C|^2}{4}\left(\frac{1}{2}H^{qT}_{4}-\xi H^{qT}_{6}\right)\right]\\&+\frac{4(1+\xi)}{\sqrt{3(1-\xi^2)}}\left(\xi H^{qT}_{9}-\frac{1}{2}H^{qT}_{10}-\frac{|C|^2}{4}H^{qT}_{7}\right)+\frac{|C|^2}{2\sqrt{3(1-\xi^2)}}[H^{qT}_{11}+H^{qT}_{12}]\\&-\frac{4[D(1-\xi^2)+\xi]}{\sqrt{3(1-\xi^2)^3}}\left[\left(\xi H^{qT}_{11}-\frac{1}{2}H^{qT}_{12}\right)\right]\\&-\frac{4[D(1-\xi^2)+\xi]}{\sqrt{3(1-\xi^2)}}\left[\left(\xi H^{qT}_{15}-\frac{1}{2}H^{qT}_{16}\right)+\frac{|C|^2}{8}\left(\frac{1}{2}H^{qT}_{4}+H^{qT}_{6}+\frac{1}{1-\xi}H^{qT}_{8}\right)\right],\end{aligned}\tag{36}$$

$$\begin{aligned}\mathcal{A}'^{q}_{(-3/2)-,(-3/2)+} = &-(1-\xi)\left(\frac{1}{2}H^{qT}_{3}+H^{qT}_{5}+\left(\xi H^{qT}_{15}-\frac{1}{2}H^{qT}_{16}\right)+\frac{|C|^2}{8}\left(\frac{1}{2}H^{qT}_{4}+H^{qT}_{6}\right)\right)\\&-\left(H^{qT}_{7}+\frac{|C|^2}{8}H^{qT}_{8}\right)-\frac{1}{(1+\xi)}\left(\xi H^{qT}_{11}-\frac{1}{2}H^{qT}_{12}\right).\end{aligned}\tag{37}$$

The other amplitudes with helicity flip $\mathcal{A}'^{q}_{i+,j-}$ can be obtained by the parity invariance (21).

In the forward limit, since $\bar{u}_{\alpha'}P^{\alpha'} = P^{\alpha}u_{\alpha} = 0$, the nonzero contributions of $H^{qT}_{1,2}(x,0,0)$ give the transversity distribution function $h_1(x)$ [21,22]. In addition, as discussed above, there is no orbital contribution, i.e., $F(\zeta \neq 0) = 0$, which constrains that the only helicity conserved amplitudes, $\mathcal{A}^{q}_{(1/2)-,(3/2)+}$, $\mathcal{A}^{q}_{(-3/2)-,(-1/2)+}$, and $\mathcal{A}^{q}_{(-1/2)-,(1/2)+}$, remain finite. Thus, the transversity distribution function $h_1(x)$ can then be given as

$$2[H^{qT}_{1}(x,0,0) - H^{qT}_{2}(x,0,0)] = h_1(x), \tag{38}$$

where the number density is from two components, spin-1/2 and spin-1, due to the fact that the Rarita-Schwinger spinor is composed by the fields of spin-1/2 and spin-1. Compared the decompositions of spin-3/2 quark transversity GPDs (10) with those of spin-1/2 [24] and of spin-1 [22], $H^{qT}_{1}(x,0,0)$ is from spin-1/2 component and $H^{qT}_{2}(x,0,0)$ from spin-1. The minus sign comes from the definition difference between (9) in this work and Eq. (17) in [22].

Recall that the transversity distribution function $h_1(x)$ can be measured in semi-inclusive deep inelastic scattering from transverse polarized targets in the scaling limit using the "Collins asymmetry" [59],

$$A_{\mathrm{Coll}} = \frac{\sum_q e_q^2 h_1^q \Delta_T^0 D_q^h}{\sum_q e_q^2 f_1^q D_q^h}, \tag{39}$$

where e_q, f_1^q, $\Delta_T^0 D_q^h$, and D_q^h are the quark charge, the unpolarized parton distribution function, the spin-dependent fragmentation function, and the spin-independent fragmentation function, respectively.

C. Gluon transversity GPDs

According to the corresponding definition (5), the gluon transversity GPDs at twist 2 can be obtained from

$$T^{gij}_{\lambda'\lambda} = -\bar{u}_{\alpha'}(p',\lambda')\mathcal{H}^{gT,ij,\alpha'\alpha}(x,\xi,t)u_{\alpha}(p,\lambda), \tag{40}$$

where

$$\begin{aligned}\mathcal{H}^{gT,ij,\alpha'\alpha} = \hat{\mathbf{S}}_{ij}\Big\{ & H_1^{gT}\frac{(\Delta^i+2\xi P^i)i\sigma^{nj}}{M(P\cdot n)}g^{\alpha'\alpha} + H_2^{gT}\frac{n^{[\alpha'}g^{\alpha]i}(\not{n}P^j - P\cdot n\gamma^j)}{(P\cdot n)^2}\\ & + H_3^{gT}\frac{(\Delta^i+2\xi P^i)n^{[\alpha'}g^{\alpha]j}}{M(P\cdot n)} + H_4^{gT}\frac{(\Delta^i+2\xi P^i)(\Delta^j+2\xi P^j)P^{\alpha'}P^{\alpha}}{M^5}\\ & + H_5^{gT}\frac{(P\cdot ng^{\alpha' i}-n^{\alpha'}P^i)(P\cdot ng^{\alpha j}-n^{\alpha}P^j)}{M(P\cdot n)^2} + H_6^{gT}\frac{(\Delta\cdot ng^{\alpha' i}-n^{\alpha'}\Delta^i)(\Delta\cdot ng^{\alpha j}-n^{\alpha}\Delta^j)}{M(P\cdot n)^2}\\ & + H_7^{gT}\frac{(\Delta^i+2\xi P^i)}{M}\frac{(\Delta\cdot nn^{\{\alpha'}g^{\alpha\}j}-2n^{\alpha'}n^{\alpha}\Delta^j)}{(P\cdot n)^2} + H_8^{gT}\frac{(\Delta^i+2\xi P^i)}{M}\frac{(P\cdot nn^{\{\alpha'}g^{\alpha\}j}-2n^{\alpha'}n^{\alpha}P^j)}{(P\cdot n)^2}\\ & + H_9^{gT}\frac{(\Delta^i+2\xi P^i)}{M^2}\frac{(\Delta\cdot nP^{[\alpha'}g^{\alpha]j}-P^{[\alpha'}n^{\alpha]}\Delta^j)}{M(P\cdot n)} + H_{10}^{gT}\frac{(\Delta^i+2\xi P^i)}{M^2}\frac{(P\cdot nP^{[\alpha'}g^{\alpha]j}-P^{[\alpha'}n^{\alpha]}P^j)}{M(P\cdot n)}\\ & + H_{11}^{gT}\frac{(P\cdot ng^{\alpha' i}-n^{\alpha'}P^i)\not{n}(P\cdot ng^{\alpha j}-n^{\alpha}P^j)}{(P\cdot n)^3} + H_{12}^{gT}\frac{(\Delta\cdot ng^{\alpha' i}-n^{\alpha'}\Delta^i)\not{n}(\Delta\cdot ng^{\alpha j}-n^{\alpha}\Delta^j)}{(P\cdot n)^3}\\ & + H_{13}^{gT}(\Delta^i+2\xi P^i)\not{n}\frac{(\Delta\cdot nn^{\{\alpha'}g^{\alpha\}j}-2n^{\alpha'}n^{\alpha}\Delta^j)}{(P\cdot n)^3} + H_{14}^{gT}(\Delta^i+2\xi P^i)\not{n}\frac{(P\cdot nn^{\{\alpha'}g^{\alpha\}j}-2n^{\alpha'}n^{\alpha}P^j)}{(P\cdot n)^3}\\ & + H_{15}^{gT}(\Delta^i+2\xi P^i)\not{n}\frac{(\Delta\cdot nP^{[\alpha'}g^{\alpha]j}-P^{[\alpha'}n^{\alpha]}\Delta^j)}{M^2(P\cdot n)^2} + H_{16}^{gT}(\Delta^i+2\xi P^i)\not{n}\frac{(P\cdot nP^{[\alpha'}g^{\alpha]j}-P^{[\alpha'}n^{\alpha]}P^j)}{M^2(P\cdot n)^2}\Big\}, \end{aligned} \quad (41)$$

where the operator $\hat{\mathbf{S}}_{ij}$ implies the symmetrization and removal of trace between indices i and j, and the variables x, ξ, t in the gluon transversity GPDs H_i^{gT} are omitted too. Similarly, the relations in Eq. (7) constrains the even or odd behavior in terms of skewness ξ as

$$\begin{aligned} & H_i^{gT}(x,\xi,t) = H_i^{gT}(x,-\xi,t) \ \text{ with } \ i=1,3\text{–}7,10\text{–}13,16,\\ & H_j^{gT}(x,\xi,t) = -H_j^{gT}(x,-\xi,t) \ \text{ with } \ j=2,8,9,14,15, \end{aligned} \quad (42)$$

where $H^{gT}_{2,8,9,14,15}$ are odd and others are even in ξ. Odd GPDs vanish at $\xi = 0$.

We also attempt to give the tensor FFs of the local current $G^{\mu\nu}(0)G^{\rho\sigma}(0)$ corresponding to the gluon transversity GPDs and

$$\begin{aligned} T^{\mu\nu,\rho\sigma} &= \frac{1}{M}\langle p',\lambda'|\mathrm{Tr}\hat{\mathbf{S}}_{\nu\rho}[G^{\mu\nu}(0)G^{\rho\sigma}(0)]|p,\lambda\rangle\\ &= -2\bar{u}_{\alpha'}(p',\lambda')\hat{\mathbf{S}}_{\nu\rho}\mathcal{F}_g^{\mu\nu,\rho\sigma,\alpha'\alpha}u_{\alpha}(p,\lambda). \end{aligned} \quad (43)$$

The gluon transversity GPDs in Eq. (41) can determine the tensor FFs $\mathcal{F}_g^{\mu\nu,\rho\sigma,\alpha'\alpha}$ by sum rules. Eliminating the $n^\rho n^\sigma n^\delta$ and $n^\rho n^\sigma n^\delta n^\nu$ terms since there are only two n's in the operator $G^{ni}(0)G^{jn}(0)$ and considering the ξ symmetry (42), there remains 6 tensor FFs defined as

$$\begin{aligned} \mathcal{F}_g^{\mu\nu,\rho\sigma,\alpha'\alpha,} = \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\Big(& G_1^{gT}(t)i\sigma^{\sigma\rho}g^{\alpha'\alpha} + G_3^{gT}(t)g^{\sigma[\alpha'}g^{\alpha]\rho}\\ & + G_4^{gT}(t)\frac{P^{[\sigma}\Delta^{\rho]}}{M^2} + G_{10}^{gT}(t)\frac{P^{[\alpha'}g^{\alpha][\rho}P^{\sigma]}}{M^2}\Big)\\ & + G_5^{gT}(t)\frac{P^{[\mu}g^{\nu]\alpha'}}{M}\frac{P^{[\sigma}g^{\rho]\alpha}}{M}\\ & + G_6^{gT}(t)\frac{\Delta^{[\mu}g^{\nu]\alpha'}}{M}\frac{\Delta^{[\sigma}g^{\rho]\alpha}}{M}, \end{aligned} \quad (44)$$

and the sum rules collecting the tensor FFs and transversity GPDs are

$$\begin{aligned} & \int_{-1}^{1}dx\,H_i^{gT}(x,\xi,t) = G_i^{gT}(t) \ \text{ with } \ i=1,3\text{–}6,10,\\ & \int_{-1}^{1}dx\,H_j^{gT}(x,\xi,t) = 0 \ \text{ with } \ j=2,7,8,9,11\text{–}16. \end{aligned} \quad (45)$$

The forward limit tells that the local current $G^{\mu\nu}(0)G^{\rho\sigma}(0)$ contains a global symmetry and contributes to the unknown gluon tensor charge $G_5^{gT}(0)$.

Analogous to the quark amplitudes with helicity flip, the ones of gluon can be derived according to the definition in Eq. (6). There is also a common factor $F(\zeta)$ defined in Eq. (19) in the expressions of the gluon amplitudes. Different from the quark case, the spin-1 gluon takes $\mu'(\mu) = \pm 1$ instead of $\pm\frac{1}{2}$. Using the same conventions in Eq. (18) and the definition of the real amplitude (20), we have the 16 gluon amplitudes with helicity flip as

$$\mathcal{A}'^{g}_{(3/2)-,(3/2)+} = -\sqrt{\frac{1+\xi}{1-\xi}}\left(\xi H_9^{gT} - \frac{1}{2}H_{10}^{gT}\right) - \sqrt{1-\xi^2}(1+\xi)\left(\xi H_{15}^{gT} - \frac{1}{2}H_{16}^{gT}\right) - \frac{|C|^2\sqrt{1-\xi^2}}{8}H_4^{gT}, \tag{46}$$

$$\begin{aligned}\mathcal{A}'^{g}_{(3/2)-,(1/2)+} = &-\frac{(1-\xi^2)}{2\sqrt{3}}\left[\left(H_9^{gT} + \frac{1}{1-\xi}H_{10}^{gT}\right) + (1+\xi)\left(H_{15}^{gT} + \frac{1}{2}H_{16}^{gT}\right) - \frac{|C|^2}{8}H_4^{gT}\right]\\ &+ \frac{[D(1-\xi^2)+\xi]}{2\sqrt{3}(1-\xi)}H_4^{gT},\end{aligned} \tag{47}$$

$$\mathcal{A}'^{g}_{(3/2)-,(-1/2)+} = \frac{\sqrt{1-\xi^2}}{8\sqrt{3}}H_4^{gT} + \frac{\sqrt{1-\xi^2}(1+\xi)}{4\sqrt{3}}\left(H_9^{gT} + \frac{1}{2}H_{10}^{gT}\right) - \frac{[D(1-\xi^2)+\xi]}{4\sqrt{3}}\sqrt{\frac{1+\xi}{1-\xi}}H_4^{gT}, \tag{48}$$

$$\mathcal{A}'^{g}_{(3/2)-,(-3/2)+} = -\frac{1-\xi^2}{16}H_4^{gT}, \tag{49}$$

$$\begin{aligned}\mathcal{A}'^{g}_{(1/2)-,(3/2)+} = &-\frac{2(1-\xi^2)}{\sqrt{3}}\left[H_1^{gT} + \frac{1}{2}H_2^{gT} - \left(\frac{1}{2}H_{11}^{gT} + 2\xi H_{12}^{gT}\right) + \frac{|C|^4}{32}H_4^{gT}\right]\\ &- \frac{4(1-\xi^2)(1-\xi)}{\sqrt{3}}\left[\left(\xi H_{13}^{gT} - \frac{1}{2}H_{14}^{gT}\right) + \frac{|C|^2}{8}\left(H_{15}^{gT} - \frac{1}{2}H_{16}^{gT}\right)\right]\\ &+ \frac{4(1-\xi)}{\sqrt{3}}\left(\frac{1}{2}H_3^{gT} - \xi H_7^{gT} + \frac{1}{2}H_8^{gT}\right) + \frac{4}{\sqrt{3}}\left(\frac{1}{4}H_5^{gT} + \xi H_6^{gT}\right) - \frac{|C|^2}{2\sqrt{3}}[(1+\xi^2)H_9^{gT} - H_{10}^{gT}]\\ &- \frac{4[D(1-\xi^2)-\xi]}{\sqrt{3}}\left[\left(\xi H_{15}^{gT} - \frac{1}{2}H_{16}^{gT}\right) + \frac{1}{(1-\xi^2)}\left(\xi H_9^{gT} - \frac{1}{2}H_{10}^{gT}\right) + \frac{|C|^2}{8(1+\xi)}H_4^{gT}\right],\end{aligned} \tag{50}$$

$$\begin{aligned}\mathcal{A}'^{g}_{(1/2)-,(1/2)+} = &\frac{2\sqrt{1-\xi^2}(1+\xi)}{3}\left(H_1^{gT} + \frac{1}{2}H_2^{gT}\right) - \frac{\sqrt{1-\xi^2}(1-\xi)}{3}\left(H_3^{gT} - 2H_6^{gT} + 2H_7^{gT} + H_8^{gT} - \xi H_{15}^{gT} - \frac{1}{2}H_{16}^{gT}\right)\\ &- \frac{2\sqrt{1-\xi^2}}{3}\left[H_3^{gT} + \frac{1}{2}H_5^{gT} + 2H_7^{gT} - \xi H_8^{gT} - \xi\left(H_{15}^{gT} - \frac{1}{2}H_{16}^{gT}\right) + \frac{5|C|^2}{16}\left(H_4^{gT} + \frac{4}{5}\xi H_9^{gT} + \frac{2}{5}H_{10}^{gT}\right)\right]\\ &+ \frac{2\sqrt{(1-\xi^2)^3}}{3}\left(H_7^{gT} - \frac{1}{4}H_{11}^{gT} + H_{12}^{gT} - 2H_{13}^{gT} + \xi H_{14}^{gT}\right)\\ &+ \frac{(1-\xi)}{3\sqrt{1-\xi^2}}\left[\left(\xi H_9^{gT} + \frac{1}{2}H_{10}^{gT}\right) + \frac{2\xi}{(1-\xi)}\left(H_9^{gT} - \frac{1}{2}H_{10}^{gT}\right) + \frac{1+\xi}{2(1-\xi)}|C|^2H_4^{gT}\right]\\ &+ \frac{2[D(1-\xi^2)-\xi]}{3\sqrt{(1-\xi^2)}}\left(\xi H_9^{gT} - \frac{3}{2}H_{10}^{gT} + \frac{|C|^2}{4}H_4^{gT}\right) - \frac{2\sqrt{(1-\xi^2)}[D(1-\xi^2)-\xi]}{3}H_{16}^{gT}\\ &+ \frac{2[D(1-\xi^2)+\xi][D(1-\xi^2)-\xi]}{3\sqrt{(1-\xi^2)^3}}H_4^{gT},\end{aligned} \tag{51}$$

$$\begin{aligned}\mathcal{A}'^{g}_{(1/2)-,(-1/2)+} = &\frac{1-\xi^2}{3}\left[H_3^{gT} + \frac{1}{4}H_5^{gT} - H_6^{gT} + 2H_7^{gT} - \xi H_8^{gT} - \left(\xi H_{15}^{gT} + \frac{1}{2}H_{16}^{gT}\right) + \frac{|C|^2}{16}H_4^{gT}\right]\\ &- \left[\frac{1}{3}H_4^{gT} + \left(\xi H_9^{gT} + \frac{1}{2}H_{10}^{gT}\right)\right] + \frac{1}{3(1-\xi)}(H_4^{gT} + (1-\xi^2)H_{10}^{gT})\\ &+ \frac{[D(1-\xi^2)-\xi]}{3(1-\xi^2)}(H_4^{gT} + (1-\xi^2)H_{10}^{gT}) - \frac{[D(1-\xi^2)+\xi][D(1-\xi^2)-\xi]}{3(1-\xi^2)}H_4^{gT},\end{aligned} \tag{52}$$

$$\mathcal{A}'^{g}_{(1/2)-,(-3/2)+} = \frac{\sqrt{1-\xi^2}}{8\sqrt{3}}[H_4^{gT} - (1-\xi)(2H_9^{gT} - H_{10}^{gT})] - \frac{[D(1-\xi^2)-\xi]}{4\sqrt{3}}\sqrt{\frac{1-\xi}{1+\xi}}H_4^{gT}, \tag{53}$$

$$\mathcal{A}'^{g}_{(-1/2)-,(3/2)+} = -\frac{2|C|^2(1-\xi)\sqrt{1-\xi^2}}{\sqrt{3}}\left[\left(H_1^{gT}-\frac{1}{2}H_3^{gT}+\xi H_7^{gT}-\frac{1}{2}H_8^{gT}\right)+\frac{|C|^2}{8}\left(H_9^{gT}-\frac{1}{2}H_{10}^{gT}\right)\right]$$
$$+\frac{2\sqrt{1-\xi^2}}{\sqrt{3}}\left[\left(\frac{2\xi}{1+\xi}H_2^{gT}-H_{11}^{gT}-4\xi^2H_{12}^{gT}\right)+\frac{|C|^2}{4}(H_5^{gT}+4\xi H_6^{gT}+4\xi H_{15}^{gT}-2H_{16}^{gT})+\frac{|C|^4}{16}(H_4^{gT})\right]$$
$$-\frac{2}{\sqrt{3(1-\xi^2)}}\left(H_5^{gT}+4\xi^2H_6^{gT}-|C|^2\left(\xi H_9^{gT}-\frac{1}{2}H_{10}^{gT}\right)\right)$$
$$-\frac{2|C|^2[D(1-\xi^2)-\xi]}{\sqrt{3(1-\xi^2)}}\left(\xi H_9^{gT}-\frac{1}{2}H_{10}^{gT}\right)-\frac{|C|^4[D(1-\xi^2)-\xi]}{4\sqrt{3}}\sqrt{\frac{1-\xi}{1+\xi}}H_4^{gT}, \quad (54)$$

$$\mathcal{A}'^{g}_{(-1/2)-,(1/2)+} = \frac{4(1-\xi^2)}{3}\left(H_1^{gT}-H_6^{gT}-2H_7^{gT}+\frac{1}{2}H_{11}^{gT}+2\xi^2H_{13}^{gT}-\xi H_{14}^{gT}\right)$$
$$-\frac{|C|^2(1-\xi^2)}{3}\left[\left(H_3^{gT}+\frac{1}{4}H_5^{gT}-H_6^{gT}+2H_7^{gT}-\xi H_8^{gT}-\xi H_{15}^{gT}-\frac{1}{2}H_{16}^{gT}\right)+\frac{|C|^2}{16}H_4^{gT}\right]$$
$$-\frac{8}{3}\xi H_2^{gT}+\frac{4}{3}H_3^{gT}+H_5^{gT}+\frac{4}{3}H_6^{gT}+\frac{8}{3}H_7^{gT}-\frac{4}{3}\xi H_8^{gT}-\frac{8}{3}\left(\xi H_{15}^{gT}-\frac{1}{2}H_{16}^{gT}\right)+\frac{|C|^2}{3}\left(\frac{\xi}{1+\xi}H_4^{gT}+2\xi H_9^{gT}\right)$$
$$-\frac{8(1+\xi)}{3}\left[H_1^{gT}-\frac{|C|^2}{8}H_{10}^{gT}-\xi H_{15}^{gT}+\frac{1}{2}H_{16}^{gT}\right]+\frac{4\xi}{3(1-\xi^2)}[2\xi H_9^{gT}-H_{10}^{gT}]$$
$$-\frac{4[D(1-\xi^2)+\xi]}{3(1-\xi^2)}\left(2\xi H_9^{gT}-H_{10}^{gT}+\frac{|C|^2}{4}H_4^{gT}\right)+\frac{8[D(1-\xi^2)+\xi]}{3}\left(H_1^{gT}-\frac{|C|^2}{8}H_{10}^{gT}-\xi H_{15}^{gT}+\frac{1}{2}H_{16}^{gT}\right)$$
$$+\frac{|C|^2[D(\xi^2-1)+\xi][D(\xi^2-1)-\xi]}{3(1-\xi^2)}H_4^{gT}, \quad (55)$$

$$\mathcal{A}'^{g}_{(-1/2)-,(-1/2)+} = \frac{2(1-\xi^2)}{3}\left[\sqrt{\frac{1-\xi}{1+\xi}}\left(H_1^{gT}-\frac{1}{2}H_2^{gT}\right)+\sqrt{\frac{1+\xi}{1-\xi}}H_6^{gT}\right]+\frac{2\sqrt{(1-\xi^2)^3}}{3}\left(H_7^{gT}-\frac{1}{4}H_{11}^{gT}+H_{12}^{gT}-2H_{13}^{gT}+\xi H_{14}^{gT}\right)$$
$$-\frac{\sqrt{1-\xi^2}(1+\xi)}{3}\left(H_3^{gT}+2H_7^{gT}-H_8^{gT}-\xi H_{15}^{gT}-\frac{1}{2}H_{16}^{gT}\right)$$
$$-\frac{2\sqrt{(1-\xi^2)}}{3}\left[H_3^{gT}+\frac{1}{2}H_5^{gT}-\xi\left(H_{15}^{gT}+\frac{1}{2}H_{16}^{gT}\right)+(2H_7^{gT}-\xi H_8^{gT})\right]$$
$$-\frac{|C|^2\sqrt{(1-\xi^2)}}{6}\left[\left(\xi H_9^{gT}+\frac{1}{2}H_{10}^{gT}+\frac{5}{4}H_4^{gT}\right)\right]$$
$$+\frac{(1+\xi)}{3\sqrt{1-\xi^2}}\left[\xi H_9^{gT}+\frac{1}{2}H_{10}^{gT}+\frac{|C|^2}{2}H_4^{g}+\frac{2\xi}{1+\xi}\left(H_9^{gT}-\frac{5}{2}H_{10}^{gT}\right)\right]$$
$$-\frac{[D(1-\xi^2)-\xi]}{\sqrt{1-\xi^2}}\left(H_{10}^{gT}-\frac{|C|^2}{6}H_4^{g}\right)+\frac{2[D(1-\xi^2)+\xi]}{3\sqrt{1-\xi^2}}(\xi H_9^{gT}-(1-\xi^2)H_{16}^{gT})$$
$$+\frac{2[D(1-\xi^2)+\xi][D(1-\xi^2)-\xi]}{3\sqrt{(1-\xi^2)^3}}H_4^{gT}, \quad (56)$$

$$\mathcal{A}'^{g}_{(-1/2)-,(-3/2)+} = \frac{(1-\xi^2)(1-\xi)}{2\sqrt{3}}\left[\left(H_{15}^{gT}-\frac{1}{2}H_{16}^{gT}\right)+\frac{1}{1-\xi}\left(H_9^{gT}+\frac{|C|^2}{8}H_4^{gT}\right)-\frac{1}{1-\xi^2}H_{10}^{gT}\right]+\frac{[D(1-\xi^2)-\xi]}{2\sqrt{3}(1+\xi)}H_4^{gT}, \quad (57)$$

$$\mathcal{A}'^{g}_{(-3/2)-,(3/2)+} = -(H_5^{gT}+4\xi^2H_6^{gT})+|C|^2\left(\xi H_9^{gT}-\frac{1}{2}H_{10}^{gT}\right)+\frac{|C|^4(1-\xi^2)}{16}H_4^{gT}, \quad (58)$$

$$\mathcal{A}'^{g}_{(-3/2)-,(1/2)+} = -\frac{2|C|^2\sqrt{1-\xi^2}(1+\xi)}{\sqrt{3}}\left[H_1^{gT} - \frac{1}{2}H_3^{gT} - \xi H_7^{gT} + \frac{1}{2}H_8^{gT} - \frac{|C|^2}{8}\left(H_9^{gT} + \frac{1}{2}H_{10}^{gT}\right)\right]$$
$$- \frac{2\sqrt{1-\xi^2}}{\sqrt{3}}\left[-\frac{2\xi}{1-\xi}H_2^{gT} + H_{11}^{gT} + 4\xi^2 H_{12}^{gT} - |C|^2\left(\frac{1}{4}H_5^{gT} - \xi H_6^{gT} + \xi H_{15}^{gT} - \frac{1}{2}H_{16}^{gT}\right) - \frac{|C|^4}{16}H_4^{gT}\right]$$
$$- \frac{2}{\sqrt{3(1-\xi^2)}}\left[H_5^{gT} + 4\xi^2 H_6^{gT} - |C|^2\left(\xi H_9^{gT} - \frac{1}{2}H_{10}^{gT}\right)\right]$$
$$- \frac{2|C|^2[D(1-\xi^2)+\xi]}{\sqrt{3(1-\xi^2)}}\left[\left(\xi H_9^{gT} - \frac{1}{2}H_{10}^{gT}\right) + (1+\xi)\frac{|C|^2}{8}H_4^{gT}\right], \tag{59}$$

$$\mathcal{A}'^{g}_{(-3/2)-,(-1/2)+} = \frac{4(1-\xi^2)(1+\xi)}{\sqrt{3}}\left(\xi H_{13}^{gT} - \frac{1}{2}H_{14}^{gT} + \frac{|C|^2}{8}\left(H_{15}^{gT} + \frac{1}{2}H_{16}^{gT}\right)\right)$$
$$- \frac{2(1-\xi^2)}{\sqrt{3}}\left(\left(H_1^{gT} - \frac{1}{2}H_2^{gT}\right) - \frac{1}{2}H_{11}^{gT} + 2\xi H_{12}^{gT} + \frac{|C|^2}{4}H_9^{gT} + \frac{|C|^4}{32}H_4^{gT}\right)$$
$$+ \frac{2(1+\xi)}{\sqrt{3}}(H_3^{gT} + 2\xi H_7^{gT} - H_8^{gT}) + \frac{1}{\sqrt{3}}\left[(H_5^{gT} - 4\xi H_6^{gT}) + |C|^2\left(H_9^{gT} + \frac{1}{2}H_{10}^{gT}\right)\right]$$
$$- \frac{4[D(1-\xi^2)+\xi]}{\sqrt{3}}\left[\frac{1}{1-\xi^2}\left(\xi H_9^{gT} - \frac{1}{2}H_{10}^{gT}\right) + \left(\xi H_{15}^{gT} - \frac{1}{2}H_{16}^{gT}\right) + \frac{|C|^2}{8(1-\xi)}H_4^{gT}\right], \tag{60}$$

$$\mathcal{A}'^{g}_{(-3/2)-,(-3/2)+} = -\frac{1-\xi}{\sqrt{1-\xi^2}}\left(\xi H_9^{gT} - \frac{1}{2}H_{10}^{gT}\right) - \sqrt{1-\xi^2}(1-\xi)\left(\xi H_{15}^{gT} - \frac{1}{2}H_{16}^{gT}\right) - \frac{|C|^2\sqrt{1-\xi^2}}{8}H_4^{gT}. \tag{61}$$

The other amplitudes $\mathcal{A}'^{g}_{i+,j-}$ can be obtained from the relation (21).

In the forward limit, similar to the discussions on quark in Sec. II B, there is no orbital contribution, i.e., $F(\zeta \neq 0) = 0$, and the only helicity conserved amplitudes $\mathcal{A}^{g}_{(-1/2)-,(3/2)+}$ and $\mathcal{A}^{g}_{(-3/2)-,(1/2)+}$ survive. The transversity distribution function of the gluon is then obtained as

$$-2[H_5^{gT}(x,0,0) + H_{11}^{gT}(x,0,0)] = x\Delta(x), \tag{62}$$

defined in Refs. [21,23] and $\Delta(x)$ has the similar interpretation with $h_1(x)$ in Eq. (38). Since $(P\cdot n)\bar{u}(p,\lambda')u(p,\lambda) = M\bar{u}(p,\lambda')\not{n}u(p,\lambda)$ in Eq. (41) is satisfied in the forward limit, $H_5^{gT}(x,0,0)$ and $H_{11}^{gT}(x,0,0)$ correspond to the same tensor structure and all the gluon transversity distributions are given by the spin-1 component. There is an obvious reason: the helicity flip of gluon leads to a change of two units of angular momentum, which is impossible for the spin-1/2 component. The minus sign in Eq. (62) is explained by the same reason as in Eq. (38).

III. SUMMARY AND DISCUSSION

In this work, the quark and gluon transversity GPDs of spin-3/2 particles are respectively derived and given for the first time. We find that there are 16 independent transversity GPDs for each parton, constrained by the parity invariance. The even or odd property with respect to ξ of the obtained transversity GPDs can help us to determine the corresponding tensor FFs that contain two tensor charges, where the quark tensor charge may describe the electric dipole moment from the quark beyond the Standard Model and the gluon tensor charge is unknown. We need more study for their physical meanings. There are seven tensor FFs for the local quark current and six for gluon from the sum rules connecting the transversity GPDs and the tensor FFs.

The amplitudes with helicity flip in terms of transversity GPDs are obtained. We conclude that all the amplitudes, including helicity nonflip and flip, have the common factor $F(\zeta)$, which carries the whole complex part and represents the transfer of the orbital angular momentum. In the forward limit, no orbital contribution, i.e., $F(\zeta \neq 0) = 0$, constrains the only helicity conserved amplitudes, which give the corresponding parton distributions, to exist and the amplitudes with helicity flip give the transversity distributions of each parton. Following the Rarita-Schwinger spinor and our explanations, the transversity distribution of the parton in the spin-3/2 system is comprised of the spin-1/2 and spin-1 system, nevertheless the gluon transversity distribution only includes the spin-1 since the gluon helicity flip changes two units of angular momentum and it leads to the violation of the angular momentum conversation in the spin-1/2 system.

Finally, the present study of the transversity GPDs will be applied to a specific spin-3/2 particle, such as Δ resonance and Ω hyperon. Such GPDs and also $N \to \Delta$ transition GPDs could be investigated experimentally. The timelike GPDs of Δ and Ω could be measured by the two-photon processes at BESIII and KEK-B, and the $N \to \Delta$ transition GPDs could be measured at JLab and J-PARC. In addition, the Δ could exist in nuclei as a small component, so that its GPDs could become important for studying specific polarization observables.

ACKNOWLEDGMENTS

We are grateful to Wim Cosyn for the very useful answer about the gluon transversity GPDs for the spin-1 particle. We are also very grateful for the support of the Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics. This work is supported by the National Key Research and Development Program of China under Contract No. 2020YFA0406300, and by the National Natural Science Foundation of China under Grants No. 11975245 and No. 12375142. This work is also supported by the Sino-German CRC 110 Symmetries and the Emergence of Structure in QCD project by NSFC under Grant No. 12070131001, and the Key Research Program of Frontier Sciences, CAS, under Grant No. Y7292610K1. This work was partially supported by the CAS president's international fellowship initiative (Grant No. 2022VMA0003) and by the Japan Society for the Promotion of Science (JSPS) Grants-in-Aid for Scientific Research (KAKENHI) Grant No. 19K03830.

APPENDIX A: THE RARITA-SCHWINGER SPINOR

The explicit form of the Rarita-Schwinger spinor of a spin-3/2 particle employed in our work is [47,60–62]

$$u^\alpha(p,\lambda) = \sum_{\rho,\sigma} C^{\frac{3}{2}\lambda}_{1\rho,\frac{1}{2}\sigma} \epsilon^\alpha(p,\rho) u(p,\sigma), \tag{A1}$$

where the coefficient in Eq. (A1) is the Clebsch-Gordan coefficient. The explicit light-front form expressions of the spin-1 [63] and $-1/2$ [64] respectively are

$$\epsilon^\alpha(p,0) = \frac{1}{M}\left(p^+, p^- - \frac{2M^2}{p^+}, \boldsymbol{\epsilon}_\perp(p,0)\right)^{\mathrm{T}} \quad \text{with} \quad \boldsymbol{\epsilon}_\perp(p,0) = (p_1, p_2), \tag{A2a}$$

$$\epsilon^\alpha(p,+1) = -\left(0, \frac{\sqrt{2}(p_1 + ip_2)}{p^+}, \boldsymbol{\epsilon}_\perp(p,+1)\right)^{\mathrm{T}} \quad \text{with} \quad \boldsymbol{\epsilon}_\perp(p,+1) = \left(\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right), \tag{A2b}$$

$$\epsilon^\alpha(p,-1) = \left(0, \frac{\sqrt{2}(p_1 - ip_2)}{p^+}, \boldsymbol{\epsilon}_\perp(p,-1)\right)^{\mathrm{T}} \quad \text{with} \quad \boldsymbol{\epsilon}_\perp(p,-1) = \left(\frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}\right), \tag{A2c}$$

and

$$u(p,\sigma) = \frac{(\not{p} + M)}{\sqrt{2p\cdot n}} \not{n} \chi_\sigma, \tag{A3}$$

where χ_σ is the rest frame spinor. The Rarita-Schwinger spinor in Eq. (A1) satisfies the Rarita-Schwinger equation, as well as the subsidiary constraint equations,

$$(\not{p} - M)u^\alpha(p,\lambda) = 0, \quad \gamma_\alpha u^\alpha(p,\lambda) = 0, \quad p_\alpha u^\alpha(p,\lambda) = 0. \tag{A4}$$

APPENDIX B: ON-SHELL IDENTITIES

Many identities and on-shell identities are proved and listed in Refs. [47,65,66] and they will not be shown in the following. Some particular and necessary identities are proved here. Here we use the abbreviation, $\mathcal{P}^\mu = 2P^\mu - 2M\gamma^\mu$, to obtain the more concise form.

From Eqs. (B13) and (B23C) in Ref. [65],

$$g^{\alpha'\alpha} i\sigma^{\mu\nu} + g^{\mu\alpha'} g^{\alpha\nu} - g^{\mu\alpha} g^{\alpha'\nu} \doteq i\epsilon^{\alpha'\mu\nu\alpha}\gamma^5, \tag{B1}$$

and $\gamma^5 \doteq \frac{\not{\Delta}\gamma^5}{2M}$ where $\doteq$ indicates on-shell equality, one can derive

$$g^{\alpha'\alpha} i\sigma^{\mu\nu} + g^{\mu[\alpha'} g^{\alpha]\nu} = g^{\alpha'\alpha} i\sigma^{\mu\nu} + g^{\mu\alpha'} g^{\alpha\nu} - g^{\mu\alpha} g^{\alpha'\nu} \doteq \frac{i\epsilon^{\alpha'\mu\nu\alpha}\Delta\gamma^5}{2M}. \tag{B2}$$

In the forward limit, $\Delta = 0$, and then

$$g^{\alpha'\alpha} i\sigma^{\mu\nu} + g^{\mu[\alpha'} g^{\alpha]\nu} \doteq 0. \tag{B3}$$

There is an equality, instead of on-shell identity, from Schouten identity [67],

$$\det\begin{bmatrix} g^{\alpha\alpha'} & g^{\beta\alpha'} & g^{\gamma\alpha'} & g^{\delta\alpha'} & g^{\varepsilon\alpha'} \\ g^{\alpha\nu} & g^{\beta\nu} & g^{\gamma\nu} & g^{\delta\nu} & g^{\varepsilon\nu} \\ g^{\alpha\rho} & g^{\beta\rho} & g^{\gamma\rho} & g^{\delta\rho} & g^{\varepsilon\rho} \\ g^{\alpha\sigma} & g^{\beta\sigma} & g^{\gamma\sigma} & g^{\delta\sigma} & g^{\varepsilon\sigma} \\ g^{\alpha\tau} & g^{\beta\tau} & g^{\gamma\tau} & g^{\delta\tau} & g^{\varepsilon\tau} \end{bmatrix} = 0, \tag{B4}$$

where if the indices α' and α are coincide with ones of the initial and final states depend on if the equality is contracted with states. Contracting this determinant with

$$P_\beta, \Delta_\gamma, n_\delta, g^i_\varepsilon, P_\nu, \Delta_\rho, n_\sigma \quad \text{and} \quad g^j_\tau, \tag{B5}$$

one can obtain

$$\det\begin{bmatrix} g^{\alpha\alpha'} & P^{\alpha'} & \Delta^{\alpha'} & n^{\alpha'} & g^{i\alpha'} \\ P^\alpha & P^2 & P\cdot\Delta & P\cdot n & P^i \\ \Delta^\alpha & P\cdot\Delta & \Delta^2 & \Delta\cdot n & \Delta^i \\ n^\alpha & P\cdot n & \Delta\cdot n & n\cdot n & n^i \\ g^{\alpha j} & P^j & \Delta^j & n^j & g^{ij} \end{bmatrix} = 0. \tag{B6}$$

Using the on-shell identities which have been shown in Refs. [47,65,66] and removing the trace term like g^{ij}, we replace the $(\Delta^i + 2\xi P^i)(\Delta^j + 2\xi P^j)g^{\alpha'\alpha}$ term by other terms surviving in Eq. (41). Similarly, the product between $\not{n}$ and Eq. (B6) explains that $(\Delta^i + 2\xi P^i)(\Delta^j + 2\xi P^j)\not{n} g^{\alpha'\alpha}$ can also be eliminated.

In addition to the vector, the matrix like γ^μ and $\sigma^{\mu\nu}$ is permitted to contract with Eq. (B4). However, it is convenient and safe to contract only one matrix with one of the lows or ranks because of the noncommutation property. One can respectively replace g^i_ε, g^i_ε, Δ_γ, and P_β in Eq. (B5) by $-i\sigma^i{}_\varepsilon$, $-i\sigma^i{}_\varepsilon$, $-i\sigma^n{}_\gamma$, and $-i\sigma^n{}_\beta$ or contract $-i\sigma^n{}_\alpha$ with Eq. (B6), therefore there are 5 equalities

$$\det\begin{bmatrix} g^{\alpha\alpha'} & P^{\alpha'} & \Delta^{\alpha'} & n^{\alpha'} & g^{i\alpha'} \\ P^\alpha & P^2 & 0 & P\cdot n & \frac{1}{2}\Delta^i \\ \Delta^\alpha & 0 & \Delta^2 & \Delta\cdot n & \mathcal{P}^i \\ n^\alpha & P\cdot n & \Delta\cdot n & 0 & -i\sigma^{in} \\ g^{\alpha j} & P^j & \Delta^j & 0 & 0 \end{bmatrix} \doteq 0, \quad \det\begin{bmatrix} g^{\alpha\alpha'} & P^{\alpha'} & \Delta^{\alpha'} & n^{\alpha'} & n^{\alpha'} \\ P^\alpha & P^2 & 0 & P\cdot n & \frac{1}{2}\Delta\cdot n \\ \Delta^\alpha & 0 & \Delta^2 & \Delta\cdot n & \mathcal{P}\cdot n \\ n^\alpha & P\cdot n & \Delta\cdot n & 0 & 0 \\ g^{\alpha j} & P^j & \Delta^j & 0 & i\sigma^{jn} \end{bmatrix} \doteq 0, \quad \det\begin{bmatrix} g^{\alpha\alpha'} & P^{\alpha'} & n^{\alpha'} & n^{\alpha'} & g^{i\alpha'} \\ P^\alpha & P^2 & \frac{1}{2}\Delta\cdot n & P\cdot n & P^i \\ \Delta^\alpha & 0 & \mathcal{P}\cdot n & \Delta\cdot n & \Delta^i \\ n^\alpha & P\cdot n & 0 & 0 & 0 \\ g^{\alpha j} & P^j & i\sigma^{jn} & 0 & 0 \end{bmatrix} \doteq 0,$$

$$\det\begin{bmatrix} g^{\alpha\alpha'} & n^{\alpha'} & \Delta^{\alpha'} & n^{\alpha'} & g^{i\alpha'} \\ P^\alpha & \frac{1}{2}\Delta\cdot n & 0 & P\cdot n & P^i \\ \Delta^\alpha & \mathcal{P}\cdot n & \Delta^2 & \Delta\cdot n & \Delta^i \\ n^\alpha & 0 & \Delta\cdot n & 0 & 0 \\ g^{\alpha j} & i\sigma^{jn} & \Delta^j & 0 & 0 \end{bmatrix} \doteq 0, \quad \det\begin{bmatrix} n^{\alpha'} & P^{\alpha'} & \Delta^{\alpha'} & n^{\alpha'} & g^{i\alpha'} \\ \frac{1}{2}\Delta\cdot n & P^2 & 0 & P\cdot n & P^i \\ \mathcal{P}\cdot n & 0 & \Delta^2 & \Delta\cdot n & \Delta^i \\ 0 & P\cdot n & \Delta\cdot n & 0 & 0 \\ i\sigma^{jn} & P^j & \Delta^j & 0 & 0 \end{bmatrix} \doteq 0, \tag{B7}$$

where we have used some known on-shell identities [47,65,66] and σ^{ij} is removed due to the symmetry from the definition (5). Analogously, respectively replacing g^j_τ, g^j_τ, Δ_ρ, and P_ν in Eq. (B5) by $-i\sigma^j{}_\tau$, $-i\sigma^n{}_\tau$, $-i\sigma^n{}_\rho$, and $-i\sigma^n{}_\nu$ or contracting $-i\sigma^n{}_{\alpha'}$ with Eq. (B6) gives

$$\det\begin{bmatrix} g^{\alpha\alpha'} & P^{\alpha'} & \Delta^{\alpha'} & n^{\alpha'} & g^{i\alpha'} \\ P^\alpha & P^2 & 0 & P\cdot n & P^i \\ \Delta^\alpha & 0 & \Delta^2 & \Delta\cdot n & \Delta^i \\ n^\alpha & P\cdot n & \Delta\cdot n & 0 & 0 \\ -g^{j\alpha} & \frac{1}{2}\Delta^j & \mathcal{P}^j & -i\sigma^{jn} & 0 \end{bmatrix} \doteq 0, \quad \det\begin{bmatrix} g^{\alpha\alpha'} & P^{\alpha'} & \Delta^{\alpha'} & n^{\alpha'} & g^{i\alpha'} \\ P^\alpha & P^2 & 0 & P\cdot n & P^i \\ \Delta^\alpha & 0 & \Delta^2 & \Delta\cdot n & \Delta^i \\ n^\alpha & P\cdot n & \Delta\cdot n & 0 & 0 \\ -n^\alpha & \frac{1}{2}\Delta\cdot n & \mathcal{P}\cdot n & 0 & i\sigma^{in} \end{bmatrix} \doteq 0, \quad \det\begin{bmatrix} g^{\alpha\alpha'} & P^{\alpha'} & \Delta^{\alpha'} & n^{\alpha'} & g^{i\alpha'} \\ P^\alpha & P^2 & 0 & P\cdot n & P^i \\ -n^\alpha & \frac{1}{2}\Delta\cdot n & \mathcal{P}\cdot n & 0 & i\sigma^{in} \\ n^\alpha & P\cdot n & \Delta\cdot n & 0 & 0 \\ g^{\alpha j} & P^j & \Delta^j & 0 & 0 \end{bmatrix} \doteq 0,$$

$$\det\begin{bmatrix} g^{\alpha\alpha'} & P^{\alpha'} & \Delta^{\alpha'} & n^{\alpha'} & g^{i\alpha'} \\ -n^\alpha & \frac{1}{2}\Delta\cdot n & \mathcal{P}\cdot n & 0 & i\sigma^{in} \\ \Delta^\alpha & 0 & \Delta^2 & \Delta\cdot n & \Delta^i \\ n^\alpha & P\cdot n & \Delta\cdot n & 0 & 0 \\ g^{\alpha j} & P^j & \Delta^j & 0 & 0 \end{bmatrix} \doteq 0, \quad \det\begin{bmatrix} -n^\alpha & \frac{1}{2}\Delta\cdot n & \mathcal{P}\cdot n & 0 & i\sigma^{in} \\ P^\alpha & P^2 & 0 & P\cdot n & P^i \\ \Delta^\alpha & 0 & \Delta^2 & \Delta\cdot n & \Delta^i \\ n^\alpha & P\cdot n & \Delta\cdot n & 0 & 0 \\ g^{\alpha j} & P^j & \Delta^j & 0 & 0 \end{bmatrix} \doteq 0. \tag{B8}$$

Solving Eqs. (B7) and (B8) can get the corresponding on-shell identities to eliminate $i\sigma^{ni} \otimes (n^{[\alpha'} g^{\alpha]j}, \Delta\cdot n n^{\{\alpha'} g^{\alpha\}j} - 2n^{\alpha'} n^\alpha \Delta^j, P\cdot n P^{[\alpha'} g^{\alpha]j} - P^{[\alpha'} n^{\alpha]} P^j$, and $\Delta\cdot n P^{\{\alpha'} g^{\alpha\}j} - P^{\{\alpha'} n^{\alpha\}} \Delta^j)$ terms.

Taking the changes, $g^i_\varepsilon \to \gamma_\varepsilon$, $n_\delta \to \gamma_\delta$, or $g^j_\tau \to \gamma_\tau$, $n_\sigma \to \gamma_\sigma$, from Eq. (B5) gives

$$\det\begin{bmatrix} g^{\alpha\alpha'} & P^{\alpha'} & \Delta^{\alpha'} & n^{\alpha'} & 0 \\ P^\alpha & P^2 & 0 & P\cdot n & M \\ \Delta^\alpha & 0 & \Delta^2 & \Delta\cdot n & 0 \\ n^\alpha & P\cdot n & \Delta\cdot n & 0 & \not{n} \\ g^{\alpha j} & P^j & \Delta^j & 0 & \gamma^j \end{bmatrix} \doteq 0, \qquad \det\begin{bmatrix} g^{\alpha\alpha'} & P^{\alpha'} & \Delta^{\alpha'} & 0 & g^{i\alpha'} \\ P^\alpha & P^2 & 0 & M & P^i \\ \Delta^\alpha & 0 & \Delta^2 & 0 & \Delta^i \\ n^\alpha & P\cdot n & \Delta\cdot n & \not{n} & 0 \\ g^{\alpha j} & P^j & \Delta^j & \gamma^j & 0 \end{bmatrix} \doteq 0, \tag{B9}$$

and

$$\det\begin{bmatrix} g^{\alpha\alpha'} & P^{\alpha'} & \Delta^{\alpha'} & n^{\alpha'} & g^{i\alpha'} \\ P^\alpha & P^2 & 0 & P\cdot n & P^i \\ \Delta^\alpha & 0 & \Delta^2 & \Delta\cdot n & \Delta^i \\ n^\alpha & P\cdot n & \Delta\cdot n & 0 & 0 \\ 0 & M & 0 & \not{n} & \gamma^i \end{bmatrix} \doteq 0, \qquad \det\begin{bmatrix} g^{\alpha\alpha'} & P^{\alpha'} & \Delta^{\alpha'} & n^{\alpha'} & g^{i\alpha'} \\ P^\alpha & P^2 & 0 & P\cdot n & P^i \\ \Delta^\alpha & 0 & \Delta^2 & \Delta\cdot n & \Delta^i \\ 0 & M & 0 & \not{n} & \gamma^i \\ g^{\alpha j} & P^j & \Delta^j & 0 & 0 \end{bmatrix} \doteq 0. \tag{B10}$$

Combining Eqs. (B9) and (B10), one can obtain that the tensor structures $(\Delta^i + 2\xi P^i)(\Delta\cdot n\gamma^j - \not{n}\Delta^j)P^{\alpha'}P^\alpha$ and $(\Delta^i + 2\xi P^i)(P\cdot n\gamma^j - \not{n}P^j)g^{\alpha'\alpha}$ can be represented by others.

[1] Dieter Müller, D. Robaschik, B. Geyer, F. M. Dittes, and J. Hořejši, Wave functions, evolution equations and evolution kernels from light ray operators of QCD, Fortschr. Phys. **42**, 101 (1994).
[2] Xiang-Dong Ji, Gauge-invariant decomposition of nucleon spin, Phys. Rev. Lett. **78**, 610 (1997).
[3] A. V. Radyushkin, Scaling limit of deeply virtual Compton scattering, Phys. Lett. B **380**, 417 (1996).
[4] A. V. Radyushkin, Asymmetric gluon distributions and hard diffractive electroproduction, Phys. Lett. B **385**, 333 (1996).
[5] Xiang-Dong Ji, Deeply virtual Compton scattering, Phys. Rev. D **55**, 7114 (1997).
[6] A. V. Radyushkin, Nonforward parton distributions, Phys. Rev. D **56**, 5524 (1997).
[7] Xiang-Dong Ji, Off forward parton distributions, J. Phys. G **24**, 1181 (1998).
[8] M. Diehl, Generalized parton distributions, Phys. Rep. **388**, 41 (2003).
[9] M. Masuda *et al.*, Study of π^0 pair production in single-tag two-photon collisions, Phys. Rev. D **93**, 032003 (2016).
[10] S. Kumano, Qin-Tao Song, and O. V. Teryaev, Hadron tomography by generalized distribution amplitudes in pion-pair production process $\gamma^*\gamma \to \pi^0\pi^0$ and gravitational form factors for pion, Phys. Rev. D **97**, 014020 (2018).
[11] R. Abdul Khalek *et al.*, Science requirements and detector concepts for the electron-ion collider: Eic yellow report, Nucl. Phys. **A1026**, 122447 (2022).
[12] Daniele P. Anderle *et al.*, Electron-ion collider in China, Front. Phys. **16**, 64701 (2021).
[13] S. Kumano, M. Strikman, and K. Sudoh, Novel two-to-three hard hadronic processes and possible studies of generalized parton distributions at hadron facilities, Phys. Rev. D **80**, 074003 (2009).
[14] Takahiro Sawada, Wen-Chen Chang, Shunzo Kumano, Jen-Chieh Peng, Shinya Sawada, and Kazuhiro Tanaka, Accessing proton generalized parton distributions and pion distribution amplitudes with the exclusive pion-induced Drell-Yan process at J-PARC, Phys. Rev. D **93**, 114034 (2016).
[15] Jian-Wei Qiu and Zhite Yu, Exclusive production of a pair of high transverse momentum photons in pion-nucleon collisions for extracting generalized parton distributions, J. High Energy Phys. 08 (2022) 103.
[16] Jian-Wei Qiu and Zhite Yu, Single diffractive hard exclusive processes for the study of generalized parton distributions, Phys. Rev. D **107**, 014007 (2023).
[17] B. Pire, L. Szymanowski, and J. Wagner, Hard exclusive neutrino production of a light meson, Phys. Rev. D **95**, 114029 (2017).
[18] Shunzo Kumano and Roberto Petti, Possible studies on generalized parton distributions and gravitational form factors in neutrino reactions, *Proc. Sci.*, NuFact2021 (2022) 092.
[19] Chen Xurong *et al.*, Pion-production cross sections in neutrino reactions for studying generalized parton distributions of the nucleon, arXiv:2401.11440.
[20] V. Barone and P. G. Ratcliffe, *Transverse Spin Physics* (World Scientific, Singapore, 2003).
[21] Pervez Hoodbhoy and Xiang-Dong Ji, Helicity flip off forward parton distributions of the nucleon, Phys. Rev. D **58**, 054006 (1998).
[22] W. Cosyn and B. Pire, Transversity generalized parton distributions for the deuteron, Phys. Rev. D **98**, 074020 (2018).

[23] R. L. Jaffe and Aneesh Manohar, Nuclear gluonometry, Phys. Lett. B **223**, 218 (1989).
[24] M. Diehl, Generalized parton distributions with helicity flip, Eur. Phys. J. C **19**, 485 (2001).
[25] John P. Ralston and Davison E. Soper, Production of dimuons from high-energy polarized proton proton collisions, Nucl. Phys. **B152**, 109 (1979).
[26] Xavier Artru and Mustapha Mekhfi, Transversely polarized parton densities, their evolution and their measurement, Z. Phys. C **45**, 669 (1990).
[27] R. L. Jaffe and Xiang-Dong Ji, Chiral odd parton distributions and polarized Drell-Yan, Phys. Rev. Lett. **67**, 552 (1991).
[28] R. L. Jaffe and Xiang-Dong Ji, Chiral odd parton distributions and Drell-Yan processes, Nucl. Phys. **B375**, 527 (1992).
[29] J. L. Cortes, B. Pire, and J. P. Ralston, Measuring the transverse polarization of quarks in the proton, Z. Phys. C **55**, 409 (1992).
[30] Xiang-Dong Ji, Probing the nucleon's transversity distribution in polarized pp, pp, and πp collisions, Phys. Lett. B **284**, 137 (1992).
[31] Vincenzo Barone, Alessandro Drago, and Philip G. Ratcliffe, Transverse polarisation of quarks in hadrons, Phys. Rep. **359**, 1 (2002).
[32] Sergio Scopetta and Vicente Vento, A quark model analysis of the transversity distribution, Phys. Lett. B **424**, 25 (1998).
[33] Katsuhiko Suzuki and Takayuki Shigetani, Chiral odd transversity spin structure function $h_1(x)$ of the nucleon in a constituent quark model, Nucl. Phys. **A626**, 886 (1997).
[34] Bo-Qiang Ma, Ivan Schmidt, and Jacques Soffer, The quark spin distributions of the nucleon, Phys. Lett. B **441**, 461 (1998).
[35] L. P. Gamberg, H. Reinhardt, and H. Weigel, Chiral odd structure functions from a chiral soliton, Phys. Rev. D **58**, 054014 (1998).
[36] I. C. Cloet, Wolfgang Bentz, and Anthony William Thomas, Transversity quark distributions in a covariant quark-diquark model, Phys. Lett. B **659**, 214 (2008).
[37] S. Kumano and Qin-Tao Song, Gluon transversity in polarized proton-deuteron Drell-Yan process, Phys. Rev. D **101**, 054011 (2020).
[38] V. Yu. Alexakhin *et al.*, First measurement of the transverse spin asymmetries of the deuteron in semi-inclusive deep inelastic scattering, Phys. Rev. Lett. **94**, 202002 (2005).
[39] R. Seidl *et al.*, Measurement of azimuthal asymmetries in inclusive production of hadron pairs in e^+e^- annihilation at $\sqrt{s} = 10.58$-GeV, Phys. Rev. D **78**, 032011 (2008); **86**, 039905(E) (2012).
[40] Andrea Moretti, Transversity and Λ polarization at COMPASS, Proc. Sci., SPIN2018 (**2018**) 138.
[41] M. G. Alexeev *et al.*, Probing transversity by measuring Λ polarisation in SIDIS, Phys. Lett. B **824**, 136834 (2022).
[42] J. Maxwell *et al.*, Search for exotic gluonic states in the nucleus, A letter of intent to jefferson lab PAC 44, arXiv:1803.11206.
[43] Zhihong Ye, Nobuo Sato, Kalyan Allada, Tianbo Liu, Jian-Ping Chen, Haiyan Gao, Zhong-Bo Kang, Alexei Prokudin, Peng Sun, and Feng Yuan, Unveiling the nucleon tensor charge at Jefferson Lab: A study of the SoLID case, Phys. Lett. B **767**, 91 (2017).
[44] A. Accardi *et al.*, Electron ion collider: The next QCD frontier: Understanding the glue that binds us all, Eur. Phys. J. A **52**, 268 (2016).
[45] M. Brooks *et al.*, target (e1039), https://twist.phys.virginia.edu/work/e1039proposal_final.pdf (2017); sinqest collaboration, https://spinquest.fnal.gov/.
[46] D. Keller, The transverse structure of the deuteron with Drell-Yan, arXiv:2205.01249.
[47] Dongyan Fu, Bao-Dong Sun, and Yubing Dong, Generalized parton distributions in spin-3/2 particles, Phys. Rev. D **106**, 116012 (2022).
[48] Edgar R. Berger, F. Cano, M. Diehl, and B. Pire, Generalized parton distributions in the deuteron, Phys. Rev. Lett. **87**, 142302 (2001).
[49] Cristiano Fanelli, Emanuele Pace, Giovanni Romanelli, Giovanni Salme, and Marco Salmistraro, Pion generalized parton distributions within a fully covariant constituent quark model, Eur. Phys. J. C **76**, 253 (2016).
[50] Markus Diehl and Peter Kroll, Nucleon form factors, generalized parton distributions and quark angular momentum, Eur. Phys. J. C **73**, 2397 (2013).
[51] Bao-Dong Sun and Yu-Bing Dong, ρ meson unpolarized generalized parton distributions with a light-front constituent quark model, Phys. Rev. D **96**, 036019 (2017).
[52] Dongyan Fu, Bao-Dong Sun, and Yubing Dong, Generalized parton distributions of Δ resonance in a diquark spectator approach, Phys. Rev. D **107**, 116021 (2023).
[53] M. Nzar and P. Hoodbhoy, Estimation of the double helicity flip deuteron structure function, Phys. Rev. D **45**, 2264 (1992).
[54] Eric Sather and Carl Schmidt, Size and scaling of the double helicity flip hadronic structure function, Phys. Rev. D **42**, 1424 (1990).
[55] Gerald A. Miller, Pionic and hidden-color, six-quark contributions to the deuteron b1 structure function, Phys. Rev. C **89**, 045203 (2014).
[56] S. Diehl *et al.*, Exploration of resonance structure with transition generalized parton distributions—status and perspectives (to be published).
[57] Maxim Pospelov and Adam Ritz, Electric dipole moments as probes of new physics, Ann. Phys. (Amsterdam) **318**, 119 (2005).
[58] Qing-Wu Wang, Si-Xue Qin, Craig D. Roberts, and Sebastian M. Schmidt, Proton tensor charges from a Poincaré-covariant Faddeev equation, Phys. Rev. D **98**, 054019 (2018).
[59] John C. Collins, Fragmentation of transversely polarized quarks probed in transverse momentum distributions, Nucl. Phys. **B396**, 161 (1993).
[60] William Rarita and Julian Schwinger, On a theory of particles with half integral spin, Phys. Rev. **60**, 61 (1941).
[61] David Lurie, *Particles and Fields* (Interscience Publishers, New York, 1968).
[62] Roger Penrose and Wolfgang Rindler, *Spinors and Space-Time*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 2011).

[63] B. D. Keister and W. N. Polyzou, Relativistic Hamiltonian dynamics in nuclear and particle physics, Adv. Nucl. Phys. **20**, 225 (1991).

[64] Cédric Lorcé, New explicit expressions for Dirac bilinears, Phys. Rev. D **97**, 016005 (2018).

[65] Sabrina Cotogno, Cédric Lorcé, Peter Lowdon, and Manuel Morales, Covariant multipole expansion of local currents for massive states of any spin, Phys. Rev. D **101**, 056016 (2020).

[66] Dongyan Fu, Bao-Dong Sun, and Yubing Dong, Electromagnetic and gravitational form factors of Δ resonance in a covariant quark-diquark approach, Phys. Rev. D **105**, 096002 (2022).

[67] Stephan Meissner, Andreas Metz, and Marc Schlegel, Generalized parton correlation functions for a spin-1/2 hadron, J. High Energy Phys. 08 (2009) 056.