Complete tree-level dictionary between simplified BSM models and SMEFT $d \leq 7$ operators

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(Received 29 January 2024; accepted 4 April 2024; published 24 May 2024)

Finding all possible UV resonances of effective operators is an important task in the bottom-up approach of effective field theory. We present all the tree-level UV resonances for the dimension-5, -6, and -7 operators in the Standard Model effective field theory (SMEFT) and then obtain the correspondence between the UV resonances and the effective operators from the relations among their Wilson coefficients, through the functional matching and operator reduction procedure. This provides a cross-dimension UV/IR dictionary for the SMEFT at tree level, and the methods used here, especially the on shell construction of general UV Lagrangian and the systematic reduction of operators, are extendable for UV resonances of $d \geq 8$ operators in SMEFT and other effective field theories.

DOI: 10.1103/PhysRevD.109.095041

I. INTRODUCTION

Being the most successful theory in particle physics, the Standard Model (SM) has been tested and verified by many experiments. However, it is not a complete theory of fundamental interactions because it fails to answer some important questions, such as the nature of the dark matter, the origin of the neutrino masses, matter-antimatter asymmetry, etc. Therefore, physicists are searching for physics beyond the SM to address these issues. Up to now, the direct searches on the Large Hadron Collider (LHC) have not found any signals beyond the Standard Model (BSM), which pushes the scale of new physics up to TeV or several TeV. Due to the considerable energy gap between the electroweak scale and the new physics scale, the effective field theory (EFT) approach provide a model independent way that parametrize the effects of the BSM physics into the Wilson coefficients of the higher dimensional operators in the EFT to probe BSM physics.

The standard model effective field theory (SMEFT) is an EFT at the electroweak scale, and it is constructed based on the fields and symmetries of the SM. The Lagrangian of the SMEFT is formulated as the sum of effective operators, including the SM Lagrangian and a series of possible higher dimensional operators according to the power counting. Among the higher dimensional operators in the SMEFT, the dimension-5 operator is first written by Weinberg [1], and since then the operator bases have been enumerated up to dimension 9 [2–10] and higher. After the complete set of the SMEFT operators is given, the Wilson coefficients of the effective operators that parametrize the deviation from the SM can be determined by analyzing the experimental data form the high energy colliders and low energy experiments, e.g., Refs. [11–13]. If the experimental data exhibit a significant difference from the SM prediction, physicists can find the corresponding effective operators that cause the difference. After that, the dictionary between the effective operators and their UV origins will greatly benefit the searching for BSM physics [14].

In the EFT framework, there are usually two ways to find the connection between the effective operators and possible UV origins. The first one is to start from an UV model and then perform the matching procedure by integrating out the

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heavy degrees of freedom. If the above matching procedure yields a nonzero Wilson coefficient of an effective operator, then the UV model should be considered as one of the UV origins of the effective operator. This method is called the top-down approach [15,16]. The advantage of this approach is that all effective operators that connect with an UV origin can be found at the same time, but one can not find all the UV origins of an effective operator without performing an exhaustive search of all possible UV origins, which is very time consuming and error prone due to the large variety of UV models. The second approach is the bottom-up approach. With this approach, one can find all UV origins of an effective operator without writing down the explicit UV interactions, by simply examining the Lorentz and gauge quantum numbers of the local on shell amplitude generated by the operator. For a partition of the external particles, one organize the on shell amplitudes into the eigenstates of angular momentum J and gauge quantum number $\mathbf{R}$, and this amplitude basis, as well as the corresponding operator basis, is called the j basis [17–20].

In this work, we focus on the tree-level dictionary between the UV origins and the effective operators in the SMEFT. The reason that we consider the tree-level dictionary is that the tree-level UV origins are typically the leading contribution to the observable. What is more, the tree-level amplitude with a heavy immediate particle behaves as a resonance and can be probed directly in the collider experiments such as the LHC. At mass dimension 5 in the SMEFT, the tree-level dictionary is straightforward since there is only one independent operator, the Weinberg operator, and the UV origins are found to be the three types of the seesaw models [21–26]. The dimension-6 tree-level dictionary is given by Ref. [27], and the complete tree-level UV resonances are also presented in Ref. [18] using the j-basis method. Here, we extend our discussion to involve the dimension-7 effective operators as well as their UV origins to give a complete dimension-5, -6, and -7 tree-level dictionary, where the cross-dimension contributions of effective operators induced by field redefinitions are included. The UV resonances that contribute to the SMEFT operators up to dimension 7 have been listed partially in Refs. [28,29] and fully in Ref. [18]. We start by utilizing the spinor-helicity formalism for both massless and massive amplitudes and the Young tableau method to generate the Lorentz structure and the gauge structure of the UV Lagrangian that involves all the UV resonances. Then, we apply the functional matching method [30–34] to integrate out the UV resonances at tree level and obtain a set of effective operators carrying all kinds of redundancies. After that, we propose a systematic reduction method inspired by the off shell amplitude formalism to reduce the set of operators to the integrated dimension-5, -6 and -7 SMEFT operator basis listed in Appendix C. The above procedures, including constructing UV Lagrangian, matching and reduction, are fully systematic and can be applied to higher-dimensional operators in the SMEFT and other EFTs.

The paper is organized as follows. In Sec. II, we briefly introduce the spinor-helicity formalism for massless and massive amplitudes, and present the independent Lorentz structures of the UV Lagrangian using the spinor-helicity formalism. We then show the construction of the full UV Lagrangian involving the UV resonances that have tree-level contributions to the dimension-5, -6, and -7 operators in Sec. III. In Sec. IV, we will use some examples to illustrate the matching and reduction procedure. We translate the result into the correspondence between the UV resonances and the IR effective operators and present the result in Sec. V. Section VI is our conclusion.

II. MASSIVE ON SHELL AMPLITUDE BASIS

In order to construct the UV Lagrangian, first we need to construct a complete and independent basis of the Lorentz structures in the UV Lagrangian. In this paper, the above basis of Lorentz structures is obtained by translating the massive amplitude basis into operator basis with the massive amplitude-operator correspondence. We will illustrate the procedure in this section.

A. Massless and massive spinors

In this section, we will briefly introduce the spinor-helicity formalism for massless [35–38] and massive spinors [39]. We start with the 4-momentum p_μ, which can be expressed by a 2×2 matrix after contracting with the $\sigma^\mu_{\alpha\dot\alpha}$ matrices,

$$p_{\alpha\dot\alpha} = p_\mu \sigma^\mu_{\alpha\dot\alpha} = \begin{pmatrix} p^0 - p^3 & -p^1 + ip^2 \\ -p^1 - ip^2 & p^0 + p^3 \end{pmatrix}. \quad (2.1)$$

It should be noted that $\det p_{\alpha\dot\alpha} = p^2 = m^2$.

For massless particles, $\det p_{\alpha\dot\alpha} = 0$. That means the matrix $p_{\alpha\dot\alpha}$ is rank-1 and thus, can be written as the direct product of two 2-vectors λ_α and $\tilde\lambda_{\dot\alpha}$,

$$p_{\alpha\dot\alpha} = \lambda_\alpha \tilde\lambda_{\dot\alpha}, \quad (2.2)$$

where the λ_α and $\tilde\lambda_{\dot\alpha}$ are independent complex dimension-2 vectors for general complex momentum while $\tilde\lambda_{\dot\alpha} = (\lambda_\alpha)^*$ for real momentum. The λ_α and $\tilde\lambda_{\dot\alpha}$ are called spinor-helicity variables, and they transform under both the Lorentz group and the little group. For a fixed momentum $p_{\alpha\dot\alpha}$, the choice of λ_α and $\tilde\lambda_{\dot\alpha}$ is not unique since we can always perform the following little group rescaling:

$$\lambda_\alpha \to \omega^{-1}\lambda_\alpha, \qquad \tilde\lambda_{\dot\alpha} \to \omega\tilde\lambda_{\dot\alpha}, \quad (2.3)$$

and keep $p_{\alpha\dot\alpha}$ invariant. Generally, ω is a complex number and the action is $GL(1)$. For real momentum, we have $\omega^{-1} = \omega^*$, and thus, $\omega = e^{i\theta}$, so the little group is $U(1)$.

The amplitudes for massless particles are functions of λ_is and $\tilde{\lambda}_i$s, where i labels the ith external particle in the amplitude, and transform under the little group scaling of the λ_i and $\tilde{\lambda}_i$ as

$$\mathcal{M}(\ldots,\omega^{-1}\lambda_i,\omega\tilde{\lambda}_i,\ldots)=\omega^{2h_i}\mathcal{M}(\ldots,\lambda_i,\tilde{\lambda}_i,\ldots), \quad (2.4)$$

where h_i is the helicity of the ith particle in the amplitude.

For massive particles, the matrix $p_{\alpha\dot{\alpha}}$ satisfies $\det p_{\alpha\dot{\alpha}}=m^2\neq 0$, so now $p_{\alpha\dot{\alpha}}$ is rank-2 instead of rank-1. Here, we adopt the massive spinor notation introduced in Ref. [39] and express the matrix $p_{\alpha\dot{\alpha}}$ as a product of two rank-1 matrices λ_α^I and $\tilde{\lambda}_{\dot{\alpha}I}$,

$$p_{\alpha\dot{\alpha}}=\lambda_\alpha^I\tilde{\lambda}_{\dot{\alpha}I}, \qquad I=1,2. \quad (2.5)$$

Now that $\det p_{\alpha\dot{\alpha}}=\det\lambda_\alpha^I\times\det\tilde{\lambda}_{\dot{\alpha}I}=m^2$, we can choose to take $\det\lambda_\alpha^I=\det\tilde{\lambda}_{\dot{\alpha}I}=m$. Similarly, the λ_α^I and $\tilde{\lambda}_{\dot{\alpha}I}$ can not be uniquely determined for a fixed momentum $p_{\alpha\dot{\alpha}}$ since we can utilize a $SL(2)$ transformation,

$$\lambda_\alpha^I\to W_J^I\lambda_\alpha^J, \qquad \tilde{\lambda}_{I\dot{\alpha}}\to (W^{-1})_I^J\tilde{\lambda}_{J\dot{\alpha}}, \quad (2.6)$$

to change λ_α^I and $\tilde{\lambda}_{\dot{\alpha}I}$ while keep $p_{\alpha\dot{\alpha}}$ invariant. For real momentum, $W_J^I=(W^{-1})_I^{J*}$, so the little group is $SU(2)$.

We can utilize the $SU(2)$ invariant tensor ϵ_{IJ} and ϵ^{IJ} to lower and raise the little group indices on λ_α^I and $\tilde{\lambda}_{\dot{\alpha}I}$, such that $p_{\alpha\dot{\alpha}}=\epsilon_{IJ}\lambda_\alpha^I\tilde{\lambda}_{\dot{\alpha}}^J$. Furthermore, taking account of $\det\lambda_\alpha^I=\det\tilde{\lambda}_{\dot{\alpha}I}=m$, we have the following relations:

$$p_{\alpha\dot{\alpha}}\tilde{\lambda}^{\dot{\alpha}I}=m\lambda_\alpha^I, \qquad p_{\alpha\dot{\alpha}}\lambda^{\alpha I}=-m\tilde{\lambda}_{\dot{\alpha}}^I, \quad (2.7)$$

which allow us to convert between λ_α^I and $\tilde{\lambda}_{\dot{\alpha}}^I$ with a coefficient $p_{\alpha\dot{\alpha}}/m$. With these relations, we can use only λ or $\tilde{\lambda}$ to present a massive particle in an amplitude, and the amplitude for massive particles must be a symmetric rank-$2S_i$ tensor $\mathcal{M}^{\{I_1\ldots I_{2S_i}\}}$ for the ith spin-S_i particle. Thus, we have

$$\mathcal{M}^{\{I_1\ldots I_{2S_i}\}}=\lambda_{\alpha_1}^{I_1}\cdots\lambda_{\alpha_{2S}}^{I_{2S}}\mathcal{M}^{\{\alpha_1\ldots\alpha_{2S_i}\}}. \quad (2.8)$$

B. Massive amplitude basis

In this subsection, we present a basis of the independent Lorentz structures of effective operators involving the SM particles and the UV states with spin $s\leq 1$. The correspondence between operators and massive amplitudes [18] indicates that constructing the Lorentz structures of an operator basis is equivalent to finding a basis of kinematically independent structures formed of spinor-helicity variables. In order to involve the massive UV state with $s=1$ in the operator basis, among the different methods of constructing amplitude basis [40–43], we adopt the method in Ref. [43] and discuss how the massive amplitude basis corresponds to an operator basis involving massive UV states.

Reference [43] provides an algorithm that can be used to find all kinematically independent massive amplitudes for certain external particles and dimension of the amplitudes. Here, we simply use the results and refer the readers to the paper mentioned above for more details of the algorithm. The correspondence between massless amplitudes and operators has been elaborated in Refs. [9,17], and here, we present the correspondence as

$$\begin{aligned} F_{\mathrm{L/R}i} &\sim \lambda_i^2/\tilde{\lambda}_i^2,\\ \psi_i/\psi_i^\dagger &\sim \lambda_i/\tilde{\lambda}_i,\\ \phi_i &\sim 1,\\ D_i &\sim -i\lambda_i\tilde{\lambda}_i, \end{aligned} \quad (2.9)$$

where i in the subscript of a field labels the ith field in an operator and i in the subscript of a covariant derivative indicates that the covariant derivative acts on the ith field, similarly hereinafter. For the correspondence between massive amplitudes and operators, the massive scalar and fermion are similar to the massless ones since the degrees of freedom of the massive and massless fields are the same. However, the massive vector has 3 degrees of freedom instead of 2, so its correspondence to the massive spinors should include the 3 degrees of freedom, that is, the three transversities. The correspondence between massive amplitudes and operators reads

$$\begin{aligned} (DV_i)_{\mathrm{L}}/V_i/(DV_i)_{\mathrm{R}} &\sim \tilde{m}_i\lambda_i^I\lambda_i^J/\lambda_i^I\tilde{\lambda}_i^J/m_i\tilde{\lambda}_i^I\tilde{\lambda}_i^J,\\ \psi_i/\psi_i^\dagger &\sim \lambda_i^I/\tilde{\lambda}_i^I,\\ \phi_i &\sim 1,\\ D_i &\sim -ip_i, \end{aligned} \quad (2.10)$$

where $(DV_i)_{\mathrm{L}}\equiv D_{\alpha\dot{\alpha}}V_\beta{}^{\dot{\alpha}}$ and $(DV_i)_{\mathrm{R}}\equiv D^\alpha{}_{\dot{\alpha}}V_{\alpha\dot{\beta}}$. As

$$\begin{aligned} D_{\alpha\dot{\alpha}}V_\beta{}^{\dot{\alpha}} &= D_\mu V^\mu\epsilon_{\beta\alpha}-iD_\mu V_\nu(\sigma^{\mu\nu})_{\alpha\beta},\\ D^\alpha{}_{\dot{\alpha}}V_{\alpha\dot{\beta}} &= D_\mu V^\mu\epsilon_{\dot{\alpha}\dot{\beta}}-iD_\mu V_\nu(\bar{\sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}}, \end{aligned} \quad (2.11)$$

and $D_\mu V^\mu$ is the EOM of V, it is equivalent to use $D_{\alpha\dot{\alpha}}V_\beta{}^{\dot{\alpha}}$ or $iD_\mu V_\nu(\sigma^{\mu\nu})_{\alpha\beta}$ to construct UV operators.

Now that we can generate the amplitude basis using the method in Ref. [43] and translate it to an UV operator basis using the amplitude-operator correspondence Eqs. (2.9) and (2.10). However, we should be careful about the fact that different amplitude bases could contribute to one UV operator basis due to Eq. (2.10). For example, if we want to find the complete and independent UV operator basis in ϕV^3D, where ϕ is considered massless and Vs are massive, we need to consider the corresponding amplitude bases where the massive vectors can be of different transversities. The complete and independent set of amplitudes that correspond to the UV operators in ϕV^3D are

$$
\begin{aligned}
&\langle \mathbf{12}\rangle\langle \mathbf{34}\rangle[\mathbf{12}][\mathbf{34}], \qquad \langle \mathbf{14}\rangle\langle \mathbf{23}\rangle[\mathbf{14}][\mathbf{23}], \qquad -\langle \mathbf{24}\rangle\langle \mathbf{3}|p_2|\mathbf{3}][\mathbf{24}], \\
&\tilde{m}_2\langle \mathbf{23}\rangle\langle \mathbf{24}\rangle[\mathbf{34}], \qquad \tilde{m}_3\langle \mathbf{23}\rangle\langle \mathbf{34}\rangle[\mathbf{24}], \qquad \tilde{m}_4\langle \mathbf{24}\rangle\langle \mathbf{34}\rangle[\mathbf{23}], \\
&m_2\langle \mathbf{34}\rangle[\mathbf{23}][\mathbf{24}], \qquad m_3\langle \mathbf{24}\rangle[\mathbf{23}][\mathbf{34}], \qquad m_4\langle \mathbf{23}\rangle[\mathbf{24}][\mathbf{34}],
\end{aligned}
\tag{2.12}
$$

where we adopt the "**BOLD**" notation instead of writing the little group indices explicitly. For the three amplitudes in the first row of Eq. (2.12), the massive vectors are of transversity 0, while the other six amplitudes in Eq. (2.12) are not. The corresponding UV operator basis is given in Eq. (2.15).

Before we end the section, we list the Lorentz structures of the UV operator basis for interactions involving massless fields with helicity $|h| \leq 1$ and massive fields with spin $s \leq 1$ in the following:

$$
\mathcal{B}_3 = \phi_1\phi_2\phi_3 + \epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\phi_1 V_{2\alpha}{}^{\dot\alpha} V_{3\beta}{}^{\dot\beta},
\tag{2.13}
$$

$$
\begin{aligned}
\mathcal{B}_4 = {} & \phi_1\phi_2\phi_3\phi_4 + \epsilon^{\alpha\beta}\phi_1\psi_{2\alpha}\psi_{3\beta} + \epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}(D_\alpha{}^{\dot\alpha}\phi_1)\phi_2 V_{3\beta}{}^{\dot\beta} + \epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\phi_1\phi_2 V_{3\alpha}{}^{\dot\alpha} V_{4\beta}{}^{\dot\beta} \\
& + \epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\psi_{1\alpha}\psi_2^{\dagger\dot\alpha} V_{3\beta}{}^{\dot\beta} + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\alpha\dot\beta} F_{\mathrm{L}1\alpha\beta} V_{2\gamma}{}^{\dot\alpha} V_{3\delta}{}^{\dot\beta} \\
& + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\beta\dot\alpha}\epsilon_{\dot\gamma\dot\delta}(D_\alpha{}^{\dot\alpha} V_{1\beta}{}^{\dot\beta}) V_{2\gamma}{}^{\dot\gamma} V_{3\delta}{}^{\dot\delta} + \epsilon^{\beta\alpha}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\gamma}\epsilon_{\dot\beta\dot\delta}(D_\alpha{}^{\dot\alpha} V_{1\beta}{}^{\dot\beta}) V_{2\gamma}{}^{\dot\gamma} V_{3\delta}{}^{\dot\delta} \\
& + \epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\delta}\epsilon_{\dot\gamma\dot\beta} V_{1\alpha}{}^{\dot\alpha}(D_\beta{}^{\dot\beta} V_{2\gamma}{}^{\dot\gamma}) V_{3\delta}{}^{\dot\delta} + \epsilon^{\alpha\delta}\epsilon^{\gamma\beta}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\gamma\dot\delta} V_{1\alpha}{}^{\dot\alpha}(D_\beta{}^{\dot\beta} V_{2\gamma}{}^{\dot\gamma}) V_{3\delta}{}^{\dot\delta} \\
& + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\delta\dot\gamma} V_{1\alpha}{}^{\dot\alpha} V_{2\beta}{}^{\dot\beta}(D_\gamma{}^{\dot\gamma} V_{3\delta}{}^{\dot\delta}) + \epsilon^{\alpha\beta}\epsilon^{\delta\gamma}\epsilon_{\dot\alpha\dot\gamma}\epsilon_{\dot\beta\dot\delta} V_{1\alpha}{}^{\dot\alpha} V_{2\beta}{}^{\dot\beta}(D_\gamma{}^{\dot\gamma} V_{3\delta}{}^{\dot\delta}) \\
& + \epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\gamma\dot\delta} V_{1\alpha}{}^{\dot\alpha} V_{2\beta}{}^{\dot\beta} V_{3\gamma}{}^{\dot\gamma} V_{4\delta}{}^{\dot\delta} + \epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\delta}\epsilon_{\dot\beta\dot\gamma} V_{1\alpha}{}^{\dot\alpha} V_{2\beta}{}^{\dot\beta} V_{3\gamma}{}^{\dot\gamma} V_{4\delta}{}^{\dot\delta} \\
& + \epsilon^{\alpha\delta}\epsilon^{\beta\gamma}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\gamma\dot\delta} V_{1\alpha}{}^{\dot\alpha} V_{2\beta}{}^{\dot\beta} V_{3\gamma}{}^{\dot\gamma} V_{4\delta}{}^{\dot\delta} + \epsilon^{\alpha\delta}\epsilon^{\beta\gamma}\epsilon_{\dot\alpha\dot\delta}\epsilon_{\dot\beta\dot\gamma} V_{1\alpha}{}^{\dot\alpha} V_{2\beta}{}^{\dot\beta} V_{3\gamma}{}^{\dot\gamma} V_{4\delta}{}^{\dot\delta},
\end{aligned}
\tag{2.14}
$$

$$
\begin{aligned}
\mathcal{B}_5 = {} & \phi_1\phi_2\phi_3\phi_4\phi_5 + \epsilon^{\alpha\beta}\phi_1\phi_2\psi_{3\alpha}\psi_{4\beta} + \epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}(D_\alpha{}^{\dot\alpha}\phi_1)\phi_2\phi_3 V_{4\beta}{}^{\dot\beta} + \epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\phi_1(D_\alpha{}^{\dot\alpha}\phi_2)\phi_3 V_{4\beta}{}^{\dot\beta} \\
& + \epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\phi_1\phi_2\phi_3 V_{4\alpha}{}^{\dot\alpha} V_{5\beta}{}^{\dot\beta} + \epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\phi_1\psi_{2\alpha}\psi_3^{\dagger\dot\alpha} V_{4\beta}{}^{\dot\beta} + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\phi_1 F_{\mathrm{L}2\alpha\beta} F_{\mathrm{L}3\gamma\delta} \\
& + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\psi_{1\alpha}\psi_{2\beta} F_{\mathrm{L}3\gamma\delta} + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\alpha\dot\beta}\phi_1 F_{\mathrm{L}2\alpha\beta}(D_\gamma{}^{\dot\alpha} V_{3\delta}{}^{\dot\beta}) + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\alpha\dot\beta}\phi_1 F_{\mathrm{L}2\alpha\beta} V_{3\gamma}{}^{\dot\alpha} V_{4\delta}{}^{\dot\beta} \\
& + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\alpha\dot\beta}\psi_{1\alpha}\psi_{2\beta}(D_\gamma{}^{\dot\alpha} V_{3\delta}{}^{\dot\beta}) + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\alpha\dot\beta}\psi_{1\alpha}\psi_{2\beta} V_{3\gamma}{}^{\dot\alpha} V_{4\delta}{}^{\dot\beta} + \epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\beta}\psi_{1\alpha}\psi_{2\beta} V_{3\gamma}{}^{\dot\alpha} V_{4\delta}{}^{\dot\beta} \\
& + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\beta\dot\alpha}\epsilon_{\dot\delta\dot\gamma}\phi_1(D_\alpha{}^{\dot\alpha} V_{2\beta}{}^{\dot\beta})(D_\gamma{}^{\dot\gamma} V_{3\delta}{}^{\dot\delta}) + \epsilon^{\beta\alpha}\epsilon^{\delta\gamma}\epsilon_{\dot\alpha\dot\gamma}\epsilon_{\dot\beta\dot\delta}\phi_1(D_\alpha{}^{\dot\alpha} V_{2\beta}{}^{\dot\beta})(D_\gamma{}^{\dot\gamma} V_{3\delta}{}^{\dot\delta}) \\
& + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\beta\dot\alpha}\epsilon_{\dot\gamma\dot\delta}\phi_1(D_\alpha{}^{\dot\alpha} V_{2\beta}{}^{\dot\beta}) V_{3\gamma}{}^{\dot\gamma} V_{4\delta}{}^{\dot\delta} + \epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\delta}\epsilon_{\dot\gamma\dot\beta}\phi_1 V_{2\alpha}{}^{\dot\alpha}(D_\beta{}^{\dot\beta} V_{3\gamma}{}^{\dot\gamma}) V_{4\delta}{}^{\dot\delta} \\
& + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\delta\dot\gamma}\phi_1 V_{2\alpha}{}^{\dot\alpha} V_{3\beta}{}^{\dot\beta}(D_\gamma{}^{\dot\gamma} V_{4\delta}{}^{\dot\delta}) + \epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\gamma\dot\delta}(D_\alpha{}^{\dot\alpha}\phi_1) V_{2\beta}{}^{\dot\beta} V_{3\gamma}{}^{\dot\gamma} V_{4\delta}{}^{\dot\delta} \\
& + \epsilon^{\alpha\delta}\epsilon^{\beta\gamma}\epsilon_{\dot\alpha\dot\delta}\epsilon_{\dot\beta\dot\gamma}(D_\alpha{}^{\dot\alpha}\phi_1) V_{2\beta}{}^{\dot\beta} V_{3\gamma}{}^{\dot\gamma} V_{4\delta}{}^{\dot\delta} + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\alpha\dot\gamma}\epsilon_{\dot\beta\dot\delta}\phi_1(D_\alpha{}^{\dot\alpha} V_{2\beta}{}^{\dot\beta}) V_{3\gamma}{}^{\dot\gamma} V_{4\delta}{}^{\dot\delta} \\
& + \epsilon^{\beta\alpha}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\gamma}\epsilon_{\dot\beta\dot\delta}\phi_1(D_\alpha{}^{\dot\alpha} V_{2\beta}{}^{\dot\beta}) V_{3\gamma}{}^{\dot\gamma} V_{4\delta}{}^{\dot\delta} + \epsilon^{\alpha\delta}\epsilon^{\gamma\beta}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\gamma\dot\delta}\phi_1 V_{2\alpha}{}^{\dot\alpha}(D_\beta{}^{\dot\beta} V_{3\gamma}{}^{\dot\gamma}) V_{4\delta}{}^{\dot\delta} \\
& + \epsilon^{\alpha\beta}\epsilon^{\delta\gamma}\epsilon_{\dot\alpha\dot\gamma}\epsilon_{\dot\beta\dot\delta}\phi_1 V_{2\alpha}{}^{\dot\alpha} V_{3\beta}{}^{\dot\beta}(D_\gamma{}^{\dot\gamma} V_{4\delta}{}^{\dot\delta}) + \epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\gamma\dot\delta}\phi_1 V_{2\alpha}{}^{\dot\alpha} V_{3\beta}{}^{\dot\beta} V_{4\gamma}{}^{\dot\gamma} V_{5\delta}{}^{\dot\delta} \\
& + \epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\delta}\epsilon_{\dot\beta\dot\gamma}\phi_1 V_{2\alpha}{}^{\dot\alpha} V_{3\beta}{}^{\dot\beta} V_{4\gamma}{}^{\dot\gamma} V_{5\delta}{}^{\dot\delta} + \epsilon^{\alpha\delta}\epsilon^{\beta\gamma}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\gamma\dot\delta}\phi_1 V_{2\alpha}{}^{\dot\alpha} V_{3\beta}{}^{\dot\beta} V_{4\gamma}{}^{\dot\gamma} V_{5\delta}{}^{\dot\delta} \\
& + \epsilon^{\alpha\delta}\epsilon^{\beta\gamma}\epsilon_{\dot\alpha\dot\delta}\epsilon_{\dot\beta\dot\gamma}\phi_1 V_{2\alpha}{}^{\dot\alpha} V_{3\beta}{}^{\dot\beta} V_{4\gamma}{}^{\dot\gamma} V_{5\delta}{}^{\dot\delta}.
\end{aligned}
\tag{2.15}
$$

III. THE GENERAL BSM MODEL

In this section, we will write down the UV Lagrangian with nonredundant operators, based on the massive basis presented above. First, we should list all possible UV resonances labeled by their representations under the SM gauge symmetry, which could be obtained by drawing Feynman diagrams and enumerating the mediating particles. We will straightforwardly use the results from Ref. [18]. After that, we build up the general Lagrangian for all the states and pick up the nonredundant terms with the help of amplitude basis and Young diagrams. The Lagrangian is presented in Appendix A with terms that could contribute to effective operators at classical level only. In this paper, the Lagrangian for SM is written as

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SM}} = & -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu H)^\dagger(D^\mu H) \\
& + \bar{q}_L i\not{D} q_L + \bar{\ell}_L i\not{D}\ell_L + \bar{u}_R i\not{D} u_R + \bar{d}_R i\not{D} d_R + \bar{e}_R i\not{D} e_R \\
& + \mu_H^2 H^\dagger H - \lambda_H (H^\dagger H)^2 - (\bar{q}_L Y_u u_R \tilde{H} + \bar{q}_L Y_d d_R H + \bar{\ell}_L Y_e e_R H + \text{H.c.}).
\end{aligned} \tag{3.1}
$$

Before digging into the details of a general model, we make some assumptions on the UV theory as follows:

(i) The UV theory follows the SM gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$, which is linearly realized.
(ii) The UV theory contains both the SM particles and new resonances.
(iii) The interactions between the new resonances and the SM particles are weakly coupled. Strong dynamics are out of the scope of this work.
(iv) The new fields decouple at the electroweak scale. That is to say, new particles do not generate nonzero vacuum expectation values (VEVs), which breaks the SM gauge symmetry, and new fermions are either vectorlike or Majorana.[1]

These assumptions enable us to describe the physics at the electroweak scale by the SMEFT instead of the more complicated one, HEFT [44–46].

We limit our discussion to the resonances that can produce tree-level contributions to the amplitudes of the SM particles. Generally, effects from BSM physics could come from loops, but tree-level origins are typically the most promising part being able to be detected. Apart from that, tree-level effects behave like a resonance and tend to be recognized on high-energy colliders. Furthermore, We only discuss renormalizable interactions in this work. In principle, the TeV physics is not required to be UV complete since the gravity will eventually come into the theory at a even higher scale. Such a theory with unrenormalizable operators is often called resonance EFT [47–49] or BSMEFT in the Ref. [27]. It is also acceptable that the UV theory contains higher spin fields like Rarita-Schwinger spinors or tensor fields. Considering that these extra contributions are further suppressed by heavy scales, we leave these possibilities to future works.

The quantum number of UV resonances can be fixed by the effective operators they contribute to. All possible partitions of the external particles of an effective operator correspond to tree Feynman diagrams whose internal lines indicate possible UV resonances. For each partition, the Poincaré and gauge Casimir eigen basis [17–19], i.e., the j-basis operators, classify the quantum number of heavy resonances by the proposed j-basis/UV correspondence [18]. Some of the selected resonances should be excluded since they only contribute to high dimensional effective operators, which can be done through dimension selection.

Tables II–IV present all the UV resonances that could contribute to effective operators in the SMEFT with mass

[1]The fields except the SM Higgs doublet are supposed to be in the broken phase if any gauge symmetry except $SU(3)_C \times SU(2)_L \times U(1)_Y$ is spontaneously broken. We are working in the "broken" phase where the SM gauge symmetry still holds. Although masses of the new particles, which are at the heavy scale, can receive contributions during the symmetry breaking, we assume that after the electroweak symmetry breaking the VEV of the SM Higgs boson contributes only a small portion to the heavy masses.

TABLE I. Dimension-3 and dimension-4 Lorentz structures in spinor indices and Lorentz indices.

Classes	Spinor notation	Lorentz notation
ϕ^3	$\phi_1\phi_2\phi_3$	$\phi_1\phi_2\phi_3$
ϕV^2	$\epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\phi_1 V_{2\alpha}{}^{\dot\alpha}V_{3\beta}{}^{\dot\beta}$	$\phi_1 V_{2\mu}V_3^\mu$
ϕ^4	$\phi_1\phi_2\phi_3\phi_4$	$\phi_1\phi_2\phi_3\phi_4$
$\phi\psi^2$	$\epsilon^{\alpha\beta}\phi_1\psi_{2\alpha}\psi_{3\beta}$	$\phi_1(\psi_2\psi_3)$
$\phi^2 VD$	$\epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}(D_\alpha{}^{\dot\alpha}\phi_1)\phi_2 V_{3\beta}{}^{\dot\beta}$	$(D_\mu\phi_1)\phi_2 V_3^\mu$
$\phi^2 V^2$	$\epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\phi_1\phi_2 V_{3\alpha}{}^{\dot\alpha}V_{4\beta}{}^{\dot\beta}$	$\phi_1\phi_2 V_{3\mu}V_4^\mu$
$\psi\psi^\dagger V$	$\epsilon^{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\psi_{1\alpha}\psi_2^{\dagger\dot\alpha}V_{3\beta}{}^{\dot\beta}$	$(\psi_1\sigma_\mu\psi_2^\dagger)V^\mu$
$F_{\mathrm{L}}V^2$	$\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\alpha\dot\beta}F_{\mathrm{L}1\alpha\beta}V_{2\gamma}{}^{\dot\alpha}V_{3\delta}{}^{\dot\beta}$	$F_{\mathrm{L}1\mu\nu}V_2^\mu V_3^\nu$
$V^3 D$	$\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\beta\dot\alpha}\epsilon_{\dot\gamma\dot\delta}(D_\alpha{}^{\dot\alpha}V_{1\beta}{}^{\dot\beta})V_{2\gamma}{}^{\dot\gamma}V_{3\delta}{}^{\dot\delta}$	$(D_\mu V_{1\nu})V_2{}^\nu V_3{}^\mu$
	$\epsilon^{\beta\alpha}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\gamma}\epsilon_{\dot\beta\dot\delta}(D_\alpha{}^{\dot\alpha}V_{1\beta}{}^{\dot\beta})V_{2\gamma}{}^{\dot\gamma}V_{3\delta}{}^{\dot\delta}$	$\epsilon^{\mu\nu\rho\lambda}(D_\mu V_{1\nu})V_{2\rho}V_{3\lambda}$
	$\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\delta}\epsilon_{\dot\gamma\dot\beta}V_{1\alpha}{}^{\dot\alpha}(D_\beta{}^{\dot\beta}V_{2\gamma}{}^{\dot\gamma})V_{3\delta}{}^{\dot\delta}$	$V_1{}^\nu(D_\mu V_{2\nu})V_3{}^\mu$
	$\epsilon^{\alpha\delta}\epsilon^{\gamma\beta}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\gamma\dot\delta}V_{1\alpha}{}^{\dot\alpha}(D_\beta{}^{\dot\beta}V_{2\gamma}{}^{\dot\gamma})V_{3\delta}{}^{\dot\delta}$	$\epsilon^{\mu\nu\rho\lambda}V_{1\mu}(D_\nu V_{2\rho})V_{3\lambda}$
	$\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\delta\dot\gamma}V_{1\alpha}{}^{\dot\alpha}V_{2\beta}{}^{\dot\beta}(D_\gamma{}^{\dot\gamma}V_{3\delta}{}^{\dot\delta})$	$V_1{}^\mu V_2{}^\nu(D_\mu V_{3\nu})$
	$\epsilon^{\alpha\beta}\epsilon^{\delta\gamma}\epsilon_{\dot\alpha\dot\gamma}\epsilon_{\dot\beta\dot\delta}V_{1\alpha}{}^{\dot\alpha}V_{2\beta}{}^{\dot\beta}(D_\gamma{}^{\dot\gamma}V_{3\delta}{}^{\dot\delta})$	$V_1{}^\nu V_2{}^\mu(D_\mu V_{3\nu})$
V^4	$\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\gamma\dot\delta}V_{1\alpha}{}^{\dot\alpha}V_{2\beta}{}^{\dot\beta}V_{3\gamma}{}^{\dot\gamma}V_{4\delta}{}^{\dot\delta}$	$V_1{}^\mu V_{2\mu}V_3{}^\nu V_{4\nu}$
	$\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\epsilon_{\dot\alpha\dot\delta}\epsilon_{\dot\beta\dot\gamma}V_{1\alpha}{}^{\dot\alpha}V_{2\beta}{}^{\dot\beta}V_{3\gamma}{}^{\dot\gamma}V_{4\delta}{}^{\dot\delta}$	$V_1{}^\mu V_2{}^\nu V_{3\mu}V_{4\nu}$
	$\epsilon^{\alpha\delta}\epsilon^{\beta\gamma}\epsilon_{\dot\alpha\dot\beta}\epsilon_{\dot\gamma\dot\delta}V_{1\alpha}{}^{\dot\alpha}V_{2\beta}{}^{\dot\beta}V_{3\gamma}{}^{\dot\gamma}V_{4\delta}{}^{\dot\delta}$	$V_1{}^\mu V_{2\nu}V_3{}^\nu V_{4\mu}$
	$\epsilon^{\alpha\delta}\epsilon^{\beta\gamma}\epsilon_{\dot\alpha\dot\delta}\epsilon_{\dot\beta\dot\gamma}V_{1\alpha}{}^{\dot\alpha}V_{2\beta}{}^{\dot\beta}V_{3\gamma}{}^{\dot\gamma}V_{4\delta}{}^{\dot\delta}$	$\epsilon^{\mu\nu\rho\lambda}V_{1\mu}V_{2\nu}V_{3\rho}V_{4\lambda}$

TABLE II. New scalars that can contribute to operators up to dimension 7 in SMEFT at classical level. The second row in each block represents their name in Ref. [27].

Notation	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
Name	$\mathcal{S}$	$\mathcal{S}_1$	$\mathcal{S}_2$	φ	Ξ	Ξ_1	Θ_1	Θ_3
Irrep	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{1})_1$	$(\mathbf{1},\mathbf{1})_2$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{1},\mathbf{3})_1$	$(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$	$(\mathbf{1},\mathbf{4})_{\frac{3}{2}}$
Notation	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}		
Name	ω_4	ω_1	ω_2	Π_1	Π_7	ζ		
Irrep	$(\mathbf{3},\mathbf{1})_{-\frac{4}{3}}$	$(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	$(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$(\mathbf{3},\mathbf{2})_{\frac{7}{6}}$	$(\mathbf{3},\mathbf{3})_{-\frac{1}{3}}$		
Notation	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}			
Name	Ω_2	Ω_1	Ω_4	Υ_1	Φ			
Irrep	$(\mathbf{6},\mathbf{1})_{-\frac{2}{3}}$	$(\mathbf{6},\mathbf{1})_{\frac{1}{3}}$	$(\mathbf{6},\mathbf{1})_{\frac{4}{3}}$	$(\mathbf{6},\mathbf{3})_{\frac{1}{3}}$	$(\mathbf{8},\mathbf{2})_{\frac{1}{2}}$			

TABLE III. New fermions that can contribute to operators up to dimension 7 in SMEFT at classical level. The second row in each block represents their name in Ref. [27], with a superscript ^{c} if they are conjugated with each other. Majorana fermions F_1 and F_5 have parity left, i.e., $F = F_L = P_L F$.

Notation	F_1	F_2	F_3	F_4	F_5	F_6	F_7
Name	N	E^c	Δ_1^c	Δ_3^c	Σ	Σ_1^c	
Irrep	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{1})_1$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{1},\mathbf{3})_1$	$(\mathbf{1},\mathbf{4})_{\frac{1}{2}}$
Notation	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
Name	D	U	Q_5	Q_1	Q_7	T_1	T_2
Irrep	$(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	$(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	$(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$(\mathbf{3},\mathbf{2})_{\frac{7}{6}}$	$(\mathbf{3},\mathbf{3})_{-\frac{1}{3}}$	$(\mathbf{3},\mathbf{3})_{\frac{2}{3}}$

TABLE IV. New vectors that can contribute to operators up to dimension 7 in SMEFT at classical level. The second row in each block represents their name in Ref. [27], with a superscript † if they are conjugated with each other.

Notation	V_1	V_2	V_3	V_4	V_5	V_6	V_7
Name	$\mathcal{B}$	$\mathcal{B}_1$	$\mathcal{L}_3^\dagger$	$\mathcal{W}$	$\mathcal{U}_2$	$\mathcal{U}_5$	$\mathcal{Q}_5$
Irrep	$(\mathbf{1},\mathbf{1})_0$	$(\mathbf{1},\mathbf{1})_1$	$(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$	$(\mathbf{1},\mathbf{3})_0$	$(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	$(\mathbf{3},\mathbf{1})_{\frac{5}{3}}$	$(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$
Notation	V_8	V_9	V_{10}	V_{11}	V_{12}	V_{13}	V_{14}
Name	$\mathcal{Q}_1$	$\mathcal{X}$	$\mathcal{Y}_1^\dagger$	$\mathcal{Y}_5^\dagger$	$\mathcal{G}$	$\mathcal{G}_1$	$\mathcal{H}$
Irrep	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$(\mathbf{3},\mathbf{3})_{\frac{2}{3}}$	$(\mathbf{6},2)_{-\frac{1}{6}}$	$(\mathbf{6},2)_{\frac{5}{6}}$	$(\mathbf{8},\mathbf{1})_0$	$(\mathbf{8},\mathbf{1})_1$	$(\mathbf{8},\mathbf{3})_0$

dimension up to 7 at the classical level. Each row in the blocks respectively represents the notation of the resonances in our paper, their names in Ref. [27] and their quantum number under $(SU(3)_C, SU(2)_L)_{U(1)_Y}$. We only consider particles with spin ≤ 1. The fields with conjugated quantum number compared to Ref. [27] are attached with a superscript † for bosons or ^{c} for fermions. The $U(1)_Y$ hypercharge Y is defined as

$$Q = Y + T_3, \qquad (3.2)$$

where Q is the electric charge after symmetry breaking and T_3 the weak isospin.

Although most resonances and terms have been listed in Ref. [27], it is necessary to point out the differences as follows:

(i) A quartet fermion F_7 is not presented in Ref. [27] since it can only generate dimension-7 operator O_{LH}.
(ii) Vector $\mathcal{L}_1$ $(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$ and vector $\mathcal{W}_1$ $(\mathbf{1},\mathbf{3})_1$ listed in Ref. [27] are not presented because operators involving $D_\mu V^\mu$ are identified as redundant operators by the massive amplitude method introduced in Sec. II. These operators can be removed by field redefinition [48,50,51]. For example, a field redefinition $V_\mu \to V_\mu - \frac{1}{M^2}(D_\mu \mathcal{O})$ transforms the original Lagrangian,

$$\mathcal{L} = -\frac{1}{2}V^\dagger_{\mu\nu}V^{\mu\nu} + M^2 V^\dagger_\mu V^\mu + V^{\dagger\mu}(D_\mu\mathcal{O}) + (D_\mu\mathcal{O})^\dagger V^\mu \tag{3.3}$$

into

$$\mathcal{L} = -\frac{1}{2}V^\dagger_{\mu\nu}V^{\mu\nu} + M^2 V^\dagger_\mu V^\mu - \frac{i}{2M^2}V^\dagger_{\mu\nu}(G^{\mu\nu}\mathcal{O}) + \frac{i}{2M^2}(G^{\mu\nu}\mathcal{O})^\dagger V_{\mu\nu} - \frac{1}{M^2}(D_\mu\mathcal{O})^\dagger(D_\mu\mathcal{O}) - \frac{1}{2M^4}(G_{\mu\nu}\mathcal{O})^\dagger(G^{\mu\nu}\mathcal{O}), \tag{3.4}$$

where $V_{\mu\nu} = (D_\mu V_\nu) - (D_\nu V_\mu)$ is the field strength tensor of V^μ, and $G_{\mu\nu} = i[D_\mu, D_\nu]$ is the gauge field tensor. Since this theory is still an effective theory of some unknown physics, we attribute the effective operators in the second line, which in principle have imprints in IR physics, to super-UV physics instead of the new vector. What is more, the interactions involved in the original Lagrangian, i.e., $\mathcal{L}_1^{\dagger\mu}(D_\mu H)$ and $\mathcal{W}_1^{\dagger\mu I} iD_\mu(\tilde{H}^\dagger\tau^I H)$, can only be generated via loop diagrams, which is out of scope of our consideration. Thus, we do not list these two resonances.

(iii) Our convention for operator basis is slightly different, such as operators containing fields $H, H^\dagger, S_5, S_5$ and $H, H^\dagger, S_6, S_6^\dagger$. The two terms, $\mathcal{D}^{pr}_{HH^\dagger S_5 S_5(1)}$ and $\mathcal{D}^{pr}_{HH^\dagger S_5 S_5(2)}$, have the same contribution only up to a factor. Therefore, a linear combination of these two terms does not contribute as is the case Ref. [27]. The case for $\mathcal{D}_{HH^\dagger S_6 S_6^\dagger(1)}$ and $\mathcal{D}_{HH^\dagger S_6 S_6^\dagger(2)}$ term is similar. We present them here for completeness.

The Lagrangian of the UV theory can be constructed from these field contents by enumerating Lorentz and gauge invariant combinations. Section II provides a general method to list all Lorentz invariant terms as in Table I with the help of massive amplitudes, while the gauge sector could be handled by the Littlewood-Richardson rule as in the SMEFT case [7]. To clarify it more clearly, take the $S_4 S_6^\dagger S_7$ term as an example. These three fields transform as **2**, **3**, and **4** representations under $SU(2)_L$ gauge transformation and are written as[2]

$$(S_4)_i \times (-1)\epsilon_{j_1 j_3}\epsilon_{j_2 j_4}(S_6^\dagger)^{j_3 j_4} \times (S_7)_{k_1 k_2 k_3}, \tag{3.5}$$

in our notation. The factor that contracts this term into a gauge singlet can be found by building a N-block-height Young tableau for $SU(N)$ group as[3]

$$\boxed{i} \xrightarrow{\boxed{j_1 j_2}} \boxed{i\ j_1\ j_2} \xrightarrow{\boxed{k_1 k_2 k_3}} \begin{array}{|c|c|c|}\hline i & j_1 & j_2 \\ \hline k_1 & k_2 & k_3 \\ \hline\end{array} \Rightarrow \epsilon^{ik_1}\epsilon^{j_1 k_2}\epsilon^{j_2 k_3}, \tag{3.6}$$

where the Young tableaux of i to the leftmost and j, k above the arrows represent the representations of $S_4, S_6^\dagger$, and S_7, respectively. Other formats to build such a rectangle Young tableau will result in vanishing factors due to the symmetry between indices. The flavor symmetry may lead to vanishing operators as well, which could be checked by imposing additional Young operators.

After dealing with Lorentz, gauge and flavor symmetries systematically, we present the results in Appendix A. The notation for the indices of fields in the Appendix is slightly different from what is used in Eq. (3.5). All fields under nonfundamental representations are attached with a single index for each group instead of repeating indices of fundamental representation. The transformation rules between the notation used in the Appendix and in Eq. (3.5) are

(i) $\mathbf{3}, \bar{\mathbf{3}}$ representations of $SU(3)_C$ remains unchanged as

$$\phi_a, \phi^{\dagger a}, \tag{3.7}$$

(ii) **8** representations of $SU(3)_C$,

$$\phi_a^b = \frac{1}{\sqrt{2}}\phi^A(\lambda^A)_a^b, \tag{3.8}$$

(iii) $\mathbf{6}, \bar{\mathbf{6}}$ representations of $SU(3)_C$,

$$\phi_{ab} = (C^{\mathfrak{c}})_{ab}\phi_{\mathfrak{c}}, \qquad \phi^{\dagger ab} = (C_{\mathfrak{c}})^{ab}\phi^{\dagger\mathfrak{c}}, \tag{3.9}$$

(iv) $\mathbf{2}, \bar{\mathbf{2}}$ representations of $SU(2)_L$ remains unchanged as

$$\phi_i, \phi^{\dagger i}, \tag{3.10}$$

[2]Subscripts and superscripts labeling the gauge components are indices of fundamental and antifundamental representation with $i_n, j_n, k_n = 1, 2$. The components are linked to the commonly used one by Clebsch-Gordan coefficients, e.g., $(S_6^\dagger)^I = \frac{1}{\sqrt{2}}(\tau^I\epsilon)_{ij}(S_6^\dagger)^{ij}$.

[3]One needs to further specify the symmetry of these (anti-)fundamental indices if calculations are performed under this notation, i.e., fields with (anti-)fundamental indices. For example, the factor is actually $\mathcal{Y}[\boxed{k_1 k_2 k_3}] \circ \mathcal{Y}[\boxed{j_1 j_2}] \circ \mathcal{Y}[\boxed{i}] \circ \epsilon^{ik_1}\epsilon^{j_1 k_2}\epsilon^{j_2 k_3} = (\epsilon^{ik_1}\epsilon^{j_1 k_2}\epsilon^{j_2 k_3} + (j_1 \leftrightarrow j_2)) + (\text{perm. of } k_1, k_2, k_3)$ in this case.

(v) **3** representations of $SU(2)_L$,

$$\phi_{ij} = \frac{1}{\sqrt{2}}\phi^I(\tau^I\epsilon)_{ij},$$
$$\phi^{\dagger ij} = (\phi_{ji})^\dagger = \frac{1}{\sqrt{2}}\phi^{\dagger I}(\tau^I)_k{}^j\epsilon^{ki} \quad \text{if } \phi \text{ is complex}, \tag{3.11}$$

(vi) $\mathbf{4}, \bar{\mathbf{4}}$ representations of $SU(2)_L$,

$$\phi_{ijk} = (C^{\mathcal{I}})_{ijk}\phi_{\mathcal{I}}, \qquad \phi^{\dagger ijk} = (C_{\mathcal{I}})^{ijk}\phi^{\dagger\mathcal{I}}. \tag{3.12}$$

The normal lowercase Latin letters stand for the indices of fundamental representations, $a, b, c, \cdots$ for $SU(3)_C$ and $i, j, k, \cdots$ for $SU(2)_L$. The corresponding capital letters are for the adjoint representations. **6** representation of $SU(3)_C$ are denoted by Gothic lowercase Latin letters $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \cdots$ in the subscripts, and **4** representation of $SU(2)_L$ are denoted by calligraphic capital letters $\mathcal{I}, \mathcal{J}, \mathcal{K}, \cdots$. Indices of conjugated representations are labeled in the superscripts. Typically, one can freely define the Clebsch-Gordan coefficients, but we suggest a normalized one as in Refs. [52–54]. In such a case, λ^A and τ^I are Gell-Mann and Pauli matrices. The Clebsch-Gordan coefficients for higher dimensional representations are $(C_{\mathfrak{a}})^{ab}$ for $\mathfrak{a} = 1, 2, \ldots, 6$ with

$$(C_1)^{ab} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad (C_2)^{ab} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad (C_3)^{ab} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$(C_4)^{ab} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad (C_5)^{ab} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad (C_6)^{ab} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.13}$$

and $(C_{\mathcal{I}})^{ijk} = \frac{1}{\sqrt{2}}(C_{\mathcal{I}})^{Jk}(\epsilon\tau^J)^{ji}$ for $\mathcal{I} = 3/2, 1/2, -1/2, -3/2$ with

$$(C_{3/2})^{Ij} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ -i & 0 \\ 0 & 0 \end{pmatrix}, \qquad (C_{1/2})^{Ij} = \frac{1}{\sqrt{6}}\begin{pmatrix} 0 & 1 \\ 0 & -i \\ -2 & 0 \end{pmatrix},$$
$$(C_{-1/2})^{Ij} = -\frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 \\ i & 0 \\ 0 & 2 \end{pmatrix}, \qquad (C_{-3/2})^{Ij} = -\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 0 & i \\ 0 & 0 \end{pmatrix}. \tag{3.14}$$

The conjugated representation are given by

$$(C^I)_{ab\cdots c} = [(C_I)^{c\cdots ba}]^*, \tag{3.15}$$

i.e., $(C^{\mathfrak{a}})_{cd} \equiv [(C_{\mathfrak{a}})^{dc}]^*$ and $(C^{\mathcal{I}})_{jkl} = [(C_{\mathcal{I}})^{lkj}]^*$. Both of them satisfy the normalization condition,

$$(C_{\mathfrak{a}})^{cd}(C^{\mathfrak{b}})_{dc} = \delta^{\mathfrak{b}}_{\mathfrak{a}}, \qquad (C_{\mathcal{I}})^{jkl}(C^{\mathcal{J}})_{lkj} = \delta^{\mathcal{J}}_{\mathcal{I}} \tag{3.16}$$

and

$$(C_{\mathfrak{a}})^{ab}(C^{\mathfrak{a}})_{c_1c_2} = \frac{1}{2}\sum_{\mathcal{P}\in S_2}\delta^a_{c_{\mathcal{P}(1)}}\delta^b_{c_{\mathcal{P}(2)}}, \qquad (C_{\mathcal{I}})^{ijk}(C^{\mathcal{I}})_{l_1l_2l_3} = \frac{1}{3!}\sum_{\mathcal{P}\in S_3}\delta^i_{l_{\mathcal{P}(1)}}\delta^j_{l_{\mathcal{P}(2)}}\delta^k_{l_{\mathcal{P}(3)}}, \tag{3.17}$$

where S_n is the permutation group on n letters.

IV. THE MATCHING PROCEDURE

The matching procedure aims to find the effective theory which contains only light degrees of freedom but still precisely describes the full theory. We have proposed some assumptions on the UV theory in Sec. III which validate SMEFT to describe the physics, and in the following, we need to find the Wilson coefficients of the effective operators. The procedure consists of two parts: deriving the effective Lagrangian and reducing the operators to a basis.

A. Tree-level matching

Amplitude matching and functional matching are two alternative ways to derive the effective theory. For convenience, we choose the latter one, which matches the effective actions of two theories. At a classical level, the effective Lagrangian can be derived from replacing the heavy fields by their classical equations of motion [32,55]. Specifically, the kinetic terms of heavy fields in the Lagrangian are

$$\Delta\mathcal{L}_{\rm kin} = -\eta\Phi^\dagger(D^2+M^2)\Phi, \qquad \text{for scalars,} \tag{4.1}$$

$$\Delta\mathcal{L}_{\rm kin} = \eta(L^\dagger_{\dot\alpha} iD^{\alpha\dot\alpha}L_\alpha + R^{\dagger\alpha}iD_{\alpha\dot\alpha}R^{\dot\alpha} - MR^{\dagger\alpha}L_\alpha - ML^\dagger_{\dot\alpha}R^{\dot\alpha}), \quad \text{for fermions,} \tag{4.2}$$

$$\Delta\mathcal{L}_{\rm kin} = \eta V^{\dagger\mu}(g_{\mu\nu}D^2 - D_\nu D_\mu + g_{\mu\nu}M^2)V^\nu, \qquad \text{for vectors,} \tag{4.3}$$

where $\eta = \frac{1}{2}(1)$ is the normalization factor for real bosonic and Majorana (complex bosonic and Dirac) fields. Note that for Majorana fermions $R^{\dot\alpha} = L^{\dagger\dot\alpha} \equiv (L_\alpha)^\dagger$. In the vector case, a term proportional to $V^{\dagger\mu}[D_\mu, D_\nu]V^\nu$ also appears as kinetic terms, which could be transformed into an interacting term since $[D_\mu, D_\nu] = -iF_{\mu\nu}$. The factor of such a term can be fixed by unitarity [32], but in this work, we treat it as a free parameter. The classical equations of motion, $\frac{\delta S}{\delta\Phi} = 0$, lead to the following equations (gauge indices are omitted):

$$\Phi = \frac{1}{M^2}\left(\frac{\delta S_{\rm int}}{\delta\Phi^\dagger} - \frac{1}{2}D_{\alpha\dot\alpha}D^{\alpha\dot\alpha}\Phi\right), \tag{4.4}$$

$$L_\alpha = -\frac{1}{M}\epsilon_{\alpha\beta}\frac{\delta S_{\rm int}}{\delta R^\dagger_\beta} + \frac{1}{M}iD_{\alpha\dot\alpha}R^{\dot\alpha}, \tag{4.5}$$

$$R^{\dot\alpha} = -\frac{1}{M}\epsilon^{\dot\alpha\dot\beta}\frac{\delta S_{\rm int}}{\delta L^{\dagger\dot\beta}} + \frac{1}{M}iD^{\alpha\dot\alpha}L_\alpha, \tag{4.6}$$

$$V_\alpha{}^{\dot\alpha} = \frac{1}{2M^2}\left(4\epsilon_{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}\frac{\delta S_{\rm int}}{\delta V^{\dagger\dot\beta}_\beta} - D_{\beta\dot\beta}D^{\beta\dot\beta}V_\alpha{}^{\dot\alpha} - D^\beta{}_{\dot\beta}D_\alpha{}^{\dot\alpha}V_\beta{}^{\dot\beta}\right). \tag{4.7}$$

To be consistent, all fields are written with 2-component spinor indices, and $SU(2)_{l(r)}$ indices are in the subscripts (superscripts). One may raise and lower the indices by the antisymmetric tensor ϵ with the index to be raised or lowered right after the ϵ. For instance, $D^{\alpha\dot\alpha}\Phi \equiv \epsilon^{\alpha\beta}D_\beta{}^{\dot\alpha}\Phi$. We choose $\epsilon^{12} = \epsilon^{\dot1\dot2} = \epsilon_{21} = \epsilon_{\dot2\dot1} = 1$ for ϵ of the Lorentz group.[4] Note that Eqs. (4.5) and (4.6) become the conjugation of each other for Majorana fermions.

Equations (4.4)–(4.7) can be solved iteratively. For every heavy field Φ, there is bound to be at least one interaction involving a single Φ. Fields not satisfying this rule can only be produced in a pair and thus, only contributes through loops. The remaining terms, e.g., terms with derivatives, are treated as perturbations due to the additional suppression from M^{-n} factors. We can stop the iteration of replacement by setting a cutoff order for M^{-1}. After we obtained a truncated solution of the equations of motion, we can replace the heavy fields in the UV Lagrangian by the solution to get the effective Lagrangian, i.e., $\mathcal{L}_{\rm EFT}[\phi] = \mathcal{L}_{\rm UV}[\phi, \Phi_c[\phi]]$, where ϕ represents light fields and Φ_c is the classical solution.

Generally, $\mathcal{L}_{\rm EFT}[\phi]$ contains operators in all kinds of forms which may not be in an on shell operator basis. In order to compare the experimental results with the Wilson coefficients, it is necessary to eliminate redundant operators in the Lagrangian since combination of redundant operators has null contribution to the S matrix, and we are not able to fix the coefficients. Operator reduction is very complicated due to various kinds of redundancy as follows.

B. Operator reduction

In this subsection, we propose a systematic method to reduce any effective operator to a given operator basis. In the reduction, not only the equations of motion (EOMs) of the SM, but the EOM terms that come from the Weinberg operator [1] are also involved, and the operator bases at mass dimension 5, 6, and 7 are integrated as one basis for a complete reduction result.

First of all, we adopt the off shell amplitude formalism introduced in Ref. [56], where a one-to-one mapping from operators to off shell amplitudes is proposed as an extension of the amplitude-operator correspondence,

$$\begin{array}{ccc} F_{{\rm L/R}i} & \sim & \lambda_{i,0}\lambda_{i,0}/\tilde\lambda_{i,0}\tilde\lambda_{i,0}, \\ \psi_i/\psi_i^\dagger & \sim & \lambda_{i,0}/\tilde\lambda_{i,0}, \\ \phi_i & \sim & 1, \\ D_{i,d_i} & \sim & -i\lambda_{i,d_i}\tilde\lambda_{i,d_i}. \end{array} \tag{4.8}$$

i in the subscript of each field labels the ith field in an operator, and i in the subscript of a covariant derivative denotes that the covariant derivative acts on the ith field.

[4] It should be noted that we use a different convention $\epsilon^{12} = \epsilon_{12} = 1$ for $SU(2)_L$ group. Thus, $\epsilon^{\alpha\beta}\epsilon_{\alpha\gamma} = \delta^\beta_\gamma$ for the Lorentz group while $\epsilon^{ij}\epsilon_{ik} = -\delta^i_k$ for $SU(2)_L$ group.

0 in the subscript of a spinor indicates that the index corresponds to a field, while d_i indicates the order of covariant derivatives acting on the ith field, $d_i \in \{1, \ldots, \hat{d}_i\}$, $\hat{d}_i \in \mathbb{Z}$. For example, $d_i = 1$ labels the first covariant derivative acting on the ith field, and $d_i = 2$ labels the second covariant derivative acting on that field, etc. We will also use x_i to label a spinor, $x_i \in \{0, 1, \ldots, \hat{d}_i\}$. The Dirac brackets of off shell spinors are defined as $\langle i_{x_i} j_{x_j} \rangle \equiv \lambda^{\alpha}_{i,x_i} \lambda_{j,x_j\alpha}$ and $[i_{x_i} j_{x_j}] \equiv \tilde{\lambda}_{i,x_i\dot{\alpha}} \tilde{\lambda}^{\dot{\alpha}}_{j,x_j}$.

With the off shell amplitude formalism, effective operators can be presented as off shell amplitudes by the map Eq. (4.8). Furthermore, the redundancy relations among these operators can be formulated in the off shell amplitude formalism. Specifically, the Integration By Part (IBP) relation for off shell amplitudes reads

$$|i_{\hat{d}_i}\rangle[i_{\hat{d}_i}| = -\sum_{j=1, j\neq i}^{N} |j_{\hat{d}_j+1}\rangle[j_{\hat{d}_j+1}|, \tag{4.9}$$

where N denotes the total number of particles in the off shell amplitudes. Equation (4.9) just means that the outermost derivative on the ith field is moved to the other $N-1$ fields by the IBP relation in operator perspective. The Schouten identity among off shell amplitudes is written as

$$\langle i_{x_i} l_{x_l}\rangle\langle j_{x_j} k_{x_k}\rangle + \langle i_{x_i} j_{x_j}\rangle\langle k_{x_k} l_{x_l}\rangle + \langle i_{x_i} k_{x_k}\rangle\langle l_{x_l} j_{x_j}\rangle = 0. \tag{4.10}$$

Utilizing the IBP relation Eq. (4.9) and the Schouten identity Eq. (4.10), any off shell amplitude of a certain operator type, where by "type" we mean that the fields and number of covariant derivatives in the operators are fixed, can be reduced into a set of independent off shell amplitudes in that operator type. What is more, off shell amplitudes that correspond to the EOM of the fields in the operator could appear during the reduction, and these off shell amplitudes would change the type. For example, we list the EOM of scalar, spinor, and gauge boson and the corresponding off shell amplitudes in the following:

$$\begin{aligned}
D^{\alpha}{}_{\dot{\alpha}} D_{\alpha}{}^{\dot{\alpha}} \phi_i &\sim \langle i_2 i_1\rangle[i_2 i_1], \\
D^{\alpha\dot{\alpha}} \psi_{i\alpha} &\sim \langle i_1 i_0\rangle|i_1], \\
D_{\alpha\dot{\alpha}} \psi_i^{\dagger\dot{\alpha}} &\sim [i_1 i_0]|i_1\rangle, \\
D^{\alpha\dot{\alpha}} F_{\mathrm{L}i\alpha\beta} &\sim \langle i_1 i_0\rangle|i_1]|i_0\rangle, \\
D_{\alpha\dot{\alpha}} F_{\mathrm{R}i}{}^{\dot{\alpha}\dot{\beta}} &\sim [i_1 i_0]|i_1\rangle|i_0].
\end{aligned} \tag{4.11}$$

For these off shell amplitudes corresponding to the EOM, we can derive the specific expressions of the EOM of the fields for a model and substitute them into the off shell amplitudes in other operator types with the EOM. In this work, the model is the SM and the EOMs include terms from the SM Lagrangian and the dimension-5 Weinberg operator. So the mass dimension of the off shell amplitude, as well as the mass dimension of the corresponding operator, may change after the substitution of the EOM. Generally, the result would be the sum of some off shell amplitudes in different types at several mass dimensions after the above reduction procedure is applied once, and the procedure should be applied repeatedly for all involved types until the result does not change any more in order to make sure the reduction is complete.

Here, we take the operator $C_{pr}(H^{\dagger} i D^{\mu} H)(L_p^{\dagger} \bar{\sigma}_{\mu} L_r)$ as a simple example to illustrate the method. Labeling the fields in the operator as $L_1 H_2 H_3^{\dagger} L_4^{\dagger}$, the corresponding off shell amplitude reads1233,

$$-C_{f_4 f_1} \delta^{i_1}_{i_4} \delta^{i_2}_{i_3} \langle 1_0 2_1\rangle[2_1 4_0], \tag{4.12}$$

and this off shell amplitude can be reduced with the IBP relation Eq. (4.9) as

$$\begin{aligned}
-C_{f_4 f_1} \delta^{i_1}_{i_4} \delta^{i_2}_{i_3} \langle 1_0 2_1\rangle[2_1 4_0] = {} & C_{f_4 f_1} \delta^{i_1}_{i_4} \delta^{i_2}_{i_3} \langle 1_0 1_1\rangle[1_1 4_0] \\
& + C_{f_4 f_1} \delta^{i_1}_{i_4} \delta^{i_2}_{i_3} \langle 1_0 3_1\rangle[3_1 4_0] \\
& + C_{f_4 f_1} \delta^{i_1}_{i_4} \delta^{i_2}_{i_3} \langle 1_0 4_1\rangle[4_1 4_0].
\end{aligned} \tag{4.13}$$

It is straightforward to see that the first term and the third term on the right-hand side of Eq. (4.13) correspond to the EOM, and can be converted to other types by substituting the EOM. For example, the first term on the right-hand side of Eq. (4.13) corresponds to the EOM of L_1, and becomes the following off shell amplitudes:

$$\begin{aligned}
\underbrace{C_{f_4 f_1} \delta^{i_1}_{i_4} \delta^{i_2}_{i_3} \langle 1_0 1_1\rangle[1_1 4_0]}_{L_1 H_2 H_3^{\dagger} L_4^{\dagger} D} \Rightarrow {} & \underbrace{-C_{f_5 p}(y_E)_{p f_4} \delta^{i_1}_{i_3} \delta^{i_2}_{i_5} [4_0 5_0]}_{H_1 H_2 H_3^{\dagger} e_4 L_5^{\dagger}} \\
& \underbrace{+ C_{f_5 p}(C_5^{\dagger})_{p f_6} \delta^{i_1}_{i_2} \epsilon_{i_5 i_4} \epsilon_{i_6 i_3} [5_0 6_0] + C_{f_5 p}(C_5^{\dagger})_{f_6 p} \delta^{i_1}_{i_2} \epsilon_{i_5 i_3} \epsilon_{i_6 i_4} [5_0 6_0]}_{H_1 H_2^{\dagger} H_3^{\dagger} H_4^{\dagger} L_5^{\dagger} L_6^{\dagger}},
\end{aligned} \tag{4.14}$$

after substituting the EOM of L_1. The first term on the right-hand side of Eq. (4.14) still corresponds to a dimension-6 operator, while the second and third terms correspond to dimension-7 operators.

As demonstrated above, we obtain the sum of a set of off shell amplitudes in different types at different mass dimensions after the reduction of one off shell amplitude in a certain type at a certain mass dimension. It is straightforward to see that each of the off shell amplitudes corresponds to an operator in the on shell operator basis since the redundancies among off shell amplitudes (operators), including the IBP, the EOM, and the Schouten identity, are removed. In fact, the correspondence can be found by simply taking the off shell amplitudes on shell [56], and the on shell basis is chosen to be the y basis [17]. However, as we are doing the reduction across types and dimensions, we should merge the y bases in different types at different dimensions to a "full" y basis, such that any off shell amplitude is reduced to this y basis.

As illustrated in Ref. [17], the f basis, instead of the y basis, is the independent and complete basis if the redundancy of the flavor structure is taken account of. After an operator is translated to an off shell amplitude and reduced to the "full" y basis, we utilize the K_{py} matrix that converts the "full" y basis to the "full" p basis to find the its coordinates on the p basis, and it is straightforward to find the its coordinates on the "full" f basis since the additional basis vectors in the p basis are related to the corresponding f basis vectors by permutations of the flavor indices. Following the idea, we can obtain the coordinates of any operator on any on shell operator basis if the basis is equivalent to the p(f) basis. In this work, the selected SMEFT operator bases are listed in Appendix C.

Here, we comment on the above operator reduction procedure. Operators related by the EOM are redundant because operators related by the field redefinitions are physically equivalent [57–59]. However, in the operator reduction, substituting the EOM of fields is equivalent to the leading-order contribution of field redefinitions [60]. For example, if one want to include the dimension-8 SMEFT operators in the cross-dimension reduction, the higher-order contribution of the field redefinitions of the dimension-6 operators should be considered.

V. THE UV-IR CORRESPONDENCE

After matching and operator reduction, we are able to project the effective Lagrangian onto a selected operator basis. The result can be translated as a correspondence between UV resonances and IR effective operators, which are listed in Tables V–IX. The complete expression for Wilson coefficients can be found in Appendix B.

The dictionary tables The relationship between UV resonances and IR effective operators is a bit complicated, so we rearrange the correspondence relationship into several tables according to the dimension of operators:

(i) The correspondence between single scalar/fermion/vector resonances and dimension-6 operators are shown in Tables V–VII, respectively. Check mark inside each box means that the operator to the left can be generated by introducing the new resonance above. For models with 2 or more kinds of UV resonances, one can just counts and combines the effective operators generated by each new resonance, except for S_1 and $\mathcal{O}_{fH}$ types of operators. $\mathcal{O}_{eH}$, $\mathcal{O}_{uH}$, and $\mathcal{O}_{dH}$ need S_1 with $S_4/F_2/F_3$, $S_4/F_9/F_{11}$, and $S_4/F_8/F_{11}$, respectively. Other than this, interaction between different types of new resonances does not result in new effective operators. Actually, the relation between dimension-6 operators and heavy field multiplets has been provided in Ref. [27], and our result is consistent with theirs.

(ii) The UV completions of operators with odd canonical dimension are listed in Tables VIII and IX, depending on whether the model preserves baryon number or not. Every box with check mark denotes that the operator above can be generated by introducing a single resonance to the left, while boxes with resonances means that the operator above needs both the resonance to the left and one of the resonance inside the box. Among single resonance extended models only three seesaw models could generate dimension-7 operators, which has been verified by Ref. [61] for type-I seesaw model. Same as above, introducing new resonance with new interactions will not change the type of effective operators shown in the tables but only the Wilson coefficients.

(iii) Note that not all models are presented in the tables. Only least requirements of resonances are listed. For example, $\mathcal{O}_{LH}$ can be generated by the model with F_5 as well as the model with F_5 and F_7, but only F_5 are marked in the column of $\mathcal{O}_{LH}$ since it has covered the latter situation.

With these identification tables, one can check what kinds of operators can be generated by a specific UV model, and also what kinds of UV resonance is required if one Wilson coefficient is measured to be nonzero. Although in most case the correspondence relationship is one-to-one, a quantitative analysis requires analytical expression of Wilson coefficients.

Notation of the Wilson coefficients The operators in the Appendix have been ordered by their dimension as well as the operator type. Dimension-6 operators are divided into bosonic, 4-fermion, 2-fermion, and baryon-number-violating operators, while dimension-7 operators are divided by whether the operator violates baryon number or not. It is

TABLE V. The correspondence between dimension-6 operators in SMEFT and single-scalar-extended UV models. Check mark inside each box means that the operator to the left can be generated by introducing the new resonance above. Note that the operators with a star mark in the S_1 column cannot generated by an S_1 extended model. $\mathcal{O}_{eH}$, $\mathcal{O}_{uH}$, and $\mathcal{O}_{uH}$ need S_1 with $S_4/F_2/F_3$, $S_4/F_9/F_{11}$, and $S_4/F_8/F_{11}$, respectively. The explicit forms of operators are listed in Table XI.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}
$\mathcal{O}_H$	✓			✓	✓	✓	✓	✓											
$\mathcal{O}_{H\Box}$	✓				✓	✓													
$\mathcal{O}_{HD}$					✓	✓													
$\mathcal{O}_{eH}$	*			✓	✓	✓													
$\mathcal{O}_{uH}$	*			✓	✓	✓													
$\mathcal{O}_{dH}$	*			✓	✓	✓													
$\mathcal{O}_{ll}$		✓				✓													
$\mathcal{O}_{qq}^{(1)}$										✓				✓		✓		✓	
$\mathcal{O}_{qq}^{(3)}$										✓				✓		✓		✓	
$\mathcal{O}_{lq}^{(1)}$										✓				✓					
$\mathcal{O}_{lq}^{(3)}$										✓				✓					
$\mathcal{O}_{ee}$			✓																
$\mathcal{O}_{uu}$									✓								✓		
$\mathcal{O}_{dd}$											✓				✓				
$\mathcal{O}_{eu}$										✓									
$\mathcal{O}_{ed}$									✓										
$\mathcal{O}_{ud}^{(1)}$										✓						✓			
$\mathcal{O}_{ud}^{(8)}$										✓						✓			
$\mathcal{O}_{le}$				✓															
$\mathcal{O}_{lu}$													✓						
$\mathcal{O}_{ld}$												✓							
$\mathcal{O}_{qe}$													✓						
$\mathcal{O}_{qu}^{(1)}$				✓															✓
$\mathcal{O}_{qu}^{(8)}$				✓															✓
$\mathcal{O}_{qd}^{(1)}$				✓															✓
$\mathcal{O}_{qd}^{(8)}$				✓															✓
$\mathcal{O}_{ledq}$				✓															
$\mathcal{O}_{quqd}^{(1)}$				✓						✓						✓			
$\mathcal{O}_{quqd}^{(8)}$										✓						✓			✓
$\mathcal{O}_{lequ}^{(1)}$				✓						✓			✓						
$\mathcal{O}_{lequ}^{(3)}$										✓			✓						
$\mathcal{O}_{duq}$										✓									
$\mathcal{O}_{qqu}$										✓									
$\mathcal{O}_{qqq}$										✓				✓					
$\mathcal{O}_{duu}$									✓	✓									

still necessary to note down the notation used in the full expression,

(i) Mass terms in the denominator of each term denote the source of this contribution, i.e., the Feynman diagram. The mediating particles are just those appearing in the subscripts of the mass terms.

(ii) The coupling parameters of the SM is the same as those in Eq. (3.1). New coupling parameters of UV models are denoted by $\mathcal{C}$ for mass-dimension-1 parameters or $\mathcal{D}$ for dimensionless parameters. The corresponding interacting particles are written in the subscript, whose flavor indices are listed in order in the superscript. For instance, $\mathcal{D}^{prs}_{F_{7L}LS_5}$ is the dimensionless coupling of the interaction between F_{7L}, L, and S_5, p, r, s are the flavor indices of F_{7L}, L, S_5 respectively.

(iii) In each term of the Wilson coefficients, the flavor indices of the Wilson coefficients are denoted by f_i, while the one summed in a pair are denoted by p_j. The subscripts of f_i are ordered by helicity and

TABLE VI. The correspondence between dimension-6 operators in SMEFT and single-fermion-extended UV models. The notation is the same as Table V.

	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}
$\mathcal{O}_{eH}$		✓	✓	✓	✓	✓								
$\mathcal{O}_{uH}$									✓		✓	✓	✓	✓
$\mathcal{O}_{dH}$								✓		✓	✓		✓	✓
$\mathcal{O}_{Hl}^{(1)}$	✓	✓			✓	✓								
$\mathcal{O}_{Hl}^{(3)}$	✓	✓			✓	✓								
$\mathcal{O}_{He}$			✓	✓										
$\mathcal{O}_{Hq}^{(1)}$								✓	✓				✓	✓
$\mathcal{O}_{Hq}^{(3)}$								✓	✓				✓	✓
$\mathcal{O}_{Hu}$											✓	✓		
$\mathcal{O}_{Hd}$										✓	✓			
$\mathcal{O}_{Hud}$											✓			

TABLE VII. The correspondence between dimension-6 operators in SMEFT and single-vector-extended UV models. The notation is the same as Table V.

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}	V_{12}	V_{13}	V_{14}
$\mathcal{O}_{H}$		✓		✓										
$\mathcal{O}_{H\Box}$	✓	✓		✓										
$\mathcal{O}_{HD}$	✓	✓		✓										
$\mathcal{O}_{eH}$	✓	✓		✓										
$\mathcal{O}_{uH}$	✓	✓		✓										
$\mathcal{O}_{dH}$	✓	✓		✓										
$\mathcal{O}_{Hl}^{(1)}$	✓													
$\mathcal{O}_{Hl}^{(3)}$				✓										
$\mathcal{O}_{He}$	✓													
$\mathcal{O}_{Hq}^{(1)}$	✓													
$\mathcal{O}_{Hq}^{(3)}$				✓										
$\mathcal{O}_{Hu}$	✓													
$\mathcal{O}_{Hd}$	✓													
$\mathcal{O}_{Hud}$		✓												
$\mathcal{O}_{ll}$	✓			✓										
$\mathcal{O}_{qq}^{(1)}$	✓											✓		✓
$\mathcal{O}_{qq}^{(3)}$				✓								✓		✓
$\mathcal{O}_{lq}^{(1)}$	✓				✓				✓					
$\mathcal{O}_{lq}^{(3)}$				✓	✓				✓					
$\mathcal{O}_{ee}$	✓													
$\mathcal{O}_{uu}$	✓											✓		
$\mathcal{O}_{dd}$	✓											✓		
$\mathcal{O}_{eu}$	✓					✓								
$\mathcal{O}_{ed}$	✓				✓									
$\mathcal{O}_{ud}^{(1)}$	✓	✓											✓	
$\mathcal{O}_{ud}^{(8)}$		✓										✓	✓	
$\mathcal{O}_{le}$	✓		✓											
$\mathcal{O}_{lu}$	✓							✓						
$\mathcal{O}_{ld}$	✓						✓							
$\mathcal{O}_{qe}$	✓						✓							

(Table continued)

TABLE VII. *(Continued)*

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}	V_{12}	V_{13}	V_{14}
$\mathcal{O}_{qu}^{(1)}$	✓						✓				✓			
$\mathcal{O}_{qu}^{(8)}$							✓				✓	✓		
$\mathcal{O}_{qd}^{(1)}$	✓							✓		✓				
$\mathcal{O}_{qd}^{(8)}$								✓		✓		✓		
$\mathcal{O}_{ledq}$					✓		✓							
$\mathcal{O}_{duq}$							✓	✓						
$\mathcal{O}_{qqu}$							✓							

also the letter of the field appearing in the operator. For example, the fields appearing in $\mathcal{O}_{dLueH} = \epsilon^{ij}(\bar{d}^a \ell_i)(u_a^T C e)H_j$ are d^c, L, u, e, H, whose helicities are $-1/2, -1/2, 1/2, 1/2, 0$, respectively. Thus, the flavor indices for the former four particles are f_1, f_2, f_5, f_4.

(iv) The flavor indices of the mass terms in the denominator are omitted. Every pair to be summed up in the numerator corresponds to a mediating particle with the flavor index, whose mass should appear in the denominator.

Example of usage To illustrate the usage of our dictionary more clearly, we will take a simple example by looking for one UV completion of neutrinoless double decay ($0\nu\beta\beta$). $0\nu\beta\beta$ receives three types of contributions at tree level: short range, long range, and neutrino mass insertion [62–64]. Among dimension-5, -6, and -7 operators in the SMEFT, $\mathcal{O}_{LHD1}$ and $\mathcal{O}_{duLLD}$ have the short-range or contact contribution while $\mathcal{O}_{LeHD}$, $\mathcal{O}_{LHW}$, $\mathcal{O}_{dLQLH1/2}$, $\mathcal{O}_{dLueH}$, and $\mathcal{O}_{QuLLH}$ have the long-range contribution. Neutrino mass insertion can be induced by $\mathcal{O}_5$ and $\mathcal{O}_{LH}$. Each operator has several UV origins according to Tables VIII–IX. We just pick the model with F_3 and V_2 as an example. Terms in the full Lagrangian that involves F_3 and V_2 can be read from Appendix A as

TABLE VIII. The correspondence between dimension-5 and -7 LNV operators in SMEFT and UV resonances. Only models conserving baryon number are listed here. Every box with check mark denotes that the operator above can be generated by introducing a single resonance to the left, while boxes with resonances means that the operator above needs both the resonance to the left and one of the resonance inside the box. Note that not all models are covered by the tables. Only least requirements of resonances are listed. The explicit forms of operators are listed in Tables X and XII.

	$\mathcal{O}_5$	$\mathcal{O}_{LH}$	$\mathcal{O}_{LeHD}$	$\mathcal{O}_{LHD1}$	$\mathcal{O}_{LHD2}$	$\mathcal{O}_{LHW}$	$\mathcal{O}_{eLLLH}$	$\mathcal{O}_{dLQLH1}$	$\mathcal{O}_{dLQLH2}$	$\mathcal{O}_{dLueH}$	$\mathcal{O}_{QuLLH}$
S_2							S_4/F_4		$S_4/F_9/F_{10}$		$S_4/F_8/F_{12}$
S_4							S_2/S_6	S_6	S_2/S_6		S_2/S_6
S_6	✓	✓	F_3	✓	✓		S_4/F_4	$S_4/F_{10}/F_{14}$	$S_4/F_{10}/F_{14}$		$S_4/F_{12}/F_{13}$
S_8		F_6									
S_{12}								F_{14}	F_9/F_{14}	F_3/F_{12}	
F_1	✓	✓	✓		✓	✓	✓	✓	✓	V_2/V_5	✓
F_3			S_6/V_2							S_{12}/V_2	
F_4							S_2/S_6				
F_5	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓
F_6		S_8									
F_8											S_2
F_9									S_2/S_{12}		
F_{10}								S_6	S_2/S_6	V_3	
F_{12}										$S_{12}/V_3/V_5$	$S_2/S_6/V_5/V_9$
F_{13}											S_6
F_{14}								S_6/S_{12}	S_6/S_{12}		
V_2			F_3/V_3							$F_1/F_3/V_3$	
V_3			V_2							$F_{10}/F_{12}/V_2$	
V_5										F_1/F_{12}	F_{12}
V_9											F_{12}

TABLE IX. The correspondence between dimension-7 operators in SMEFT and UV resonances. Models listed here all violate baryon number. Models with resonances marked with bold can generate dimension-6 baryon number violating operators. Other notations are the same as Table VIII.

	$\mathcal{O}_{dLQLH1}$	$\mathcal{O}_{dLQLH2}$	$\mathcal{O}_{dLueH}$	$\mathcal{O}_{QuLLH}$	$\mathcal{O}_{LdudH}$	$\mathcal{O}_{LdddH}$	$\mathcal{O}_{eQddH}$	$\mathcal{O}_{LdQQH}$
$\mathbf{S_{10}}$	$S_{12}/F_1/F_{10}$	$S_{12}/F_1/F_{10}$	$S_{12}/F_1/F_{10}$		$S_{12}/F_1/F_{10}$			$S_{12}/F_1/F_{10}$
S_{11}					$S_{13}/F_1/F_{11}$	$S_{12}/F_2/F_{11}$	$S_{13}/F_3/F_8$	
S_{12}	$\mathbf{S_{10}}/\mathbf{S_{14}}$	$\mathbf{S_{10}}/\mathbf{S_{14}}$	$\mathbf{S_{10}}$		$\mathbf{S_{10}}/F_{10}/F_{11}$	S_{11}/F_{11}		$\mathbf{S_{10}}/\mathbf{S_{14}}/F_8/F_{13}$
S_{13}					S_{11}/F_{10}		S_{11}/F_{10}	
$\mathbf{S_{14}}$	$S_{12}/F_5/F_{10}$	$S_{12}/F_5/F_{10}$						$S_{12}/F_5/F_{10}$
F_1	$\mathbf{S_{10}}$	$\mathbf{S_{10}}$	$\mathbf{S_{10}}$	$\mathbf{V_8}$	$\mathbf{S_{10}}/S_{11}$			$\mathbf{S_{10}}/\mathbf{V_8}$
F_2						S_{11}		
F_3			$\mathbf{V_8}$				$S_{11}/\mathbf{V_8}$	
F_5	$\mathbf{S_{14}}$	$\mathbf{S_{14}}$		$\mathbf{V_8}$				$\mathbf{S_{14}}/\mathbf{V_8}$
F_8				$\mathbf{V_8}$			S_{11}/V_5	$S_{12}/V_5/\mathbf{V_8}$
F_{10}	$\mathbf{S_{10}}/\mathbf{S_{14}}$	$\mathbf{S_{10}}/\mathbf{S_{14}}$	$\mathbf{S_{10}}/\mathbf{V_8}$		$\mathbf{S_{10}}/S_{12}/S_{13}$		$S_{13}/V_5/\mathbf{V_8}$	$\mathbf{S_{10}}/\mathbf{S_{14}}/V_5/V_9$
F_{11}					S_{11}/S_{12}	S_{11}/S_{12}		
F_{13}				$\mathbf{V_8}$				$S_{12}/\mathbf{V_8}/V_9$
V_5			$\mathbf{V_8}$	$\mathbf{V_8}$			$F_8/F_{10}/\mathbf{V_8}$	$F_8/F_{10}/\mathbf{V_8}$
$\mathbf{V_8}$			$F_3/F_{10}/V_5$	$F_1/F_5/F_8/F_{13}/V_5/V_9$			$F_3/F_{10}/V_5$	$F_1/F_5/F_8/F_{13}/V_5/V_9$
V_9				$\mathbf{V_8}$				$F_{10}/F_{13}/\mathbf{V_8}$

$$\Delta\mathcal{L}_{\rm UV} = \sum_p [(\bar{F}_{3p})^i i\not{D}(F_{3p})_i - M_{F_3}^p(\bar{F}_{3p})^i(F_{3p})_i + V_{2p}^{\dagger\mu}(g_{\mu\nu}D^2 - D_\nu D_\mu + g_{\mu\nu}(M_{V_2}^p)^2)V_{2p}^\nu]$$
$$+\Big[-\mathcal{D}^{rp}_{e^\dagger F^\dagger_{3R}H^\dagger}\epsilon_{ij}[\bar{e}_r(F_{3p})^i]H^{\dagger j} - 2\mathcal{D}^{rsp}_{d^\dagger uV_2^\dagger}[(\bar{d}_r)^a\gamma_\mu(u_s)_a]V_{2p}^{\dagger\mu}$$
$$-2\mathcal{D}^p_{HHV_2^\dagger D}\epsilon^{ij}[D_\mu H_i]H_j V_{2p}^{\dagger\mu} + \mathcal{D}^{psr}_{F_{3L}L^\dagger V_2^\dagger}\delta^i_j[(\bar{l}_s)^j\gamma_\mu(F_{3p})_i]V_{2r}^{\dagger\mu} + \text{H.c.}\Big]. \tag{5.1}$$

From the F_3 row of Table VIII, it can be verified that the model with F_3 and V_2 will generate both $\mathcal{O}_{LeHD}$ and $\mathcal{O}_{dLueH}$. Their Wilson coefficients are listed in Appendix B as

$$C_{LeHD}^{f_1f_5} = \sum_{p_1,p_2}\frac{2i\mathcal{D}^{f_5p_1*}_{e^\dagger F^\dagger_{3R}H^\dagger}\mathcal{D}^{p_1f_1p_2*}_{F_{3L}L^\dagger V_2^\dagger}\mathcal{D}^{p_2}_{HHV_2^\dagger D}}{M_{F_3}^{p_1}(M_{V_2}^{p_2})^2}, \tag{5.2}$$

$$C_{dLueH}^{f_1f_2f_5f_4} = \sum_{p_1,p_2}\frac{4\mathcal{D}^{f_1f_5p_1}_{d^\dagger uV_2^\dagger}\mathcal{D}^{f_4p_2*}_{e^\dagger F^\dagger_{3R}H^\dagger}\mathcal{D}^{p_2f_2p_1*}_{F_{3L}L^\dagger V_2^\dagger}}{M_{F_3}^{p_1}(M_{V_2}^{p_2})^2}. \tag{5.3}$$

We have attached the flavor indices to the Wilson coefficients. The indices are sorted by helicities and letters of the composing fields. For $\mathcal{O}_{LeHD}$, the helicities of the composing fields ℓ, e, H, H, H are $-1/2, 1/2, 0, 0, 0$, so ℓ, e are labeled by f_1 and f_5. The same rule applies for $\mathcal{O}_{dLueH}$, which has been mentioned above. Thus, the effective Lagrangian is

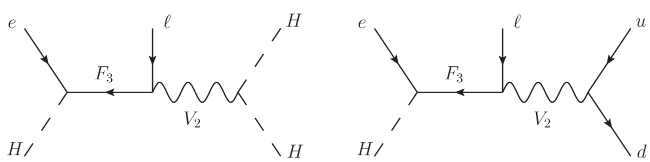

FIG. 1. Contributions to dimension-7 operators $\mathcal{O}_{LeHD}$ and $\mathcal{O}_{dLueH}$ from the model with F_3 and V_2.

$$\Delta\mathcal{L}_{\rm EFT} = C_{LeHD}^{f_1f_5}\times\epsilon^{ij}\epsilon^{kl}(\ell^T_{if_1}C\gamma^\mu e_{f_5})H_jH_k(iD_\mu H_l)$$
$$+ C_{dLueH}^{f_1f_2f_5f_4}\times\epsilon^{ij}(\bar{d}^a_{f_1}\ell_{if_2})(u^T_{af_5}Ce_{f_4})H_j. \tag{5.4}$$

Constraints on the couplings in Eq. (5.1) can be deduced from current experimental constraints on the Wilson coefficients.

One can also draw the corresponding Feynman diagrams from the Wilson coefficients. The mediating propagators F_3 and V_2 are encoded in the denominator, while the vertices are just presented as the couplings in the numerator. The Feynman diagrams shows as Fig. 1.

VI. SUMMARY

The EFT approach provides a systematic way to parametrize the BSM physics in terms of a series of Wilson coefficients of effective operators. One could get knowledge of UV physics by measuring the related low-energy experimental observables and determining the Wilson coefficients. In order not to miss any possibilities, a systematic bottom-up approach to link the UV physics and Wilson coefficients is needed. In this work, we present a correspondence between different UV resonances and dimension-5, -6, and -7 SMEFT operators in Tables V–IX. Information about the UV resonances is encoded in the relation among Wilson coefficients of effective operators, including same dimension and cross-dimension relation. With the help of the dictionary tables, a pattern of nonzero Wilson coefficients measured by experiments can be utilized to determine the resonance that possibly exists. The complete expression of the Wilson coefficients is presented in Appendix B.

Following Ref. [18], the UV resonances that have tree-level contributions to the effective operator can be enumerated by finding the eigenstates of the Casimir operators. We use spinor helicity formalism with massive amplitude to generate the renormalizable Lagrangian for UV physics containing all possible resonances. The Lorentz structures are listed in Table I. The gauge structures as well as the Clebsch-Gordan coefficients can be found by Young tableau formalism. After writing down the complete Lagrangian, we use the functional matching method to integrate out the heavy fields at the classical level. In order to reduce numerous effective operators to one operator basis, we provide a systematic method by the off shell amplitude formalism, which can be applied to any redundant operators. Our enumeration, matching, and reduction procedure is also applicable for all kinds of EFT-guided physics search.

The UV-IR dictionary listed in Tables V–IX and Appendix B can be used in two ways: one may check what kinds of effective operators can be generated for one UV model, and if several effective coefficients are measured to be nonzero, he can also check which heavy resonance has the most possibilities to exist. The complete expression of the Wilson coefficients is also presented in the Appendix for qualitative analysis. Our result could provide an EFT-guided UV resonance searches in the future collider experiments.

ACKNOWLEDGMENTS

We would like to thank Yu-Han Ni and Ming-Lei Xiao for helpful discussions. This work is supported by the National Science Foundation of China under Grants No. 12022514, No. 11875003, and No. 12047503, and National Key Research and Development Program of China Grants No. 2020YFC2201501, No. 2021YFA0718304, and CAS Project for Young Scientists in Basic Research YSBR-006, the Key Research Program of the CAS Grant No. XDPB15.

APPENDIX A: THE UV LAGRANGIAN

In this appendix, we provide the relevant UV Lagrangian. Some descriptions are given as follows:

(i) New couplings with mass dimension 1 and 0 are denoted by $\mathcal{C}$ and $\mathcal{D}$, respectively. The subscript marks the interacting fields and the flavor indices are written in the superscript.

(ii) Every field has its gauge indices ($a, A, \mathfrak{a}, \ldots$ or $i, I, \mathcal{I}, \ldots$) and flavor indices ($p, r, s, t, \ldots$).

(iii) The transpose marks ^{T} of fermion fields are omitted for convenience. $f_1^T C f_2 = \bar{f}_1^c f_2$ and $\bar{f}_1 C \bar{f}_2^T = \bar{f}_1 f_2^c$ are denoted by $f_1 C f_2$ and $\bar{f}_1 C \bar{f}_2$, respectively.

Detailed description can be found in Sec. III.

1. Kinetic terms

The kinetic terms have been presented in Sec. IV. For completeness, we list the terms here,

$$\Delta\mathcal{L}_{\text{kin}} = -\eta \Phi^\dagger (D^2 + M^2)\Phi, \qquad \text{for scalars,} \tag{A1}$$

$$\Delta\mathcal{L}_{\text{kin}} = \bar{F} i \not{D} F - \frac{1}{2} M (\bar{F}^c F + \bar{F} F^c), \quad \text{for Majorana fermions,} \tag{A2}$$

$$\Delta\mathcal{L}_{\text{kin}} = \bar{F} i \not{D} F - M \bar{F} F, \qquad \text{for Dirac fermions,} \tag{A3}$$

$$\Delta\mathcal{L}_{\text{kin}} = \eta V^{\dagger\mu} (g_{\mu\nu} D^2 - D_\nu D_\mu + g_{\mu\nu} M^2) V^\nu, \quad \text{for vectors,} \tag{A4}$$

where $\eta = \frac{1}{2}(1)$ for real(complex) bosonic fields. Note that Majorana fermions have parity left; i.e., $F = F_L = P_L F$, P_L is the projection operator. Contraction of gauge indices are straightforward. For example,

$$\Delta\mathcal{L}_{\text{kin}} = -S_{15}^{\dagger\mathfrak{a}} (D^2 + M_{S_{15}}^2) S_{15\mathfrak{a}}, \tag{A5}$$

for S_{15} and

$$\Delta\mathcal{L}_{\text{kin}} = \bar{F}_5^I i \not{D} F_5^I - \frac{1}{2} M_{F_5} (\bar{F}_5^{cI} F_5^I + \bar{F}_5^I F_5^{cI}), \tag{A6}$$

for F_5.

2. Interacting terms

As we construct the UV Lagrangian with the two-component spinors, initially the fermions in the UV Lagrangian are all two-component Weyl spinors, and then we translate them to four-component Dirac spinors for readers' convenience, through the following relations:

$$q = \begin{pmatrix} Q \\ 0 \end{pmatrix}, \qquad u = \begin{pmatrix} 0 \\ u_{(R)} \end{pmatrix}, \qquad d = \begin{pmatrix} 0 \\ d_{(R)} \end{pmatrix}, \qquad l = \begin{pmatrix} L \\ 0 \end{pmatrix}, \qquad e = \begin{pmatrix} 0 \\ e_{(R)} \end{pmatrix}, \tag{A7}$$

$$\bar{q} = (0, Q^\dagger), \qquad \bar{u} = (u^\dagger_{(R)}, 0), \qquad \bar{d} = (d^\dagger_{(R)}, 0), \qquad \bar{l} = (0, L^\dagger), \qquad \bar{e} = (e^\dagger_{(R)}, 0), \tag{A8}$$

for SM fermions,

$$F = \begin{pmatrix} F_{\mathrm{L}} \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} F_{\mathrm{L}} \\ F_{\mathrm{R}} \end{pmatrix}, \tag{A9}$$

for UV Majorana and Dirac fermions.

a. New scalars

$$\begin{aligned}
\Delta\mathcal{L}^{(d=3)}_{\mathrm{UV,S}} = {} & \mathcal{C}^{p}_{HH^\dagger S_1} H_i H^{\dagger i} S_{1p} - \frac{\mathcal{C}^{p}_{HH^\dagger S_5}}{\sqrt{2}} (\tau^I)^i_j H_i H^{\dagger j} (S_{5p})^I + \Bigg[\frac{\mathcal{C}^{p}_{HHS_6^\dagger}}{\sqrt{2}} \epsilon^{kj} (\tau^I)^i_k H_i H_j (S^\dagger_{6p})^I \\
& + \mathcal{C}^{pr}_{HS_1S_4^\dagger} H_i (S^\dagger_{4r})^i S_{1p} + \mathcal{C}^{rp}_{HS_2^\dagger S_4} \epsilon^{ji} H_j (S_{4r})_i S^\dagger_{2p} - \frac{\mathcal{C}^{pr}_{H^\dagger S_4 S_5}}{\sqrt{2}} (\tau^I)^i_j H^{\dagger j} (S_{4p})_i (S_{5r})^I \\
& + \frac{\mathcal{C}^{pr}_{HS_4S_6^\dagger}}{\sqrt{2}} \epsilon^{ki} (\tau^I)^j_k H_j (S_{4p})_i (S^\dagger_{6r})^I + \frac{\mathcal{C}^{pr}_{HS_5S_7^\dagger}}{\sqrt{2}} \epsilon_{jl} C^{ikl}_{\mathfrak{i}} (\tau^I)^j_k H_i (S_{5p})^I (S^\dagger_{7r})^{\mathfrak{i}} \\
& + \frac{\mathcal{C}^{rp}_{HS_6^\dagger S_7}}{\sqrt{2}} \epsilon^{im} \epsilon^{jl} C^{\mathfrak{i}}_{klm} (\tau^I)^k_j H_i (S^\dagger_{6p})^I (S_{7r})^{\mathfrak{i}} + \frac{\mathcal{C}^{pr}_{HS_6S_8^\dagger}}{\sqrt{2}} \epsilon_{jl} C^{ikl}_{\mathfrak{i}} (\tau^I)^j_k H_i (S_{6p})^I (S^\dagger_{8r})^{\mathfrak{i}} \\
& + 2\mathcal{C}^{pr}_{HS_{10}S_{12}^\dagger} H_i (S_{10p})_a (S^\dagger_{12r})^{ai} + 2\mathcal{C}^{rp}_{HS_{11}^\dagger S_{12}} \epsilon^{ji} H_j (S^\dagger_{11p})^a (S_{12r})_{ai} \\
& + 2\mathcal{C}^{pr}_{HS_{11}S_{13}^\dagger} H_i (S_{11p})_a (S^\dagger_{13r})^{ai} - \sqrt{2}\mathcal{C}^{rp}_{HS_{12}^\dagger S_{14}} (\tau^I)^j_i H_j (S^\dagger_{12p})^{ai} (S_{14r})^I_a + \mathrm{H.c.} \Bigg] \\
& + \mathcal{C}^{prs}_{S_1S_1S_1} S_{1p} S_{1r} S_{1s} - \mathcal{C}^{prs}_{S_1S_5S_5} (S_{5r})^I (S_{5s})^I S_{1p} + \mathcal{C}^{prs}_{S_1S_6S_6^\dagger} (S_{6r})^I (S^\dagger_{6s})^I S_{1p} \\
& + \Big[-\mathcal{C}^{spr}_{S_2^\dagger S_5 S_6} (S_{5r})^I (S_{6s})^I S^\dagger_{2p} - \mathcal{C}^{prs}_{S_3 S_6^\dagger S_6^\dagger} (S^\dagger_{6r})^I (S^\dagger_{6s})^I S_{3p} + \mathrm{H.c.} \Big] \\
& - \frac{i\mathcal{C}^{prs}_{S_5S_6S_6^\dagger}}{\sqrt{2}} \epsilon^{IJK} (S_{5p})^I (S_{6r})^J (S^\dagger_{6s})^K
\end{aligned} \tag{A10}$$

$$\begin{aligned}
\Delta\mathcal{L}^{(d=4)}_{\mathrm{UV,S}} = & \Big[-\mathcal{D}^{rsp}_{LLS_2}\epsilon^{ij}[(l_r)_i C(l_s)_j]S_{2p} - \mathcal{D}^{rsp}_{e^\dagger e^\dagger S_3^\dagger}[\bar{e}_r C\bar{e}_s]S^\dagger_{3p} - 2\mathcal{D}^{rsp}_{d^\dagger Q S_4^\dagger}[(\bar{d}_r)^a(q_s)_{ai}](S^\dagger_{4p})^i \\
& - \mathcal{D}^{rsp}_{e^\dagger L S_4^\dagger}[\bar{e}_r(l_s)_i](S^\dagger_{4p})^i - 2\mathcal{D}^{rps}_{QS_4u^\dagger}\epsilon^{ji}[(\bar{u}_s)^a(q_r)_{aj}](S_{4p})_i \\
& - \frac{\mathcal{D}^{rsp}_{LLS_6}}{\sqrt{2}}\epsilon^{jk}(\tau^I)^i_k[(l_r)_i C(l_s)_j](S_{6p})^I - 2\mathcal{D}^{rsp}_{d^\dagger e^\dagger S_9}[(\bar{d}_r)^a C\bar{e}_s](S_{9p})_a \\
& - 4\mathcal{D}^{prs}_{S_9^\dagger u^\dagger u^\dagger}\epsilon^{abc}[(\bar{u}_r)^b C(\bar{u}_s)^c](S^\dagger_{9p})^a - 4\mathcal{D}^{rps}_{d^\dagger S_{10}^\dagger u^\dagger}\epsilon^{bac}[(\bar{d}_r)^b C(\bar{u}_s)^c](S^\dagger_{10p})^a \\
& - 2\mathcal{D}^{rps}_{e^\dagger S_{10}u^\dagger}[\bar{e}_r C(\bar{u}_s)^a](S_{10p})_a - 2\mathcal{D}^{rsp}_{LQS_{10}^\dagger}\epsilon^{ij}[(l_r)_i C(q_s)_{aj}](S^\dagger_{10p})^a \\
& - \mathcal{D}^{rsp}_{QQS_{10}}\epsilon^{bca}\epsilon^{ij}[(q_r)_{bi}C(q_s)_{cj}](S_{10p})_a - 4\mathcal{D}^{rsp}_{d^\dagger d^\dagger S_{11}^\dagger}\epsilon^{bca}[(\bar{d}_r)^b C(\bar{d}_s)^c](S^\dagger_{11p})^a \\
& - 2\mathcal{D}^{rsp}_{d^\dagger L S_{12}}\epsilon^{ji}[(\bar{d}_r)^a(l_s)_j](S_{12p})_{ai} - 2\mathcal{D}^{rsp}_{e^\dagger Q S_{13}^\dagger}[\bar{e}_r(q_s)_{ai}](S^\dagger_{13p})^{ai} \\
& - 2\mathcal{D}^{rps}_{LS_{13}u^\dagger}\epsilon^{ji}[(\bar{u}_s)^a(l_r)_j](S_{13p})_{ai} - \sqrt{2}\mathcal{D}^{rsp}_{LQS_{14}^\dagger}\epsilon^{kj}(\tau^I)^i_k[(l_r)_i C(q_s)_{aj}](S^\dagger_{14p})^{aI} \\
& - \frac{\mathcal{D}^{rsp}_{QQS_{14}}}{\sqrt{2}}\epsilon^{bca}\epsilon^{jk}(\tau^I)^i_k[(q_r)_{bi}C(q_s)_{cj}](S_{14p})^I_a - 4\mathcal{D}^{rsp}_{d^\dagger d^\dagger S_{15}}C^{\mathfrak{a}}_{ab}[(\bar{d}_r)^a C(\bar{d}_s)^b](S_{15p})^{\mathfrak{a}} \\
& - 4\mathcal{D}^{rps}_{d^\dagger S_{16}u^\dagger}C^{\mathfrak{a}}_{ab}[(\bar{d}_r)^a C(\bar{u}_s)^b](S_{16p})^{\mathfrak{a}} - 4\mathcal{D}^{rsp}_{QQS_{16}^\dagger}\epsilon^{ij}C^{ab}_{\mathfrak{a}}[(q_r)_{ai}C(q_s)_{bj}](S^\dagger_{16p})^{\mathfrak{a}} \\
& - 4\mathcal{D}^{prs}_{S_{17}u^\dagger u^\dagger}C^{\mathfrak{a}}_{ab}[(\bar{u}_r)^a C(\bar{u}_s)^b](S_{17p})^{\mathfrak{a}} \\
& - 2\sqrt{2}\mathcal{D}^{rsp}_{QQS_{18}^\dagger}\epsilon^{kj}C^{ab}_{\mathfrak{a}}(\tau^I)^i_k[(q_r)_{ai}C(q_s)_{bj}](S^\dagger_{18p})^{\mathfrak{a}I} \\
& - 2\sqrt{2}\mathcal{D}^{rsp}_{d^\dagger Q S_{19}^\dagger}(\lambda^A)^b_a[(\bar{d}_r)^a(q_s)_{bi}](S^\dagger_{19p})^{Ai} \\
& - \sqrt{2}\mathcal{D}^{rps}_{QS_{19}u^\dagger}\epsilon^{ji}(\lambda^A)^a_b[(\bar{u}_s)^b(q_r)_{aj}](S_{19p})^A_i - \mathcal{D}^p_{HH^\dagger H^\dagger S_4}\delta^i_k\delta^j_l H_j H^{\dagger k}H^{\dagger l}(S_{4p})_i \\
& + \mathcal{D}^p_{HHH^\dagger S_7^\dagger}\epsilon_{kl}C^{ijl}_{\mathfrak{i}}H_iH_jH^{\dagger k}(S^\dagger_{7p})^{\mathfrak{i}} + \mathcal{D}^p_{HHHS_8^\dagger}C^{ijk}_{\mathfrak{i}}H_iH_jH_k(S^\dagger_{8p})^{\mathfrak{i}} + \mathrm{H.c.}\Big] \\
& + \mathcal{D}^{pr}_{HH^\dagger S_1S_1}\delta^i_jH_iH^{\dagger j}S_{1p}S_{1r} - \frac{\mathcal{D}^{pr}_{HH^\dagger S_1S_5}}{\sqrt{2}}(\tau^I)^i_jH_iH^{\dagger j}(S_{5r})^I S_{1p} \\
& + \Bigg[\frac{\mathcal{D}^{pr}_{HHS_1S_6^\dagger}}{\sqrt{2}}\epsilon^{kj}(\tau^I)^i_kH_iH_j(S^\dagger_{6r})^I S_{1p} + \frac{\mathcal{D}^{rp}_{HHS_2^\dagger S_5}}{\sqrt{2}}\epsilon^{jk}(\tau^I)^i_kH_iH_j(S_{5r})^I S^\dagger_{2p} \\
& + \frac{\mathcal{D}^{pr}_{HH^\dagger S_2S_6^\dagger}}{\sqrt{2}}(\tau^I)^i_jH_iH^{\dagger j}(S^\dagger_{6r})^I S_{2p} + \frac{\mathcal{D}^{rp}_{HHS_3^\dagger S_6}}{\sqrt{2}}\epsilon^{jk}(\tau^I)^i_kH_iH_j(S_{6r})^I S^\dagger_{3p} + \mathrm{H.c.}\Bigg] \\
& - \frac{\mathcal{D}^{pr}_{HH^\dagger S_5S_5(1)}}{2}(\tau^I)^i_k(\tau^J)^k_jH_iH^{\dagger j}(S_{5p})^I(S_{5r})^J \\
& - \mathcal{D}^{pr}_{HH^\dagger S_5S_5(2)}\delta^{IJ}\delta^i_jH_iH^{\dagger j}(S_{5p})^I(S_{5r})^J \\
& + \Bigg[\frac{\mathcal{D}^{pr}_{HHS_5S_6^\dagger}}{2}\epsilon^{kl}(\tau^I)^i_l(\tau^J)^j_kH_iH_j(S_{5p})^I(S^\dagger_{6r})^J + \mathrm{H.c.}\Bigg] \\
& + \frac{\mathcal{D}^{pr}_{HH^\dagger S_6S_6^\dagger(1)}}{2}(\tau^I)^i_k(\tau^J)^k_jH_iH^{\dagger j}(S_{6p})^I(S^\dagger_{6r})^J \\
& + \mathcal{D}^{pr}_{HH^\dagger S_6S_6^\dagger(2)}\delta^{IJ}\delta^i_jH_iH^{\dagger j}(S_{6p})^I(S^\dagger_{6r})^J
\end{aligned} \tag{A11}$$

b. New fermions

$$
\begin{aligned}
\Delta\mathcal{L}^{(d=4)}_{\mathrm{UV,F}} = & -\mathcal{D}^{pr}_{F_1HL}\epsilon^{ji}[F_{1p}C(l_r)_i]H_j + \mathcal{D}^{pr}_{F_{2L}H^\dagger L}[F_{2p}C(l_r)_i]H^{\dagger i} \\
& - \mathcal{D}^{rp}_{e^\dagger F^\dagger_{3R}H^\dagger}\epsilon_{ij}[\bar{e}_rC(\bar{F}_{3p})^i]H^{\dagger j} + \mathcal{D}^{rp}_{e^\dagger F^\dagger_{4R}H}[\bar{e}_rC(\bar{F}_{4p})^i]H_i \\
& + \frac{\mathcal{D}^{pr}_{F_5HL}}{\sqrt{2}}\epsilon^{kj}(\tau^I)^i_k[(F_{5p})^IC(l_r)_i]H_j + \frac{\mathcal{D}^{pr}_{F_{6L}H^\dagger L}}{\sqrt{2}}(\tau^I)^i_j[(F_{6p})^IC(l_r)_i]H^{\dagger j} \\
& + 2\mathcal{D}^{pr}_{F^\dagger_{8R}H^\dagger Q}[(\bar{F}_{8p})^a(q_r)_{ai}]H^{\dagger i} - 2\mathcal{D}^{pr}_{F^\dagger_{9R}HQ}\epsilon^{ji}[(\bar{F}_{9p})^a(q_r)_{ai}]H_j \\
& - 2\mathcal{D}^{rp}_{d^\dagger F_{10L}H}\epsilon^{ij}[(\bar{d}_r)^a(F_{10p})_{ai}]H_j - 2\mathcal{D}^{rp}_{d^\dagger F_{11L}H^\dagger}[(\bar{d}_r)^a(F_{11p})_{ai}]H^{\dagger i} \\
& - 2\mathcal{D}^{pr}_{F_{11L}Hu^\dagger}\epsilon^{ij}[(\bar{u}_r)^a(F_{11p})_{ai}]H_j - 2\mathcal{D}^{pr}_{F_{12L}H^\dagger u^\dagger}[(\bar{u}_r)^a(F_{12p})_{ai}]H^{\dagger i} \\
& - \sqrt{2}\mathcal{D}^{pr}_{F^\dagger_{13R}H^\dagger Q}(\tau^I)^i_j[(\bar{F}_{13p})^{aI}(q_r)_{ai}]H^{\dagger j} \\
& - \sqrt{2}\mathcal{D}^{pr}_{F^\dagger_{14R}HQ}\epsilon^{kj}(\tau^I)^i_k[(\bar{F}_{14p})^{aI}(q_r)_{ai}]H_j - \mathcal{D}^{pr}_{F_1F_{3L}H^\dagger}\delta^i_j[F_{1p}C(F_{3r})_i]H^{\dagger j} \\
& - \mathcal{D}^{pr}_{F_1F^\dagger_{3R}H}\delta^j_i[(\bar{F}_{3r})^iF_{1p}]H_j - \mathcal{D}^{rp}_{F^\dagger_{2R}F_{3L}H}\epsilon^{ij}[\bar{F}_{2p}(F_{3r})_i]H_j \\
& - \mathcal{D}^{rp}_{F^\dagger_{2R}F_{4L}H^\dagger}\delta^i_j[\bar{F}_{2p}(F_{4r})_i]H^{\dagger j} - \frac{\mathcal{D}^{pr}_{F_{3L}F_5H^\dagger}}{\sqrt{2}}(\tau^I)^i_j[(F_{3p})_iC(F_{5r})^I]H^{\dagger j} \\
& - \frac{\mathcal{D}^{rp}_{F^\dagger_{3R}F_5H}}{\sqrt{2}}(\tau^I)^j_i[(\bar{F}_{3p})^i(F_{5r})^I]H_j - \frac{\mathcal{D}^{pr}_{F_{3L}F^\dagger_{6R}H}}{\sqrt{2}}\epsilon^{kj}(\tau^I)^i_k[(\bar{F}_{6r})^I(F_{3p})_i]H_j \\
& - \frac{\mathcal{D}^{pr}_{F_{4L}F^\dagger_{6R}H^\dagger}}{\sqrt{2}}(\tau^I)^i_j[(\bar{F}_{6r})^I(F_{4p})_i]H^{\dagger j} + \frac{\mathcal{D}^{pr}_{F_5F_{7L}H^\dagger}}{\sqrt{2}}\epsilon^{lk}C^{\mathfrak{i}}_{ijk}(\tau^I)^j_l[(F_{5p})^IC(F_{7r})^{\mathfrak{i}}]H^{\dagger i} \\
& - \frac{\mathcal{D}^{pr}_{F_5F^\dagger_{7R}H}}{\sqrt{2}}\epsilon_{jl}C^{kli}_{\mathfrak{i}}(\tau^I)^j_k[(\bar{F}_{7r})^{\mathfrak{i}}(F_{5p})^I]H_i \\
& - \frac{\mathcal{D}^{rp}_{F^\dagger_{6R}F_{7L}H}}{\sqrt{2}}\epsilon^{jl}\epsilon^{mi}C^{\mathfrak{i}}_{klm}(\tau^I)^k_j[(\bar{F}_{6p})^I(F_{7r})^{\mathfrak{i}}]H_i \\
& - 2\mathcal{D}^{rp}_{F^\dagger_{10R}F_{8L}H^\dagger}\epsilon_{ij}\delta^a_b[(\bar{F}_{10p})^{ai}(F_{8r})_b]H^{\dagger j} + 2\mathcal{D}^{rp}_{F^\dagger_{11R}F_{8L}H}\delta^a_b\delta^j_i[(\bar{F}_{11p})^{ai}(F_{8r})_b]H_j \\
& - 2\mathcal{D}^{rp}_{F^\dagger_{11R}F_{9L}H^\dagger}\epsilon_{ij}\delta^a_b[(\bar{F}_{11p})^{ai}(F_{9r})_b]H^{\dagger j} + 2\mathcal{D}^{rp}_{F^\dagger_{12R}F_{9L}H}\delta^a_b\delta^j_i[(\bar{F}_{12p})^{ai}(F_{9r})_b]H_j \\
& + \sqrt{2}\mathcal{D}^{rp}_{F^\dagger_{10R}F_{13L}H^\dagger}\epsilon_{kj}\delta^a_b(\tau^I)^k_i[(\bar{F}_{10p})^{ai}(F_{13r})^I_b]H^{\dagger j} \\
& - \sqrt{2}\mathcal{D}^{rp}_{F^\dagger_{11R}F_{13L}H}\delta^a_b(\tau^I)^j_i[(\bar{F}_{11p})^{ai}(F_{13r})^I_b]H_j \\
& + \sqrt{2}\mathcal{D}^{rp}_{F^\dagger_{11R}F_{14L}H^\dagger}\epsilon_{kj}\delta^a_b(\tau^I)^k_i[(\bar{F}_{11p})^{ai}(F_{14r})^I_b]H^{\dagger j} \\
& - \sqrt{2}\mathcal{D}^{rp}_{F^\dagger_{12R}F_{14L}H}\delta^a_b(\tau^I)^j_i[(\bar{F}_{12p})^{ai}(F_{14r})^I_b]H_j + \mathrm{H.c.}
\end{aligned}
\tag{A12}
$$

c. New vectors

$$
\begin{aligned}
\Delta\mathcal{L}^{(d=3)}_{\mathrm{UV,V}} = & -2\mathcal{C}^{pr}_{HV_2V^\dagger_3}H_i(V^\dagger_{3r})^{i\mu}V_{2p\mu} - 4\mathcal{C}^{rp}_{HV^\dagger_5V_8}\epsilon^{ji}H_j(V^\dagger_{5p})^a_\mu(V_{8r})^\mu_{ai} \\
& - 2\sqrt{2}\mathcal{C}^{pr}_{HV_8V^\dagger_9}\epsilon^{ki}(\tau^I)^j_kH_j(V_{8p})_{ai\mu}(V^\dagger_{9r})^{aI\mu} + \mathrm{H.c.}
\end{aligned}
\tag{A13}
$$

$$
\begin{aligned}
\Delta\mathcal{L}^{(d=4)}_{\mathrm{UV,V}} = & -2\mathcal{D}^{rsp}_{d^\dagger dV_1}[(\bar{d}_r)^a\gamma_\mu(d_s)_a]V^\mu_{1p} - \mathcal{D}^{rsp}_{e^\dagger eV_1}[\bar{e}_r\gamma_\mu e_s]V^\mu_{1p} \\
& + [-2\mathcal{D}^{p}_{HH^\dagger V_1D}[D_\mu H_i]H^{\dagger i}V^\mu_{1p} + \mathrm{H.c.}] + \mathcal{D}^{rsp}_{LL^\dagger V_1}[(\bar{l}_s)^i\gamma_\mu(l_r)_i]V^\mu_{1p} \\
& + 2\mathcal{D}^{rsp}_{QQ^\dagger V_1}[(\bar{q}_s)^{ai}\gamma_\mu(q_r)_{ai}]V^\mu_{1p} - 2\mathcal{D}^{rsp}_{u^\dagger uV_1}[(\bar{u}_r)^a\gamma_\mu(u_s)_a]V^\mu_{1p} \\
& + [-2\mathcal{D}^{rsp}_{d^\dagger uV_2^\dagger}[(\bar{d}_r)^a\gamma_\mu(u_s)_a]V^{\dagger\mu}_{2p} - 2\mathcal{D}^{p}_{HHV_2^\dagger D}\epsilon^{ij}[D_\mu H_i]H_jV^{\dagger\mu}_{2p} \\
& - \mathcal{D}^{rsp}_{e^\dagger L^\dagger V_3^\dagger}\epsilon_{ji}[\bar{e}_r\gamma_\mu C(\bar{l}_s)^j](V^\dagger_{3p})^{i\mu} + \mathrm{H.c.}] \\
& + [\sqrt{2}\mathcal{D}^{p}_{HH^\dagger V_4D}(\tau^I)^i_j[D_\mu H_i]H^{\dagger j}(V_{4p})^{I\mu} + \mathrm{H.c.}] \\
& - \frac{\mathcal{D}^{rsp}_{LL^\dagger V_4}}{\sqrt{2}}(\tau^I)^i_j[(\bar{l}_s)^j\gamma_\mu(l_r)_i](V_{4p})^{I\mu} - \sqrt{2}\mathcal{D}^{rsp}_{QQ^\dagger V_4}(\tau^I)^i_j[(\bar{q}_s)^{aj}\gamma_\mu(q_r)_{ai}](V_{4p})^{I\mu} \\
& + [-2\mathcal{D}^{rsp}_{d^\dagger eV_5}[(\bar{d}_r)^a\gamma_\mu e_s](V_{5p})^\mu_a + 2\mathcal{D}^{srp}_{L^\dagger QV_5^\dagger}[(\bar{l}_s)^i\gamma_\mu(q_r)_{ai}](V^\dagger_{5p})^{a\mu} \\
& + 2\mathcal{D}^{srp}_{eu^\dagger V_6}[(\bar{u}_r)^a\gamma_\mu e_s](V_{6p})^\mu_a + 2\mathcal{D}^{rsp}_{d^\dagger L^\dagger V_7}[(\bar{d}_r)^a\gamma_\mu C(\bar{l}_s)^i](V_{7p})^\mu_{ai} \\
& + 2\mathcal{D}^{srp}_{eQV_7^\dagger}[(q_r)_{ai}C\gamma_\mu e_s](V^\dagger_{7p})^{ai\mu} + \mathcal{D}^{rsp}_{QuV_7}\epsilon^{bca}\epsilon^{ij}[(q_r)_{bj}C\gamma_\mu(u_s)_c](V_{7p})^\mu_{ai} \\
& - \mathcal{D}^{srp}_{dQV_8}\epsilon^{cba}\epsilon^{ij}[(q_r)_{bj}C\gamma_\mu(d_s)_c](V_{8p})^\mu_{ai} - 2\mathcal{D}^{srp}_{L^\dagger u^\dagger V_8}[(\bar{u}_r)^a\gamma_\mu C(\bar{l}_s)^i](V_{8p})^\mu_{ai} \\
& - \sqrt{2}\mathcal{D}^{srp}_{L^\dagger QV_9^\dagger}(\tau^I)^i_j[(\bar{l}_s)^j\gamma_\mu(q_r)_{ai}](V^\dagger_{9p})^{aI\mu} + 4\mathcal{D}^{srp}_{dQV_{10}^\dagger}C^{ba}_{\mathfrak{a}}[(q_r)_{ai}C\gamma_\mu(d_s)_b](V^\dagger_{10p})^{\mathfrak{a}i\mu} \\
& - 4\mathcal{D}^{rsp}_{QuV_{11}^\dagger}C^{ab}_{\mathfrak{a}}[(q_r)_{ai}C\gamma_\mu(u_s)_b](V^\dagger_{11p})^{\mathfrak{a}i\mu} + \mathrm{H.c.}] \\
& - 2\sqrt{2}\mathcal{D}^{rsp}_{d^\dagger dV_{12}}(\lambda^A)^b_a[(\bar{d}_r)^a\gamma_\mu(d_s)_b](V_{12p})^{A\mu} \\
& + \sqrt{2}\mathcal{D}^{rsp}_{QQ^\dagger V_{12}}(\lambda^A)^a_b[(\bar{q}_s)^{bi}\gamma_\mu(q_r)_{ai}](V_{12p})^{A\mu} \\
& - 2\sqrt{2}\mathcal{D}^{rsp}_{u^\dagger uV_{12}}(\lambda^A)^b_a[(\bar{u}_r)^a\gamma_\mu(u_s)_b](V_{12p})^{A\mu} \\
& + [-2\sqrt{2}\mathcal{D}^{rsp}_{d^\dagger uV_{13}^\dagger}(\lambda^A)^b_a[(\bar{d}_r)^a\gamma_\mu(u_s)_b](V^\dagger_{13p})^{A\mu} + \mathrm{H.c.}] \\
& - \mathcal{D}^{rsp}_{QQ^\dagger V_{14}}(\lambda^A)^a_b(\tau^I)^i_j[(\bar{q}_s)^{bj}\gamma_\mu(q_r)_{ai}](V_{14p})^{AI\mu}
\end{aligned}
\tag{A14}
$$

d. Mixed terms

$$
\begin{aligned}
\Delta\mathcal{L}^{(d=4)}_{\mathrm{UV,SF}} = \Bigg[& -\mathcal{D}^{spr}_{e^\dagger F^\dagger_{2R}S_1}[\bar{e}_sF_{2p}]S_{1r} - \mathcal{D}^{psr}_{F_{3L}LS_1}\epsilon^{ij}[(F_{3p})_iC(l_s)_j]S_{1r} \\
& - 2\mathcal{D}^{spr}_{d^\dagger F_{8L}S_1}\delta^a_b[(\bar{d}_s)^b(F_{8p})_a]S_{1r} - 2\mathcal{D}^{prs}_{F_{9L}S_1u^\dagger}\delta^a_b[(\bar{u}_s)^b(F_{9p})_a]S_{1r} \\
& + 2\mathcal{D}^{psr}_{F^\dagger_{11R}QS_1}\delta^b_a\delta^j_i[(F_{11p})^{ai}C(q_s)_{bj}]S_{1r} - \mathcal{D}^{spr}_{e^\dagger F_1S_2^\dagger}[\bar{e}_sF_{1p}]S^\dagger_{2r} \\
& - \mathcal{D}^{psr}_{F_{4L}LS_2^\dagger}\epsilon^{ij}[(F_{4p})_iC(l_s)_j]S^\dagger_{2r} - 2\mathcal{D}^{prs}_{F_{8L}S_2u^\dagger}\delta^a_b[(\bar{u}_s)^b(F_{8p})_a]S_{2r} \\
& - 2\mathcal{D}^{spr}_{d^\dagger F_{9L}S_2^\dagger}\delta^a_b[(\bar{d}_s)^b(F_{9p})_a]S^\dagger_{2r} + 2\mathcal{D}^{psr}_{F^\dagger_{10R}QS_2^\dagger}\delta^b_a\delta^j_i[(F_{10p})^{ai}C(q_s)_{bj}]S^\dagger_{2r} \\
& + 2\mathcal{D}^{psr}_{F^\dagger_{12R}QS_2}\delta^b_a\delta^j_i[(F_{12p})^{ai}C(q_s)_{bj}]S_{2r} - \mathcal{D}^{psr}_{F_{1L}S_4}\epsilon^{ji}[F_{1p}C(l_s)_j](S_{4r})_i \\
& + \frac{\mathcal{D}^{psr}_{F_{5L}LS_4}}{\sqrt{2}}\epsilon^{kj}(\tau^I)^i_k[(F_{5p})^IC(l_s)_j](S_{4r})_i - \frac{\mathcal{D}^{psr}_{F_{3L}LS_5}}{\sqrt{2}}\epsilon^{jk}(\tau^I)^i_k[(F_{3p})_iC(l_s)_j](S_{5r})^I
\end{aligned}
$$

$$
\begin{aligned}
&-\mathcal{D}^{spr}_{e^\dagger F^\dagger_{6R}S_5}\delta^{IJ}[\bar{e}_s(F_{6p})^I](S_{5r})^J-\frac{\mathcal{D}^{psr}_{F_{7L}LS_5}}{\sqrt{2}}\epsilon^{li}\epsilon^{mj}C^{\mathfrak{i}}_{klm}(\tau^I)^k_j[(F_{7p})^{\mathfrak{i}}C(l_s)_i](S_{5r})^I\\
&+\sqrt{2}\mathcal{D}^{psr}_{F^\dagger_{11R}QS_5}\delta^b_a(\tau^I)^j_i[(F_{11p})^{ai}C(q_s)_{bj}](S_{5r})^I\\
&+2\mathcal{D}^{spr}_{d^\dagger F_{13L}S_5}\delta^{IJ}\delta^a_b[(\bar{d}_s)^b(F_{13p})^I_a](S_{5r})^J+2\mathcal{D}^{prs}_{F_{14L}S_5u^\dagger}\delta^{IJ}\delta^a_b[(\bar{u}_s)^b(F_{14p})^I_a](S_{5r})^J\\
&+\frac{\mathcal{D}^{psr}_{F^\dagger_{3R}LS_6}}{\sqrt{2}}(\tau^I)^j_i[(F_{3p})^iC(l_s)_j](S_{6r})^I-\frac{\mathcal{D}^{psr}_{F_{4L}LS^\dagger_6}}{\sqrt{2}}\epsilon^{kj}(\tau^I)^i_k[(F_{4p})_iC(l_s)_j](S^\dagger_{6r})^I\\
&-\mathcal{D}^{spr}_{e^\dagger F_5S^\dagger_6}\delta^{IJ}[\bar{e}_s(F_{5p})^I](S^\dagger_{6r})^J+\frac{\mathcal{D}^{psr}_{F^\dagger_{7R}LS_6}}{\sqrt{2}}\epsilon_{jl}C^{ikl}_{\mathfrak{i}}(\tau^I)^j_k[(F_{7p})^{\mathfrak{i}}C(l_s)_i](S_{6r})^I\\
&-\sqrt{2}\mathcal{D}^{psr}_{F^\dagger_{10R}QS^\dagger_6}\delta^b_a(\tau^I)^j_i[(F_{10p})^{ai}C(q_s)_{bj}](S^\dagger_{6r})^I\\
&+\sqrt{2}\mathcal{D}^{psr}_{F^\dagger_{12R}QS_6}\delta^b_a(\tau^I)^j_i[(F_{12p})^{ai}C(q_s)_{bj}](S_{6r})^I\\
&+2\mathcal{D}^{prs}_{F_{13L}S_6u^\dagger}\delta^{IJ}\delta^a_b[(\bar{u}_s)^b(F_{13p})^I_a](S_{6r})^J-2\mathcal{D}^{spr}_{d^\dagger F_{14L}S^\dagger_6}\delta^{IJ}\delta^a_b[(\bar{d}_s)^b(F_{14p})^I_a](S^\dagger_{6r})^J\\
&+\frac{\mathcal{D}^{psr}_{F_{5L}S_7}}{\sqrt{2}}\epsilon^{il}\epsilon^{mk}C^{\mathfrak{i}}_{jkl}(\tau^I)^j_m[(F_{5p})^IC(l_s)_i](S_{7r})^{\mathfrak{i}}\\
&-\frac{\mathcal{D}^{psr}_{F^\dagger_{6R}LS_8}}{\sqrt{2}}\epsilon^{im}\epsilon^{jl}C^{\mathfrak{i}}_{klm}(\tau^I)^k_j[(F_{6p})^IC(l_s)_i](S_{8r})^{\mathfrak{i}}-2\mathcal{D}^{spr}_{d^\dagger F_1S_{10}}\delta^a_b[(\bar{d}_s)^bF_{1p}](S_{10r})_a\\
&+2\mathcal{D}^{psr}_{F^\dagger_{10R}LS_{10}}\delta^b_a\delta^j_i[(F_{10p})^{ai}C(l_s)_j](S_{10r})_b-2\mathcal{D}^{prs}_{F_1S_{11}u^\dagger}\delta^a_b[(\bar{u}_s)^bF_{1p}](S_{11r})_a\\
&-2\mathcal{D}^{spr}_{d^\dagger F^\dagger_{2R}S_{11}}\delta^a_b[(\bar{d}_s)^bF_{2p}](S_{11r})_a-2\mathcal{D}^{psr}_{F_{3L}QS^\dagger_{11}}\epsilon^{ij}\delta^b_a[(F_{3p})_iC(q_s)_{bj}](S^\dagger_{11r})^a\\
&-2\mathcal{D}^{spr}_{e^\dagger F_{8L}S^\dagger_{11}}\delta^a_b[\bar{e}_s(F_{8p})_a](S^\dagger_{11r})^b+2\mathcal{D}^{psr}_{F^\dagger_{11R}LS_{11}}\delta^b_a\delta^j_i[(F_{11p})^{ai}C(l_s)_j](S_{11r})_b\\
&-2\mathcal{D}^{psr}_{F_1QS^\dagger_{12}}\delta^b_a\delta^j_i[F_{1p}C(q_s)_{bj}](S^\dagger_{12r})^{ai}-2\mathcal{D}^{prs}_{F_{3L}S_{12}u^\dagger}\epsilon^{ij}\delta^a_b[(\bar{u}_s)^b(F_{3p})_i](S_{12r})_{aj}\\
&+\sqrt{2}\mathcal{D}^{psr}_{F_5QS^\dagger_{12}}\delta^b_a(\tau^I)^j_i[(F_{5p})^IC(q_s)_{bj}](S^\dagger_{12r})^{ai}\\
&-\mathcal{D}^{psr}_{F_{8L}QS_{12}}\epsilon^{acb}\epsilon^{ji}[(F_{8p})_aC(q_s)_{cj}](S_{12r})_{bi}-2\mathcal{D}^{psr}_{F_{9L}LS^\dagger_{12}}\delta^a_b\delta^j_i[(F_{9p})_aC(l_s)_j](S^\dagger_{12r})^{bi}\\
&-4\mathcal{D}^{prs}_{F^\dagger_{10R}S^\dagger_{12}u^\dagger}\epsilon_{ij}\epsilon^{abc}[(\bar{u}_s)^c(F_{10p})^{ai}](S^\dagger_{12r})^{bj}\\
&-4\mathcal{D}^{spr}_{d^\dagger F^\dagger_{11R}S^\dagger_{12}}\epsilon_{ij}\epsilon^{cab}[(\bar{d}_s)^c(F_{11p})^{ai}](S^\dagger_{12r})^{bj}\\
&+2\mathcal{D}^{spr}_{e^\dagger F^\dagger_{12R}S_{12}}\delta^a_b\delta^j_i[\bar{e}_s(F_{12p})^{ai}](S_{12r})_{bj}\\
&-\frac{\mathcal{D}^{psr}_{F_{13L}QS_{12}}}{\sqrt{2}}\epsilon^{acb}\epsilon^{ik}(\tau^I)^j_k[(F_{13p})^I_aC(q_s)_{cj}](S_{12r})_{bi}\\
&+\sqrt{2}\mathcal{D}^{psr}_{F_{14L}LS^\dagger_{12}}\delta^a_b(\tau^I)^j_i[(F_{14p})^I_aC(l_s)_j](S^\dagger_{12r})^{bi}\\
&-4\mathcal{D}^{spr}_{d^\dagger F^\dagger_{10R}S^\dagger_{13}}\epsilon_{ij}\epsilon^{cab}[(\bar{d}_s)^c(F_{10p})^{ai}](S^\dagger_{13r})^{bj}\\
&+2\mathcal{D}^{spr}_{d^\dagger F_5S_{14}}\delta^{IJ}\delta^a_b[(\bar{d}_s)^b(F_{5p})^I](S_{14r})^J_a\\
&+\sqrt{2}\mathcal{D}^{psr}_{F^\dagger_{10R}LS_{14}}\delta^b_a(\tau^I)^j_i[(F_{10p})^{ai}C(l_s)_j](S_{14r})^I_b+\mathrm{H.c.}\Big]-\mathcal{D}^{prs}_{F_1F_1S_1}[F_{1p}CF_{1r}]S_{1s}\\
&+\mathcal{D}^{prs}_{F_5F_5S_1}\delta^{IJ}[(F_{5p})^IC(F_{5r})^J]S_{1s}+\mathcal{D}^{prs}_{F_1F_5S_5}\delta^{IJ}[F_{1p}C(F_{5r})^I](S_{5s})^J
\end{aligned}
$$

$$
-\frac{i\mathcal{D}^{prs}_{F_5F_5S_5}}{\sqrt{2}}\epsilon^{IJK}[(F_{5p})^I C(F_{5r})^J](S_{5s})^K + \Big[-\mathcal{D}^{prs}_{F_1F^\dagger_{6R}S_6}\delta^{IJ}[F_{1p}C(F_{6r})^I](S_{6s})^J + \mathcal{D}^{rps}_{F^\dagger_{2R}F_5S_6}\delta^{IJ}[F_{2p}C(F_{5r})^I](S_{6s})^J - \frac{\mathcal{D}^{prs}_{F_{3L}F_{3L}S^\dagger_6}}{\sqrt{2}}\epsilon^{kj}(\tau^I)^i_k[(F_{3p})_i C(F_{3r})_j](S^\dagger_{6s})^I + \frac{i\mathcal{D}^{prs}_{F_5F^\dagger_{6R}S_6}}{\sqrt{2}}\epsilon^{IJK}[(F_{5p})^I C(F_{6r})^J](S_{6s})^K + \text{H.c.}\Big] \tag{A15}
$$

$$
\Delta\mathcal{L}^{(d=4)}_{\text{UV,SV}} = -\sqrt{2}\mathcal{D}^{pr}_{HS_6V^\dagger_3D}(\tau^I)^j_i[D_\mu H_j](S_{6p})^I(V^\dagger_{3r})^{i\mu} + \text{H.c.} \tag{A16}
$$

$$
\begin{aligned}
\Delta\mathcal{L}^{(d=4)}_{\text{UV,FV}} = {}& \mathcal{D}^{spr}_{eF_1V_2}[F_{1p}C\gamma_\mu e_s]V^\mu_{2r} - 2\mathcal{D}^{psr}_{F^\dagger_1u^\dagger V_5}\delta^a_b[(\bar{u}_s)^b\gamma_\mu C\bar{F}_{1p}](V_{5r})^\mu_a \\
&+ 2\mathcal{D}^{psr}_{F^\dagger_1QV^\dagger_8}\delta^b_a\delta^j_i[\bar{F}_{1p}\gamma_\mu(q_s)_{bj}](V^\dagger_{8r})^{ai\mu} + \mathcal{D}^{psr}_{F_{3L}L^\dagger V^\dagger_2}\delta^i_j[(\bar{l}_s)^j\gamma_\mu(F_{3p})_i]V^{\dagger\mu}_{2r} \\
&- 2\mathcal{D}^{spr}_{d^\dagger F^\dagger_{3L}V_8}\delta^a_b\delta^j_i[(\bar{d}_s)^b\gamma_\mu C(\bar{F}_{3p})^i](V_{8r})^\mu_{aj} + \sqrt{2}\mathcal{D}^{psr}_{F^\dagger_5QV^\dagger_8}\delta^b_a(\tau^I)^j_i[(\bar{F}_{5p})^I\gamma_\mu(q_s)_{bj}](V^\dagger_{8r})^{ai\mu} \\
&- 2\mathcal{D}^{psr}_{F^\dagger_5u^\dagger V_9}\delta^{IJ}\delta^a_b[(\bar{u}_s)^b\gamma_\mu C(\bar{F}_{5p})^I](V_{9r})^{J\mu}_a \\
&+ 4\mathcal{D}^{spr}_{d^\dagger F^\dagger_{8L}V^\dagger_5}\epsilon^{cab}[(\bar{d}_s)^c\gamma_\mu C(\bar{F}_{8p})^a](V^\dagger_{5r})^{b\mu} + 2\mathcal{D}^{psr}_{F_{8L}L^\dagger V^\dagger_8}\epsilon_{ji}\delta^a_b[(\bar{l}_s)^j\gamma_\mu(F_{8p})_a](V^\dagger_{8r})^{bi\mu} \\
&- 2\mathcal{D}^{psr}_{F^\dagger_{10R}uV^\dagger_3}\epsilon_{ij}\delta^b_a[(F_{10p})^{ai}C\gamma_\mu(u_s)_b](V^\dagger_{3r})^{j\mu} - 4\mathcal{D}^{psr}_{F^\dagger_{10R}Q^\dagger V^\dagger_5}\epsilon_{ij}\epsilon^{abc}[(\bar{q}_s)^{cj}\gamma_\mu(F_{10p})^{ai}](V^\dagger_{5r})^{b\mu} \\
&- 2\mathcal{D}^{spr}_{eF^\dagger_{10R}V_8}\delta^b_a\delta^j_i[(F_{10p})^{ai}C\gamma_\mu e_s](V_{8r})^\mu_{bj} + 2\sqrt{2}\mathcal{D}^{psr}_{F^\dagger_{10R}Q^\dagger V^\dagger_9}\epsilon_{kj}\epsilon^{abc}(\tau^I)^k_i[(\bar{q}_s)^{cj}\gamma_\mu(F_{10p})^{ai}](V^\dagger_{9r})^{bI\mu} \\
&- 2\mathcal{D}^{spr}_{dF^\dagger_{12R}V_3}\delta^b_a\delta^j_i[(F_{12p})^{ai}C\gamma_\mu(d_s)_b](V_{3r})^\mu_j + 2\mathcal{D}^{psr}_{F^\dagger_{12R}L^\dagger V_5}\epsilon_{ij}\delta^b_a[(\bar{l}_s)^j\gamma_\mu(F_{12p})^{ai}](V_{5r})^\mu_b \\
&+ \sqrt{2}\mathcal{D}^{psr}_{F^\dagger_{12R}L^\dagger V_9}\epsilon_{kj}\delta^b_a(\tau^I)^k_i[(\bar{l}_s)^j\gamma_\mu(F_{12p})^{ai}](V_{9r})^{I\mu}_b + \sqrt{2}\mathcal{D}^{psr}_{F_{13L}L^\dagger V^\dagger_8}\epsilon_{kj}\delta^a_b(\tau^I)^k_i[(\bar{l}_s)^j\gamma_\mu(F_{13p})^I_a](V^\dagger_{8r})^{bi\mu} \\
&+ \mathcal{D}^{spr}_{dF_{13L}V_9}\epsilon^{cba}\delta^{IJ}[(F_{13p})^I_a C\gamma_\mu(d_s)_c](V_{9r})^{J\mu}_b + \text{H.c.}
\end{aligned} \tag{A17}
$$

APPENDIX B: THE COMPLETE WILSON COEFFICIENTS OF MATCHING RESULT

In this appendix, we provide the complete expression of Wilson coefficients. Some descriptions are given as follows:

(i) y, μ_H, λ_H are SM Yukawa, Higgs quadratic, quartic couplings as in Sec. III. $\mathcal{C}$ and $\mathcal{D}$ are new couplings with mass dimension 1 and 0 as described in Appendix A.

(ii) Flavor indices of every effective operator and Wilson coefficients are marked by $f_1, f_2, \ldots$ in the order of the helicities and the letters of the composing fields. For example, C_{dLueH} is the Wilson coefficients of $\mathcal{O}^{f_1f_2f_5f_4}_{dLueH} = \epsilon^{ij}(\bar{d}^a_{f_1}\ell_{if_2})(u^T_{af_5}Ce_{f_4})H_j$, since the helicities of the fields $\bar{d}, \ell, u, e, H$ are $-1/2, -1/2, 1/2, 1/2, 0$, respectively.

(iii) C_5 denotes the coefficient of the Weinberg operator. Since it depends on the model, we just leave in the expressions.

Detailed description of usage can be found in Sec. V.

1. Dimension-5

$$
C_5 = -\frac{\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_1f_2}_{F_1HL}\mu^2_H}{2M^3_{F_1}} - \frac{\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_2}_{F_5HL}\mu^2_H}{4M^3_{F_5}} - \frac{\mathcal{D}^{f_1f_2p_1}_{LLS_6}\mathcal{C}^{p_1}_{HHS^\dagger_6}}{M^2_{S_6}} + \frac{\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_1f_2}_{F_1HL}}{2M_{F_1}} + \frac{\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_2}_{F_5HL}}{4M_{F_5}} \tag{B1}
$$

2. Dimension-6

a. Bosonic operators

$$
\begin{aligned}
C_H ={}& \frac{2\lambda_H(\mathcal{C}^{p_1}_{HH^\dagger S_5})^2}{M^4_{S_5}} + \frac{\mathcal{D}^{p_1*}_{HHHS_8^\dagger}\mathcal{D}^{p_1}_{HHHS_8^\dagger}}{M^2_{S_8}} + \frac{\mathcal{D}^{p_1*}_{HHH^\dagger S_7^\dagger}\mathcal{D}^{p_1}_{HHH^\dagger S_7^\dagger}}{3M^2_{S_7}} + \frac{4\lambda_H\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_1}_{HHS_6^\dagger}}{M^4_{S_6}} - \frac{8\lambda_H\mathcal{D}^{p_1*}_{HHV_2^\dagger D}\mathcal{D}^{p_1}_{HHV_2^\dagger D}}{M^2_{V_2}} \\
&+ \frac{\mathcal{D}^{p_1*}_{HH^\dagger H^\dagger S_4}\mathcal{D}^{p_1}_{HH^\dagger H^\dagger S_4}}{M^2_{S_4}} + \frac{\mathcal{D}^{p_1p_2}_{HHS_1S_6^\dagger}\mathcal{C}^{p_2*}_{HHS_6^\dagger}\mathcal{C}^{p_1}_{HH^\dagger S_1}}{M^2_{S_1}M^2_{S_6}} + \frac{\mathcal{D}^{p_1p_2*}_{HHS_1S_6^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{C}^{p_1}_{HH^\dagger S_1}}{M^2_{S_1}M^2_{S_6}} + \frac{\mathcal{C}^{p_1}_{HH^\dagger S_1}\mathcal{C}^{p_2}_{HH^\dagger S_1}\mathcal{D}^{p_1p_2}_{HH^\dagger S_1S_1}}{M^4_{S_1}} \\
&- \frac{\mathcal{D}^{p_1p_2}_{HHS_5S_6^\dagger}\mathcal{C}^{p_2*}_{HHS_6^\dagger}\mathcal{C}^{p_1}_{HH^\dagger S_5}}{2M^2_{S_5}M^2_{S_6}} + \frac{\mathcal{D}^{p_1p_2*}_{HHS_5S_6^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{C}^{p_1}_{HH^\dagger S_5}}{2M^2_{S_5}M^2_{S_6}} + \frac{\mathcal{C}^{p_1}_{HH^\dagger S_1}\mathcal{D}^{p_1p_2}_{HH^\dagger S_1S_5}\mathcal{C}^{p_2}_{HH^\dagger S_5}}{2M^2_{S_1}M^2_{S_5}} - \frac{\mathcal{C}^{p_1}_{HH^\dagger S_5}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{D}^{p_2p_1}_{HH^\dagger S_5S_5(1)}}{4M^4_{S_5}} \\
&- \frac{\mathcal{C}^{p_1}_{HH^\dagger S_5}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{D}^{p_2p_1}_{HH^\dagger S_5S_5(2)}}{2M^4_{S_5}} + \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{D}^{p_2p_1}_{HH^\dagger S_6S_6^\dagger(2)}}{M^4_{S_6}} + \frac{8\lambda_H\mathcal{D}^{p_1*}_{HH^\dagger V_4D}\mathcal{D}^{p_1}_{HH^\dagger V_4D}}{M^2_{V_4}} - \frac{\mathcal{D}^{p_1*}_{HH^\dagger H^\dagger S_4}\mathcal{C}^{p_2}_{HH^\dagger S_1}\mathcal{C}^{p_2p_1*}_{HS_1S_4^\dagger}}{M^2_{S_1}M^2_{S_4}} \\
&+ \frac{\mathcal{C}^{p_1}_{HH^\dagger S_1}\mathcal{C}^{p_2}_{HH^\dagger S_1}\mathcal{C}^{p_2p_3*}_{HS_1S_4^\dagger}\mathcal{C}^{p_1p_3}_{HS_1S_4^\dagger}}{M^4_{S_1}M^2_{S_4}} - \frac{\mathcal{D}^{p_1}_{HH^\dagger H^\dagger S_4}\mathcal{C}^{p_2}_{HH^\dagger S_1}\mathcal{C}^{p_2p_1}_{HS_1S_4^\dagger}}{M^2_{S_1}M^2_{S_4}} - \frac{\mathcal{C}^{p_1}_{HHS_6^\dagger}\mathcal{D}^{p_2}_{HH^\dagger H^\dagger S_4}\mathcal{C}^{p_2p_1*}_{HS_4S_6^\dagger}}{M^2_{S_4}M^2_{S_6}} + \frac{\mathcal{C}^{p_1}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_1}\mathcal{C}^{p_2p_3*}_{HS_1S_4^\dagger}\mathcal{C}^{p_3p_1*}_{HS_4S_6^\dagger}}{M^2_{S_1}M^2_{S_4}M^2_{S_6}} \\
&- \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{D}^{p_2*}_{HH^\dagger H^\dagger S_4}\mathcal{C}^{p_2p_1}_{HS_4S_6^\dagger}}{M^2_{S_4}M^2_{S_6}} + \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_1}\mathcal{C}^{p_2p_3}_{HS_1S_4^\dagger}\mathcal{C}^{p_3p_1}_{HS_4S_6^\dagger}}{M^2_{S_1}M^2_{S_4}M^2_{S_6}} + \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{C}^{p_3p_2*}_{HS_4S_6^\dagger}\mathcal{C}^{p_3p_1}_{HS_4S_6^\dagger}}{M^2_{S_4}M^4_{S_6}} + \frac{\mathcal{D}^{p_1}_{HHH^\dagger S_7^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_2p_1*}_{HS_5S_7^\dagger}}{3M^2_{S_5}M^2_{S_7}} \\
&- \frac{2\mathcal{C}^{p_1}_{HH^\dagger S_5}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_2p_3*}_{HS_5S_7^\dagger}\mathcal{C}^{p_1p_3}_{HS_5S_7^\dagger}}{3M^4_{S_5}M^2_{S_7}} - \frac{\mathcal{D}^{p_1*}_{HHH^\dagger S_7^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_2p_1}_{HS_5S_7^\dagger}}{3M^2_{S_5}M^2_{S_7}} + \frac{\mathcal{C}^{p_1}_{HH^\dagger S_5}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_1p_3*}_{HS_5S_7^\dagger}\mathcal{C}^{p_2p_3}_{HS_5S_7^\dagger}}{3M^4_{S_5}M^2_{S_7}} + \frac{\mathcal{D}^{p_1}_{HHHS_8^\dagger}\mathcal{C}^{p_2*}_{HHS_6^\dagger}\mathcal{C}^{p_2p_1*}_{HS_6S_8^\dagger}}{M^2_{S_6}M^2_{S_8}} \\
&+ \frac{\mathcal{D}^{p_1*}_{HHHS_8^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{C}^{p_2p_1}_{HS_6S_8^\dagger}}{M^2_{S_6}M^2_{S_8}} + \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{C}^{p_1p_3*}_{HS_6S_8^\dagger}\mathcal{C}^{p_2p_3}_{HS_6S_8^\dagger}}{M^4_{S_6}M^2_{S_8}} + \frac{\mathcal{C}^{p_1}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_2p_3*}_{HS_5S_7^\dagger}\mathcal{C}^{p_1p_3*}_{HS_6^\dagger S_7}}{3M^2_{S_5}M^2_{S_6}M^2_{S_7}} + \frac{\mathcal{D}^{p_1*}_{HHH^\dagger S_7^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{C}^{p_2p_1*}_{HS_6^\dagger S_7}}{3M^2_{S_6}M^2_{S_7}} \\
&- \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_2p_3}_{HS_5S_7^\dagger}\mathcal{C}^{p_1p_3}_{HS_6^\dagger S_7}}{3M^2_{S_5}M^2_{S_6}M^2_{S_7}} + \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{C}^{p_2p_3*}_{HS_6^\dagger S_7}\mathcal{C}^{p_1p_3}_{HS_6^\dagger S_7}}{3M^4_{S_6}M^2_{S_7}} + \frac{\mathcal{D}^{p_1}_{HHH^\dagger S_7^\dagger}\mathcal{C}^{p_2*}_{HHS_6^\dagger}\mathcal{C}^{p_2p_1}_{HS_6^\dagger S_7}}{3M^2_{S_6}M^2_{S_7}} + \frac{\mathcal{D}^{p_1}_{HH^\dagger H^\dagger S_4}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_1p_2*}_{H^\dagger S_4S_5}}{2M^2_{S_4}M^2_{S_5}} \\
&- \frac{\mathcal{C}^{p_1}_{HH^\dagger S_1}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_1p_3*}_{HS_1S_4^\dagger}\mathcal{C}^{p_3p_2*}_{H^\dagger S_4S_5}}{2M^2_{S_1}M^2_{S_4}M^2_{S_5}} - \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_3p_1}_{HS_4S_6^\dagger}\mathcal{C}^{p_3p_2*}_{H^\dagger S_4S_5}}{2M^2_{S_4}M^2_{S_5}M^2_{S_6}} - \frac{\mathcal{D}^{p_1*}_{HH^\dagger H^\dagger S_4}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_1p_2}_{H^\dagger S_4S_5}}{2M^2_{S_4}M^2_{S_5}} - \frac{\mathcal{C}^{p_1}_{HH^\dagger S_5}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_3p_2*}_{H^\dagger S_4S_5}\mathcal{C}^{p_3p_1}_{H^\dagger S_4S_5}}{4M^2_{S_4}M^4_{S_5}} \\
&+ \frac{\mathcal{C}^{p_1}_{HH^\dagger S_1}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_1p_3}_{HS_1S_4^\dagger}\mathcal{C}^{p_3p_2}_{H^\dagger S_4S_5}}{2M^2_{S_1}M^2_{S_4}M^2_{S_5}} + \frac{\mathcal{C}^{p_1}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_3p_1*}_{HS_4S_6^\dagger}\mathcal{C}^{p_3p_2}_{H^\dagger S_4S_5}}{2M^2_{S_4}M^2_{S_5}M^2_{S_6}} + \frac{\mathcal{C}^{p_1}_{HH^\dagger S_1}\mathcal{C}^{p_2}_{HH^\dagger S_1}\mathcal{C}^{p_3}_{HH^\dagger S_1}\mathcal{C}^{p_1p_2p_3}_{S_1S_1S_1}}{M^6_{S_1}} \\
&- \frac{\mathcal{C}^{p_1}_{HH^\dagger S_1}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_3}_{HH^\dagger S_5}\mathcal{C}^{p_1p_3p_2}_{S_1S_5S_5}}{2M^2_{S_1}M^4_{S_5}} + \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{C}^{p_3}_{HH^\dagger S_1}\mathcal{C}^{p_3p_2p_1}_{S_1S_6S_6^\dagger}}{M^2_{S_1}M^4_{S_6}} + \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{C}^{p_3}_{HH^\dagger S_5}\mathcal{C}^{p_3p_2p_1}_{S_5S_6S_6^\dagger}}{2M^2_{S_5}M^4_{S_6}}
\end{aligned}
\tag{B2}
$$

$$
\begin{aligned}
C_{H\Box} ={}& -\frac{(\mathcal{C}^{p_1}_{HH^\dagger S_1})^2}{2M^4_{S_1}} + \frac{(\mathcal{C}^{p_1}_{HH^\dagger S_5})^2}{4M^4_{S_5}} + \frac{(\mathcal{D}^{p_1}_{HH^\dagger V_1D})^2}{M^2_{V_1}} + \frac{(\mathcal{D}^{p_1*}_{HH^\dagger V_1D})^2}{M^2_{V_1}} + \frac{(\mathcal{D}^{p_1}_{HH^\dagger V_4D})^2}{2M^2_{V_4}} \\
&+ \frac{(\mathcal{D}^{p_1*}_{HH^\dagger V_4D})^2}{2M^2_{V_4}} + \frac{\mathcal{C}^{p_1}_{HHS_6^\dagger}\mathcal{C}^{p_1*}_{HHS_6^\dagger}}{M^4_{S_6}} - \frac{2\mathcal{D}^{p_1}_{HHV_2^\dagger D}\mathcal{D}^{p_1*}_{HHV_2^\dagger D}}{M^2_{V_2}} + \frac{2\mathcal{D}^{p_1}_{HH^\dagger V_4D}\mathcal{D}^{p_1*}_{HH^\dagger V_4D}}{M^2_{V_4}}
\end{aligned}
\tag{B3}
$$

$$C_{HD} = -\frac{(\mathcal{C}^{p_1}_{HH^\dagger S_5})^2}{M^4_{S_5}} + \frac{2(\mathcal{D}^{p_1}_{HH^\dagger V_1 D})^2}{M^2_{V_1}} + \frac{2(\mathcal{D}^{p_1*}_{HH^\dagger V_1 D})^2}{M^2_{V_1}} + \frac{(\mathcal{D}^{p_1}_{HH^\dagger V_4 D})^2}{M^2_{V_4}} + \frac{(\mathcal{D}^{p_1*}_{HH^\dagger V_4 D})^2}{M^2_{V_4}} + \frac{2\mathcal{C}^{p_1}_{HHS_6^\dagger}\mathcal{C}^{p_1*}_{HHS_6^\dagger}}{M^4_{S_6}} + \frac{4\mathcal{D}^{p_1}_{HHV_2^\dagger D}\mathcal{D}^{p_1*}_{HHV_2^\dagger D}}{M^2_{V_2}} - \frac{4\mathcal{D}^{p_1}_{HH^\dagger V_1 D}\mathcal{D}^{p_1*}_{HH^\dagger V_1 D}}{M^2_{V_1}} - \frac{2\mathcal{D}^{p_1}_{HH^\dagger V_4 D}\mathcal{D}^{p_1*}_{HH^\dagger V_4 D}}{M^2_{V_4}} \tag{B4}$$

$$\begin{aligned} C_{eH} = {} & \frac{(\mathcal{C}^{p_1}_{HH^\dagger S_5})^2 y_e^{f_5 f_4}}{2M^4_{S_5}} - \frac{(\mathcal{D}^{p_1*}_{HH^\dagger V_1 D})^2 y_e^{f_5 f_4}}{M^2_{V_1}} + \frac{(\mathcal{D}^{p_1}_{HH^\dagger V_1 D})^2 y_e^{f_5 f_4}}{M^2_{V_1}} - \frac{(\mathcal{D}^{p_1*}_{HH^\dagger V_4 D})^2 y_e^{f_5 f_4}}{2M^2_{V_4}} \\
& + \frac{(\mathcal{D}^{p_1}_{HH^\dagger V_4 D})^2 y_e^{f_5 f_4}}{2M^2_{V_4}} + \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_1}_{HHS_6^\dagger} y_e^{f_5 f_4}}{M^4_{S_6}} - \frac{2\mathcal{D}^{p_1*}_{HHV_2^\dagger D}\mathcal{D}^{p_1}_{HHV_2^\dagger D} y_e^{f_5 f_4}}{M^2_{V_2}} \\
& + \frac{2\mathcal{D}^{p_1*}_{HH^\dagger V_4 D}\mathcal{D}^{p_1}_{HH^\dagger V_4 D} y_e^{f_5 f_4}}{M^2_{V_4}} + \frac{y_e^{f_5 p_1}\mathcal{D}^{f_4 p_2*}_{e^\dagger F^\dagger_{3R} H^\dagger}\mathcal{D}^{p_1 p_2}_{e^\dagger F^\dagger_{3R} H^\dagger}}{2M^2_{F_3}} + \frac{y_e^{f_5 p_1}\mathcal{D}^{f_4 p_2*}_{e^\dagger F^\dagger_{4R} H}\mathcal{D}^{p_1 p_2}_{e^\dagger F^\dagger_{4R} H}}{2M^2_{F_4}} \\
& + \frac{y_e^{p_1 f_4}\mathcal{D}^{p_2 f_5*}_{F_{2L} H^\dagger L}\mathcal{D}^{p_2 p_1}_{F_{2L} H^\dagger L}}{2M^2_{F_2}} + \frac{\mathcal{D}^{f_4 p_1*}_{e^\dagger F^\dagger_{3R} H^\dagger}\mathcal{D}^{p_2 f_5*}_{F_{2L} H^\dagger L}\mathcal{D}^{p_2 p_1*}_{F^\dagger_{2R} F_{3L} H}}{M_{F_2} M_{F_3}} \\
& - \frac{\mathcal{D}^{f_4 p_1*}_{e^\dagger F^\dagger_{4R} H}\mathcal{D}^{p_2 f_5*}_{F_{2L} H^\dagger L}\mathcal{D}^{p_2 p_1*}_{F^\dagger_{2R} F_{4L} H^\dagger}}{M_{F_2} M_{F_4}} + \frac{\mathcal{D}^{f_4 p_1*}_{e^\dagger F^\dagger_{3R} H^\dagger}\mathcal{D}^{p_1 p_2*}_{F_{3L} F_5 H^\dagger}\mathcal{D}^{p_2 f_5*}_{F_5 HL}}{M_{F_3} M_{F_5}} + \frac{y_e^{p_1 f_4}\mathcal{D}^{p_2 f_5*}_{F_5 HL}\mathcal{D}^{p_2 p_1}_{F_5 HL}}{2M^2_{F_5}} \\
& + \frac{\mathcal{D}^{f_4 p_1*}_{e^\dagger F^\dagger_{3R} H^\dagger}\mathcal{D}^{p_1 p_2*}_{F_{3L} F^\dagger_{6R} H}\mathcal{D}^{p_2 f_5*}_{F_{6L} H^\dagger L}}{2M_{F_3} M_{F_6}} + \frac{\mathcal{D}^{f_4 p_1*}_{e^\dagger F^\dagger_{4R} H}\mathcal{D}^{p_1 p_2*}_{F_{4L} F^\dagger_{6R} H^\dagger}\mathcal{D}^{p_2 f_5*}_{F_{6L} H^\dagger L}}{2M_{F_4} M_{F_6}} \\
& + \frac{y_e^{p_1 f_4}\mathcal{D}^{p_2 f_5*}_{F_{6L} H^\dagger L}\mathcal{D}^{p_2 p_1}_{F_{6L} H^\dagger L}}{4M^2_{F_6}} - \frac{\mathcal{D}^{f_4 p_1*}_{e^\dagger F^\dagger_{4R} H}\mathcal{D}^{p_1 f_5 p_2*}_{F_{4L} L S_6^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}}{M_{F_4} M^2_{S_6}} - \frac{\mathcal{D}^{f_4 p_1 p_2*}_{e^\dagger F_5 S_6^\dagger}\mathcal{D}^{p_1 f_5*}_{F_5 HL}\mathcal{C}^{p_2}_{HHS_6^\dagger}}{M_{F_5} M^2_{S_6}} \\
& + \frac{\mathcal{D}^{f_4 f_5 p_1*}_{e^\dagger L S_4^\dagger}\mathcal{D}^{p_1*}_{HH^\dagger H^\dagger S_4}}{M^2_{S_4}} - \frac{\mathcal{D}^{f_4 p_1 p_2*}_{e^\dagger F^\dagger_{2R} S_1}\mathcal{D}^{p_1 f_5*}_{F_{2L} H^\dagger L}\mathcal{C}^{p_2}_{HH^\dagger S_1}}{M_{F_2} M^2_{S_1}} + \frac{\mathcal{D}^{f_4 p_1*}_{e^\dagger F^\dagger_{3R} H^\dagger}\mathcal{D}^{p_1 f_5 p_2*}_{F_{3L} L S_1}\mathcal{C}^{p_2}_{HH^\dagger S_1}}{M_{F_3} M^2_{S_1}} \\
& + \frac{\mathcal{D}^{f_4 p_1*}_{e^\dagger F^\dagger_{3R} H^\dagger}\mathcal{D}^{p_1 f_5 p_2*}_{F_{3L} L S_5}\mathcal{C}^{p_2}_{HH^\dagger S_5}}{2M_{F_3} M^2_{S_5}} - \frac{\mathcal{D}^{f_4 p_1 p_2*}_{e^\dagger F^\dagger_{6R} S_5}\mathcal{D}^{p_1 f_5*}_{F_{6L} H^\dagger L}\mathcal{C}^{p_2}_{HH^\dagger S_5}}{2M_{F_6} M^2_{S_5}} + \frac{i y_e^{f_5 p_1}\mathcal{D}^{p_1 f_4 p_2}_{e^\dagger e V_1}\mathcal{D}^{p_2*}_{HH^\dagger V_1 D}}{M^2_{V_1}} \\
& + \frac{i y_e^{f_5 p_1}\mathcal{D}^{p_1 f_4 p_2}_{e^\dagger e V_1}\mathcal{D}^{p_2}_{HH^\dagger V_1 D}}{M^2_{V_1}} - \frac{\mathcal{D}^{f_4 f_5 p_1*}_{e^\dagger L S_4^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_1}\mathcal{C}^{p_2 p_1}_{HS_1 S_4^\dagger}}{M^2_{S_1} M^2_{S_4}} - \frac{\mathcal{D}^{f_4 f_5 p_1*}_{e^\dagger L S_4^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{C}^{p_1 p_2*}_{HS_4 S_6^\dagger}}{M^2_{S_4} M^2_{S_6}} \\
& + \frac{\mathcal{D}^{f_4 f_5 p_1*}_{e^\dagger L S_4^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_1 p_2*}_{H^\dagger S_4 S_5}}{2M^2_{S_4} M^2_{S_5}} + \frac{i y_e^{p_1 f_4}\mathcal{D}^{p_2*}_{HH^\dagger V_1 D}\mathcal{D}^{p_1 f_5 p_2}_{LL^\dagger V_1}}{M^2_{V_1}} + \frac{i y_e^{p_1 f_4}\mathcal{D}^{p_2}_{HH^\dagger V_1 D}\mathcal{D}^{p_1 f_5 p_2}_{LL^\dagger V_1}}{M^2_{V_1}} \\
& - \frac{i y_e^{p_1 f_4}\mathcal{D}^{p_2*}_{HH^\dagger V_4 D}\mathcal{D}^{p_1 f_5 p_2}_{LL^\dagger V_4}}{2M^2_{V_4}} + \frac{i y_e^{p_1 f_4}\mathcal{D}^{p_2}_{HH^\dagger V_4 D}\mathcal{D}^{p_1 f_5 p_2}_{LL^\dagger V_4}}{2M^2_{V_4}} \end{aligned} \tag{B5}$$

$$
\begin{aligned}
C_{uH} ={}& \frac{(\mathcal{C}^{p_1}_{HH^\dagger S_5})^2 y_u^{f_4 f_5}}{2M_{S_5}^4} + \frac{(\mathcal{D}^{p_1*}_{HH^\dagger V_1 D})^2 y_u^{f_4f_5}}{M_{V_1}^2} - \frac{(\mathcal{D}^{p_1}_{HH^\dagger V_1 D})^2 y_u^{f_4f_5}}{M_{V_1}^2} + \frac{(\mathcal{D}^{p_1*}_{HH^\dagger V_4 D})^2 y_u^{f_4f_5}}{2M_{V_4}^2} \\
& - \frac{(\mathcal{D}^{p_1}_{HH^\dagger V_4 D})^2 y_u^{f_4f_5}}{2M_{V_4}^2} + \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_1}_{HHS_6^\dagger} y_u^{f_4f_5}}{M_{S_6}^4} - \frac{2\mathcal{D}^{p_1*}_{HHV_2^\dagger D}\mathcal{D}^{p_1}_{HHV_2^\dagger D} y_u^{f_4f_5}}{M_{V_2}^2} \\
& + \frac{2\mathcal{D}^{p_1*}_{HH^\dagger V_4 D}\mathcal{D}^{p_1}_{HH^\dagger V_4 D} y_u^{f_4f_5}}{M_{V_4}^2} + \frac{2y_u^{f_4p_1}\mathcal{D}^{p_2f_5*}_{F_{11L}Hu^\dagger}\mathcal{D}^{p_2p_1}_{F_{11L}Hu^\dagger}}{M_{F_{11}}^2} \\
& + \frac{2y_u^{f_4p_1}\mathcal{D}^{p_2f_5*}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_2p_1}_{F_{12L}H^\dagger u^\dagger}}{M_{F_{12}}^2} - \frac{8\mathcal{D}^{p_1f_5*}_{F_{11L}Hu^\dagger}\mathcal{D}^{p_1p_2*}_{F_{11R}^\dagger F_{13L}H}\mathcal{D}^{p_2f_4*}_{F_{13R}^\dagger H^\dagger Q}}{M_{F_{11}}M_{F_{13}}} \\
& + \frac{2y_u^{p_1f_5}\mathcal{D}^{p_2f_4*}_{F_{13R}^\dagger H^\dagger Q}\mathcal{D}^{p_2p_1}_{F_{13R}^\dagger H^\dagger Q}}{M_{F_{13}}^2} + \frac{4\mathcal{D}^{p_1f_5*}_{F_{11L}Hu^\dagger}\mathcal{D}^{p_1p_2*}_{F_{11R}^\dagger F_{14L}H^\dagger}\mathcal{D}^{p_2f_4*}_{F_{14R}^\dagger HQ}}{M_{F_{11}}M_{F_{14}}} \\
& + \frac{4\mathcal{D}^{p_1f_5*}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_1p_2*}_{F_{12R}^\dagger F_{14L}H}\mathcal{D}^{p_2f_4*}_{F_{14R}^\dagger HQ}}{M_{F_{12}}M_{F_{14}}} + \frac{y_u^{p_1f_5}\mathcal{D}^{p_2f_4*}_{F_{14R}^\dagger HQ}\mathcal{D}^{p_2p_1}_{F_{14R}^\dagger HQ}}{M_{F_{14}}^2} \\
& + \frac{8\mathcal{D}^{p_1f_5*}_{F_{11L}Hu^\dagger}\mathcal{D}^{p_1p_2*}_{F_{11R}^\dagger F_{9L}H^\dagger}\mathcal{D}^{p_2f_4*}_{F_{9R}^\dagger HQ}}{M_{F_{11}}M_{F_9}} - \frac{8\mathcal{D}^{p_1f_5*}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_1p_2*}_{F_{12R}^\dagger F_{9L}H}\mathcal{D}^{p_2f_4*}_{F_{9R}^\dagger HQ}}{M_{F_{12}}M_{F_9}} \\
& + \frac{2y_u^{p_1f_5}\mathcal{D}^{p_2f_4*}_{F_{9R}^\dagger HQ}\mathcal{D}^{p_2p_1}_{F_{9R}^\dagger HQ}}{M_{F_9}^2} - \frac{4\mathcal{D}^{p_1f_5*}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_1f_4p_2*}_{F_{12R}^\dagger QS_6}\mathcal{C}^{p_2*}_{HHS_6^\dagger}}{M_{F_{12}}M_{S_6}^2} \\
& - \frac{4\mathcal{D}^{p_1p_2f_5*}_{F_{13L}S_6u^\dagger}\mathcal{D}^{p_1f_4*}_{F_{13R}^\dagger H^\dagger Q}\mathcal{C}^{p_2*}_{HHS_6^\dagger}}{M_{F_{13}}M_{S_6}^2} - \frac{4\mathcal{D}^{p_1f_5*}_{F_{11L}Hu^\dagger}\mathcal{D}^{p_1f_4p_2*}_{F_{11R}^\dagger QS_1}\mathcal{C}^{p_2}_{HH^\dagger S_1}}{M_{F_{11}}M_{S_1}^2} \\
& - \frac{4\mathcal{D}^{p_1p_2f_5*}_{F_{9L}S_1u^\dagger}\mathcal{D}^{p_1f_4*}_{F_{9R}^\dagger HQ}\mathcal{C}^{p_2}_{HH^\dagger S_1}}{M_{F_9}M_{S_1}^2} + \frac{2\mathcal{D}^{p_1f_5*}_{F_{11L}Hu^\dagger}\mathcal{D}^{p_1f_4p_2*}_{F_{11R}^\dagger QS_5}\mathcal{C}^{p_2}_{HH^\dagger S_5}}{M_{F_{11}}M_{S_5}^2} \\
& + \frac{2\mathcal{D}^{p_1p_2f_5*}_{F_{14L}S_5u^\dagger}\mathcal{D}^{p_1f_4*}_{F_{14R}^\dagger HQ}\mathcal{C}^{p_2}_{HH^\dagger S_5}}{M_{F_{14}}M_{S_5}^2} + \frac{2iy_u^{p_1f_5}\mathcal{D}^{p_2*}_{HH^\dagger V_1 D}\mathcal{D}^{p_1f_4p_2}_{QQ^\dagger V_1}}{M_{V_1}^2} \\
& + \frac{2iy_u^{p_1f_5}\mathcal{D}^{p_2}_{HH^\dagger V_1D}\mathcal{D}^{p_1f_4p_2}_{QQ^\dagger V_1}}{M_{V_1}^2} + \frac{iy_u^{p_1f_5}\mathcal{D}^{p_2*}_{HH^\dagger V_4D}\mathcal{D}^{p_1f_4p_2}_{QQ^\dagger V_4}}{M_{V_4}^2} - \frac{iy_u^{p_1f_5}\mathcal{D}^{p_2}_{HH^\dagger V_4D}\mathcal{D}^{p_1f_4p_2}_{QQ^\dagger V_4}}{M_{V_4}^2} \\
& + \frac{2\mathcal{D}^{p_1}_{HH^\dagger H^\dagger S_4}\mathcal{D}^{f_4p_1f_5*}_{QS_4u^\dagger}}{M_{S_4}^2} - \frac{2\mathcal{C}^{p_1}_{HH^\dagger S_1}\mathcal{C}^{p_1p_2*}_{HS_1S_4^\dagger}\mathcal{D}^{f_4p_2f_5*}_{QS_4u^\dagger}}{M_{S_1}^2M_{S_4}^2} - \frac{2\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_2p_1}_{HS_4S_6^\dagger}\mathcal{D}^{f_4p_2f_5*}_{QS_4u^\dagger}}{M_{S_4}^2M_{S_6}^2} \\
& - \frac{\mathcal{C}^{p_1}_{HH^\dagger S_5}\mathcal{C}^{p_2p_1}_{H^\dagger S_4S_5}\mathcal{D}^{f_4p_2f_5*}_{QS_4u^\dagger}}{M_{S_4}^2M_{S_5}^2} + \frac{2iy_u^{f_4p_1}\mathcal{D}^{p_2*}_{HH^\dagger V_1D}\mathcal{D}^{p_1f_5p_2}_{u^\dagger uV_1}}{M_{V_1}^2} + \frac{2iy_u^{f_4p_1}\mathcal{D}^{p_2}_{HH^\dagger V_1D}\mathcal{D}^{p_1f_5p_2}_{u^\dagger uV_1}}{M_{V_1}^2}
\end{aligned} \tag{B6}
$$

$$\begin{aligned}
C_{dH} = {} & \frac{(\mathcal{C}^{p_1}_{HH^\dagger S_5})^2 y_d^{f_5 f_4}}{2M_{S_5}^4} - \frac{(\mathcal{D}^{p_1*}_{HH^\dagger V_1 D})^2 y_d^{f_5f_4}}{M_{V_1}^2} + \frac{(\mathcal{D}^{p_1}_{HH^\dagger V_1 D})^2 y_d^{f_5f_4}}{M_{V_1}^2} - \frac{(\mathcal{D}^{p_1*}_{HH^\dagger V_4 D})^2 y_d^{f_5f_4}}{2M_{V_4}^2} \\
& + \frac{(\mathcal{D}^{p_1}_{HH^\dagger V_4 D})^2 y_d^{f_5f_4}}{2M_{V_4}^2} + \frac{\mathcal{C}^{p_1*}_{HHS_6^\dagger}\mathcal{C}^{p_1}_{HHS_6^\dagger} y_d^{f_5f_4}}{M_{S_6}^4} - \frac{2\mathcal{D}^{p_1*}_{HHV_2^\dagger D}\mathcal{D}^{p_1}_{HHV_2^\dagger D} y_d^{f_5f_4}}{M_{V_2}^2} \\
& + \frac{2\mathcal{D}^{p_1*}_{HH^\dagger V_4 D}\mathcal{D}^{p_1}_{HH^\dagger V_4 D} y_d^{f_5f_4}}{M_{V_4}^2} + \frac{2 y_d^{f_5 p_1}\mathcal{D}^{f_4 p_2*}_{d^\dagger F_{10L} H}\mathcal{D}^{p_1p_2}_{d^\dagger F_{10L}H}}{M_{F_{10}}^2} \\
& + \frac{2y_d^{f_5p_1}\mathcal{D}^{f_4p_2*}_{d^\dagger F_{11L}H^\dagger}\mathcal{D}^{p_1p_2}_{d^\dagger F_{11L}H^\dagger}}{M_{F_{11}}^2} - \frac{4\mathcal{D}^{f_4p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1p_2*}_{F_{10R}^\dagger F_{13L}H^\dagger}\mathcal{D}^{p_2f_5*}_{F_{13R}^\dagger H^\dagger Q}}{M_{F_{10}}M_{F_{13}}} \\
& - \frac{4\mathcal{D}^{f_4p_1*}_{d^\dagger F_{11L}H^\dagger}\mathcal{D}^{p_1p_2*}_{F_{11R}^\dagger F_{13L}H}\mathcal{D}^{p_2f_5*}_{F_{13R}^\dagger H^\dagger Q}}{M_{F_{11}}M_{F_{13}}} + \frac{y_d^{p_1f_4}\mathcal{D}^{p_2f_5*}_{F_{13R}^\dagger H^\dagger Q}\mathcal{D}^{p_2p_1}_{F_{13R}^\dagger H^\dagger Q}}{M_{F_{13}}^2} \\
& + \frac{8\mathcal{D}^{f_4p_1*}_{d^\dagger F_{11L}H^\dagger}\mathcal{D}^{p_1p_2*}_{F_{11R}^\dagger F_{14L}H^\dagger}\mathcal{D}^{p_2f_5*}_{F_{14R}^\dagger HQ}}{M_{F_{11}}M_{F_{14}}} + \frac{2y_d^{p_1f_4}\mathcal{D}^{p_2f_5*}_{F_{14R}^\dagger HQ}\mathcal{D}^{p_2p_1}_{F_{14R}^\dagger HQ}}{M_{F_{14}}^2} \\
& + \frac{8\mathcal{D}^{f_4p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1p_2*}_{F_{10R}^\dagger F_{8L}H^\dagger}\mathcal{D}^{p_2f_5*}_{F_{8R}^\dagger H^\dagger Q}}{M_{F_{10}}M_{F_8}} - \frac{8\mathcal{D}^{f_4p_1*}_{d^\dagger F_{11L}H^\dagger}\mathcal{D}^{p_1p_2*}_{F_{11R}^\dagger F_{8L}H}\mathcal{D}^{p_2f_5*}_{F_{8R}^\dagger H^\dagger Q}}{M_{F_{11}}M_{F_8}} \\
& + \frac{2y_d^{p_1f_4}\mathcal{D}^{p_2f_5*}_{F_{8R}^\dagger H^\dagger Q}\mathcal{D}^{p_2p_1}_{F_{8R}^\dagger H^\dagger Q}}{M_{F_8}^2} + \frac{4\mathcal{D}^{f_4p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_5p_2*}_{F_{10R}^\dagger QS_6^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}}{M_{F_{10}}M_{S_6}^2} \\
& + \frac{4\mathcal{D}^{f_4p_1p_2*}_{d^\dagger F_{14L}S_6^\dagger}\mathcal{D}^{p_1f_5*}_{F_{14R}^\dagger HQ}\mathcal{C}^{p_2}_{HHS_6^\dagger}}{M_{F_{14}}M_{S_6}^2} + \frac{2\mathcal{D}^{f_4f_5p_1*}_{d^\dagger QS_4^\dagger}\mathcal{D}^{p_1*}_{HH^\dagger H^\dagger S_4}}{M_{S_4}^2} - \frac{4\mathcal{D}^{f_4p_1*}_{d^\dagger F_{11L}H^\dagger}\mathcal{D}^{p_1f_5p_2*}_{F_{11R}^\dagger QS_1}\mathcal{C}^{p_2}_{HH^\dagger S_1}}{M_{F_{11}}M_{S_1}^2} \\
& - \frac{4\mathcal{D}^{f_4p_1p_2*}_{d^\dagger F_{8L}S_1}\mathcal{D}^{p_1f_5*}_{F_{8R}^\dagger H^\dagger Q}\mathcal{C}^{p_2}_{HH^\dagger S_1}}{M_{F_8}M_{S_1}^2} - \frac{2\mathcal{D}^{f_4p_1*}_{d^\dagger F_{11L}H^\dagger}\mathcal{D}^{p_1f_5p_2*}_{F_{11R}^\dagger QS_5}\mathcal{C}^{p_2}_{HH^\dagger S_5}}{M_{F_{11}}M_{S_5}^2} \\
& - \frac{2\mathcal{D}^{f_4p_1p_2*}_{d^\dagger F_{13L}S_5}\mathcal{D}^{p_1f_5*}_{F_{13R}^\dagger H^\dagger Q}\mathcal{C}^{p_2}_{HH^\dagger S_5}}{M_{F_{13}}M_{S_5}^2} + \frac{2iy_d^{f_5p_1}\mathcal{D}^{p_1f_4p_2}_{d^\dagger dV_1}\mathcal{D}^{p_2*}_{HH^\dagger V_1 D}}{M_{V_1}^2} \\
& + \frac{2iy_d^{f_5p_1}\mathcal{D}^{p_1f_4p_2}_{d^\dagger dV_1}\mathcal{D}^{p_2}_{HH^\dagger V_1D}}{M_{V_1}^2} - \frac{2\mathcal{D}^{f_4f_5p_1*}_{d^\dagger QS_4^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_1}\mathcal{C}^{p_2p_1}_{HS_1S_4^\dagger}}{M_{S_1}^2M_{S_4}^2} - \frac{2\mathcal{D}^{f_4f_5p_1*}_{d^\dagger QS_4^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{C}^{p_1p_2*}_{HS_4S_6^\dagger}}{M_{S_4}^2M_{S_6}^2} \\
& + \frac{\mathcal{D}^{f_4f_5p_1*}_{d^\dagger QS_4^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{C}^{p_1p_2*}_{H^\dagger S_4S_5}}{M_{S_4}^2M_{S_5}^2} + \frac{2iy_d^{p_1f_4}\mathcal{D}^{p_2*}_{HH^\dagger V_1D}\mathcal{D}^{p_1f_5p_2}_{QQ^\dagger V_1}}{M_{V_1}^2} + \frac{2iy_d^{p_1f_4}\mathcal{D}^{p_2}_{HH^\dagger V_1D}\mathcal{D}^{p_1f_5p_2}_{QQ^\dagger V_1}}{M_{V_1}^2} \\
& - \frac{iy_d^{p_1f_4}\mathcal{D}^{p_2*}_{HH^\dagger V_4D}\mathcal{D}^{p_1f_5p_2}_{QQ^\dagger V_4}}{M_{V_4}^2} + \frac{iy_d^{p_1f_4}\mathcal{D}^{p_2}_{HH^\dagger V_4D}\mathcal{D}^{p_1f_5p_2}_{QQ^\dagger V_4}}{M_{V_4}^2}
\end{aligned} \tag{B7}$$

b. Mixed operators

$$\begin{aligned}
C_{Hl}^{(1)} = {} & -\frac{i\mathcal{D}^{f_1f_4p_1}_{LL^\dagger V_1}\mathcal{D}^{p_1}_{HH^\dagger V_1D}}{M_{V_1}^2} + \frac{i\mathcal{D}^{f_1f_4p_1}_{LL^\dagger V_1}\mathcal{D}^{p_1*}_{HH^\dagger V_1D}}{M_{V_1}^2} + \frac{\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_1f_4*}_{F_1HL}}{4M_{F_1}^2} - \frac{\mathcal{D}^{p_1f_1}_{F_{2L}H^\dagger L}\mathcal{D}^{p_1f_4*}_{F_{2L}H^\dagger L}}{4M_{F_2}^2} \\
& + \frac{3\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_4*}_{F_5HL}}{8M_{F_5}^2} - \frac{3\mathcal{D}^{p_1f_1}_{F_{6L}H^\dagger L}\mathcal{D}^{p_1f_4*}_{F_{6L}H^\dagger L}}{8M_{F_6}^2}
\end{aligned} \tag{B8}$$

$$C_{Hl}^{(3)} = -\frac{i\mathcal{D}^{f_1 f_4 p_1}_{LL^\dagger V_4}\mathcal{D}^{p_1}_{HH^\dagger V_4 D}}{2M_{V_4}^2} - \frac{i\mathcal{D}^{f_1 f_4 p_1}_{LL^\dagger V_4}\mathcal{D}^{p_1 *}_{HH^\dagger V_4 D}}{2M_{V_4}^2} - \frac{\mathcal{D}^{p_1 f_1}_{F_1 HL}\mathcal{D}^{p_1 f_4 *}_{F_1 HL}}{4M_{F_1}^2} - \frac{\mathcal{D}^{p_1 f_1}_{F_{2L} H^\dagger L}\mathcal{D}^{p_1 f_4 *}_{F_{2L} H^\dagger L}}{4M_{F_2}^2} + \frac{\mathcal{D}^{p_1 f_1}_{F_5 HL}\mathcal{D}^{p_1 f_4 *}_{F_5 HL}}{8M_{F_5}^2} + \frac{\mathcal{D}^{p_1 f_1}_{F_{6L} H^\dagger L}\mathcal{D}^{p_1 f_4 *}_{F_{6L} H^\dagger L}}{8M_{F_6}^2} \tag{B9}$$

$$C_{He} = \frac{i\mathcal{D}^{f_1 f_4 p_1}_{e^\dagger e V_1}\mathcal{D}^{p_1}_{HH^\dagger V_1 D}}{M_{V_1}^2} - \frac{i\mathcal{D}^{f_1 f_4 p_1}_{e^\dagger e V_1}\mathcal{D}^{p_1 *}_{HH^\dagger V_1 D}}{M_{V_1}^2} + \frac{\mathcal{D}^{f_1 p_1}_{e^\dagger F^\dagger_{3R} H^\dagger}\mathcal{D}^{f_4 p_1 *}_{e^\dagger F^\dagger_{3R} H^\dagger}}{2M_{F_3}^2} - \frac{\mathcal{D}^{f_1 p_1}_{e^\dagger F^\dagger_{4R} H}\mathcal{D}^{f_4 p_1 *}_{e^\dagger F^\dagger_{4R} H}}{2M_{F_4}^2} \tag{B10}$$

$$C_{Hq}^{(1)} = -\frac{3\mathcal{D}^{p_1 f_1}_{F^\dagger_{13R} H^\dagger Q}\mathcal{D}^{p_1 f_4 *}_{F^\dagger_{13R} H^\dagger Q}}{2M_{F_{13}}^2} + \frac{3\mathcal{D}^{p_1 f_1}_{F^\dagger_{14R} HQ}\mathcal{D}^{p_1 f_4 *}_{F^\dagger_{14R} HQ}}{2M_{F_{14}}^2} - \frac{2i\mathcal{D}^{f_1 f_4 p_1}_{QQ^\dagger V_1}\mathcal{D}^{p_1}_{HH^\dagger V_1 D}}{M_{V_1}^2} + \frac{2i\mathcal{D}^{f_1 f_4 p_1}_{QQ^\dagger V_1}\mathcal{D}^{p_1 *}_{HH^\dagger V_1 D}}{M_{V_1}^2} - \frac{\mathcal{D}^{p_1 f_1}_{F^\dagger_{8R} H^\dagger Q}\mathcal{D}^{p_1 f_4 *}_{F^\dagger_{8R} H^\dagger Q}}{M_{F_8}^2} + \frac{\mathcal{D}^{p_1 f_1}_{F^\dagger_{9R} HQ}\mathcal{D}^{p_1 f_4 *}_{F^\dagger_{9R} HQ}}{M_{F_9}^2} \tag{B11}$$

$$C_{Hq}^{(3)} = \frac{\mathcal{D}^{p_1 f_1}_{F^\dagger_{13R} H^\dagger Q}\mathcal{D}^{p_1 f_4 *}_{F^\dagger_{13R} H^\dagger Q}}{2M_{F_{13}}^2} + \frac{\mathcal{D}^{p_1 f_1}_{F^\dagger_{14R} HQ}\mathcal{D}^{p_1 f_4 *}_{F^\dagger_{14R} HQ}}{2M_{F_{14}}^2} - \frac{i\mathcal{D}^{f_1 f_4 p_1}_{QQ^\dagger V_4}\mathcal{D}^{p_1}_{HH^\dagger V_4 D}}{M_{V_4}^2} - \frac{i\mathcal{D}^{f_1 f_4 p_1}_{QQ^\dagger V_4}\mathcal{D}^{p_1 *}_{HH^\dagger V_4 D}}{M_{V_4}^2} - \frac{\mathcal{D}^{p_1 f_1}_{F^\dagger_{8R} H^\dagger Q}\mathcal{D}^{p_1 f_4 *}_{F^\dagger_{8R} H^\dagger Q}}{M_{F_8}^2} - \frac{\mathcal{D}^{p_1 f_1}_{F^\dagger_{9R} HQ}\mathcal{D}^{p_1 f_4 *}_{F^\dagger_{9R} HQ}}{M_{F_9}^2} \tag{B12}$$

$$C_{Hu} = -\frac{2\mathcal{D}^{p_1 f_1}_{F_{11L} Hu^\dagger}\mathcal{D}^{p_1 f_4 *}_{F_{11L} Hu^\dagger}}{M_{F_{11}}^2} + \frac{2\mathcal{D}^{p_1 f_1}_{F_{12L} H^\dagger u^\dagger}\mathcal{D}^{p_1 f_4 *}_{F_{12L} H^\dagger u^\dagger}}{M_{F_{12}}^2} + \frac{2i\mathcal{D}^{f_1 f_4 p_1}_{u^\dagger u V_1}\mathcal{D}^{p_1}_{HH^\dagger V_1 D}}{M_{V_1}^2} - \frac{2i\mathcal{D}^{f_1 f_4 p_1}_{u^\dagger u V_1}\mathcal{D}^{p_1 *}_{HH^\dagger V_1 D}}{M_{V_1}^2} \tag{B13}$$

$$C_{Hd} = \frac{2i\mathcal{D}^{f_1 f_4 p_1}_{d^\dagger d V_1}\mathcal{D}^{p_1}_{HH^\dagger V_1 D}}{M_{V_1}^2} - \frac{2i\mathcal{D}^{f_1 f_4 p_1}_{d^\dagger d V_1}\mathcal{D}^{p_1 *}_{HH^\dagger V_1 D}}{M_{V_1}^2} - \frac{2\mathcal{D}^{f_1 p_1}_{d^\dagger F_{10L} H}\mathcal{D}^{f_4 p_1 *}_{d^\dagger F_{10L} H}}{M_{F_{10}}^2} + \frac{2\mathcal{D}^{f_1 p_1}_{d^\dagger F_{11L} H^\dagger}\mathcal{D}^{f_4 p_1 *}_{d^\dagger F_{11L} H^\dagger}}{M_{F_{11}}^2} \tag{B14}$$

$$C_{Hud} = \frac{4\mathcal{D}^{f_4 p_1 *}_{d^\dagger F_{11L} H^\dagger}\mathcal{D}^{p_1 f_1}_{F_{11L} Hu^\dagger}}{M_{F_{11}}^2} + \frac{4i\mathcal{D}^{f_4 f_1 p_1 *}_{d^\dagger u V_2^\dagger}\mathcal{D}^{p_1}_{HHV_2^\dagger D}}{M_{V_2}^2} \tag{B15}$$

c. 4-fermion operators

$$C_{ll} = -\frac{\mathcal{D}^{f_1 f_2 p_1}_{LLS_2}\mathcal{D}^{f_4 f_3 p_1 *}_{LLS_2}}{2M_{S_2}^2} + \frac{\mathcal{D}^{f_1 f_2 p_1}_{LLS_6}\mathcal{D}^{f_4 f_3 p_1 *}_{LLS_6}}{4M_{S_6}^2} - \frac{\mathcal{D}^{f_1 f_3 p_1}_{LL^\dagger V_1}\mathcal{D}^{f_2 f_4 p_1}_{LL^\dagger V_1}}{2M_{V_1}^2} + \frac{\mathcal{D}^{f_1 f_3 p_1}_{LL^\dagger V_4}\mathcal{D}^{f_2 f_4 p_1}_{LL^\dagger V_4}}{4M_{V_4}^2} - \frac{\mathcal{D}^{f_1 f_4 p_1}_{LL^\dagger V_4}\mathcal{D}^{f_2 f_3 p_1}_{LL^\dagger V_4}}{2M_{V_4}^2} + \frac{\mathcal{D}^{f_2 f_1 p_1}_{LLS_2}\mathcal{D}^{f_4 f_3 p_1 *}_{LLS_2}}{2M_{S_2}^2} + \frac{\mathcal{D}^{f_2 f_1 p_1}_{LLS_6}\mathcal{D}^{f_4 f_3 p_1 *}_{LLS_6}}{4M_{S_6}^2} \tag{B16}$$

$$C_{qq}^{(1)} = \frac{\mathcal{D}_{QQS_{10}}^{f_1f_2p_1}\mathcal{D}_{QQS_{10}}^{f_4f_3p_1*}}{4M_{S_{10}}^2} - \frac{3\mathcal{D}_{QQS_{14}}^{f_1f_2p_1}\mathcal{D}_{QQS_{14}}^{f_4f_3p_1*}}{8M_{S_{14}}^2} - \frac{2\mathcal{D}_{QQS_{16}^\dagger}^{f_1f_2p_1}\mathcal{D}_{QQS_{16}^\dagger}^{f_4f_3p_1*}}{M_{S_{16}}^2} + \frac{3\mathcal{D}_{QQS_{18}^\dagger}^{f_1f_2p_1}\mathcal{D}_{QQS_{18}^\dagger}^{f_4f_3p_1*}}{M_{S_{18}}^2} - \frac{2\mathcal{D}_{QQ^\dagger V_1}^{f_1f_3p_1}\mathcal{D}_{QQ^\dagger V_1}^{f_2f_4p_1}}{M_{V_1}^2} + \frac{2\mathcal{D}_{QQ^\dagger V_{12}}^{f_1f_3p_1}\mathcal{D}_{QQ^\dagger V_{12}}^{f_2f_4p_1}}{3M_{V_{12}}^2} - \frac{\mathcal{D}_{QQ^\dagger V_{12}}^{f_1f_4p_1}\mathcal{D}_{QQ^\dagger V_{12}}^{f_2f_3p_1}}{M_{V_{12}}^2} - \frac{3\mathcal{D}_{QQ^\dagger V_{14}}^{f_1f_4p_1}\mathcal{D}_{QQ^\dagger V_{14}}^{f_2f_3p_1}}{2M_{V_{14}}^2} + \frac{\mathcal{D}_{QQS_{10}}^{f_2f_1p_1}\mathcal{D}_{QQS_{10}}^{f_4f_3p_1*}}{4M_{S_{10}}^2} + \frac{3\mathcal{D}_{QQS_{14}}^{f_2f_1p_1}\mathcal{D}_{QQS_{14}}^{f_4f_3p_1*}}{8M_{S_{14}}^2} + \frac{2\mathcal{D}_{QQS_{16}^\dagger}^{f_2f_1p_1}\mathcal{D}_{QQS_{16}^\dagger}^{f_4f_3p_1*}}{M_{S_{16}}^2} + \frac{3\mathcal{D}_{QQS_{18}^\dagger}^{f_2f_1p_1}\mathcal{D}_{QQS_{18}^\dagger}^{f_4f_3p_1*}}{M_{S_{18}}^2} \tag{B17}$$

$$C_{qq}^{(3)} = -\frac{\mathcal{D}_{QQS_{10}}^{f_1f_2p_1}\mathcal{D}_{QQS_{10}}^{f_4f_3p_1*}}{4M_{S_{10}}^2} - \frac{\mathcal{D}_{QQS_{14}}^{f_1f_2p_1}\mathcal{D}_{QQS_{14}}^{f_4f_3p_1*}}{8M_{S_{14}}^2} + \frac{2\mathcal{D}_{QQS_{16}^\dagger}^{f_1f_2p_1}\mathcal{D}_{QQS_{16}^\dagger}^{f_4f_3p_1*}}{M_{S_{16}}^2} + \frac{\mathcal{D}_{QQS_{18}^\dagger}^{f_1f_2p_1}\mathcal{D}_{QQS_{18}^\dagger}^{f_4f_3p_1*}}{M_{S_{18}}^2} + \frac{\mathcal{D}_{QQ^\dagger V_{14}}^{f_1f_3p_1}\mathcal{D}_{QQ^\dagger V_{14}}^{f_2f_4p_1}}{3M_{V_{14}}^2} - \frac{\mathcal{D}_{QQ^\dagger V_4}^{f_1f_3p_1}\mathcal{D}_{QQ^\dagger V_4}^{f_2f_4p_1}}{M_{V_4}^2} - \frac{\mathcal{D}_{QQ^\dagger V_{12}}^{f_1f_4p_1}\mathcal{D}_{QQ^\dagger V_{12}}^{f_2f_3p_1}}{M_{V_{12}}^2} + \frac{\mathcal{D}_{QQ^\dagger V_{14}}^{f_1f_4p_1}\mathcal{D}_{QQ^\dagger V_{14}}^{f_2f_3p_1}}{2M_{V_{14}}^2} - \frac{\mathcal{D}_{QQS_{10}}^{f_2f_1p_1}\mathcal{D}_{QQS_{10}}^{f_4f_3p_1*}}{4M_{S_{10}}^2} + \frac{\mathcal{D}_{QQS_{14}}^{f_2f_1p_1}\mathcal{D}_{QQS_{14}}^{f_4f_3p_1*}}{8M_{S_{14}}^2} - \frac{2\mathcal{D}_{QQS_{16}^\dagger}^{f_2f_1p_1}\mathcal{D}_{QQS_{16}^\dagger}^{f_4f_3p_1*}}{M_{S_{16}}^2} + \frac{\mathcal{D}_{QQS_{18}^\dagger}^{f_2f_1p_1}\mathcal{D}_{QQS_{18}^\dagger}^{f_4f_3p_1*}}{M_{S_{18}}^2} \tag{B18}$$

$$C_{lq}^{(1)} = \frac{\mathcal{D}_{LQS_{10}^\dagger}^{f_1f_2p_1}\mathcal{D}_{LQS_{10}^\dagger}^{f_3f_4p_1*}}{M_{S_{10}}^2} + \frac{3\mathcal{D}_{LQS_{14}^\dagger}^{f_1f_2p_1}\mathcal{D}_{LQS_{14}^\dagger}^{f_3f_4p_1*}}{2M_{S_{14}}^2} - \frac{2\mathcal{D}_{LL^\dagger V_1}^{f_1f_3p_1}\mathcal{D}_{QQ^\dagger V_1}^{f_2f_4p_1}}{M_{V_1}^2} - \frac{2\mathcal{D}_{L^\dagger QV_5^\dagger}^{f_1f_4p_1*}\mathcal{D}_{L^\dagger QV_5^\dagger}^{f_3f_2p_1}}{M_{V_5}^2} - \frac{3\mathcal{D}_{L^\dagger QV_9^\dagger}^{f_1f_4p_1*}\mathcal{D}_{L^\dagger QV_9^\dagger}^{f_3f_2p_1}}{M_{V_9}^2} \tag{B19}$$

$$C_{lq}^{(3)} = -\frac{\mathcal{D}_{LQS_{10}^\dagger}^{f_1f_2p_1}\mathcal{D}_{LQS_{10}^\dagger}^{f_3f_4p_1*}}{M_{S_{10}}^2} + \frac{\mathcal{D}_{LQS_{14}^\dagger}^{f_1f_2p_1}\mathcal{D}_{LQS_{14}^\dagger}^{f_3f_4p_1*}}{2M_{S_{14}}^2} - \frac{\mathcal{D}_{LL^\dagger V_4}^{f_1f_3p_1}\mathcal{D}_{QQ^\dagger V_4}^{f_2f_4p_1}}{M_{V_4}^2} - \frac{2\mathcal{D}_{L^\dagger QV_5^\dagger}^{f_1f_4p_1*}\mathcal{D}_{L^\dagger QV_5^\dagger}^{f_3f_2p_1}}{M_{V_5}^2} + \frac{\mathcal{D}_{L^\dagger QV_9^\dagger}^{f_1f_4p_1*}\mathcal{D}_{L^\dagger QV_9^\dagger}^{f_3f_2p_1}}{M_{V_9}^2} \tag{B20}$$

$$C_{ee} = \frac{\mathcal{D}_{e^\dagger e^\dagger S_3^\dagger}^{f_1f_2p_1}\mathcal{D}_{e^\dagger e^\dagger S_3^\dagger}^{f_4f_3p_1*}}{2M_{S_3}^2} - \frac{\mathcal{D}_{e^\dagger eV_1}^{f_1f_3p_1}\mathcal{D}_{e^\dagger eV_1}^{f_2f_4p_1}}{2M_{V_1}^2} \tag{B21}$$

$$C_{uu} = -\frac{2\mathcal{D}_{u^\dagger uV_1}^{f_1f_3p_1}\mathcal{D}_{u^\dagger uV_1}^{f_2f_4p_1}}{M_{V_1}^2} + \frac{8\mathcal{D}_{u^\dagger uV_{12}}^{f_1f_3p_1}\mathcal{D}_{u^\dagger uV_{12}}^{f_2f_4p_1}}{3M_{V_{12}}^2} - \frac{8\mathcal{D}_{u^\dagger uV_{12}}^{f_1f_4p_1}\mathcal{D}_{u^\dagger uV_{12}}^{f_2f_3p_1}}{M_{V_{12}}^2} + \frac{4\mathcal{D}_{S_{17}u^\dagger u^\dagger}^{p_1f_1f_2}\mathcal{D}_{S_{17}u^\dagger u^\dagger}^{p_1f_4f_3*}}{M_{S_{17}}^2} + \frac{4\mathcal{D}_{S_{17}u^\dagger u^\dagger}^{p_1f_2f_1}\mathcal{D}_{S_{17}u^\dagger u^\dagger}^{p_1f_4f_3*}}{M_{S_{17}}^2} - \frac{8\mathcal{D}_{S_9^\dagger u^\dagger u^\dagger}^{p_1f_1f_2}\mathcal{D}_{S_9^\dagger u^\dagger u^\dagger}^{p_1f_4f_3*}}{M_{S_9}^2} + \frac{8\mathcal{D}_{S_9^\dagger u^\dagger u^\dagger}^{p_1f_2f_1}\mathcal{D}_{S_9^\dagger u^\dagger u^\dagger}^{p_1f_4f_3*}}{M_{S_9}^2} \tag{B22}$$

$$C_{dd} = -\frac{2\mathcal{D}_{d^\dagger dV_1}^{f_1f_3p_1}\mathcal{D}_{d^\dagger dV_1}^{f_2f_4p_1}}{M_{V_1}^2} + \frac{8\mathcal{D}_{d^\dagger dV_{12}}^{f_1f_3p_1}\mathcal{D}_{d^\dagger dV_{12}}^{f_2f_4p_1}}{3M_{V_{12}}^2} - \frac{8\mathcal{D}_{d^\dagger dV_{12}}^{f_1f_4p_1}\mathcal{D}_{d^\dagger dV_{12}}^{f_2f_3p_1}}{M_{V_{12}}^2} - \frac{8\mathcal{D}_{d^\dagger d^\dagger S_{11}^\dagger}^{f_1f_2p_1}\mathcal{D}_{d^\dagger d^\dagger S_{11}^\dagger}^{f_4f_3p_1*}}{M_{S_{11}}^2} + \frac{8\mathcal{D}_{d^\dagger d^\dagger S_{11}^\dagger}^{f_2f_1p_1}\mathcal{D}_{d^\dagger d^\dagger S_{11}^\dagger}^{f_4f_3p_1*}}{M_{S_{11}}^2} + \frac{4\mathcal{D}_{d^\dagger d^\dagger S_{15}}^{f_1f_2p_1}\mathcal{D}_{d^\dagger d^\dagger S_{15}}^{f_4f_3p_1*}}{M_{S_{15}}^2} + \frac{4\mathcal{D}_{d^\dagger d^\dagger S_{15}}^{f_2f_1p_1}\mathcal{D}_{d^\dagger d^\dagger S_{15}}^{f_4f_3p_1*}}{M_{S_{15}}^2} \tag{B23}$$

$$C_{eu} = -\frac{4\mathcal{D}_{eu^\dagger V_6}^{f_1f_4p_1*}\mathcal{D}_{eu^\dagger V_6}^{f_3f_2p_1}}{M_{V_6}^2} - \frac{2\mathcal{D}_{e^\dagger eV_1}^{f_1f_3p_1}\mathcal{D}_{u^\dagger uV_1}^{f_2f_4p_1}}{M_{V_1}^2} + \frac{2\mathcal{D}_{e^\dagger S_{10}u^\dagger}^{f_1p_1f_2}\mathcal{D}_{e^\dagger S_{10}u^\dagger}^{f_3p_1f_4*}}{M_{S_{10}}^2} \tag{B24}$$

$$C_{ed} = -\frac{2\mathcal{D}^{f_1f_3p_1}_{d^\dagger dV_1}\mathcal{D}^{f_2f_4p_1}_{e^\dagger eV_1}}{M^2_{V_1}} - \frac{4\mathcal{D}^{f_1f_4p_1}_{d^\dagger eV_5}\mathcal{D}^{f_3f_2p_1*}_{d^\dagger eV_5}}{M^2_{V_5}} + \frac{2\mathcal{D}^{f_1f_2p_1}_{d^\dagger e^\dagger S_9}\mathcal{D}^{f_3f_4p_1*}_{d^\dagger e^\dagger S_9}}{M^2_{S_9}} \tag{B25}$$

$$C^{(1)}_{ud} = -\frac{4\mathcal{D}^{f_1f_3p_1}_{d^\dagger dV_1}\mathcal{D}^{f_2f_4p_1}_{u^\dagger uV_1}}{M^2_{V_1}} + \frac{16\mathcal{D}^{f_1p_1f_2}_{d^\dagger S^\dagger_{10}u^\dagger}\mathcal{D}^{f_3p_1f_4*}_{d^\dagger S^\dagger_{10}u^\dagger}}{3M^2_{S_{10}}} + \frac{16\mathcal{D}^{f_1p_1f_2}_{d^\dagger S_{16}u^\dagger}\mathcal{D}^{f_3p_1f_4*}_{d^\dagger S_{16}u^\dagger}}{3M^2_{S_{16}}} - \frac{128\mathcal{D}^{f_1f_4p_1}_{d^\dagger uV^\dagger_{13}}\mathcal{D}^{f_3f_2p_1*}_{d^\dagger uV^\dagger_{13}}}{9M^2_{V_{13}}} - \frac{4\mathcal{D}^{f_1f_4p_1}_{d^\dagger uV^\dagger_2}\mathcal{D}^{f_3f_2p_1*}_{d^\dagger uV^\dagger_2}}{3M^2_{V_2}} \tag{B26}$$

$$C^{(8)}_{ud} = -\frac{32\mathcal{D}^{f_1f_3p_1}_{d^\dagger dV_{12}}\mathcal{D}^{f_2f_4p_1}_{u^\dagger uV_{12}}}{M^2_{V_{12}}} - \frac{16\mathcal{D}^{f_1p_1f_2}_{d^\dagger S^\dagger_{10}u^\dagger}\mathcal{D}^{f_3p_1f_4*}_{d^\dagger S^\dagger_{10}u^\dagger}}{M^2_{S_{10}}} + \frac{8\mathcal{D}^{f_1p_1f_2}_{d^\dagger S_{16}u^\dagger}\mathcal{D}^{f_3p_1f_4*}_{d^\dagger S_{16}u^\dagger}}{M^2_{S_{16}}} + \frac{32\mathcal{D}^{f_1f_4p_1}_{d^\dagger uV^\dagger_{13}}\mathcal{D}^{f_3f_2p_1*}_{d^\dagger uV^\dagger_{13}}}{3M^2_{V_{13}}} - \frac{8\mathcal{D}^{f_1f_4p_1}_{d^\dagger uV^\dagger_2}\mathcal{D}^{f_3f_2p_1*}_{d^\dagger uV^\dagger_2}}{M^2_{V_2}} \tag{B27}$$

$$C_{le} = \frac{\mathcal{D}^{f_1f_3p_1}_{e^\dagger eV_1}\mathcal{D}^{f_2f_4p_1}_{LL^\dagger V_1}}{M^2_{V_1}} - \frac{\mathcal{D}^{f_1f_2p_1}_{e^\dagger LS^\dagger_4}\mathcal{D}^{f_3f_4p_1*}_{e^\dagger LS^\dagger_4}}{2M^2_{S_4}} + \frac{\mathcal{D}^{f_1f_4p_1}_{e^\dagger L^\dagger V^\dagger_3}\mathcal{D}^{f_3f_2p_1*}_{e^\dagger L^\dagger V^\dagger_3}}{M^2_{V_3}} \tag{B28}$$

$$C_{lu} = \frac{2\mathcal{D}^{f_1f_3p_1}_{LL^\dagger V_1}\mathcal{D}^{f_2f_4p_1}_{u^\dagger uV_1}}{M^2_{V_1}} + \frac{4\mathcal{D}^{f_1f_4p_1*}_{L^\dagger u^\dagger V_8}\mathcal{D}^{f_3f_2p_1}_{L^\dagger u^\dagger V_8}}{M^2_{V_8}} - \frac{2\mathcal{D}^{f_1p_1f_2}_{LS_{13}u^\dagger}\mathcal{D}^{f_3p_1f_4*}_{LS_{13}u^\dagger}}{M^2_{S_{13}}} \tag{B29}$$

$$C_{ld} = \frac{2\mathcal{D}^{f_1f_3p_1}_{d^\dagger dV_1}\mathcal{D}^{f_2f_4p_1}_{LL^\dagger V_1}}{M^2_{V_1}} - \frac{2\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{D}^{f_3f_4p_1*}_{d^\dagger LS_{12}}}{M^2_{S_{12}}} + \frac{4\mathcal{D}^{f_1f_4p_1}_{d^\dagger L^\dagger V_7}\mathcal{D}^{f_3f_2p_1*}_{d^\dagger L^\dagger V_7}}{M^2_{V_7}} \tag{B30}$$

$$C_{qe} = \frac{4\mathcal{D}^{f_1f_4p_1*}_{eQV^\dagger_7}\mathcal{D}^{f_3f_2p_1}_{eQV^\dagger_7}}{M^2_{V_7}} + \frac{2\mathcal{D}^{f_1f_3p_1}_{e^\dagger eV_1}\mathcal{D}^{f_2f_4p_1}_{QQ^\dagger V_1}}{M^2_{V_1}} - \frac{2\mathcal{D}^{f_1f_2p_1}_{e^\dagger QS^\dagger_{13}}\mathcal{D}^{f_3f_4p_1*}_{e^\dagger QS^\dagger_{13}}}{M^2_{S_{13}}} \tag{B31}$$

$$C^{(1)}_{qu} = \frac{4\mathcal{D}^{f_1f_3p_1}_{QQ^\dagger V_1}\mathcal{D}^{f_2f_4p_1}_{u^\dagger uV_1}}{M^2_{V_1}} + \frac{32\mathcal{D}^{f_1f_4p_1}_{QuV^\dagger_{11}}\mathcal{D}^{f_3f_2p_1*}_{QuV^\dagger_{11}}}{3M^2_{V_{11}}} + \frac{2\mathcal{D}^{f_1f_4p_1}_{QuV_7}\mathcal{D}^{f_3f_2p_1*}_{QuV_7}}{3M^2_{V_7}} - \frac{16\mathcal{D}^{f_1p_1f_2}_{QS_{19}u^\dagger}\mathcal{D}^{f_3p_1f_4*}_{QS_{19}u^\dagger}}{9M^2_{S_{19}}} - \frac{2\mathcal{D}^{f_1p_1f_2}_{QS_4u^\dagger}\mathcal{D}^{f_3p_1f_4*}_{QS_4u^\dagger}}{3M^2_{S_4}} \tag{B32}$$

$$C^{(8)}_{qu} = \frac{16\mathcal{D}^{f_1f_3p_1}_{QQ^\dagger V_{12}}\mathcal{D}^{f_2f_4p_1}_{u^\dagger uV_{12}}}{M^2_{V_{12}}} + \frac{16\mathcal{D}^{f_1f_4p_1}_{QuV^\dagger_{11}}\mathcal{D}^{f_3f_2p_1*}_{QuV^\dagger_{11}}}{M^2_{V_{11}}} - \frac{2\mathcal{D}^{f_1f_4p_1}_{QuV_7}\mathcal{D}^{f_3f_2p_1*}_{QuV_7}}{M^2_{V_7}} + \frac{4\mathcal{D}^{f_1p_1f_2}_{QS_{19}u^\dagger}\mathcal{D}^{f_3p_1f_4*}_{QS_{19}u^\dagger}}{3M^2_{S_{19}}} - \frac{4\mathcal{D}^{f_1p_1f_2}_{QS_4u^\dagger}\mathcal{D}^{f_3p_1f_4*}_{QS_4u^\dagger}}{M^2_{S_4}} \tag{B33}$$

$$C^{(1)}_{qd} = \frac{32\mathcal{D}^{f_1f_4p_1*}_{dQV^\dagger_{10}}\mathcal{D}^{f_3f_2p_1}_{dQV^\dagger_{10}}}{3M^2_{V_{10}}} + \frac{2\mathcal{D}^{f_1f_4p_1*}_{dQV_8}\mathcal{D}^{f_3f_2p_1}_{dQV_8}}{3M^2_{V_8}} + \frac{4\mathcal{D}^{f_1f_3p_1}_{d^\dagger dV_1}\mathcal{D}^{f_2f_4p_1}_{QQ^\dagger V_1}}{M^2_{V_1}} - \frac{64\mathcal{D}^{f_1f_2p_1}_{d^\dagger QS^\dagger_{19}}\mathcal{D}^{f_3f_4p_1*}_{d^\dagger QS^\dagger_{19}}}{9M^2_{S_{19}}} - \frac{2\mathcal{D}^{f_1f_2p_1}_{d^\dagger QS^\dagger_4}\mathcal{D}^{f_3f_4p_1*}_{d^\dagger QS^\dagger_4}}{3M^2_{S_4}} \tag{B34}$$

$$C_{qd}^{(8)} = \frac{16\mathcal{D}_{dQV_{10}^\dagger}^{f_1f_4p_1*}\mathcal{D}_{dQV_{10}^\dagger}^{f_3f_2p_1}}{M_{V_{10}}^2} - \frac{2\mathcal{D}_{dQV_8}^{f_1f_4p_1*}\mathcal{D}_{dQV_8}^{f_3f_2p_1}}{M_{V_8}^2} + \frac{16\mathcal{D}_{d^\dagger dV_{12}}^{f_1f_3p_1}\mathcal{D}_{QQ^\dagger V_{12}}^{f_2f_4p_1}}{M_{V_{12}}^2} + \frac{16\mathcal{D}_{d^\dagger QS_{19}^\dagger}^{f_1f_2p_1}\mathcal{D}_{d^\dagger QS_{19}^\dagger}^{f_3f_4p_1*}}{3M_{S_{19}}^2} - \frac{4\mathcal{D}_{d^\dagger QS_4^\dagger}^{f_1f_2p_1}\mathcal{D}_{d^\dagger QS_4^\dagger}^{f_3f_4p_1*}}{M_{S_4}^2} \tag{B35}$$

$$C_{ledq} = -\frac{8\mathcal{D}_{d^\dagger eV_5}^{f_1f_3p_1}\mathcal{D}_{L^\dagger QV_5^\dagger}^{f_4f_2p_1}}{M_{V_5}^2} - \frac{8\mathcal{D}_{d^\dagger L^\dagger V_7}^{f_1f_4p_1}\mathcal{D}_{eQV_7^\dagger}^{f_3f_2p_1}}{M_{V_7}^2} + \frac{2\mathcal{D}_{d^\dagger QS_4^\dagger}^{f_1f_2p_1}\mathcal{D}_{e^\dagger LS_4^\dagger}^{f_3f_4p_1*}}{M_{S_4}^2} \tag{B36}$$

$$C_{quqd}^{(1)} = \frac{4\mathcal{D}_{d^\dagger QS_4^\dagger}^{f_1f_3p_1*}\mathcal{D}_{QS_4u^\dagger}^{f_2p_1f_4*}}{M_{S_4}^2} - \frac{8\mathcal{D}_{d^\dagger S_{10}^\dagger u^\dagger}^{f_1p_1f_4*}\mathcal{D}_{QQS_{10}}^{f_2f_3p_1*}}{3M_{S_{10}}^2} - \frac{8\mathcal{D}_{d^\dagger S_{10}^\dagger u^\dagger}^{f_1p_1f_4*}\mathcal{D}_{QQS_{10}}^{f_3f_2p_1*}}{3M_{S_{10}}^2} - \frac{32\mathcal{D}_{d^\dagger S_{16}u^\dagger}^{f_1p_1f_4*}\mathcal{D}_{QQS_{16}^\dagger}^{f_2f_3p_1*}}{3M_{S_{16}}^2} + \frac{32\mathcal{D}_{d^\dagger S_{16}u^\dagger}^{f_1p_1f_4*}\mathcal{D}_{QQS_{16}^\dagger}^{f_3f_2p_1*}}{3M_{S_{16}}^2} \tag{B37}$$

$$C_{quqd}^{(8)} = \frac{16\mathcal{D}_{d^\dagger QS_{19}^\dagger}^{f_1f_3p_1*}\mathcal{D}_{QS_{19}u^\dagger}^{f_2p_1f_4*}}{M_{S_{19}}^2} + \frac{8\mathcal{D}_{d^\dagger S_{10}^\dagger u^\dagger}^{f_1p_1f_4*}\mathcal{D}_{QQS_{10}}^{f_2f_3p_1*}}{M_{S_{10}}^2} + \frac{8\mathcal{D}_{d^\dagger S_{10}^\dagger u^\dagger}^{f_1p_1f_4*}\mathcal{D}_{QQS_{10}}^{f_3f_2p_1*}}{M_{S_{10}}^2} - \frac{16\mathcal{D}_{d^\dagger S_{16}u^\dagger}^{f_1p_1f_4*}\mathcal{D}_{QQS_{16}^\dagger}^{f_2f_3p_1*}}{M_{S_{16}}^2} + \frac{16\mathcal{D}_{d^\dagger S_{16}u^\dagger}^{f_1p_1f_4*}\mathcal{D}_{QQS_{16}^\dagger}^{f_3f_2p_1*}}{M_{S_{16}}^2} \tag{B38}$$

$$C_{lequ}^{(1)} = -\frac{2\mathcal{D}_{e^\dagger LS_4^\dagger}^{f_1f_2p_1*}\mathcal{D}_{QS_4u^\dagger}^{f_3p_1f_4*}}{M_{S_4}^2} - \frac{2\mathcal{D}_{e^\dagger QS_{13}^\dagger}^{f_1f_3p_1*}\mathcal{D}_{LS_{13}u^\dagger}^{f_2p_1f_4*}}{M_{S_{13}}^2} - \frac{2\mathcal{D}_{e^\dagger S_{10}u^\dagger}^{f_1p_1f_4*}\mathcal{D}_{LQS_{10}^\dagger}^{f_2f_3p_1*}}{M_{S_{10}}^2} \tag{B39}$$

$$C_{lequ}^{(3)} = \frac{\mathcal{D}_{e^\dagger S_{10}u^\dagger}^{f_1p_1f_4*}\mathcal{D}_{LQS_{10}^\dagger}^{f_2f_3p_1*}}{2M_{S_{10}}^2} - \frac{\mathcal{D}_{e^\dagger QS_{13}^\dagger}^{f_1f_3p_1*}\mathcal{D}_{LS_{13}u^\dagger}^{f_2p_1f_4*}}{2M_{S_{13}}^2} \tag{B40}$$

d. B-violating operators

$$C_{duq} = -\frac{4\mathcal{D}_{dQV_8}^{f_3f_2p_1}\mathcal{D}_{L^\dagger u^\dagger V_8}^{f_1f_4p_1*}}{M_{V_8}^2} - \frac{4\mathcal{D}_{d^\dagger L^\dagger V_7}^{f_3f_1p_1*}\mathcal{D}_{QuV_7}^{f_2f_4p_1}}{M_{V_7}^2} + \frac{8\mathcal{D}_{d^\dagger S_{10}^\dagger u^\dagger}^{f_3p_1f_4*}\mathcal{D}_{LQS_{10}^\dagger}^{f_1f_2p_1}}{M_{S_{10}}^2} \tag{B41}$$

$$C_{qqu} = \frac{2\mathcal{D}_{e^\dagger S_{10}u^\dagger}^{f_3p_1f_4*}\mathcal{D}_{QQS_{10}}^{f_1f_2p_1}}{M_{S_{10}}^2} - \frac{4\mathcal{D}_{eQV_7^\dagger}^{f_3f_1p_1}\mathcal{D}_{QuV_7}^{f_2f_4p_1}}{M_{V_7}^2} \tag{B42}$$

$$C_{qqq} = \frac{2\mathcal{D}_{LQS_{10}^\dagger}^{f_1f_4p_1}\mathcal{D}_{QQS_{10}}^{f_2f_3p_1}}{M_{S_{10}}^2} + \frac{\mathcal{D}_{LQS_{14}^\dagger}^{f_1f_4p_1}\mathcal{D}_{QQS_{14}}^{f_2f_3p_1}}{M_{S_{14}}^2} + \frac{2\mathcal{D}_{LQS_{10}^\dagger}^{f_1f_4p_1}\mathcal{D}_{QQS_{10}}^{f_3f_2p_1}}{M_{S_{10}}^2} - \frac{\mathcal{D}_{LQS_{14}^\dagger}^{f_1f_4p_1}\mathcal{D}_{QQS_{14}}^{f_3f_2p_1}}{M_{S_{14}}^2} \tag{B43}$$

$$C_{duu} = -\frac{8\mathcal{D}_{d^\dagger e^\dagger S_9}^{f_1f_2p_1*}\mathcal{D}_{S_9^\dagger u^\dagger u^\dagger}^{p_1f_3f_4*}}{M_{S_9}^2} + \frac{8\mathcal{D}_{d^\dagger e^\dagger S_9}^{f_1f_2p_1*}\mathcal{D}_{S_9^\dagger u^\dagger u^\dagger}^{p_1f_4f_3*}}{M_{S_9}^2} - \frac{8\mathcal{D}_{d^\dagger S_{10}^\dagger u^\dagger}^{f_1p_1f_3*}\mathcal{D}_{e^\dagger S_{10}u^\dagger}^{f_2p_1f_4*}}{M_{S_{10}}^2} \tag{B44}$$

3. Dimension-7

a. B-conserving operators

$$
\begin{aligned}
C_{LH} = &-\frac{C_5^{f_1 f_2}(\mathcal{C}^{p_1}_{HH^\dagger S_5})^2}{M_{S_5}^4} + \frac{2C_5^{f_1 f_2}(\mathcal{D}^{p_1 *}_{HH^\dagger V_1 D})^2}{M_{V_1}^2} - \frac{2C_5^{f_1 f_2}(\mathcal{D}^{p_1}_{HH^\dagger V_1 D})^2}{M_{V_1}^2} \\
&+ \frac{C_5^{f_1 f_2}(\mathcal{D}^{p_1 *}_{HH^\dagger V_4 D})^2}{M_{V_4}^2} - \frac{C_5^{f_1 f_2}(\mathcal{D}^{p_1}_{HH^\dagger V_4 D})^2}{M_{V_4}^2} + \frac{\lambda_H \mathcal{D}^{p_1 f_1}_{F_1 HL}\mathcal{D}^{p_1 f_2}_{F_1 HL}}{M_{F_1}^3} \\
&- \frac{C_5^{f_2 p_1}\mathcal{D}^{p_2 p_1 *}_{F_1 HL}\mathcal{D}^{p_2 f_1}_{F_1 HL}}{2M_{F_1}^2} - \frac{C_5^{p_1 f_2}\mathcal{D}^{p_2 p_1 *}_{F_1 HL}\mathcal{D}^{p_2 f_1}_{F_1 HL}}{2M_{F_1}^2} - \frac{\mathcal{D}^{p_1 p_2}_{F_1 F_{3L} H^\dagger}\mathcal{D}^{p_3 p_2}_{F_1 F_{3R}^\dagger H}\mathcal{D}^{p_1 f_2}_{F_1 HL}\mathcal{D}^{p_3 f_1}_{F_1 HL}}{M_{F_1}^2 M_{F_3}} \\
&+ \frac{\lambda_H \mathcal{D}^{p_1 f_1}_{F_5 HL}\mathcal{D}^{p_1 f_2}_{F_5 HL}}{2M_{F_5}^3} - \frac{C_5^{f_2 p_1}\mathcal{D}^{p_2 p_1 *}_{F_5 HL}\mathcal{D}^{p_2 f_1}_{F_5 HL}}{4M_{F_5}^2} - \frac{C_5^{p_1 f_2}\mathcal{D}^{p_2 p_1 *}_{F_5 HL}\mathcal{D}^{p_2 f_1}_{F_5 HL}}{4M_{F_5}^2} \\
&- \frac{\mathcal{D}^{p_1 p_2}_{F_1 F_{3R}^\dagger H}\mathcal{D}^{p_1 f_2}_{F_1 HL}\mathcal{D}^{p_2 p_3}_{F_{3L} F_5 H^\dagger}\mathcal{D}^{p_3 f_1}_{F_5 HL}}{2M_{F_1} M_{F_3} M_{F_5}} + \frac{\mathcal{D}^{p_1 p_2}_{F_5 F_{7L} H^\dagger}\mathcal{D}^{p_3 p_2}_{F_5 F_{7R}^\dagger H}\mathcal{D}^{p_1 f_2}_{F_5 HL}\mathcal{D}^{p_3 f_1}_{F_5 HL}}{3M_{F_5}^2 M_{F_7}} \\
&+ \frac{\mathcal{D}^{p_1 p_2}_{F_{3L} F_5 H^\dagger}\mathcal{D}^{p_1 p_3}_{F_{3R}^\dagger F_5 H}\mathcal{D}^{p_2 f_2}_{F_5 HL}\mathcal{D}^{p_3 f_1}_{F_5 HL}}{4M_{F_3} M_{F_5}^2} + \frac{\mathcal{D}^{p_1 p_2}_{F_1 F_{3R}^\dagger H}\mathcal{D}^{p_1 f_1}_{F_1 HL}\mathcal{D}^{p_2 p_3}_{F_{3L} F_5 H^\dagger}\mathcal{D}^{p_3 f_2}_{F_5 HL}}{M_{F_1} M_{F_3} M_{F_5}} \\
&- \frac{\mathcal{D}^{p_1 p_2}_{F_1 F_{3L} H^\dagger}\mathcal{D}^{p_1 f_1}_{F_1 HL}\mathcal{D}^{p_2 p_3}_{F_{3R}^\dagger F_5 H}\mathcal{D}^{p_3 f_2}_{F_5 HL}}{2M_{F_1} M_{F_3} M_{F_5}} - \frac{\mathcal{D}^{p_1 p_2}_{F_{3L} F_{6R}^\dagger H}\mathcal{D}^{p_1 p_3}_{F_{3R}^\dagger F_5 H}\mathcal{D}^{p_3 f_2}_{F_5 HL}\mathcal{D}^{p_2 f_1}_{F_{6L} H^\dagger L}}{4M_{F_3} M_{F_5} M_{F_6}} \\
&- \frac{C_5^{f_2 p_1}\mathcal{D}^{p_2 p_1 *}_{F_{6L} H^\dagger L}\mathcal{D}^{p_2 f_1}_{F_{6L} H^\dagger L}}{2M_{F_6}^2} - \frac{C_5^{p_1 f_2}\mathcal{D}^{p_2 p_1 *}_{F_{6L} H^\dagger L}\mathcal{D}^{p_2 f_1}_{F_{6L} H^\dagger L}}{2M_{F_6}^2} \\
&+ \frac{3\mathcal{D}^{p_1 p_2}_{F_{3L} F_{6R}^\dagger H}\mathcal{D}^{p_1 p_3}_{F_{3R}^\dagger F_5 H}\mathcal{D}^{p_3 f_1}_{F_5 HL}\mathcal{D}^{p_2 f_2}_{F_{6L} H^\dagger L}}{4M_{F_3} M_{F_5} M_{F_6}} - \frac{3\mathcal{D}^{p_1 p_2}_{F_1 F_{3R}^\dagger H}\mathcal{D}^{p_1 f_2}_{F_1 HL}\mathcal{D}^{p_2 p_3}_{F_{3L} F_{6R}^\dagger H}\mathcal{D}^{p_3 f_1}_{F_{6L} H^\dagger L}}{2M_{F_1} M_{F_3} M_{F_6}} \\
&+ \frac{5\mathcal{D}^{p_1 p_2}_{F_1 F_{3R}^\dagger H}\mathcal{D}^{p_1 f_1}_{F_1 HL}\mathcal{D}^{p_2 p_3}_{F_{3L} F_{6R}^\dagger H}\mathcal{D}^{p_3 f_2}_{F_{6L} H^\dagger L}}{2M_{F_1} M_{F_3} M_{F_6}} + \frac{\mathcal{D}^{p_1 p_2}_{F_5 F_{7R}^\dagger H}\mathcal{D}^{p_1 f_2}_{F_5 HL}\mathcal{D}^{p_3 f_1}_{F_{6L} H^\dagger L}\mathcal{D}^{p_3 p_2}_{F_{6R}^\dagger F_{7L} H}}{6M_{F_5} M_{F_6} M_{F_7}} \\
&- \frac{\mathcal{D}^{p_1 p_2}_{F_5 F_{7R}^\dagger H}\mathcal{D}^{p_1 f_1}_{F_5 HL}\mathcal{D}^{p_3 f_2}_{F_{6L} H^\dagger L}\mathcal{D}^{p_3 p_2}_{F_{6R}^\dagger F_{7L} H}}{2M_{F_5} M_{F_6} M_{F_7}} - \frac{\mathcal{D}^{p_1 f_2}_{F_{6L} H^\dagger L}\mathcal{D}^{p_1 f_1 p_2}_{F_{6R}^\dagger L S_8}\mathcal{D}^{p_2}_{HHHS_8^\dagger}}{M_{F_6} M_{S_8}^2} \\
&- \frac{\mathcal{D}^{p_1 f_2}_{F_5 HL}\mathcal{D}^{p_1 f_1 p_2}_{F_{5L} S_7}\mathcal{D}^{p_2}_{HHH^\dagger S_7^\dagger}}{6M_{F_5} M_{S_7}^2} - \frac{\mathcal{D}^{p_1 f_1}_{F_5 HL}\mathcal{D}^{p_1 f_2 p_2}_{F_{5L} S_7}\mathcal{D}^{p_2}_{HHH^\dagger S_7^\dagger}}{6M_{F_5} M_{S_7}^2} - \frac{2C_5^{f_1 f_2}\mathcal{C}^{p_1 *}_{HHS_6^\dagger}\mathcal{C}^{p_1}_{HHS_6^\dagger}}{M_{S_6}^4} \\
&- \frac{\mathcal{D}^{p_1 p_2}_{F_1 F_{3L} H^\dagger}\mathcal{D}^{p_1 f_2}_{F_1 HL}\mathcal{D}^{p_2 f_1 p_3}_{F_{3R}^\dagger L S_6}\mathcal{C}^{p_3}_{HHS_6^\dagger}}{2M_{F_1} M_{F_3} M_{S_6}^2} - \frac{\mathcal{D}^{p_1 p_2}_{F_1 F_{3L} H^\dagger}\mathcal{D}^{p_1 f_1}_{F_1 HL}\mathcal{D}^{p_2 f_2 p_3}_{F_{3R}^\dagger L S_6}\mathcal{C}^{p_3}_{HHS_6^\dagger}}{2M_{F_1} M_{F_3} M_{S_6}^2} \\
&- \frac{\mathcal{D}^{p_1 p_2}_{F_{3L} F_5 H^\dagger}\mathcal{D}^{p_1 f_2 p_3}_{F_{3R}^\dagger L S_6}\mathcal{D}^{p_2 f_1}_{F_5 HL}\mathcal{C}^{p_3}_{HHS_6^\dagger}}{4M_{F_3} M_{F_5} M_{S_6}^2} + \frac{3\mathcal{D}^{p_1 p_2}_{F_{3L} F_5 H^\dagger}\mathcal{D}^{p_1 f_1 p_3}_{F_{3R}^\dagger L S_6}\mathcal{D}^{p_2 f_2}_{F_5 HL}\mathcal{C}^{p_3}_{HHS_6^\dagger}}{4M_{F_3} M_{F_5} M_{S_6}^2} \\
&+ \frac{\mathcal{D}^{p_1 p_2 p_3}_{F_1 F_{6R}^\dagger S_6}\mathcal{D}^{p_1 f_2}_{F_1 HL}\mathcal{D}^{p_2 f_1}_{F_{6L} H^\dagger L}\mathcal{C}^{p_3}_{HHS_6^\dagger}}{2M_{F_1} M_{F_6} M_{S_6}^2} - \frac{\mathcal{D}^{p_1 p_2}_{F_{3L} F_{6R}^\dagger H}\mathcal{D}^{p_1 f_2 p_3}_{F_{3R}^\dagger L S_6}\mathcal{D}^{p_2 f_1}_{F_{6L} H^\dagger L}\mathcal{C}^{p_3}_{HHS_6^\dagger}}{M_{F_3} M_{F_6} M_{S_6}^2}
\end{aligned}
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\begin{aligned}
&-\frac{\mathcal{D}^{p_1p_2p_3}_{F_5F^\dagger_{6R}S_6}\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_2f_1}_{F_{6L}H^\dagger L}\mathcal{C}^{p_3}_{HHS^\dagger_6}}{4M_{F_5}M_{F_6}M^2_{S_6}}-\frac{3\mathcal{D}^{p_1p_2p_3}_{F_1F^\dagger_{6R}S_6}\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_2f_2}_{F_{6L}H^\dagger L}\mathcal{C}^{p_3}_{HHS^\dagger_6}}{2M_{F_1}M_{F_6}M^2_{S_6}}\\
&+\frac{2\mathcal{D}^{p_1p_2}_{F_{3L}F^\dagger_{6R}H}\mathcal{D}^{p_1f_1p_3}_{F^\dagger_{3R}LS_6}\mathcal{D}^{p_2f_2}_{F_{6L}H^\dagger L}\mathcal{C}^{p_3}_{HHS^\dagger_6}}{M_{F_3}M_{F_6}M^2_{S_6}}+\frac{3\mathcal{D}^{p_1p_2p_3}_{F_5F^\dagger_{6R}S_6}\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_2f_2}_{F_{6L}H^\dagger L}\mathcal{C}^{p_3}_{HHS^\dagger_6}}{4M_{F_5}M_{F_6}M^2_{S_6}}\\
&+\frac{\mathcal{D}^{p_1p_2}_{F_5F_{7L}H^\dagger}\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_2f_1p_3}_{F^\dagger_{7R}LS_6}\mathcal{C}^{p_3}_{HHS^\dagger_6}}{6M_{F_5}M_{F_7}M^2_{S_6}}-\frac{\mathcal{D}^{p_1f_2}_{F_{6L}H^\dagger L}\mathcal{D}^{p_1p_2}_{F^\dagger_{6R}F_{7L}H}\mathcal{D}^{p_2f_1p_3}_{F^\dagger_{7R}LS_6}\mathcal{C}^{p_3}_{HHS^\dagger_6}}{3M_{F_6}M_{F_7}M^2_{S_6}}\\
&+\frac{\mathcal{D}^{p_1p_2}_{F_5F_{7L}H^\dagger}\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_2f_2p_3}_{F^\dagger_{7R}LS_6}\mathcal{C}^{p_3}_{HHS^\dagger_6}}{6M_{F_5}M_{F_7}M^2_{S_6}}+\frac{4C^{f_1f_2}_5\mathcal{D}^{p_1*}_{HHV^\dagger_2D}\mathcal{D}^{p_1}_{HHV^\dagger_2D}}{M^2_{V_2}}\\
&+\frac{\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{p_1f_1p_2}_{F_{1L}S_4}\mathcal{D}^{p_2*}_{HH^\dagger H^\dagger S_4}}{2M_{F_1}M^2_{S_4}}+\frac{\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_1f_2p_2}_{F_{1L}S_4}\mathcal{D}^{p_2*}_{HH^\dagger H^\dagger S_4}}{2M_{F_1}M^2_{S_4}}-\frac{\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_1p_2}_{F_{5L}S_4}\mathcal{D}^{p_2*}_{HH^\dagger H^\dagger S_4}}{4M_{F_5}M^2_{S_4}}\\
&-\frac{\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_2p_2}_{F_{5L}S_4}\mathcal{D}^{p_2*}_{HH^\dagger H^\dagger S_4}}{4M_{F_5}M^2_{S_4}}+\frac{\mathcal{D}^{p_1f_2p_2}_{F_{3L}LS_1}\mathcal{D}^{p_1p_3}_{F^\dagger_{3R}F_5H}\mathcal{D}^{p_3f_1}_{F_5HL}\mathcal{C}^{p_2}_{HH^\dagger S_1}}{4M_{F_3}M_{F_5}M^2_{S_1}}\\
&+\frac{\mathcal{D}^{p_1f_1p_2}_{F_{3L}LS_1}\mathcal{D}^{p_1p_3}_{F^\dagger_{3R}F_5H}\mathcal{D}^{p_3f_2}_{F_5HL}\mathcal{C}^{p_2}_{HH^\dagger S_1}}{4M_{F_3}M_{F_5}M^2_{S_1}}+\frac{\mathcal{D}^{p_1f_2p_2}_{F_{3L}LS_1}\mathcal{D}^{p_1f_1p_3}_{F^\dagger_{3R}LS_6}\mathcal{C}^{p_3}_{HHS^\dagger_6}\mathcal{C}^{p_2}_{HH^\dagger S_1}}{M_{F_3}M^2_{S_1}M^2_{S_6}}\\
&-\frac{\mathcal{D}^{p_1p_2p_3}_{F_1F_1S_1}\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_2f_2}_{F_1HL}\mathcal{C}^{p_3}_{HH^\dagger S_1}}{M^2_{F_1}M^2_{S_1}}-\frac{\mathcal{D}^{p_1p_2}_{F_1F^\dagger_{3R}H}\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{p_2f_1p_3}_{F_{3L}LS_1}\mathcal{C}^{p_3}_{HH^\dagger S_1}}{2M_{F_1}M_{F_3}M^2_{S_1}}\\
&+\frac{3\mathcal{D}^{p_1p_2}_{F_1F^\dagger_{3R}H}\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_2f_2p_3}_{F_{3L}LS_1}\mathcal{C}^{p_3}_{HH^\dagger S_1}}{2M_{F_1}M_{F_3}M^2_{S_1}}+\frac{\mathcal{D}^{p_1p_2p_3}_{F_5F_5S_1}\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_2f_2}_{F_5HL}\mathcal{C}^{p_3}_{HH^\dagger S_1}}{2M^2_{F_5}M^2_{S_1}}\\
&+\frac{\mathcal{D}^{p_1f_2p_2}_{F_{3L}LS_5}\mathcal{D}^{p_1p_3}_{F^\dagger_{3R}F_5H}\mathcal{D}^{p_3f_1}_{F_5HL}\mathcal{C}^{p_2}_{HH^\dagger S_5}}{8M_{F_3}M_{F_5}M^2_{S_5}}+\frac{\mathcal{D}^{p_1f_1p_2}_{F_{3L}LS_5}\mathcal{D}^{p_1p_3}_{F^\dagger_{3R}F_5H}\mathcal{D}^{p_3f_2}_{F_5HL}\mathcal{C}^{p_2}_{HH^\dagger S_5}}{8M_{F_3}M_{F_5}M^2_{S_5}}\\
&+\frac{\mathcal{D}^{p_1f_2p_2}_{F_{3L}LS_5}\mathcal{D}^{p_1f_1p_3}_{F^\dagger_{3R}LS_6}\mathcal{C}^{p_3}_{HHS^\dagger_6}\mathcal{C}^{p_2}_{HH^\dagger S_5}}{2M_{F_3}M^2_{S_5}M^2_{S_6}}-\frac{\mathcal{D}^{p_1f_2p_2}_{F_{7L}LS_5}\mathcal{D}^{p_1f_1p_3}_{F^\dagger_{7R}LS_6}\mathcal{C}^{p_3}_{HHS^\dagger_6}\mathcal{C}^{p_2}_{HH^\dagger S_5}}{3M_{F_7}M^2_{S_5}M^2_{S_6}}\\
&-\frac{\mathcal{D}^{p_1p_2}_{F_1F^\dagger_{3R}H}\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{p_2f_1p_3}_{F_{3L}LS_5}\mathcal{C}^{p_3}_{HH^\dagger S_5}}{4M_{F_1}M_{F_3}M^2_{S_5}}+\frac{3\mathcal{D}^{p_1p_2}_{F_1F^\dagger_{3R}H}\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_2f_2p_3}_{F_{3L}LS_5}\mathcal{C}^{p_3}_{HH^\dagger S_5}}{4M_{F_1}M_{F_3}M^2_{S_5}}\\
&+\frac{\mathcal{D}^{p_1p_2p_3}_{F_1F_5S_5}\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_2f_2}_{F_5HL}\mathcal{C}^{p_3}_{HH^\dagger S_5}}{2M_{F_1}M_{F_5}M^2_{S_5}}+\frac{\mathcal{D}^{p_1p_2}_{F_5F^\dagger_{7R}H}\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_2f_1p_3}_{F_{7L}LS_5}\mathcal{C}^{p_3}_{HH^\dagger S_5}}{6M_{F_5}M_{F_7}M^2_{S_5}}\\
&-\frac{\mathcal{D}^{p_1p_2}_{F_5F^\dagger_{7R}H}\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_2f_2p_3}_{F_{7L}LS_5}\mathcal{C}^{p_3}_{HH^\dagger S_5}}{2M_{F_5}M_{F_7}M^2_{S_5}}-\frac{4C^{f_1f_2}_5\mathcal{D}^{p_1*}_{HH^\dagger V_4D}\mathcal{D}^{p_1}_{HH^\dagger V_4D}}{M^2_{V_4}}\\
&-\frac{\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{p_1f_1p_2}_{F_{1L}S_4}\mathcal{C}^{p_3}_{HH^\dagger S_1}\mathcal{C}^{p_3p_2}_{HS_1S^\dagger_4}}{2M_{F_1}M^2_{S_1}M^2_{S_4}}-\frac{\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_1f_2p_2}_{F_{1L}S_4}\mathcal{C}^{p_3}_{HH^\dagger S_1}\mathcal{C}^{p_3p_2}_{HS_1S^\dagger_4}}{2M_{F_1}M^2_{S_1}M^2_{S_4}}
\end{aligned}
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\begin{aligned}
&+\frac{\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_1p_2}_{F_{5L}S_4}\mathcal{C}^{p_3}_{HH^\dagger S_1}\mathcal{C}^{p_3p_2}_{HS_1S_4^\dagger}}{4M_{F_5}M_{S_1}^2M_{S_4}^2}+\frac{\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_2p_2}_{F_{5L}S_4}\mathcal{C}^{p_3}_{HH^\dagger S_1}\mathcal{C}^{p_3p_2}_{HS_1S_4^\dagger}}{4M_{F_5}M_{S_1}^2M_{S_4}^2}\\
&-\frac{\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{p_1f_1p_2}_{F_{1L}S_4}\mathcal{C}^{p_3}_{HHS_6^\dagger}\mathcal{C}^{p_2p_3*}_{HS_4S_6^\dagger}}{2M_{F_1}M_{S_4}^2M_{S_6}^2}-\frac{\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_1f_2p_2}_{F_{1L}S_4}\mathcal{C}^{p_3}_{HHS_6^\dagger}\mathcal{C}^{p_2p_3*}_{HS_4S_6^\dagger}}{2M_{F_1}M_{S_4}^2M_{S_6}^2}\\
&+\frac{\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_1p_2}_{F_{5L}S_4}\mathcal{C}^{p_3}_{HHS_6^\dagger}\mathcal{C}^{p_2p_3*}_{HS_4S_6^\dagger}}{4M_{F_5}M_{S_4}^2M_{S_6}^2}+\frac{\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_2p_2}_{F_{5L}S_4}\mathcal{C}^{p_3}_{HHS_6^\dagger}\mathcal{C}^{p_2p_3*}_{HS_4S_6^\dagger}}{4M_{F_5}M_{S_4}^2M_{S_6}^2}\\
&+\frac{\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_1p_2}_{F_{5L}S_7}\mathcal{C}^{p_3}_{HH^\dagger S_5}\mathcal{C}^{p_3p_2}_{HS_5S_7^\dagger}}{6M_{F_5}M_{S_5}^2M_{S_7}^2}+\frac{\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_2p_2}_{F_{5L}S_7}\mathcal{C}^{p_3}_{HH^\dagger S_5}\mathcal{C}^{p_3p_2}_{HS_5S_7^\dagger}}{6M_{F_5}M_{S_5}^2M_{S_7}^2}\\
&-\frac{\mathcal{D}^{p_1f_2}_{F_{6L}H^\dagger L}\mathcal{D}^{p_1f_1p_2}_{F_{6R}^\dagger LS_8}\mathcal{C}^{p_3}_{HHS_6^\dagger}\mathcal{C}^{p_3p_2}_{HS_6S_8^\dagger}}{M_{F_6}M_{S_6}^2M_{S_8}^2}-\frac{\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_1p_2}_{F_{5L}S_7}\mathcal{C}^{p_3}_{HHS_6^\dagger}\mathcal{C}^{p_3p_2*}_{HS_6^\dagger S_7}}{6M_{F_5}M_{S_6}^2M_{S_7}^2}\\
&-\frac{\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_2p_2}_{F_{5L}S_7}\mathcal{C}^{p_3}_{HHS_6^\dagger}\mathcal{C}^{p_3p_2*}_{HS_6^\dagger S_7}}{6M_{F_5}M_{S_6}^2M_{S_7}^2}+\frac{\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{p_1f_1p_2}_{F_{1L}S_4}\mathcal{C}^{p_3}_{HH^\dagger S_5}\mathcal{C}^{p_2p_3*}_{H^\dagger S_4S_5}}{4M_{F_1}M_{S_4}^2M_{S_5}^2}\\
&+\frac{\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_1f_2p_2}_{F_{1L}S_4}\mathcal{C}^{p_3}_{HH^\dagger S_5}\mathcal{C}^{p_2p_3*}_{H^\dagger S_4S_5}}{4M_{F_1}M_{S_4}^2M_{S_5}^2}-\frac{\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_1p_2}_{F_{5L}S_4}\mathcal{C}^{p_3}_{HH^\dagger S_5}\mathcal{C}^{p_2p_3*}_{H^\dagger S_4S_5}}{8M_{F_5}M_{S_4}^2M_{S_5}^2}\\
&-\frac{\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_2p_2}_{F_{5L}S_4}\mathcal{C}^{p_3}_{HH^\dagger S_5}\mathcal{C}^{p_2p_3*}_{H^\dagger S_4S_5}}{8M_{F_5}M_{S_4}^2M_{S_5}^2}-\frac{\mathcal{D}^{p_1p_2}_{HHS_1S_6^\dagger}\mathcal{C}^{p_1}_{HH^\dagger S_1}\mathcal{D}^{f_1f_2p_2}_{LLS_6}}{M_{S_1}^2M_{S_6}^2}\\
&+\frac{\mathcal{D}^{p_1p_2}_{HHS_5S_6^\dagger}\mathcal{C}^{p_1}_{HH^\dagger S_5}\mathcal{D}^{f_1f_2p_2}_{LLS_6}}{2M_{S_5}^2M_{S_6}^2}-\frac{\mathcal{C}^{p_1}_{HHS_6^\dagger}\mathcal{D}^{p_1p_2}_{HH^\dagger S_6S_6^\dagger(2)}\mathcal{D}^{f_1f_2p_2}_{LLS_6}}{M_{S_6}^4}\\
&+\frac{\mathcal{D}^{p_1*}_{HH^\dagger H^\dagger S_4}\mathcal{C}^{p_1p_2}_{HS_4S_6^\dagger}\mathcal{D}^{f_1f_2p_2}_{LLS_6}}{M_{S_4}^2M_{S_6}^2}-\frac{\mathcal{D}^{p_1}_{HHHS_8^\dagger}\mathcal{C}^{p_2p_1*}_{HS_6S_8^\dagger}\mathcal{D}^{f_1f_2p_2}_{LLS_6}}{M_{S_6}^2M_{S_8}^2}\\
&-\frac{\mathcal{C}^{p_1}_{HHS_6^\dagger}\mathcal{C}^{p_2p_3*}_{HS_6S_8^\dagger}\mathcal{C}^{p_1p_3}_{HS_6S_8^\dagger}\mathcal{D}^{f_1f_2p_2}_{LLS_6}}{M_{S_6}^4M_{S_8}^2}-\frac{\mathcal{D}^{p_1}_{HHH^\dagger S_7^\dagger}\mathcal{C}^{p_2p_1}_{HS_6^\dagger S_7}\mathcal{D}^{f_1f_2p_2}_{LLS_6}}{3M_{S_6}^2M_{S_7}^2}\\
&-\frac{\mathcal{C}^{p_1}_{HH^\dagger S_1}\mathcal{C}^{p_1p_2}_{HS_1S_4^\dagger}\mathcal{C}^{p_2p_3}_{HS_4S_6^\dagger}\mathcal{D}^{f_1f_2p_3}_{LLS_6}}{M_{S_1}^2M_{S_4}^2M_{S_6}^2}-\frac{\mathcal{C}^{p_1}_{HHS_6^\dagger}\mathcal{C}^{p_2p_1*}_{HS_4S_6^\dagger}\mathcal{C}^{p_2p_3}_{HS_4S_6^\dagger}\mathcal{D}^{f_1f_2p_3}_{LLS_6}}{M_{S_4}^2M_{S_6}^4}\\
&+\frac{\mathcal{C}^{p_1}_{HH^\dagger S_5}\mathcal{C}^{p_1p_2}_{HS_5S_7^\dagger}\mathcal{C}^{p_3p_2}_{HS_6^\dagger S_7}\mathcal{D}^{f_1f_2p_3}_{LLS_6}}{3M_{S_5}^2M_{S_6}^2M_{S_7}^2}-\frac{\mathcal{C}^{p_1}_{HHS_6^\dagger}\mathcal{C}^{p_1p_2*}_{HS_6^\dagger S_7}\mathcal{C}^{p_3p_2}_{HS_6^\dagger S_7}\mathcal{D}^{f_1f_2p_3}_{LLS_6}}{3M_{S_6}^4M_{S_7}^2}\\
&+\frac{\mathcal{C}^{p_1}_{HH^\dagger S_5}\mathcal{C}^{p_2p_3}_{HS_4S_6^\dagger}\mathcal{C}^{p_2p_1*}_{H^\dagger S_4S_5}\mathcal{D}^{f_1f_2p_3}_{LLS_6}}{2M_{S_4}^2M_{S_5}^2M_{S_6}^2}+\frac{iC_5^{f_2p_1}\mathcal{D}^{p_2*}_{HH^\dagger V_1D}\mathcal{D}^{f_1p_1p_2}_{LL^\dagger V_1}}{M_{V_1}^2}\\
&+\frac{iC_5^{p_1f_2}\mathcal{D}^{p_2*}_{HH^\dagger V_1D}\mathcal{D}^{f_1p_1p_2}_{LL^\dagger V_1}}{M_{V_1}^2}+\frac{iC_5^{f_2p_1}\mathcal{D}^{p_2}_{HH^\dagger V_1D}\mathcal{D}^{f_1p_1p_2}_{LL^\dagger V_1}}{M_{V_1}^2}+\frac{iC_5^{p_1f_2}\mathcal{D}^{p_2}_{HH^\dagger V_1D}\mathcal{D}^{f_1p_1p_2}_{LL^\dagger V_1}}{M_{V_1}^2}
\end{aligned}
$$

$$
+\frac{iC_5^{f_2p_1}\mathcal{D}^{p_2*}_{HH^\dagger V_4D}\mathcal{D}^{f_1p_1p_2}_{LL^\dagger V_4}}{2M_{V_4}^2}+\frac{iC_5^{p_1f_2}\mathcal{D}^{p_2*}_{HH^\dagger V_4D}\mathcal{D}^{f_1p_1p_2}_{LL^\dagger V_4}}{2M_{V_4}^2}-\frac{iC_5^{f_2p_1}\mathcal{D}^{p_2}_{HH^\dagger V_4D}\mathcal{D}^{f_1p_1p_2}_{LL^\dagger V_4}}{2M_{V_4}^2}
-\frac{iC_5^{p_1f_2}\mathcal{D}^{p_2}_{HH^\dagger V_4D}\mathcal{D}^{f_1p_1p_2}_{LL^\dagger V_4}}{2M_{V_4}^2}-\frac{\mathcal{C}^{p_1}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_1}\mathcal{D}^{f_1f_2p_3}_{LLS_6}\mathcal{C}^{p_2p_1p_3}_{S_1S_6S_6^\dagger}}{M_{S_1}^2M_{S_6}^4}
-\frac{\mathcal{C}^{p_1}_{HHS_6^\dagger}\mathcal{C}^{p_2}_{HH^\dagger S_5}\mathcal{D}^{f_1f_2p_3}_{LLS_6}\mathcal{C}^{p_2p_1p_3}_{S_5S_6S_6^\dagger}}{2M_{S_5}^2M_{S_6}^4} \tag{B45}
$$

$$
C_{LeHD}=-\frac{y_e^{p_1f_5}\mathcal{D}^{p_2f_1}_{F_1HL}\mathcal{D}^{p_2p_1}_{F_1HL}}{2M_{F_1}^3}-\frac{y_e^{p_1f_5}\mathcal{D}^{f_1p_1p_2}_{LLS_6}\mathcal{C}^{p_2}_{HHS_6^\dagger}}{M_{S_6}^4}-\frac{y_e^{p_1f_5}\mathcal{D}^{p_2f_1}_{F_5HL}\mathcal{D}^{p_2p_1}_{F_5HL}}{4M_{F_5}^3}
+\frac{y_e^{p_1f_5}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{D}^{p_1f_1p_2}_{LLS_6}}{M_{S_6}^4}+\frac{2i\mathcal{D}^{f_5p_1p_2}_{eF_1V_2}\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_2}_{HHV_2^\dagger D}}{M_{F_1}M_{V_2}^2}-\frac{\mathcal{D}^{f_5p_1*}_{e^\dagger F_{3R}^\dagger H^\dagger}\mathcal{D}^{p_2p_1}_{F_1F_{3R}^\dagger H}\mathcal{D}^{p_2f_1}_{F_1HL}}{M_{F_1}M_{F_3}^2}
-\frac{\mathcal{D}^{f_5p_1*}_{e^\dagger F_{3R}^\dagger H^\dagger}\mathcal{D}^{p_1p_2*}_{F_{3L}F_5H^\dagger}\mathcal{D}^{p_2f_1}_{F_5HL}}{M_{F_3}M_{F_5}^2}+\frac{2i\mathcal{D}^{f_5p_1*}_{e^\dagger F_{3R}^\dagger H^\dagger}\mathcal{D}^{p_1f_1p_2*}_{F_{3L}L^\dagger V_2^\dagger}\mathcal{D}^{p_2}_{HHV_2^\dagger D}}{M_{F_3}M_{V_2}^2}
-\frac{\mathcal{D}^{f_5p_1*}_{e^\dagger F_{3R}^\dagger H^\dagger}\mathcal{D}^{p_1p_2}_{F_{3R}^\dagger F_5H}\mathcal{D}^{p_2f_1}_{F_5HL}}{2M_{F_3}^2M_{F_5}}-\frac{\mathcal{D}^{f_5p_1*}_{e^\dagger F_{3R}^\dagger H^\dagger}\mathcal{D}^{p_1f_1p_2}_{F_{3R}^\dagger LS_6}\mathcal{C}^{p_2}_{HHS_6^\dagger}}{M_{F_3}^2M_{S_6}^2}+\frac{\mathcal{D}^{f_5p_1p_2*}_{e^\dagger F_5S_6^\dagger}\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{C}^{p_2}_{HHS_6^\dagger}}{M_{F_5}^2M_{S_6}^2}
-\frac{2i\mathcal{D}^{f_5f_1p_1*}_{e^\dagger L^\dagger V_3^\dagger}\mathcal{C}^{p_2}_{HHS_6^\dagger}\mathcal{D}^{p_2p_1}_{HS_6V_3^\dagger D}}{M_{S_6}^2M_{V_3}^2}-\frac{4i\mathcal{D}^{f_5f_1p_1*}_{e^\dagger L^\dagger V_3^\dagger}\mathcal{D}^{p_2}_{HHV_2^\dagger D}\mathcal{C}^{p_2p_1}_{HV_2V_3^\dagger}}{M_{V_2}^2M_{V_3}^2} \tag{B46}
$$

$$
C_{LHD1}=\frac{2\mathcal{D}^{f_1f_2p_1}_{LLS_6}\mathcal{C}^{p_1}_{HHS_6^\dagger}}{M_{S_6}^4}+\frac{\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_2}_{F_5HL}}{M_{F_5}^3} \tag{B47}
$$

$$
C_{LHD2}=-\frac{4\mathcal{D}^{f_1f_2p_1}_{LLS_6}\mathcal{C}^{p_1}_{HHS_6^\dagger}}{M_{S_6}^4}-\frac{\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_1f_2}_{F_1HL}}{M_{F_1}^3}-\frac{\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_2}_{F_5HL}}{2M_{F_5}^3} \tag{B48}
$$

$$
C_{LHB}=-\frac{ig'\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{p_1f_3}_{F_1HL}}{8M_{F_1}^3}-\frac{ig'\mathcal{D}^{f_2f_3p_1}_{LLS_6}\mathcal{C}^{p_1}_{HHS_6^\dagger}}{2M_{S_6}^4}-\frac{ig'\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_3}_{F_5HL}}{16M_{F_5}^3} \tag{B49}
$$

$$
C_{LHW}=-\frac{ig\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{p_1f_3}_{F_1HL}}{8M_{F_1}^3}-\frac{ig\mathcal{D}^{f_2f_3p_1}_{LLS_6}\mathcal{C}^{p_1}_{HHS_6^\dagger}}{4M_{S_6}^4}+\frac{ig\mathcal{D}^{f_3f_2p_1}_{LLS_6}\mathcal{C}^{p_1}_{HHS_6^\dagger}}{4M_{S_6}^4}+\frac{3ig\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_3}_{F_5HL}}{16M_{F_5}^3} \tag{B50}
$$

$$
\begin{aligned}
C_{eLLLH} = &-\frac{y_e^{f_1f_2*}\mathcal{D}^{p_1f_3}_{F_1HL}\mathcal{D}^{p_1f_4}_{F_1HL}}{2M_{F_1}^3} - \frac{3\mathcal{D}^{f_1p_1p_2}_{e^\dagger F_1S_2^\dagger}\mathcal{D}^{f_2f_3p_2}_{LLS_2}\mathcal{D}^{p_1f_4}_{F_1HL}}{2M_{F_1}M_{S_2}^2} + \frac{3\mathcal{D}^{f_1p_1p_2}_{e^\dagger F_1S_2^\dagger}\mathcal{D}^{f_3f_2p_2}_{LLS_2}\mathcal{D}^{p_1f_4}_{F_1HL}}{2M_{F_1}M_{S_2}^2} \\
&-\frac{\mathcal{D}^{f_1f_2p_1}_{e^\dagger LS_4^\dagger}\mathcal{D}^{p_2f_4}_{F_1HL}\mathcal{D}^{p_2f_3p_1}_{F_{1L}S_4}}{M_{F_1}M_{S_4}^2} - \frac{y_e^{f_1f_2*}\mathcal{D}^{p_1f_3}_{F_5HL}\mathcal{D}^{p_1f_4}_{F_5HL}}{4M_{F_5}^3} - \frac{\mathcal{D}^{f_1f_4p_1}_{e^\dagger LS_4^\dagger}\mathcal{D}^{p_2f_3}_{F_5HL}\mathcal{D}^{p_2f_2p_1}_{F_{5L}S_4}}{M_{F_5}M_{S_4}^2} \\
&+\frac{\mathcal{D}^{f_1f_3p_1}_{e^\dagger LS_4^\dagger}\mathcal{D}^{p_2f_4}_{F_5HL}\mathcal{D}^{p_2f_2p_1}_{F_{5L}S_4}}{2M_{F_5}M_{S_4}^2} + \frac{\mathcal{D}^{f_1f_4p_1}_{e^\dagger LS_4^\dagger}\mathcal{D}^{p_2f_2}_{F_5HL}\mathcal{D}^{p_2f_3p_1}_{F_{5L}S_4}}{2M_{F_5}M_{S_4}^2} + \frac{\mathcal{D}^{f_1f_3p_1}_{e^\dagger LS_4^\dagger}\mathcal{D}^{p_2f_2}_{F_5HL}\mathcal{D}^{p_2f_4p_1}_{F_{5L}S_4}}{2M_{F_5}M_{S_4}^2} \\
&+\frac{\mathcal{D}^{f_1p_1}_{e^\dagger F_{4R}^\dagger H}\mathcal{D}^{p_1f_4p_2}_{F_{4L}LS_2^\dagger}\mathcal{D}^{f_2f_3p_2}_{LLS_2}}{M_{F_4}M_{S_2}^2} + \frac{2\mathcal{D}^{f_1f_4p_1}_{e^\dagger LS_4^\dagger}\mathcal{C}^{p_2p_1}_{HS_2^\dagger S_4}\mathcal{D}^{f_2f_3p_2}_{LLS_2}}{M_{S_2}^2M_{S_4}^2} \\
&-\frac{\mathcal{D}^{f_1p_1}_{e^\dagger F_{4R}^\dagger H}\mathcal{D}^{p_1f_4p_2}_{F_{4L}LS_2^\dagger}\mathcal{D}^{f_3f_2p_2}_{LLS_2}}{M_{F_4}M_{S_2}^2} - \frac{2\mathcal{D}^{f_1f_4p_1}_{e^\dagger LS_4^\dagger}\mathcal{C}^{p_2p_1}_{HS_2^\dagger S_4}\mathcal{D}^{f_3f_2p_2}_{LLS_2}}{M_{S_2}^2M_{S_4}^2} + \frac{\mathcal{D}^{f_1p_1p_2}_{e^\dagger F_1S_2^\dagger}\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{f_3f_4p_2}_{LLS_2}}{2M_{F_1}M_{S_2}^2} \\
&-\frac{\mathcal{D}^{f_1f_2p_1}_{e^\dagger LS_4^\dagger}\mathcal{C}^{p_2p_1}_{HS_2^\dagger S_4}\mathcal{D}^{f_3f_4p_2}_{LLS_2}}{M_{S_2}^2M_{S_4}^2} - \frac{\mathcal{D}^{f_1p_1p_2}_{e^\dagger F_1S_2^\dagger}\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{f_4f_3p_2}_{LLS_2}}{2M_{F_1}M_{S_2}^2} + \frac{\mathcal{D}^{f_1f_2p_1}_{e^\dagger LS_4^\dagger}\mathcal{C}^{p_2p_1}_{HS_2^\dagger S_4}\mathcal{D}^{f_4f_3p_2}_{LLS_2}}{M_{S_2}^2M_{S_4}^2} \\
&-\frac{\mathcal{D}^{f_1p_1}_{e^\dagger F_{4R}^\dagger H}\mathcal{D}^{p_1f_4p_2}_{F_{4L}LS_6^\dagger}\mathcal{D}^{f_2f_3p_2}_{LLS_6}}{2M_{F_4}M_{S_6}^2} - \frac{3\mathcal{D}^{f_1p_1p_2}_{e^\dagger F_5S_6^\dagger}\mathcal{D}^{p_1f_4}_{F_5HL}\mathcal{D}^{f_2f_3p_2}_{LLS_6}}{4M_{F_5}M_{S_6}^2} - \frac{\mathcal{D}^{f_1f_4p_1}_{e^\dagger LS_4^\dagger}\mathcal{C}^{p_1p_2}_{HS_4S_6^\dagger}\mathcal{D}^{f_2f_3p_2}_{LLS_6}}{2M_{S_4}^2M_{S_6}^2} \\
&+\frac{\mathcal{D}^{f_1p_1}_{e^\dagger F_{4R}^\dagger H}\mathcal{D}^{p_1f_4p_2}_{F_{4L}LS_6^\dagger}\mathcal{D}^{f_3f_2p_2}_{LLS_6}}{2M_{F_4}M_{S_6}^2} + \frac{3\mathcal{D}^{f_1p_1p_2}_{e^\dagger F_5S_6^\dagger}\mathcal{D}^{p_1f_4}_{F_5HL}\mathcal{D}^{f_3f_2p_2}_{LLS_6}}{4M_{F_5}M_{S_6}^2} + \frac{\mathcal{D}^{f_1f_4p_1}_{e^\dagger LS_4^\dagger}\mathcal{C}^{p_1p_2}_{HS_4S_6^\dagger}\mathcal{D}^{f_3f_2p_2}_{LLS_6}}{2M_{S_4}^2M_{S_6}^2} \\
&+\frac{\mathcal{D}^{f_1p_1p_2}_{e^\dagger F_5S_6^\dagger}\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{f_3f_4p_2}_{LLS_6}}{4M_{F_5}M_{S_6}^2} - \frac{\mathcal{D}^{f_1p_1}_{e^\dagger F_{4R}^\dagger H}\mathcal{D}^{p_1f_2p_2}_{F_{4L}LS_6^\dagger}\mathcal{D}^{f_4f_3p_2}_{LLS_6}}{M_{F_4}M_{S_6}^2} - \frac{5\mathcal{D}^{f_1p_1p_2}_{e^\dagger F_5S_6^\dagger}\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{f_4f_3p_2}_{LLS_6}}{4M_{F_5}M_{S_6}^2} \\
&-\frac{\mathcal{D}^{f_1f_2p_1}_{e^\dagger LS_4^\dagger}\mathcal{C}^{p_1p_2}_{HS_4S_6^\dagger}\mathcal{D}^{f_4f_3p_2}_{LLS_6}}{M_{S_4}^2M_{S_6}^2}
\end{aligned}
\tag{B51}
$$

$$
\begin{aligned}
C_{dLQLH1} = &-\frac{y_d^{f_1f_4*}\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{p_1f_3}_{F_1HL}}{M_{F_1}^3} - \frac{y_d^{f_1f_4*}\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_3}_{F_5HL}}{2M_{F_5}^3} + \frac{8\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_2p_2}_{F_{10R}^\dagger LS_{10}}\mathcal{D}^{f_3f_4p_2}_{LQS_{10}^\dagger}}{M_{F_{10}}M_{S_{10}}^2} \\
&+\frac{4\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_2p_2}_{F_{10R}^\dagger LS_{14}}\mathcal{D}^{f_3f_4p_2}_{LQS_{14}^\dagger}}{M_{F_{10}}M_{S_{14}}^2} + \frac{4\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_4p_2}_{F_{10R}^\dagger QS_6^\dagger}\mathcal{D}^{f_2f_3p_2}_{LLS_6}}{M_{F_{10}}M_{S_6}^2} \\
&+\frac{4\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_4p_2}_{F_{10R}^\dagger QS_6^\dagger}\mathcal{D}^{f_3f_2p_2}_{LLS_6}}{M_{F_{10}}M_{S_6}^2} + \frac{4\mathcal{D}^{f_1p_1p_2}_{d^\dagger F_{14L}S_6^\dagger}\mathcal{D}^{p_1f_4}_{F_{14R}^\dagger HQ}\mathcal{D}^{f_2f_3p_2}_{LLS_6}}{M_{F_{14}}M_{S_6}^2} \\
&+\frac{4\mathcal{D}^{f_1p_1p_2}_{d^\dagger F_{14L}S_6^\dagger}\mathcal{D}^{p_1f_4}_{F_{14R}^\dagger HQ}\mathcal{D}^{f_3f_2p_2}_{LLS_6}}{M_{F_{14}}M_{S_6}^2} + \frac{4\mathcal{D}^{f_1p_1p_2}_{d^\dagger F_1S_{10}}\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{f_3f_4p_2}_{LQS_{10}^\dagger}}{M_{F_1}M_{S_{10}}^2} \\
&-\frac{2\mathcal{D}^{f_1p_1p_2}_{d^\dagger F_5S_{14}}\mathcal{D}^{f_3f_4p_2}_{LQS_{14}^\dagger}\mathcal{D}^{p_1f_2}_{F_5HL}}{M_{F_5}M_{S_{14}}^2} + \frac{8\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{D}^{p_2f_3p_1}_{F_{14L}LS_{12}^\dagger}\mathcal{D}^{p_2f_4}_{F_{14R}^\dagger HQ}}{M_{F_{14}}M_{S_{12}}^2}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{4\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{D}^{p_2f_3}_{F_1HL}\mathcal{D}^{p_2f_4p_1}_{F_1QS^\dagger_{12}}}{M_{F_1}M^2_{S_{12}}}+\frac{8\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{D}^{f_3f_4p_2}_{LQS^\dagger_{10}}\mathcal{C}^{p_2p_1}_{HS_{10}S^\dagger_{12}}}{M^2_{S_{10}}M^2_{S_{12}}}\\
&+\frac{4\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{D}^{f_3f_4p_2}_{LQS^\dagger_{14}}\mathcal{C}^{p_1p_2}_{HS^\dagger_{12}S_{14}}}{M^2_{S_{12}}M^2_{S_{14}}}+\frac{2\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{D}^{p_2f_3}_{F_5HL}\mathcal{D}^{p_2f_4p_1}_{F_5QS^\dagger_{12}}}{M_{F_5}M^2_{S_{12}}}\\
&-\frac{2\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{D}^{p_2f_2}_{F_1HL}\mathcal{D}^{p_2f_3p_1}_{F_{1L}S_4}}{M_{F_1}M^2_{S_4}}-\frac{2\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{D}^{p_2f_3}_{F_1HL}\mathcal{D}^{p_2f_2p_1}_{F_{1L}S_4}}{M_{F_1}M^2_{S_4}}-\frac{2\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{D}^{f_2f_3p_2}_{LLS_6}\mathcal{C}^{p_1p_2}_{HS_4S^\dagger_6}}{M^2_{S_4}M^2_{S_6}}\\
&-\frac{2\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{D}^{f_3f_2p_2}_{LLS_6}\mathcal{C}^{p_1p_2}_{HS_4S^\dagger_6}}{M^2_{S_4}M^2_{S_6}}+\frac{\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{D}^{p_2f_2}_{F_5HL}\mathcal{D}^{p_2f_3p_1}_{F_{5L}S_4}}{M_{F_5}M^2_{S_4}}+\frac{\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{D}^{p_2f_3}_{F_5HL}\mathcal{D}^{p_2f_2p_1}_{F_{5L}S_4}}{M_{F_5}M^2_{S_4}}
\end{aligned}
\tag{B52}
$$

$$
\begin{aligned}
C_{dLQLH2}=&-\frac{4\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{D}^{p_2f_3p_1}_{F_{14L}LS^\dagger_{12}}\mathcal{D}^{p_2f_4}_{F^\dagger_{14R}HQ}}{M_{F_{14}}M^2_{S_{12}}}+\frac{y_d^{f_1f_4*}\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{p_1f_3}_{F_1HL}}{2M^3_{F_1}}\\
&+\frac{2\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{D}^{p_2f_2}_{F_1HL}\mathcal{D}^{p_2f_3p_1}_{F_{1L}S_4}}{M_{F_1}M^2_{S_4}}+\frac{y_d^{f_1f_4*}\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_3}_{F_5HL}}{4M^3_{F_5}}-\frac{2\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{D}^{p_2f_3}_{F_5HL}\mathcal{D}^{p_2f_2p_1}_{F_{5L}S_4}}{M_{F_5}M^2_{S_4}}\\
&+\frac{\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{D}^{p_2f_2}_{F_5HL}\mathcal{D}^{p_2f_3p_1}_{F_{5L}S_4}}{M_{F_5}M^2_{S_4}}-\frac{4\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{D}^{p_2f_3}_{F_5HL}\mathcal{D}^{p_2f_4p_1}_{F_5QS^\dagger_{12}}}{M_{F_5}M^2_{S_{12}}}\\
&+\frac{8\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{D}^{p_2f_3p_1}_{F_{9L}LS^\dagger_{12}}\mathcal{D}^{p_2f_4}_{F^\dagger_{9R}HQ}}{M_{F_9}M^2_{S_{12}}}-\frac{4\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_4p_2}_{F^\dagger_{10R}QS^\dagger_2}\mathcal{D}^{f_2f_3p_2}_{LLS_2}}{M_{F_{10}}M^2_{S_2}}\\
&-\frac{4\mathcal{D}^{f_1p_1p_2}_{d^\dagger F_{9L}S^\dagger_2}\mathcal{D}^{p_1f_4}_{F^\dagger_{9R}HQ}\mathcal{D}^{f_2f_3p_2}_{LLS_2}}{M_{F_9}M^2_{S_2}}+\frac{2\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{C}^{p_2p_1}_{HS^\dagger_2S_4}\mathcal{D}^{f_2f_3p_2}_{LLS_2}}{M^2_{S_2}M^2_{S_4}}\\
&+\frac{4\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_4p_2}_{F^\dagger_{10R}QS^\dagger_2}\mathcal{D}^{f_3f_2p_2}_{LLS_2}}{M_{F_{10}}M^2_{S_2}}+\frac{4\mathcal{D}^{f_1p_1p_2}_{d^\dagger F_{9L}S^\dagger_2}\mathcal{D}^{p_1f_4}_{F^\dagger_{9R}HQ}\mathcal{D}^{f_3f_2p_2}_{LLS_2}}{M_{F_9}M^2_{S_2}}\\
&-\frac{2\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{C}^{p_2p_1}_{HS^\dagger_2S_4}\mathcal{D}^{f_3f_2p_2}_{LLS_2}}{M^2_{S_2}M^2_{S_4}}-\frac{2\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_4p_2}_{F^\dagger_{10R}QS^\dagger_6}\mathcal{D}^{f_2f_3p_2}_{LLS_6}}{M_{F_{10}}M^2_{S_6}}\\
&-\frac{2\mathcal{D}^{f_1p_1p_2}_{d^\dagger F_{14L}S^\dagger_6}\mathcal{D}^{p_1f_4}_{F^\dagger_{14R}HQ}\mathcal{D}^{f_2f_3p_2}_{LLS_6}}{M_{F_{14}}M^2_{S_6}}+\frac{\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{C}^{p_1p_2}_{HS_4S^\dagger_6}\mathcal{D}^{f_2f_3p_2}_{LLS_6}}{M^2_{S_4}M^2_{S_6}}\\
&-\frac{2\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_4p_2}_{F^\dagger_{10R}QS^\dagger_6}\mathcal{D}^{f_3f_2p_2}_{LLS_6}}{M_{F_{10}}M^2_{S_6}}-\frac{2\mathcal{D}^{f_1p_1p_2}_{d^\dagger F_{14L}S^\dagger_6}\mathcal{D}^{p_1f_4}_{F^\dagger_{14R}HQ}\mathcal{D}^{f_3f_2p_2}_{LLS_6}}{M_{F_{14}}M^2_{S_6}}\\
&+\frac{\mathcal{D}^{f_1f_4p_1}_{d^\dagger QS^\dagger_4}\mathcal{C}^{p_1p_2}_{HS_4S^\dagger_6}\mathcal{D}^{f_3f_2p_2}_{LLS_6}}{M^2_{S_4}M^2_{S_6}}-\frac{8\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_2p_2}_{F^\dagger_{10R}LS_{10}}\mathcal{D}^{f_3f_4p_2}_{LQS^\dagger_{10}}}{M_{F_{10}}M^2_{S_{10}}}\\
&-\frac{4\mathcal{D}^{f_1p_1p_2}_{d^\dagger F_1S_{10}}\mathcal{D}^{p_1f_2}_{F_1HL}\mathcal{D}^{f_3f_4p_2}_{LQS^\dagger_{10}}}{M_{F_1}M^2_{S_{10}}}-\frac{8\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{C}^{p_2p_1}_{HS_{10}S^\dagger_{12}}\mathcal{D}^{f_3f_4p_2}_{LQS^\dagger_{10}}}{M^2_{S_{10}}M^2_{S_{12}}}\\
&+\frac{4\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_2p_2}_{F^\dagger_{10R}LS_{14}}\mathcal{D}^{f_3f_4p_2}_{LQS^\dagger_{14}}}{M_{F_{10}}M^2_{S_{14}}}-\frac{2\mathcal{D}^{f_1p_1p_2}_{d^\dagger F_5S_{14}}\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{f_3f_4p_2}_{LQS^\dagger_{14}}}{M_{F_5}M^2_{S_{14}}}+\frac{4\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{C}^{p_1p_2}_{HS^\dagger_{12}S_{14}}\mathcal{D}^{f_3f_4p_2}_{LQS^\dagger_{14}}}{M^2_{S_{12}}M^2_{S_{14}}}
\end{aligned}
\tag{B53}
$$

$$
\begin{aligned}
C_{dLueH} = {} & \frac{8\mathcal{D}^{f_1p_1p_2*}_{dF^\dagger_{12R}V_3}\mathcal{D}^{f_4f_2p_2*}_{e^\dagger L^\dagger V_3^\dagger}\mathcal{D}^{p_1f_5*}_{F_{12L}H^\dagger u^\dagger}}{M_{F_{12}}M_{V_3}^2} - \frac{16\mathcal{D}^{f_1f_4p_1}_{d^\dagger eV_5}\mathcal{D}^{p_2f_5*}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_2f_2p_1*}_{F^\dagger_{12R}L^\dagger V_5}}{M_{F_{12}}M_{V_5}^2} \\
& + \frac{8\mathcal{D}^{f_1f_4p_1}_{d^\dagger eV_5}\mathcal{D}^{p_2f_2}_{F_1HL}\mathcal{D}^{p_2f_5p_1*}_{F_1^\dagger u^\dagger V_5}}{M_{F_1}M_{V_5}^2} + \frac{32\mathcal{D}^{f_1f_4p_1}_{d^\dagger eV_5}\mathcal{D}^{f_2f_5p_2*}_{L^\dagger u^\dagger V_8}\mathcal{C}^{p_1p_2}_{HV_5^\dagger V_8}}{M_{V_5}^2M_{V_8}^2} \\
& + \frac{16\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{f_4p_1p_2}_{eF^\dagger_{10R}V_8}\mathcal{D}^{f_2f_5p_2*}_{L^\dagger u^\dagger V_8}}{M_{F_{10}}M_{V_8}^2} + \frac{8\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{f_4f_2p_2*}_{e^\dagger L^\dagger V_3^\dagger}\mathcal{D}^{p_1f_5p_2}_{F^\dagger_{10R}uV_3^\dagger}}{M_{F_{10}}M_{V_3}^2} \\
& + \frac{8\mathcal{D}^{f_1p_1}_{d^\dagger F_{10L}H}\mathcal{D}^{f_4p_2f_5*}_{e^\dagger S_{10}u^\dagger}\mathcal{D}^{p_1f_2p_2}_{F^\dagger_{10R}LS_{10}}}{M_{F_{10}}M_{S_{10}}^2} + \frac{4\mathcal{D}^{f_1p_1p_2}_{d^\dagger F_1S_{10}}\mathcal{D}^{f_4p_2f_5*}_{e^\dagger S_{10}u^\dagger}\mathcal{D}^{p_1f_2}_{F_1HL}}{M_{F_1}M_{S_{10}}^2} \\
& - \frac{8\mathcal{D}^{f_1p_1p_2}_{d^\dagger F^\dagger_{3L}V_8}\mathcal{D}^{f_4p_1*}_{e^\dagger F^\dagger_{3R}H^\dagger}\mathcal{D}^{f_2f_5p_2*}_{L^\dagger u^\dagger V_8}}{M_{F_3}M_{V_8}^2} + \frac{8\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{D}^{f_4p_2p_1*}_{e^\dagger F^\dagger_{12R}S_{12}}\mathcal{D}^{p_2f_5*}_{F_{12L}H^\dagger u^\dagger}}{M_{F_{12}}M_{S_{12}}^2} \\
& - \frac{4\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{D}^{f_4p_2*}_{e^\dagger F^\dagger_{3R}H^\dagger}\mathcal{D}^{p_2p_1f_5*}_{F_{3L}S_{12}u^\dagger}}{M_{F_3}M_{S_{12}}^2} + \frac{8\mathcal{D}^{f_1f_2p_1}_{d^\dagger LS_{12}}\mathcal{D}^{f_4p_2f_5*}_{e^\dagger S_{10}u^\dagger}\mathcal{C}^{p_2p_1}_{HS_{10}S^\dagger_{12}}}{M_{S_{10}}^2M_{S_{12}}^2} \\
& + \frac{4\mathcal{D}^{f_1f_5p_1}_{d^\dagger uV_2^\dagger}\mathcal{D}^{f_4p_2p_1}_{eF_1V_2}\mathcal{D}^{p_2f_2}_{F_1HL}}{M_{F_1}M_{V_2}^2} + \frac{4\mathcal{D}^{f_1f_5p_1}_{d^\dagger uV_2^\dagger}\mathcal{D}^{f_4p_2*}_{e^\dagger F^\dagger_{3R}H^\dagger}\mathcal{D}^{p_2f_2p_1*}_{F_{3L}L^\dagger V_2^\dagger}}{M_{F_3}M_{V_2}^2} - \frac{8\mathcal{D}^{f_1f_5p_1}_{d^\dagger uV_2^\dagger}\mathcal{D}^{f_4f_2p_2*}_{e^\dagger L^\dagger V_3^\dagger}\mathcal{C}^{p_1p_2}_{HV_2V_3^\dagger}}{M_{V_2}^2M_{V_3}^2}
\end{aligned}
\tag{B54}
$$

$$
\begin{aligned}
C_{QuLLH} = {} & \frac{y_u^{f_4f_5}\mathcal{D}^{p_1f_1}_{F_1HL}\mathcal{D}^{p_1f_2}_{F_1HL}}{2M_{F_1}^3} + \frac{8\mathcal{D}^{p_1f_5p_2*}_{F_1^\dagger u^\dagger V_5}\mathcal{D}^{f_1f_4p_2*}_{L^\dagger QV_5^\dagger}\mathcal{D}^{p_1f_2}_{F_1HL}}{M_{F_1}M_{V_5}^2} + \frac{8\mathcal{D}^{p_1f_4p_2*}_{F_1^\dagger QV_8^\dagger}\mathcal{D}^{f_1f_5p_2*}_{L^\dagger u^\dagger V_8}\mathcal{D}^{p_1f_2}_{F_1HL}}{M_{F_1}M_{V_8}^2} \\
& + \frac{2\mathcal{D}^{p_1f_1p_2}_{F_{1L}S_4}\mathcal{D}^{f_4p_2f_5*}_{QS_4u^\dagger}\mathcal{D}^{p_1f_2}_{F_1HL}}{M_{F_1}M_{S_4}^2} + \frac{y_u^{f_4f_5}\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_2}_{F_5HL}}{4M_{F_5}^3} - \frac{4\mathcal{D}^{p_1f_5*}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_1f_4p_2*}_{F^\dagger_{12R}QS_2}\mathcal{D}^{f_1f_2p_2}_{LLS_2}}{M_{F_{12}}M_{S_2}^2} \\
& - \frac{4\mathcal{D}^{p_1p_2f_5*}_{F_{8L}S_2u^\dagger}\mathcal{D}^{p_1f_4*}_{F^\dagger_{8R}H^\dagger Q}\mathcal{D}^{f_1f_2p_2}_{LLS_2}}{M_{F_8}M_{S_2}^2} + \frac{4\mathcal{D}^{p_1f_5*}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_1f_4p_2*}_{F^\dagger_{12R}QS_2}\mathcal{D}^{f_2f_1p_2}_{LLS_2}}{M_{F_{12}}M_{S_2}^2} \\
& + \frac{4\mathcal{D}^{p_1p_2f_5*}_{F_{8L}S_2u^\dagger}\mathcal{D}^{p_1f_4*}_{F^\dagger_{8R}H^\dagger Q}\mathcal{D}^{f_2f_1p_2}_{LLS_2}}{M_{F_8}M_{S_2}^2} + \frac{2\mathcal{D}^{p_1f_5*}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_1f_4p_2*}_{F^\dagger_{12R}QS_6}\mathcal{D}^{f_1f_2p_2}_{LLS_6}}{M_{F_{12}}M_{S_6}^2} \\
& + \frac{2\mathcal{D}^{p_1p_2f_5*}_{F_{13L}S_6u^\dagger}\mathcal{D}^{p_1f_4*}_{F^\dagger_{13R}H^\dagger Q}\mathcal{D}^{f_1f_2p_2}_{LLS_6}}{M_{F_{13}}M_{S_6}^2} + \frac{2\mathcal{D}^{p_1f_5*}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_1f_4p_2*}_{F^\dagger_{12R}QS_6}\mathcal{D}^{f_2f_1p_2}_{LLS_6}}{M_{F_{12}}M_{S_6}^2} \\
& + \frac{2\mathcal{D}^{p_1p_2f_5*}_{F_{13L}S_6u^\dagger}\mathcal{D}^{p_1f_4*}_{F^\dagger_{13R}H^\dagger Q}\mathcal{D}^{f_2f_1p_2}_{LLS_6}}{M_{F_{13}}M_{S_6}^2} - \frac{16\mathcal{D}^{p_1f_5*}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_1f_2p_2*}_{F^\dagger_{12R}L^\dagger V_5}\mathcal{D}^{f_1f_4p_2*}_{L^\dagger QV_5^\dagger}}{M_{F_{12}}M_{V_5}^2} \\
& + \frac{8\mathcal{D}^{p_1f_5*}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_1f_2p_2*}_{F^\dagger_{12R}L^\dagger V_9}\mathcal{D}^{f_1f_4p_2*}_{L^\dagger QV_9^\dagger}}{M_{F_{12}}M_{V_9}^2} + \frac{4\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_5p_2*}_{F_5^\dagger u^\dagger V_9}\mathcal{D}^{f_1f_4p_2*}_{L^\dagger QV_9^\dagger}}{M_{F_5}M_{V_9}^2}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{16\mathcal{D}^{p_1f_5*}_{F_{12L}H^\dagger u^\dagger}\mathcal{D}^{p_1f_1p_2*}_{F^\dagger_{12R}L^\dagger V_9}\mathcal{D}^{f_2f_4p_2*}_{L^\dagger QV_9^\dagger}}{M_{F_{12}}M^2_{V_9}}-\frac{8\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_5p_2*}_{F_5^\dagger u^\dagger V_9}\mathcal{D}^{f_2f_4p_2*}_{L^\dagger QV_9^\dagger}}{M_{F_5}M^2_{V_9}}\\
&+\frac{32\mathcal{C}^{p_1p_2}_{HV_8V_9^\dagger}\mathcal{D}^{f_2f_4p_2*}_{L^\dagger QV_9^\dagger}\mathcal{D}^{f_1f_5p_1*}_{L^\dagger u^\dagger V_8}}{M^2_{V_8}M^2_{V_9}}-\frac{8\mathcal{D}^{p_1f_2p_2*}_{F_{13L}L^\dagger V_8^\dagger}\mathcal{D}^{p_1f_4*}_{F^\dagger_{13R}H^\dagger Q}\mathcal{D}^{f_1f_5p_2*}_{L^\dagger u^\dagger V_8}}{M_{F_{13}}M^2_{V_8}}\\
&-\frac{4\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_4p_2*}_{F_5^\dagger QV_8^\dagger}\mathcal{D}^{f_1f_5p_2*}_{L^\dagger u^\dagger V_8}}{M_{F_5}M^2_{V_8}}-\frac{16\mathcal{D}^{p_1f_2p_2*}_{F_{8L}L^\dagger V_8^\dagger}\mathcal{D}^{p_1f_4*}_{F^\dagger_{8R}H^\dagger Q}\mathcal{D}^{f_1f_5p_2*}_{L^\dagger u^\dagger V_8}}{M_{F_8}M^2_{V_8}}\\
&-\frac{16\mathcal{C}^{p_1p_2}_{HV_8V_9^\dagger}\mathcal{D}^{f_1f_4p_2*}_{L^\dagger QV_9^\dagger}\mathcal{D}^{f_2f_5p_1*}_{L^\dagger u^\dagger V_8}}{M^2_{V_8}M^2_{V_9}}-\frac{8\mathcal{D}^{p_1f_1p_2*}_{F_{13L}L^\dagger V_8^\dagger}\mathcal{D}^{p_1f_4*}_{F^\dagger_{13R}H^\dagger Q}\mathcal{D}^{f_2f_5p_2*}_{L^\dagger u^\dagger V_8}}{M_{F_{13}}M^2_{V_8}}\\
&+\frac{8\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_4p_2*}_{F_5^\dagger QV_8^\dagger}\mathcal{D}^{f_2f_5p_2*}_{L^\dagger u^\dagger V_8}}{M_{F_5}M^2_{V_8}}+\frac{16\mathcal{D}^{p_1f_1p_2*}_{F_{8L}L^\dagger V_8^\dagger}\mathcal{D}^{p_1f_4*}_{F^\dagger_{8R}H^\dagger Q}\mathcal{D}^{f_2f_5p_2*}_{L^\dagger u^\dagger V_8}}{M_{F_8}M^2_{V_8}}\\
&+\frac{32\mathcal{C}^{p_1p_2}_{HV_5^\dagger V_8}\mathcal{D}^{f_1f_4p_1*}_{L^\dagger QV_5^\dagger}\mathcal{D}^{f_2f_5p_2*}_{L^\dagger u^\dagger V_8}}{M^2_{V_5}M^2_{V_8}}+\frac{\mathcal{C}^{p_1p_2}_{HS_4S_6^\dagger}\mathcal{D}^{f_1f_2p_2}_{LLS_6}\mathcal{D}^{f_4p_1f_5*}_{QS_4u^\dagger}}{M^2_{S_4}M^2_{S_6}}\\
&+\frac{\mathcal{C}^{p_1p_2}_{HS_4S_6^\dagger}\mathcal{D}^{f_2f_1p_2}_{LLS_6}\mathcal{D}^{f_4p_1f_5*}_{QS_4u^\dagger}}{M^2_{S_4}M^2_{S_6}}+\frac{\mathcal{D}^{p_1f_2}_{F_5HL}\mathcal{D}^{p_1f_1p_2}_{F_{5L}S_4}\mathcal{D}^{f_4p_2f_5*}_{QS_4u^\dagger}}{M_{F_5}M^2_{S_4}}-\frac{2\mathcal{D}^{p_1f_1}_{F_5HL}\mathcal{D}^{p_1f_2p_2}_{F_{5L}S_4}\mathcal{D}^{f_4p_2f_5*}_{QS_4u^\dagger}}{M_{F_5}M^2_{S_4}}\\
&-\frac{2\mathcal{C}^{p_1p_2}_{HS_2^\dagger S_4}\mathcal{D}^{f_1f_2p_1}_{LLS_2}\mathcal{D}^{f_4p_2f_5*}_{QS_4u^\dagger}}{M^2_{S_2}M^2_{S_4}}+\frac{2\mathcal{C}^{p_1p_2}_{HS_2^\dagger S_4}\mathcal{D}^{f_2f_1p_1}_{LLS_2}\mathcal{D}^{f_4p_2f_5*}_{QS_4u^\dagger}}{M^2_{S_2}M^2_{S_4}}
\end{aligned}
\tag{B55}
$$

b. B-violating operators

$$
\begin{aligned}
C_{LdudH}={}&\frac{16\mathcal{D}^{f_2f_3p_1*}_{d^\dagger d^\dagger S_{11}^\dagger}\mathcal{D}^{p_2f_5*}_{F_{11L}Hu^\dagger}\mathcal{D}^{p_2f_4p_1*}_{F^\dagger_{11R}LS_{11}}}{M_{F_{11}}M^2_{S_{11}}}-\frac{16\mathcal{D}^{f_3f_2p_1*}_{d^\dagger d^\dagger S_{11}^\dagger}\mathcal{D}^{p_2f_5*}_{F_{11L}Hu^\dagger}\mathcal{D}^{p_2f_4p_1*}_{F^\dagger_{11R}LS_{11}}}{M_{F_{11}}M^2_{S_{11}}}\\
&+\frac{8\mathcal{D}^{f_2f_3p_1*}_{d^\dagger d^\dagger S_{11}^\dagger}\mathcal{D}^{p_2f_4*}_{F_1HL}\mathcal{D}^{p_2p_1f_5*}_{F_1S_{11}u^\dagger}}{M_{F_1}M^2_{S_{11}}}-\frac{8\mathcal{D}^{f_3f_2p_1*}_{d^\dagger d^\dagger S_{11}^\dagger}\mathcal{D}^{p_2f_4*}_{F_1HL}\mathcal{D}^{p_2p_1f_5*}_{F_1S_{11}u^\dagger}}{M_{F_1}M^2_{S_{11}}}\\
&+\frac{16\mathcal{D}^{f_2f_3p_1*}_{d^\dagger d^\dagger S_{11}^\dagger}\mathcal{D}^{f_4p_2f_5*}_{LS_{13}u^\dagger}\mathcal{C}^{p_1p_2*}_{HS_{11}S_{13}^\dagger}}{M^2_{S_{11}}M^2_{S_{13}}}-\frac{16\mathcal{D}^{f_3f_2p_1*}_{d^\dagger d^\dagger S_{11}^\dagger}\mathcal{D}^{f_4p_2f_5*}_{LS_{13}u^\dagger}\mathcal{C}^{p_1p_2*}_{HS_{11}S_{13}^\dagger}}{M^2_{S_{11}}M^2_{S_{13}}}\\
&+\frac{16\mathcal{D}^{f_2p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{f_3p_1p_2*}_{d^\dagger F^\dagger_{10R}S_{13}^\dagger}\mathcal{D}^{f_4p_2f_5*}_{LS_{13}u^\dagger}}{M_{F_{10}}M^2_{S_{13}}}-\frac{16\mathcal{D}^{f_3p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{f_2p_1p_2*}_{d^\dagger F^\dagger_{10R}S_{13}^\dagger}\mathcal{D}^{f_4p_2f_5*}_{LS_{13}u^\dagger}}{M_{F_{10}}M^2_{S_{13}}}\\
&-\frac{16\mathcal{D}^{f_3p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{f_2f_4p_2*}_{d^\dagger LS_{12}}\mathcal{D}^{p_1p_2f_5*}_{F^\dagger_{10R}S_{12}^\dagger u^\dagger}}{M_{F_{10}}M^2_{S_{12}}}+\frac{16\mathcal{D}^{f_2p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{f_3p_2f_5*}_{d^\dagger S_{10}^\dagger u^\dagger}\mathcal{D}^{p_1f_4p_2*}_{F^\dagger_{10R}LS_{10}}}{M_{F_{10}}M^2_{S_{10}}}\\
&+\frac{16\mathcal{D}^{f_3p_1p_2*}_{d^\dagger F^\dagger_{11R}S_{12}^\dagger}\mathcal{D}^{f_2f_4p_2*}_{d^\dagger LS_{12}}\mathcal{D}^{p_1f_5*}_{F_{11L}Hu^\dagger}}{M_{F_{11}}M^2_{S_{12}}}+\frac{8\mathcal{D}^{f_2p_1p_2*}_{d^\dagger F_1S_{10}}\mathcal{D}^{f_3p_2f_5*}_{d^\dagger S_{10}^\dagger u^\dagger}\mathcal{D}^{p_1f_4*}_{F_1HL}}{M_{F_1}M^2_{S_{10}}}+\frac{16\mathcal{D}^{f_2f_4p_1*}_{d^\dagger LS_{12}}\mathcal{D}^{f_3p_2f_5*}_{d^\dagger S_{10}^\dagger u^\dagger}\mathcal{C}^{p_2p_1*}_{HS_{10}S_{12}^\dagger}}{M^2_{S_{10}}M^2_{S_{12}}}
\end{aligned}
\tag{B56}
$$

$$\begin{aligned}C_{LdddH} =& -\frac{16\mathcal{D}^{f_2 f_3 p_1 *}_{d^\dagger d^\dagger S^\dagger_{11}}\mathcal{D}^{f_4 p_2 *}_{d^\dagger F_{11L} H^\dagger}\mathcal{D}^{p_2 f_5 p_1 *}_{F^\dagger_{11R} L S_{11}}}{M_{F_{11}} M^2_{S_{11}}} + \frac{16\mathcal{D}^{f_3 f_2 p_1 *}_{d^\dagger d^\dagger S^\dagger_{11}}\mathcal{D}^{f_4 p_2 *}_{d^\dagger F_{11L} H^\dagger}\mathcal{D}^{p_2 f_5 p_1 *}_{F^\dagger_{11R} L S_{11}}}{M_{F_{11}} M^2_{S_{11}}} \\ &- \frac{8\mathcal{D}^{f_2 f_3 p_1 *}_{d^\dagger d^\dagger S^\dagger_{11}}\mathcal{D}^{f_4 p_2 p_1 *}_{d^\dagger F^\dagger_{2R} S_{11}}\mathcal{D}^{p_2 f_5 *}_{F_{2L} H^\dagger L}}{M_{F_2} M^2_{S_{11}}} + \frac{8\mathcal{D}^{f_3 f_2 p_1 *}_{d^\dagger d^\dagger S^\dagger_{11}}\mathcal{D}^{f_4 p_2 p_1 *}_{d^\dagger F^\dagger_{2R} S_{11}}\mathcal{D}^{p_2 f_5 *}_{F_{2L} H^\dagger L}}{M_{F_2} M^2_{S_{11}}} \\ &- \frac{32\mathcal{D}^{f_2 f_3 p_1 *}_{d^\dagger d^\dagger S^\dagger_{11}}\mathcal{D}^{f_4 f_5 p_2 *}_{d^\dagger L S_{12}}\mathcal{C}^{p_1 p_2}_{H S^\dagger_{11} S_{12}}}{M^2_{S_{11}} M^2_{S_{12}}} + \frac{16\mathcal{D}^{f_4 f_3 p_1 *}_{d^\dagger d^\dagger S^\dagger_{11}}\mathcal{D}^{f_2 f_5 p_2 *}_{d^\dagger L S_{12}}\mathcal{C}^{p_1 p_2}_{H S^\dagger_{11} S_{12}}}{M^2_{S_{11}} M^2_{S_{12}}} \\ &+ \frac{32\mathcal{D}^{f_3 f_2 p_1 *}_{d^\dagger d^\dagger S^\dagger_{11}}\mathcal{D}^{f_4 f_5 p_2 *}_{d^\dagger L S_{12}}\mathcal{C}^{p_1 p_2}_{H S^\dagger_{11} S_{12}}}{M^2_{S_{11}} M^2_{S_{12}}} - \frac{16\mathcal{D}^{f_2 p_1 *}_{d^\dagger F_{11L} H^\dagger}\mathcal{D}^{f_3 p_1 p_2 *}_{d^\dagger F^\dagger_{11R} S^\dagger_{12}}\mathcal{D}^{f_4 f_5 p_2 *}_{d^\dagger L S_{12}}}{M_{F_{11}} M^2_{S_{12}}} + \frac{16\mathcal{D}^{f_3 p_1 *}_{d^\dagger F_{11L} H^\dagger}\mathcal{D}^{f_2 p_1 p_2 *}_{d^\dagger F^\dagger_{11R} S^\dagger_{12}}\mathcal{D}^{f_4 f_5 p_2 *}_{d^\dagger L S_{12}}}{M_{F_{11}} M^2_{S_{12}}} \end{aligned} \tag{B57}$$

$$\begin{aligned}C_{eQddH} =& \frac{16\mathcal{D}^{f_4 f_2 p_2}_{dQV_8}\mathcal{D}^{f_5 f_1 p_1 *}_{d^\dagger e V_5}\mathcal{C}^{p_1 p_2 *}_{H V^\dagger_5 V_8}}{M^2_{V_5} M^2_{V_8}} + \frac{8\mathcal{D}^{f_4 f_2 p_2}_{dQV_8}\mathcal{D}^{f_5 p_1 *}_{d^\dagger F_{10L} H}\mathcal{D}^{f_1 p_1 p_2 *}_{e F^\dagger_{10R} V_8}}{M_{F_{10}} M^2_{V_8}} \\ &- \frac{4\mathcal{D}^{f_4 f_2 p_2}_{dQV_8}\mathcal{D}^{f_5 p_1 p_2 *}_{d^\dagger F^\dagger_{3L} V_8}\mathcal{D}^{f_1 p_1}_{e^\dagger F^\dagger_{3R} H^\dagger}}{M_{F_3} M^2_{V_8}} - \frac{8\mathcal{D}^{f_5 f_4 p_1 *}_{d^\dagger d^\dagger S^\dagger_{11}}\mathcal{D}^{f_1 p_2}_{e^\dagger F^\dagger_{3R} H^\dagger}\mathcal{D}^{p_2 f_2 p_1}_{F_{3L} Q S^\dagger_{11}}}{M_{F_3} M^2_{S_{11}}} \\ &+ \frac{16\mathcal{D}^{f_5 f_4 p_1 *}_{d^\dagger d^\dagger S^\dagger_{11}}\mathcal{D}^{f_1 p_2 p_1}_{e^\dagger F_{8L} S^\dagger_{11}}\mathcal{D}^{p_2 f_2}_{F^\dagger_{8R} H^\dagger Q}}{M_{F_8} M^2_{S_{11}}} + \frac{16\mathcal{D}^{f_5 f_4 p_1 *}_{d^\dagger d^\dagger S^\dagger_{11}}\mathcal{D}^{f_1 f_2 p_2}_{e^\dagger Q S^\dagger_{13}}\mathcal{C}^{p_1 p_2 *}_{H S_{11} S^\dagger_{13}}}{M^2_{S_{11}} M^2_{S_{13}}} \\ &- \frac{32\mathcal{D}^{f_4 f_1 p_1 *}_{d^\dagger e V_5}\mathcal{D}^{f_5 p_2 *}_{d^\dagger F_{10L} H}\mathcal{D}^{p_2 f_2 p_1 *}_{F^\dagger_{10R} Q^\dagger V^\dagger_5}}{M_{F_{10}} M^2_{V_5}} + \frac{32\mathcal{D}^{f_4 f_1 p_1 *}_{d^\dagger e V_5}\mathcal{D}^{f_5 p_2 p_1 *}_{d^\dagger F^\dagger_{8L} V^\dagger_5}\mathcal{D}^{p_2 f_2}_{F^\dagger_{8R} H^\dagger Q}}{M_{F_8} M^2_{V_5}} - \frac{16\mathcal{D}^{f_4 p_1 *}_{d^\dagger F_{10L} H}\mathcal{D}^{f_5 p_1 p_2 *}_{d^\dagger F^\dagger_{10R} S^\dagger_{13}}\mathcal{D}^{f_1 f_2 p_2}_{e^\dagger Q S^\dagger_{13}}}{M_{F_{10}} M^2_{S_{13}}} \end{aligned} \tag{B58}$$

$$\begin{aligned}C_{LdQQH} =& -\frac{8\mathcal{D}^{f_4 f_2 p_1}_{dQV_8}\mathcal{D}^{p_2 f_5 p_1}_{F_{13L} L^\dagger V^\dagger_8}\mathcal{D}^{p_2 f_1}_{F^\dagger_{13R} H^\dagger Q}}{M_{F_{13}} M^2_{V_8}} - \frac{4\mathcal{D}^{f_4 f_5 p_1 *}_{d^\dagger L S_{12}}\mathcal{D}^{p_2 f_2 p_1}_{F_{13L} Q S_{12}}\mathcal{D}^{p_2 f_1}_{F^\dagger_{13R} H^\dagger Q}}{M_{F_{13}} M^2_{S_{12}}} \\ &- \frac{4\mathcal{D}^{f_4 f_1 p_1}_{dQV_8}\mathcal{D}^{p_2 f_5 p_1}_{F_{13L} L^\dagger V^\dagger_8}\mathcal{D}^{p_2 f_2}_{F^\dagger_{13R} H^\dagger Q}}{M_{F_{13}} M^2_{V_8}} - \frac{2\mathcal{D}^{f_4 f_5 p_1 *}_{d^\dagger L S_{12}}\mathcal{D}^{p_2 f_1 p_1}_{F_{13L} Q S_{12}}\mathcal{D}^{p_2 f_2}_{F^\dagger_{13R} H^\dagger Q}}{M_{F_{13}} M^2_{S_{12}}} \\ &+ \frac{4\mathcal{D}^{f_4 f_2 p_1}_{dQV_8}\mathcal{D}^{p_2 f_5 *}_{F_1 H L}\mathcal{D}^{p_2 f_1 p_1}_{F^\dagger_1 Q V^\dagger_8}}{M_{F_1} M^2_{V_8}} + \frac{4\mathcal{D}^{f_4 f_1 p_1}_{dQV_8}\mathcal{D}^{p_2 f_5 *}_{F_1 H L}\mathcal{D}^{p_2 f_2 p_1}_{F^\dagger_1 Q V^\dagger_8}}{M_{F_1} M^2_{V_8}} + \frac{2\mathcal{D}^{f_4 f_2 p_1}_{dQV_8}\mathcal{D}^{p_2 f_5 *}_{F_5 H L}\mathcal{D}^{p_2 f_1 p_1}_{F^\dagger_5 Q V^\dagger_8}}{M_{F_5} M^2_{V_8}} \\ &- \frac{2\mathcal{D}^{f_4 f_1 p_1}_{dQV_8}\mathcal{D}^{p_2 f_5 *}_{F_5 H L}\mathcal{D}^{p_2 f_2 p_1}_{F^\dagger_5 Q V^\dagger_8}}{M_{F_5} M^2_{V_8}} - \frac{8\mathcal{D}^{f_4 f_1 p_1}_{dQV_8}\mathcal{D}^{p_2 f_5 p_1}_{F_{8L} L^\dagger V^\dagger_8}\mathcal{D}^{p_2 f_2}_{F^\dagger_{8R} H^\dagger Q}}{M_{F_8} M^2_{V_8}} \\ &+ \frac{4\mathcal{D}^{f_4 f_5 p_1 *}_{d^\dagger L S_{12}}\mathcal{D}^{p_2 f_1 p_1}_{F_{8L} Q S_{12}}\mathcal{D}^{p_2 f_2}_{F^\dagger_{8R} H^\dagger Q}}{M_{F_8} M^2_{S_{12}}} + \frac{32\mathcal{D}^{f_4 p_1 *}_{d^\dagger F_{10L} H}\mathcal{D}^{p_1 f_2 p_2 *}_{F^\dagger_{10R} Q^\dagger V^\dagger_5}\mathcal{D}^{f_5 f_1 p_2}_{L^\dagger Q V^\dagger_5}}{M_{F_{10}} M^2_{V_5}}\end{aligned}$$

$$
\begin{aligned}
&-\frac{32\mathcal{D}^{f_4p_1p_2*}_{d^\dagger F^\dagger_{8L}V^\dagger_5}\mathcal{D}^{p_1f_2}_{F^\dagger_{8R}H^\dagger Q}\mathcal{D}^{f_5f_1p_2}_{L^\dagger QV^\dagger_5}}{M_{F_8}M^2_{V_5}}+\frac{16\mathcal{D}^{f_4f_2p_1}_{dQV_8}\mathcal{C}^{p_2p_1*}_{HV^\dagger_5V_8}\mathcal{D}^{f_5f_1p_2}_{L^\dagger QV^\dagger_5}}{M^2_{V_5}M^2_{V_8}}\\
&+\frac{16\mathcal{D}^{f_4p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_2p_2*}_{F^\dagger_{10R}Q^\dagger V^\dagger_9}\mathcal{D}^{f_5f_1p_2}_{L^\dagger QV^\dagger_9}}{M_{F_{10}}M^2_{V_9}}+\frac{4\mathcal{D}^{f_4p_1p_2}_{dF_{13L}V_9}\mathcal{D}^{p_1f_2}_{F^\dagger_{13R}H^\dagger Q}\mathcal{D}^{f_5f_1p_2}_{L^\dagger QV^\dagger_9}}{M_{F_{13}}M^2_{V_9}}\\
&+\frac{8\mathcal{D}^{f_4f_2p_1}_{dQV_8}\mathcal{C}^{p_1p_2*}_{HV_8V^\dagger_9}\mathcal{D}^{f_5f_1p_2}_{L^\dagger QV^\dagger_9}}{M^2_{V_8}M^2_{V_9}}+\frac{32\mathcal{D}^{f_4p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_1p_2*}_{F^\dagger_{10R}Q^\dagger V^\dagger_9}\mathcal{D}^{f_5f_2p_2}_{L^\dagger QV^\dagger_9}}{M_{F_{10}}M^2_{V_9}}\\
&+\frac{8\mathcal{D}^{f_4p_1p_2}_{dF_{13L}V_9}\mathcal{D}^{p_1f_1}_{F^\dagger_{13R}H^\dagger Q}\mathcal{D}^{f_5f_2p_2}_{L^\dagger QV^\dagger_9}}{M_{F_{13}}M^2_{V_9}}+\frac{16\mathcal{D}^{f_4f_1p_1}_{dQV_8}\mathcal{C}^{p_1p_2*}_{HV_8V^\dagger_9}\mathcal{D}^{f_5f_2p_2}_{L^\dagger QV^\dagger_9}}{M^2_{V_8}M^2_{V_9}}\\
&-\frac{4\mathcal{D}^{f_4p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_5p_2*}_{F^\dagger_{10R}LS_{10}}\mathcal{D}^{f_1f_2p_2}_{QQS_{10}}}{M_{F_{10}}M^2_{S_{10}}}-\frac{2\mathcal{D}^{f_4p_1p_2*}_{d^\dagger F_1S_{10}}\mathcal{D}^{p_1f_5*}_{F_1HL}\mathcal{D}^{f_1f_2p_2}_{QQS_{10}}}{M_{F_1}M^2_{S_{10}}}\\
&-\frac{4\mathcal{D}^{f_4f_5p_1*}_{d^\dagger LS_{12}}\mathcal{C}^{p_2p_1*}_{HS_{10}S^\dagger_{12}}\mathcal{D}^{f_1f_2p_2}_{QQS_{10}}}{M^2_{S_{10}}M^2_{S_{12}}}-\frac{4\mathcal{D}^{f_4p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_5p_2*}_{F^\dagger_{10R}LS_{10}}\mathcal{D}^{f_2f_1p_2}_{QQS_{10}}}{M_{F_{10}}M^2_{S_{10}}}\\
&-\frac{2\mathcal{D}^{f_4p_1p_2*}_{d^\dagger F_1S_{10}}\mathcal{D}^{p_1f_5*}_{F_1HL}\mathcal{D}^{f_2f_1p_2}_{QQS_{10}}}{M_{F_1}M^2_{S_{10}}}-\frac{4\mathcal{D}^{f_4f_5p_1*}_{d^\dagger LS_{12}}\mathcal{C}^{p_2p_1*}_{HS_{10}S^\dagger_{12}}\mathcal{D}^{f_2f_1p_2}_{QQS_{10}}}{M^2_{S_{10}}M^2_{S_{12}}}\\
&+\frac{2\mathcal{D}^{f_4p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_5p_2*}_{F^\dagger_{10R}LS_{14}}\mathcal{D}^{f_1f_2p_2}_{QQS_{14}}}{M_{F_{10}}M^2_{S_{14}}}-\frac{\mathcal{D}^{f_4p_1p_2*}_{d^\dagger F_5S_{14}}\mathcal{D}^{p_1f_5*}_{F_5HL}\mathcal{D}^{f_1f_2p_2}_{QQS_{14}}}{M_{F_5}M^2_{S_{14}}}\\
&+\frac{2\mathcal{D}^{f_4f_5p_1*}_{d^\dagger LS_{12}}\mathcal{C}^{p_1p_2*}_{HS^\dagger_{12}S_{14}}\mathcal{D}^{f_1f_2p_2}_{QQS_{14}}}{M^2_{S_{12}}M^2_{S_{14}}}-\frac{2\mathcal{D}^{f_4p_1*}_{d^\dagger F_{10L}H}\mathcal{D}^{p_1f_5p_2*}_{F^\dagger_{10R}LS_{14}}\mathcal{D}^{f_2f_1p_2}_{QQS_{14}}}{M_{F_{10}}M^2_{S_{14}}}\\
&+\frac{\mathcal{D}^{f_4p_1p_2*}_{d^\dagger F_5S_{14}}\mathcal{D}^{p_1f_5*}_{F_5HL}\mathcal{D}^{f_2f_1p_2}_{QQS_{14}}}{M_{F_5}M^2_{S_{14}}}-\frac{2\mathcal{D}^{f_4f_5p_1*}_{d^\dagger LS_{12}}\mathcal{C}^{p_1p_2*}_{HS^\dagger_{12}S_{14}}\mathcal{D}^{f_2f_1p_2}_{QQS_{14}}}{M^2_{S_{12}}M^2_{S_{14}}}
\end{aligned}
\tag{B59}
$$

4. Examples of comparison with previous dim-6 results

In this subsection, we show two examples, C_5 and C_{uu}, to help readers to compare our results with existing dimension-6 matching results in Ref. [27]. The transformation rules, Eqs. (3.7)–(3.17), are used to compare two different notations here and also applicable to other cases.

C_5 Three UV resonances can contribute to the Weinberg operator, known as S_6, F_1, F_5 or Ξ_1, N, Σ in Ref. [27]. Related interaction vertices are

$$
\begin{aligned}
\Delta\mathcal{L}=&\frac{\mathcal{C}^{p}_{HHS^\dagger_6}}{\sqrt{2}}\epsilon^{kj}(\tau^I)^i_kH_iH_j(S^\dagger_{6p})^I-\frac{\mathcal{D}^{rsp}_{LLS_6}}{\sqrt{2}}\epsilon^{jk}(\tau^I)^i_k[(l_r)_iC(l_s)_j](S_{6p})^I\\
&-\mathcal{D}^{pr}_{F_1HL}\epsilon^{ji}[F_{1p}C(l_r)_i]H_j+\frac{\mathcal{D}^{pr}_{F_5HL}}{2\sqrt{2}}\epsilon^{kj}(\tau^I)^i_k[(F_{5p})^IC(l_r)_i]H_j\\
&+\frac{\mathcal{D}^{pr}_{F_5HL}}{2\sqrt{2}}\epsilon^{ki}(\tau^I)^j_k[(F_{5p})^IC(l_r)_i]H_j+\text{H.c.},
\end{aligned}
\tag{B60}
$$

or

$$
\begin{aligned}
-\Delta\mathcal{L}=&(y_{\Xi_1})_{rij}\Xi^{a\dagger}_{1r}\bar{l}_{Li}\sigma^ai\sigma_2l^c_{Lj}+(\kappa_{\Xi_1})_r\Xi^{a\dagger}_{1r}(\tilde{\phi}^\dagger\sigma^a\phi)\\
&+(\lambda_N)_{ri}\bar{N}_{Rr}\tilde{\phi}^\dagger l_{Li}+\frac{1}{2}(\lambda_\Sigma)_{ri}\bar{\Sigma}^a_{Rr}\tilde{\phi}^\dagger\sigma^al_{Li}+\text{H.c.}
\end{aligned}
\tag{B61}
$$

in the notation of Ref. [27]. The relation between two notations can be derived from comparing Lagrangians as

$$(\kappa_{\Xi_1})_r = -\frac{1}{\sqrt{2}}\mathcal{C}^r_{HHS_6^\dagger}, \qquad (y_{\Xi_1})_{rji} = -\frac{1}{\sqrt{2}}(\mathcal{D}^{ijr}_{LLS_6})^*, \tag{B62}$$

$$(\lambda_N)_{ri} = -\mathcal{D}^{ri}_{F_1HL}, \qquad (\lambda_\Sigma)_{ri} = -\sqrt{2}\mathcal{D}^{ri}_{F_5HL}, \tag{B63}$$

which leads to a transformation between our matching results for C_5 at $O(v^2/M^2)$ order. Note that last two terms involving F_5 in Eq. (B60) are identical due to the symmetric tensor $\epsilon\tau^I$.

C_{uu} Similar result can be derived from comparing

$$\begin{aligned}\Delta\mathcal{L} = [&-4\mathcal{D}^{prs}_{S_9^\dagger u^\dagger u^\dagger}\epsilon^{abc}[(\bar{u}_r)^b(u_s^c)^c](S_{9p}^\dagger)^a - 4\mathcal{D}^{prs}_{S_{17}u^\dagger u^\dagger}C^{\mathfrak{a}}_{ab}[(\bar{u}_r)^a(u_s^c)^b](S_{17p})^{\mathfrak{a}} + \text{H.c.}] \\ &- 2\mathcal{D}^{rsp}_{u^\dagger uV_1}[(\bar{u}_r)^a\gamma_\mu(u_s)_a]V^\mu_{1p} - 2\sqrt{2}\mathcal{D}^{rsp}_{u^\dagger uV_{12}}(\lambda^A)^b_a[(\bar{u}_r)^a\gamma_\mu(u_s)_b](V_{12p})^{A\mu}\end{aligned} \tag{B64}$$

and

$$\begin{aligned}-\Delta\mathcal{L} = \{&(y^{uu}_{\omega_4})_{rij}\omega^{A\dagger}_{4r}\epsilon_{ABC}\bar{u}^B_{Ri}u^{cC}_{Rj} + (y_{\Omega_4})_{rij}\Omega^{AB\dagger}_{4r}\bar{u}^{c(A|}_{Ri}u^{|B)}_{Rj} + \text{H.c.}\} \\ &+ (g^u_{\mathcal{B}})_{rij}\mathcal{B}^\mu_r\bar{u}_{Li}\gamma_\mu u_{Lj} + (g^u_{\mathcal{G}})_{rij}\mathcal{G}^{\mu A}_r\bar{u}_{Li}\gamma_\mu T_A u_{Rj},\end{aligned} \tag{B65}$$

which gives

$$(y^{uu}_{\omega_4})_{rki} = 4\mathcal{D}^{pki}_{S_9^\dagger u^\dagger u^\dagger}, \qquad (y_{\Omega_4})_{rjl} = 4(\mathcal{D}^{rjl}_{S_{17}u^\dagger u^\dagger})^*, \tag{B66}$$

$$(g^u_{\mathcal{B}})_{rij} = 2\mathcal{D}^{ijr}_{u^\dagger u^\dagger V_1}, \qquad (g^u_{\mathcal{G}})_{rij} = 4\sqrt{2}\mathcal{D}^{ijr}_{u^\dagger uV_{12}}. \tag{B67}$$

Note that two terms involving S_9 in Eq. (B22) are identical due to the antisymmetry of the indices, and the same to S_{17}.

APPENDIX C: DIM-5, 6 WARSAW AND DIM-7 GREEN BASIS

TABLE X. The Weinberg operator.

Type: ψ^2H^2	
$\mathcal{O}_5$	$\epsilon^{ik}\epsilon^{jl}(\ell_i^TC\ell_j)H_kH_l$

TABLE XI. The Warsaw basis [3].

Type: X^3		Type: H^4D^2		Type: ψ^2H^3	
$\mathcal{O}_G$	$f^{ABC}G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$\mathcal{O}_{eH}$	$(H^\dagger H)(\bar{\ell}eH)$
$\mathcal{O}_{\tilde{G}}$	$f^{ABC}\tilde{G}^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^*(H^\dagger D^\mu H)$	$\mathcal{O}_{uH}$	$(H^\dagger H)(\bar{q}u\tilde{H})$
$\mathcal{O}_W$	$\epsilon^{IJK}W^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$	Type: H^6		$\mathcal{O}_{dH}$	$(H^\dagger H)(\bar{q}dH)$
$\mathcal{O}_{\tilde{W}}$	$\epsilon^{ABC}\tilde{W}^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$	$\mathcal{O}_H$	$(H^\dagger H)^3$		
Type: X^2H^2		Type: ψ^2XH		Type: ψ^2H^2D	
$\mathcal{O}_{HG}$	$H^\dagger HG^A_{\mu\nu}G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{\ell}\sigma^{\mu\nu}e)\tau^IHW^I_{\mu\nu}$	$\mathcal{O}^{(1)}_{H\ell}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{\ell}\gamma^\mu\ell)$
$\mathcal{O}_{H\tilde{G}}$	$H^\dagger H\tilde{G}^A_{\mu\nu}G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{\ell}_L\sigma^{\mu\nu}e_R)HB_{\mu\nu}$	$\mathcal{O}^{(3)}_{H\ell}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{\ell}\tau^I\gamma^\mu\ell)$

(Table continued)

TABLE XI. *(Continued)*

Type: X^3		Type: H^4D^2		Type: $\psi^2 H^3$	
$\mathcal{O}_{HW}$	$H^\dagger H W^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}\sigma^{\mu\nu}T^A u)\tilde{H}G^A_{\mu\nu}$	$\mathcal{O}_{He}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e)$
$\mathcal{O}_{H\tilde{W}}$	$H^\dagger H \tilde{W}^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}\sigma^{\mu\nu}u)\tau^I \tilde{H}W^I_{\mu\nu}$	$\mathcal{O}^{(1)}_{Hq}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q)$
$\mathcal{O}_{HB}$	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}\sigma^{\mu\nu}u)\tilde{H}B_{\mu\nu}$	$\mathcal{O}^{(3)}_{Hq}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}\tau^I\gamma^\mu q)$
$\mathcal{O}_{H\tilde{B}}$	$H^\dagger H \tilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}\sigma^{\mu\nu}T^A d)HG^A_{\mu\nu}$	$\mathcal{O}_{Hu}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u)$
$\mathcal{O}_{HWB}$	$H^\dagger \tau^I H W^I_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}\sigma^{\mu\nu}d)\tau^I HW^I_{\mu\nu}$	$\mathcal{O}_{Hd}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)$
$\mathcal{O}_{H\tilde{W}B}$	$H^\dagger \tau^I H \tilde{W}^I_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}\sigma^{\mu\nu}d)HB^I_{\mu\nu}$	$\mathcal{O}_{Hud}$	$i(\tilde{H}^\dagger iD_\mu H)(\bar{u}\gamma^\mu d)$
Type: $(\bar{L}L)(\bar{L}L)$		Type: $(\bar{R}R)(\bar{R}R)$		Type: $(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{\ell\ell}$	$(\bar{\ell}\gamma^\mu \ell)(\bar{\ell}\gamma_\mu \ell)$	$\mathcal{O}_{ee}$	$(\bar{e}\gamma^\mu e)(\bar{e}\gamma_\mu e)$	$\mathcal{O}_{\ell e}$	$(\bar{\ell}\gamma^\mu \ell)(\bar{e}\gamma_\mu e)$
$\mathcal{O}^{(1)}_{qq}$	$(\bar{q}\gamma^\mu q)(\bar{q}\gamma_\mu q)$	$\mathcal{O}_{uu}$	$(\bar{u}\gamma^\mu u)(\bar{u}\gamma_\mu u)$	$\mathcal{O}_{\ell u}$	$(\bar{\ell}\gamma^\mu \ell)(\bar{u}\gamma_\mu u)$
$\mathcal{O}^{(3)}_{qq}$	$(\bar{q}\gamma^\mu \tau^I q)(\bar{q}\gamma_\mu \tau^I q)$	$\mathcal{O}_{dd}$	$(\bar{d}\gamma^\mu d)(\bar{d}\gamma_\mu d)$	$\mathcal{O}_{\ell d}$	$(\bar{\ell}\gamma^\mu \ell)(\bar{d}\gamma_\mu d)$
$\mathcal{O}^{(1)}_{\ell q}$	$(\bar{\ell}\gamma^\mu \ell)(\bar{q}\gamma_\mu q)$	$\mathcal{O}_{eu}$	$(\bar{e}\gamma^\mu e)(\bar{u}\gamma_\mu u)$	$\mathcal{O}_{qe}$	$(\bar{q}\gamma^\mu q)(\bar{e}\gamma_\mu e)$
$\mathcal{O}^{(3)}_{\ell q}$	$(\bar{\ell}\gamma^\mu \tau^I \ell)(\bar{q}\gamma_\mu \tau^I q)$	$\mathcal{O}_{ed}$	$(\bar{e}\gamma^\mu e)(\bar{d}\gamma_\mu d)$	$\mathcal{O}^{(1)}_{qu}$	$(\bar{q}\gamma^\mu q)(\bar{u}\gamma_\mu u)$
		$\mathcal{O}^{(1)}_{ud}$	$(\bar{u}\gamma^\mu u)(\bar{d}\gamma_\mu d)$	$\mathcal{O}^{(8)}_{qu}$	$(\bar{q}\gamma^\mu T^A q)(\bar{u}\gamma_\mu T^A u)$
Type: $(\bar{L}R)(\bar{R}L)$		$\mathcal{O}^{(8)}_{ud}$	$(\bar{u}\gamma^\mu T^A u)(\bar{d}\gamma_\mu T^A d)$	$\mathcal{O}^{(1)}_{qd}$	$(\bar{q}\gamma^\mu q)(\bar{d}\gamma_\mu d)$
$\mathcal{O}_{\ell edq}$	$(\bar{\ell}e)(\bar{d}q)$			$\mathcal{O}^{(8)}_{qd}$	$(\bar{q}\gamma^\mu T^A q)(\bar{d}\gamma_\mu T^A d)$
Type: $(\bar{L}R)(\bar{L}R)$		Type: B-violating			
$\mathcal{O}^{(1)}_{quqd}$	$(\bar{q}^j u)\epsilon_{jk}(\bar{q}^k d)$	$\mathcal{O}_{duq}$	$\epsilon^{abc}\epsilon^{jk}(d^T_a C u_b)(q^T_{cj} C\ell_k)$		
$\mathcal{O}^{(8)}_{quqd}$	$(\bar{q}^j T^A u)\epsilon_{jk}(\bar{q}^k T^A d)$	$\mathcal{O}_{qqu}$	$\epsilon^{abc}\epsilon_{jk}(q^T_{aj} C q_{bk})(u^T_c C e)$		
$\mathcal{O}^{(1)}_{\ell equ}$	$(\bar{\ell}^j e)\epsilon_{jk}(\bar{q}^k u)$	$\mathcal{O}_{qqq}$	$\epsilon^{abc}\epsilon_{jn}\epsilon_{km}(q^T_{aj} C q_{bk})(q^T_{cm} C\ell_n)$		
$\mathcal{O}^{(3)}_{\ell equ}$	$(\bar{\ell}^j \sigma_{\mu\nu} e)\epsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u)$	$\mathcal{O}_{duu}$	$\epsilon^{abc}(d^T_a C u_b)(u^T_c C e)$		

TABLE XII. The dimension-7 Green basis. The redundant operators $\mathcal{R}_i$ are marked in bold. A similar basis is also presented in Ref. [65].

Only L-violating			
Type: $\psi^2 H^4$		Type: $\psi^2 H^3 D$	
$\mathcal{O}_{LH}$	$\epsilon^{ik}\epsilon^{jl}(\ell^T_i C\ell_j)H_k H_l(H^\dagger H)$	$\mathcal{O}_{LeHD}$	$\epsilon^{ij}\epsilon^{kl}(\ell^T_i C\gamma^\mu e)H_j H_k(iD_\mu H_l)$
Type: $\psi^2 H^2 D^2$		Type: $\psi^2 H^2 X$	
$\mathcal{O}_{LDH1}$	$\epsilon^{ij}\epsilon^{kl}(\ell^T_i CD_\mu \ell_j)(H_k D^\mu H_l)$	$\mathcal{O}_{LHW}$	$\epsilon^{ik}(\epsilon\tau^I)^{jl}(\ell^T_i Ci\sigma^{\mu\nu}\ell_j)H_k H_l W^I_{\mu\nu}$
$\mathcal{O}_{LDH2}$	$\epsilon^{ik}\epsilon^{jl}(\ell^T_i CD_\mu \ell_j)(H_k D^\mu H_l)$	$\mathcal{O}_{LHB}$	$\epsilon^{ik}\epsilon^{jl}(\ell^T_i Ci\sigma^{\mu\nu}\ell_j)H_k H_l B_{\mu\nu}$
$\boldsymbol{\mathcal{R}_{LDH3}}$	$\boldsymbol{\epsilon^{ik}\epsilon^{jl}(\ell^T_i C\ell_j)(D_\mu H_k)(D^\mu H_l)}$		
$\boldsymbol{\mathcal{R}_{LDH4}}$	$\boldsymbol{\epsilon^{ik}\epsilon^{jl}(\ell^T_i C\ell_j)(H_k D_\mu D^\mu H_l)}$		
$\boldsymbol{\mathcal{R}_{LDH5}}$	$\boldsymbol{\epsilon^{ik}\epsilon^{jl}(\ell^T_i Ci\sigma^{\mu\nu}\ell_j)(D_\mu H_k)(D_\nu H_l)}$		
$\boldsymbol{\mathcal{R}_{LDH6}}$	$\boldsymbol{\epsilon^{ik}\epsilon^{jl}(\ell^T_i Ci\sigma^{\mu\nu}D_\mu \ell_j)(H_k D_\nu H_l)}$		
Type: $\psi^4 D$		Type: $\psi^4 H$	
$\mathcal{O}_{duLLD}$	$\epsilon^{ij}(\bar{d}^a\gamma^\mu u_a)(\ell^T_i CiD_\mu \ell_j)$	$\mathcal{O}_{eLLLH}$	$\epsilon^{ij}\epsilon^{kl}(\bar{e}\ell_i)(\ell^T_j C\ell_k)H_l$
$\boldsymbol{\mathcal{R}_{duLLD2}}$	$\boldsymbol{\epsilon^{ij}(\bar{d}^a iD_\mu \ell_j)(\ell^T_i C\gamma^\mu u_a)}$	$\mathcal{O}_{dLQLH1}$	$\epsilon^{ij}\epsilon^{kl}(\bar{d}^a\ell_i)(q^T_{aj} C\ell_k)H_l$
$\boldsymbol{\mathcal{R}_{duLLD3}}$	$\boldsymbol{\epsilon^{ij}(\bar{d}^a \ell_i)(\ell^T_j CiDu_a)}$	$\mathcal{O}_{dLQLH2}$	$\epsilon^{ik}\epsilon^{jl}(\bar{d}^a\ell_i)(q^T_{aj} C\ell_k)H_l$
		$\mathcal{O}_{dLueH}$	$\epsilon^{ij}(\bar{d}^a\ell_i)(u^T_a Ce)H_j$
		$\mathcal{O}_{QuLLH}$	$\epsilon^{ij}(\bar{q}^{ak}u_a)(\ell^T_k C\ell_i)H_j$

(Table continued)

TABLE XII. *(Continued)*

Only L-violating			
Type: $\psi^2 H^4$		Type: $\psi^2 H^3 D$	
L- and B-violating			
Type: $\psi^4 D$		Type: $\psi^4 H$	
$\mathcal{O}_{LQddD}$	$\epsilon^{abc}(\overline{\ell}^i\gamma^\mu q_{ai})(d_b^T C i D_\mu d_c)$	$\mathcal{O}_{LdudH}$	$\epsilon^{abc}\epsilon_{ij}(\overline{\ell}^i d_a)(u_b^T C d_c)H^{*j}$
$\boldsymbol{\mathcal{R}_{LQddD2}}$	$\boldsymbol{\epsilon^{abc}(q_{ai}^T C\gamma_\mu d_b)(\overline{\ell}^i i D^\mu d_c)}$	$\mathcal{O}_{LdddH}$	$\epsilon^{abc}(\overline{\ell}^i d_a)(d_b^T C d_c)H_i$
$\boldsymbol{\mathcal{R}_{LQddD3}}$	$\boldsymbol{\epsilon^{abc}(q_{ai}^T C i \overleftarrow{\not{D}} d_b)(\overline{\ell}^i d_c)}$	$\mathcal{O}_{eQddH}$	$-\epsilon^{abc}(\bar{e}Q_{ai})(d_b^T C d_c)H^{*i}$
$\mathcal{O}_{edddD}$	$\epsilon^{abc}(\bar{e}\gamma^\mu d_a)(d_b^T C i D_\mu d_c)$	$\mathcal{O}_{LdQQH}$	$-\epsilon^{abc}(\overline{\ell}^k d_a)(q_{bk}^T C q_{ci})H^{*i}$

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