Assisted baryon number violation in $4k+2$ dimensions

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Proton decay in six-dimensions orbifolded on square T^2/Z_2 is highly suppressed at tree level. This is because baryon number violating (BNV) operators containing only the zero mode of bulk fermions must satisfy the selection rule $\frac{3}{2}\Delta B \pm \frac{1}{2}\Delta L = 0 \bmod 4$. In this article, we show that the above relation does not prohibit mass dimension-six BNV operators containing Kaluza-Klein partners of the bulk fermions. Together with a "spinless" adjoint scalar partner of hypercharge gauge boson (the dark matter candidate), these novel operators generate dark matter assisted proton decay at mass dimension eight. Here, with explicit examples of scalar and vector baryon number violating interactions, we discuss the importance of such $\Delta B = 1 = \Delta L$ and $\Delta B = 2 = \Delta L$ operators and derive the limit on new physics.

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I. INTRODUCTION

Baryon-antibaryon asymmetry in the Universe has been one of the most intriguing mysteries in Nature. With the partial lifetime of a proton confirmed to beyond 10^{34} years [1], it is impossible for simple baron number violating new physics models to exist at $\mathcal{O}(1\ \text{TeV})$. On the other hand, next order effects like neutron-antineuron ($n-\bar{n}$) oscillations [2], $nn \to \bar{\nu}\bar{\nu}$, hydrogen-antihydrogen ($H-\bar{H}$) oscillations, or double proton annihilation ($pp \to e^+e^+$), which violate baryon number by two units, have been interesting due to the much lower constraint on the new physics scale. Such rare processes that violate these accidental symmetries of the Standard Model (SM) have been very powerful probes to exploring physics beyond the Standard Model. Thus, if detected, it will be of fundamental importance in particle physics and cosmology. New experiments [3] are being devised to detect these rare events. On the other hand, the lack of any conclusive observation of such events on terrestrial experiments poses an indomitable challenge for new physics models. In an effective field theory, since both scalar and vector operators lead to $\Delta B = 1$ and $\Delta B = 2$ processes, it is not possible to predict observable $\Delta B = 2$ processes without suppressing $\Delta B = 1$ with discrete symmetries or additional quantum numbers. The solution to these problems could lie in some dynamical process which suppresses baryon number violating currents on Earth, but have had significant contributions to baryogengesis in the evolution of the Universe.

Though baryon and lepton numbers are accidental symmetries of the Standard Model at the classical level, quantum effects break them nonperturbatively [4] to $\mathrm{U}(1)_{B-L}$. There is no *a priori* reason for these symmetries to be preserved in beyond Standard Model scenarios. Nevertheless, to describe new physics with a minimal SM-like gauge structure and representations, it is suggestive to keep the $\mathrm{U}(1)_{B-L}$ symmetry to be intact. With the proton decay suppressed, various new physics models [5–15] can accommodate baryon number violation by two units. These processes are highly sensitive to new physics at an intermediate energy scale $\sim\mathcal{O}(1\text{–}100\ \text{TeV})$. The strongest constraint on this intermediate scale arises from neutron-antineutron oscillation $\sim$500 TeV [11] in four-dimensions. On the other hand, embedding the model in six dimensions with nested warping [15] has proven to substantially relax this constraint to $\sim$3 TeV. More interestingly, though $n-\bar{n}$ oscillation is usually understood to be baryon number violating by two units, with a suitably extended Higgs sector [16] that spontaneously breaks global $B-L$ symmetry, this process also violates the lepton number. With the neutrino mass $m_\nu \lesssim 10^{-1}$ eV, this model accommodates a much relaxed new physics scale $\sim$1 TeV.

The identification of 11 possible candidates with an expected background of 9.3 ± 2.7 events at Super-Kamiokande [17], 0.37 megaton-year exposure, and the prospect of observing the neutron-antineutron oscillation at Hyper-Kamiokande [18] and HIBEAM/NNBAR [19] with much improved sensitivity has reignited the interest in $\Delta B = 2$ processes. Moreover, the predictions of hydrogen-antihydrogen oscillation and proton-proton annihilation ($pp \to e^+e^+$) [20] are other possible signatures of the

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new physics. At the quark-level, $H-\bar{H}/pp \to e^+e^+$ is given by the dimension-12 operator,

$$C_{H-\bar{H}}(uud)^2(\bar{e}^c e). \tag{1}$$

At the scale of the measurement (~2 GeV), it is convenient to construct hadron-level effective field theory as

$$\mathcal{O}_{ppee} = \frac{1}{\Lambda^2_{ppee}}(\bar{p}^c\Gamma_p p)(\bar{e}^c\Gamma_e e)$$
$$\text{where } \Gamma_{p,e} = (1, i\gamma^5, \gamma^\mu\gamma^5). \tag{2}$$

The quark-level effective operator could then be compared with the low-energy effective field theory operator made up of leptons and hadrons as

$$\Lambda_{ppee}(0.22m_p)^3 = 1/\sqrt{C_{H-\bar{H}}}, \tag{3}$$

where we have used $\langle 0|uud|p\rangle = \sqrt{2m_p}\beta_H$, with the hadronic parameter $\beta_H = 0.014\ \text{GeV}^3$ determined by lattice methods [21].

With large densities of atomic hydrogen present at the interstellar medium (ISM), the search for $\Delta B = 2, \Delta L = 2$ process in the oscillation-induced diffused γ-rays survey, by Fermi LAT, constrains the upper-limit of the oscillation matrix element to $\delta = 2\langle pe^-|H|\bar{p}e^+\rangle \lesssim 6\times 10^{-17}s^{-1}$ [22]. On the other hand, search for $pp \to e^+e^+$ (proton annihilation rate in oxygen nuclei), at Super-Kamiokande [1], places a much stronger upper bound of $\delta \lesssim 10^{-21}s^{-1}$. This limit constrains the scale of new physics to be $\gtrsim$2 TeV.

In $4k+2$ dimensions, the operator in Eq. (1) does not remain invariant under Lorentz transformations. This is because the charge conjugation operators in $4k$ and $4k+2$ dimensions behave differently. In even dimensions, it is understood that the Lorentz group is reducible and there exists a chiral projection operator. While in $4k$ dimensions, the charge conjugation operator anticommutes with the chirality projection operator, in $4k+2$ dimensions they commute. Thus, it is not straightforward to realize a $4k$-dimensional model by compactifying from $4k+2$ dimensions, in the presence of currents with charge conjugate fields. In this article, we discuss the correct manner to address the baryon number violating operators in $4k+2$ dimensions, in particular in six dimensions. We conduct a model independent effective field theory analysis with scalar, vector, and mixed operators, generated through the interactions of a scalar and vector bilinear of spinor fields that transform under the full $4k+2$-dimensional Lorentz symmetry. An interesting scenario arises with operators containing KK-1 modes at the lowest order.

Note that in generic d dimensions, the gauge boson has $d-2$ polarizations. After compactification, a combination of $d-4$ broken polarization in the Kaluza-Klein (KK) spectrum becomes the "spinless adjoint scalar field." One such combination is "eaten" by the KK towers to become massive, while other combinations survive. In six dimensions, the surviving combination of the broken polarizations of the hypercharge gauge boson forms the "spinless adjoint scalar," which, with the degeneracy of the KK-mode masses lifted at one loop [23,24], becomes the lightest stable particle and thus the dark matter (DM) [25]. Limits from the WMAP data [26] constrains the mass of this adjoint scalar to be ~2 TeV [27], but it can be relaxed by allowing additional resonant annihilation and coannihilation channels. One such possibility is to embed the model in higher dimensional space-time with warping [28].

With the spinless adjoint scalar becoming the dark matter candidate, its interactions with the KK-1 fermion can influence the aforementioned operators leading to dark matter assisted baryon number violating currents. These operators influenced by the dark matter, in $4k+2$ dimensions, can predict large baryon number violation near superdense dark matter clumps [29,30]. This can also explain the absence of any observation yet at the terrestrial experiments. Moreover, this operator also provides an interesting annihilation channel for the dark matter.

In literature, four-dimensional models that predict such dark matter influenced baryon number violation [31–34] are discussed usually in the context of asymmetric dark matter carrying a net antibaryon number which can describe both dark and baryonic matter origin through a unified phylogenesis mechanism. These antibaryonic dark matter can cause induced nucleon decay with ~1 GeV meson in the final sate and provide a novel signature in the terrestrial nucleon decay experiments. Models with hidden MeV dark matter [35] can also contribute to dark matter induced processes like $\bar{f}p \to e^+n$ and have interesting signatures at Super-Kamiokande. They are constrained by dark matter relic density and supernova cooling, and for Majorana type dark matter, Super-Kamiokande strongly rules them out up to the scale ~100 TeV.

A minimal $4k+2$ extra-dimensional construction assumes six dimensions, such that the six-dimensional Lorentz symmetry is broken to four dimensions by orbifolding on T^2/Z_2. This construction, with Standard Model-like bulk fermions transforming under a $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ gauge group, boasts a rich phenomenology [36–39], provides a viable cold dark matter candidate [25,40], predicts the number of chiral generations [41], and can lead to a small cosmological constant [42] naturally. Upon compactification, the six-dimensional Lorentz symmetry $\mathrm{SO}(5,1)$ breaks to the four-dimensional Lorentz symmetry $\mathrm{SO}(3,1)$ and a residual $\mathrm{U}(1)_{45}$ symmetry which generates rotation in the $x_4 - x_5$ plane. Thus in addition to the four-dimensional Lorentz transformations, the fermions also transform under the $\mathrm{U}(1)_{45}$ symmetry. This brings in an additional charge to the fermions and leads to the selection rule $\frac{3}{2}\Delta B \pm \frac{1}{2}\Delta L = 0 \bmod 4$ for the baryon and lepton number violating operator constructed only of zero mode of fermions [43]. This selection rule suppresses the proton decay to very large orders at tree level, and saves the model from tight constraints.

In this article, we explicitly show that this selection rule does not hold true with operators containing Kaluza-Klein partners of the SM fermions. These new sets of operators, although inconsequential on their own, become interesting when we include their interaction with the "spinless" adjoint scalar field. The interaction of KK-1 fermions with the spinless adjoint scalar field can readily convert it to a SM fermion. Thus, here, we show that the proton can decay faster than what was discussed in [43], albeit in the presence of dark matter (spinless adjoint scalar field). This process then can explain the rarity of proton decay on Earth with the lack of enough dark matter density.

This article is organized as follows. In the next section and its subsection, we will derive some of the relevant properties of Clifford algebra and fields $4k+2$ dimensions, particularly in six dimensions and also discuss the field content in our model. In Sec. III, we discuss the possible baryon number violating interactions of scalar and vector new physics fields and resultant operators and in Sec. IV, we discuss the proton decay and assisted proton decay. The $\Delta B = 2 = \Delta L$ term is discussed in Sec. V and we also derive limits on these operators from various processes like $pp \to e^+e^+$, $DM + p \to DM + \bar{p} + e^+ + e^+$, and dark matter initiated hydrogen-antihydrogen oscillation. We conclude our analysis in Sec. VI.

II. CLIFFORD ALGEBRA IN $4k+2$ DIMENSIONS

In a general d-dimensional vector space, with the basis generated by Γ^M, over the field of complex numbers, the Clifford algebra is given by [44,45]

$$\{\Gamma_M, \Gamma_N\} = 2\eta_{MN}, \tag{4}$$

where $\eta_{MN} = \mathrm{diag}(-1, +1, +1, +1, \ldots, d-1 \text{ times})$ and $M, N = (0, 1, 2, 3, \ldots, d-1)$. When d is even, the Clifford algebra falls apart into two simple sets, whose representations we call Weyl spinors.

The Lorentz group generators in this geometry become

$$\Sigma^{MN} = -\frac{i}{2}[\Gamma^M, \Gamma^N]. \tag{5}$$

In even dimensions, the Lorentz symmetry also supports an extra Gamma matrix that anticommutes with all the other Γ^M as,

$$\Gamma^{4k+3} = \alpha \Gamma^0 \Gamma^1 \Gamma^2 \ldots . \Gamma^{4k+1}, \tag{6}$$

where $\alpha = 1$ in $4k+2$ dimensions, chosen to satisfy $(\Gamma^{4k+3})^2 = 1$. Using this, we can define a chiral projection operator $P_\pm = \frac{1}{2}(1 \pm \Gamma^{4k+3})$, such that every Dirac fermion (ψ) can be projected into two irreducible Weyl representations ($\psi_\pm$) by

$$\psi_\pm = P_\pm \psi. \tag{7}$$

We name the chiralities in $4k+2$ dimensions to be $+$ and $-$ to distinguish from the chiralities in $4k$ dimensions where they are called left and right.

Moreover, for gamma matrices Γ^M, there exists similarity transformation that relates them to $-\Gamma^{M*}$. Given this transformation, we can define a charge conjugation operator that acts on the fermion field as

$$\psi^c = C\psi \equiv (C\Gamma^0)\psi^*, \tag{8}$$

such that the ψ and ψ^c have the same Lorentz transformation, satisfying $[C\Gamma^0, \Sigma^{MN}] = 0$. Further, the transformation of the Gamma matrix under this operator is given by

$$\begin{aligned}\Gamma^M &= -(C\Gamma^0)\Gamma^{M*}(C\Gamma^0)^{-1} \\ &= -C(\Gamma^M)^T C^{-1}.\end{aligned} \tag{9}$$

Now, from Eq. (6), we see that $[C\Gamma^0, \Gamma^{4k+3}] = 0$ in $4k+2$ dimensions. Unlike in four dimensions, since the charge conjugation operator $(C\Gamma^0)$ commutes with Γ^{4k+3}, the charge conjugate fermion representation in six dimensions must satisfy the same Weyl condition as the original spinor field did. Due to this, the charge conjugation operator does not flip chirality in $4k+2$ dimensions.

For illustrating the arguments above, we will work with Standard Model fermions in six dimensions and describe the relevant Lorentz symmetry properties below. In six dimensions, the spin-half representation of the Lorentz group is defined by six 8×8 gamma matrices that satisfy the relation in Eq. (4). In particular, we choose to work in the representation of the algebra defined by

$$\Gamma^\mu = \gamma^\mu \otimes \sigma^1, \qquad \Gamma^4 = \gamma^5 \otimes \sigma^1, \qquad \Gamma^5 = \mathbb{1} \otimes \sigma^2. \tag{10}$$

In the above relations, γ^μ denotes the four-dimensional Dirac matrices and γ^5 the chirality projection operator four dimensions. The Lorentz algebra for the spinor field is now generated by

$$\begin{aligned}\Sigma_{\mu\nu} &= \frac{i}{2}[\Gamma_\mu, \Gamma_\nu], \qquad \Sigma_{\mu 4} = \frac{i}{2}[\Gamma_\mu, \Gamma_4], \\ \Sigma_{\mu 5} &= \frac{i}{2}[\Gamma_\mu, \Gamma_5], \qquad \Sigma_{45} = \frac{i}{2}[\Gamma_4, \Gamma_5],\end{aligned} \tag{11}$$

with spinors transforming as $\Psi \to e^{\frac{i}{4}\Sigma_{MN}\theta^{MN}}\Psi$. As we discussed previously, the Lorentz group in six dimensions admits irreducible chiral representations $\Psi_\pm = \frac{1}{2}(1 \pm \Gamma^7)\Psi$, where the chiral projection operator is given by

$$\Gamma^7 = \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5 = \mathbb{1} \otimes \sigma^3. \tag{12}$$

Using Eq. (9), along with the gamma matrices given in Eq. (12), the charge conjugation operator C can be seen to anticommute with Γ^0, Γ^2, Γ^4 and commute with Γ^1, Γ^3, Γ^5. Therefor the charge conjugation operator is given by

$$\begin{aligned} C &= i\Gamma^4\Gamma^2\Gamma^0 \\ &= \gamma^5\gamma^2\gamma^0 \otimes \sigma^1. \end{aligned} \tag{13}$$

TABLE I. Six-dimensional Standard Model fermions and their charges under $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ gauge group.

Fermions	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_W$	$\mathrm{U}(1)_Y$
$\mathcal{Q}_+(x^M)$	3	2	1/3
$\mathcal{U}_-(x^M)$	3	1	4/3
$\mathcal{D}_-(x^M)$	3	1	−2/3
$L_+(x^M)$	1	2	−1
$\mathcal{E}_-(x^M)$	1	1	−2
$\mathcal{N}_-(x^M)$	1	1	0

A. Standard Model fermions in six dimensions

Let us now consider bulk Standard Model fermions in six dimensions that transform under the $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ gauge group. These fermions are denoted by $\mathcal{Q}_+$, $\mathcal{U}_-$, $\mathcal{D}_-$ for quarks and L_+, $\mathcal{E}_-$, $\mathcal{N}_-$ for leptons, where $\pm$ are chiralities defined by the chirality projection operator defined as $P_\pm = \frac{1}{2}(1 \pm \Gamma^7)$. These fields are set to satisfy the gauge quantum numbers given in Table I.

Unlike in four dimensions, in six dimensions the gauge anomalies are give by box diagrams. The aforementioned fermion chiralities and gauge charges are assigned such that the irreducible $[\mathrm{SU}(3)_c]^3\mathrm{U}(1)_Y$ and mixed gauge-gravitational anomalies vanish exactly. But, the nonvanishing reducible anomalies, $[\mathrm{SU}(2)_W]^4$, $[\mathrm{SU}(2)_W]^2[\mathrm{SU}(3)_c]^2$, $[\mathrm{SU}(2)_W]^2[\mathrm{U}(1)]_Y^2$, and $[\mathrm{SU}(3)_c]^2[\mathrm{U}(1)]_Y^2$ are cancelled via the Green-Schwarz mechanism [46].

On compactifying the six-dimensional geometry on a torus T^2, the Lorentz generators in Eq. (11) break to $\Sigma_{\mu\nu}$ and Σ_{45}. Note that, Σ_{45} generates rotation in the $x_4 - x_5$ plane. Hence, along with the four-dimensional Lorentz transformation, $\Psi_\pm \to e^{\frac{i}{4}\Sigma_{\mu\nu}\theta^{\mu\nu}}\Psi_\pm$, the fermions also transform under Σ_{45} as $\Psi_\pm \to e^{\frac{i}{4}\Sigma_{45}\theta^{45}}\Psi_\pm$. This residual $U(1)_{45} = e^{i\Sigma^{45}\theta_{45}}$ symmetry, where θ_{45} is an arbitrary rotation in the (x_4, x_5) plane, is broken to its discrete subgroups upon orbifolding. The T^2/Z_2 orbifold on a rectangle, in general, is now invariant under a rotation through π, thus preserving the Z_2 subgroup of $U(1)_{45}$. Whereas, a square T^2/Z_2 possesses a Z_4 symmetry since it is invariant under $\pi/2$ rotations.

Orbifolding six dimensions on T^2/Z_2 breaks the six-dimensional fermion $\Psi_\pm(x^M)$ to its Fourier mode $\Psi_\pm(x^M) = \sum_n \psi^n_{\pm l}(x^\mu)\chi^n_l(x^4,x^5) + \psi^n_{\pm r}(x^\mu)\chi^n_r(x^4,x^5)$, where l and r are four-dimensional chiralities given by $\psi_{\pm l} = P_L\Psi_\pm = \frac{1}{2}(1+\gamma^5)\Psi_\pm$ and $\psi_{\pm r} = P_R\Psi_\pm = \frac{1}{2}(1-\gamma^5)\Psi_\pm$, where $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. Since the residual Z_4 subgroup of $\mathrm{U}(1)_{45}$ is preserved under orbifolding, the fermion $\psi_{\pm l}$ gets a charge $\pm 1/2$ and $\psi_{\pm r}$ gets charge $\mp 1/2$ under this symmetry [43]. Thus, all the operators originating from six-dimensional geometry are bound to preserve this quantum charge.

To understand what this means for six dimensions with the bulk Standard Model, let us consider their Kaluza-Klein decomposition,

$$\begin{aligned}
\mathcal{Q}_+(x^\mu, x_a) &= \frac{\sqrt{2}}{(2\pi R)}\left\{q^{(0,0)}_{+l}(x^\mu) + \sqrt{2}\sum_{m,n}\left[P_L\mathcal{Q}^{(m,n)}_{+l}(x^\mu)\cos\left(\frac{1}{R}(mx_4+nx_5)\right) + P_R\mathcal{Q}^{(m,n)}_{+r}(x^\mu)\sin\left(\frac{1}{R}(mx_4+nx_5)\right)\right]\right\}, \\
\mathcal{U}_-(x^\mu, x_a) &= \frac{\sqrt{2}}{(2\pi R)}\left\{u^{(0,0)}_{-r}(x^\mu) + \sqrt{2}\sum_{m,n}\left[P_R\mathcal{U}^{(m,n)}_{-r}(x^\mu)\cos\left(\frac{1}{R}(mx_4+nx_5)\right) + P_L\mathcal{U}^{(m,n)}_{-l}(x^\mu)\sin\left(\frac{1}{R}(mx_4+nx_5)\right)\right]\right\} \\
\mathcal{D}_-(x^\mu, x_a) &= \frac{\sqrt{2}}{(2\pi R)}\left\{d^{(0,0)}_{-r}(x^\mu) + \sqrt{2}\sum_{m,n}\left[P_R\mathcal{D}^{(m,n)}_{-r}(x^\mu)\cos\left(\frac{1}{R}(mx_4+nx_5)\right) + P_L\mathcal{D}^{(m,n)}_{-l}(x^\mu)\sin\left(\frac{1}{R}(mx_4+nx_5)\right)\right]\right\} \\
L_+(x^\mu, x_a) &= \frac{\sqrt{2}}{(2\pi R)}\left\{\ell^{(0,0)}_{+l}(x^\mu) + \sqrt{2}\sum_{m,n}\left[P_L L^{(m,n)}_{+l}(x^\mu)\cos\left(\frac{1}{R}(mx_4+nx_5)\right) + P_R L^{(m,n)}_{+r}(x^\mu)\sin\left(\frac{1}{R}(mx_4+nx_5)\right)\right]\right\}, \\
\mathcal{E}_-(x^\mu, x_a) &= \frac{\sqrt{2}}{(2\pi R)}\left\{e^{(0,0)}_{-r}(x^\mu) + \sqrt{2}\sum_{m,n}\left[P_R\mathcal{E}^{(m,n)}_{-r}(x^\mu)\cos\left(\frac{1}{R}(mx_4+nx_5)\right) + P_L\mathcal{E}^{(m,n)}_{-l}(x^\mu)\sin\left(\frac{1}{R}(mx_4+nx_5)\right)\right]\right\}, \\
\mathcal{N}_-(x^\mu, x_a) &= \frac{\sqrt{2}}{(2\pi R)}\left\{n^{(0,0)}_{-r}(x^\mu) + \sqrt{2}\sum_{m,n}\left[P_R\mathcal{N}^{(m,n)}_{-r}(x^\mu)\cos\left(\frac{1}{R}(mx_4+nx_5)\right) + P_L\mathcal{N}^{(m,n)}_{-l}(x^\mu)\sin\left(\frac{1}{R}(mx_4+nx_5)\right)\right]\right\}
\end{aligned} \tag{14}$$

TABLE II. Resultant charges of four-dimensional fermions after breaking the six-dimensional Lorentz symmetry, SO(5, 1), to $SO(3,1)\times U(1)_{45}$.

$6d$ fermions	$4d$ fermions	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$	$U(1)_{45}$
$\mathcal{Q}_+(x^M)$	$q_{+l}^{(0,0)}(x^\mu)$, $\mathcal{Q}_{+l}^{(m,n)}(x^\mu)$	3	2	1/3	1/2
	$\mathcal{Q}_{+r}^{(m,n)}(x^\mu)$				−1/2
$\mathcal{U}_-(x^M)$	$u_{-r}^{(0,0)}(x^\mu)$, $\mathcal{U}_{-r}^{(m,n)}(x^\mu)$	3	1	4/3	1/2
	$\mathcal{U}_{-l}^{(m,n)}(x^\mu)$				−1/2
$\mathcal{D}_-(x^M)$	$d_{-r}^{(0,0)}(x^\mu)$, $\mathcal{D}_{-r}^{(m,n)}(x^\mu)$	3	1	−2/3	1/2
	$\mathcal{D}_{-l}^{(m,n)}(x^\mu)$				−1/2
$L_+(x^M)$	$\ell_{+l}^{(0,0)}(x^\mu)$, $L_{+l}^{(m,n)}(x^\mu)$	1	2	−1	1/2
	$L_{+r}^{(m,n)}(x^\mu)$				−1/2
$\mathcal{E}_-(x^M)$	$e_{-r}^{(0,0)}(x^\mu)$, $\mathcal{E}_{-r}^{(m,n)}(x^\mu)$	1	1	−2	1/2
	$\mathcal{E}_{-l}^{(m,n)}(x^\mu)$				−1/2
$\mathcal{N}_-(x^M)$	$n_{-r}^{(0,0)}(x^\mu)$, $\mathcal{N}_{-r}^{(m,n)}(x^\mu)$	1	1	0	1/2
	$\mathcal{N}_{-l}^{(m,n)}(x^\mu)$				−1/2

where $q_{+l}^{(0,0)}$, $u_{-r}^{(0,0)}$, and $d_{-r}^{(0,0)}$ are the zero modes and are identified with the four-dimensional Standard Model quarks. Similarly, $\ell_{+l}^{(0,0)}$, $e_{-r}^{(0,0)}$, and $n_{-r}^{(0,0)}$ are identified with the Standard Model leptons and the right-handed neutrino. The rest of the states are Kaluza-Klein partners of the Standard Model fermions and carry the same gauge quantum charge. As mentioned before, the fermions are also charged under the residual $U(1)_{45}$ symmetry. Since $\Sigma_{45}=\gamma^5\otimes\sigma^3$, from Eq. (11), left- and right-handed partners of the same fermion carry opposite charge. The full set of charges that four-dimensional fermion fields carry are given in Table II.

B. New physics scalar and vector bosons in six dimensions

The Higgs field and $SU(3)_c\times SU(2)_W\times U(1)_Y$ gauge bosons are assumed to propagate in the bulk of six dimensions. Upon compactification and orbifolding, they break to a KK state. For brevity, we will not discuss the Standard Model scalar and vector fields, but refer the reader to [47]. On the other hand, since the spinless adjoint scalar partner of the hypercharge gauge boson is of importance, we will discuss the Abelian gauge theory in six dimensions in the Appendix.

Let us now consider the new physics scalar and vector bosons that mediate the baryon number violating currents. Their charges under the Standard Model gauge group are given in Table III. We will assume that unlike Standard Model fields these new physics bosons satisfy the Dirichlet boundary condition at orbifold fixed points. The reason for this will be clear when we write their interaction terms.

To that end, let us begin with the Lagrangian of a complex scalar boson $[\Phi(x_\mu,x_4,x_5)]$ in six dimensions given by

$$\mathcal{L}_S=(D_M\Phi)^\dagger(D^M\Phi)-m_\Phi^2\Phi^\dagger\Phi, \tag{15}$$

where D_M is the covariant derivate. Orbifolding the geometry and applying the Dirichlet boundary condition, Φ can be expanded in its Fourier modes as

$$\Phi(x_\mu,x_4,x_5)=\sum_{m,n=0}\Phi^{(m,n)}(x_\mu)\sin\left(\frac{mx_4}{R}+\frac{nx_5}{R}\right). \tag{16}$$

With this, after integrating out the extra dimensions, the four-dimensional Lagrangian density becomes,

$$\mathcal{L}_{S4D}=\int dx_4dx_5\mathcal{L}_S=(D_\mu\Phi^{(m,n)})^\dagger(D^\mu\Phi^{(m,n)}) -\left(\frac{1}{R^2}(m^2+n^2)+m_\Phi^2\right)\Phi^{(m,n)\dagger}\Phi^{(m,n)}. \tag{17}$$

Note that the lightest mode in the above Lagrangian has mass $(\frac{1}{R^2}+m_\Phi^2)^{1/2}$.

Similarly, the Lagrangian density of the vector boson (A_M) in six dimensions is given by

TABLE III. Six-dimensional scalar and vector color representations in addition to Standard Model fields.

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$\Phi^\dagger$	3	2	5/3
A_M	$\bar{6}$	3	2/3
V_M^*	3	1, 3	2/3
U_M	3	1	2/3

$$\begin{aligned}\mathcal{L}_A &= -\frac{1}{4}F_{MN}F^{MN} - m_A^2 A_M A^M \\ &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}F_{\mu 4}F^{\mu 4} - \frac{1}{2}F_{\mu 5}F^{\mu 5} - \frac{1}{2}F_{45}F^{45} - m_A^2 A_\mu A^\mu - m_A^2 A_4 A^4 - m_A^2 A_5 A^5,\end{aligned} \tag{18}$$

where $F_{\mu 4} = \partial_\mu A_4 - \partial_4 A_\mu$, $F_{\mu 5} = \partial_\mu A_5 - \partial_5 A_\mu$, and $F_{45} = \partial_4 A_5 - \partial_5 A_4$. For simplicity we have considered Abelian fields here, but the derivation can be extended to non-Abelian fields as well.

Upon orbifolding, we again consider a Dirichlet boundary condition for the A_μ component of the vector field, while A_4 and A_5 are set to satisfy the Neumann boundary condition. The components of the 6D vector field that satisfy the above boundary condition in the orbifolded geometry, now can be Fourier expanded as

$$\begin{aligned}A_\mu(x_\mu, x_4, x_5) &= \sum_{m,n\neq 0} A_\mu^{(m,n)}(x_\mu)\sin\left(\frac{mx_4}{R} + \frac{nx_5}{R}\right), \\ A_4(x_\mu, x_4, x_5) &= \sum_{m,n=0} A_4^{(m,n)}(x_\mu)\cos\left(\frac{mx_4}{R} + \frac{nx_5}{R}\right), \\ A_5(x_\mu, x_4, x_5) &= \sum_{m,n=0} A_5^{(m,n)}(x_\mu)\cos\left(\frac{mx_4}{R} + \frac{nx_5}{R}\right).\end{aligned} \tag{19}$$

The Lagrangian density in Eq. (18), in generalized R_ζ gauge-fixing, after integrating over the x_4 and x_5 directions, becomes

$$\begin{aligned}\mathcal{L}_A &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}F_{\mu 4}F^{\mu 4} - \frac{1}{2}F_{\mu 5}F^{\mu 5} - \frac{1}{2}F_{45}F^{45} - m_A^2 A_\mu A^\mu - m_A^2 A_4 A^4 - m_A^2 A_5 A^5 - \frac{1}{2\zeta}(\partial_\mu A^\mu + \zeta(\partial_4 A^4 + \partial_5 A_5))^2 \\ &= \sum_{m,n\neq 0} -\frac{1}{4}F_{\mu\nu}^{(m,n)}F^{(m,n)\mu\nu} - \frac{1}{2}\left(\left(\frac{m}{R}\right)^2 + \left(\frac{n}{R}\right)^2\right)A_\mu^{(m,n)}A^{(m,n)\mu} - m_A^2 A_\mu^{(m,n)}A^{(m,n)\mu} - \frac{1}{2\zeta}(\partial_\mu A^{(m,n)\mu})^2 \\ &\quad + \sum_{m,n=0} -\frac{1}{2}(\partial_\mu A_4^{(m,n)})^2 - \frac{1}{2}(\partial_\mu A_5^{(m,n)})^2 \\ &\quad - \frac{1}{2}\left(\frac{n}{R}A_4^{(m,n)} - \frac{m}{R}A_5^{(m,n)}\right)^2 - \frac{\zeta}{2}\left(\frac{m}{R}A_4^{(m,n)} + \frac{n}{R}A_5^{(m,n)}\right)^2 - m_A^2 A_4^{(m,n)2} - m_A^2 A_5^{(m,n)2}.\end{aligned} \tag{20}$$

Since in this article we are only interested in the first few heavy modes of the field, the dynamics of zeroth and first KK sates, represented by $(m = 0, n = 0)$ and $(m = 1, n = 0)$ is given by

$$\begin{aligned}\mathcal{L}_A &= \sum_{0<m\leq 1} -\frac{1}{4}F_{\mu\nu}^{(m,0)}F^{(m,0)\mu\nu} - \frac{1}{2}\left(\frac{m}{R}\right)^2 A_\mu^{(m,0)}A^{(m,0)\mu} - m_A^2 A_\mu^{(m,0)}A^{(m,0)\mu} - \frac{1}{2\zeta}(\partial_\mu A^{(m,0)\mu})^2 \\ &\quad \times \sum_{0\leq m\leq 1} -\frac{1}{2}(\partial_\mu A_4^{(n,0)})^2 - \tilde{m}_{\zeta Am}A_4^{(m,0)2} - \frac{1}{2}(\partial_\mu A_5^{(m,0)})^2 - \tilde{m}_{Am}^2 A_5^{(m,0)2},\end{aligned} \tag{21}$$

where $\tilde{m}_{\zeta Am} = \zeta(\frac{m}{R})^2 + m_A^2$ and $\tilde{m}_{Am} = (\frac{m}{R})^2 + m_A^2$.

C. Baryon number violating interactions of the new physics scalar and vector boson

In this subsection, we discuss the baryon number violating interactions of the new physics described above. To start with, let us consider the interaction of the scalar boson. The interaction terms in the Lagrangian density is given by

$$\begin{aligned}(\mathcal{L}_{\rm int})_\Phi &= \bar{\mathcal{Q}}_+^c \mathcal{U}_- \Phi + \bar{\mathcal{E}}_-^c \mathcal{Q}_+ \Phi^\dagger \\ &= \mathcal{Q}_+^T C \mathcal{U}_- \Phi(x_\mu, x_4, x_5) + \mathcal{E}_-^T C \mathcal{Q}_+ \Phi^\dagger(x_\mu, x_4, x_5),\end{aligned} \tag{22}$$

where $\bar{\mathcal{Q}}_+^c = \overline{C\bar{\mathcal{Q}}_+^T} = (C\bar{\mathcal{Q}}_+^T)^\dagger \Gamma^0 = \bar{\mathcal{Q}}_+^* C^\dagger \Gamma^0 = \mathcal{Q}_+^T \Gamma^0 C^\dagger \Gamma^0 = \mathcal{Q}_+^T C$. Here, we have used the property $\Gamma^{M\dagger} = \Gamma^0 \Gamma^M \Gamma^0$ and the charge conjugation operator C is defined in Eq. (13).

Using the Fourier decompositions given in Eq. (14) and Eq. (16), after integrating out the x_4 and x_5 directions, the above interaction contains

$$\int dx_4 dx_5 (\mathcal{L}_{\rm int})_\Phi \supset \sum_{n,m\neq 0} q_{+l}^{(0,0)T}\gamma^2\gamma^0\mathcal{U}_{-l}^{(m,n)}\Phi^{(m,n)}(x_\mu) + \sum_{n,m\neq 0}\mathcal{E}_{-l}^{(m,n)T}\gamma^2\gamma^0 q_{+l}^{(0,0)}\Phi^{(m,n)\dagger}(x_\mu). \tag{23}$$

The rest of the terms are not of interest to us.

Similarly, for the vector bosons, the six-dimensional interaction is given by

$$\begin{aligned}(\mathcal{L}_{\rm int})_U &= \bar{\mathcal{U}}_-^c\Gamma^M\mathcal{D}_-U_M = \mathcal{U}_-^T C\Gamma^M\mathcal{D}_-U_M \\ &= \mathcal{U}_-^T C\Gamma^\mu\mathcal{D}_-U_\mu(x_\mu,x_4,x_5) + \mathcal{U}_-^T C\Gamma^4\mathcal{D}_-U_4(x_\mu,x_4,x_5) + \mathcal{U}_-^T C\Gamma^5\mathcal{D}_-U_5(x_\mu,x_4,x_5).\end{aligned} \tag{24}$$

Here, we have used the interaction terms of $U_M(x^N)$ as an example.

Using the KK decomposition of the fermions and vector boson given in Eq. (14) and Eq. (19), after integrating out the x_4 and x_5 directions, the nonvanishing contributions of the fermion KK states contain

$$\int dx_4 dx_5 (\mathcal{L}_{\rm int})_U \supset -\sum_{m,n\neq 0} u_{-r}^{(0,0)T}\gamma^2\gamma^0\gamma^\mu\mathcal{D}_{-l}^{(m,n)}U_\mu^{(m,n)}(x_\mu) \tag{25}$$

$$\supset -u_{-r}^{(0,0)T}\gamma^2\gamma^0\gamma^\mu\mathcal{D}_{-l}^{(1,0)}U_\mu^{(1,0)}(x_\mu). \tag{26}$$

Since we are only interested in the lightest KK partner of the vector boson, the most dominant contribution to the baryon number violating operator arises from the interaction term,

$$\int dx_4 dx_5 (\mathcal{L}_{\rm int})_U \supset -u_{-r}^{(0,0)T}\gamma^2\gamma^0\gamma^\mu\mathcal{D}_{-l}^{(1,0)}U_\mu^{(1,0)}(x_\mu). \tag{27}$$

This analysis can be generalized to other vector fields given in Table III and we get

$$\int dx_4 dx_5 (\mathcal{L}_{\rm int})_A \supset -q_{+l}^{(0,0)T}\gamma^2\gamma^0\gamma^\mu\mathcal{Q}_{+r}^{(1,0)}A_\mu^{(1,0)}(x_\mu), \tag{28}$$

$$\int dx_4 dx_5 (\mathcal{L}_{\rm int})_{V^*} \supset -q_{+l}^{(0,0)T}\gamma^2\gamma^0\gamma^\mu L_{+r}^{(1,0)}V_\mu^{(1,0)*}(x_\mu), \tag{29}$$

$$\int dx_4 dx_5 (\mathcal{L}_{\rm int})_{U^*} \supset -u_{-r}^{(0,0)T}\gamma^2\gamma^0\gamma^\mu\mathcal{E}_{-l}^{(1,0)}U_\mu^{(1,0)*}(x_\mu). \tag{30}$$

Integrating out these heavy bosons generates the operators that violate baryon number violation. But these operators contain KK-1 modes of the Standard Model fermions. As an example, from Eq. (23), integrating out $\Phi^{(1,0)}(x_\mu)$ we obtain the operator,

$$(q_{+l}^{T(0,0)}\gamma^0\gamma^2 U_{-l}^{(1,0)})(\mathcal{E}_{-l}^{T(1,0)}\gamma^0\gamma^2 q_{+l}^{(0,0)}).$$

And from Eq. (27) and Eq. (30), on integrating out $U_\mu^{(1,0)}(x_\mu)$, we get the corresponding vector operator. One important observation is that there are no direct proton decay operators at tree level.

But, before we conclude this section, it is important that we discuss the interactions of the spinless adjoint scalar.

D. Interaction of spinless adjoint scalar ($V_B^{(1,0)}$) with fermions

For illustration, let us consider the interaction term in the six-dimensional Lagrangian density for the spinless adjoint scalar with quark field. Details required for this subsection are given in the Appendix.

The kinetic term of the quarks are given by

$$\mathcal{L}_f = \bar{\mathcal{Q}}_+(x^M)\Gamma^M D_M\mathcal{Q}_+(x^M) + \bar{\mathcal{U}}_-(x^M)\Gamma^M D_M\mathcal{U}_-(x^M), \tag{31}$$

where Γ^M are the six-dimensional gamma matrices and the covariant derivatives are defined as $D_M\mathcal{Q}_+ = (\partial_M - \frac{ig}{2}\tau^i W_M^i - i\frac{g_Y}{2}y_+B_M)\mathcal{Q}_+$ and $D_M\mathcal{U}_- = (\partial_M - \frac{ig_Y}{2}y_-B_M)\mathcal{U}_-$. The hypercharge interaction term in the above Lagrangian is given by

$$\mathcal{L}_I = \frac{ig_Y}{2}(y_+\bar{\mathcal{Q}}_+\Gamma^M B_M\mathcal{Q}_+ + y_-\bar{\mathcal{U}}_-\Gamma^M B_M\mathcal{U}_-). \tag{32}$$

Using the KK expansions given in Eqs. (14) and (A4) for quarks and Standard Model gauge bosons, and integrating out the extra dimensions, the above interaction term contains the term

$$\mathcal{L}_I \supset \frac{ig_Y}{2}(B_4^{(1,0)}(y_q\bar{\mathcal{Q}}_{+r}^{(1,0)}\gamma^5 q_{+l} + y_u\bar{\mathcal{U}}_{-l}^{(1,0)}\gamma^5 u_{-r}) + B_5^{(1,0)}(y_q\bar{\mathcal{Q}}_{+r}^{(1,0)} q_{+l} - y_u\bar{\mathcal{U}}_{-l}^{(1,0)} u_{-r}). \quad (33)$$

On diagonalizing using Eq. (A8), for $(m = 1, n = 0)$, the term becomes

$$\mathcal{L}_I \supset \frac{ig_Y}{2}(V_1^{(1,0)}(y_q\bar{\mathcal{Q}}_{+r}^{(1,0)} q_{+l} - y_u\bar{\mathcal{U}}_{-l}^{(1,0)} u_{-r}) + V_2^{(1,0)}(y_q\bar{\mathcal{Q}}_{+r}^{(1,0)} q_{+l} - y_u\bar{\mathcal{U}}_{-l}^{(1,0)} u_{-r}). \quad (34)$$

Since the $V_1^{(1,0)}$ field is nondynamical, in unitary gauge, the interaction of the spinless adjoint is given by

$$\mathcal{L}_I \supset V_2^{(1,0)}(y_q\bar{\mathcal{Q}}_{+r}^{(1,0)} q_{+l} - y_u\bar{\mathcal{U}}_{-l}^{(1,0)} u_{-r}). \quad (35)$$

Upon identifying the $V_2^{(1,0)}$ with the dark matter and renaming it to $V_B^{(1,0)}$, we see that the nonvanishing terms in the interaction between the $V_B^{(1,0)}$ and quarks take the form

$$\mathcal{L}_I = \frac{ig_Y}{2} V_B^{(1,0)}(y_q\bar{\mathcal{Q}}_{+r}^{(1,0)} q_{+l} - y_u\bar{\mathcal{U}}_{-l}^{(1,0)} u_{-r}) \quad (36)$$

Similarly, for leptons, the interaction becomes

$$\mathcal{L}_I = \frac{ig_Y}{2} V_B^{(1,0)}(y_\ell\bar{L}_{+r}^{(1,0)} \ell_{+l} - y_e\bar{\mathcal{E}}_{-l}^{(1,0)} e_{-r}). \quad (37)$$

The terms shown here satisfy the quantum charge of $U(1)_{45}$ given in Table II.

III. BARYON NUMBER VIOLATING OPERATORS

In this section, we discuss the baryon number violating operators generated by integrating out the new physics scalar and vector bosons discussed in Sec. II B. Though we work with six dimensions to illustrate our arguments, the operators and results can be generalized to any $4k + 2$ dimensions. The operators to be discussed here are a consequence of the interactions discussed in Eq. (23) and Eqs. (27) through (30).

Before, going to the main discussion of this article, for completeness, let us first discuss why there are no tree-level proton decay operators [43], containing only the zero modes of fermions. Since each quark carries $1/3$ baryon number, the operator mediating baryon number violation contains $3\Delta B$ quarks. From Table II, note the zero mode of quarks carry an additional $1/2$ charge under $\mathrm{U}(1)_{45}$. Hence, a baryon number violating operator, under this residual symmetry, has charge $\frac{3}{2}\Delta B$. Assuming that the lepton number is also violated in the process, the operator carries $\frac{3}{2}\Delta B \pm \frac{1}{2}\Delta L$ charge under $\mathrm{U}(1)_{45}$. Since the T^2/Z_2 orbifold breaks the $\mathrm{U}(1)_{45}$ symmetry down to a discrete subgroup, the operators constructed only with the zero modes now must satisfy the selection rule,

$$\frac{3}{2}\Delta B \pm \frac{1}{2}\Delta L = 0 \bmod 4. \quad (38)$$

This makes sure that baryon number violating operators appear only at dimension 15 [43].

On the other hand, the quantum charges in Table II clearly show that this relation does not hold true once KK modes are introduced in the external legs. Below, we will derive the baryon number violating operators with KK modes and we will show that such novel operators become relevant and interesting on introducing interactions with spinless adjoint scalar partners of the hypercharge gauge boson.

Using the interaction terms given in Eqs. (23), (27), (28), (29), (30), the four-dimensional Lorentz invariant scalar and vector operators that generate dominant contribution to baryon and lepton number violations can be obtained and are given in Table IV. Indeed, the C_1^S and C_2^S Wilson coefficients are generated by scalar new physics, whereas, C_1^V and C_2^V are generated by vectors. For simplicity, we have kept only the dominant term containing zero mode of doublet Standard Model fermions among these four-dimensional operators. The rest of the contribution can be derived similarly. The dimension-eight operators require additional quartic interactions of scalar bosons but we do not get into the details here.

On the other hand, using the interactions of a spinless adjoint partner of the hypercharge boson, given in Eqs. (36) and (37), the above operators generate operators given in Table V. We will analyze the phenomenology of these operators in the coming sections.

TABLE IV. The list of dominant four-dimensional Lorentz invariant baryon number violating operators that originate from the interactions in Eq. (23), and (27) through (30).

Operators	$\Delta B = 1 = \Delta L$	$\Delta B = 2 = \Delta L$
Scalar	$\frac{C_1^S}{\Lambda_4^2}(q_{+l}^{(0,0)T}\gamma^2\gamma^0\mathcal{U}_{-l}^{(1,0)})(\mathcal{E}_{-l}^{T(1,0)}\gamma^2\gamma^0 q_{+l}^{(0,0)})$	$\frac{C_2^S}{\Lambda_4^8}(q_{+l}^{(0,0)T}\gamma^2\gamma^0\mathcal{U}_{-l}^{(1,0)})^2(\mathcal{E}_{-l}^{T(1,0)}\gamma^2\gamma^0 q_{+l}^{(0,0)})^2$
Vector	$\frac{C_1^V}{\Lambda_4^2}(u_{-r}^{(0,0)T}\gamma^2\gamma^0\gamma^\mu\mathcal{D}_{-l}^{(1,0)})(u_{-r}^{(0,0)T}\gamma^2\gamma^0\gamma_\mu\mathcal{E}_{-l}^{(1,0)})$	$\frac{C_2^V}{\Lambda_4^8}(u_{-r}^{(0,0)T}\gamma^2\gamma^0\gamma^\mu\mathcal{D}_{-l}^{(1,0)})^2(u_{-r}^{(0,0)T}\gamma^2\gamma^0\gamma_\mu\mathcal{E}_{-l}^{(1,0)})^2$

TABLE V. The baryon number violating operators with Standard Model zero mode fermions in the external legs, generated after including the interaction of spinless adjoint scalar field, given in Eq. (36) and Eq. (37).

Operators	$\Delta B = 1 = \Delta L$
Scalar	$y_u y_e g_Y^2 \frac{C_1^S}{\Lambda_4^2} \frac{1}{M_{KK}^2} (q_{+l}^{T(0,0)} \gamma^2 \gamma^0 u_{-r}^{(0,0)})(e_{-r}^{T(0,0)} \gamma^2 \gamma^0 q_{+l}^{(0,0)}) V_B^{(1,0)} V_B^{(1,0)}$
Vector	$y_u y_e g_Y^2 \frac{C_1^V}{\Lambda_4^2} \frac{1}{M_{KK}^2} (u_{-r}^{(0,0)T} \gamma^2 \gamma^0 \gamma^\mu d_{-r}^{(0,0)})(u_{-r}^{(0,0)T} \gamma^2 \gamma^0 \gamma_\mu e_{-r}^{(0,0)}) V_B^{(1,0)} V_B^{(1,0)}$

Operators	$\Delta B = 2 = \Delta L$
Scalar	$y_u^2 y_e^2 g_Y^4 \frac{C_2^S}{\Lambda_4^8} \frac{1}{M_{KK}^4} (q_{+l}^{(0,0)T} \gamma^2 \gamma^0 u_{-r}^{(0,0)})^2 (e_{-r}^{T(0,0)} \gamma^2 \gamma^0 q_{+l}^{(0,0)})^2 V_B^{(1,0)} V_B^{(1,0)} V_B^{(1,0)} V_B^{(1,0)}$
Vector	$y_u^2 y_e^2 g_Y^4 \frac{C_2^V}{\Lambda_4^8} \frac{1}{M_{KK}^4} (u_{-r}^{(0,0)T} \gamma^2 \gamma^0 \gamma^\mu d_{-r}^{(0,0)})^2 (u_{-r}^{(0,0)T} \gamma^2 \gamma^0 \gamma_\mu e_{-r}^{(0,0)})^2 V_B^{(1,0)} V_B^{(1,0)} V_B^{(1,0)} V_B^{(1,0)}$

IV. (ASSISTED) PROTON DECAY

After orbifolding and integrating out the extra dimensions, the operators that contribute to the baryon number violation by one unit, in Table IV, are

$$\mathcal{O}_1 = \frac{C_1^S}{\Lambda_4^2} (q_{+l}^{(0,0)T} \gamma^2 \gamma^0 \mathcal{U}_{-l}^{(1,0)})(\mathcal{E}_{-l}^{T(1,0)} \gamma^2 \gamma^0 q_{+l}^{(0,0)}) + \frac{C_1^V}{\Lambda_4^2} (u_{-r}^{(0,0)T} \gamma^2 \gamma^0 \gamma^\mu \mathcal{D}_{-l}^{(1,0)})(u_{-r}^{(0,0)T} \gamma^2 \gamma^0 \gamma_\mu \mathcal{E}_{-l}^{(1,0)}). \quad (39)$$

The scalar operator generates an exotic baryon number violating current, where the KK-1 partner of the right-handed up quark decays to two SM quarks and a lepton KK-1 mode. Such decays are allowed due to the $\sim$20 GeV mass split [23–25] between the KK-1 modes of the up quark and the lepton [23].

Along with the adjoint scalar interactions, the first term in Eq. (39) becomes

$$C_{\rm AND} \mathcal{O}_{\rm AND} = y_u y_e g_Y^2 \frac{C_1^S}{\Lambda_4^2} \frac{1}{M_{KK}^2} (q_{+l}^{T(0,0)} \gamma^2 \gamma^0 u_{-r}^{(0,0)}) \times (e_{-r}^{T(0,0)} \gamma^2 \gamma^0 q_{+l}^{(0,0)}) V_B^{(1,0)} V_B^{(1,0)}, \quad (40)$$

where g_Y is the coupling for the KK-1 hypercharge spinless adjoint scalar field and $y_u = 4/3$, $y_e = -2$. The processes generated by this operator are shown in Fig. 1. With SM-like interactions, the hypercharge coupling is given by $g_Y^2 = \frac{4M_z^2 Sin\theta_W^2}{v^2} \simeq 0.14$. This operator leads to the assisted proton decay process in the early epochs of the Universe, and also provides a new annihilation channel for the dark matter.

Note that at one loop, as shown in Fig. 2(b), this operator contributes to the direct proton decay. The effective operator in Eq. (40) for the process, after integrating out the loop, becomes

$$C_{p\to e} \mathcal{O}_d^{(2)} = y_u y_e g_Y^2 \frac{C_1^S}{16\pi^2 \Lambda_4^2} \left(\frac{M_s}{M_{KK}}\right)^4 \times (u_{+l}^{T(0,0)} \gamma^2 \gamma^0 d_{+l}^{(0,0)})(e_{-r}^{T(0,0)} \gamma^2 \gamma^0 u_{-r}^{(0,0)}), \quad (41)$$

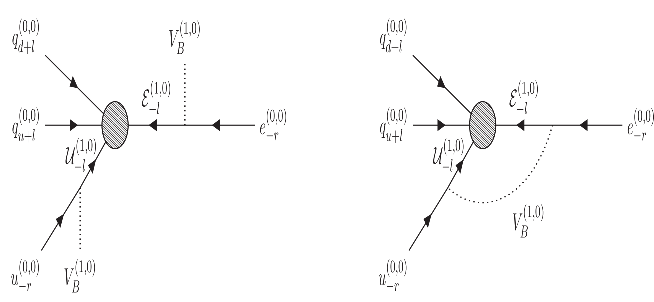
where M_s is the loop momentum and the Wilson coefficient for the decay can be read off as

$$C_{p\to e} = y_u y_e g_Y^2 \frac{C_1^S}{16\pi^2 \Lambda_4^2} \left(\frac{M_s}{M_{KK}}\right)^4. \quad (42)$$

After matching the quark level operator in Eq. (41) with the nucleon decay matrix element using χPT [21], the hadronic operator generates the decay width,

$$\Gamma_{p\to e} = \frac{1}{2\times 10^{34}} \left| \frac{C_{p\to e}}{(3\times 10^{15}\ \text{GeV})^{-2}} \right|^2. \quad (43)$$

Then, from the above relation, assuming $C_1^S = 0.01$, $M_{KK} = 10$ TeV and $M_s = m_p$, the new physics that contributes to the proton decay can be constrained to be $\gtrsim$140 TeV. This scale can be relaxed further to $\sim$40 TeV,



(a) (b)

FIG. 1. (a) The process $DM + p \to DM + e^+ + \pi^0$ generated through the scalar operator in Eq. (40). The stable spinless adjoint scalar field, $V_B^{(1,0)}$, is the dark matter candidate. (b) The process $p \to e^+ + \pi^0$ generated at one loop.

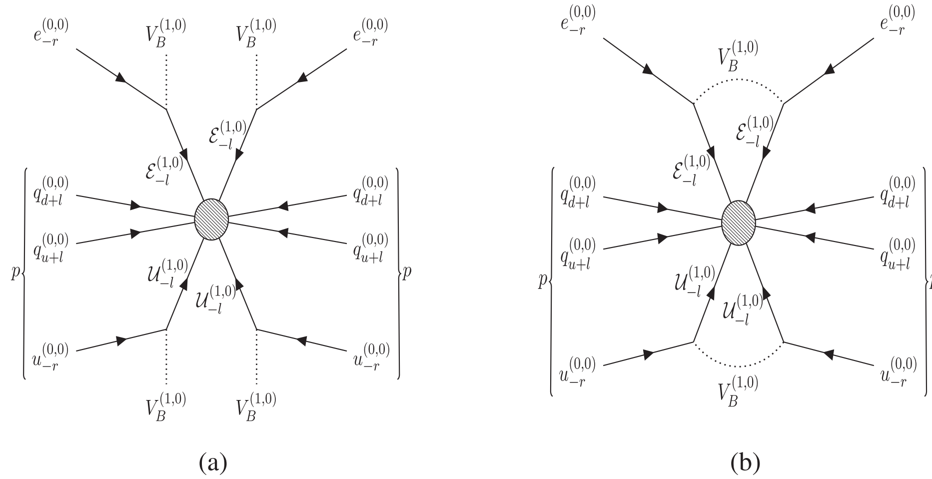


FIG. 2. (a) The processes $\Delta B = 2$, $\Delta L = 2$ given in Eq. (48). The stable adjoint scalar field, $V_B^{(1,0)}$, is the dark matter candidate. (b) The process generated at two loops.

which is within the reach of possible future 100 TeV hadron collider [48] for Wilson Coefficient $C_1^S \sim \mathcal{O}(10^{-3})$.

Getting back to the assisted nucleon decay (AND), in the scenario in which the spinless adjoint scalar becomes DM, the effective rate can be written as

$$\Gamma_{\text{AND}} = n_{\text{DM}}(\sigma v)_{\text{AND}}, \tag{44}$$

where n_{DM} is the dark matter density and $(\sigma v)_{\text{AND}}$ is the cross section for the assisted proton decay process. Using Eq. (40), the assisted proton decay cross section can be computed as

$$(\sigma v)_{\text{AND}} \sim \frac{1}{16\pi}\left|\frac{y_u y_e g_Y^2 C_1^S}{\Lambda_4^2 M_{KK}^2}\right|^2 m_p^6. \tag{45}$$

The lifetime of this process is then given by

$$\tau_{\text{AND}} = \frac{1}{\Gamma_{\text{AND}}} = \frac{M_{KK}}{\rho_{\text{DM}}(\sigma v)_{\text{AND}}}, \tag{46}$$

where the number density of DM has been replaced with the mass density $\rho_{\text{DM}} = M_{KK} n_{\text{DM}} = 0.3\ \text{GeV/cm}^3$. Using the scale of the operator previously computed, $\Lambda_4 \sim 140$ TeV, the time period for the assisted proton decay, given in Eq. (46) and assuming $C_1^S = 0.01$, becomes $\tau_{\text{AND}} \gg 1.4 \times 10^{34}$ years, satisfying the constraint from Super-Kamiokande [1]. Thus, the model suggests that the rareness of dark matter density on Earth results in the assisted nucleon decay time period much beyond the observational sensitivity of terrestrial experiments. Nevertheless, note that this process produces a striking signature with a highly collimated pion and positron Cherenkov rings. Moreover, the assisted proton decay can be much more enhanced near large dark matter densities like center of the galaxy. And since the process preserves the dark matter number density, this will play a major role in the baryon number violation near very heavy astrophysical objects.

V. (ASSISTED) $\Delta B = 2$, $\Delta L = 2$ PROCESS

After orbifolding and integrating out the extra dimensions, the operators that contribute to violation of baryon number and lepton number by two units, given in Table IV, are

$$\begin{aligned}\mathcal{O}_2 = {}& \frac{C_2^S}{\Lambda_4^8}(q_{+l}^{(0,0)T}\gamma^2\gamma^0\mathcal{U}_{-l}^{(1,0)})^2(\mathcal{E}_{-l}^{T(1,0)}\gamma^2\gamma^0 q_{+l}^{(0,0)})^2 \\ & + \frac{C_2^V}{\Lambda_4^8}(u_{-r}^{(0,0)T}\gamma^2\gamma^0\gamma^\mu\mathcal{D}_{-l}^{(1,0)})^2(u_{-r}^{(0,0)T}\gamma^2\gamma^0\gamma_\mu\mathcal{E}_{-l}^{(1,0)})^2.\end{aligned} \tag{47}$$

These operators, along with the spinless adjoint scalar interaction term given in Eq. (36), generates the assisted nucleon nucleon annihilation (ANNA). An example of this process, generated from the scalar operator, is given by the effective Lagrangian term,

$$\begin{aligned}C_{\text{ANNA}}\mathcal{O}_{\text{ANNA}} = {}& y_u^2 y_e^2 g_Y^4 \frac{C_2^S}{\Lambda_4^8 M_{KK}^4}(q_{+l}^{(0,0)T}\gamma^2\gamma^0 u_{-r}^{(0,0)})^2 \\ & \times (e_{-r}^{T(0,0)}\gamma^2\gamma^0 q_{+l}^{(0,0)})^2 V_B^{(1,0)}V_B^{(1,0)}V_B^{(1,0)}V_B^{(1,0)}.\end{aligned} \tag{48}$$

If the adjoint scalar becomes the dark matter, this interaction generates $DM + p + DM + p \to DM + e^+ + DM + e^+$ and assisted hydrogen-antihydrogen oscillation as shown in Fig. 2(a). Though the probability of these processes is very small on Earth, they can be substantial near dark matter clumps. These processes can be interesting to study in the context of the observed positron excess in cosmic rays [49–54].

At loop level there exists processes like $p+p\to e^{+}+e^{+}$, $DM+p\to DM+\bar{p}+e^{+}+e^{+}$, and $(DM+e)+p\to(DM+e^{+})+\bar{p}$, but except for the first, current experiments do not constrain the rest directly. Nevertheless, we rederive the bounds on these processes using the results from $pp\to e^{+}e^{+}$ below.

A. $p+p\to e^{+}+e^{+}$

At two loops, the diagram given in Fig. 2(b) generates processes like hydrogen-antihydrogen oscillation and $p+p\to\ell^{+}+\ell^{+}$, where $\ell^{+}=(e^{+},\mu^{+})$. For the proton-proton annihilation process, the hadron-level effective operator becomes

$$\frac{1}{\Lambda_{ppee}^{2}}(\bar{p}^{c}\gamma^{5}p)(\bar{e}^{c}\gamma^{5}e). \tag{49}$$

At Super-Kamiokande, this term is constrained by studying the process ${}^{16}\mathrm{O}\to{}^{14}\mathrm{C}+\ell^{+}+\ell^{+}$, with same-sign dilepton back-to-back Cherenkov rings with no hadrons. With a fiducial mass of 22.5 kilotons containing $\sim10^{34}$ nucleons, the time period of the decay per oxygen nucleus is constrained to $T_{pp\to e^{+}e^{+}}=4.2\times10^{33}$ years [1]. Comparing Eq. (49) with the first term in Eq. (47), and using the well-known nuclear matrix element, we get

$$\frac{1}{\Lambda_{ppee}^{2}}=\frac{C_{2}^{S}}{\Lambda_{4}^{8}}*(0.22m_{p})^{6}\left(\frac{1}{16\pi^{2}}\right)^{2}\left(\frac{Ms}{M_{KK}}\right)^{4}. \tag{50}$$

Assuming a new physics scale $\Lambda_{4}\sim2$ TeV, with $C_{2}^{S}\sim\mathcal{O}(1)$ and $M_{KK}=10$ TeV, the hadronic effective operator gets highly suppressed beyond the current sensitivity of experiments.

Moreover, the hydrogen-anti hydrogen oscillation process $pe\to\bar{p}e^{+}$, obtained by Fierz transforming the relation in Eq. (49), can be studied by measuring the γ rays from the annihilation of antihydrogen at the interstellar medium [22], and annihilation of proton in a nucleus with an electron in the inner shell of oxygen at Super-Kamiokande [1].

B. $DM+p\to DM+\bar{p}+e^{+}+e^{+}$

Interestingly, the term in Eq. (47) generates baryon number violating processes in which dark matter interacts with the proton legs. The most constraining among these processes is the one in which dark matter scatters with one proton producing same-sign dileptons, an antiproton ($\bar{p}$) and dark matter at rest ($DM+p\to DM+\bar{p}+e^{+}+e^{+}$). With the $\bar{p}$ annihilating with a another proton, the results of $p+p\to e^{+}+e^{+}$ can be recasted here. Since this is initiated by a heavy dark matter, the dileptons produced will be highly energetic. They can be searched for at Super-Kamiokande through two collinear Cherenkov rings. To analyze this, let us write the hadron-level operator,

$$\mathcal{O}_{VVppee}=\frac{1}{(\Lambda_{VVppee})^{4}}(\bar{p}^{c}p)(\bar{e}^{c}e)V_{B}^{(1,0)}V_{B}^{(1,0)}, \tag{51}$$

where, comparing with Eq. (48), we get

$$\frac{1}{(\Lambda_{VVppee})^{4}}=y_{u}y_{e}g_{Y}^{2}\frac{C_{2}^{S}}{\Lambda_{4}^{8}}\frac{1}{M_{KK}^{2}}(0.22m_{p})^{6}\frac{1}{16\pi^{2}}\left(\frac{M_{s}}{M_{KK}}\right)^{2}. \tag{52}$$

The effective decay width for the nucleon, then, becomes

$$\Gamma_{\mathrm{ANNA}}=n_{\mathrm{DM}}(\sigma v)_{\mathrm{ANNA}}, \tag{53}$$

where n_{DM} is the dark matter density and $(\sigma v)_{\mathrm{ANNA}}\sim\frac{1}{16\pi}|\frac{1}{(\Lambda_{VVppee})^{4}}|^{2}m_{p}^{6}$ is the cross section for the assisted double nucleon decay process given by the operator in Eq. (51). The lifetime of this process is then given by

$$\tau_{\mathrm{ANNA}}=\frac{1}{\Gamma_{\mathrm{ANNA}}}=\frac{M_{KK}}{\rho_{\mathrm{DM}}(\sigma v)_{\mathrm{ANNA}}}. \tag{54}$$

In the above equation, we have replaced the number density of DM with the mass density $\rho_{\mathrm{DM}}=M_{KK}n_{\mathrm{DM}}=0.3\ \mathrm{GeV\,cm^{-3}}$. For dark matter of mass $M_{KK}=2$ TeV, from the galactic center, with speed ~100 km/s, colliding with an ${}^{16}O$ atom in the experiment, the average transfer momentum can be computed to be $P_{t}=m_{V}v\sim600$ MeV. In this process, $DM+p\to DM+\bar{p}+e^{+}+e^{+}$, since $m_{e}\ll P_{t}<m_{p}\ll m_{V}$, and we can safely assume that the DM and antiproton are produced at rest. Thus, for all practical considerations, this is a $2\to2$ process with a proton in the incoming leg at rest in the lab frame.

Using the limit on double proton decay time period $\tau_{pp\to e^{+}e^{+}}\gtrsim4.2\times10^{33}$ years [1], we get

$$\tau_{\mathrm{ANNA}}=5\times10^{33}\mathrm{years}\left(\frac{\Lambda_{VVppee}}{300\ \mathrm{GeV}}\right)^{8}. \tag{55}$$

This is a very weak limit for the new physics model, thus terrestrial experiments are not very sensitive to the assisted nucleon-nucleon decay yet. On the other hand, the clean and unique signal for this event is very interesting, in case a dark matter interacts in the upcoming Hyper-Kamiokande [18] experiment. The constraint on this operator is weak due to the rarity of dark matter density on Earth, whereas in primordial superdense cosmological dark matter clumps [29,30] with large gravitating mass, this may not be the case. Such assisted double nucleon decays can be a very large source of baryon number violation in cosmology. The processes such as assisted hydrogen oscillation $[DM(pe)\to DM(\bar{p}e^{+})]$ can also be searched for at the interstellar medium by Fermi-Lat and can give complementary measurements for the operator.

This process also generates baryon number violation in which the dark matter interacts with the lepton legs. Though this process is kinematically prohibited at low energies, it can be probed in high energy collider experiments. Such processes can be constrained by the CMS [55] study where they consider finals states with two same sign leptons, two or more hadronic jets, and missing energy. Unfortunately, since the operator is at mass dimension 10, it is highly suppressed and moreover, at a high energy collider, the patrons will probe the insides of the effective operator. Though the best constraint on this operator might arise from the collider experiments, this will not be a model independent result.

C. $(DM + e) + p \to (DM + e^+) + \bar{p}$

On the other hand, $(DM + e) + p \to (DM + e^+) + \bar{p}$, or DM assisted hydrogen-antihydrogen oscillation can be a better probe to study this operator. The relevant constraint emanates from the nonobeservation of hydrogen-antihydrogen annihilation γ rays from the ISM surveyed by Fermi LAT.

We consider the scenario where the hydrogen atoms, in its ground state, are influenced by dark matter. Then the oscillation Hamiltonian,

$$H_{\mathrm{osc}} = \frac{1}{\Lambda_{AH\bar{H}}^4} (\bar{p}^c e)(\bar{p}^c e) V_B^{(1,0)} V_B^{(1,0)}, \qquad (56)$$

generates small amounts of antihydrogen. Since the conversion is enabled by dark matter number density (n_{DM}) in the ISM, the rate of antihydrogen production is given by

$$\tau_{\mathrm{osc}} = \frac{1}{\Gamma_{\mathrm{osc}}} = \frac{1}{n_{\mathrm{DM}} (\sigma v)_{\mathrm{osc}}}, \qquad (57)$$

where $(\sigma v)_{\mathrm{osc}} = \frac{1}{16\pi} \left(\frac{m_p^6}{\Lambda_{AH\bar{H}}^8}\right)$ is the cross section for the process.

Then, the rate of assisted hydrogen oscillation can be computed by studying the production rate of γ rays due to the annihilation of the antihydrogen with hydrogen [20]. For a new physics, presumably at $\Lambda_{AH\bar{H}}$ of $\mathcal{O}(1\ \mathrm{TeV})$, the width of the process can be computed to be $\Gamma_{\mathrm{osc}} \sim 7 \times 10^{-44} s^{-1}$. Thus, the constraint placed by the analysis of γ-ray data from Fermi LAT in the range 100 MeV–9.05 GeV [22] does not constrain this operator. A more reliable bound can be obtained from 14 TeV or 100 TeV LHC, but that requires a UV complete model. Nevertheless, this process also depends on the dark matter density. Thus, galaxy cluster centers can be a good source of assisted hydrogen-antihydrogen oscillation.

VI. CONCLUSION

The phenomenology in $4k + 2$ dimensions, in particular six dimensions, is very interesting. Vanishing of the Witten anomaly in six dimensions, arising from the nontrivial winding of the spacetime on $\mathrm{SU}(2)_W$ given by $\pi_6(\mathrm{SU}(2)_W) = Z_{12}$, correctly predicts the fermion generations charged under the $\mathrm{SU}(2)_w$ gauge group to be multiples of 3. The gauge bosons in dimensions $d \geq 6$ also exhibit interesting properties. In an uncompactified geometry, these gauge bosons have $d - 2$ polarization vectors. Upon compactification, $d - 4$ of them break and one combination among them is "eaten" by the KK modes making them heavy. The rest of the broken polarizations become the spinless adjoint scalar fields and remain in a KK spectrum with mass $\sim 1/R$, where R is the compactification radius. In general, due to mass corrections, at one loop, the degeneracy among all the lightest KK masses is lifted leading to a mass hierarchy, and the spinless adjoint scalar of the hypercharge gauge boson becomes the lightest. Including the KK-partiy conservation, this KK-1 scalar is stable and hence is the dark matter candidate in the model. The relic density constraint places a bound of ~2 TeV on the compactifications scale, assuming that the adjoint scalar makes up the entire dark matter density. This limit can be relaxed by introducing additional resonant annihilation and coannihilation channels, either by including additional fields or embedding the six dimensions in a seven-dimensional space-time with only gravity allowed to propagate in the bulk [56].

The spinor properties in $4k + 2$ dimensions are different from $4k$ dimension. In $4k + 2$ dimensions, the charge conjugation operator commutes with the chiral projection operator, thus keeping the chirality unchanged under charge conjugation of the fermion. And its consequences are interesting in the context of baryon number and lepton number violating currents. In this article, we analyze such effective operators in six dimensions, as a minimal extension. For simplicity and clarity, we explicitly work with Standard Model and new physics (scalar and vector bosons) in the bulk. Though these new physics fields, in four dimensions, generate the usual baryon number violation, in orbifolded six dimensions they generate novel operators given in Table IV. In six dimensions, the tree-level proton decay operator with only KK zero modes in their external legs have been shown to be highly suppressed [43] due to the emergent selection criteria $3/2\Delta B \pm 1/2\Delta L = 0 \bmod 4$. Here, we show that this selection criteria can be circumvented if we allow higher Kaluza-Klein modes in the operators. Moreover, these novel operators generate dark matter assisted baryon number violating processes at mass dimension eight and higher upon including the interaction of the spinless adjoint scalar field. These processes, among others, contain assisted proton decay $V_B^{(1,0)} + p \to V_B^{(1,0)} + e^+ + \pi^0$ and assisted hydrogen-antihydrogen oscillation $V_B^{(1,0)} + p + e \to V_B^{(1,0)} + \bar{p} + e^+$. We show that the proton decay data from Super-Kamiokande constrains the assisted proton decay operator to

$\gtrsim$1400 TeV, for $1/R = 10$ TeV and $C_1^S = \mathcal{O}(1)$. Though the kinematics of this $2\to 3$ process are different, they can be identified in the water Cherenkov detector with rings corresponding to a positron and a pion [34]. The lower bound on the new physics can indeed be brought within the reach of a 100 TeV collider in the weak coupling limit, $C_1^S \lesssim 10^{-3}$. But in the upcoming HyperKamiokande experiment, with its better sensitivity to the proton decay process, the scale of new physics could be constrained by a factor $\mathcal{O}(10)$.

Unlike the models with baryon number violating scalar and vector bosons in four dimensions, here we show that the rarity of the processes at terrestrial experiments could be explained by the lack of enough dark matter density on Earth. On the other hand, they have larger probability to occur near dark matter clusters and can be uniquely identified by studying the positron fluxes emerging from such clusters. Such a scenario could also arise from dark matter accumulation in the Sun.

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APPENDIX: STANDARD MODEL HYPERCHARGE GAUGE FIELD AND ITS KK MODES

The Lagrangian density of an Abelian gauge field in six dimensions is given by

$$\mathcal{L} = \frac{1}{4}F^{MN}F_{MN} + \mathcal{L}_{GF}, \tag{A1}$$

where $M = 0, 1, 2, 3, 4, 5$ and $\mathcal{L}_{GF}$ is the appropriate gauge-fixing term. Since the six-dimensional geometry is orbifolded on T^2/Z_2, we use the generalized R_ξ gauge,

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}(\partial_\mu \mathcal{A}_\mu - \xi(\partial_4\mathcal{A}_4 + \partial_5\mathcal{A}_5))^2. \tag{A2}$$

Expanding the terms we get

$$\mathcal{L} = \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}((\partial_\mu\mathcal{A}_4 - \partial_4\mathcal{A}_\mu)^2 + (\partial_\mu\mathcal{A}_5 - \partial_5\mathcal{A}_\mu)^2 + (\partial_5\mathcal{A}_4 - \partial_4\mathcal{A}_5)^2) - \frac{1}{2\xi}(\partial_\mu\mathcal{A}_\mu)^2$$
$$- \frac{\xi}{2}((\partial_4\mathcal{A}_4)^2 + (\partial_5\mathcal{A}_5)^2 + 2(\partial_4\mathcal{A}_4)(\partial_5\mathcal{A}_5)) - (\partial_\mu\mathcal{A}_\mu)(\partial_4\mathcal{A}_4) - (\partial_\mu\mathcal{A}_\mu)(\partial_5\mathcal{A}_5). \tag{A3}$$

Upon orbifolding, the $\mathcal{A}_\mu(x^\mu, x^4, x^5)$ is set to satisfy the Neumann boundary condition and $\mathcal{A}_4(x^\mu, x^4, x^5)$, $\mathcal{A}_5(x^\mu, x^4, x^5)$ are set to satisfy the Dirichlet boundary condition at both the brane positions. Hence, they get decomposed in terms of their KK modes as

$$\mathcal{A}_\mu(x^\mu, x^4, x^5) = {\mathcal{A}_\mu}^{(0,0)}(x^\mu) + \sqrt{2}\sum_{m,n}\mathcal{A}_\mu^{(m,n)}(x^\mu)\cos\left[\frac{1}{R}(mx_4 + nx_5)\right],$$
$$\mathcal{A}_4(x^\mu, x^4, x^5) = \sum_{m,n}\mathcal{A}_4^{(m,n)}(x^\mu)\sin\left[\frac{1}{R}(mx_4 + nx_5)\right],$$
$$\mathcal{A}_5(x^\mu, x^4, x^5) = \sum_{m,n}\mathcal{A}_5^{(m,n)}(x^\mu)\sin\left[\frac{1}{R}(mx_4 + nx_5)\right]. \tag{A4}$$

The zero mode of $\mathcal{A}_\mu(x^\mu, x^4, x^5)$ is identified with the four-dimensional Standard Model gauge field, while $\mathcal{A}_4$ and $\mathcal{A}_5$ are adjoint scalar fields that arise from the two broken polarizations of the six-dimensional gauge field. Note that these adjoint scalar fields do not possess zero modes.

After integrating out the x_4, x_5 coordinates, the partial derivatives ∂_4, ∂_5 are replaced by $\frac{m}{R}$ and $\frac{n}{R}$ respectively. The simplified four-dimensional Lagrangian density is given by

$$\mathcal{L} = \frac{1}{4}F^{(m,n)\mu\nu}F^{(m,n)}_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu\mathcal{A}_\mu^{(m,n)})^2 + (\partial_\mu\mathcal{A}_4^{(m,n)})^2 + (\partial_\mu\mathcal{A}_5^{(m,n)})^2 + \frac{1}{2}\left(\frac{n}{R}\mathcal{A}_4^{(m,n)} - \frac{m}{R}\mathcal{A}_5^{(m,n)}\right)^2$$
$$- \frac{\xi}{2R^2}(m^2\mathcal{A}_4^{(m,n)2} + n^2\mathcal{A}_5^{(m,n)2} + 2mn\mathcal{A}_4^{(m,n)}\mathcal{A}_5^{(m,n)}). \tag{A5}$$

In the above equation, the Lagrangian term with $\frac{\xi}{2}$ can be written in matrix form,

$$\mathcal{L}_\xi = \left[\mathcal{A}_4^{(m,n)} \mathcal{A}_5^{(m,n)}\right] \frac{1}{R} \begin{bmatrix} m^2 & mn \\ mn & n^2 \end{bmatrix} \frac{1}{R} \begin{bmatrix} \mathcal{A}_4^{(m,n)} \\ \mathcal{A}_5^{(m,n)} \end{bmatrix}. \quad \text{(A6)}$$

After diagonalizing this matrix we get

$$\mathcal{L}_\xi = \left[V_1^{(m,n)} V_2^{(m,n)}\right] \frac{1}{R} \begin{bmatrix} m^2+n^2 & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{R} \begin{bmatrix} V_1^{(m,n)} \\ V_2^{(m,n)} \end{bmatrix}, \quad \text{(A7)}$$

where $V_1^{(m,n)}$ and $V_2^{(m,n)}$ are given by

$$V_1^{(m,n)} = \frac{m}{\sqrt{m^2+n^2}} \mathcal{A}_4^{(m,n)} + \frac{n}{\sqrt{m^2+n^2}} \mathcal{A}_5^{(m,n)},$$
$$V_2^{(m,n)} = \frac{-n}{\sqrt{m^2+n^2}} \mathcal{A}_4^{(m,n)} + \frac{m}{\sqrt{m^2+n^2}} \mathcal{A}_5^{(m,n)}. \quad \text{(A8)}$$

The fields $\mathcal{A}_4^{(m,n)}$ and $\mathcal{A}_5^{(m,n)}$ are replaced by its scalar adjoint $V_1^{(m,n)}$ and $V_2^{(m,n)}$ in Eq. (A3),

$$\mathcal{L} = \frac{1}{4} F^{(m,n)\mu\nu} F^{(m,n)}_{\mu\nu} - \frac{1}{2\xi} (\partial_\mu \mathcal{A}_\mu^{(m,n)})^2 + (\partial_\mu V_1^{(m,n)})^2 + \frac{\xi}{2R^2} (m^2+n^2) V_1^{(m,n)2} + (\partial_\mu V_2^{(m,n)})^2 + (m^2+n^2) V_2^{(m,n)2}. \quad \text{(A9)}$$

Gauge invariance implies that ξ must drop out from any calculation of physical observables. The limit $\xi \to \infty$, unitary gauge, V_1 becomes nondynamical. Hence, only the adjoint $V_2^{(m,n)}$ remains as a physical spin-0 particle. The lightest stable spinless adjoint partner of hypercharge gauge boson, $V_2^{(1,0)}$, becomes the dark matter candidate. We refer to it as $V_B^{(1,0)}$ throughout this article.

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