Tension between neutrino masses and gauge coupling unification in natural grand unified theories

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The natural grand unified theories (GUT) solve various problems of the supersymmetric GUT and give realistic quark and lepton mass matrices under the natural assumption that all terms allowed by the symmetry are introduced with $O(1)$ coefficients. However, because of the natural assumption, it is difficult to achieve the gauge coupling unification without tuning, while keeping neutrino masses at realistic values. In this paper, we try to avoid this tension between the neutrino masses and the gauge coupling unification, by introducing suppression factors for several terms. These suppression factors can be understood by approximate symmetries in some of the solutions. We show that one of the most important results in the natural GUT scenario, that the nucleon decay mediated by superheavy gauge fields is enhanced due to a smaller unification scale while the nucleon decay mediated by superheavy colored Higgs is suppressed, may change in those models proposed in this paper.

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I. INTRODUCTION

The grand unified theory (GUT) [1] is one of the most promising models beyond the standard model (SM). The GUT not only unifies three of the four forces in nature, except gravity, but also unifies quarks and leptons. Furthermore, the GUT already has been supported by experiments for each unification. With respect to the force unification, it has been shown that by introducing supersymmetry (SUSY) [2,3], the three running gauge couplings of the SM coincide at the grand unified scale $\Lambda_G \sim 2 \times 10^{16}$ GeV [4]. As for the unification of matter, various hierarchies of quark and lepton masses and mixings measured in numerous experiments [5] can be reasonably explained by the $SU(5)$ GUT, in which quarks and leptons of each generation can be unified into $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations. In the $SU(5)$ unification, an assumption that $\mathbf{10}$ fields induce stronger hierarchies in Yukawa couplings than $\bar{\mathbf{5}}$ fields explains not only various mass hierarchies of quarks and leptons (where up-type quarks and neutrinos have the strongest and the weakest mass hierarchy,[1] respectively) but also simultaneously describes that the quark mixings are smaller than the lepton mixings. This is a nontrivial result, and hence this can be understood as an experimental support for the matter unification in $SU(5)$ unification. Unfortunately, SUSY GUT [3] has two main problems. The first is that the unification of matter also unifies the mass matrices of quarks and leptons, which are inconsistent with the observed values [5]. The second problem is that the mass of the SM Higgs partner (which is called colored Higgs or triplet Higgs) must be sufficiently large compared to the SM Higgs mass to obtain a sufficiently stable proton, which is difficult to achieve without fine-tuning. The later problem is one of the most serious problems in the SUSY GUT scenario and is called the doublet-triplet (DT) splitting problem [6].

The natural GUTs [7–9] solve those SUSY GUT problems under a natural assumption that all terms allowed by the symmetry are introduced with $O(1)$ coefficients. As a result, we obtain a natural GUT which becomes the SM at low energy. Unfortunately, if we take the $O(1)$ coefficients for the mass terms of all superheavy particles to be 1, three gauge coupling constants do not unify when the measured values are used as boundary conditions. In other words, the

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[1]Here, we assume the normal hierarchy for the neutrino mass spectrum, and we do not take account of the lightest neutrino mass which has not been observed yet.

running gauge coupling constants at low energies will differ from the measured values when the equal gauge coupling constants at the unified scale are used as boundary conditions. To achieve the unification of the gauge coupling constants, many $O(1)$ coefficients in the range of 0.5 to 2 must be artificially chosen, which is in a sense a fine-tuning [7]. As will be discussed in detail in this paper, this issue is related with the measured neutrino masses. That is, under the assumptions in the natural GUT, there is a tension between the unification of gauge couplings and the measured neutrino masses.

In this paper, we discuss how to resolve this tension. In particular, we consider the possibility of solving this problem by considering cases in which small suppression factors are applied to several particular terms rather than $O(1)$ coefficients. After building the models, we discuss the origin of these small suppression factors, such as approximate symmetries. Moreover, we discuss the predictions on the nucleon decay in those models. Interestingly, the predictions of the natural GUT that the nucleon decay mediated by the superheavy gauge fields (the main decay mode is usually considered to be $P \to e^+\pi^0$ [10,11]) is enhanced due to a smaller unification scale while that mediated by the superheavy colored Higgs (the main decay mode is usually considered to be $P \to K^+\bar{\nu}$ [10,11]) is suppressed can change in those scenarios.[2]

After this Introduction, we review the natural GUT in Sec. II. In Sec. III, we discuss the resolutions of this tension and approximate symmetries to make this solution natural. We also build explicit natural GUT models and discuss the nucleon decay therein. Section IV is devoted to discussions and summary.

II. NATURAL GUT AND ITS PROBLEMS

In this section, we review the natural $SO(10)$ GUT following the papers [7,8] and its problem, which also appears in the natural E_6 GUT [9].

One of the most important features in the natural GUT is that all terms allowed by the symmetry are introduced with $O(1)$ coefficients. Because of this feature, once we fix the symmetry, the model can be defined except $O(1)$ coefficients. Under the symmetry $SO(10) \times U(1)_A \times Z_2$, typical quantum numbers of the field content is given in Table I. In this model, the specific charges differ from those in the papers [7,8]. This model minimizes the tuning of the $O(1)$ coefficients needed to achieve gauge coupling unification among models that achieve realistic quark and lepton masses and mixing angles while solving the DT splitting problem naturally.

TABLE I. Field content of natural $SO(10)$ GUT with $U(1)_A$ charges. $\pm$ labels the Z_2 parity. The half-integer $U(1)_A$ charges play the same role as R-parity.

$SO(10)$	Negatively charged fields	Positively charged fields	Matter fields
45	$A(a=-1,-)$	$A'(a'=3,-)$	
16	$C(c=-4,+)$	$C'(c'=3,-)$	$\Psi_i(\psi_1=\frac{9}{2},\psi_2=\frac{7}{2},\psi_3=\frac{3}{2},+)$
$\overline{\mathbf{16}}$	$\bar{C}(\bar{c}=-1,+)$	$\bar{C}'(\bar{c}'=7,-)$	
10	$H(h=-3,+)$	$H'(h'=4,-)$	$T(t=\frac{5}{2},+)$
1	$\Theta(\theta=-1,+)$, $Z(z=-2,-)$, $\bar{Z}(\bar{z}=-2,-)$	$S(s=5,+)$	

In this paper, we use large characters for fields or operators and small characters for their $U(1)_A$ charges. The $U(1)_A$ [13] has gauge anomalies that are canceled by the Green-Schwarz mechanism [14],[3] and the Fayet-Iliopoulos (FI) term [16] $\xi^2 \int d^2\theta V_A$ is assumed, where V_A and ξ are a vector multiplet of $U(1)_A$ and a constant parameter, respectively. It is surprising that various problems in SUSY GUT scenarios, including the DT splitting problem, can be solved in this model with the above natural feature. Unfortunately, there is a tension between the neutrino masses and the unification of the gauge couplings. Let us explain them in detail in the review of the natural GUT below.

A. Anomalous $U(1)_A$ gauge symmetry

First, for simplicity, we consider a simpler model in which we have only three matter fields Ψ_i, two negatively charged fields H, and Θ as shown in Table I. The superpotential invariant under $U(1)_A$ is given as

$$W_Y = c_{ij} \sum_{i,j=1,2,3} \left(\frac{\Theta}{\Lambda}\right)^{\psi_i+\psi_j+h} \Psi_i \Psi_j H, \tag{1}$$

where Λ and c_{ij} are the cutoff of the model and the $O(1)$ coefficients, respectively. If we assume that only Θ has a nonvanishing vacuum expectation value (VEV), which is determined by the D-flatness condition of $U(1)_A$ as $\langle\Theta\rangle = \xi \equiv \lambda\Lambda$, the interaction terms in the above superpotential become the hierarchical Yukawa interactions as

$$W_Y = c_{ij}\lambda^{\psi_i+\psi_j+h}\Psi_i\Psi_j H, \tag{2}$$

[2]The proton decay mediated by superheavy gauge fields is called the proton decay via dimension-six operators because it is caused by an effective interaction in dimension six with four fermions. On the other hand, the proton decay mediated by a superheavy colored Higgs field is called the proton decay via dimension-five operators because it is caused by the effective interaction in dimension five with two fermions and two scalar fermions [11,12].

[3]Strictly, the fields in Table I alone may not satisfy the conditions for the anomaly cancellation by shifting the dilaton [15], but the arbitrariness of the normalization of $U(1)_A$ and the introduction of a few new $SO(10)$ singlet fields can satisfy those conditions.

when $\lambda < 1$. In this paper, we take $\lambda \sim 0.22$, which is approximately the Cabibbo angle. Unless otherwise noted, the $O(1)$ coefficients are omitted and we take $\Lambda = 1$ in this paper. The realization of the hierarchical structure of Yukawa couplings by higher dimensional effective interactions by developing the VEV of some fields, which breaks a (flavor) symmetry, is often called the Froggatt-Nielsen mechanism [17]. The important point is that the Yukawa hierarchy can be reproduced under the natural assumption that all terms allowed by the symmetry are introduced with $O(1)$ coefficients, including higher dimensional terms.

However, it is quite rare to adopt this natural assumption even in the GUT Higgs sector in which the GUT group is spontaneously broken into the SM gauge group, mainly because it is difficult to control an infinite number of higher dimensional terms. Within the same theory, it is not reasonable for the Yukawa sector to adopt this natural assumption and the Higgs sector not. The natural GUTs are the theories in which this natural assumption is adopted in the GUT Higgs sector as well as in the Yukawa sector.

Note that the Higgs mass term $\lambda^{2h}H^2$ is forbidden when $h < 0$ because of the holomorphic feature of the superpotential, which is called the SUSY zero mechanism, or the holomorphic zero mechanism. The SUSY zero mechanism plays important roles in controlling the infinite number of higher dimensional terms and in solving the DT splitting problem. We will explain them in the next subsection.

B. Higgs sector in natural GUT

In this subsection, we will briefly review the GUT Higgs sector in the natural GUT, which breaks $SO(10)$ into $G_{\rm SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$ and solves the DT splitting problem under the natural assumption.

One of the most important assumptions is that all positively $U(1)_A$ charged fields have vanishing VEVs. This assumption not only allows the SUSY zero mechanism to work but also provides control over an infinite number of higher dimensional terms. Under this assumption, it is easy to show that the F-flatness conditions for negatively charged fields are automatically satisfied. The F-flatness conditions of positively charged fields determine the VEVs of negatively charged fields. As a result, we obtain the VEVs of the composite operators O, which are invariant under the GUT gauge group, with $U(1)_A$ charge o as [7]

$$\langle O \rangle \sim \begin{cases} \lambda^{-o} & (o \leq 0) \\ 0 & (o > 0) \end{cases}. \tag{3}$$

Thus, ignoring the D-flatness conditions, if the number of positively charged fields equals that of negatively charged fields, the VEVs of all negatively charged fields can in principle be determined.

To break $SO(10)$ into the SM gauge group $G_{\rm SM}$, an adjoint Higgs $\mathbf{45}_A$ and one pair of spinor $\mathbf{16}_C$ and antispinor $\overline{\mathbf{16}}_{\bar{C}}$ are minimally required [18]. In addition, to include the SM Higgs, $\mathbf{10}_H$ is needed. These fields must have negative $U(1)_A$ charges because these fields have nonvanishing VEVs. Moreover, the same number of positively charged fields are introduced as in Table I to fix the VEVs of these negatively charged fields. It is nontrivial that simply introducing the smallest number of fields necessary in $SO(10)$ GUT as in Table I can solve various problems, including the DT splitting problem. Note that the terms which include two or more positively charged fields have no effects in fixing these VEVs of negatively charged fields under the assumption. Therefore, only those which include one positively charged field are important to fix the VEVs. The superpotentials for fixing the VEVs are

$$W = W_{H'} + W_{A'} + W_S + W_{C'} + W_{\bar{C}'}, \tag{4}$$

where W_X denotes the terms linear in the X field. Each W_X includes a finite number of terms because of the SUSY zero mechanism. Note that only a finite number of terms is important to fix the VEVs although an infinite number of higher dimensional terms is introduced.

Now we discuss how to determine the VEVs via W_X. First, we consider $W_{A'}$, which is given as

$$W_{A'} = \lambda^{a'+a}A'A + \lambda^{a'+3a}((A'A)_{\mathbf{1}}(A^2)_{\mathbf{1}} + (A'A)_{\mathbf{54}}(A^2)_{\mathbf{54}}), \tag{5}$$

where the subscripts $\mathbf{1}$ and $\mathbf{54}$ denote the representation of the composite operators under the $SO(10)$ gauge symmetry. The F-flatness condition of A' fixes the VEV of A. One of the 6 vacua[4] takes the Dimopoulos-Wilczek (DW) form [19] as $\langle A \rangle = i\tau_2 \times \mathrm{diag}(v, v, v, 0, 0)$, which breaks $SO(10)$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Note that the v is determined by the $U(1)_A$ charge of A as $v \sim \lambda^{-a}$. This VEV of A plays an important role in solving the DT splitting problem. Actually, through

$$W_{H'} = \lambda^{h+a+h'}H'AH \tag{6}$$

the triplet Higgses become massive while the doublet ones remain massless. One pair of doublet Higgses becomes massive through the mass term $\lambda^{2h'}H'^2$. (Note that to determine the mass spectrum, the terms that include two positively charged fields must be considered.) Then, only one pair of doublet Higgses becomes massless, and therefore, the DT splitting problem can be solved. The effective colored Higgs mass related to the nucleon decay

[4]Without loss of generality, the VEV is written as $\langle A \rangle = i\tau_2 \times \mathrm{diag}(x_1, x_2, x_3, x_4, x_5)$. The vacua are classified by the number of 0 because the F-flatness condition of A' gives the solution $x_i = 0, v$. Therefore, the number of vacua becomes 6.

becomes λ^{2h}, which is larger than the cutoff scale because $h < 0$. Note that Z_2 parity has been introduced to forbid the $H'H$ term which gives the GUT scale mass to the doublet Higgs and therefore spoils the DT splitting.

The VEVs of C and $\bar{C}$, which are important to break $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ into $G_{\rm SM}$, are induced by the F-flatness condition of S by the superpotential

$$W_S = \lambda^s S\left(1 + \lambda^{c+\bar{c}}\bar{C}C + \sum_k \lambda^{2ka}A^{2k}\right). \tag{7}$$

Since $\lambda^{2ka}\langle A^{2k}\rangle \sim 1$, the last term in Eq. (7) does not basically change the following result. The F-flatness condition of S gives $\langle \bar{C}C\rangle \sim \lambda^{-(c+\bar{c})}$, and thus the D-flatness condition of $SO(10)$ leads to $|\langle C\rangle| = |\langle \bar{C}\rangle| \sim \lambda^{-(c+\bar{c})/2}$. Note that the VEVs of C and $\bar{C}$ are again determined by their charges. The F-flatness conditions of C' and $\bar{C}'$ realize the alignment of the VEVs $\langle C\rangle$, $\langle \bar{C}\rangle$, and $\langle A\rangle$, and impart masses to the pseudo–Nambu-Goldstone fields.[5] This mechanism proposed by Barr and Raby [20] is naturally embedded in the natural GUT. $W_{C'}$ and $W_{\bar{C}'}$ are given as

$$W_{C'} = \bar{C}(\lambda^{\bar{c}+c'+a}A + \lambda^{\bar{c}+c'+\bar{z}}\bar{Z})C', \tag{8}$$

$$W_{\bar{C}'} = \bar{C}'(\lambda^{\bar{c}'+c+a}A + \lambda^{\bar{c}'+c+z}Z)C. \tag{9}$$

Since the VEV of A is proportional to the $B-L$ generator Q_{B-L}, only one of the four component fields $(\mathbf{3},\mathbf{2},\mathbf{1})_{1/3}$, $(\bar{\mathbf{3}},\mathbf{1},\mathbf{2})_{-1/3}$, $(\mathbf{1},\mathbf{2},\mathbf{1})_{-1}$, and $(\mathbf{1},\mathbf{1},\mathbf{2})_1$ under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, which are obtained by the decomposition of **16** of $SO(10)$, has a nonvanishing VEV. When the component $(\mathbf{1},\mathbf{1},\mathbf{2})_1$ has a nonvanishing VEV, $G_{\rm SM}$ can be obtained.

C. Mass spectra of superheavy particles

Since all the interactions are determined by the symmetry, the mass spectra of superheavy particles are also fixed except the $O(1)$ coefficients in the natural GUTs. The mass spectra are important in calculating the renormalization group equations (RGEs) of the gauge couplings. Note that we have to consider also the terms which include two positively charged fields in order to examine the mass spectra.

The spinor **16**, the vector **10**, and the adjoint **45** of $SO(10)$ are decomposed under $SO(10) \supset SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$ as

$$\mathbf{16} \to \underbrace{[Q + U^c + E^c]}_{\mathbf{10}} + \underbrace{[D^c + L]}_{\bar{\mathbf{5}}} + \underbrace{N^c}_{\mathbf{1}}, \tag{10}$$

$$\mathbf{10} \to \underbrace{[D^c + L]}_{\bar{\mathbf{5}}} + \underbrace{[\bar{D}^c + \bar{L}]}_{\mathbf{5}}, \tag{11}$$

$$\mathbf{45} \to \underbrace{[G + W + X + \bar{X} + N^c]}_{\mathbf{24}} + \underbrace{[Q + U^c + E^c]}_{\mathbf{10}} + \underbrace{[\bar{Q} + \bar{U}^c + \bar{E}^c]}_{\overline{\mathbf{10}}} + \underbrace{N^c}_{\mathbf{1}}, \tag{12}$$

where the quantum numbers of $G_{\rm SM}$ are explicitly written as $Q(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$, $U^c(\bar{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$, $D^c(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$, $L(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$, $E^c(\mathbf{1},\mathbf{1})_1$, $N^c(\mathbf{1},\mathbf{1})_0$, $X(\mathbf{3},\mathbf{2})_{-\frac{5}{6}}$, $G(\mathbf{8},\mathbf{1})_0$, and $W(\mathbf{1},\mathbf{3})_0$.

First, let us consider the mass spectra of **5** and $\bar{\mathbf{5}}$ of $SU(5)$. The mass matrices M_I $[I = D^c(H_T), L(H_D)]$ can be written as

$$M_I = \begin{array}{c} \\ \bar{I}_H \\ \bar{I}_{H'} \\ \bar{I}_{\bar{C}} \\ \bar{I}_{\bar{C}'} \end{array} \begin{array}{c} \begin{array}{cccc} I_H & I_{H'} & I_C & I_{C'} \end{array} \\ \begin{pmatrix} 0 & \lambda^{h+h'}\alpha_I & 0 & 0 \\ \lambda^{h+h'}\alpha_I & \lambda^{2h'} & 0 & \lambda^{h'+c'+\frac{1}{2}(c-\bar{c})} \\ 0 & \lambda^{h'+\frac{3}{2}\bar{c}-\frac{1}{2}c} & 0 & \lambda^{\bar{c}+c'}\beta_I \\ \lambda^{h+\bar{c}'-\frac{1}{2}(c-\bar{c})} & \lambda^{h'+\bar{c}'-\frac{1}{2}(c-\bar{c})} & \lambda^{c+\bar{c}'}\beta_I & \lambda^{c'+\bar{c}'} \end{pmatrix} \end{array}. \tag{13}$$

where $\alpha_{L(H_D)}$ is vanishing and $\alpha_{D^c(H_T)} \sim O(1)$, while $\beta_I = \frac{3}{2}((B-L)_I - 1)$; that is, $\beta_L = -3$ and $\beta_{D^c} = -1$. Only one pair of doublet Higgs becomes massless, which comes from

$$\mathbf{5}_H, \qquad \bar{\mathbf{5}}_H + \lambda^{h-c+\frac{1}{2}(\bar{c}-c)}\bar{\mathbf{5}}_C. \tag{14}$$

Next, we consider the mass matrices for **10** of $SU(5)$, which are given by

[5]Without $W_{C'}$ and $W_{\bar{C}'}$, the superpotential fixing $\langle A\rangle$ becomes independent of that fixing the VEVs $\langle C\rangle$, $\langle \bar{C}\rangle$. It means that an accidental global symmetry appears, and as a result, pseudo–Nambu-Goldstone fields appear by breaking the global symmetry.

$$M_I = \begin{array}{c} \\ \bar{I}_A \\ \bar{I}_{A'} \\ \bar{I}_{\bar{C}} \\ \bar{I}_{\bar{C}'} \end{array} \begin{array}{c} \begin{array}{cccc} I_A & I_{A'} & I_C & I_{C'} \end{array} \\ \begin{pmatrix} 0 & \lambda^{a'+a}\alpha_I & 0 & \lambda^{c'-\frac{1}{2}(c-\bar{c})+a} \\ \lambda^{a+a'}\alpha_I & \lambda^{2a'} & 0 & \lambda^{c'-\frac{1}{2}(c-\bar{c})+a'} \\ 0 & 0 & 0 & \lambda^{\bar{c}+c'}\beta_I \\ \lambda^{\bar{c}'+\frac{1}{2}(c-\bar{c})+a} & \lambda^{\bar{c}'+\frac{1}{2}(c-\bar{c})+a'} & \lambda^{c+\bar{c}'}\beta_I & \lambda^{c'+\bar{c}'} \end{pmatrix} \end{array}. \tag{15}$$

Here, α_Q and α_{U^c} are vanishing because these are Nambu-Goldstone modes, but $\alpha_{E^c} \sim O(1)$. Also $\beta_Q = -1$, $\beta_{U^c} = -2$, and $\beta_{E^c} = 0$. Thus, each 4×4 matrix has one vanishing eigenvalue. The mass spectra of the remaining three modes is $(\lambda^{c+\bar{c}'}, \lambda^{c'+\bar{c}}, \lambda^{2a'})$ for Q and U^c, and $(\lambda^{a+a'}, \lambda^{a+a'}, \lambda^{c'+\bar{c}'})$ or $(\lambda^{\bar{c}'+\frac{1}{2}(c-\bar{c})+a}, \lambda^{c'-\frac{1}{2}(c-\bar{c})+a}, \lambda^{2a'})$ for E^c.

Finally, we consider the mass spectrum for **24** of $SU(5)$. The mass matrices $M_I (I = G, W, X)$ are given by

$$M_I = \begin{array}{c} \\ \bar{I}_A \\ \bar{I}_{A'} \end{array} \begin{array}{c} \begin{array}{cc} I_A & I_{A'} \end{array} \\ \begin{pmatrix} 0 & \lambda^{a+a'}\alpha_I \\ \lambda^{a+a'}\alpha_I & \lambda^{2a'} \end{pmatrix} \end{array}. \tag{16}$$

The mass spectra for G and W are $(\lambda^{a'+a}, \lambda^{a'+a})$, while for X it becomes $(0, \lambda^{2h'})$. The massless mode of X is eaten by the Higgs mechanism.

D. Gauge coupling unification

Since all symmetry breaking scales and all mass spectra of superheavy particles are fixed by anomalous $U(1)_A$ charges, we can calculate the running gauge couplings and discuss their unification. In the following, we study the running gauge couplings obtained by one-loop RGEs. Note that superheavy particles of the matter sector which is discussed in the next subsection are complete multiplets of $SU(5)$, and therefore, they do not affect the conditions for the unification of the gauge coupling constants.

In the natural $SO(10)$ GUT, $SO(10)$ is broken by the VEV $\langle A \rangle \equiv \Lambda_A \sim \lambda^{-a}$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, which is broken by the VEVs $|\langle C \rangle| = |\langle \bar{C} \rangle| \equiv \Lambda_C \sim \lambda^{-(c+\bar{c})/2}$ into G_{SM}.

Now let us discuss the conditions of the gauge coupling unification

$$\alpha_3(\Lambda_A) = \alpha_2(\Lambda_A) = \frac{3}{5}\alpha_Y(\Lambda_A) \equiv \alpha_1(\Lambda_A), \tag{17}$$

where $\alpha_1^{-1}(\mu > \Lambda_C) \equiv \frac{3}{5}\alpha_R^{-1}(\mu > \Lambda_C) + \frac{2}{5}\alpha_{B-L}^{-1}(\mu > \Lambda_C)$ with the renormalization scale μ. Here $\alpha_X \equiv \frac{g_X^2}{4\pi}$ and $g_X (X = 3, 2, R, B-L, Y)$ are the gauge couplings of $SU(3)_C$, $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$, and $U(1)_Y$, respectively. Since the model has the left-right symmetry above Λ_C, we expect $g_2 = g_R$ at $\mu > \Lambda_C$.

The gauge couplings at the scale Λ_A are obtained by one-loop RGEs as

$$\alpha_1^{-1}(\Lambda_A) = \alpha_1^{-1}(M_{SB}) + \frac{1}{2\pi}\left(b_1 \ln\left(\frac{M_{SB}}{\Lambda_A}\right) + \Sigma_i \Delta b_{1i} \ln\left(\frac{m_i}{\Lambda_A}\right) - \frac{12}{5}\ln\left(\frac{\Lambda_C}{\Lambda_A}\right)\right), \tag{18}$$

$$\alpha_2^{-1}(\Lambda_A) = \alpha_2^{-1}(M_{SB}) + \frac{1}{2\pi}\left(b_2 \ln\left(\frac{M_{SB}}{\Lambda_A}\right) + \Sigma_i \Delta b_{2i} \ln\left(\frac{m_i}{\Lambda_A}\right)\right), \tag{19}$$

$$\alpha_3^{-1}(\Lambda_A) = \alpha_3^{-1}(M_{SB}) + \frac{1}{2\pi}\left(b_3 \ln\left(\frac{M_{SB}}{\Lambda_A}\right) + \Sigma_i \Delta b_{3i} \ln\left(\frac{m_i}{\Lambda_U}\right)\right), \tag{20}$$

where M_{SB} is the SUSY breaking scale. Here, $(b_1, b_2, b_3) = (33/5, 1, -3)$ represent the renormalization group coefficients for the minimal SUSY standard model (MSSM) and Δb_{ai} $(a = 1, 2, 3)$ denote the corrections to these coefficients arising from the massive fields with mass m_i, which can be read from the following table:

I	$Q+\bar{Q}$	$U^c+\bar{U}^c$	$E^c+\bar{E}^c$	$D^c+\bar{D}^c$	$L+\bar{L}$	G	W	$X+\bar{X}$
Δb_{1I}	$\frac{1}{5}$	$\frac{8}{5}$	$\frac{6}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	0	0	5
Δb_{2I}	3	0	0	0	1	0	2	3
Δb_{3I}	2	1	0	1	0	3	0	2

The last term in Eq. (18) is caused by the breaking $SU(2)_R \times U(1)_{B-L} \to U(1)_Y$ due to the VEV $\langle C \rangle$. The gauge couplings at the SUSY breaking scale M_{SB} can be obtained by the success of the gauge coupling unification in the MSSM as

$$\alpha_1^{-1}(M_{SB}) = \alpha_G^{-1}(\Lambda_G) + \frac{1}{2\pi}\left(b_1 \ln\left(\frac{\Lambda_G}{M_{SB}}\right)\right), \tag{21}$$

$$\alpha_2^{-1}(M_{SB}) = \alpha_G^{-1}(\Lambda_G) + \frac{1}{2\pi}\left(b_2 \ln\left(\frac{\Lambda_G}{M_{SB}}\right)\right), \tag{22}$$

$$\alpha_3^{-1}(M_{SB}) = \alpha_G^{-1}(\Lambda_G) + \frac{1}{2\pi}\left(b_3 \ln\left(\frac{\Lambda_G}{M_{SB}}\right)\right), \tag{23}$$

where $\alpha_G^{-1}(\Lambda_G) \sim 25$ and $\Lambda_G \sim 2\times 10^{16}$ GeV. The above conditions for the unification are rewritten as

$$\left(\frac{\Lambda_A}{\Lambda_G}\right)^{14}\left(\frac{\Lambda_C}{\Lambda_A}\right)^{6}\left(\frac{\det \bar{M}_L}{\det \bar{M}_{D^c}}\right)\left(\frac{\det \bar{M}_Q}{\det \bar{M}_U}\right)^{4}\left(\frac{\det \bar{M}_Q}{\det \bar{M}_{E^c}}\right)^{3}\left(\frac{\det \bar{M}_W}{\det \bar{M}_X}\right)^{5} = \Lambda_A^{-\bar{r}_{D^c}+\bar{r}_L-4\bar{r}_{U^c}-3\bar{r}_{E^c}+7\bar{r}_Q-5\bar{r}_X+5\bar{r}_W}, \tag{24}$$

$$\left(\frac{\Lambda_A}{\Lambda_G}\right)^{16}\left(\frac{\Lambda_C}{\Lambda_A}\right)^{4}\left(\frac{\det \bar{M}_{D^c}}{\det \bar{M}_L}\right)\left(\frac{\det \bar{M}_Q}{\det \bar{M}_U}\right)\left(\frac{\det \bar{M}_Q}{\det \bar{M}_{E^c}}\right)^{2}\left(\frac{\det \bar{M}_G}{\det \bar{M}_X}\right)^{5} = \Lambda_A^{-\bar{r}_L+\bar{r}_{D^c}-\bar{r}_{U^c}-2\bar{r}_{E^c}+3\bar{r}_Q-5\bar{r}_X+5\bar{r}_G}, \tag{25}$$

$$\left(\frac{\Lambda_A}{\Lambda_G}\right)^{4}\left(\frac{\det \bar{M}_{D^c}}{\det \bar{M}_L}\right)\left(\frac{\det \bar{M}_U}{\det \bar{M}_Q}\right)\left(\frac{\det \bar{M}_G}{\det \bar{M}_W}\right)^{2}\left(\frac{\det \bar{M}_G}{\det \bar{M}_X}\right) = \Lambda_A^{-\bar{r}_L+\bar{r}_{D^c}-\bar{r}_Q+\bar{r}_U-2\bar{r}_W-\bar{r}_X+3\bar{r}_G}, \tag{26}$$

where $\bar{M}_I$ are the reduced mass matrices where massless modes are omitted from the original mass matrices and $\bar{r}_I$ are rank of the reduced mass matrices. In our scenario, the symmetry breaking scales $\Lambda_A \sim \lambda^{-a}$, $\Lambda_C \sim \lambda^{-\frac{1}{2}(c+\bar{c})}$, and the determinants of the reduced mass matrices are determined by the anomalous $U(1)_A$ charges:

$$\det \bar{M}_Q \sim \det \bar{M}_{U^c} \sim \lambda^{2a'+c+\bar{c}+c'+\bar{c}'}, \tag{27}$$

$$\det \bar{M}_{E^c} \sim \lambda^{2a+2a'+c'+\bar{c}'}, \tag{28}$$

$$\det M_{D^c} \sim \lambda^{2h+2h'+c+\bar{c}+c'+\bar{c}'}, \tag{29}$$

$$\det \bar{M}_L \sim \lambda^{2h'+c+\bar{c}+c'+\bar{c}'}, \tag{30}$$

$$\det M_G \sim \det M_W \sim \lambda^{2a+2a'}, \tag{31}$$

$$\det \bar{M}_X \sim \lambda^{2a'}. \tag{32}$$

The unification conditions $\alpha_1(\Lambda_A) = \alpha_2(\Lambda_A)$, $\alpha_1(\Lambda_A) = \alpha_3(\Lambda_A)$, and $\alpha_2(\Lambda_A) = \alpha_3(\Lambda_A)$ lead to $\Lambda \sim \lambda^{\frac{h}{7}}\Lambda_G$, $\Lambda \sim \lambda^{-\frac{h}{8}}\Lambda_G$, and $\Lambda \sim \lambda^{-\frac{h}{2}}\Lambda_G$, respectively. Finally, the unification conditions become

$$h \sim 0, \tag{33}$$

$$\Lambda \sim \Lambda_G. \tag{34}$$

Surprisingly, the above unification conditions do not depend on the anomalous $U(1)_A$ charges other than h. This can be shown to be a general result in the GUT with the anomalous $U(1)_A$ [8]. It is important that the cutoff scale in the natural GUT is taken to be around the usual GUT scale. It means that the true GUT scale $\Lambda_A \equiv \langle A\rangle \sim \lambda^{-a}\Lambda$ becomes smaller than Λ_G. Therefore, the nucleon decay via superheavy gauge field exchange is enhanced, and it may be seen in near future experiments.

Unfortunately, in the natural GUT model in Table I, we take $h=-3$, and not $h=0$. Of course, to forbid the explicit SM Higgs mass term H^2, h must be negative. But only because of that, we can take larger h, for example, $h=-1$. We take $h=-3$ in order to obtain realistic neutrino masses. In other words, if we take $h\sim 0$, the neutrino masses become too small. We will explain them in the next subsection.

E. Matter sector in natural $SO(10)$ GUT

In this subsection, we will briefly review how to obtain realistic quark and lepton masses and mixings in the natural $SO(10)$ GUT. Especially, neutrino masses will be explained in detail because we will introduce a tension between the neutrino masses and gauge coupling unification conditions later.

If the Yukawa interactions have been obtained only from the superpotential in Eq. (1), the model would be unrealistic because of the unrealistic $SO(10)$ GUT relations

$$Y_u = Y_d = Y_e = Y_{\nu_D}, \tag{35}$$

where Y_u, Y_d, Y_e, and Y_{ν_D} are 3×3 Yukawa matrices of up-type quarks, down-type quarks, charged leptons, and Dirac type neutrinos, respectively. The easiest way to avoid these unrealistic $SO(10)$ GUT relations is to introduce **10** of $SO(10)$ as a matter field in addition to three **16** as in Table I. The model has four $\bar{\mathbf{5}}$ and one **5** of $SU(5)$ as matter fields since **16** and **10** of $SO(10)$ are decomposed under $SU(5)$ as $\mathbf{16} = \mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}$ and $\mathbf{10} = \mathbf{5} + \bar{\mathbf{5}}$. One of the four $\bar{\mathbf{5}}$ s becomes superheavy with a **5** field through the interactions

$$W = \lambda^{\psi_i+t+c}\Psi_i TC + \lambda^{2t}T^2. \tag{36}$$

The main modes of three massless $\bar{\mathbf{5}}$ fields become $(\bar{\mathbf{5}}_1, \bar{\mathbf{5}}_2, \bar{\mathbf{5}}_3) \sim (\bar{\mathbf{5}}_{\Psi_1}, \bar{\mathbf{5}}_T, \bar{\mathbf{5}}_{\Psi_2})$, and the Yukawa matrices are obtained as

$$Y_u = \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d \sim Y_e^T \sim Y_{\nu_D}^T \sim \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^{3.5} & \lambda^3 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \\ \lambda^1 & \lambda^{0.5} & 1 \end{pmatrix}, \tag{37}$$

when $t-\psi_3-\frac{1}{2}(c-\bar{c})\equiv\Delta=\frac{5}{2}$. Here, $\bar{\mathbf{5}}_2$ has the Yukawa couplings through the Yukawa interactions in Eq. (36) with the $\bar{5}_C$ Higgs and the Yukawa couplings through the mixing with $\bar{\mathbf{5}}_{\Psi_3}$ because $\bar{\mathbf{5}}_2\sim\bar{\mathbf{5}}_T+\lambda^{\Delta}\bar{\mathbf{5}}_{\Psi_3}$. Note that the higher dimensional interactions, $\lambda^{\psi_i+\psi_j+c+\bar{c}+h}\Psi_i\Psi_j\bar{C}CH$ and $\lambda^{\psi_i+\psi_j+2La+h}\Psi_i\Psi_jA^{2L}H$ with the positive integer L, give the same order contributions to these Yukawa couplings as $\lambda^{\psi_i+\psi_j+h}\Psi_i\Psi_jH$ after developing the VEVs, $\langle\bar{C}C\rangle\sim\lambda^{-(c+\bar{c})}$ and $\langle A\rangle\sim\lambda^{-a}$. Because of this feature, the $SU(5)$ GUT relation $Y_d=Y_e^T$ can naturally be avoided in the natural GUT. Thus, we can obtain the Cabibbo-Kobayashi-Maskawa (CKM) [21] matrix as

$$U_{\rm CKM}=\begin{pmatrix}1 & \lambda & \lambda^3\\ \lambda & 1 & \lambda^2\\ \lambda^3 & \lambda^2 & 1\end{pmatrix}, \tag{38}$$

which is consistent with the experimental value if we choose $\lambda\sim0.22$.[6] Note that we have assumed that $\psi_1=\psi_3+3$, $\psi_2=\psi_3+2$, and $\psi_3=-h/2$ to obtain realistic quark and lepton masses and mixings.

The right-handed neutrino masses are obtained from the interactions

$$\lambda^{\psi_i+\psi_j+2\bar{c}}\Psi_i\Psi_j\bar{C}\,\bar{C} \tag{39}$$

as

$$M_R=\lambda^{\psi_i+\psi_j+2\bar{c}}\langle\bar{C}\rangle^2=\lambda^{2\psi_3+\bar{c}-c}\begin{pmatrix}\lambda^6 & \lambda^5 & \lambda^3\\ \lambda^5 & \lambda^4 & \lambda^2\\ \lambda^3 & \lambda^2 & 1\end{pmatrix}. \tag{40}$$

Thus, we obtain the neutrino mass matrix

$$M_\nu=M_{\nu_D}M_R^{-1}M_{\nu_D}^T=\lambda^{4-2\psi_3+c-\bar{c}}\begin{pmatrix}\lambda^2 & \lambda^{1.5} & \lambda\\ \lambda^{1.5} & \lambda & \lambda^{0.5}\\ \lambda & \lambda^{0.5} & 1\end{pmatrix}\langle H_u\rangle^2, \tag{41}$$

where $M_{\nu_D}=Y_{\nu_D}\langle H_u\rangle$. The Maki-Nakagawa-Sakata (MNS) matrix [22] is obtained from Y_e in Eq. (37) and M_ν in Eq. (41) as

$$U_{\rm MNS}=\begin{pmatrix}1 & \lambda^{0.5} & \lambda\\ \lambda^{0.5} & 1 & \lambda^{0.5}\\ \lambda & \lambda^{0.5} & 1\end{pmatrix}, \tag{42}$$

which is consistent with observed neutrino data [5]. To obtain the observed neutrino masses [5],

$$\lambda^{4-2\psi_3+c-\bar{c}}\frac{\langle H_u\rangle^2}{\Lambda}\sim m_{\nu_\tau}\sim0.05\ \text{eV} \tag{43}$$

is required. When $\Lambda\sim\Lambda_G\sim2\times10^{16}$ GeV, this condition is rewritten as

$$h+c-\bar{c}\sim-6, \tag{44}$$

which is satisfied by the natural GUT in Table I. Here we use $h+2\psi_3=0$, which is required to obtain $O(1)$ top Yukawa coupling. It is difficult to obtain larger h by smaller c and/or larger $\bar{c}$, because several conditions are required to obtain realistic natural GUT models as follows:

(1) $2\psi_3+h=0$: to obtain an $O(1)$ top Yukawa coupling, i.e., to obtain the term $\lambda^0\Psi_3\Psi_3H$.
(2) $t-\psi_3-\frac{1}{2}(c-\bar{c})=\frac{5}{2}$: To obtain the MNS matrix as in Eq. (42), which is consistent with the observations.
(3) $\psi_3+t+c\geq0$: To allow the term Ψ_3TC which makes $\bar{\mathbf{5}}_{\Psi_3}$ superheavy.
(4) $\psi_3+\psi_1+2\bar{c}\geq0$: To allow the term $\Psi_3\Psi_1\bar{C}^2$ which makes the right-handed neutrino mass matrix's rank three.
(5) $c+\bar{c}+a'+a<0$: To forbid the term $\bar{C}A'AC$ which destabilizes the DW type VEV.
(6) $c+\bar{c}\geq-6$: To obtain realistic quark masses and mixings.

Let us explain the last two new conditions. It would be unnatural to obtain the DW form of the VEV of A, which is important in solving the DT splitting problem, if $W_{A'}$ has included the spinor Higgs C and $\bar{C}$, which break $SO(10)$ into $SU(5)$. This is because the DW form of the VEV can be obtained naturally in $SO(10)$ but not in $SU(5)$. Therefore, all the terms that include C and $\bar{C}$, for example, $\bar{C}A'AC$, must be forbidden in $W_{A'}$. This gives the fifth condition. The last condition is required to obtain the sufficiently small up quark mass. Without the term $\Psi_1\bar{C}C\Psi_1H$, the up and down quark masses cannot be the measured values simultaneously.

Note that due to the conditions 2, 3, and 5 in addition to the condition (44), it becomes difficult to approach zero for h. The largest value for h is -3, and typical $U(1)_A$ charges are given in Table I. It is worth noting that this model provides realistic quark and lepton masses and mixing angles, and realizes the DT splitting. However, the gauge couplings at low energies do not match the measured values when all $O(1)$ coefficients for the masses of the superheavy particles are taken to one. Nonetheless, if we artificially select the $O(1)$ coefficients to be between 0.5 and 2, the gauge couplings can match the measured values because the models have a lot of superheavy particles. In this sense, this model is a realistic GUT model. However, such artificial selections of the $O(1)$ coefficients amount to fine-tuning, making the model unnatural. This is the tension between the neutrino masses and the gauge

[6]Here, it is also important that the massless Higgs doublet comes from $\bar{5}_H+\lambda^{h-c+\frac{1}{2}(\bar{c}-c)}\bar{5}_C$.

coupling unification in the natural GUT scenario. The above six conditions are considered to build explicit natural GUT models with suppression factors in the next section.

III. SOLUTIONS FOR TENSION BETWEEN NEUTRINO MASSES AND GAUGE COUPLING UNIFICATION

In this section, we examine several possibilities to avoid the tension between the neutrino masses and the gauge coupling unification. Since this tension is strongly dependent on the basic assumption that all terms allowed by the symmetry are introduced with $O(1)$ coefficients, we explore the possibilities in which some of the terms have much smaller coefficients than 1. We introduce the terms with small coefficients that maintain the VEV relations (3) because the VEV relations play critical roles in the natural GUT scenario.

After identifying sets of terms with small coefficients that avoid this tension, we discuss the reason for their smallness, such as an approximate symmetry.

Furthermore, we build concrete natural GUT models which avoid the tension between the neutrino masses and the gauge coupling unification. And we discuss the nucleon decay within these models.

A. Model 1: Suppression factor for terms related to the masses of right-handed neutrinos

One of the easiest ways to avoid the tension is to introduce small coefficients proportional to $\epsilon_\nu \ll 1$ for the terms which give the right-handed neutrino masses as

$$\epsilon_\nu \lambda^{\psi_i+\psi_j+2\bar{c}} \Psi_i \Psi_j \bar{C}\,\bar{C}, \tag{45}$$

where we omit the $O(1)$ coefficients. Since the right-handed neutrino masses become smaller, the (left-handed) neutrino masses become larger. The heaviest neutrino mass can be given as

$$m_{\nu_\tau} = \frac{1}{\epsilon_\nu} \lambda^{4+h+c-\bar{c}} \frac{\langle H_u \rangle^2}{\Lambda}, \tag{46}$$

which must be the observed value $m_{\nu_\tau} \sim 0.05$ eV. Note that this suppression factor does not change the VEV relation (3) and the mass spectra of superheavy particles except right-handed neutrinos. This means that the beta functions do not change, and therefore, the gauge coupling unification conditions remain unchanged as $h \sim 0$ and $\Lambda = \Lambda_G$. Since $h = 0$ allows the Higgs mass term H^2 which spoils the DT splitting, we take $h = -1$. A concrete natural GUT model with $h = -1$ is given in Table II. Note that the half integer $U(1)_A$ charges for matter fields play the same role as the R-parity, and all requirements listed at the end of the previous section are satisfied in this model.

TABLE II. In the case of $(t, c, \bar{c}) = (\frac{5}{2}, -3, -2)$ in model 1.

$SO(10)$	Negatively charged fields	Positively charged fields	Matter fields
45	$A(a=-1,-)$	$A'(a'=3,-)$	
16	$C(c=-3,+)$	$C'(c'=4,-)$	$\Psi_i(\psi_1=\frac{7}{2}, \psi_2=\frac{5}{2}, \psi_3=\frac{1}{2},+)$
$\overline{\mathbf{16}}$	$\bar{C}(\bar{c}=-2,+)$	$\bar{C}'(\bar{c}'=5,-)$	
10	$H(h=-1,+)$	$H'(h'=2,-)$	$T(t=\frac{5}{2},+)$
1	$\Theta(\theta=-1,+)$, $Z(z=-2,-)$, $\bar{Z}(\bar{z}=-2,-)$	$S(s=5,+)$	

From Eq. (46), the heaviest neutrino mass is given by $m_{\nu_\tau} = \frac{1}{\epsilon_\nu} \lambda^2 \frac{\langle H_u \rangle^2}{\Lambda} \sim 0.05$ eV, which determines the suppression factor $\epsilon_\nu \sim 10^{-3}$.

The effective colored Higgs mass for the nucleon decay becomes $m_{H_C}^{\rm eff} \sim \lambda^{2h} \Lambda \sim 10^{18}$ GeV, which results in a sufficient suppression of the nucleon decay via colored Higgs mediation. On the other hand, since the nucleon decay via gauge boson mediation is enhanced, it may be seen in near future experiments as in the usual natural GUT scenario.

Unfortunately, we have not found any approximate symmetry to understand this suppression factor. We need other reasoning for this suppression factor.

B. Suppression factors for terms with positively charged fields

In the natural GUT, terms linear in positively charged fields play an important role in determining the VEVs of fields. Therefore, if a common suppression factor for terms with positively charged fields is introduced, the VEV relations of Eq. (3) do not change. And terms that include two positively charged fields are important to determine the mass spectra of superheavy particles. Therefore, if we introduce an independent suppression factor for terms with a certain two positively charged fields, the gauge coupling unification conditions may change. In the following subsections, we consider this possibility.

Concretely, we introduce the following suppression factors:

$$W_{A'} = \epsilon_{A'} (\lambda^{a'+a} A' A + \lambda^{a'+3a} A' A^3), \tag{47}$$

$$W_{C'} = \epsilon_{C'} \bar{C} (\lambda^{\bar{c}+c'+a} A + \lambda^{\bar{c}+c'+\bar{z}} \bar{Z}) C', \tag{48}$$

$$W_{\bar{C}'} = \epsilon_{\bar{C}'} \bar{C}' (\lambda^{\bar{c}'+c+a} A + \lambda^{\bar{c}'+c+z} Z) C, \tag{49}$$

$$W_{H'} = \epsilon_{H'} \lambda^{h+h'+a} H' A H, \tag{50}$$

$$W_{X'Y'} = \sum_{X',Y'=A',C',\bar{C}',H'} \epsilon_{X'Y'} \lambda^{x'+y'} X' Y', \tag{51}$$

where the F-flatness conditions of the first four superpotentials determine the VEVs of negatively charged fields, while the last superpotential is important to fix the mass spectra of superheavy particles. The mass matrices of **5** and $\bar{\mathbf{5}}$ of $SU(5)$ become

$$M_I = \begin{array}{c} \\ \bar{I}_H \\ \bar{I}_{H'} \\ \bar{I}_{\bar{C}} \\ \bar{I}_{\bar{C}'} \end{array} \begin{array}{c} \begin{array}{cccc} I_H & I_{H'} & I_C & I_{C'} \end{array} \\ \begin{pmatrix} 0 & \epsilon_{H'}\lambda^{h+h'}\alpha_I & 0 & 0 \\ \epsilon_{H'}\lambda^{h+h'}\alpha_I & \epsilon_{H'H'}\lambda^{2h'} & 0 & \epsilon_{H'C'}\lambda^{h'+c'+\frac{1}{2}(c-\bar{c})} \\ 0 & \epsilon_{H'}\lambda^{h'+\frac{3}{2}\bar{c}-\frac{1}{2}c} & 0 & \epsilon_{C'}\lambda^{\bar{c}+c'}\beta_I \\ \epsilon_{\bar{C}'}\lambda^{h+\bar{c}'-\frac{1}{2}(c-\bar{c})} & \epsilon_{H'\bar{C}'}\lambda^{h'+\bar{c}'-\frac{1}{2}(c-\bar{c})} & \epsilon_{\bar{C}'}\lambda^{c+\bar{c}'}\beta_I & \epsilon_{\bar{C}'C'}\lambda^{c'+\bar{c}'} \end{pmatrix} \end{array}. \tag{52}$$

The determinants of reduced mass matrices, which are important to obtain the RGEs, are written as

$$\det \bar{M}_{D^c} \sim \lambda^{2h+2h'+c+\bar{c}+c'+\bar{c}'}\epsilon_{H'}^2\epsilon_{C'}\epsilon_{\bar{C}'}, \tag{53}$$

$$\det \bar{M}_L \sim \lambda^{2h'+c+\bar{c}+c'+\bar{c}'} \max\,[\epsilon_{H'H'}\epsilon_{C'}\epsilon_{\bar{C}'}, \epsilon_{H'}\epsilon_{\bar{C}'}\epsilon_{H'C'}, \epsilon_{H'}\epsilon_{\bar{C}'}\epsilon_{H'C'}\lambda^{h-\frac{3}{2}c+\frac{1}{2}\bar{c}}]. \tag{54}$$

Here, max[A, B, C] means the largest number among A, B, and C.

Similarly, the mass matrices of **10** of $SU(5)$ become

$$M_I = \begin{array}{c} \\ \bar{I}_A \\ \bar{I}_{A'} \\ \bar{I}_{\bar{C}} \\ \bar{I}_{\bar{C}'} \end{array} \begin{array}{c} \begin{array}{cccc} I_A & I_{A'} & I_C & I_{C'} \end{array} \\ \begin{pmatrix} 0 & \epsilon_{A'}\lambda^{a'+a}\alpha_I & 0 & \epsilon_{C'}\lambda^{c'-\frac{1}{2}(c-\bar{c})+a} \\ \epsilon_{A'}\lambda^{a+a'}\alpha_I & \epsilon_{A'A'}\lambda^{2a'} & 0 & \epsilon_{A'C'}\lambda^{c'-\frac{1}{2}(c-\bar{c})+a'} \\ 0 & 0 & 0 & \epsilon_{C'}\lambda^{\bar{c}+c'}\beta_I \\ \epsilon_{\bar{C}'}\lambda^{\bar{c}'+\frac{1}{2}(c-\bar{c})+a} & \epsilon_{A'\bar{C}'}\lambda^{\bar{c}'+\frac{1}{2}(c-\bar{c})+a'} & \epsilon_{\bar{C}'}\lambda^{c+\bar{c}'}\beta_I & \epsilon_{\bar{C}'C'}\lambda^{c'+\bar{c}'} \end{pmatrix} \end{array}, \tag{55}$$

and the determinants of reduced mass matrices are given as

$$\det \bar{M}_Q \sim \det \bar{M}_{U^c} \sim \lambda^{2a'+c+\bar{c}+c'+\bar{c}'}\epsilon_{A'A'}\epsilon_{C'}\epsilon_{\bar{C}'}, \tag{56}$$

$$\det \bar{M}_{E^c} \sim \lambda^{2a+2a'+c'+\bar{c}'} \max\,[\epsilon_{A'}^2\epsilon_{\bar{C}'C'}, \epsilon_{A'A'}\epsilon_{C'}\epsilon_{\bar{C}'}, \epsilon_{A'}\epsilon_{\bar{C}'}\epsilon_{A'C'}, \epsilon_{A'}\epsilon_{C'}\epsilon_{A'\bar{C}'}]. \tag{57}$$

For the adjoint fields G, W, X, and $\bar{X}$, the mass matrices are given by

$$M_I = \begin{array}{c} \\ \bar{I}_A \\ \bar{I}_{A'} \end{array} \begin{array}{c} \begin{array}{cc} I_A & I_{A'} \end{array} \\ \begin{pmatrix} 0 & \epsilon_{A'}\lambda^{a+a'}\alpha_I \\ \epsilon_{A'}\lambda^{a+a'}\alpha_I & \epsilon_{A'A'}\lambda^{2a'} \end{pmatrix} \end{array}, \tag{58}$$

and the determinants of the reduced mass matrices are

$$\det \bar{M}_G \sim \det \bar{M}_W \sim \lambda^{2a+2a'}\epsilon_{A'}^2, \tag{59}$$

$$\det \bar{M}_X \sim \lambda^{2a'}\epsilon_{A'A'}. \tag{60}$$

When we define the suppression parameters as

$$D_{D^c} = \epsilon_{H'}^2\epsilon_{C'}\epsilon_{\bar{C}'}, \tag{61}$$

$$D_L = \max\,[\epsilon_{H'H'}\epsilon_{C'}\epsilon_{\bar{C}'}, \epsilon_{H'}\epsilon_{\bar{C}'}\epsilon_{H'C'}, \epsilon_{H'}\epsilon_{\bar{C}'}\epsilon_{H'C'}\lambda^{h-\frac{3}{2}c+\frac{1}{2}\bar{c}}], \tag{62}$$

$$D_{Q,U^c} = \epsilon_{A'A'}\epsilon_{C'}\epsilon_{\bar{C}'}, \tag{63}$$

$$D_{E^c} = \max\,[\epsilon_{A'}^2\epsilon_{\bar{C}'C'}, \epsilon_{A'A'}\epsilon_{C'}\epsilon_{\bar{C}'}, \epsilon_{A'}\epsilon_{\bar{C}'}\epsilon_{A'C'}, \epsilon_{A'}\epsilon_{C'}\epsilon_{A'\bar{C}'}], \tag{64}$$

$$D_{G,W} = \epsilon_{A'}^2, \tag{65}$$

$$D_X = \epsilon_{A'A'}, \tag{66}$$

the gauge coupling unification conditions Eqs. (24)–(26) can be rewritten as

$$\Lambda = \Lambda_G\left(\frac{D_{E^c}}{D_{Q,U^c}}\right)^{\frac{1}{6}}\left(\frac{D_X}{D_{G,W}}\right)^{\frac{1}{3}}, \tag{67}$$

$$\lambda^{2h} = \left(\frac{D_L}{D_{D^c}}\right)\left(\frac{D_{Q,U^c}}{D_{E^c}}\right)^{\frac{2}{3}}\left(\frac{D_{G,W}}{D_X}\right)^{\frac{1}{3}}. \quad (68)$$

Furthermore, we obtain the heaviest neutrino mass

$$m_{\nu_\tau} = \lambda^{4+h+c-\bar{c}}\frac{\langle H_u\rangle^2}{\Lambda} = 3\times 10^{-6}\lambda^{c-\bar{c}}\left(\frac{D_L D_{Q,U^c} D_{G,W}}{D_{D^c} D_{E^c} D_X}\right)^{\frac{1}{2}} \text{ eV}. \quad (69)$$

In the next subsection, using the above results, we discuss several possibilities to avoid the tension between neutrino masses and the gauge coupling unification.

C. Models

In this subsection, we build several explicit natural GUT models that have no tension between the neutrino masses and the gauge coupling unification by introducing various suppression factors as discussed in the previous subsection.

First, let us explain the features common to the natural GUT models built in this paper. They have no tension between the neutrino masses and the gauge coupling unification while they have all the advantages of the usual natural GUTs except the basic principle that all terms allowed by symmetry are introduced with $O(1)$ coefficients. We fix $a = -1$ and $a' = 3$, which allow the terms $A'A$ and $A'A^3$, and forbid $A'A^5$ and more higher dimensional terms to obtain the DW type VEV naturally, although we have other options to take $a = -1/2$ and $a = 3/2$ which predict a longer lifetime for nucleon decay mediated by superheavy gauge fields because of the larger unification scale. To obtain the realistic natural GUTs, they must satisfy the conditions listed at the end of the previous section, which can be rewritten as

$$t + \frac{1}{2}h - \frac{1}{2}(c-\bar{c}) = \frac{5}{2}, \quad (70)$$

$$-\frac{1}{2}h + t + c \geq 0, \quad (71)$$

$$-h + 3 + 2\bar{c} \geq 0, \quad (72)$$

$$-6 \leq c + \bar{c} < -2, \quad (73)$$

in addition to the three relations (67)–(69) we obtained at the end of the last subsection.

1. Model 2: $\varepsilon_{H'} \ll 1$, others $\sim O(1)$

In this model, we assume that $\varepsilon_{H'} \ll 1$ while the others are $O(1)$. The $\varepsilon_{H'}$ dependence of the determinants of the reduced mass matrices becomes

$$D_{D^c} \sim \varepsilon_{H'}^2, \quad D_L \sim D_{Q,U^c} \sim D_{E^c} \sim D_{G,W} \sim D_X \sim 1. \quad (74)$$

Equations (67)–(69) are rewritten as

$$\Lambda \sim \Lambda_G, \quad (75)$$

$$\lambda^{2h} \sim \varepsilon_{H'}^{-2}, \quad (76)$$

$$m_{\nu_\tau} \sim 3\times 10^{-6}\lambda^{c-\bar{c}}\varepsilon_{H'}^{-1} \text{ eV} \sim 0.05 \text{ eV}. \quad (77)$$

From the last two relations given above, we obtain

$$h = -(c-\bar{c}) - 6. \quad (78)$$

Then, condition (70) becomes

$$t + h = -\frac{1}{2}. \quad (79)$$

Among several solutions that satisfy all the conditions, two solutions $(h,t,c,\bar{c}) = (-3,\frac{5}{2},-4,-1)$ and $(h,t,c,\bar{c}) = (-3,\frac{5}{2},-3,0)$ have the largest h and an interesting feature that the half integer $U(1)_A$ charges play the same role as the R-parity. An example of $U(1)_A$ charges for the former solution is nothing but the model given in Table I. In these models, the suppression factor becomes

$$\varepsilon_{H'} \sim \lambda^{-h} \sim \lambda^3 \sim 10^{-2}. \quad (80)$$

The origin of this suppression factor can be understood by an approximate Z_2 symmetry under which H' is the unique field with the odd Z_2 parity. When this approximate symmetry is imposed, the other suppression factor $\varepsilon_{H'C'} \sim \varepsilon_{H'}$ appears, but this additional suppression factor does not change the physical results.

Since the effective colored Higgs mass is calculated as

$$m_{H_T,\text{eff}} \sim \frac{(\varepsilon_{H'}\lambda^{h+h'})^4}{(\varepsilon_{H'}\lambda^{h+h'})^2\lambda^{2h'}} \sim 1, \quad (81)$$

which equals cutoff scale $\Lambda = \Lambda_G = 2\times 10^{16}$ GeV, the signal for the nucleon decay mediated by colored Higgs (see Fig. 1) may be seen in future experiments. However,

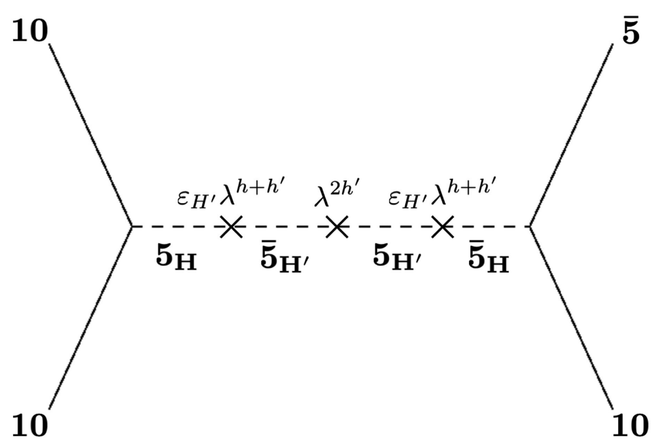

FIG. 1. Proton decay mediated by colored Higgs.

the predictions depend on the SUSY breaking scale and the explicit structure of Yukawa couplings. For example, the natural GUT with spontaneous SUSY breaking predicts quite large sfermion masses as $m_{\tilde{f}}^2 \sim (10^{3-4}\ \mathrm{TeV})^2$ [23], and therefore the proton decay mediated by colored Higgs is suppressed.

2. Model 3: $\epsilon_{H'} \sim \lambda^{\delta}\epsilon_{A'} \ll 1$, others $\sim O(1)$

Here, we try to build a natural GUT in which not only the tension is avoided but also the cutoff scale becomes larger than Λ_G since the cutoff scale is quite important in predicting the nucleon lifetime.

In addition to $\epsilon_{H'} \ll 1$, we introduce $\epsilon_{A'} \sim \lambda^{-\delta}\epsilon_{H'} \lesssim 1$ ($\delta > 0$) in order to change the cutoff. Since

$$D_{D^c} \sim \epsilon_{H'}^2, \qquad D_{G,W} \sim \epsilon_{A'}^2 \sim \lambda^{-2\delta}\epsilon_{H'}^2, \\ D_L \sim D_{Q,U^c} \sim D_{E^c} \sim D_X \sim 1. \tag{82}$$

Equations (67)–(69) become

$$\Lambda \sim \Lambda_G \lambda^{\frac{2}{3}\delta}\epsilon_{H'}^{-\frac{2}{3}}, \tag{83}$$

$$\lambda^{2h} \sim \lambda^{-\frac{2}{3}\delta}\epsilon_{H'}^{-\frac{4}{3}}, \tag{84}$$

$$m_{\nu_\tau} \sim 3\times 10^{-6}\lambda^{c-\bar{c}}\lambda^{-\delta}\ \mathrm{eV} \sim 0.05\ \mathrm{eV}. \tag{85}$$

Note that if $\lambda^{\delta} = \epsilon_{H'}$, the above conditions become nothing but those in the previous model. Here, for simplicity, we assume that the cutoff is around the reduced Planck scale, $\Lambda \sim M_{\mathrm{Planck}} \sim 2\times 10^{18}$ GeV. As a result, the first two equations (83) and (84) become

$$\epsilon_{H'} \sim \lambda^{\delta+\frac{9}{2}}, \tag{86}$$

$$h \sim -\delta - 3. \tag{87}$$

The condition (85) can be rewritten as

$$c - \bar{c} = \delta - 6 = -h - 9. \tag{88}$$

Substituting this relation into Eq. (70), we obtain

$$t = -h - 2. \tag{89}$$

Among the solutions that satisfy conditions (71)–(73) in addition to (89), the solution with the largest h has ($h = -4, t = 2, c = -4, \bar{c} = 1$), resulting in $\delta = 1$. Typical $U(1)_A$ charges are shown in Table III.

The suppression factors are determined as

$$\epsilon_{H'} \sim \lambda^{4.5+\delta} = \lambda^{5.5}, \tag{90}$$

$$\epsilon_{A'} \sim \lambda^{-\delta}\epsilon_{H'} \sim \lambda^{4.5}. \tag{91}$$

TABLE III. $U(1)_A$ charge assignment in model 3.

$SO(10)$	Negatively charged fields	Positively charged fields	Matter fields
45	$A(a=-1,-)$	$A'(a'=3,-)$	
16	$C(c=-4,+)$	$C'(c'=1,-)$	$\Psi_i(\psi_1=5,\psi_2=4,\psi_3=2,+)$
$\overline{\mathbf{16}}$	$\bar{C}(\bar{c}=-1,+)$	$\bar{C}'(\bar{c}'=6,-)$	
10	$H(h=-4,+)$	$H'(h'=5,-)$	$T(t=2,+)$
1	$\Theta(\theta=-1,+)$, $Z(z=-2,-)$, $\bar{Z}(\bar{z}=-2,-)$	$S(s=3,+)$	

Such suppression factors can be naturally realized by the approximate symmetries $Z_{2H'}$ and $Z_{2A'}$, where Z_{2X} is a Z_2 symmetry under which only the X field has odd parity. When these approximate symmetries are imposed, the other suppression factors appear, for example, $\epsilon_{A'\bar{C}'}$, but these additional suppression factors do not change the physical results.

The effective colored Higgs mass related to the proton decay becomes

$$m_{H_T,\mathrm{eff}} \sim \Lambda\epsilon^2\lambda^{2h} \sim \Lambda_G \sim 2\times 10^{16}\ \mathrm{GeV}, \tag{92}$$

which means that the proton decay mediated by colored Higgs may be seen in future experiments although the predictions of the lifetime depend on the Yukawa structure and the SUSY breaking scale.

It is a rather general result that the effective colored Higgs mass becomes close to Λ_G. Actually, it is shown only by Eqs. (83)–(85). In the next subsubsection, we try to build the natural GUT with a larger effective colored Higgs mass.

3. Model 4: $\epsilon_{H'H'} \ll \epsilon_{H'} \ll 1$, others $\sim O(1)$

Generically, the effective colored Higgs mass can be obtained as

$$m_{H_T,\mathrm{eff}} \sim \frac{(\epsilon_{H'}\lambda^{h+h'})^4}{(\epsilon_{H'}\lambda^{h+h'})^2\epsilon_{H'H'}\lambda^{2h'}}\Lambda \sim \epsilon_{H'}^2\epsilon_{H'H'}^{-1}\lambda^{2h}\Lambda \sim \epsilon_{H'}^2\epsilon_{H'H'}^{-1} \\ \times \left(\frac{D_L}{D_{D^c}}\right)\left(\frac{D_{Q,U^c}}{D_{E^c}}\right)^{\frac{1}{2}}\Lambda_G, \tag{93}$$

where the last similarity is shown by Eqs. (67) and (68). Obviously, $\frac{D_L}{D_{D^c}} \geq \frac{\epsilon_{H'H'}}{\epsilon_{H'}^2}$ and $\frac{D_{Q,U^c}}{D_{E^c}} \leq 1$ due to the definitions of D_{D^c}, D_L, D_{Q,U^c}, and D_{E^c} (61)–(64). Therefore, to obtain a larger effective colored Higgs mass than Λ_G, $\frac{D_L}{D_{D^c}} > \frac{\epsilon_{H'H'}}{\epsilon_{H'}^2}$ is necessary. In addition, if $\frac{D_{Q,U^c}}{D_{E^c}} \sim 1$ is satisfied, $m_{H_T,\mathrm{eff}} > \Lambda_G$ is realized. These sufficient conditions are fulfilled when $\epsilon_{H'H'} \ll \epsilon_{H'} \ll 1$, while the others are $\sim O(1)$. Since

$$D_{D^c} \sim \varepsilon_{H'}^2, \quad D_L \sim \varepsilon_{H'}, \quad D_{Q,U^c} \sim D_{E^c} \sim D_{G,W} \sim D_X \sim 1, \tag{94}$$

we obtain the effective colored Higgs mass as

$$m_{H_T,\mathrm{eff}} \sim \frac{\varepsilon_{H'}}{\varepsilon_{H'H'}} \Lambda_G > \Lambda_G. \tag{95}$$

The explicit $U(1)_A$ charge assignment is the same as what is shown in Table I. From Eqs. (67) and (68), we obtain

$$\Lambda = \Lambda_G, \tag{96}$$

$$\varepsilon_{H'} = \lambda^{-2h} = 10^{-4}. \tag{97}$$

Since the unification scale $\lambda^{-a}\Lambda < \Lambda_G$, the nucleon decay mediated by superheavy gauge fields may be seen in future experiments, while the proton decay mediated by colored Higgs is suppressed although it strongly depends on $\varepsilon_{H'H'}$.

Unfortunately, these suppression factors cannot be realized by an approximate symmetry. For example, in an approximate symmetry where only H' has a nontrivial charge, $\varepsilon_{H'C'}$ is replaced by $\varepsilon_{H'}$. This leads to the following relations:

$$D_{D^c} \sim \varepsilon_{H'}^2, \quad D_L \sim \varepsilon_{H'H'}, \quad D_{Q,U^c} \sim D_{E^c} \sim D_{G,W} \sim D_X \sim 1. \tag{98}$$

As a result, the effective colored Higgs mass $m_{H_T,\mathrm{eff}}$ becomes Λ_G. Here, we assumed inequalities $\varepsilon_{H'}^2 \lesssim \varepsilon_{H'H'}$ commonly found in approximate symmetries. However, if there have been any symmetries that can satisfy the relation $\varepsilon_{H'}^2 \gg \varepsilon_{H'H'}$, the symmetries would explain the suppression factors.

IV. DISCUSSION AND SUMMARY

Under the natural assumption that all terms allowed by symmetry are introduced with $O(1)$ coefficients, the natural GUT solves various problems of SUSY GUT and gives a GUT that leads to the Standard Model, which is consistent with almost all observations and experiments. Unfortunately, the natural GUT has an unsatisfactory point that many $O(1)$ coefficients must be artificially chosen between 0.5 and 2 to achieve the unification of the gauge coupling constants. This problem is due to the fact that the neutrino masses become too small to satisfy the measured values under the conditions of the unification of the gauge coupling constants without the above artificial choice of the $O(1)$ coefficients.

In this paper we discussed how to avoid the tension between the unification of gauge coupling constants and neutrino masses in the natural GUT. In particular, we considered the possibilities that the tension could be eliminated by assuming that, for some reason, some terms have suppression factors in addition to the suppression factors determined by the $U(1)_A$ symmetry. We found several solutions and explicitly built natural GUT models. For some solutions (models 2 and 3), we also found that their additional suppression factors can be understood naturally with approximate symmetries.

We focused on how the nucleon decay, which is an important prediction of GUT, changes in these solutions. In the original SUSY GUT scenario, the nucleon decay mediated by colored Higgs is important, while the nucleon decay mediated by superheavy gauge fields is suppressed because of the larger unification scale. In the original natural GUT scenario, the nucleon decay mediated by superheavy gauge fields becomes interesting because the unification scale becomes generally lower, while the nucleon decay mediated by colored Higgs is strongly suppressed because the effective colored Higgs mass becomes $\lambda^{2h}\Lambda_G$ with negative h. This is an important prediction of the natural GUT. In the natural GUT with suppression factors, which is discussed in this paper to avoid the tension between the gauge coupling unification and the neutrino masses, the predictions on nucleon decay have changed in some models. The model with suppression factors of terms for right-handed neutrino masses gives similar predictions on nucleon decays as the original natural GUT because the colored Higgs mass becomes $\lambda^{2h}\Lambda_G$ with $h = -1$. In the models with suppression factors explained by the approximate symmetry for terms with positively charged fields, the predictions on the nucleon decay mediated by colored Higgs becomes more important generically, while the nucleon decay mediated by superheavy gauge fields can be suppressed. This is an important observation of this paper, although we also showed that in model 4 the nucleon decay mediated by colored Higgs may be suppressed in a natural GUT with suppression factors which are not understood by an approximate symmetry.

Note that in the natural GUT, in which the suppression factors can be understood by approximate symmetry, the suppression factor discussed above cannot be understood in terms of the spontaneous breaking of the symmetry under the VEV relations (3). For example, if we try to explain the suppression factor of the $H'H$ term by the VEV of the Z_2 odd and $U(1)_A$ negatively charged field Z_- from a symmetric term $\lambda^{z_-+h'+h}Z_-H'H$. Here, H' and H have an odd and an even Z_2 parity, respectively. However, the suppression factor does not appear at all because the VEV relation $\langle Z_- \rangle \sim \lambda^{-z_-}$ is canceled by the enhancement factor λ^{z_-}. If the VEV of Z_- is much smaller than the value fixed by the VEV relation λ^{-z_-}, the approximate symmetry can be understood by spontaneous symmetry breaking. Such a small VEV may be possible if the direction of the VEV of Z_- is a flat direction. Building models in this direction is beyond the scope of this paper. Indeed, the approximate symmetries that may appear in the natural GUT with suppression factors can be understood by other reasons, for example, extradimension, additional $U(1)_A'$ symmetry, or other stringy reasons. We hope that our consideration may be a hint to find the model beyond the natural GUT.

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