Associated production of heavy quarkonia and D mesons in the improved color evaporation model with KaTie

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In this article, we study associated production of prompt $J/\psi(\Upsilon)$ and D mesons in the improved color evaporation model using the high-energy factorization approach as it is realized in the Monte Carlo event generator KaTie. The modified Kimber-Martin-Ryskin-Watt model for unintegrated parton distribution functions is used. We predict cross sections for associated $J/\psi(\Upsilon)$ and D meson hadroproduction via the single and double parton scattering mechanisms using the set of model parameters which has been fixed early for description of prompt single and pair heavy quarkonium production at the LHC energies. We found the results of calculations agree with the LHCb Collaboration data at the energies $\sqrt{s} = 7$, 8 TeV and we present theoretical predictions for the energy $\sqrt{s} = 13$ TeV.

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I. INTRODUCTION

Measurements of the $J/\psi + D$ and $\Upsilon + D$ (here and below $\Upsilon = \Upsilon(1S)$) associated production by the LHCb Collaboration [1,2] at the energies $\sqrt{s} = 7$, 8 TeV clearly demonstrate a dominant role of the double parton scattering (DPS) mechanism of the parton model compared with the conventional single parton scattering (SPS) scenario. The average value of the DPS parameter σ_{eff} extracted in the $\Upsilon + D$ pair production is about $\sigma_{\mathrm{eff}} = 18.0 \pm 1.8$ mb and in the $J/\psi + D$ it is about $\sigma_{\mathrm{eff}} = 15.8 \pm 2.4$ mb [1,2]. The extractions of parameter σ_{eff}, based on the DPS pocket formula [3], have been obtained in different experiments. Values of $\sigma_{\mathrm{eff}} = 2$–25 mb have been derived, though with large errors, with a simple average giving $\sigma_{\mathrm{eff}} = 15$ mb [4].

Theoretical calculations performed in the leading order (LO) in α_S of the collinear parton model within the color singlet model [5,6] and within the approach of the nonrelativistic quantum chromodynamics (NRQCD) [7] predict very small values of the SPS cross sections for $J/\psi + D$ [8] and $\Upsilon + D$ [9,10] pair production. In Ref. [11], the associated $\Upsilon + D$ pair production was studied in the k_T-factorization [12–14] and NRQCD, and it was demonstrated that SPS contribution to the cross section is also sufficiently smaller than the experimental data from the LHCb collaboration [2]. The new LHCb measurements of $J/\psi + J/\psi$ and $J/\psi + \Upsilon$ pair production cross sections [15,16] motivate making predictions for $J/\psi + D$ and $\Upsilon + D$ pair production at the energies $\sqrt{s} = 7$–13 TeV. Instead of previous theoretical studies for such processes, we use the improved color evaporation model (ICEM) [17] to describe hadronization of heavy quark and antiquark pairs into a final quarkonium. The ICEM was successfully used recently to describe single $J/\psi(\Upsilon)$ production both in the collinear parton model [18,19] and in the k_T-factorization [20,21]. The pair quarkonium production in the ICEM using the k_T-factorization was studied in Refs. [22,23]. The D meson production at the LHC energies was described in the k_T-factorization and the fragmentation approach using $c \to D$ nonperturbative fragmentation function $D_{c\to D}(z)$ in Refs. [24,25].

In the study, we calculate the cross section for associated $J/\psi + D$ and $\Upsilon + D$ production in the proton-proton collisions in the k_T-factorization [12–14] using the Monte Carlo (MC) event generator KaTie [26]. Following the parton Reggeization approach (PRA) [27,28], which is a gauge-invariant version of the k_T-factorization, the modified Kimber-Martin-Ryskin-Watt (mKMRW) model [29,30] for unintegrated parton distribution functions (uPDFs) is used [28].

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II. EVENT GENERATOR KATIE AND UNINTEGRATED PARTON DISTRIBUTION FUNCTIONS

We apply a fully numerical method of the calculation using the parton level event generator KaTie [26]. The approach to obtaining gauge invariant amplitudes with off-shell initial state partons in scattering at high-energy multi-Regge kinematics was proposed in Refs. [31,32]. The method is based on the use of spinor amplitudes formalism and recurrent relations of the Britto-Cachazo-Feng-Witten (BCFW) type. This formalism [26,31,32] for numerical amplitude generation is equivalent to analytical amplitude building accordingly to the Feynman rules of the Lipatov effective field theory [33] at the level of tree diagrams [34,35]. The accuracy of numerical calculations using KaTie for total proton-proton cross sections is taking equal 0.1%.

In the high-energy factorization or k_T-factorization, the cross section for the hard process $p + p \to \mathcal{Q} + X$ in the multi-Regge kinematics is calculated as integral convolution of the parton cross section $d\hat{\sigma}(i + j \to \mathcal{Q} + X)$ and the uPDFs by the factorization formula

$$d\sigma = \sum_{i,\bar{j}} \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \Phi_i(x_1, t_1, \mu^2) \times \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \Phi_j(x_2, t_2, \mu^2) \cdot d\hat{\sigma}, \qquad (1)$$

where $t_{1,2} = +\mathbf{q}_{1,2T}^2$, $q_{1,2}^\mu = x_{1,2} P_{1,2}^\mu + q_{1,2T}^\mu$, $q_{1,2T} = (0, \mathbf{q}_{1,2T}, 0)$, $P_{1,2}^\mu = \frac{\sqrt{s}}{2}(1, 0, 0, \pm 1)$, the cross section of the subprocess with off-shell initial partons, which are treated as Reggeized partons [33,36], $d\hat{\sigma}$ is expressed in terms of squared Reggeized amplitudes $\overline{|\mathcal{A}_{\rm PRA}|^2}$ in a standard way [34].

The uPDFs can be written as follows from the KMRW model [29,30]:

$$\Phi_i(x, t, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} \frac{T_i(t, \mu^2, x)}{t} \sum_{j=q,\bar{q},g} \int_x^1 dz P_{ij}(z) F_j\left(\frac{x}{z}, t\right) \times \theta(\Delta(t, \mu^2) - z), \qquad (2)$$

where $F_i(x, \mu^2) = x f_i(x, \mu^2)$. To resolve infrared divergence, the following cutoff on $z_{1,2}$ can be derived: $z_{1,2} < 1 - \Delta_{\rm KMR}(t_{1,2}, \mu^2)$, where $\Delta_{\rm KMR}(t, \mu^2) = \sqrt{t}/(\sqrt{\mu^2} + \sqrt{t})$ is the KMR-cutoff function [29]. To resolve the collinear divergence problem, we require that in the modified KMRW (mKMRW) uPDFs $\Phi_i(x, t, \mu^2)$ should be satisfied by normalization condition:

$$\int_0^\mu dt \Phi_i(x, t, \mu^2) = F_i(x, \mu^2), \qquad (3)$$

which is equivalent to:

$$\Phi_i(x, t, \mu^2) = \frac{d}{dt}[T_i(t, \mu^2, x) F_i(x, t)], \qquad (4)$$

where $T_i(t, \mu^2, x)$ is referred to as Sudakov form factor, satisfying the boundary conditions $T_i(t = 0, \mu^2, x) = 0$ and $T_i(t = \mu^2, \mu^2, x) = 1$.

The solution for Sudakov form factor in Eq. (4) has been obtained in Ref. [28]:

$$T_i(t, \mu^2, x) = \exp\left[-\int_t^{\mu^2} \frac{dt'}{t'} \frac{\alpha_s(t')}{2\pi} (\tau_i(t', \mu^2) + \Delta\tau_i(t', \mu^2, x))\right] \qquad (5)$$

with

$$\tau_i(t, \mu^2) = \sum_j \int_0^1 dz\, z P_{ji}(z) \theta(\Delta(t, \mu^2) - z),$$

$$\Delta\tau_i(t, \mu^2, x) = \sum_j \int_0^1 dz\, \theta(z - \Delta(t, \mu^2)) \times \left[z P_{ji}(z) - \frac{F_j(\frac{x}{z}, t)}{F_i(x, t)} P_{ij}(z) \theta(z - x)\right].$$

In our mKMRW model, the Sudakov form factor (5) contains the x-depended $\Delta\tau_i$-term in the exponent which is needed to preserve the exact normalization condition for arbitrary x and μ. There is a numerically important difference that in mKMRW uPDFs the rapidity-ordering condition is imposed both on quarks and gluons, while in the original KMRW approach it is imposed only on gluons.

There is a certain amount of uPDF sets, some of them are collected in the TMDlib2 library [37]. We compare the most typical of them with our mKMRW gluon uPDF in Figs. 1 and 2. The uPDF `ccfm-JH-2013-set2` [38] is obtained as a solution of Ciafaloni-Catani-Fiorani-Marchesini (CCFM) evolution equations [39,40], which incorporate using some approximation effects from the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation (large x) and the Balitski-Fadin-Kuraev-Lipatov evolution equation (small x) together. The uPDFs `PB-NLO-HERAI+II-2018-set1` [41] are derived as a Monte Carlo solution of a system of evolution equations constructed in such a way that k_T-integrated uPDF satisfies usual DGLAP equations, while transverse momentum dependence of uPDF is essentially determined from the

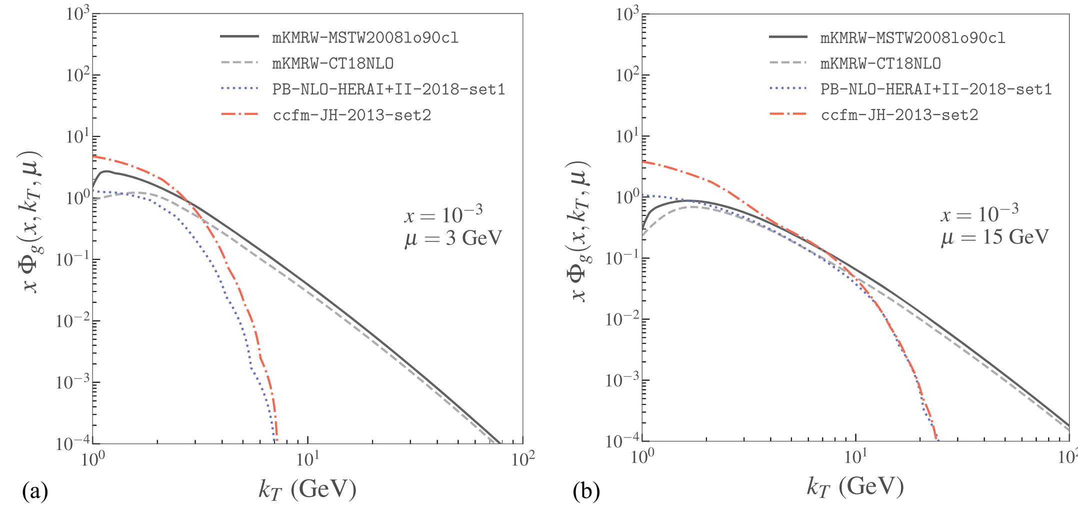

FIG. 1. Comparison of different gluon uPDFs as a function of k_T at the $x = 10^{-3}$; (a) $\mu = 3$ GeV and (b) $\mu = 15$ GeV. Curve description is presented in the text.

ambiguity in definition of "nonresolved" parton branchings by means of a suitable cutoff function and several scale-choices in the definition of Sudakov form factor and branching probability. The uPDFs `mKMRW-MSTW2008lo90cl` and `mKMRW-CT18NLO` are obtained by the formula (2) of the mKMRW model with different LO [42] and next-to-leading-order (NLO) [43] collinear inputs.

First of all, Figs. 1 and 2 demonstrate nonsufficient dependence of mKMRW gluon uPDFs from the collinear input which can be ignored taking in mind strong dependence on the choice of the hard scale parameter μ. The uPDF `mKMRW-MSTW2008lo90cl` will be used during our current calculations as well as it has been used in our recent study of single and double heavy quarkonium production to obtain parameters $F^{\psi,\Upsilon}$ of the ICEM and parameter σ_{eff} of the DPS model [22,23].

The important differences in the x or k_T dependence between mKMRW uPDFs and `ccfm-JH-2013-set2` and `PB-NLO-HERAI+II-2018-set1` gluon uPDFs are illuminated for the region of $k_T \gg \mu$. The mKMRW gluon uPDFs exhibit a powerlike tail, while `ccfm-JH-2013-set2` and `PB-NLO-HERAI+II-2018-set1` gluon uPDFs drop exponentially. This difference is extremely important for the description of the large p_T production region, where calculation of the last mentioned PDFs will significantly underestimate the cross section. To overcome this problem, authors of recent Ref. [44] attempt to match the LO calculation with `PB-NLO-HERAI+II-2018-set1` uPDFs with NLO QCD corrections obtained via the standard implementation of the MC@NLO method [45]. However the standard NLO calculation in the CPM is not designed to properly take into account the off-shell

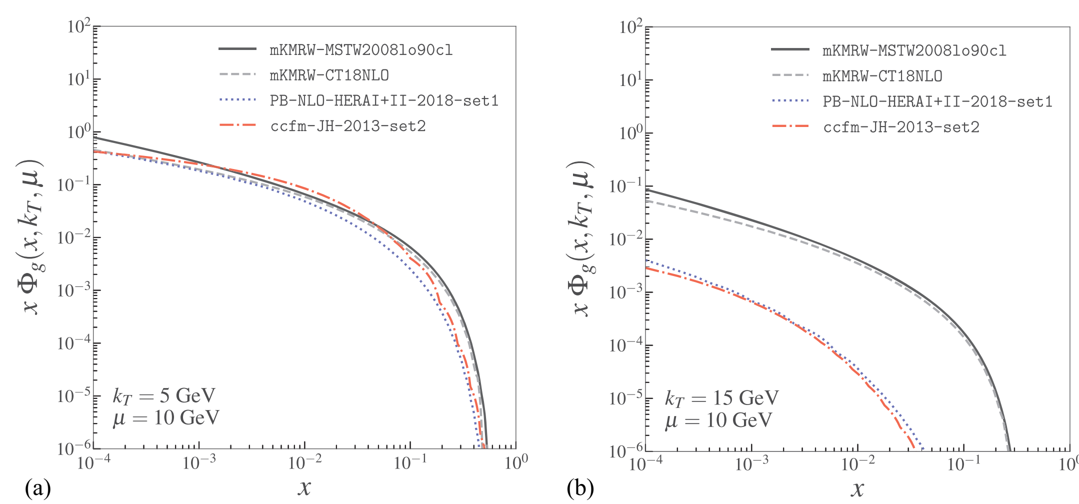

FIG. 2. Comparison of different gluon uPDFs as a function of x at the $\mu = 10$ GeV; (a) $k_T = 5$ GeV and (b) $k_T = 15$ GeV. Curve description is presented in the text.

initial state partons and hence its application together with uPDFs is hard to justify. The consistent formalism of NLO calculations with off-shell initial state partons is currently under development [46,47]. Moreover, taking into account the powerlike tail of mKMRW uPDF, together with effects of amplitude Reggeization, we can extend the range of applicability of k_T-factorization for particle production with large transverse momenta.

III. ICEM

In the ICEM, the cross section for the production of heavy quarkonium $\mathcal{Q} = J/\psi, \Upsilon$ is related to the cross section for the production of $q\bar{q}$—pair as follows ($q = c,\, b$):

$$\sigma(p + p \to \mathcal{Q} + X) = \mathcal{F}^{\mathcal{Q}} \times \int_{m_{\mathcal{Q}}}^{2m_{D,B}} \frac{d\sigma(p + p \to q + \bar{q} + X)}{dM} dM, \quad (6)$$

where M is the invariant mass of the $q\bar{q}$-pair with 4-momentum $p^{\mu}_{q\bar{q}} = p^{\mu}_{q} + p^{\mu}_{\bar{q}}$, $m_{\mathcal{Q}}$ is the mass of the quarkonium, and $m_{D,B}$ are the masses of the lightest D and B mesons. ICEM takes into account an important kinematical effect consists in the difference between final quarkonium mass and proto-quarkonium state via matching condition between 4–momenta: $p_{\mathcal{Q}} = (M_{\mathcal{Q}}/M) p_{q\bar{q}}$. Parameter $\mathcal{F}^{\mathcal{Q}}$ is considered as a probability of transformation of the $q\bar{q}$-pair with invariant mass $m_{\mathcal{Q}} < M < 2m_{D,B}$ into the quarkonium $\mathcal{Q}$.

The cross section for the associated production of quarkonium $\mathcal{Q}$ and D meson in the ICEM is related to the cross section for the associated production of $q\bar{q}$-pair and D meson in the SPS as follows:

$$\sigma^{\rm SPS}(p + p \to \mathcal{Q} + D + X) = \mathcal{F}^{\mathcal{Q}} \times \int_{m_{\mathcal{Q}}}^{2m_{D,B}} \frac{d\sigma(p + p \to q + \bar{q} + D + X)}{dM} dM. \quad (7)$$

In the DPS scenario, the cross section for the associated production of a quarkonium $\mathcal{Q}$ and D meson is expressed in terms of the cross sections of two independent subprocesses

$$\sigma^{\rm DPS}(p + p \to \mathcal{Q} + D + X) = \frac{\sigma^{\rm SPS}(p + p \to \mathcal{Q} + X_1) \times \sigma^{\rm SPS}(p + p \to D + X_2)}{\sigma_{\rm eff}}, \quad (8)$$

where parameter $\sigma_{\rm eff}$ controls the contribution of the DPS mechanism. To calculate $J/\psi + D$ and $\Upsilon + D$ associated productions we take parameters $\mathcal{F}^{\psi} \simeq \mathcal{F}^{\Upsilon} = 0.02$, as was early obtained by the fit of the LHCb data for the single prompt J/ψ production cross section [22] and single prompt Υ production [23] using the ICEM and event generator KaTie. The DPS parameter is taken equal $\sigma_{\rm eff} = 11$ mb as it follows from the fit of J/ψ and Υ pair production cross sections and spectra using the same approaches [22,23].

In Figs. 3 and 4, we plot theoretical predictions for transverse momentum spectra (p_T) of prompt Υ and J/ψ obtained within the ICEM and k_T-factorization using generator KaTie [26] and modified mKMRW uPDFs [28]. The good agreement with the LHCb data [48,49] at the $\sqrt{s} = 7$ is founded. Here and in the figures below the shadow bounds demonstrate theoretical uncertainty following the choice of the hard scale μ, which is taken as $\mu = \xi\sqrt{M^2_{\mathcal{Q}} + p^2_T}$ with $\xi = \frac{1}{2}, 1, 2$.

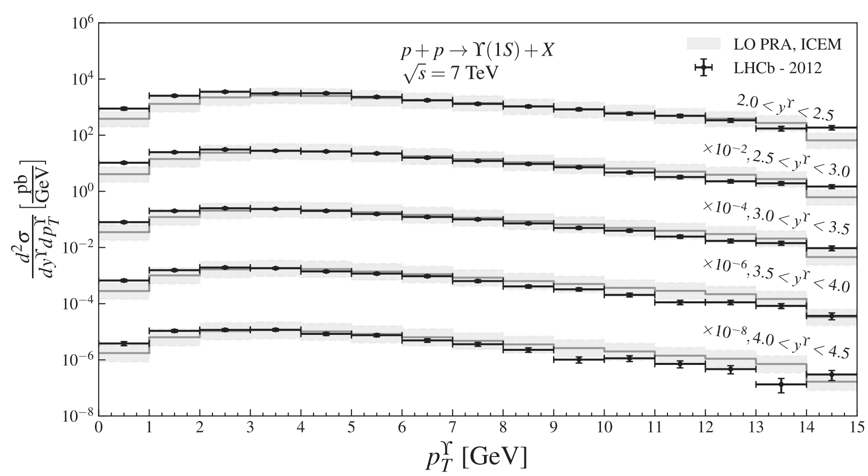

FIG. 3. Prompt $\Upsilon(1S)$ production cross section as a function of transverse momenta at the $\sqrt{s} = 7$ TeV at the different ranges of rapidity. The data are from the LHCb collaboration [48].

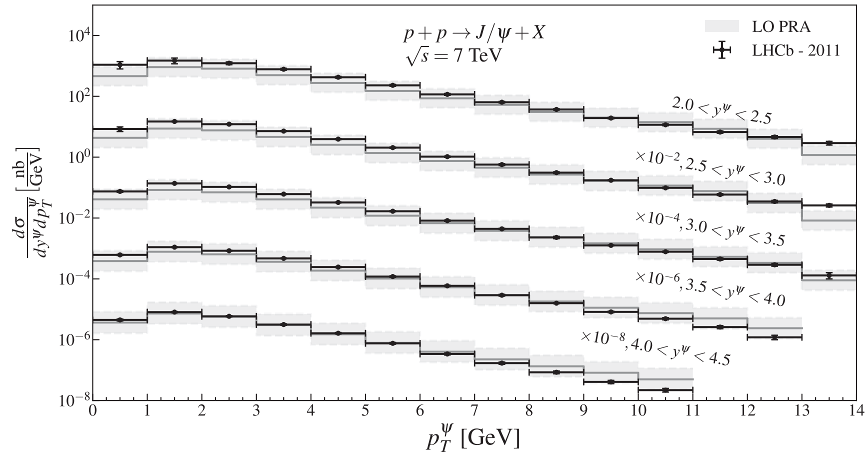

FIG. 4. Prompt J/ψ production cross section as a function of transverse momenta at the $\sqrt{s} = 7$ TeV at the different ranges of rapidity. The data are from the LHCb collaboration [49].

IV. FRAGMENTATION APPROACH

For the description of the inclusive production of an open charm meson it is often used the fragmentation approach in which the cross section for the production of D meson is related to the $c\bar{c}$ pair production by the following way:

$$\frac{d\sigma^{\text{SPS}}(p+p \to D+X)}{d^2 p_{TD} dy_D} = \int_{z_{\text{cut}}}^{1} dz \mathcal{D}_{c\to D}(z) \frac{d\sigma^{\text{SPS}}(p+p \to c+\bar{c}+X)}{d^2 p_{Tc} dy_c}, \quad (9)$$

where $\mathcal{D}_{c\to D}(z)$ is a fragmentation function (FF) of the c-quark into the D meson, p_c is the c-quark four-momentum expressed in terms of D meson four-momentum p_D through parameter z, which is defined as

$$z = \frac{E_D + |\mathbf{p}_D|}{E_c + |\mathbf{p}_c|}.$$

The minimal value of parameter $z_{\text{cut}} = m_D/(E_c + |\mathbf{p}_c|)$ cuts out the nonphysical region, where $E_c < m_D$, and we apply collinear fragmentation approximation, $\mathbf{p}_D/|\mathbf{p}_D| = \mathbf{p}_c/|\mathbf{p}_c|$. In our calculations, we use so-called Peterson FF

$$\mathcal{D}_{c\to D}(z) = \mathcal{N} \frac{z(1-z)^2}{[(1-z)^2 + \epsilon z]^2} \quad (10)$$

with parameter $\epsilon = 0.06$. FF is normalized such that

$$\int_0^1 dz\, \mathcal{D}_{c\to D}(z) = P_{c\to D}, \quad (11)$$

where $P_{c\to D^+} = 0.225$ and $P_{c\to D^0} = 0.542$. [50].

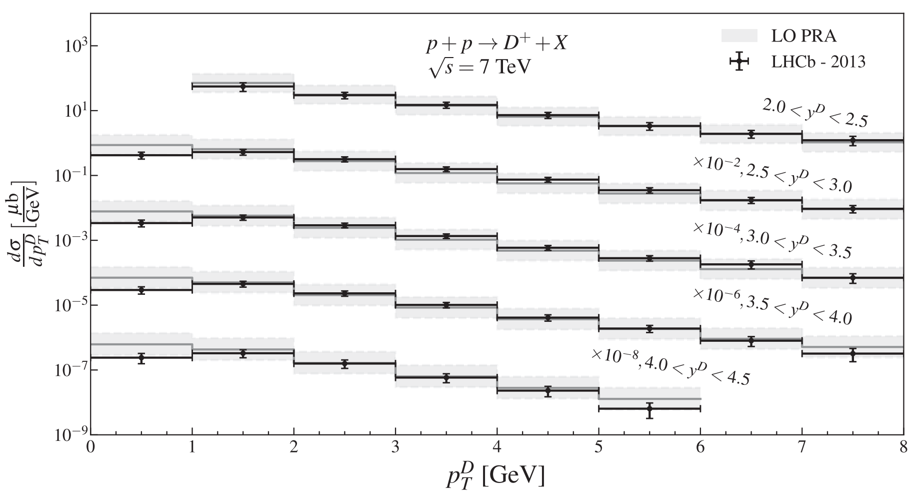

FIG. 5. D^+ meson production cross sections as functions of transverse momenta at the $\sqrt{s} = 7$ TeV. The data are from the LHCb collaboration [51].

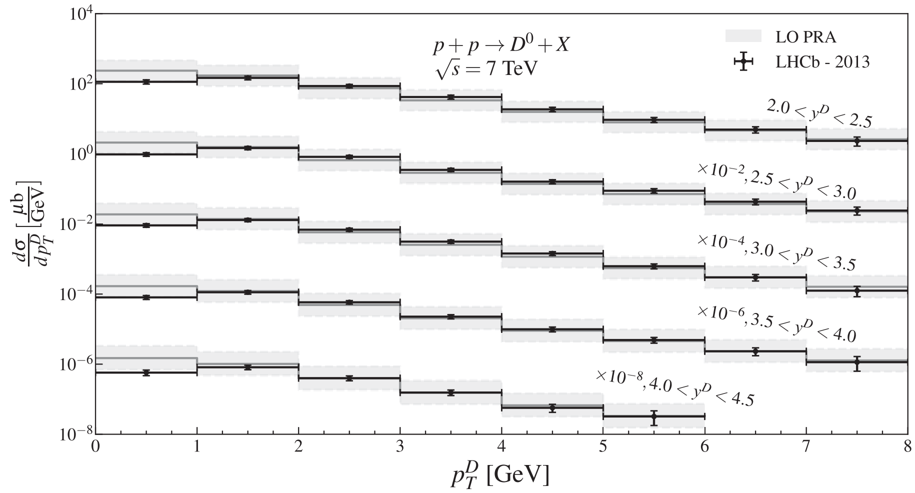

FIG. 6. D^0 meson production cross sections as functions of transverse momenta at the $\sqrt{s}=7$ TeV. The data are from the LHCb collaboration [51].

To test the fragmentation approach, we performed calculations for $D^{+,0}$ meson transverse momentum spectra and compare obtained results with the relevant LHCb data [51] at the energy $\sqrt{s}=7$ TeV, as it is shown in the Figs. 5 and 6.

In such a way, we demonstrate the applicability of our approach based on the k_T-factorization with the mKMRW gluon uPDF, the ICEM, and the fragmentation model, for description of single heavy quarkonium and single D meson production. Now, we are in position to study associated $J/\psi(\Upsilon)+D$ production.

V. ASSOCIATED $J/\psi(\Upsilon)+D$ PRODUCTION

In the case of $J/\psi+D$ associated production via the SPS, we take into account contributions of the following parton subprocesses:

$$R+R\to c+\bar{c}+c+\bar{c}, \quad (12)$$

$$Q_q+\bar{Q}_q\to c+\bar{c}+c+\bar{c}, \quad (13)$$

where R is a Reggeized gluon, $Q_q(\bar{Q}_q)$ as a Reggeized quark (antiquark), and $q=u, d, s, c$. In the DPS approach, the subprocesses of the $J/\psi+D$ associated production are the following:

$$R+R\to c+\bar{c}, \quad (14)$$

$$Q_q+\bar{Q}_q\to c+\bar{c} \quad (15)$$

in both groups.

In case of $\Upsilon+D$ associated production via the SPS, we take into account contributions of the following parton subprocesses:

$$R+R\to b+\bar{b}+c+\bar{c}, \quad (16)$$

$$Q_q+\bar{Q}_q\to b+\bar{b}+c+\bar{c}. \quad (17)$$

In the DPS approach, the processes of $\Upsilon+D$ associated production are the following:

$$R+R\to b+\bar{b}, \quad (18)$$

$$Q_q+\bar{Q}_q\to b+\bar{b} \quad (19)$$

$$R+R\to c+\bar{c}, \quad (20)$$

$$Q_q+\bar{Q}_q\to c+\bar{c}. \quad (21)$$

Using the KaTie, we can do calculations up to four particles in a final state that it is enough for our purposes. We put masses of quarks, heavy mesons, and heavy quarkonia during the calculations equal to $m_c=1.3$ GeV, $m_b=4.5$ GeV, $m_{D^+}=1.87$ GeV, $m_{D^0}=1.86$ GeV, $m_{B^+}=5.28$ GeV, $m_{J/\psi}=3.097$ GeV, and $m_\Upsilon=9.46$ GeV. As it was already mentioned in Sec. II, we use mKMRW gluon and quark uPDFs [28], which were used previously in our calculations for $J/\psi(\Upsilon)$ pair production using event generator KaTie [22,23].

The results of our calculations for associated $J/\psi+D^{+,0}$ production are presented in the Figs. 7 and 8 and for associated $\Upsilon+D^{+,0}$ production in the Figs. 9 and 10. We have obtained a quite satisfactory agreement between data and our calculations for all spectra in $J/\psi+D$ and $\Upsilon+D$ associated production. The following spectra are calculated: D meson transverse momentum, $J/\psi(\Upsilon)$ transverse momentum, D meson rapidity, $J/\psi(\Upsilon)$ rapidity, $J/\psi(\Upsilon)+D$ transverse momentum, $J/\psi(\Upsilon)+D$ rapidity, $J/\psi(\Upsilon)+D$ invariant mass, $\Delta y^{\Upsilon D}$ rapidity difference, $\Delta\phi_{\Upsilon D}$

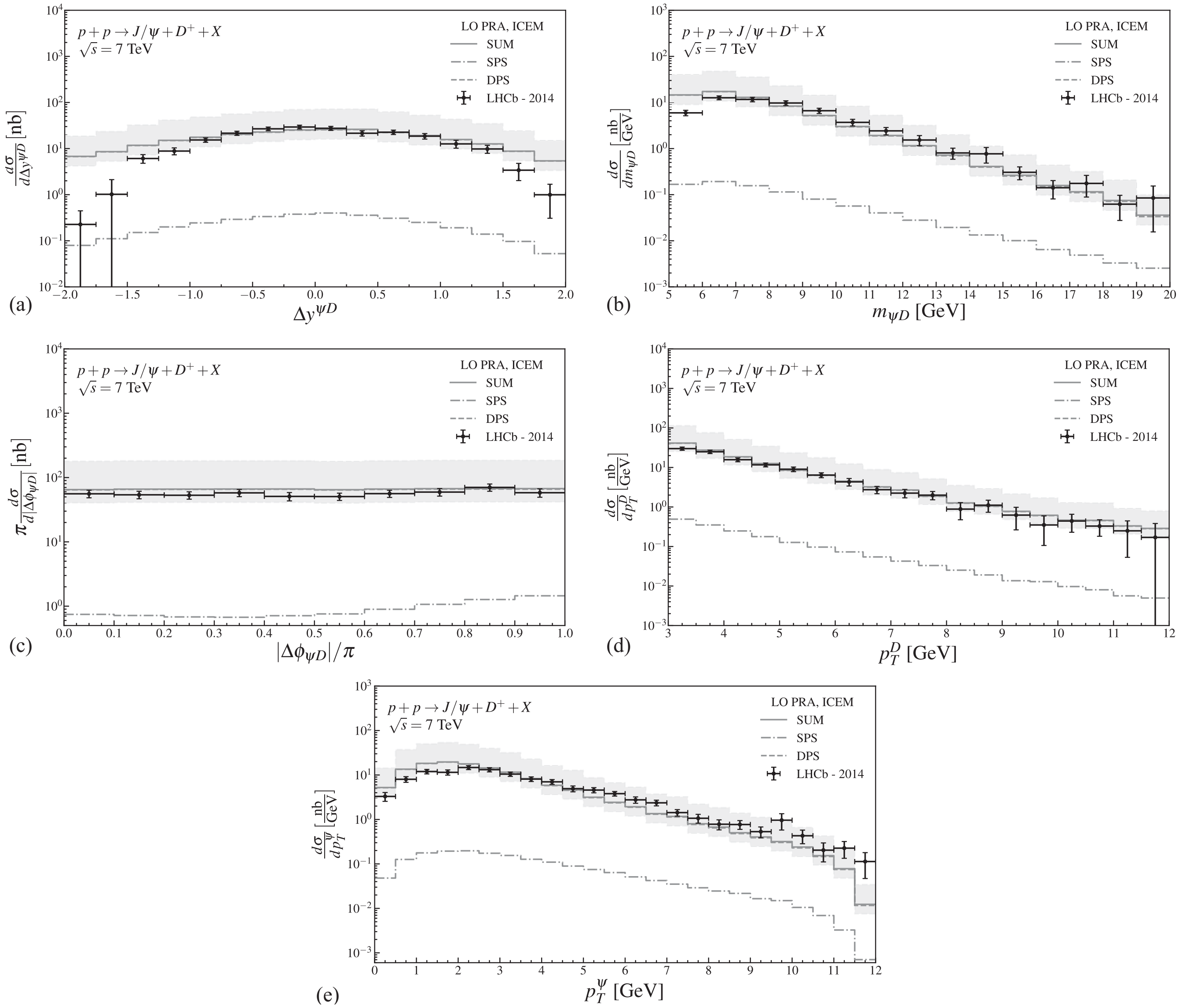

FIG. 7. Spectra for associated $J/\psi + D^+$ production at the $\sqrt{s} = 7$ TeV: (a) rapidity difference, (b) invariant mass, (c) azimuthal angle difference, (d) D^+ transverse momentum, and (e) J/ψ transverse momentum. The dashed line is the DPS contribution, the dashed-dotted line is the SPS contribution, and the solid line is their sum. Cross section times dilepton branching ratio of quarkonium. The data are from the LHCb collaboration [1].

azimuthal angle difference, and transverse momentum difference $\mathcal{A}_T^{\Upsilon D} = (|\mathbf{p}_T^\Upsilon| - |\mathbf{p}_T^D|)(|\mathbf{p}_T^\Upsilon| + |\mathbf{p}_T^D|)$.

There are disagreements only in the rapidity difference spectra, especially in $J/\psi + D$ production, at the large values of rapidity difference $|\Delta y^{\psi D}|$. We find theoretical calculations overestimate LHCb data at the $|\Delta y^{\psi D}| \simeq$ 1.75–2.0 about one order of magnitude.

In Table I we collect our theoretical predictions for associated $J/\psi(\Upsilon) + D$ production cross sections at the energies $\sqrt{s} = 7$ and 8 TeV. The predictions are compared with the experimental data from the LHCb collaboration [1,2]. We find a quite good agreement between data and theoretical calculations, which correspond to the default choice of the hard scale $\mu = \frac{1}{2}(m_T^\mathcal{Q} + m_T^D)$, where $m_T^\mathcal{Q} = \sqrt{m_\mathcal{Q}^2 + p_{T\mathcal{Q}}^2}$ and $m_T^D = \sqrt{m_D^2 + p_{TD}^2}$. The ratio of SPS to DPS averaged over bins is equal to $\sim 1/13$ and $\sim 1/10$ in case of $J/\psi + D$ and $\Upsilon + D$ associated productions respectively. The variation of hard scale by factor 2 around the default value gives an estimation of uncertainty of the LO k_T-factorization calculation. In case of the DPS production, such uncertainty may be very large, about 100% at the upper limit, instead of the SPS production mechanism. It is due to our having the product of four uPDFs in the DPS calculation, each of them also sufficiently depends on the choice of hard scale μ. In the

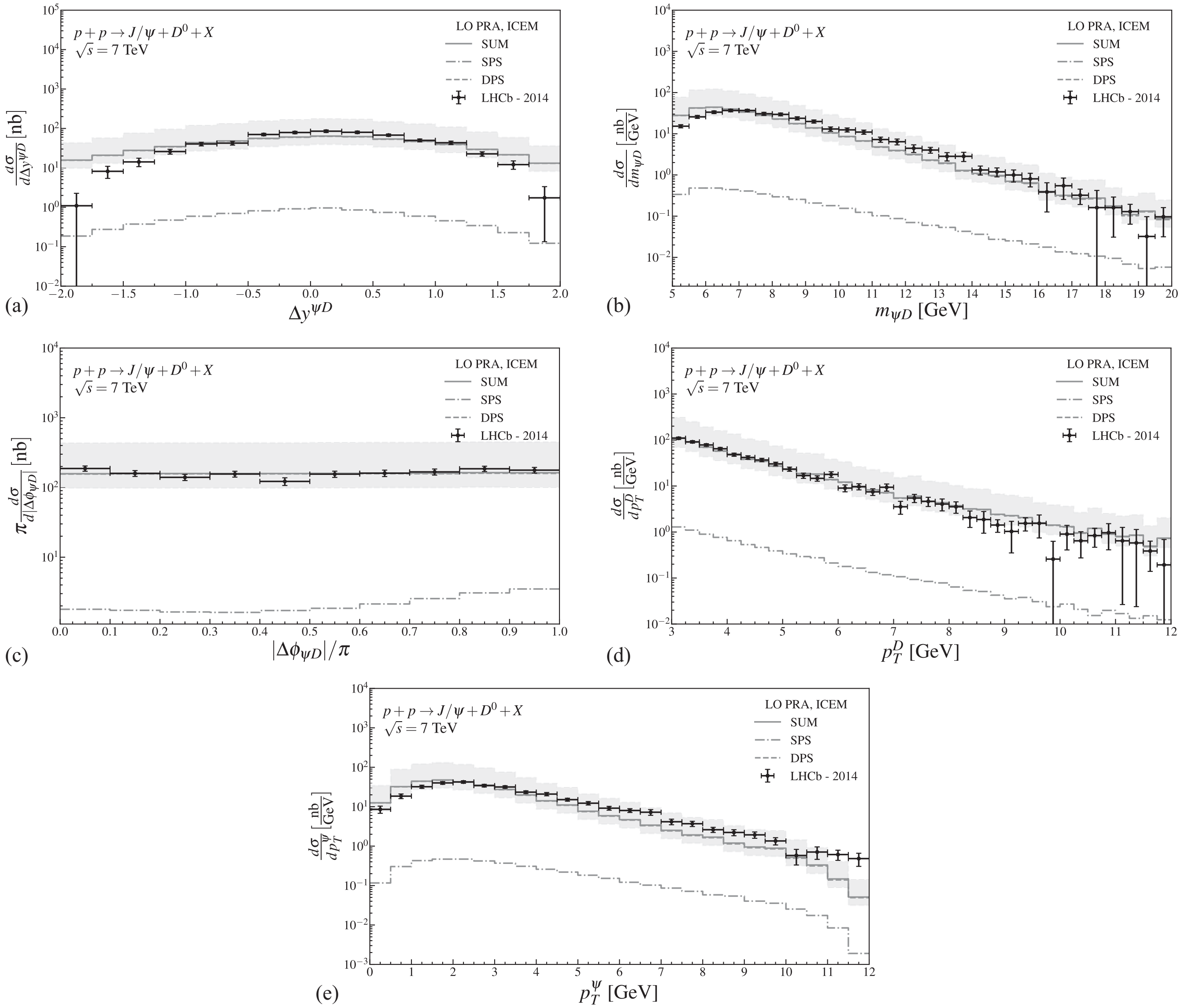

FIG. 8. Spectra for associated $J/\psi + D^0$ production at the $\sqrt{s} = 7$ TeV: (a) rapidity difference, (b) invariant mass, (c) azimuthal angle difference, (d) D^0 transverse momentum, and (e) J/ψ transverse momentum. The dashed line is the DPS contribution, the dashed-dotted line is the SPS contribution, and the solid line is their sum. Cross section times dilepton branching ratio of quarkonium. The data are from the LHCb collaboration [1].

KMRW model, strong scale dependence of the uPDF is determined by scale dependence of the relevant collinear PDFs (2) and, additionally, by the scale dependence of Sudakov form factor $T_i(t, \mu^2, x)$ (4). We may expect, as in the collinear parton model, the scale dependence will be reduced at the NLO level of accuracy in the k_T-factorization.

To estimate the uncertainty of our predictions following from the choice of the gluon uPDF parametrization, we have calculated contributions to cross sections which come from the regions $t_{1,2} > \mu^2$, when we have found sufficient differences in the values and k_T-dependence for different uPDFs. In the calculations with mKMRW parametrization [28], we obtain that regions $t_{1,2} > \mu^2$ give only 10–20% to total cross sections. In such a way, it is sufficiently smaller uncertainty that follows from the choice of hard scale μ as it was discussed above.

We also obtain predictions for the relevant total cross sections at the energy $\sqrt{s} = 13$ TeV under the similar to the experiments [1,2] kinematical conditions:

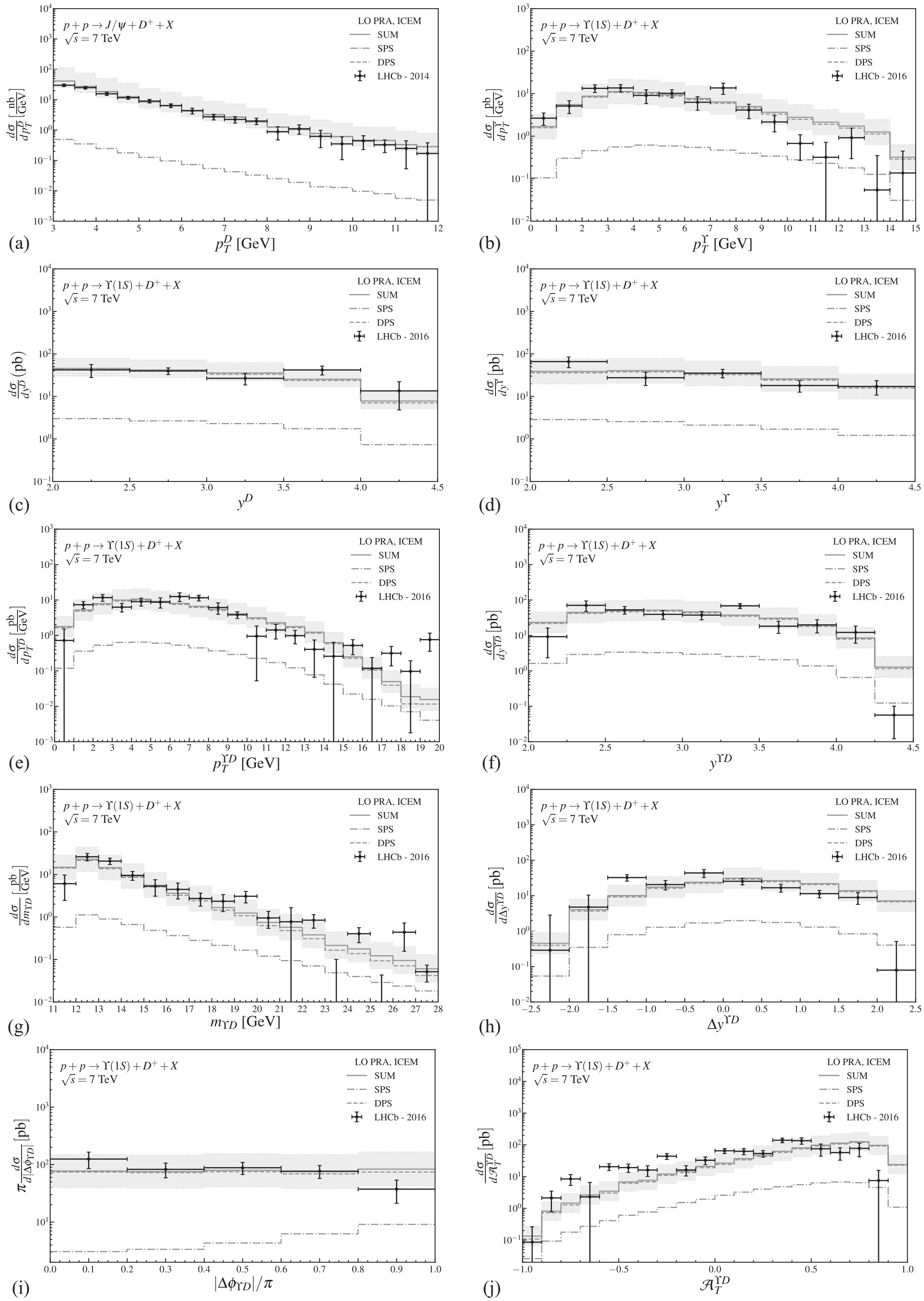

FIG. 9. Spectra for associated $\Upsilon + D^+$ production at the $\sqrt{s} = 7$ TeV: (a) D^+ transverse momentum, (b) Υ transverse momentum, (c) D^+ rapidity, (d) Υ rapidity, (e) $\Upsilon + D^+$ transverse momentum, (f) $\Upsilon + D^+$ rapidity, (g) invariant mass, (h) rapidity difference, (i) azimuthal angle difference, and (j) transverse momentum difference. The dashed line is the DPS contribution, the dashed-dotted line is the SPS contribution, and the solid line is their sum. Cross section times dilepton branching ratio of the quarkonium. The data are from the LHCb collaboration [2].

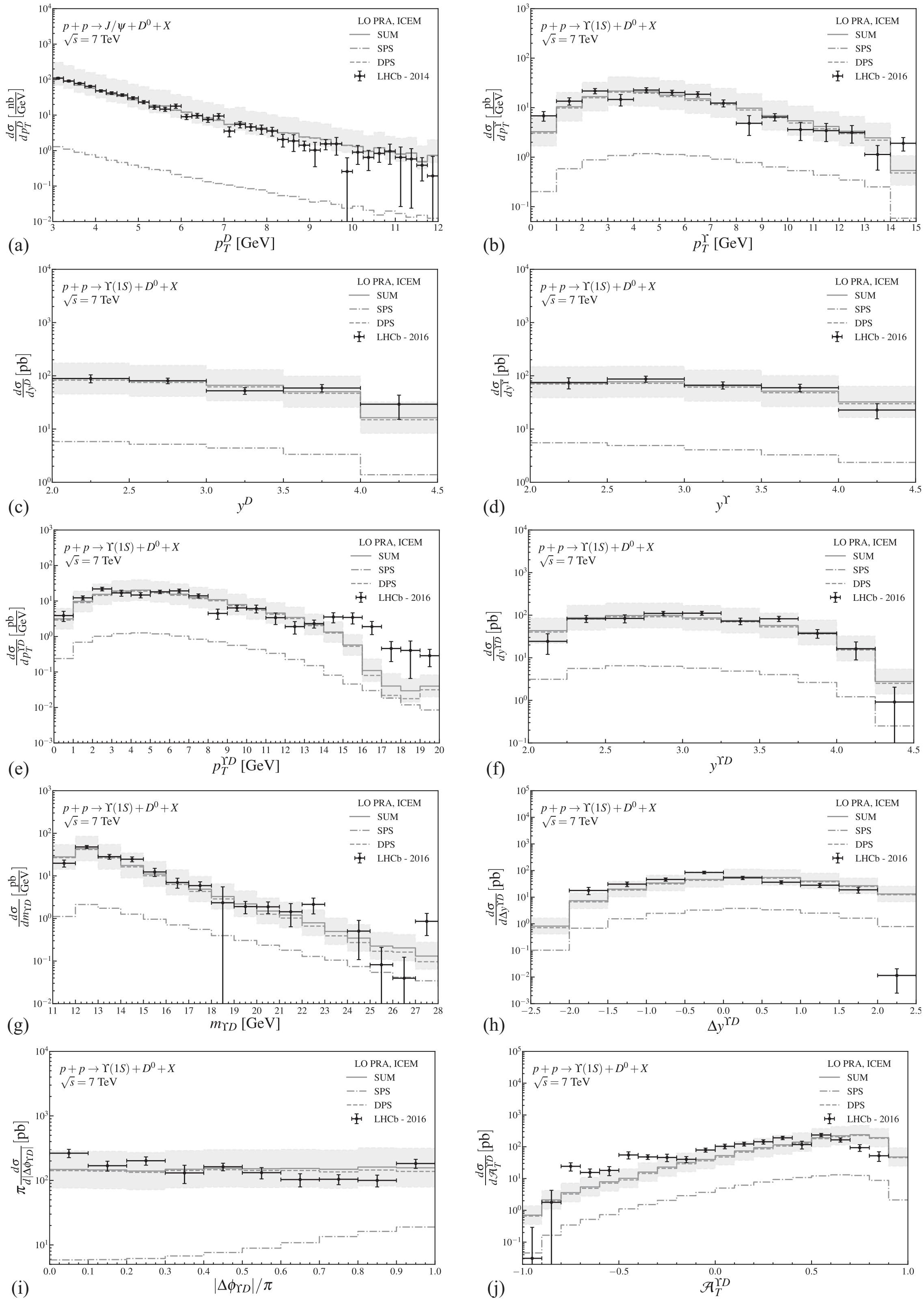

FIG. 10. Spectra for associated $\Upsilon + D^0$ production at the $\sqrt{s} = 7$ TeV: (a) D^0 transverse momentum, (b) Υ transverse momentum, (c) D^0 rapidity, (d) Υ rapidity, (e) $\Upsilon + D^0$ transverse momentum, (f) $\Upsilon + D^0$ rapidity, (g) invariant mass, (h) rapidity difference, (i) azimuthal angle difference, and (j) transverse momentum difference. The dashed line is the DPS contribution, the dashed-dotted line is the SPS contribution, and the solid line is their sum. Cross section times dilepton branching ratio of the quarkonium. The data are from the LHCb collaboration [2].

TABLE I. Comparison of theoretical and experimental total cross sections for associated $J/\psi(\Upsilon) + D^{+,0}$ production. The theoretical uncertainties include the variation of the hard scale by factors $\xi = 1/2$, 1, 2 from its default value $\mu = \frac{1}{2}(m_T^Q + m_T^D)$.

Final state	Energy	Exp ± (stat) ± (syst)	SPS ± Δ_{SPS}	DPS ± Δ_{DPS}	SUM
$J/\psi + D^0$	$\sqrt{s} = 7$ TeV	$9.7 \pm 0.2 \pm 0.7$	$0.13^{+0.25}_{-0.08}$	$9.44^{+10.54}_{-4.91}$	$9.57^{+10.79}_{-4.99}$ nb
$J/\psi + D^+$	$\sqrt{s} = 7$ TeV	$3.4 \pm 0.1 \pm 0.4$	$0.05^{+0.11}_{-0.03}$	$3.89^{+6.86}_{-1.47}$	$3.94^{+6.97}_{-1.50}$ nb
$\Upsilon + D^0$	$\sqrt{s} = 7$ TeV	$155 \pm 21 \pm 7$	$10.06^{+16.89}_{-5.99}$	$134.65^{+94.17}_{-46.18}$	$144.71^{+111.06}_{-52.17}$ pb
$\Upsilon + D^+$	$\sqrt{s} = 7$ TeV	$82 \pm 19 \pm 5$	$5.22^{+8.76}_{-3.11}$	$72.99^{+48.87}_{-23.97}$	$78.21^{+57.63}_{-27.08}$ pb
$\Upsilon + D^0$	$\sqrt{s} = 8$ TeV	$250 \pm 28 \pm 11$	$17.18^{+16.23}_{-10.17}$	$237.73^{+125.21}_{-71.09}$	$254.91^{+141.44}_{-81.26}$ pb
$\Upsilon + D^+$	$\sqrt{s} = 8$ TeV	$80 \pm 16 \pm 5$	$5.73^{+5.41}_{-3.39}$	$79.21^{+41.75}_{-23.71}$	$84.94^{+47.16}_{-27.10}$ pb

$$2\sigma^{\mathrm{SPS+DPS}}_{J/\psi+D^0}(p_T^\psi < 12\ \mathrm{GeV}, 3 < p_T^D < 12\ \mathrm{GeV}, 2 < y^{\psi,D} < 4) = 34^{+24}_{-11}\ \mathrm{nb}$$
$$\sigma^{\mathrm{SPS+DPS}}_{J/\psi+D^+}(p_T^\psi < 12\ \mathrm{GeV}, 3 < p_T^D < 12\ \mathrm{GeV}, 2 < y^{\psi,D} < 4) = 14^{+10}_{-5}\ \mathrm{nb}$$
$$\sigma^{\mathrm{SPS+DPS}}_{\Upsilon+D^0}(p_T^\psi < 15\ \mathrm{GeV}, 1 < p_T^D < 20\ \mathrm{GeV}, 2 < y^{\Upsilon,D} < 4.5) = 475^{+207}_{-123}\ \mathrm{pb}$$
$$\sigma^{\mathrm{SPS+DPS}}_{\Upsilon+D^+}(p_T^\psi < 15\ \mathrm{GeV}, 1 < p_T^D < 20\ \mathrm{GeV}, 2 < y^{\Upsilon,D} < 4.5) = 197^{+86}_{-48}\ \mathrm{pb}.$$

We should compare the obtained using ICEM and k_T-factorization results for associated $\Upsilon + D$ production with the early obtained predictions using NRQCD and k_T-factorization [11], where some evidence in favor of the SPS mechanism in Υ and D associated production at the LHC was justified. There are no contradictions between ICEM and NRQCD predictions in the inclusive Υ production in the k_T-factorization approach [23,52,53]. In Ref. [53], the SPS contribution in the associated $\Upsilon + D$ production was overestimated due to the use of gluon to D meson fragmentation mechanism, which dominates over c-quark fragmentation at the large transverse momenta of D mesons, $p_T^D \gg m_D$. More accurate calculation taking into account mass effects during the gluon to D meson fragmentation leads to reduction of SPS contribution in the Υ and D associated production and we find the absolutely dominant role of DPS production scenario.

VI. CONCLUSIONS

Working in the ICEM and the k_T-factorization, taking into account the SPS and DPS mechanisms, we have obtained self agreement description of the LHCb data for prompt heavy quarkonium $(J/\psi, \Upsilon)$ production, inclusive $D^{+,0}$ meson production, heavy quarkonium pair production $(J/\psi J/\psi, \Upsilon\Upsilon, J/\psi\Upsilon)$ [22,23], and heavy quarkonium plus $D^{+,0}$ meson associated production in the high-energy proton-proton collisions. In the study, we confirm early obtained numerical values for parameters of the ICEM, $F^\psi \simeq F^\Upsilon \simeq 0.02$, and the DPS pocket formula, $\sigma_{\mathrm{eff}} \simeq 11$ mb, which do not contradict results obtained previously by the different authors within the ICEM, the k_T-factorization, and the DPS model. We did not find any differences in the description of J/ψ or Υ production within the ICEM as well as specific effects for different D^0 or D^+ production within the fragmentation model.

It is shown that the k_T-factorization, which involves into consideration high-order QCD corrections effectively included in uPDFs and off-shell (Reggeized) hard parton cross sections, may be a powerful tool to calculate inclusive multiparticle production cross sections and spectra in the multi-Regge kinematics regime. It is very important in case of four or more particle production processes for which, in the collinear parton model, exact high order calculations in a strong coupling constant including real and virtual corrections are extremely difficult. The efficiency of the event generator KaTie for calculations of multiparticle production cross sections in the k_T-factorization approach is demonstrated once more.

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