Magnetic moments and axial charges of the octet hidden-charm molecular pentaquark family

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In this work, we calculate the magnetic moment and axial charge of the octet hidden-charm molecular pentaquark family in quark model. The Coleman-Glashow sum rule for the magnetic moments of pentaquark family is always fulfilled independently of $SU(3)$ symmetry breaking. In the 8_{2f} flavor representation, the magnetic moments of hidden-charm molecular pentaquark states with spin configuration $J^P = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$ are all equal to $\mu_c = 0.38\mu_N$. The axial charges of octet hidden-charm molecular pentaquark states are quite small compared to the axial charge of nucleon. The axial charges of the pentaquark states with the spin configuration $J^P = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$ in 8_{2f} flavor representation are all zero.

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I. INTRODUCTION

In order to classify hadrons constantly discovered in experiments, Gell-Mann [1] proposed a set of correct and effective classification in 1964, called the quark model. The model divides hadrons into singlet, octet, and decuplets under $SU(3)$ symmetry. Theorists predicted the existence of multiquark states after the establishment of the quark model. So far, many multiquark states including pentaquark and tetraquark states have been observed in different experiments [2–10]. We use the new naming scheme for the representation of pentaquarks in this paper [11].

In 2015, the hidden-charm pentaquark state was firstly observed by LHCb Collaboration in the $\Lambda_b^0 \to J/\psi K^- p$ decay. Two candidates of the hidden-charm pentaquark states were announced as $P_\psi^N(4380)^+$ and $P_\psi^N(4450)^+$ [6]. In 2019, three new pentaquark states $P_\psi^N(4312)^+$, $P_\psi^N(4440)^+$, and $P_\psi^N(4457)^+$, were reported in the updated analyses of the LHCb Collaboration [7]. The $P_\psi^N(4450)^+$ reported earlier is split into two peaks corresponding to the $P_\psi^N(4440)^+$ and the $P_\psi^N(4457)^+$. In 2020, the LHCb Collaboration observed the hidden-charm strange pentaquark state $P_{\psi s}^\Lambda(4459)^0$ in the $J/\psi\Lambda$ mass spectrum through an amplitude analysis of the $\Xi_b^- \to J/\psi \Lambda K^-$ decay [8]. In 2021, a new pentaquark structure $P_\psi^N(4337)^+$ was observed by the LHCb Collaboration in the $B_s^0 \to J/\psi p\bar{p}$ decay [9]. Recently, the LHCb collaboration observed a new structure $P_{\psi s}^\Lambda(4338)^0$ in the $B^- \to J/\psi \Lambda \bar{p}$ decay [10].

With the discovery of the hidden-charm pentaquark states in the experiments, many theorists have predicted the spin-parity quantum numbers of the hidden-charm pentaquark states [12–21], although the reliable information about their spin-parity has been unavailable until now. The near-threshold behaviors of these states in Table I indicate that those candidates of the hidden-charm pentaquark states can be explained as the molecular state belonging to the octet in the quark model. It is reasonable to speculate that the remaining hidden-charm pentaquark states will be observed in the coming future.

In hadron physics, it is interesting and important to investigate the properties of the hidden-charm pentaquark family in molecular models, which provides useful clues about the internal structure of the pentaquark states. The electromagnetic properties of hadrons are crucial for understanding their strong interactions and structure, which help to gain insight into QCD in the low-energy regime. The magnetic moment of hadrons encodes important details about the internal charge and magnetization distributions. In Refs. [22–24], the authors calculated the magnetic moments of hidden-charm pentaquark states with $J^P = \frac{1}{2}^\pm, \frac{3}{2}^\pm, \frac{5}{2}^\pm$, and $\frac{7}{2}^+$ in quark model. In Refs. [21,25,26], within the framework of QCD light cone sum rules, the authors extracted the magnetic dipole moment of the $P_\psi^N(4312)^+$, $P_\psi^N(4440)^+$, $P_\psi^N(4457)^+$, $P_\psi^N(4380)^+$,

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TABLE I. Properties of the hidden-charm pentaquark states.

Pentaquarks	Masses (MeV)	Widths (MeV)	Near thresholds
$P_\psi^N(4380)^+$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$\Sigma_c^* \bar{D}$
$P_\psi^N(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	$\Sigma_c \bar{D}$
$P_\psi^N(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	$\Sigma_c \bar{D}^*$
$P_\psi^N(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	$\Sigma_c \bar{D}^*$
$P_\psi^N(4337)^+$	$4337.3^{+7}_{-4}{}^{+2}_{-2}$	$29^{+26}_{-12}{}^{+14}_{-14}$	$\Lambda_c \bar{D}^*$
$P_{\psi s}^\Lambda(4459)^0$	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$	$\Xi_c D^*$
$P_{\psi s}^\Lambda(4338)^0$	$4338.2 \pm 0.7 \pm 0.4$	$7.0 \pm 1.2 \pm 1.3$	$\Xi_c D$

and $P_{\psi s}^\Lambda(4459)^0$ pentaquark states in the molecular and diquark-diquark-antiquark models. In Ref. [27], the authors obtained the ground state of hidden-charm pentaquark states with $J^P = \frac{3}{2}^-$ quantum numbers as well as the related magnetic dipole moment and electromagnetic coupling constant, which is significant for quark-model-driven pentaquark photoproduction experiments.

The axial charge g_A of baryons is a fundamental quantity for understanding the electroweak and strong interactions in standard model [28,29]. It not only controls weak interaction processes, such as β decay, but also intertwines weak and strong interactions. The Goldberger-Treiman relation [30], $g_A = f_\pi g_{\pi NN}/M_N$, reflects this most clearly. When the π decay constant f_π and nucleon mass M_N are given, the πNN coupling constant $g_{\pi NN}$ is proportional to g_A. Therefore, the correlation between π degrees of freedom and axial charges is closely related to hadron physics. g_A also represent an indicator of nonperturbative QCD chiral symmetry breaking, thus, the axial charge is an important parameter in low-energy effective theories. Therefore, investigating the axial charge of the octet hidden-charm pentaquark family can serve as another effective approach independent of the magnetic moment.

Studying the magnetic moment and axial charge of pentaquark hadrons is so important that they provide insight into the internal structure and the interactions between constituent quarks, which help to understand the fundamental strong force that binds quarks together to hadrons. With the continuous advancement of the experimental process in the future, discoveries of the octet hidden-charm molecular pentaquark family are no longer far away; researchers will be more eager to explore information about internal structure of the pentaquark states, and research on magnetic moment and axial charge can provide a large amount of data reference for this. Therefore, in this paper, we calculate the magnetic moment and axial charge of the octet hidden-charm molecular pentaquark states with quark model.

The remainder of this paper is organized as follows. In Sec. II, we introduce the wave function in molecular model, while in Sec. III, we calculate the magnetic moment of octet hidden-charm molecular pentaquark family. In Sec. IV, we calculate the axial charge of octet hidden-charm molecular pentaquark family. Section V summarizes this work.

II. WAVE FUNCTIONS

The wave function Ψ of the hadron state is a prerequisite for exploring its electromagnetic properties. The overall wave function of the hadron state is composed of the flavor wave function ψ_{flavor}, spin wave function χ_{spin}, color wave function ξ_{color} and space wave function η_{space} composition. The specific representation is

$$\Psi = \psi_{\text{flavor}} \otimes \chi_{\text{spin}} \otimes \xi_{\text{color}} \otimes \eta_{\text{space}}. \tag{1}$$

The Fermi statistic requires the overall wave function to be antisymmetric. Due to the color wave function ξ_{color} is antisymmetric and the space wave function η_{space} is symmetric in the ground state, we need to consider the symmetry requirement of the flavor-spin wave function when calculating the magnetic moment.

The hidden-charm pentaquark family in the molecular model is composed of the corresponding singly charmed baryons and anticharmed mesons. Singly charmed baryons can be obtained from light baryons under $SU(3)$ symmetry by substituting a light quark for a charm quark. The two light quarks in the singly charmed baryons can be symmetrical or antisymmetrical. In the former case, the flavor of the singly charmed baryons belongs to 6_f, and it can form 10_f and 8_{1f} with the anticharmed meson(3_f). In the latter case, the flavor of the singly charmed baryons belongs to $\bar{3}_f$, which can form 8_{2f} and 1_f with anticharm meson (3_f). We obtain the direct product $3 \otimes 3 \otimes 3 = 1 \oplus 8_1 \oplus 8_2 \oplus 10$. The hidden-charm pentaquark states which have been observed so far are likely to belong to the octet states. Therefore, it is very meaningful to study the properties of the octet hidden-charm molecular pentaquark states. These octet hidden-charm molecular pentaquark states have three spin configurations $J^P(J_b^{P_b} \otimes J_m^{P_m})$: $\frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$, $\frac{1}{2}^-(\frac{1}{2}^+ \otimes 1^-)$, and $\frac{3}{2}^-(\frac{1}{2}^+ \otimes 1^-)$, where J^P is

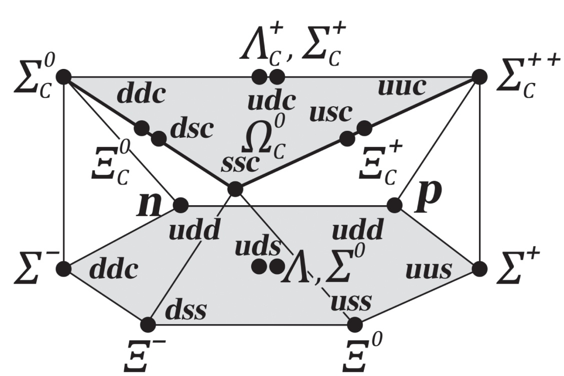

FIG. 1. The octet baryons with $J^P = \frac{1}{2}^+$ under $SU(3)$ symmetry become singly charmed baryons by replacing a light quark with a charmed quark.

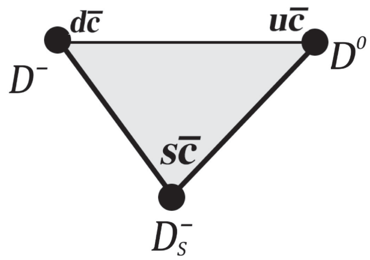

FIG. 2. The anticharmed mesons are composed of a anticharm quark and a light quark.

the total spin of pentaquark states, and $J_b^{P_b} \otimes J_m^{P_m}$ are corresponding to the angular momentum and parity of baryon and meson, respectively.

In Fig. 1, we show that the octet baryons with $J^P = \frac{1}{2}^+$ become singly charmed baryons by replacing a light quark with a charmed quark. In Fig. 2, anticharmed mesons are composed of a anticharm quark and a light quark. In Table II, we show that the flavor wave functions $\psi_{\rm flavor}$ and the spin wave functions $\chi_{\rm spin}$ of the S-wave singly charmed baryons. In Table III, we show that the flavor wave

TABLE II. The flavor wave functions $\psi_{\rm flavor}$ and the spin wave functions $\chi_{\rm spin}$ of the S-wave singly charmed baryon. Here, S and S_3 are the spin and its third component, while the arrow denotes the third component of the quark spin.

Baryons	$\vert S, S_3\rangle$	$\psi_{\rm flavor} \otimes \chi_{\rm spin}$
Σ_c^{++}	$\vert\frac{1}{2},\frac{1}{2}\rangle$	$uuc \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$
	$\vert\frac{1}{2},-\frac{1}{2}\rangle$	$uuc \otimes \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$
Σ_c^{+}	$\vert\frac{1}{2},\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(udc + duc) \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$
	$\vert\frac{1}{2},-\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(udc + duc) \otimes \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$
Σ_c^{0}	$\vert\frac{1}{2},\frac{1}{2}\rangle$	$ddc \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$
	$\vert\frac{1}{2},-\frac{1}{2}\rangle$	$ddc \otimes \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$
$\Xi_c^{\prime+}$	$\vert\frac{1}{2},\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(usc + suc) \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$
	$\vert\frac{1}{2},-\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(usc + suc) \otimes \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$
$\Xi_c^{\prime0}$	$\vert\frac{1}{2},\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(dsc + sdc) \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$
	$\vert\frac{1}{2},-\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(dsc + sdc) \otimes \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$
Ω_c^{0}	$\vert\frac{1}{2},\frac{1}{2}\rangle$	$ssc \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)$
	$\vert\frac{1}{2},-\frac{1}{2}\rangle$	$ssc \otimes \frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow - 2\downarrow\downarrow\uparrow)$
Ξ_c^{+}	$\vert\frac{1}{2},\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(usc - suc) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$
	$\vert\frac{1}{2},-\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(usc - suc) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow)$
Ξ_c^{0}	$\vert\frac{1}{2},\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(dsc - sdc) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$
	$\vert\frac{1}{2},-\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(dsc - sdc) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow)$
Λ_c^{+}	$\vert\frac{1}{2},\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(udc - duc) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$
	$\vert\frac{1}{2},-\frac{1}{2}\rangle$	$\frac{1}{\sqrt{2}}(udc - duc) \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow)$

TABLE III. The flavor wave functions $\psi_{\rm flavor}$ and the spin wave functions $\chi_{\rm spin}$ of the S-wave anticharmed mesons. Here, S and S_3 are the spin and its third component, while the arrow denotes the third component of the quark spin.

Mesons	$\vert S, S_3\rangle$	$\psi_{\rm flavor} \otimes \chi_{\rm spin}$
$\bar{D}^0$	$\vert 0,0\rangle$	$\bar{c}u \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$
D^-	$\vert 0,0\rangle$	$\bar{c}d \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$
D_s^-	$\vert 0,0\rangle$	$\bar{c}s \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$
$\bar{D}^{*0}$	$\vert 1,1\rangle$	$\bar{c}u \otimes \uparrow\uparrow$
	$\vert 1,0\rangle$	$\bar{c}u \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$
	$\vert 1,-1\rangle$	$\bar{c}u \otimes \downarrow\downarrow$
D^{*-}	$\vert 1,1\rangle$	$\bar{c}d \otimes \uparrow\uparrow$
	$\vert 1,0\rangle$	$\bar{c}d \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$
	$\vert 1,-1\rangle$	$\bar{c}d \otimes \downarrow\downarrow$
D_s^{*-}	$\vert 1,1\rangle$	$\bar{c}s \otimes \uparrow\uparrow$
	$\vert 1,0\rangle$	$\bar{c}s \otimes \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$
	$\vert 1,-1\rangle$	$\bar{c}s \otimes \downarrow\downarrow$

TABLE IV. The expressions for different flavor wave function of the octet hidden-charm molecular pentaquark states under $SU(3)$ symmetry in the molecular model.

States	Flavor	Wave functions
$P_\psi^{N^+}$	8_{1f}	$-\sqrt{\frac{1}{3}}\Sigma_c^+\bar{D}^{(*)0} + \sqrt{\frac{2}{3}}\Sigma_c^{++}D^{(*)-}$
	8_{2f}	$\Lambda_c^+\bar{D}^{(*)0}$
$P_\psi^{N^0}$	8_{1f}	$\sqrt{\frac{1}{3}}\Sigma_c^+D^{(*)-} - \sqrt{\frac{2}{3}}\Sigma_c^0\bar{D}^{(*)0}$
	8_{2f}	$\Lambda_c^+D^{(*)-}$
$P_{\psi s}^{\Sigma^+}$	8_{1f}	$\sqrt{\frac{1}{3}}\Xi_c^{\prime+}\bar{D}^{(*)0} - \sqrt{\frac{2}{3}}\Sigma_c^{++}D_s^{(*)-}$
	8_{2f}	$\Xi_c^+\bar{D}^{(*)0}$
$P_{\psi s}^{\Sigma^0}$	8_{1f}	$\sqrt{\frac{1}{6}}\Xi_c^{\prime+}D^{(*)-} + \sqrt{\frac{1}{6}}\Xi_c^{\prime0}\bar{D}^{(*)0} - \sqrt{\frac{2}{3}}\Sigma_c^+D_s^{(*)-}$
	8_{2f}	$\sqrt{\frac{1}{2}}\Xi_c^+D^{(*)-} + \sqrt{\frac{1}{2}}\Xi_c^0\bar{D}^{(*)0}$
$P_{\psi s}^{\Lambda^0}$	8_{1f}	$\sqrt{\frac{1}{2}}\Xi_c^{\prime+}D^{(*)-} - \sqrt{\frac{1}{2}}\Xi_c^{\prime0}\bar{D}^{(*)0}$
	8_{2f}	$\sqrt{\frac{1}{6}}\Xi_c^+D^{(*)-} - \sqrt{\frac{1}{6}}\Xi_c^0\bar{D}^{(*)0} - \sqrt{\frac{2}{3}}\Lambda_c^+D_s^{(*)-}$
$P_{\psi s}^{\Sigma^-}$	8_{1f}	$\sqrt{\frac{1}{3}}\Xi_c^{\prime0}D^{(*)-} - \sqrt{\frac{2}{3}}\Sigma_c^0D_s^{(*)-}$
	8_{2f}	$\Xi_c^0D^{(*)-}$
$P_{\psi ss}^{N^0}$	8_{1f}	$\sqrt{\frac{1}{3}}\Xi_c^{\prime+}D_s^{(*)-} - \sqrt{\frac{2}{3}}\Omega_c^0\bar{D}^{(*)0}$
	8_{2f}	$\Xi_c^+D_s^{(*)-}$
$P_{\psi ss}^{N^-}$	8_{1f}	$\sqrt{\frac{1}{3}}\Xi_c^{\prime0}D_s^{(*)-} - \sqrt{\frac{2}{3}}\Omega_c^0D^{(*)-}$
	8_{2f}	$\Xi_c^0D_s^{(*)-}$

functions ψ_{flavor} and the spin wave functions χ_{spin} of the S-wave anticharmed mesons. We list the expression for the flavor wave function of the hidden-charm molecular pentaquark under $SU(3)$ symmetry in Table IV.

III. MAGNETIC MOMENTS OF THE OCTET HIDDEN-CHARM MOLECULAR PENTAQUARK FAMILY

In this section, we adopt the constituent quark model to obtain the magnetic moment of hadrons. The constituent quark model was extensively applied to study various properties of the hadronic states in the past decades [31–33], while the magnetic moments of the hadronic molecular states are focused [34–41]. Magnetic moments of the hadron states are composed of two parts; the spin magnetic moment μ_{spin} and the orbital magnetic moment μ_{orbital}

$$\mu = \mu_{\text{spin}} + \mu_{\text{orbital}}. \tag{2}$$

In this work, we focus on the S-wave pentaquark states composed of a singly charmed baryon and an anticharmed meson in the molecular model. Thence, the orbital magnetic moment μ_{orbital} between baryons and mesons will not be considered in the following calculations.

At the quark level, the operators of the magnetic moments are

$$\hat{\mu}_{\text{spin}} = \sum_i \frac{Q_i}{2M_i}\hat{\sigma}_i, \tag{3}$$

where Q_i, M_i, and $\hat{\sigma}_i$ denote charge, mass, and Pauli's spin matrix of the ith quark, respectively.

The magnetic moments of the hidden-charm molecular pentaquark states are closely related to the magnetic moments of the singly charmed baryons and the magnetic moments of the anticharmed mesons. The magnetic moment of the S-wave molecular pentaquark states include the sum of the baryon spin magnetic moment and the meson spin magnetic moment,

$$\hat{\mu} = \hat{\mu}_B + \hat{\mu}_M, \tag{4}$$

where the subscripts B and M represent baryon and meson respectively. At quark level, the magnetic moment of quark can be obtained by the following matrix elements:

$$\langle q\uparrow|\hat{\mu}_z|q\uparrow\rangle = \frac{Q_q}{2M_q}, \tag{5}$$

$$\langle q\downarrow|\hat{\mu}_z|q\downarrow\rangle = -\frac{Q_q}{2M_q}, \tag{6}$$

the arrow stands for the third component of the quark spin. As an example, we derive the magnetic moment of the Σ_c^{++} baryon as follows:

TABLE V. The magnetic moments of the singly charmed baryons and anticharmed mesons, in unit of the nuclear magnetic moment μ_N.

Flavor	Hadrons	Expressions	Results
6_f	Σ_c^{++}	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_c$	2.36
	Σ_c^{+}	$\frac{2}{3}\mu_u + \frac{2}{3}\mu_d - \frac{1}{3}\mu_c$	0.49
	Σ_c^{0}	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_c$	−1.37
	$\Xi_c^{\prime +}$	$\frac{2}{3}\mu_u + \frac{2}{3}\mu_s - \frac{1}{3}\mu_c$	0.73
	$\Xi_c^{\prime 0}$	$\frac{2}{3}\mu_d + \frac{2}{3}\mu_s - \frac{1}{3}\mu_c$	−1.13
	Ω_c^{0}	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_c$	−0.90
$\bar{3}_f$	Ξ_c^{+}	μ_c	0.38
	Ξ_c^{0}	μ_c	0.38
	Λ_c^{+}	μ_c	0.38
3_f	$\bar{D}^{*0}$	$\mu_u + \mu_{\bar{c}}$	1.48
	D^{*-}	$\mu_d + \mu_{\bar{c}}$	−1.31
	D_s^{*-}	$\mu_s + \mu_{\bar{c}}$	−0.96

$$\begin{aligned}\mu_{\Sigma_c^{++}} &= \Big\langle uuc \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow) \\ &\quad \times |\hat{\mu}_{\text{spin}}| uuc \otimes \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow)\Big\rangle \\ &= \frac{4}{3}\mu_u - \frac{1}{3}\mu_c.\end{aligned} \tag{7}$$

In Table V, we collect the magnetic moment expressions and numerical results for singly charmed baryons and anticharmed mesons, which are expressed as the combination of the magnetic moments of their constituent quarks. In the numerical analysis, we take the constituent quark masses as input [42],

$$\begin{aligned} m_u &= m_d = 0.336\ \text{GeV}, \\ m_s &= 0.540\ \text{GeV}, \\ m_c &= 1.660\ \text{GeV}. \end{aligned}$$

Based on the magnetic moments of the singly charmed baryons and anticharmed mesons, we obtain the magnetic moments of the octet hidden-charm molecular pentaquark states. In Table VI, we present the expressions and numerical results of the magnetic moments of the S-wave octet hidden-charm molecular pentaquark states.

The magnetic moment expressions of the hidden-charm molecular pentaquark family are composed of the magnetic moments of singly charmed baryons and anticharmed mesons. Different angular momentum couplings have different Clebsch-Gordan coefficients to distinguish them. The magnetic moment corresponding to the same S-wave molecular pentaquark states in the 8_{1f} flavor representation and 8_{2f} flavor representation is

TABLE VI. The magnetic moments of the S-wave octet hidden-charm pentaquark family. The $J_b^{P_b} \otimes J_m^{P_m}$ are corresponding to the angular momentum and parity of baryon and meson, respectively. The magnetic moment is in unit of the nuclear magnetic moment μ_N.

States	$J_b^{P_b} \otimes J_m^{P_m}$	$I(J^P)$	Expressions (8_{1f})	Results	Expressions (8_{2f})	Results
$P_{\psi}^{N^+}$	$\frac{1}{2}^+ \otimes 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{2}{3}\mu_{\Sigma_c^{++}} + \frac{1}{3}\mu_{\Sigma_c^{+}}$	1.74	$\mu_{\Lambda_c^+}$	0.38
	$\frac{1}{2}^+ \otimes 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{2}{9}\mu_{\Sigma_c^{++}} - \frac{1}{9}\mu_{\Sigma_c^{+}} + \frac{4}{9}\mu_{D^{*-}} + \frac{2}{9}\mu_{\bar{D}^{*0}}$	−0.83	$-\frac{1}{3}\mu_{\Lambda_c^+} + \frac{2}{3}\mu_{\bar{D}^{*0}}$	0.86
		$\frac{1}{2}(\frac{3}{2})^-$	$\frac{2}{3}\mu_{\Sigma_c^{++}} + \frac{1}{3}\mu_{\Sigma_c^{+}} + \frac{2}{3}\mu_{D^{*-}} + \frac{1}{3}\mu_{\bar{D}^{*0}}$	1.36	$\mu_{\Lambda_c^+} + \mu_{\bar{D}^{*0}}$	1.86
$P_{\psi}^{N^0}$	$\frac{1}{2}^+ \otimes 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{2}{3}\mu_{\Sigma_c^{0}} + \frac{1}{3}\mu_{\Sigma_c^{+}}$	−0.75	$\mu_{\Lambda_c^+}$	0.38
	$\frac{1}{2}^+ \otimes 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{2}{9}\mu_{\Sigma_c^{0}} - \frac{1}{9}\mu_{\Sigma_c^{+}} + \frac{4}{9}\mu_{\bar{D}^{*0}} + \frac{2}{9}\mu_{D^{*-}}$	0.62	$-\frac{1}{3}\mu_{\Lambda_c^+} + \frac{2}{3}\mu_{D^{*-}}$	−1.00
		$\frac{1}{2}(\frac{3}{2})^-$	$\frac{2}{3}\mu_{\Sigma_c^{0}} + \frac{1}{3}\mu_{\Sigma_c^{+}} + \frac{2}{3}\mu_{\bar{D}^{*0}} + \frac{1}{3}\mu_{D^{*-}}$	−0.19	$\mu_{\Lambda_c^+} + \mu_{D^{*-}}$	−0.93
$P_{\psi s}^{\Sigma^+}$	$\frac{1}{2}^+ \otimes 0^-$	$1(\frac{1}{2})^-$	$\frac{2}{3}\mu_{\Sigma_c^{++}} + \frac{1}{3}\mu_{\Xi_c'^{+}}$	1.81	$\mu_{\Xi_c^+}$	0.38
	$\frac{1}{2}^+ \otimes 1^-$	$1(\frac{1}{2})^-$	$-\frac{2}{9}\mu_{\Sigma_c^{++}} - \frac{1}{9}\mu_{\Xi_c'^{+}} + \frac{4}{9}\mu_{D_s^{*-}} + \frac{2}{9}\mu_{\bar{D}^{*0}}$	−0.70	$-\frac{1}{3}\mu_{\Xi_c^+} + \frac{2}{3}\mu_{\bar{D}^{*0}}$	0.86
		$1(\frac{3}{2})^-$	$\frac{2}{3}\mu_{\Sigma_c^{++}} + \frac{1}{3}\mu_{\Xi_c'^{+}} + \frac{2}{3}\mu_{D_s^{*-}} + \frac{1}{3}\mu_{\bar{D}^{*0}}$	1.67	$\mu_{\Xi_c^+} + \mu_{\bar{D}^{*0}}$	1.86
$P_{\psi s}^{\Sigma^0}$	$\frac{1}{2}^+ \otimes 0^-$	$1(\frac{1}{2})^-$	$\frac{1}{6}\mu_{\Xi_c'^{+}} + \frac{1}{6}\mu_{\Xi_c'^{0}} + \frac{2}{3}\mu_{\Sigma_c^{+}}$	0.26	$\frac{1}{2}\mu_{\Xi_c^+} + \frac{1}{2}\mu_{\Xi_c^0}$	0.38
	$\frac{1}{2}^+ \otimes 1^-$	$1(\frac{1}{2})^-$	$-\frac{2}{9}\mu_{\Sigma_c^{+}} - \frac{1}{18}\mu_{\Xi_c'^{+}} - \frac{1}{18}\mu_{\Xi_c'^{0}} + \frac{4}{9}\mu_{D_s^{*-}} + \frac{1}{9}\mu_{\bar{D}^{*0}} + \frac{1}{9}\mu_{D^{*-}}$	−0.49	$-\frac{1}{6}\mu_{\Xi_c^+} - \frac{1}{6}\mu_{\Xi_c^0} + \frac{1}{3}\mu_{D^{*-}} + \frac{1}{3}\mu_{\bar{D}^{*0}}$	−0.07
		$1(\frac{3}{2})^-$	$\frac{2}{3}\mu_{\Sigma_c^{+}} + \frac{1}{6}\mu_{\Xi_c'^{+}} + \frac{1}{6}\mu_{\Xi_c'^{0}} + \frac{2}{3}\mu_{D_s^{*-}} + \frac{1}{6}\mu_{\bar{D}^{*0}} + \frac{1}{6}\mu_{D^{*-}}$	−0.35	$\frac{1}{2}\mu_{\Xi_c^+} + \frac{1}{2}\mu_{\Xi_c^0} + \frac{1}{2}\mu_{D^{*-}} + \frac{1}{2}\mu_{\bar{D}^{*0}}$	0.47
$P_{\psi s}^{\Sigma^-}$	$\frac{1}{2}^+ \otimes 0^-$	$1(\frac{1}{2})^-$	$\frac{2}{3}\mu_{\Sigma_c^{0}} + \frac{1}{3}\mu_{\Xi_c'^{0}}$	−1.29	$\mu_{\Xi_c^0}$	0.38
	$\frac{1}{2}^+ \otimes 1^-$	$1(\frac{1}{2})^-$	$-\frac{2}{9}\mu_{\Sigma_c^{0}} - \frac{1}{9}\mu_{\Xi_c'^{0}} + \frac{4}{9}\mu_{D_s^{*-}} + \frac{2}{9}\mu_{D^{*-}}$	−0.29	$-\frac{1}{3}\mu_{\Xi_c^0} + \frac{2}{3}\mu_{D^{*-}}$	−1.00
		$1(\frac{3}{2})^-$	$\frac{2}{3}\mu_{\Sigma_c^{0}} + \frac{1}{3}\mu_{\Xi_c'^{0}} + \frac{2}{3}\mu_{D_s^{*-}} + \frac{1}{3}\mu_{D^{*-}}$	−2.36	$\mu_{\Xi_c^0} + \mu_{D^{*-}}$	−0.93
$P_{\psi s}^{\Lambda^0}$	$\frac{1}{2}^+ \otimes 0^-$	$0(\frac{1}{2})^-$	$\frac{1}{2}\mu_{\Xi_c'^{+}} + \frac{1}{2}\mu_{\Xi_c'^{0}}$	−0.20	$\frac{1}{6}\mu_{\Xi_c^+} + \frac{1}{6}\mu_{\Xi_c^0} + \frac{2}{3}\mu_{\Lambda_c^+}$	0.38
	$\frac{1}{2}^+ \otimes 1^-$	$0(\frac{1}{2})^-$	$-\frac{1}{6}\mu_{\Xi_c'^{+}} - \frac{1}{6}\mu_{\Xi_c'^{0}} + \frac{1}{3}\mu_{D^{*-}} + \frac{1}{3}\mu_{\bar{D}^{*0}}$	0.13	$-\frac{2}{9}\mu_{\Lambda_c^+} - \frac{1}{18}\mu_{\Xi_c^+} - \frac{1}{18}\mu_{\Xi_c^0} + \frac{4}{9}\mu_{D_s^{*-}} + \frac{1}{9}\mu_{\bar{D}^{*0}} + \frac{1}{9}\mu_{D^{*-}}$	−0.53
		$0(\frac{3}{2})^-$	$\frac{1}{2}\mu_{\Xi_c'^{+}} + \frac{1}{2}\mu_{\Xi_c'^{0}} + \frac{1}{2}\mu_{D^{*-}} + \frac{1}{2}\mu_{\bar{D}^{*0}}$	−0.11	$\frac{2}{3}\mu_{\Lambda_c^+} + \frac{1}{6}\mu_{\Xi_c^+} + \frac{1}{6}\mu_{\Xi_c^0} + \frac{2}{3}\mu_{D_s^{*-}} + \frac{1}{6}\mu_{\bar{D}^{*0}} + \frac{1}{6}\mu_{D^{*-}}$	−0.23
$P_{\psi ss}^{N^0}$	$\frac{1}{2}^+ \otimes 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{2}{3}\mu_{\Omega_c^{0}} + \frac{1}{3}\mu_{\Xi_c'^{+}}$	−0.36	$\mu_{\Xi_c^+}$	0.38
	$\frac{1}{2}^+ \otimes 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{2}{9}\mu_{\Omega_c^{0}} - \frac{1}{9}\mu_{\Xi_c'^{+}} + \frac{4}{9}\mu_{\bar{D}^{*0}} + \frac{2}{9}\mu_{D_s^{*-}}$	0.57	$-\frac{1}{3}\mu_{\Xi_c^+} + \frac{2}{3}\mu_{D_s^{*-}}$	−0.76
		$\frac{1}{2}(\frac{3}{2})^-$	$\frac{2}{3}\mu_{\Omega_c^{0}} + \frac{1}{3}\mu_{\Xi_c'^{+}} + \frac{2}{3}\mu_{\bar{D}^{*0}} + \frac{1}{3}\mu_{D_s^{*-}}$	0.32	$\mu_{\Xi_c^+} + \mu_{D_s^{*-}}$	−0.58
$P_{\psi ss}^{N^-}$	$\frac{1}{2}^+ \otimes 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{2}{3}\mu_{\Omega_c^{0}} + \frac{1}{3}\mu_{\Xi_c'^{0}}$	−0.98	$\mu_{\Xi_c^0}$	0.38
	$\frac{1}{2}^+ \otimes 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{2}{9}\mu_{\Omega_c^{0}} - \frac{1}{9}\mu_{\Xi_c'^{0}} + \frac{4}{9}\mu_{D^{*-}} + \frac{2}{9}\mu_{D_s^{*-}}$	−0.47	$-\frac{1}{3}\mu_{\Xi_c^0} + \frac{2}{3}\mu_{D_s^{*-}}$	−0.76
		$\frac{1}{2}(\frac{3}{2})^-$	$\frac{2}{3}\mu_{\Omega_c^{0}} + \frac{1}{3}\mu_{\Xi_c'^{0}} + \frac{2}{3}\mu_{D^{*-}} + \frac{1}{3}\mu_{D_s^{*-}}$	−2.17	$\mu_{\Xi_c^0} + \mu_{D_s^{*-}}$	−0.58

completely different. In the 8_{2f} flavor representation, the magnetic moments of pentaquark states with spin configuration $J^P = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$ are all equal to $\mu_c = 0.38\mu_N$. We also notice that the smaller the number of charges, the smaller magnetic moments of octet hidden-charm molecular pentaquark states with the same strangeness number. For a more intuitive expression, we have shown the magnetic moments of the S-wave molecular pentaquark states with 8_{1f} flavor representation and 8_{2f} flavor representation in Figs. 3 and 4, respectively.

The 8_{1f} and 8_{2f} states can be coupled by strong interaction and electromagnetic interaction. Considering the coupled channel effect with two channels 8_{1f} and 8_{2f} states, the magnetic moments of pentaquark state can be derived by

$$\sum_{i,j}\mu_{1_i\to 1_j}\langle\phi_{1_j}|\phi_{1_i}\rangle + \sum_{i,j}\mu_{2_i\to 2_j}\langle\phi_{2_j}|\phi_{2_i}\rangle + \sum_{i,j}\mu_{2_i\to 1_j}\langle\phi_{1_j}|\phi_{2_i}\rangle + \sum_{i,j}\mu_{1_i\to 2_j}\langle\phi_{2_j}|\phi_{1_i}\rangle, \quad (8)$$

where 1 and 2 represent 8_{1f} and 8_{2f} pentaquark states, while ϕ_i represents the spatial wave function of the corresponding ith channel. The magnetic moments of the pentaquark states depend on the relevant mixing channel components $\langle\phi_{1_j}|\phi_{2_i}\rangle$ and $\langle\phi_{2_j}|\phi_{1_i}\rangle$ during the 8_{1f} and 8_{2f} mixing analysis. These components are

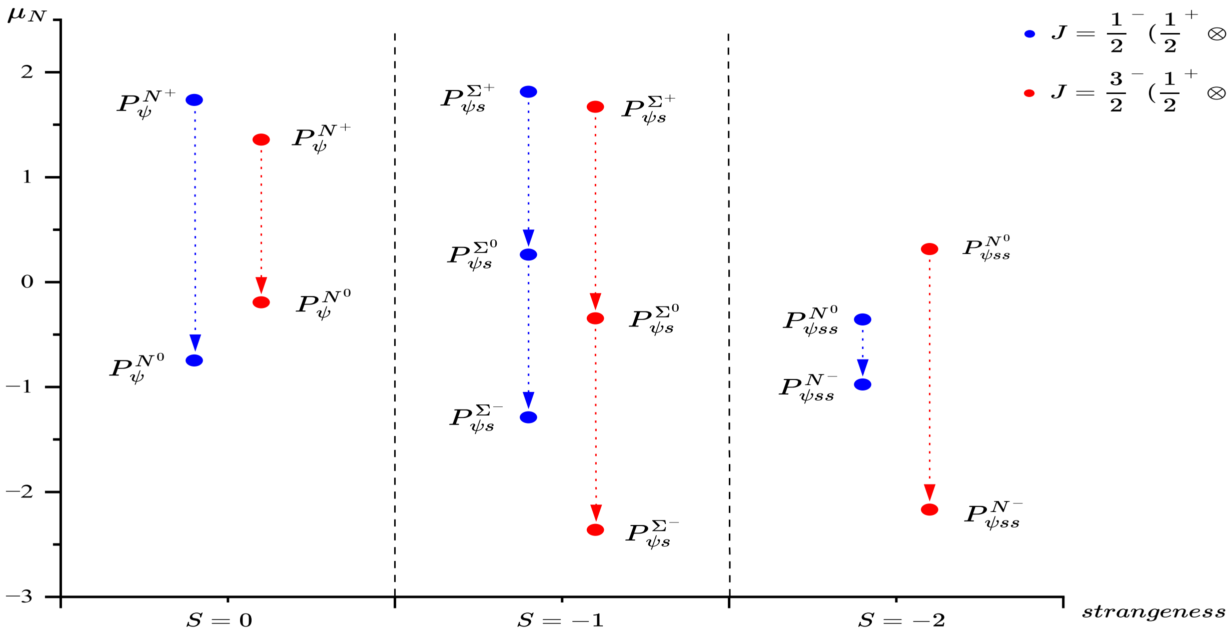

FIG. 3. The magnetic moments of the S-wave hidden-charm molecular pentaquark states with the 8_{1f} flavor representation. Here, the blue arrow points to the lower magnetic moment in the hidden-charm molecular pentaquark states with quantum number $J = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$, and the red arrow points to the lower magnetic moment in the hidden-charm molecular pentaquark states with quantum number $J = \frac{3}{2}^-(\frac{1}{2}^+ \otimes 1^-)$.

associated with the binding energies of 8_{1f} and 8_{2f} pentaquark states. Since these pentaquark states have yet to be observed experimentally, we are temporally unable to calculate the magnetic moment with coupled channel effects. We will study pentaquark states more comprehensively by considering coupling effects in the following work when enough experimental data is delivered.

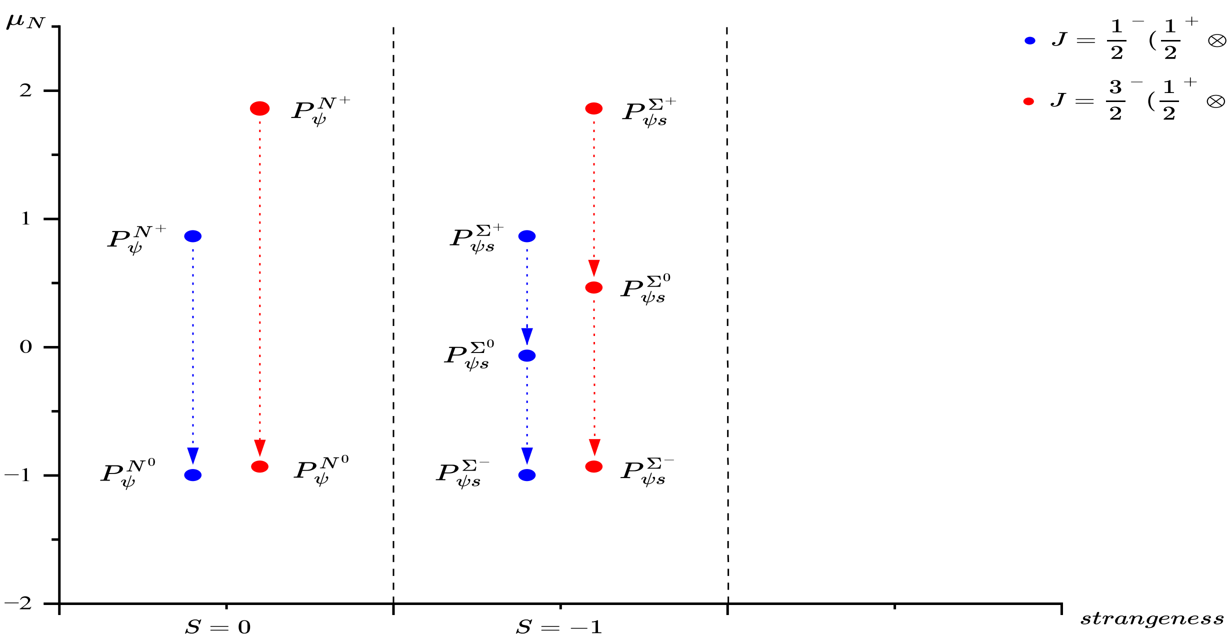

FIG. 4. The magnetic moments of the S-wave hidden-charm molecular pentaquark states with the 8_{2f} flavor representation. Here, the blue arrow points to the lower magnetic moment in the hidden-charm molecular pentaquark states with quantum number $J = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 1^-)$, and the red arrow points to the lower magnetic moment in the hidden-charm molecular pentaquark states with quantum number $J = \frac{3}{2}^-(\frac{1}{2}^+ \otimes 1^-)$.

In this work, we calculate the magnetic moments of pure 8_{1f} and 8_{2f} states and find the Coleman-Glashow sum rule for the magnetic moments of the same spin flavor configuration pentaquark states. Coleman-Glashow sum rule was initially applied to express the magnetic moments of all the octet baryons in terms of those of the neutron and proton in the limit of $SU(3)$ flavor symmetry [43–45]. The Coleman-Glashow sum rule for the magnetic moments of the octet baryons turns out to hold also for the magnetic moments of the same spin flavor configuration pentaquark states,

$$\mu_{P_{\psi}^{N+}} - \mu_{P_{\psi}^{N0}} + \mu_{P_{\psi s}^{\Sigma-}} - \mu_{P_{\psi s}^{\Sigma+}} + \mu_{P_{\psi ss}^{N0}} - \mu_{P_{\psi ss}^{N-}} = 0.$$

The rules that the magnetic moments satisfy provide us with new directions for exploring the inner structures of hidden-charm pentaquark states. Our prediction without the coupled channel effects will be valuable for exploring the inner structures of hidden-charm pentaquark states and serves as a reference for subsequent works.

IV. THE AXIAL CHARGE OF THE OCTET HIDDEN-CHARM MOLECULAR PENTAQUARK FAMILY

The axial charge plays a fundamental role in both the electroweak and strong interactions within the standard model. Moreover, it serves as an essential indicator of the spontaneous breaking of chiral symmetry in nonperturbative QCD. The exploration of axial charges has remained continuous endeavor [46–52]. Examining the axial charges of the newly discovered hidden-charm pentaquark states presents an effective approach to gain a deeper understanding of these phenomena.

At the quark level, the pion-quark interaction reads,

$$\mathcal{L}_Q = \frac{1}{2} g_q \bar{\psi}_q \gamma^{\mu} \gamma_5 \partial_{\mu} \phi \psi_q, \tag{9}$$

considering only the z-component, the above equation can be written as

$$\begin{aligned} \mathcal{L}_{\text{quark}} &= \frac{1}{2} g_q \bar{\psi}_q \sigma_z \partial_z \phi \psi_q \\ &= \frac{1}{2} \frac{g_q}{f_\pi} (\bar{u} \sigma_z \partial_z \pi_0 u - \bar{d} \sigma_z \partial_z \pi_0 d) \\ &\quad + \frac{\sqrt{2}}{2} \frac{g_q}{f_\pi} (\bar{u} \sigma_z \partial_z \pi^+ d + \bar{d} \sigma_z \partial_z \pi^- u), \end{aligned} \tag{10}$$

where g_q is the coupling constant at the quark level, σ_z is the Pauli matrix. The pion decay constant $f_\pi = 92$ MeV. ϕ is the pseudoscalar meson field,

$$\phi = \sum_{i=1}^{3} \tau_i \phi_i \equiv \begin{pmatrix} \pi_0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi_0 \end{pmatrix}. \tag{11}$$

The nucleon-pion Lagrangian at hadron level reads

$$\mathcal{L}_E^N = \frac{1}{2} g_A \bar{N} \gamma^{\mu} \gamma_5 \partial_{\mu} \phi N, \tag{12}$$

considering only the z-component, the above equation can be written as

$$\mathcal{L}_{\text{eff}}^N = \frac{g_A}{f_\pi} \bar{N} \frac{\Sigma_{Nz}}{2} \partial_z \pi_0 N, \tag{13}$$

where g_A is the axial charge of the nucleon,

$$\left\langle N, j_3 = \frac{1}{2}; \pi_0 | \mathcal{L}_{\text{eff}}^N | N, j_3 = \frac{1}{2} \right\rangle = \frac{1}{2} \frac{q_z}{f_\pi} g_A, \tag{14}$$

where q_z is the external momentum of π_0. At the quark level,

$$\left\langle N, j_3 = +\frac{1}{2}; \pi_0 | \mathcal{L}_{\text{quark}} | N, j_3 = +\frac{1}{2} \right\rangle = \frac{5}{6} \frac{q_z}{f_\pi} g_q. \tag{15}$$

From Eq. (14) and Eq. (15), we obtain $g_q = \frac{3}{5} g_A$ under the quark-hadron duality space.

The $SU(3)$ invariant Lagrangian of pentaquark state for $J^P = \frac{1}{2}^- (\frac{1}{2}^+ \otimes 0^-)$ in 8_{1f} flavor representation reads,

$$\mathcal{L}_1^{\frac{1}{2}} = \mathrm{Tr}(g_1 \bar{P} \gamma_\mu \gamma^5 \{\partial^\mu \Phi, P\} + f_1 \bar{P} \gamma_\mu \gamma^5 [\partial^\mu \Phi, P]), \tag{16}$$

where g_1 and f_1 are independent axial coupling constants of pentaquark state for $J^P = \frac{1}{2}^- (\frac{1}{2}^+ \otimes 0^-)$ in 8_{1f} flavor representation. Φ represents the pseudoscalar meson field in $SU(3)$ flavor symmetry,

$$\Phi = \sum_{a=1}^{8} \lambda_a \Phi_a \equiv \begin{pmatrix} \pi_0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi_0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}. \tag{17}$$

P represents the octet hidden-charm molecular pentaquark states,

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}} P_{\psi s}^{\Sigma^0} + \frac{1}{\sqrt{6}} P_{\psi s}^{\Lambda^0} & P_{\psi s}^{\Sigma^+} & P_{\psi}^{N^+} \\ P_{\psi s}^{\Sigma^-} & -\frac{1}{\sqrt{2}} P_{\psi s}^{\Sigma^0} + \frac{1}{\sqrt{6}} P_{\psi s}^{\Lambda^0} & P_{\psi}^{N^0} \\ P_{\psi ss}^{N^-} & P_{\psi ss}^{N^+} & \frac{2}{\sqrt{3}} P_{\psi s}^{\Lambda^0} \end{pmatrix}. \tag{18}$$

After bringing the matrix representation of Φ and P into Eq. (16) and keep π_0 meson term, we obtain

$$\begin{aligned}\mathcal{L}_1^{\frac{1}{2}}=&\frac{2}{\sqrt{3}}\frac{1}{f_\pi}g_1(\bar{P}_{\psi s}^{\Sigma^0}\Sigma_z\partial_z\pi^0P_{\psi s}^{\Lambda^0}+\bar{P}_{\psi s}^{\Lambda^0}\Sigma_z\partial_z\pi^0P_{\psi s}^{\Sigma^0})\\&-\frac{1}{f_\pi}(f_1+g_1)\bar{P}_\psi^{N^0}\Sigma_z\partial_z\pi^0P_\psi^{N0}\\&+\frac{1}{f_\pi}(g_1+f_1)\bar{P}_\psi^{N^+}\Sigma_z\partial_z\pi^0P_\psi^{N^+}\\&+\frac{1}{f_\pi}(g_1-f_1)\bar{P}_{\psi ss}^{N^-}\Sigma_z\partial_z\pi^0P_{\psi ss}^{N^-}\\&+\frac{1}{f_\pi}(f_1-g_1)\bar{P}_{\psi ss}^{N^0}\Sigma_z\partial_z\pi^0P_{\psi ss}^{N^0}+\frac{1}{f_\pi}2f_1\bar{P}_{\psi s}^{\Sigma^+}\Sigma_z\partial_z\pi^0P_{\psi s}^{\Sigma^+}\\&-\frac{1}{f_\pi}2f_1\bar{P}_{\psi s}^{\Sigma}{}^-\Sigma_z\partial_z\pi^0P_{\psi s}^{\Sigma^-}.\end{aligned}\tag{19}$$

We can obtain the axial coupling constant of the octet hidden-charm molecular pentaquark state with the coupling constants g_1 and f_1. While Eq. (19) solely presents the components associated with the decay of the π_0 meson, the axial coupling constants for decays involving other mesons can also be derived from the constants g_1 and f_1.

To determine the coupling constants g_1 and f_1, similar to the procedure employed for the nucleon, we consider the π_0 meson decay of $P_\psi^{N^+}$ and $P_{\psi ss}^{N^0}$ in Eq. (19). The Lagrangian for the π_0 decay of $P_\psi^{N^+}$ with the spin configuration $J^P=\frac{1}{2}^-(\frac{1}{2}^+\otimes 0^-)$ at the hadron level reads,

$$\mathcal{L}^{\frac{1}{2}}_{P_\psi^{N^+}}=\frac{g_1+f_1}{f_\pi}\bar{P}_\psi^{N^+}\frac{\Sigma_z}{2}\partial_z\pi_0P_\psi^{N^+}.\tag{20}$$

The Lagrangian for $P_{\psi ss}^{N^0}$ with the spin configuration $J^P=\frac{1}{2}^-(\frac{1}{2}^+\otimes 0^-)$ reads,

$$\mathcal{L}^{\frac{1}{2}}_{P_{\psi ss}^{N^0}}=\frac{f_1-g_1}{f_\pi}\bar{P}_{\psi ss}^{N^0}\frac{\Sigma_z}{2}\partial_z\pi_0P_{\psi ss}^{N},\tag{21}$$

where $\frac{\Sigma_z}{2}$ is the spin operator of the hidden-charm pentaquark states. At the hadron level, the axial charges read,

$$\left\langle P_\psi^{N^+};\pi_0\right|\frac{g_1+f_1}{f_\pi}\bar{P}_\psi^{N+}\frac{\Sigma_z}{2}\partial_z\pi_0P_\psi^{N+}\left|P_\psi^{N+}\right\rangle=\frac{g_1+f_1}{2}\frac{q_z}{f_\pi}.\tag{22}$$

$$\left\langle P_{\psi ss}^{N^0};\pi_0\right|\frac{f_1-g_1}{f_\pi}\bar{P}_{\psi ss}^{N^0}\frac{\Sigma_z}{2}\partial_z\pi_0P_{\psi ss}^{N^0}\left|P_{\psi ss}^{N^0}\right\rangle=\frac{f_1-g_1}{2}\frac{q_z}{f_\pi}.\tag{23}$$

At the quark level, the axial charge of $P_\psi^{N^+}$ and $P_{\psi ss}^{N^0}$ with the spin configuration $J^P=\frac{1}{2}^-(\frac{1}{2}^+\otimes 0^-)$ read,

$$\left\langle P_\psi^{N^+},+\frac{1}{2};\pi_0\right|\mathcal{L}_{\text{quark}}\left|P_\psi^{N^+},+\frac{1}{2}\right\rangle=\frac{4}{9}\frac{q_z}{f_\pi}g_q,\tag{24}$$

$$\left\langle P_{\psi ss}^{N^0},+\frac{1}{2};\pi_0\right|\mathcal{L}_{\text{quark}}\left|P_{\psi ss}^{N^0},+\frac{1}{2}\right\rangle=\frac{1}{9}\frac{q_z}{f_\pi}g_q.\tag{25}$$

Compare this with the axial charge of the nucleon,

$$\frac{\frac{1}{2}g_A}{\frac{5}{6}g_q}=\frac{\frac{g_1+f_1}{2}}{\frac{4}{9}g_q}=\frac{\frac{f_1-g_1}{2}}{\frac{1}{9}g_q}.\tag{26}$$

We obtain $f_1=\frac{1}{3}g_A$ and $g_1=\frac{1}{5}g_A$. Similarly, we can obtain the axial charge of the pentaquark states with other flavor-spin configurations.

The Lagrangian of pentaquark state for $J^P=\frac{1}{2}^-(\frac{1}{2}^+\otimes 1^-)$ in 8_{1f} flavor representation reads,

$$\mathcal{L}_2^{\frac{1}{2}}=\mathrm{Tr}(g_2\bar{P}\gamma_\mu\gamma^5\{\partial^\mu\Phi,P\}+f_2\bar{P}\gamma_\mu\gamma^5[\partial^\mu\Phi,P]).\tag{27}$$

The Lagrangian of pentaquark state for $J^P=\frac{3}{2}^-(\frac{1}{2}^+\otimes 1^-)$ in 8_{1f} flavor representation reads,

$$\mathcal{L}_3^{\frac{3}{2}}=\mathrm{Tr}(g_3\bar{P}^\nu\gamma_\mu\gamma^5\{\partial^\mu\Phi,P_\nu\}+f_3\bar{P}^\nu\gamma_\mu\gamma^5[\partial^\mu\Phi,P_\nu]).\tag{28}$$

The Lagrangian of pentaquark state for $J^P=\frac{1}{2}^-(\frac{1}{2}^+\otimes 0^-)$ in 8_{2f} flavor representation reads,

$$\mathcal{L}_4^{\frac{1}{2}}=\mathrm{Tr}(g_4\bar{P}\gamma_\mu\gamma^5\{\partial^\mu\Phi,P\}+f_4\bar{P}\gamma_\mu\gamma^5[\partial^\mu\Phi,P]).\tag{29}$$

The Lagrangian of pentaquark state for $J^P=\frac{1}{2}^-(\frac{1}{2}^+\otimes 1^-)$ in 8_{2f} flavor representation reads,

$$\mathcal{L}_5^{\frac{1}{2}}=\mathrm{Tr}(g_5\bar{P}\gamma_\mu\gamma^5\{\partial^\mu\Phi,P\}+f_5\bar{P}\gamma_\mu\gamma^5[\partial^\mu\Phi,P]).\tag{30}$$

The Lagrangian of pentaquark state for $J^P=\frac{3}{2}^-(\frac{1}{2}^+\otimes 1^-)$ in 8_{2f} flavor representation reads,

$$\mathcal{L}_6^{\frac{3}{2}}=\mathrm{Tr}(g_6\bar{P}^\nu\gamma_\mu\gamma^5\{\partial^\mu\Phi,P_\nu\}+f_6\bar{P}^\nu\gamma_\mu\gamma^5[\partial^\mu\Phi,P_\nu]).\tag{31}$$

The numerical results for the coupling constants f_i and g_i of pentaquark states with different flavor-spin configurations are presented in Table VII. The axial charge of the octet hidden-charm molecular family in 8_{1f} and 8_{2f} flavor

TABLE VII. Coupling constants f_i and g_i of the pentaquark states with different flavor-spin configurations.

Constants	Values	Constants	Values
f_1	$\frac{1}{3}g_A$	g_1	$\frac{1}{5}g_A$
f_2	$-\frac{2}{45}g_A$	g_2	$-\frac{12}{45}g_A$
f_3	$\frac{13}{90}g_A$	g_3	$-\frac{1}{30}g_A$
f_4	0	g_4	0
f_5	$\frac{1}{5}g_A$	g_5	$\frac{1}{5}g_A$
f_6	$\frac{1}{10}g_A$	g_6	$\frac{1}{10}g_A$

TABLE VIII. The axial charges of the octet hidden-charm pentaquark family in 8_{1f} flavor representation. The $J_b^{P_b} \otimes J_m^{P_m}$ are corresponding to the angular momentum and parity of baryon and meson, respectively.

Couplings	Coefficients	Wave functions	$J_b^{P_b} \otimes J_m^{P_m}$	$I(J^P)$	Results
$P_\psi^{N^+} P_\psi^{N^+} \pi_0$	f_1+g_1	$P_\psi^{N^+}:\ -\sqrt{\frac{1}{3}}\Sigma_c^+\bar{D}^{(*)0}+\sqrt{\frac{2}{3}}\Sigma_c^{++}D^{(*)-}$	$\frac{1}{2}^+\times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{8}{15}g_A$
	f_2+g_2		$\frac{1}{2}^+\times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{14}{45}g_A$
	f_3+g_3			$\frac{1}{2}(\frac{3}{2})^-$	$\frac{1}{9}g_A$
$P_\psi^{N^0} P_\psi^{N^0} \pi_0$	$-g_1-f_1$	$P_\psi^{N^0}:\ \sqrt{\frac{1}{3}}\Sigma_c^+D^{(*)-}-\sqrt{\frac{2}{3}}\Sigma_c^0\bar{D}^{(*)0}$	$\frac{1}{2}^+\times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{8}{15}g_A$
	$-g_2-f_2$		$\frac{1}{2}^+\times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{14}{45}g_A$
	$-g_3-f_3$			$\frac{1}{2}(\frac{3}{2})^-$	$-\frac{1}{9}g_A$
$P_{\psi s}^{\Sigma^+} P_{\psi s}^{\Sigma^+} \pi_0$	$2f_1$	$P_{\psi s}^{\Sigma^+}:\ \sqrt{\frac{1}{3}}\Xi_c'^+\bar{D}^{(*)0}-\sqrt{\frac{2}{3}}\Sigma_c^{++}D_s^{(*)-}$	$\frac{1}{2}^+\times 0^-$	$1(\frac{1}{2})^-$	$\frac{2}{3}g_A$
	$2f_2$		$\frac{1}{2}^+\times 1^-$	$1(\frac{1}{2})^-$	$-\frac{4}{45}g_A$
	$2f_3$			$1(\frac{3}{2})^-$	$\frac{13}{45}g_A$
$P_{\psi s}^{\Sigma^0} P_{\psi s}^{\Lambda^0} \pi_0$	$\frac{2}{\sqrt{3}}g_1$	$P_{\psi s}^{\Sigma^0}:\ \sqrt{\frac{1}{6}}\Xi_c'^+D^{(*)-}+\sqrt{\frac{1}{6}}\Xi_c'^0\bar{D}^{(*)0}-\sqrt{\frac{2}{3}}\Sigma_c^+D_s^{(*)-}$	$\frac{1}{2}^+\times 0^-$	$1(\frac{1}{2})^-$	$\frac{2\sqrt{3}}{15}g_A$
	$\frac{2}{\sqrt{3}}g_2$		$\frac{1}{2}^+\times 1^-$	$1(\frac{1}{2})^-$	$-\frac{8\sqrt{3}}{45}g_A$
	$\frac{2}{\sqrt{3}}g_3$			$1(\frac{3}{2})^-$	$-\frac{\sqrt{3}}{45}g_A$
	$\frac{2}{\sqrt{3}}g_1$	$P_{\psi s}^{\Lambda^0}:\ -\sqrt{\frac{1}{2}}\Xi_c'^+D^{(*)-}+\sqrt{\frac{1}{2}}\Xi_c'^0\bar{D}^{(*)0}$	$\frac{1}{2}^+\times 0^-$	$0(\frac{1}{2})^-$	$\frac{2\sqrt{3}}{15}g_A$
	$\frac{2}{\sqrt{3}}g_2$		$\frac{1}{2}^+\times 1^-$	$0(\frac{1}{2})^-$	$-\frac{8\sqrt{3}}{45}g_A$
	$\frac{2}{\sqrt{3}}g_3$			$0(\frac{3}{2})^-$	$-\frac{\sqrt{3}}{45}g_A$
$P_{\psi s}^{\Sigma^-} P_{\psi s}^{\Sigma^-} \pi_0$	$-2f_1$	$P_{\psi s}^{\Sigma^-}:\ \sqrt{\frac{1}{3}}\Xi_c'^0D^{(*)-}-\sqrt{\frac{2}{3}}\Sigma_c^0D_s^{(*)-}$	$\frac{1}{2}^+\times 0^-$	$1(\frac{1}{2})^-$	$-\frac{2}{3}g_A$
	$-2f_2$		$\frac{1}{2}^+\times 1^-$	$1(\frac{1}{2})^-$	$\frac{4}{45}g_A$
	$-2f_3$			$1(\frac{3}{2})^-$	$-\frac{13}{45}g_A$
$P_{\psi ss}^{N^0} P_{\psi ss}^{N^0} \pi_0$	f_1-g_1	$P_{\psi ss}^{N^0}:\ \sqrt{\frac{1}{3}}\Xi_c'^+D_s^{(*)-}-\sqrt{\frac{2}{3}}\Omega_c^0\bar{D}^{(*)0}$	$\frac{1}{2}^+\times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{2}{15}g_A$
	f_2-g_2		$\frac{1}{2}^+\times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{2}{9}g_A$
	f_3-g_3			$\frac{1}{2}(\frac{3}{2})^-$	$\frac{8}{45}g_A$
$P_{\psi ss}^{N^-} P_{\psi ss}^{N^-} \pi_0$	g_1-f_1	$P_{\psi ss}^{N^-}:\ \sqrt{\frac{1}{3}}\Xi_c'^0D_s^{(*)-}-\sqrt{\frac{2}{3}}\Omega_c^0D^{(*)-}$	$\frac{1}{2}^+\times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{2}{15}g_A$
	g_2-f_2		$\frac{1}{2}^+\times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{2}{9}g_A$
	g_3-f_3			$\frac{1}{2}(\frac{3}{2})^-$	$-\frac{8}{45}g_A$

TABLE IX. The axial charges of the octet hidden-charm pentaquark family in 8_{2f} flavor representation. The $J_b^{P_b} \otimes J_m^{P_m}$ are corresponding to the angular momentum and parity of baryon and meson, respectively.

Couplings	Constants	Wave functions	$J_b^{P_b} \otimes J_m^{P_m}$	$I(J^P)$	Results
$P_\psi^{N^+} P_\psi^{N^+} \pi_0$	f_4+g_4	$P_\psi^{N^+}:\ \Lambda_c^+\bar{D}^{(*)0}$	$\frac{1}{2}^+\times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	0
	f_5+g_5		$\frac{1}{2}^+\times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$\frac{2}{5}g_A$
	f_6+g_6			$\frac{1}{2}(\frac{3}{2})^-$	$\frac{1}{5}g_A$
$P_\psi^{N^0} P_\psi^{N^0} \pi_0$	$-g_4-f_4$	$P_\psi^{N^0}:\ \Lambda_c^+D^{(*)-}$	$\frac{1}{2}^+\times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	0
	$-g_5-f_5$		$\frac{1}{2}^+\times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	$-\frac{2}{5}g_A$
	$-g_6-f_6$			$\frac{1}{2}(\frac{3}{2})^-$	$-\frac{1}{5}g_A$
$P_{\psi s}^{\Sigma^+} P_{\psi s}^{\Sigma^+} \pi_0$	$2f_4$	$P_{\psi s}^{\Sigma^+}:\ \Xi_c^+\bar{D}^{(*)0}$	$\frac{1}{2}^+\times 0^-$	$1(\frac{1}{2})^-$	0
	$2f_5$		$\frac{1}{2}^+\times 1^-$	$1(\frac{1}{2})^-$	$\frac{2}{5}g_A$
	$2f_6$			$1(\frac{3}{2})^-$	$\frac{1}{5}g_A$

(Table continued)

TABLE IX. *(Continued)*

Couplings	Constants	Wave functions	$J_b^{P_b} \otimes J_m^{P_m}$	$I(J^P)$	Results
$P_{\psi s}^{\Lambda^0} P_{\psi s}^{\Sigma^0} \pi_0$	$\frac{2}{\sqrt{3}} g_4$	$P_{\psi s}^{\Sigma^0}$: $\sqrt{\frac{1}{2}} \Xi_c^+ D^{(*)-} + \sqrt{\frac{1}{2}} \Xi_c^0 \bar{D}^{(*)0}$	$\frac{1}{2}^+ \times 0^-$	$1(\frac{1}{2})^-$	0
	$\frac{2}{\sqrt{3}} g_5$		$\frac{1}{2}^+ \times 1^-$	$1(\frac{1}{2})^-$	$\frac{2\sqrt{3}}{15} g_A$
	$\frac{2}{\sqrt{3}} g_6$			$1(\frac{3}{2})^-$	$\frac{\sqrt{3}}{15} g_A$
	$\frac{2}{\sqrt{3}} g_4$	$P_{\psi s}^{\Lambda^0}$: $\sqrt{\frac{1}{6}} \Xi_c^+ D^{(*)-} - \sqrt{\frac{1}{6}} \Xi_c^0 \bar{D}^{(*)0} - \sqrt{\frac{2}{3}} \Lambda_c^+ D_s^{(*)-}$	$\frac{1}{2}^+ \times 0^-$	$0(\frac{1}{2})^-$	0
	$\frac{2}{\sqrt{3}} g_5$		$\frac{1}{2}^+ \times 1^-$	$0(\frac{1}{2})^-$	$\frac{2\sqrt{3}}{15} g_A$
	$\frac{2}{\sqrt{3}} g_6$			$0(\frac{3}{2})^-$	$\frac{\sqrt{3}}{15} g_A$
$P_{\psi s}^{\Sigma^-} P_{\psi s}^{\Sigma^-} \pi_0$	$-2f_4$	$P_{\psi s}^{\Sigma^-}$: $\Xi_c^0 D^{(*)-}$	$\frac{1}{2}^+ \times 0^-$	$1(\frac{1}{2})^-$	0
	$-2f_5$		$\frac{1}{2}^+ \times 1^-$	$1(\frac{1}{2})^-$	$-\frac{2}{5} g_A$
	$-2f_6$			$1(\frac{3}{2})^-$	$-\frac{1}{5} g_A$
$P_{\psi ss}^{N^0} P_{\psi ss}^{N^0} \pi_0$	$f_4 - g_4$	$P_{\psi ss}^{N^0}$: $\Xi_c^+ D_s^{(*)-}$	$\frac{1}{2}^+ \times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	0
	$f_5 - g_5$		$\frac{1}{2}^+ \times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	0
	$f_6 - g_6$			$\frac{1}{2}(\frac{3}{2})^-$	0
$P_{\psi ss}^{N^-} P_{\psi ss}^{N^-} \pi_0$	$g_4 - f_4$	$P_{\psi ss}^{N^-}$: $\Xi_c^0 D_s^{(*)-}$	$\frac{1}{2}^+ \times 0^-$	$\frac{1}{2}(\frac{1}{2})^-$	0
	$g_5 - f_5$		$\frac{1}{2}^+ \times 1^-$	$\frac{1}{2}(\frac{1}{2})^-$	0
	$g_6 - f_6$			$\frac{1}{2}(\frac{3}{2})^-$	0

representations are listed in Table VIII and Table IX, respectively.

As presented in Table VIII and Table IX, the axial charges of the octet hidden-charm pentaquark family are generally lower compared to that of the nucleon. In Table IX, the axial charges of the pentaquark states with the spin configuration $J^P = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$ in 8_{2f} flavor representation are all zero, because both f_4 and g_4 are all zero in Table VII. Due to $f_5 = g_5$ and $f_6 = g_6$ in Table VII, the axial charges of both $P_{\psi ss}^{N^0}$ and $P_{\psi ss}^{N^-}$ in 8_{2f} flavor representation are all zero as shown in Table VII.

The axial charges at the hadron level exhibits a high sensitivity to the flavor-spin configurations of the pentaquark states. This sensitivity results in numerical discrepancies within the quark-hadron dual space, which proves advantageous for distinguishing the strong decay processes of the pentaquark states with different structures in the future. Moreover, the axial charges of the pentaquark states in Table VIII and Table IX will be very beneficial for the theorical calculation of the pentaquark states in chiral perturbation theory (details in our another work in Ref. [53]).

V. SUMMARY

With the discovery of more and more exotic hadron states comprising multiple quarks in experiments, significant progress has been made in the theoretical study of these exotic hadrons. Magnetic moments and axial charges of these exotic hadrons are intrinsic properties that offer valuable insights into their quark constituents.

In this work, we investigate the magnetic moment of the octet hidden-charm molecular pentaquark family with quark model. Our numerical results show that the magnetic moments of the S-wave molecular pentaquark states in the 8_{1f} flavor representation and 8_{2f} flavor representation are completely different. In the 8_{2f} flavor representation, the magnetic moments of pentaquark states with spin configuration $J^P = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$ are all equal to $\mu_c = 0.38\mu_N$. We notice an interesting phenomenon that the smaller the number of charges, the smaller magnetic moments of octet hidden-charm molecular pentaquark states with the same strangeness number. The Coleman-Glashow sum rule turns out to hold for the magnetic moments of pentaquark family. The improvement of the Coleman-Glashow sum rule requires the inclusion of a realistic $SU(3)$-breaking mechanism, we improve the Coleman-Glashow sum rule for the magnetic moments of the pentaquark states with chiral perturbation theory in Ref. [53].

We also calculate the axial charge of the octet hidden-charm molecular pentaquark family. Numerical results show that the axial charges of the octet hidden-charm pentaquark family are generally lower compared to that of the nucleon. The axial charges of the pentaquark states with the spin configuration $J^P = \frac{1}{2}^-(\frac{1}{2}^+ \otimes 0^-)$ in 8_{2f} flavor representation are all zero. Our calculation of the pentaquark axial charges will greatly contribute to the theoretical calculation of the pentaquark states in chiral perturbation theory.

Magnetic moments and axial charges are important physical quantities. As the lifetime of pentaquark state is very short, experimenters can measure the magnetic moment of pentaquark state by using the Thomas precession of particles with magnetic moment in a uniform magnetic field. The measurement of the axial coupling of the pentaquark state is relatively simple, we can extract the axial coupling from the decay width of the strong decay of the pentaquark state. We hope our present study would enhance our understanding of the mechanism of hadronic molecules.

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