Study of the $B^- \to K^- \eta\eta_c$ decay due to the $D\bar{D}$ bound state

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We study the $B^- \to K^- \eta\eta_c$ decay by taking into account the S-wave contributions from the pseudoscalar meson-pseudoscalar meson interactions within the unitary coupled-channel approach, where the $D\bar{D}$ bound state is dynamically generated. In addition, the contribution from the intermediate resonance $K_0^*(1430)^-$, with $K_0^*(1430)^- \to K^-\eta$, is also considered. Our results show that there is a clear peak around 3720 MeV in the $\eta\eta_c$ invariant mass distribution, which could be associated with the $D\bar{D}$ bound state. The future precise measurements of the $B^- \to K^- \eta\eta_c$ process at the Belle II and LHCb experiments could be, therefore, used to check the existence of the $D\bar{D}$ bound state, and to deepen our understanding of the hadron-hadron interactions.

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I. INTRODUCTION

Since the discovery of $X(3872)$ by the Belle Collaboration in 2003 [1], many exotic states, which do not fit into the expectations of conventional quark models, have been observed experimentally during the past two decades [2]. Many of these exotic states, especially the ones observed in the charmonium sector, are observed around the threshold of a pair of heavy hadrons; some of them, such as $X(3872)$ [3], $Z_c(3900)$ [4], and $X(4160)$ [5], can be explained as the hadronic molecules. However, the hadronic molecular states with mass near the $D\bar{D}$ threshold have not yet been observed experimentally, and further detailed studies are therefore required both theoretically and experimentally [6].

In Ref. [7], by taking into account the $\pi\pi$, $K\bar{K}$, $D\bar{D}$, $D_s\bar{D}_s$, $\eta\eta$, and $\eta\eta_c$ coupled channels, the authors predicted a narrow hidden charm resonance with quantum numbers $I(J^{PC}) = 0(0^{++})$ and mass around 3700 MeV, which will be denoted as $X(3700)$ throughout this paper, within the unitary coupled-channel approach. Furthermore, by considering the η_c as a pure $c\bar{c}$ state and the η–η' mixing, together with the same parameters as used in Ref. [7], the pole of the new $X(3700)$ state was predicted to be $\sqrt{s} = (3722 - i18)$ MeV within the unitary coupled-channel approach [8]. The mass of the $D\bar{D}$ bound state predicted by other different models is also basically around the $D\bar{D}$ threshold [9–16], and the theoretical studies of the experimentally measured processes $e^+e^- \to J/\psi D\bar{D}$ [17–19], $B^+ \to D^0\bar{D}^0K^+$ [20] and $\gamma\gamma \to D\bar{D}$ [21–24] all support the existence of such a $D\bar{D}$ bound state. Meanwhile, some processes like $\psi(3770) \to \gamma X(3700) \to \gamma\eta\eta'$, $\psi(4040) \to \gamma X(3700) \to \gamma\eta\eta'$, $e^+e^- \to J/\psi X(3700) \to J/\psi\eta\eta'$ [25], $\psi(3770) \to \gamma D\bar{D}$ [26], $\Lambda_b \to \Lambda D\bar{D}$ [27], and $B^+ \to K^+\eta\eta$ [28] have also been suggested to search for the $D\bar{D}$ bound state. It is worth mentioning that the BESIII Collaboration has recently searched for the $X(3700)$ in the $\psi(3770) \to \gamma\eta\eta'$ decay for the first time, observing however no significant signals due to the low detection efficiencies of the photons [29].

Although the $D\bar{D}$ bound state $X(3700)$ couples mainly to the $D\bar{D}$ and $D_s\bar{D}_s$ channels, it is not easy to search for any signals of the state in these systems. This is due to the fact that, since its mass is a little bit lower than the $D\bar{D}$ threshold, the $X(3700)$ state would manifest itself as a near-threshold enhancement in the $D\bar{D}$ invariant mass distribution, which may be difficult to identify due to the low detection efficiencies near the threshold [27,30]. On the other hand, the $X(3700)$ state has also a sizeable coupling to the $\eta\eta_c$ channel, as observed in Refs. [7,8]. Since the $\eta\eta_c$

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threshold is about 200 MeV lower than the predicted mass of $X(3700)$, one expects that, if the $D\bar{D}$ bound state exists, a clear peak near the $D\bar{D}$ threshold would appear in the $\eta\eta_c$ invariant mass distribution of some processes with large phase space.

As is well known, the three-body weak decays of the B mesons involve much more complicated dynamics than do the two-body decays and can, therefore, provide a wealth of information about the meson-meson interactions and the hadron resonances [31–35] (see, e.g., Ref. [36] for a recent review). For instance, the $B \to K + X/Y/Z$ decay is an ideal process to produce the charmoniumlike hadronic molecular states [11,37–40], and many exotic states have been observed experimentally through the B-meson weak decays during the past few years, such as $Z_{cs}(4000)$, $Z_{cs}(4220)$ [41], and $X(4140)$ [42,43] in $B^+ \to J/\psi\phi K^+$, as well as $X_0(2900)$ and $X_1(2900)$ in $B^+ \to D^+D^-K^+$ decay [44,45]. In this paper, we propose to search for the $D\bar{D}$ bound state $X(3700)$ in the $B^- \to K^-\eta\eta_c$ decay. It is worth mentioning that the Belle Collaboration has already searched for the process in 2015 based on 772×10^6 $B\bar{B}$ pairs collected at the $\Upsilon(4S)$ resonance [46], but no significant signal of the $D\bar{D}$ bound state was observed due to insufficient statistics. However, the Belle II Collaboration will accumulate about 50 times the Belle dataset [47,48] and is expected to make further precise measurements of the $B^- \to K^-\eta\eta_c$ decay, which will shed more light on the existence of the $D\bar{D}$ bound state in this process. In addition, the authors of Ref. [49] have suggested to search for the $D\bar{D}$ bound state in the $\eta\eta_c$ mass distribution of the $B^+ \to K^+\eta\eta_c$ decay and predicted the branching ratio of $\mathcal{B}(B^+ \to X_{q\bar{q}}(\to\eta_c\eta)K^+) = (0.9 - 6.7) \times 10^{-4}$.

In this paper, motivated by the observations made above, we will study the $B^- \to K^-\eta\eta_c$ decay by taking into account the pseudoscalar meson-pseudoscalar meson interactions within the chiral unitary approach, from where the $D\bar{D}$ bound state is generated dynamically. On the other hand, the $B^- \to K^-\eta\eta_c$ decay can also proceed through the subsequent decay of the intermediate resonance $K_0^*(1430)$, i.e., $K_0^*(1430) \to K\eta$, whose contribution will be considered in this paper too. We will demonstrate that, besides a peak of $K_0^*(1430)$ in the $K^-\eta$ invariant mass distribution, there is a clear peak around 3720 MeV in the $\eta\eta_c$ invariant mass distribution, which could be associated with the $D\bar{D}$ bound state. Therefore, future precise measurements of the $B^- \to K^-\eta\eta_c$ decay at the Belle II and LHCb experiments could be used to check the existence of the $D\bar{D}$ bound state, and to deepen our understanding of the hadron-hadron interactions.

This paper is organized as follows. In Sec. II, we will firstly introduce our formalism for the $B^- \to K^-\eta\eta_c$ decay. Our numerical results and discussions are then presented in Sec. III. In Sec. IV, we give our final conclusion.

II. FORMALISM

In analogy to the discussions made in Refs. [27,50–52], the $B^- \to K^-\eta\eta_c$ decay proceeds via the following three steps: the weak decay, the hadronization, and the final-state interactions. Explicitly, the b quark of the B^- meson firstly decays into a c quark and a virtual W^- boson, and then the W^- boson turns into a $\bar{c}s$ pair. In order to give rise to the $K^-\eta\eta_c$ final state, the $\bar{u}$ antiquark of the initial B^- meson and the $\bar{c}s$ pair from the W^- subsequent decay have to hadronize together with the $\bar{q}q$ ($\equiv \bar{u}u + \bar{d}d + \bar{s}s$) created from the vacuum with the quantum numbers $J^{PC} = 0^{++}$. The relevant quark-level diagrams can be classified as the internal and external W^- emission mechanisms, as depicted in Figs. 1(a)–1(d), respectively. Here, we have neglected all the Cabbibo-Kobayashi-Maskawa (CKM) suppressed diagrams that are proportional to the CKM element V_{ub}.

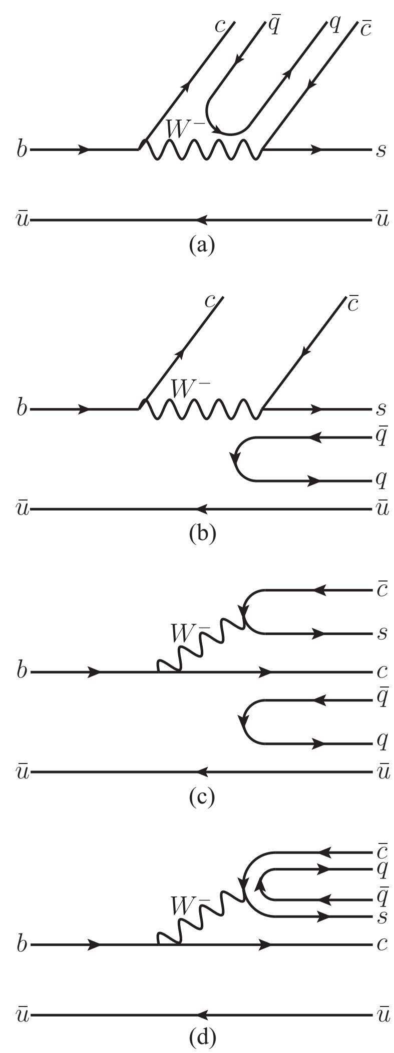

FIG. 1. The dominant quark-level diagrams for the $B^- \to K^-\eta\eta_c$ decay, where (a)–(d) refer to the internal and external W^- emission mechanisms, respectively.

The meson-meson systems formed by the hadronization of q_i, $\bar{q}_j$ and $\bar{q}_k q_k$ are given by

$$\sum_{k=1}^{3} q_i(\bar{q}_k q_k)\bar{q}_j = \sum_{k=1}^{3} M_{ik}M_{kj} = (M^2)_{ij}, \tag{1}$$

with the $q\bar{q}$ matrix defined as

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} & u\bar{c} \\ d\bar{u} & d\bar{d} & d\bar{s} & d\bar{c} \\ s\bar{u} & s\bar{d} & s\bar{s} & s\bar{c} \\ c\bar{u} & c\bar{d} & c\bar{s} & c\bar{c} \end{pmatrix}, \tag{2}$$

which could be expressed in terms of the physical pseudoscalar mesons as [33],

$$M = \begin{pmatrix} \frac{\eta}{\sqrt{3}}+\frac{\pi^0}{\sqrt{2}}+\frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & \frac{\eta}{\sqrt{3}}-\frac{\pi^0}{\sqrt{2}}+\frac{\eta'}{\sqrt{6}} & K^0 & D^- \\ K^- & \bar{K}^0 & \sqrt{\frac{2}{3}}\eta'-\frac{\eta}{\sqrt{3}} & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}. \tag{3}$$

Thus, by isolating the meson K^-, one could easily obtain the components of the meson systems for Figs. 1(a) and 1(b) as follows:

$$\begin{aligned} |H\rangle^{\rm a} &= V_p V_{cb} V_{cs}^* c(\bar{u}u + \bar{d}d + \bar{s}s)\bar{c}s\bar{u} \\ &= V_p V_{cb} V_{cs}^* (M^2)_{44} K^- \\ &= V_p V_{cb} V_{cs}^* \times (D^0\bar{D}^0 + D^+D^- + D_s^+D_s^-)K^-, \end{aligned} \tag{4}$$

$$\begin{aligned} |H\rangle^{\rm b} &= V_p V_{cb} V_{cs}^* c\bar{c}s(\bar{u}u + \bar{d}d + \bar{s}s)\bar{u} \\ &= V_p V_{cb} V_{cs}^* (M^2)_{31}\eta_c \\ &= V_p V_{cb} V_{cs}^* \times \left(\frac{1}{\sqrt{2}}K^-\pi^0 + \frac{3}{\sqrt{6}}K^-\eta'\right)\eta_c, \end{aligned} \tag{5}$$

where $V_{cb} = 0.04182$ and $V_{cs}^* = 0.97349$ are the CKM matrix elements, and V_p encodes all the remaining factors arising from the production vertex. Then, the final-state interactions of $D\bar{D}$, $D_s\bar{D}_s$, and $\eta'\eta_c$ will dynamically generate the $D\bar{D}$ bound state, which could decay into the $\eta\eta_c$ system. Here we do not consider the component $K^-\pi^0\eta_c$, since the isospin of the $\pi^0\eta_c$ system is $I = 1$.

Similarly, we can write the hadron components for Figs. 1(c) and 1(d) that could couple to the $K^-\eta\eta_c$ system as follows:

$$|H\rangle^{\rm c} = V_p V_{cb} V_{cs}^* \times C \times (K^- D_s^+)D_s^-, \tag{6}$$

$$|H\rangle^{\rm d} = V_p V_{cb} V_{cs}^* \times C \times (K^- \bar{D}^0)D^0, \tag{7}$$

where we have introduced the color factor C to account for the relative weight of the external W^- emission mechanism with respect to the internal W^- emission mechanism and will take $C = 3$ in the case of color number $N_C = 3$, as done in Refs. [53–55].

According to the above discussions, the $K^-\eta\eta_c$ final state could not be produced directly through the tree-level diagrams of the B^- decay but can via the final-state interactions of the coupled channels $D^0\bar{D}^0$, D^+D^-, $D_s^+D_s^-$, and $\eta'\eta_c$, which could then generate the $D\bar{D}$ bound state, as shown in Fig. 2. The total amplitude of Fig. 2 can be expressed as

$$\begin{aligned} \mathcal{T}_X = V_p V_{cb} V_{cs}^* \Big[& G_{D^+D^-} t_{D^+D^- \to \eta\eta_c} \\ & + (1 + C) \times G_{D^0\bar{D}^0} t_{D^0\bar{D}^0 \to \eta\eta_c} \\ & + (1 + C) \times G_{D_s^+D_s^-} t_{D_s^+D_s^- \to \eta\eta_c} \\ & + \frac{3}{\sqrt{6}} \times G_{\eta'\eta_c} t_{\eta'\eta_c \to \eta\eta_c} \Big], \end{aligned} \tag{8}$$

where G_l is the loop function for the two-meson propagation in the lth channel, and its explicit expression is given by [7]

$$\begin{aligned} G_l &= i\int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_1^2 + i\epsilon} \frac{1}{(P-q)^2 - m_2^2 + i\epsilon} \\ &= \frac{1}{16\pi^2}\left[\alpha_l + \ln\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}\ln\frac{m_2^2}{m_1^2} \right. \\ &\quad + \frac{p}{\sqrt{s}} \times \left(\ln\frac{s - m_2^2 + m_1^2 + 2p\sqrt{s}}{-s + m_2^2 - m_1^2 + 2p\sqrt{s}} \right. \\ &\quad \left.\left. + \ln\frac{s + m_2^2 - m_1^2 + 2p\sqrt{s}}{-s - m_2^2 + m_1^2 + 2p\sqrt{s}}\right)\right], \end{aligned} \tag{9}$$

with the subtraction constant $\alpha_l = -1.3$ for the coupled channels D^+D^-, $D^0\bar{D}^0$, $D_s^+D_s^-$, and $\eta'\eta_c$, and $\mu = 1500$ MeV, being the same as used in Ref. [8]. $\sqrt{s} = M_{\eta\eta_c}$ is the invariant mass of the two mesons in the lth channel, and m_1 and m_2 are the masses of these two mesons. P is the total four-momentum of the two mesons in the lth channel, and p is

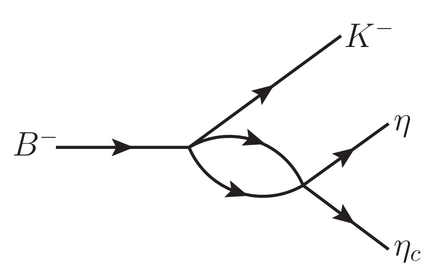

FIG. 2. The final-state interactions of the coupled channels $D^0\bar{D}^0$, D^+D^-, $D_s^+D_s^-$, and $\eta'\eta_c$.

the magnitude of the three-momentum of each meson in the meson-meson center of mass frame, with

$$p = \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{2\sqrt{s}}, \tag{10}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ is the Källen function. The transition amplitudes in Eq. (8) are obtained by solving the Bethe-Salpeter equation in coupled channels [7,8],

$$t = [1 - VG]^{-1}V, \tag{11}$$

where the matrix V is the potential constructed at the tree level for each one of the possible channels. Here, we take into account the channels of $\pi^+\pi^-$, $\pi^0\pi^0$, K^+K^-, $K^0\bar{K}^0$, $\eta\eta$, $\eta\eta_c$, D^+D^-, $D^0\bar{D}^0$, $D_s^+D_s^-$, $\eta\eta'$, $\eta'\eta'$, as well as $\eta'\eta_c$.

On the other hand, the $B^- \to K^-\eta\eta_c$ decay could also proceed via the intermediate excited kaon mesons. According to the Dalitz plot shown in Fig. 3, one can see that only the well-established resonance $K_0^*(1430)$ could contribute to this process, since the $K_0^*(1430)$ couples to the channel $K^-\eta$ in S-wave with a branching fraction $\mathcal{B}(K_0^*(1430) \to K\eta) = (8.6^{+2.7}_{-3.4})\%$ [2]. Therefore, in this paper, we neglect all the other excited kaon mesons, and only take into account the contribution from the intermediate $K_0^*(1430)$ resonance as shown by Fig. 4, whose amplitude can be expressed as

$$\mathcal{T}_{K_0^*} = \frac{V_p \times \beta \times e^{i\varphi} \times M^2_{K_0^*(1430)}}{M^2_{K^-\eta} - M^2_{K_0^*(1430)} + iM_{K_0^*(1430)}\Gamma_{K_0^*(1430)}}, \tag{12}$$

where the parameter β accounts for the relative weight of the $K_0^*(1430)$ contribution with respect to that of the $D\bar{D}$ bound state $X(3700)$, and the phase factor $e^{i\varphi}$ is introduced

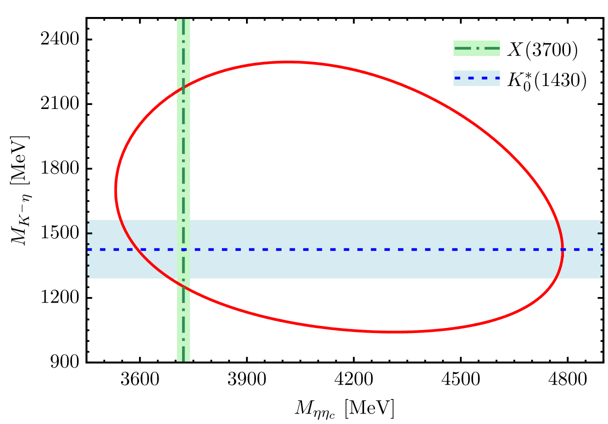

FIG. 3. The Dalitz plot for the $B^- \to K^-\eta\eta_c$ decay. The green dash-dotted line and band stand for the mass and width of $X(3700)$, while the blue dashed line and band for the mass and width of the well-established resonance $K_0^*(1430)$.

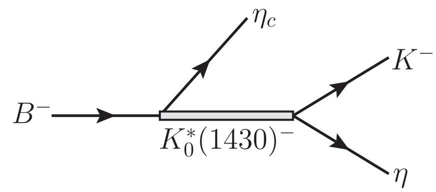

FIG. 4. The diagram for the $B^- \to K^-\eta\eta_c$ decay via the intermediate $K_0^*(1430)$ resonance.

to describe the interference between the amplitudes from the $D\bar{D}$ bound state and the $K_0^*(1430)$ resonance. $M_{K^-\eta}$ is the invariant mass of the $K^-\eta$ system. We will take as input $M_{K_0^*(1430)} = 1425$ MeV and $\Gamma_{K_0^*(1430)} = 270$ MeV [2].

With the amplitudes given by Eqs. (8) and (12) at hand, the doubly differential decay width of the $B^- \to K^-\eta\eta_c$ process can be written as

$$\frac{d^2\Gamma}{dM_{\eta\eta_c}dM_{K^-\eta}} = \frac{1}{(2\pi)^3}\frac{M_{\eta\eta_c}M_{K^-\eta}}{8M^3_{B^-}}|\mathcal{T}_X + \mathcal{T}_{K_0^*}|^2, \tag{13}$$

$$\frac{d^2\Gamma}{dM_{\eta\eta_c}dM_{K^-\eta_c}} = \frac{1}{(2\pi)^3}\frac{M_{\eta\eta_c}M_{K^-\eta_c}}{8M^3_{B^-}}|\mathcal{T}_X + \mathcal{T}_{K_0^*}|^2. \tag{14}$$

One could obtain the invariant mass distributions $d\Gamma/dM_{\eta\eta_c}$, $d\Gamma/dM_{K^-\eta}$, and $d\Gamma/dM_{K^-\eta_c}$ by integrating Eqs. (13) and (14) over each of the invariant mass variables. For instance, the differential decay width $d\Gamma/dM_{\eta\eta_c}$ can then be obtained by integrating Eq. (13) over the $K^-\eta$ invariant mass $M_{K^-\eta}$, with the final result given by

$$\frac{d\Gamma}{dM_{\eta\eta_c}} = \int dM_{K^-\eta}\frac{1}{(2\pi)^3}\frac{M_{\eta\eta_c}M_{K^-\eta}}{8M^3_{B^-}}|\mathcal{T}_X + \mathcal{T}_{K_0^*}|^2. \tag{15}$$

Here, the integration range is given by

$$(M^2_{K^-\eta})_{\min} = (E^*_{K^-} + E^*_\eta)^2 - \left(\sqrt{E^{*2}_\eta - m^2_\eta} + \sqrt{E^{*2}_{K^-} - m^2_{K^-}}\right)^2, \tag{16}$$

$$(M^2_{K^-\eta})_{\max} = (E^*_{K^-} + E^*_\eta)^2 - \left(\sqrt{E^{*2}_\eta - m^2_\eta} - \sqrt{E^{*2}_{K^-} - m^2_{K^-}}\right)^2, \tag{17}$$

where $E^*_{K^-}$ and E^*_η are the energies of K^- and η in the $\eta\eta_c$ rest frame, respectively. Explicitly, we have

$$E^*_{K^-} = \frac{M^2_{B^-} - M^2_{\eta\eta_c} - M^2_{K^-}}{2M_{\eta\eta_c}}, \tag{18}$$

$$E^*_\eta = \frac{M^2_{\eta\eta_c} - M^2_{\eta_c} + M^2_\eta}{2M_{\eta\eta_c}}. \tag{19}$$

Here, all the meson masses involved are taken from Ref. [2].

III. RESULTS AND DISCUSSION

In our model, we have three free parameters, V_p, β, and φ. The parameter V_p is a global factor and its value does not affect the shapes of the $\eta\eta_c$, $K^-\eta$, and $K^-\eta_c$ invariant mass distributions, and thus, we take $V_p = 1$ for simplicity. The parameter β represents the relative weight of the $K_0^*(1430)$ contribution with respect to that of $X(3700)$, and the parameter φ is the relative phase between these two amplitudes.

As indicated by the current data on the branching fractions of B-meson decays [2],

$$\mathcal{B}(B^0 \to K_0^*(1430)^0\eta_c) = (1.8 \pm 0.4) \times 10^{-4},$$
$$\mathcal{B}(B^0 \to K^0 D^+ D^-) = (7.5 \pm 1.7) \times 10^{-4},$$
$$\mathcal{B}(B^0 \to K^0 D^0 \bar{D}^0) = (2.7 \pm 1.1) \times 10^{-4},$$
$$\mathcal{B}(B^+ \to K^+ D^+ D^-) = (2.2 \pm 0.7) \times 10^{-4},$$
$$\mathcal{B}(B^+ \to K^+ D^0 \bar{D}^0) = (1.45 \pm 0.33) \times 10^{-3},$$

the branching fractions of the processes $B^0 \to K_0^*(1430)^0\eta_c$ and $B^0 \to K^0 D\bar{D}$ are of the same order of magnitude. Thus, the contributions from the $D\bar{D}$ bound state and the $K_0^*(1430)$ resonance are expected to be of similar magnitudes. By integrating the differential decay width over the corresponding invariant mass, one can estimate the partial decay widths $\Gamma(B^- \to K_0^*(1430)^-\eta_c \to K^-\eta\eta_c)$ and $\Gamma(B^- \to K^- X(3700) \to K^-\eta\eta_c)$. It is found numerically that, with $\beta = 0.012$, the values of $\Gamma(B^- \to K_0^*(1430)^-\eta_c \to K^-\eta\eta_c)$ and $\Gamma(B^- \to K^- X(3700) \to K^-\eta\eta_c)$ are of the same order of magnitude. Therefore, in this work, we take the parameter $\beta = 0.012$ and also discuss our results with different values of β later.

Firstly, we show in Fig. 5 the $\eta\eta_c$, $K^-\eta$, and $K^-\eta_c$ invariant mass distributions with $\beta = 0.012$ and $\varphi = 0$. One can see a clear peak around 3720 MeV in the $\eta\eta_c$ invariant mass distribution, which should be associated with the $D\bar{D}$ bound state $X(3700)$. At the same time, a cusp structure appears around 3930 MeV in the same invariant mass distribution, which is due to the strong coupling of the $D\bar{D}$ bound state to the $D_s\bar{D}_s$ channel. In addition, a $K_0^*(1430)$ signal appears in the $K^-\eta$ invariant mass distribution but gives rise to a smooth shape in the $\eta\eta_c$ invariant mass distribution and thus, does not affect the

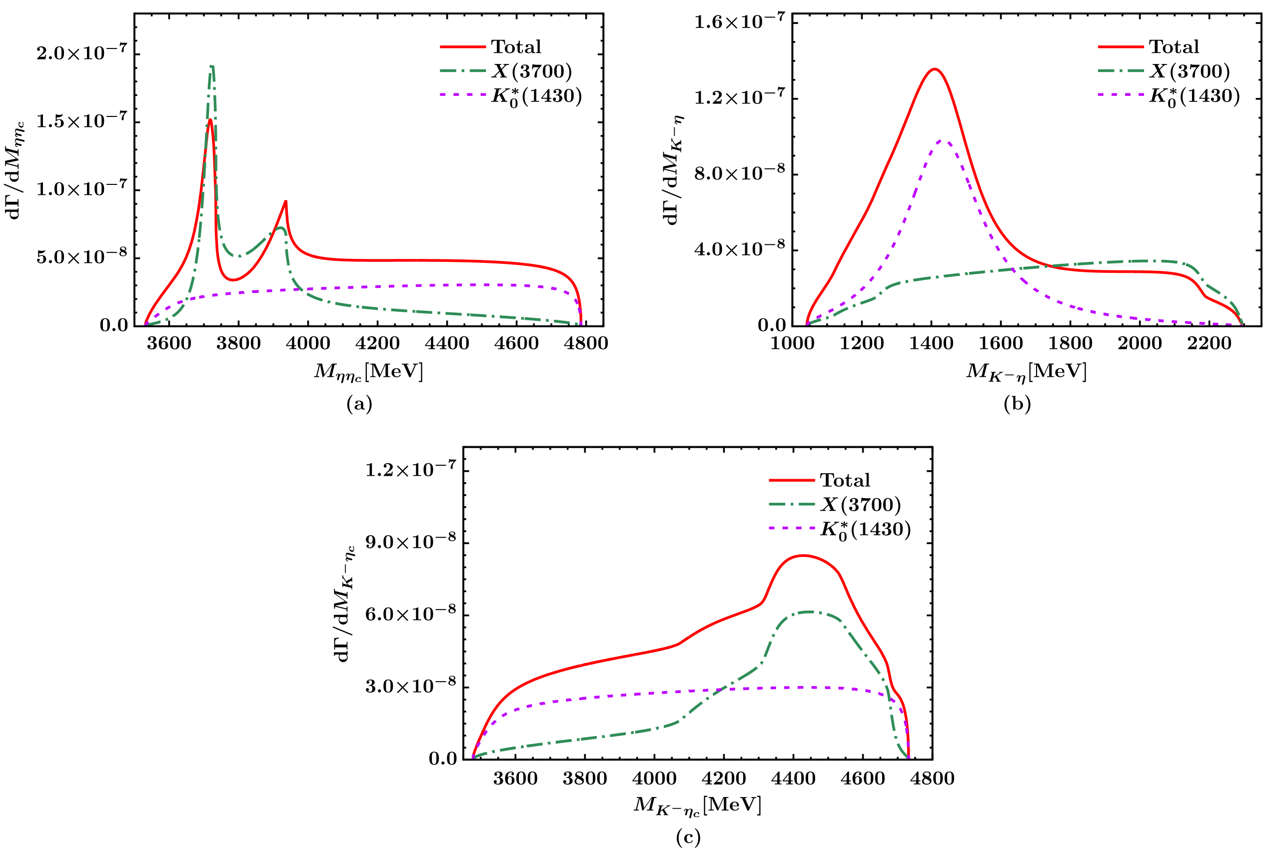

FIG. 5. The $\eta\eta_c$ (a), $K^-\eta$ (b) and $K^-\eta_c$ (c) invariant mass distributions of the $B^- \to K^-\eta\eta_c$ decay with $\beta = 0.012$, $\varphi = 0$, and $C = 3.0$. The green dash-dotted, the magenta dashed, and the red solid curves represent the contributions from $X(3700)$, $K_0^*(1430)$, and the total contributions, respectively.

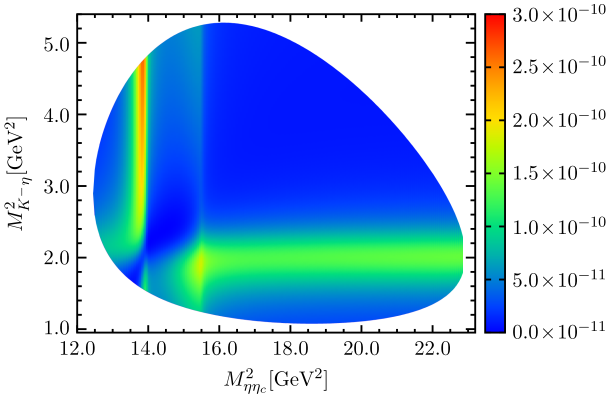

FIG. 6. The doubly differential decay width $\mathrm{d}^2\Gamma/(\mathrm{d}M_{\eta\eta_c}\mathrm{d}M_{K^-\eta})$ of the $B^- \to K^-\eta\eta_c$ decay in the $(M^2_{\eta\eta_c}, M^2_{K^-\eta})$ plane, where the $X(3700)$ and $K_0^*(1430)$ resonances can be clearly seen.

peak structure of the $X(3700)$ significantly. It should be stressed that the line shape of the $X(3700)$ in the $\eta\eta_c$ invariant mass distribution is different from that of a Breit-Wigner form, which is a typical feature of the $D\bar{D}$ molecular state. On the other hand, one bump structure appears around 4400 MeV in the $K^-\eta_c$ invariant mass distribution, which is due to the $D\bar{D}$ interaction and hence, should not be associated with any resonance.

It is worth mentioning that one narrow state $\chi_{c0}(3930)$, with mass around 3930 MeV and the quantum numbers $J^{PC} = 0^{++}$, was observed in the process $B^+ \to D^+D^-K^+$ process by the LHCb Collaboration [45]. In addition, the LHCb Collaboration has discovered a peaking structure near the $D_s^+D_s^-$ threshold, denoted as $X(3960)$ with $M = 3956 \pm 5 \pm 10$ MeV, $\Gamma = 43 \pm 13 \pm 8$ MeV, and $J^{PC} = 0^{++}$, in the process $B^+ \to D_s^+D_s^-K^+$ [56]. Some studies suggest that the near-threshold structure, $X(3960)$, may come from the $D_s^+D_s^-$ bound state below the $D_s^+D_s^-$ threshold, which can be associated with the $\chi_{c0}(3930)$ state [13,57–64]. Taking into account the fact that the $\chi_{c0}(3930)$ and $X(3960)$ states favor the same quantum number $J^{PC} = 0^{++}$, and both of them can decay into the $\eta\eta_c$ final state, the cusp structure around 3930 MeV in the $\eta\eta_c$ invariant mass distribution could be associated with the resonance $X(3930)$. Therefore, the future precise measurements of this process could be used to search for the $\chi_{c0}(3930)$ and $X(3960)$ states.

We also show in Fig. 6 the doubly differential decay width $\mathrm{d}^2\Gamma/(\mathrm{d}M_{\eta\eta_c}\mathrm{d}M_{K^-\eta})$ for the $B^- \to K^-\eta\eta_c$ decay in the $(M^2_{\eta\eta_c}, M^2_{K^-\eta})$ plane, where one can see two clear bands corresponding to the $X(3700)$ and $K_0^*(1430)$ resonances, respectively.

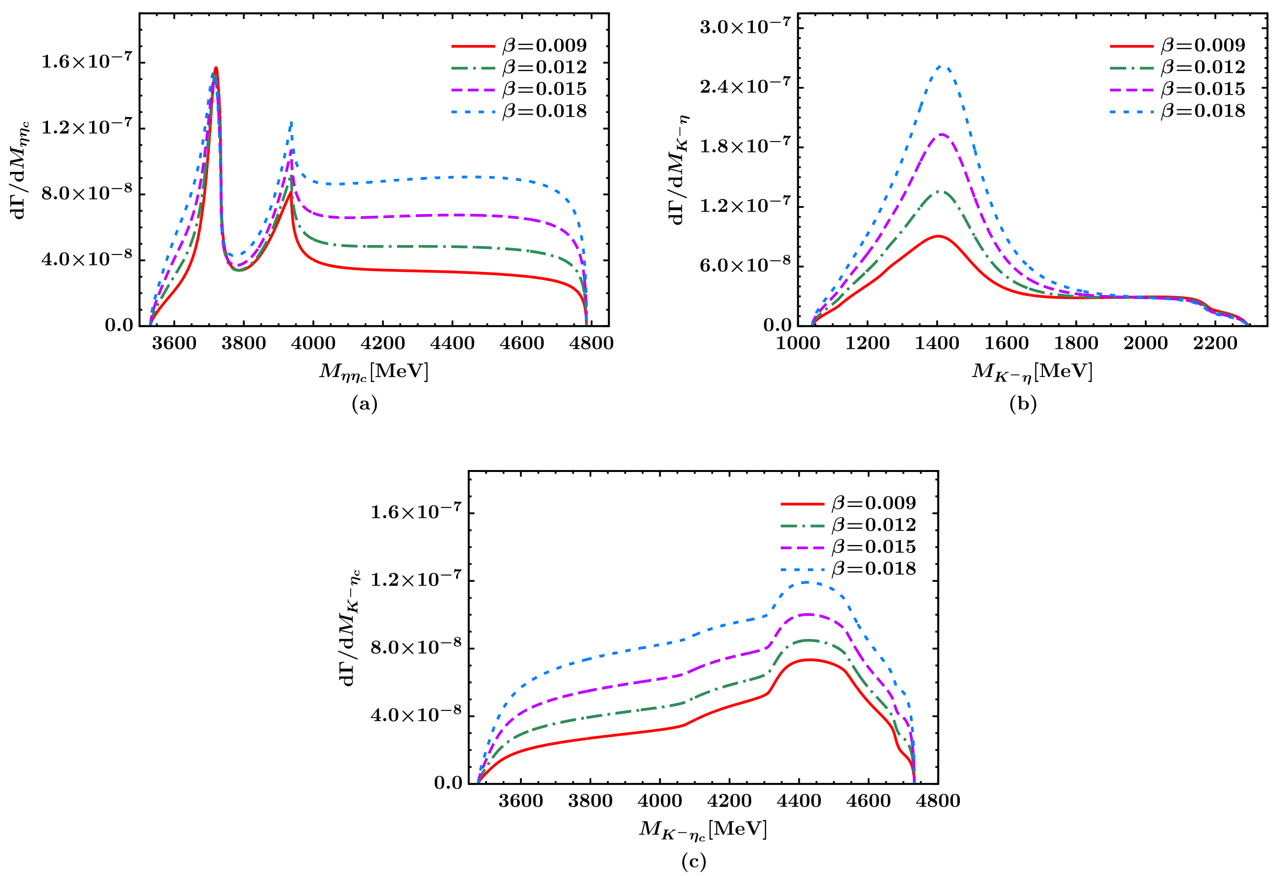

FIG. 7. The $\eta\eta_c$ (a), $K^-\eta$ (b) and $K^-\eta_c$ (c) invariant mass distributions of the $B^- \to K^-\eta\eta_c$ decay with $\varphi = 0$, $C = 3.0$, as well as four different values of $\beta = 0.009$ (red solid), 0.012 (green dash-dotted), 0.015 (magenta long-dashed), and 0.018 (blue dashed).

The parameters β and φ are unknown in our model, and their values could be determined if the precise experimental measurements of the $B^- \to K^-\eta\eta_c$ decay are available in the future. In order to study the dependence of our results on β and φ, we have calculated the $\eta\eta_c$, $K^-\eta$, and $K^-\eta_c$ invariant mass distributions with different values of β and φ, which are shown in Figs. 7 and 8, respectively. From Fig. 7, one can see that the peak of the $K_0^*(1430)$ resonance in the $K^-\eta$ invariant mass distribution becomes more significant when the value of β increases. From Fig. 8, on the other hand, the peak of $K_0^*(1430)$ moves a little bit for different values of φ. However, the peak of the $D\bar{D}$ bound state $X(3700)$ is always clear in the $\eta\eta_c$ invariant mass distribution.

Finally, we should note that the value of the color factor C, which represents the relative weight of the external W^- emission mechanism with respect to the internal W^- emission mechanism, could vary around 3 in order to account for the potential nonfactorizable contributions [65]. To this end, we show in Fig. 9 the $\eta\eta_c$, $K^-\eta$, and $K^-\eta_c$ invariant mass distributions of the $B^- \to K^-\eta\eta_c$ decay by taking three different values of $C = 3.0$, 2.5, 2.0. One can see that, although the peak of the $X(3700)$ state in the $\eta\eta_c$ invariant mass distribution becomes weaker when the value of C decreases, its signal is still clear and can be easily distinguished from the background contribution. Meanwhile, the peak of the $K_0^*(1430)$ resonance in the $K^-\eta$ invariant mass distribution has little changes for these three different values of C, because the contribution from the $D\bar{D}$ bound state is smooth around the peak of $K_0^*(1430)$ in the $K^-\eta$ invariant mass distribution, as observed already in Fig. 5.

From the above analyses, one can conclude that, within the variation ranges of the three free parameters, there is always a clear peak around 3720 MeV in the $\eta\eta_c$ invariant mass distribution, which corresponds to the $D\bar{D}$ bound state. Thus, we strongly suggest our experimental colleagues to perform more precise measurements of the $B^- \to K^-\eta\eta_c$ decay at the Belle II and LHCb experiments in the future, which is very important for confirming the existence of the predicted $D\bar{D}$ bound state.

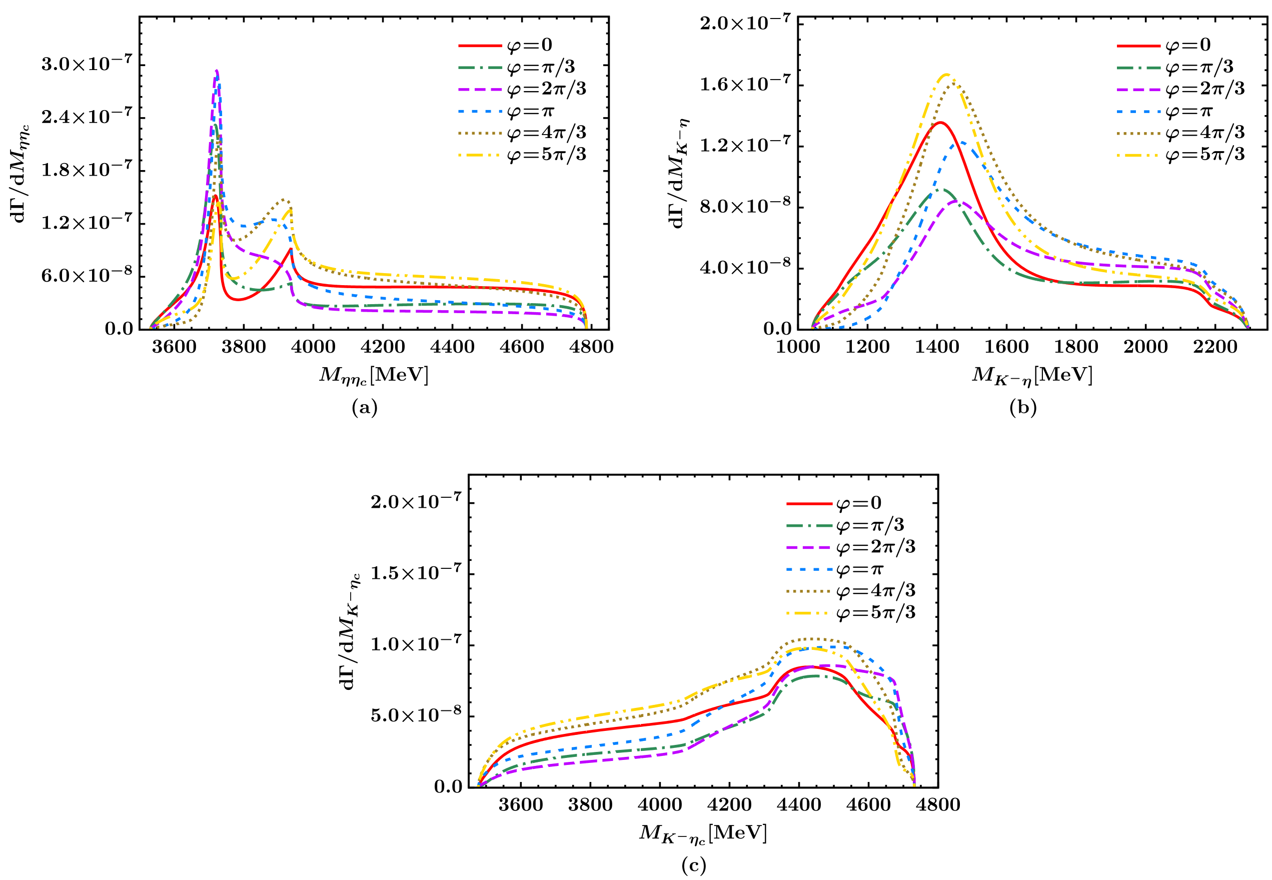

FIG. 8. The $\eta\eta_c$ (a), $K^-\eta$ (b) and $K^-\eta_c$ (c) invariant mass distributions of the $B^- \to K^-\eta\eta_c$ decay with $\beta = 0.012$, $C = 3.0$, as well as six different values of $\varphi = 0$ (red solid), $\pi/3$ (green dash-dotted), $2\pi/3$ (magenta long-dashed), π (blue dashed), $4\pi/3$ (olive dotted), and $5\pi/3$ (yellow dash-dot-dotted).

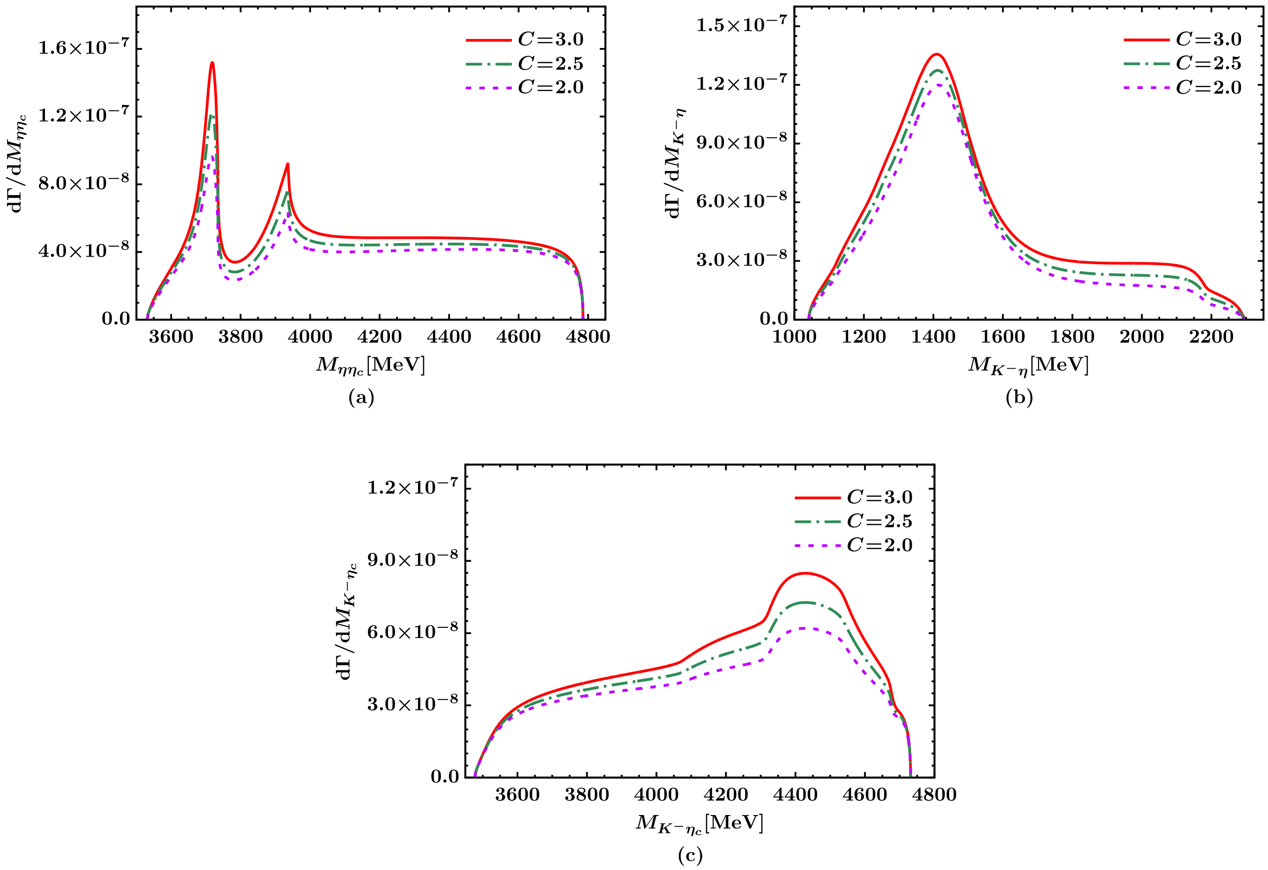

FIG. 9. The $\eta\eta_c$ (a), $K^-\eta$ (b) and $K^-\eta_c$ (c) invariant mass distributions of the $B^- \to K^-\eta\eta_c$ decay with $\beta = 0.012$, $\varphi = 0$, as well as three different values of $C = 3.0$ (red solid), 2.5 (green dash-dotted), and 2.0 (magenta dashed).

IV. CONCLUSIONS

In this paper, motivated by the theoretical predictions for the $D\bar{D}$ bound state $X(3700)$, we propose to search for this state in the $B^- \to K^-\eta\eta_c$ decay. To this end, we have investigated the process within the unitary coupled-channel approach, by taking into account the contributions from the S-wave pseudoscalar meson-pseudoscalar meson interactions, which can dynamically generate the $D\bar{D}$ bound state $X(3700)$. We have also taken into account the contribution from the intermediate resonance $K_0^*(1430)$, since it couples to the $K\eta$ channel in S-wave with a branching fraction of $\mathcal{B}(K_0^*(1430) \to K\eta) = (8.6^{+2.7}_{-3.4})\%$.

Our results show that a clear peak appears around 3720 MeV in the $\eta\eta_c$ invariant mass distribution, which should be associated with the $D\bar{D}$ bound state. It should be stressed that the line shape of the $D\bar{D}$ bound state is significantly different from that of a Breit-Winger form, which is a typical feature of the $D\bar{D}$ molecular state. On the other hand, one can also find the peak of the resonance $K_0^*(1430)$ in the $K^-\eta$ invariant mass distribution, and the resonance gives a smooth contribution in the $\eta\eta_c$ invariant mass distribution.

In summary, we strongly encourage our experimental colleagues to perform a more precise measurement of the $B^- \to K^-\eta\eta_c$ decay at the Belle II and LHCb experiments in the future, which will be very helpful to confirm the existence of the predicted $D\bar{D}$ bound state, as well as to deepen our understanding of the hadron-hadron interactions.

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