

Nonleptonic three-body charmed baryon weak decays with $H(\mathbf{15})$

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We study the nonleptonic three-body charmed baryon weak decays of $\mathbf{B}_c \rightarrow \mathbf{B}_n PP'$ under the $SU(3)_F$ flavor symmetry, where \mathbf{B}_c denotes the antitriplet charmed baryon, comprising $(\Xi_c^0, -\Xi_c^+, \Lambda_c^+)$, and \mathbf{B}_n and $P(P')$ represent octet baryon and pseudoscalar meson states, respectively. In addition to 12 parameters from the contributions of the color-antisymmetric part of the effective Hamiltonian, denoted as $H(\bar{\mathbf{6}})$, there are 4 parameters from the color-symmetric one, $H(\mathbf{15})$, which were not included in the previous study. With 16 parameters in total and 28 experimental data points, we obtain the minimal χ^2 over degree of freedom of $\chi^2/\text{d.o.f} = 1.5$, which is a great improvement comparing to that without $H(\mathbf{15})$. With the better fitting values, we evaluate the branching ratios and up-down asymmetries of $\mathbf{B}_c \rightarrow \mathbf{B}_n PP'$, which present some interesting results such as $\mathcal{B}(\Lambda_c^+ \rightarrow (\Xi(1690)^0 \rightarrow \Sigma^+ K^-) K^+) \equiv (1.5 \pm 0.4) \times 10^{-3}$ and potential $SU(3)$ breaking effects in $\Xi_c^+ \rightarrow p\pi^+ K^-$ and $\Lambda_c^+ \rightarrow \Sigma^+ \pi^- K^+$ to be verified by the experiments at BESIII, Belle-II, and LHCb.

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I. INTRODUCTION

The nonleptonic three-body charmed baryon weak decays of $\mathbf{B}_c \rightarrow \mathbf{B}_n PP'$ have been searched continuously by Belle-II [1–5], BESIII [6–12], and LHCb [13,14] Collaborations with increasing high precision, where $\mathbf{B}_c \equiv (\Xi_c^0, -\Xi_c^+, \Lambda_c^+)$ represents the antitriplet charmed baryon, while \mathbf{B}_n and $P(P')$ denote octet baryon and pseudoscalar meson states, respectively. A systematic study of these three-body charmed baryon decays is crucial due to their rich spin structures, providing insights into P , CP , or T -violating spin correlations, and aiding in understanding the complete dynamics of baryonic decay processes.

In contrast to the wealth of observational data, calculating charm quark three-body weak decays into light quarks has proven challenging. This is attributed to the large mass of the charm quark, making the $SU(4)_F$ flavor symmetry ineffective, and the failure of the heavy quark expansion

due to the insufficiently large m_c . The increasing complexity of these decays further causes the ineffectiveness of factorization methods [15]. To overcome these challenges, alternative approaches for charmed hadron decays have been explored in various studies [16–22]. These approaches recognize the necessity of considering nonfactorizable effects. On the other hand, the $SU(3)_F$ flavor symmetry method has been tested as a useful tool both in the beauty and charmed hadron decays. Its feasibility has been established in charmed baryon two-body and three-body semileptonic charmed baryon weak decays [23–39].

To investigate the nonresonant weak decays of $\mathbf{B}_c \rightarrow \mathbf{B}_n PP'$, we make the assumption that the final state configurations of the pseudoscalar meson-pairs are predominantly characterized by S-wave ones. We express the decay amplitudes in terms of parity-conserving and violating components under $SU(3)_F$, following a similar framework as outlined in Ref. [38]. Building upon a thorough discussion [34,40] of the contribution from the color-antisymmetric part of the effective Hamiltonian associated with the irreducible representation $\bar{\mathbf{6}}$ under $SU(3)_f$ and incorporating additional experimental data, we extend our analysis to consider complete effective Hamiltonian contributions related to both $\bar{\mathbf{6}}$ and $\mathbf{15}$ representations, resulting in 16 real parameters to be fitted with 28 available

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experiment data points. Furthermore, we discuss the possible error sources and some interesting findings of the new fit.

Our paper is organized as follows. We interpret the formalism and give the explicit amplitudes of all decay channels of $\mathbf{B}_c \rightarrow \mathbf{B}_n PP'$ under the $SU(3)_F$ flavor symmetry in Sec. II. In Sec. III, we present our numerical fitting results and discussions. Our conclusion is given in Sec. IV.

II. FORMALISM

The nonleptonic three-body charmed baryon weak decays of $\mathbf{B}_c \rightarrow \mathbf{B}_n PP'$ can be proceeded through the charmed quark decays of $c \rightarrow su\bar{d}$, $c \rightarrow ud\bar{d}(us\bar{s})$ and $c \rightarrow du\bar{s}$. Accordingly, the effective Hamiltonian at tree level is given by [41]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i=-,+} [V_{cs}V_{ud}^*c_i O_i^{ds} + V_{cd}V_{ud}^*c_i (O_i^{dd} - O_i^{ss}) + V_{cd}V_{us}^*c_i O_i^{sd}] \quad (1)$$

with the four-quark operators written as:

$$O_{\pm}^{q_1 q_2} = \frac{1}{2} [(\bar{u}q_1)_{V-A}(\bar{q}_2c)_{V-A} \pm (\bar{q}_2q_1)_{V-A}(\bar{u}c)_{V-A}] \quad (2)$$

where G_F is Fermi constant and c_i represent the Wilson coefficients. The four quark operators O_{\pm}^{ds} , $O_{\pm}^{dd} - O_{\pm}^{ss}$, O_{\pm}^{sd} are classified into so-called Cabibbo-favored(CF), singly Cabibbo-suppressed(CS), and doubly Cabibbo-suppressed (DCS) processes, respectively.

The three modes of charmed quark decays can be written as $c \rightarrow q^i q^j \bar{q}_k$ with $q_i = (u, d, s)$ is the triplet of light quarks under the $SU(3)_F$ flavor symmetry. The form of $q^i q^j \bar{q}_k$ can be decomposed as the irreducible representations of $\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{15} \oplus \bar{\mathbf{6}} \oplus \mathbf{3} \oplus \bar{\mathbf{3}}$, in which $\mathbf{15}$ and $\bar{\mathbf{6}}$ correspond to the color-symmetric operator $O_{+}^{q_1 q_2}$ and the color-antisymmetric operator $O_{-}^{q_1 q_2}$ [23,24], respectively. Consequently, the effective Hamiltonian can be divided into the symmetric part $H(\mathbf{15})$ and antisymmetric part $H(\bar{\mathbf{6}})$, defined as

$$\begin{aligned} A(\mathbf{B}_c \rightarrow \mathbf{B}_n PP') &= a_1(\bar{\mathbf{B}}_n)_i^k(\bar{P})_l^m(\bar{P})_m^l H(\bar{\mathbf{6}})_{jk} T^{ij} + a_2(\bar{\mathbf{B}}_n)_i^k(\bar{P})_j^m(\bar{P})_m^l H(\bar{\mathbf{6}})_{kl} T^{ij} + a_3(\bar{\mathbf{B}}_n)_i^k(\bar{P})_k^m(\bar{P})_m^l H(\bar{\mathbf{6}})_{jl} T^{ij} \\ &+ a_4(\bar{\mathbf{B}}_n)_i^k(\bar{P})_j^l(\bar{P})_k^m H(\bar{\mathbf{6}})_{lm} T^{ij} + a_5(\bar{\mathbf{B}}_n)_l^k(\bar{P})_j^m(\bar{P})_m^k H(\bar{\mathbf{6}})_{il} T^{ij} + a_6(\bar{\mathbf{B}}_n)_l^k(\bar{P})_j^m(\bar{P})_l^k H(\bar{\mathbf{6}})_{im} T^{ij} \\ &+ a_7(\bar{\mathbf{B}}_n)_{ijl}(\bar{P})_m^n(\bar{P})_n^k H(\mathbf{15})_{lk}^{lm} T^{ij} + a_8(\bar{\mathbf{B}}_n)_{ijn}(\bar{P})_m^k(\bar{P})_l^n H(\mathbf{15})_{lk}^{lm} T^{ij} \\ &+ a_9(\bar{\mathbf{B}}_n)_{iln}(\bar{P})_j^k(\bar{P})_n^m H(\mathbf{15})_{km}^{lm} T^{ij} + a_{10}(\bar{\mathbf{B}}_n)_{iln}(\bar{P})_m^k(\bar{P})_j^n H(\mathbf{15})_{km}^{lm} T^{ij} \end{aligned}$$

$$B(\mathbf{B}_c \rightarrow \mathbf{B}_n PP') = A(\mathbf{B}_c \rightarrow \mathbf{B}_n PP') \{a_i \rightarrow b_i\} \quad (5)$$

where $(\bar{\mathbf{B}}_n)_{ijl} = \epsilon_{ijk}(\bar{\mathbf{B}}_n)_l^k$. We observe that in Figs. 1(a) and 1(b), \mathbf{B}_n is composed of the two spectator quarks from \mathbf{B}_c , whereas in Figs. 1(c₁) and 1(c₂), it consists of only one

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} (c_+ H(\mathbf{15})_{jk}^{ij} + c_- H(\bar{\mathbf{6}})_{lk} \epsilon^{lij}) (\bar{q}_i q^k)_{V-A} (\bar{q}_j c)_{V-A}. \quad (3)$$

Under $SU(3)_F$, the three lowest-lying charmed baryon states of \mathbf{B}_c form antitriplet charmed baryon states, and \mathbf{B}_n and P belong to octet baryon and pseudoscalar meson states. In this work, we adopt the same convention for the $SU(3)_F$ tensors as those in Refs. [34,40].

We assume that S-wave($L=0$) pseudoscalar meson pairs dominate in the nonresonant amplitudes. The decay amplitude can be written as

$$\begin{aligned} \mathcal{M}(\mathbf{B}_c \rightarrow \mathbf{B}_n PP') &= \langle P' P \mathbf{B}_n | \mathcal{H}_{\text{eff}} | \mathbf{B}_c \rangle \\ &= i\bar{u}_{\mathbf{B}_n} (A - B\gamma_5) u_{\mathbf{B}_c}, \end{aligned} \quad (4)$$

where $u_{\mathbf{B}_c}$ are Dirac spinors of baryons, and A and B represent the parity conserving and parity violating parts, respectively. Assuming the dominance of $H(\bar{\mathbf{6}})$ over $H(\mathbf{15})$, $\langle \mathbf{B}_n PP' | \mathcal{H}_{\text{eff}} | \mathbf{B}_c \rangle$ contains six $SU(3)_F$ parameters in A and B amplitudes to be fitted with data [40]. Due to the limitation of data points, we assume that final-state interactions (FSIs) are negligible between nonresonance states, and the parameters A and B are considered to be relatively real.

However, the $H(\bar{\mathbf{6}})$ fitting [40] has presented large deviations with updated experiment data, such as $\Lambda_c^+ \rightarrow \Lambda^0 K^+ \pi^0$ [12], and $\Xi_c^0 \rightarrow \Xi^0 K^+ K^-$ [4], indicating the $H(\mathbf{15})$ contribution is of great significance in the $SU(3)_F$ fitting. In this study, we first incorporate $H(\mathbf{15})$ into the analysis of charmed baryon three-body weak decay. We list all possible topological diagrams contributed by $H(\mathbf{15})$ before integrating the W boson in Fig. 1. Focusing on $H(\mathbf{15})$, the quarks represented by q_c and i of the W -exchange part in Fig. 1(e) are symmetric in color. Meanwhile, q_c and i originate from \mathbf{B}_c , where the color of the quarks is totally antisymmetric. We conclude that the topology of W -exchange processes like Fig. 1(e) does not contribute to $H(\mathbf{15})$, owing to the Körner-Pati-Woo theorem [42,43]. From these diagrams we can get complete A and B $SU(3)_F$ amplitudes, given by

spectator quark. Given that the light quarks in \mathbf{B}_c and \mathbf{B}_n have similar wave functions in the quark model, it is reasonable to assume that the contributions from (c₁) and

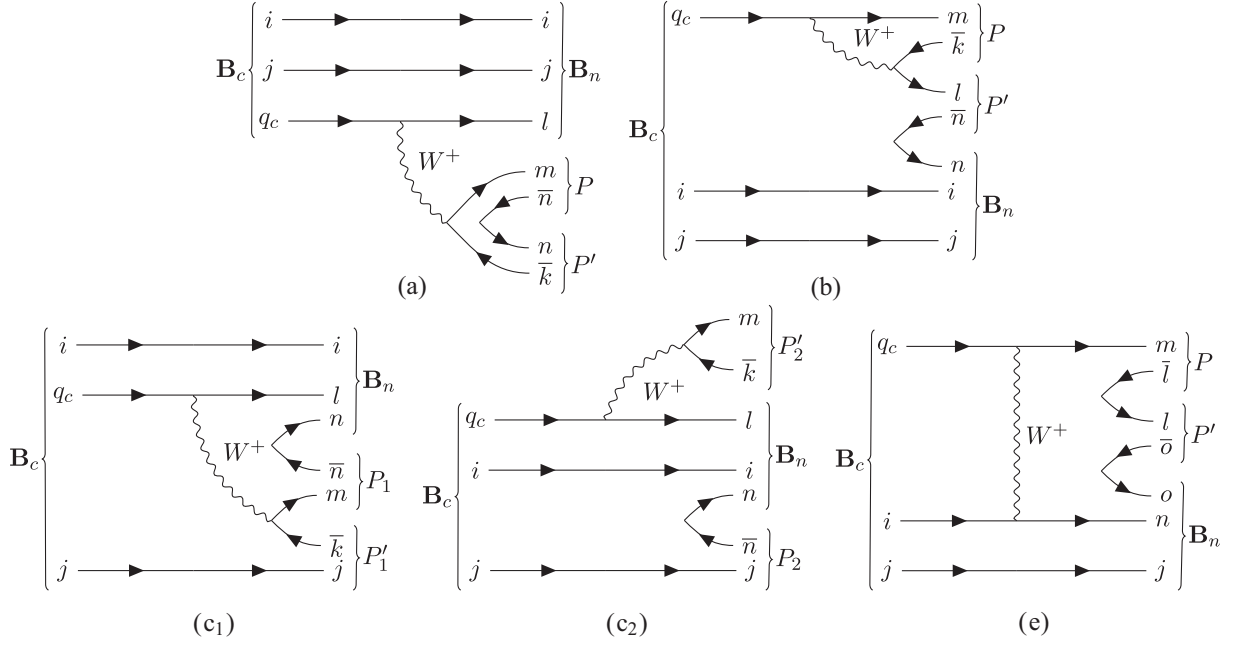


FIG. 1. Topology diagrams contributed by $H(\mathbf{15})$ before integrating the W boson, where (a), (b), (c₁), and (c₂) are W -emission processes, while (e) is an example of W -exchange processes. The parameters a_7, a_8, a_9, a_{10} are given by (a), (b), (c₁), and (c₂), respectively.

(c₂) are smaller than those from (a) and (b). Hence, we take $a_{9,10}$ and $b_{9,10}$, which come from the topology of Figs. 1(c₁) and 1(c₂), to be zero in the following.

The explicit full expansions of $A(\Lambda_c^+ \rightarrow \mathbf{B}_n PP')$, $A(\Xi_c^+ \rightarrow \mathbf{B}_n PP')$, $A(\Xi_c^0 \rightarrow \mathbf{B}_n PP')$ within 16 parameters are presented in Tables XI–XIV in the Appendix, while $B(\mathbf{B}_c \rightarrow \mathbf{B}_n PP')$ can be simply acquired by the replacement of $A(\mathbf{B}_c \rightarrow \mathbf{B}_n PP')\{a_i \rightarrow b_i, i = 1 \sim 8\}$. As we only consider the physical quantities after integrating over the phase space, we assume the amplitudes of a_i and b_i to be independent of m_{23}^2 , which can be justified in the limit of the $SU(3)_f$ flavor symmetry.¹ By introducing the kinematic correction $\kappa(m_{23}^2)$, the differential decay width and averaged up-down asymmetry of $\mathbf{B}_c \rightarrow \mathbf{B}_n PP'$ can be derived as [40]

$$\Gamma = \int_{m_{12}^2} \int_{m_{23}^2} \frac{1}{(2\pi)^3} \frac{A^2 + \kappa^2(m_{23}^2)B^2}{32m_{\mathbf{B}_c}^3} dm_{12}^2 dm_{23}^2, \quad (6)$$

and

$$\langle \alpha \rangle \equiv \frac{2 \int_{m_{12}^2} \int_{m_{23}^2} \kappa(m_{23}^2) AB dm_{12}^2 dm_{23}^2}{\int_{m_{12}^2} \int_{m_{23}^2} A^2 + \kappa^2(m_{23}^2) B^2 dm_{12}^2 dm_{23}^2}, \quad (7)$$

¹We have examined the simplest first-order of m_{23}^2 dependency in the form of $a_i = a_i^0 [1 + (m_{23}^2/m_{\mathbf{B}_c}^2)r]$. The fitting result closely mirrors that presented in this paper with $r = 0.09$.

respectively. The kinematic correction $\kappa(m_{23}^2)$ is defined as

$$\kappa(m_{23}^2) = \sqrt{\frac{(m_{\mathbf{B}_c} - m_{\mathbf{B}_n})^2 - m_{23}^2}{(m_{\mathbf{B}_c} + m_{\mathbf{B}_n})^2 - m_{23}^2}} \quad (8)$$

with m_{23} is the sum of the 3-momentum of the two pseudoscalar mesons in the rest frame.

III. NUMERICAL RESULTS

We make use of the minimum χ^2 fit in the numerical analysis to obtain the values of 16 parameters a_i and b_i in Eq. (5) under $SU(3)_F$ for $\mathbf{B}_c \rightarrow \mathbf{B}_n PP'$. The validity can be tested via $\chi^2/\text{d.o.f.}$ The minimum χ^2 fit approach is given by

$$\chi^2 = \sum_i \left(\frac{\mathcal{B}_{SU(3)}^i - \mathcal{B}_{\text{data}}^i}{\sigma_{\text{data}}^i} \right)^2 \quad (9)$$

in which $\mathcal{B}_{SU(3)}^i$ is the i th decay branching ratio from $SU(3)_F$ fitting predictions, $\mathcal{B}_{\text{data}}^i$ represents the i th experiment data, and σ_{data}^i stands for the i th experiment error, while $i = 1, 2, \dots, 28$ for 28 experiment measured channels in Table I.

TABLE I. The experimental data from Refs. [1–5,7–14,44–50] and reproductions for $\mathcal{B}(\mathbf{B}_c \rightarrow \mathbf{B}_n PP')$.

Channels	Data	Our fittings	Channels	Data	Our fittings
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow p\pi^+K^-)$	3.4 ± 0.4	3.4 ± 0.4	$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+\pi^0\pi^0)$	1.3 ± 0.1	1.3 ± 0.1
$10^3 \mathcal{B}(\Lambda_c^+ \rightarrow \Lambda^0 K^+ \bar{K}^0)$	5.6 ± 1.1	5.9 ± 1.0	$10^4 \mathcal{B}(\Lambda_c^+ \rightarrow p\pi^- K^+)$	1.0 ± 0.1	1.0 ± 0.1
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Lambda^0 \pi^+ \eta^0)$	1.8 ± 0.3	1.9 ± 0.3	$10^4 \mathcal{B}(\Lambda_c^+ \rightarrow nK^+ \bar{K}^0)$	$8.6^{+3.8}_{-3.0}$	6.5 ± 2.2
$10^3 \mathcal{B}(\Lambda_c^+ \rightarrow \Lambda^0 \pi^0 K^+)$	1.5 ± 0.3	1.4 ± 0.3	$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow p\pi^0 K_S^0)$	1.9 ± 0.1	1.9 ± 0.1
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \pi^+ \pi^-)$	2.9 ± 0.5	2.8 ± 0.5	$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow n\pi^+ K_S^0)$	1.9 ± 0.1	1.9 ± 0.1
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^- \pi^+ \pi^+)$	1.9 ± 0.2	2.0 ± 0.2	$10^2 \mathcal{B}(\Xi_c^+ \rightarrow \Sigma^+ \pi^+ K^-)$	2.6 ± 1.2	3.9 ± 0.4
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+ \pi^0)$	2.2 ± 0.8	1.0 ± 0.1	$10^2 \mathcal{B}(\Xi_c^+ \rightarrow \Xi^0 \pi^+ \pi^0)$	6.7 ± 3.5	1.0 ± 0.3
$10^3 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+ \eta^0)$	8.2 ± 0.9	8.3 ± 0.8	$10^3 \mathcal{B}(\Xi_c^+ \rightarrow \Sigma^+ \pi^+ \pi^-)$	14.0 ± 8.0	6.5 ± 1.6
$10^3 \mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \pi^- K^+)$	2.0 ± 0.4	1.6 ± 0.3	$10^3 \mathcal{B}(\Xi_c^+ \rightarrow \Sigma^- \pi^+ \pi^+)$	5.1 ± 3.4	6.9 ± 2.3
$10^3 \mathcal{B}(\Lambda_c^+ \rightarrow \Xi^- \pi^+ K^+)$	3.3 ± 0.9	1.5 ± 0.5	$10^3 \mathcal{B}(\Xi_c^+ \rightarrow \Sigma^+ K^+ K^-)$	4.2 ± 2.5	0.4 ± 0.2
$10^3 \mathcal{B}(\Lambda_c^+ \rightarrow p\pi^+ \pi^-)$	4.7 ± 0.3	4.6 ± 0.3	$10^2 \mathcal{B}(\Xi_c^0 \rightarrow \Lambda^0 \pi^+ K^-)$	1.2 ± 0.4	1.3 ± 0.3
$10^4 \mathcal{B}(\Lambda_c^+ \rightarrow pK^+ K^-)$	5.2 ± 1.2	4.8 ± 1.0	$10^4 \mathcal{B}(\Xi_c^0 \rightarrow \Lambda^0 K^+ K^-)$	5.1 ± 1.9	4.5 ± 0.7
$10^2 \mathcal{B}(\Lambda_c^+ \rightarrow p\bar{K}^0 \eta^0)$	0.8 ± 0.2	0.7 ± 0.1	$10^3 \mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 K^+ K^-)$	0.7 ± 0.2	1.0 ± 0.1
$10^3 \mathcal{B}(\Lambda_c^+ \rightarrow \Xi^0 \pi^0 K^+)$	7.8 ± 1.6	8.0 ± 1.5	$10^2 \mathcal{B}(\Xi_c^+ \rightarrow \Xi^- \pi^+ \pi^+)$	2.9 ± 1.3	4.0 ± 1.1

We now discuss the data input in Table I. Most importantly, the contributions from two-body resonances are excluded from these 28 data in the table. Aside from the usage of nonresonant experimental data, we also take $\Lambda_c^+ \rightarrow \Sigma^+(\rho^0 \rightarrow \pi^+ \pi^-)$ [51,52] and $\Lambda_c^+ \rightarrow (\Xi(1530)^+ \rightarrow \Xi^- \pi^+)K^+$ [6] into consideration for subtracting the resonant contributions. To obtain absolute branching ratios, we incorporate specific branching ratio measurements, $\mathcal{B}(\Lambda_c^+ \rightarrow p\pi^+ K^-) = (6.8 \pm 0.3)\%$ [46] and $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+) = (1.8 \pm 0.5)\%$ [2], provided by Belle. These measurements serve as crucial data to convert relative branching ratios into absolute values.

The fitted 16 parameters are collected in Table II with $\chi^2/\text{d.o.f} = 1.5$ and $\text{d.o.f} = 12$ standing for the degree of freedom. In Table I, the branching ratios of 28 input data have been reproduced, and we can see that the $SU(3)_F$ fittings are in good agreement with the data. We list our all numerical fitting results for the branching ratios of $\Lambda_c^+ \rightarrow \mathbf{B}_n PP'$, $\Xi_c^+ \rightarrow \mathbf{B}_n PP'$ and $\Xi_c^0 \rightarrow \mathbf{B}_n PP'$ in Tables III–VI, respectively.

Our predictions include the branching ratio of $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ K^+ K^-) = (4.6 \pm 0.4) \times 10^{-4}$, which is notably

TABLE II. Fitting values for a_i and b_i in unit of $\frac{G_F}{\sqrt{2}}$, GeV^2 .

a_i	Result	b_i	Result
a_1	3.12 ± 0.99	b_1	8.08 ± 4.15
a_2	0.06 ± 0.61	b_2	2.20 ± 1.88
a_3	-4.91 ± 1.09	b_3	-18.04 ± 7.15
a_4	2.05 ± 0.44	b_4	-3.10 ± 3.72
a_5	6.36 ± 0.76	b_5	-12.83 ± 8.58
a_6	-1.09 ± 0.89	b_6	12.15 ± 2.91
a_7	1.69 ± 1.40	b_7	-30.06 ± 3.18
a_8	-5.22 ± 0.37	b_8	-2.85 ± 6.09

TABLE III. Numerical results for $\mathcal{B}(\Lambda_c^+ \rightarrow \mathbf{B}_n PP')$.

CF mode	$10^2 \mathcal{B}$	CF mode	$10^2 \mathcal{B}$
$\Lambda^0 \pi^+ \eta^0$	1.93 ± 0.27	$p\pi^0 \bar{K}^0$	3.90 ± 0.29
$\Lambda^0 K^+ \bar{K}^0$	0.59 ± 0.10	$p\bar{K}^0 \eta^0$	0.74 ± 0.11
$\Sigma^0 \pi^+ \pi^0$	1.01 ± 0.10	$p\pi^+ K^-$	3.42 ± 0.36
$\Sigma^0 \pi^+ \eta^0$	0.83 ± 0.08	$n\pi^+ \bar{K}^0$	3.67 ± 0.22
$\Sigma^0 K^+ \bar{K}^0$	$(2.34 \pm 0.83) \times 10^{-2}$	$\Sigma^+ \pi^0 \pi^0$	1.33 ± 0.10
$\Sigma^- \pi^+ \pi^+$	1.95 ± 0.20	$\Sigma^+ \pi^0 \eta^0$	0.87 ± 0.09
$\Xi^- \pi^+ K^+$	0.15 ± 0.05	$\Sigma^+ \eta^0 \eta^0$	$(4.29 \pm 4.04) \times 10^{-6}$
$\Xi^0 \pi^0 K^+$	0.80 ± 0.15	$\Sigma^+ \pi^+ \pi^-$	2.81 ± 0.47
$\Xi^0 K^+ \eta^0$	$(1.45 \pm 0.32) \times 10^{-2}$	$\Sigma^+ K^+ K^-$	$(4.58 \pm 0.38) \times 10^{-2}$
$\Xi^0 \pi^+ K^0$	0.97 ± 0.33	$\Sigma^+ K^0 \bar{K}^0$	$(0.86^{+1.18}_{-0.86}) \times 10^{-2}$
CS mode	$10^4 \mathcal{B}$	CS mode	$10^4 \mathcal{B}$
$\Lambda^0 \pi^0 K^+$	13.6 ± 2.5	$p\eta^0 \eta^0$	4.46 ± 1.18
$\Lambda^0 K^+ \eta^0$	0.59 ± 0.24	$p\pi^+ \pi^-$	46.4 ± 3.0
$\Lambda^0 \pi^+ K^0$	39.3 ± 7.6	$pK^+ K^-$	4.82 ± 1.05
$\Sigma^0 \pi^0 K^+$	8.24 ± 1.41	$pK^0 \bar{K}^0$	3.53 ± 1.68
$\Sigma^0 K^+ \eta^0$	$(4.29 \pm 1.23) \times 10^{-2}$	$n\pi^+ \pi^0$	25.3 ± 5.5
$\Sigma^0 \pi^+ K^0$	3.31 ± 0.97	$n\pi^+ \eta^0$	45.2 ± 12.1
$\Sigma^- \pi^+ K^+$	3.73 ± 1.11	$nK^+ \bar{K}^0$	6.54 ± 2.17
$\Xi^- K^+ K^+$	$(3.33^{+4.23}_{-3.33}) \times 10^{-3}$	$\Sigma^+ \pi^0 K^0$	3.46 ± 1.01
$\Xi^0 K^+ K^0$	$(1.24^{+1.62}_{-1.24}) \times 10^{-3}$	$\Sigma^+ K^0 \eta^0$	$(8.38 \pm 2.40) \times 10^{-2}$
$p\pi^0 \pi^0$	39.6 ± 9.2	$\Sigma^+ \pi^- K^+$	16.1 ± 3.0
$p\pi^0 \eta^0$	25.8 ± 6.1		
DCS mode	$10^6 \mathcal{B}$	DCS mode	$10^6 \mathcal{B}$
$\Lambda^0 K^+ K^0$	7.22 ± 1.08	$p\pi^- K^+$	96.1 ± 9.2
$\Sigma^0 K^+ K^0$	1.10 ± 0.42	$n\pi^0 K^+$	79.1 ± 28.8

(Table continued)

TABLE III. (Continued)

DCS mode	$10^6\mathcal{B}$	DCS mode	$10^6\mathcal{B}$
$\Sigma^- K^+ K^+$	1.08 ± 0.41	$nK^+\eta^0$	$0.85^{+1.08}_{-0.85}$
$p\pi^0 K^0$	56.8 ± 15.8	$n\pi^+ K^0$	153 ± 61
$pK^0\eta^0$	13.6 ± 2.6	$\Sigma^+ K^0 K^0$	1.09 ± 0.42

 TABLE IV. Numerical results for $\mathcal{B}(\Xi_c^+ \rightarrow \mathbf{B}_n PP')$.

CF mode	$10^2\mathcal{B}$	CF mode	$10^2\mathcal{B}$
$\Lambda^0\pi^+\bar{K}^0$	0.61 ± 0.60	$\Xi^0 K^+\bar{K}^0$	0.18 ± 0.08
$\Sigma^0\pi^+\bar{K}^0$	7.33 ± 1.21	$p\bar{K}^0\bar{K}^0$	2.46 ± 0.69
$\Xi^-\pi^+\pi^+$	3.95 ± 1.10	$\Sigma^+\pi^0\bar{K}^0$	5.96 ± 1.08
$\Xi^0\pi^+\pi^0$	1.01 ± 0.28	$\Sigma^+\bar{K}^0\eta^0$	$(3.02 \pm 1.78) \times 10^{-2}$
$\Xi^0\pi^+\eta^0$	1.18 ± 0.56	$\Sigma^+\pi^+K^-$	3.86 ± 0.44

CS mode	$10^3\mathcal{B}$	CS mode	$10^3\mathcal{B}$
$\Lambda^0\pi^+\pi^0$	0.85 ± 0.17	$p\pi^0\bar{K}^0$	7.39 ± 1.77
$\Lambda^0\pi^+\eta^0$	1.82 ± 0.93	$p\bar{K}^0\eta^0$	1.18 ± 0.44
$\Lambda^0 K^+\bar{K}^0$	0.73 ± 0.37	$p\pi^+K^-$	28.3 ± 2.6
$\Sigma^0\pi^+\pi^0$	6.70 ± 1.70	$n\pi^+\bar{K}^0$	4.80 ± 1.76
$\Sigma^0\pi^+\eta^0$	5.33 ± 1.38	$\Sigma^+\pi^0\pi^0$	17.9 ± 2.59
$\Sigma^0 K^+\bar{K}^0$	2.07 ± 0.37	$\Sigma^+\pi^0\eta^0$	1.14 ± 0.46
$\Sigma^-\pi^+\pi^+$	6.91 ± 2.27	$\Sigma^+\eta^0\eta^0$	0.18 ± 0.07
$\Xi^-\pi^+K^+$	0.95 ± 0.28	$\Sigma^+\pi^+\pi^-$	6.53 ± 1.59
$\Xi^0\pi^0 K^+$	1.94 ± 0.54	$\Sigma^+K^+K^-$	0.37 ± 0.15
$\Xi^0 K^+\eta^0$	0.27 ± 0.04	$\Sigma^+K^0\bar{K}^0$	0.45 ± 0.21
$\Xi^0\pi^+K^0$	1.93 ± 0.60		

DCS mode	$10^4\mathcal{B}$	DCS mode	$10^4\mathcal{B}$
$\Lambda^0\pi^0 K^+$	0.53 ± 0.20	$p\pi^0\eta^0$	2.72 ± 0.86
$\Lambda^0 K^+\eta^0$	$(9.02 \pm 2.74) \times 10^{-2}$	$p\eta^0\eta^0$	$(3.76 \pm 3.30) \times 10^{-2}$
$\Lambda^0\pi^+K^0$	2.13 ± 0.47	$p\pi^+\pi^-$	27.9 ± 2.4
$\Sigma^0\pi^0 K^+$	3.31 ± 0.33	pK^+K^-	1.11 ± 0.19
$\Sigma^0 K^+\eta^0$	$(4.58 \pm 1.05) \times 10^{-2}$	$pK^0\bar{K}^0$	$0.14^{+0.19}_{-0.14}$
$\Sigma^0\pi^+K^0$	4.18 ± 0.80	$n\pi^+\eta^0$	5.38 ± 1.70
$\Sigma^-\pi^+K^+$	0.69 ± 0.21	$nK^+\bar{K}^0$	2.00 ± 0.70
$\Xi^-K^+K^+$	$(2.45 \pm 1.09) \times 10^{-2}$	$\Sigma^+\pi^0 K^0$	2.85 ± 0.31
$\Xi^0 K^+K^0$	0.12 ± 0.03	$\Sigma^+K^0\eta^0$	$(4.75 \pm 1.67) \times 10^{-2}$
$p\pi^0\pi^0$	14.1 ± 1.2	$\Sigma^+\pi^-K^+$	1.14 ± 0.20

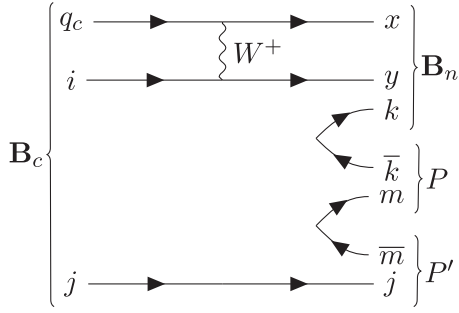
 TABLE V. Numerical results for $\mathcal{B}(\Xi_c^0 \rightarrow \mathbf{B}_n PP')$.

CF mode	$10^2\mathcal{B}$	CF mode	$10^2\mathcal{B}$
$\Lambda^0\pi^0\bar{K}^0$	1.18 ± 0.30	$\Xi^0\pi^0\eta^0$	0.33 ± 0.09
$\Lambda^0\bar{K}^0\eta^0$	0.28 ± 0.05	$\Xi^0\eta^0\eta^0$	$(7.50 \pm 5.18) \times 10^{-3}$
$\Lambda^0\pi^+K^-$	1.25 ± 0.30	$\Xi^0\pi^+\pi^-$	$0.28^{+0.38}_{-0.28}$
$\Sigma^0\pi^0\bar{K}^0$	2.26 ± 0.51	$\Xi^0 K^+K^-$	$(9.57 \pm 0.77) \times 10^{-2}$
$\Sigma^0\bar{K}^0\eta^0$	$(9.61 \pm 1.61) \times 10^{-2}$	$\Xi^0 K^0\bar{K}^0$	0.11 ± 0.02
$\Sigma^0\pi^+K^-$	2.85 ± 0.61	$pK^-\bar{K}^0$	1.01 ± 0.34
$\Sigma^-\pi^+\bar{K}^0$	4.20 ± 0.75	$n\bar{K}^0\bar{K}^0$	1.14 ± 0.13
$\Xi^-\pi^+\pi^0$	0.34 ± 0.09	$\Sigma^+\pi^0 K^-$	2.38 ± 0.49
$\Xi^-\pi^+\eta^0$	1.14 ± 0.22	$\Sigma^+K^-\eta^0$	0.13 ± 0.02
$\Xi^-K^+\bar{K}^0$	0.43 ± 0.14	$\Sigma^+\pi^-\bar{K}^0$	2.63 ± 0.91
$\Xi^0\pi^0\pi^0$	0.89 ± 0.43		

CS mode	$10^3\mathcal{B}$	CS mode	$10^3\mathcal{B}$
$\Lambda^0\pi^0\pi^0$	2.25 ± 0.66	$\Xi^-\pi^0 K^+$	1.99 ± 0.26
$\Lambda^0\pi^0\eta^0$	0.68 ± 0.14	$\Xi^-K^+\eta^0$	$(9.07 \pm 2.99) \times 10^{-2}$
$\Lambda^0\eta^0\eta^0$	0.24 ± 0.05	$\Xi^-\pi^+K^0$	1.33 ± 0.22
$\Lambda^0\pi^+\pi^-$	2.34 ± 0.93	$\Xi^0\pi^0 K^0$	0.63 ± 0.10
$\Lambda^0 K^+K^-$	0.45 ± 0.07	$\Xi^0 K^0\eta^0$	$(1.05 \pm 0.14) \times 10^{-2}$
$\Lambda^0 K^0\bar{K}^0$	0.38 ± 0.05	$\Xi^0\pi^-K^+$	1.03 ± 0.14
$\Sigma^0\pi^0\pi^0$	1.04 ± 0.29	$p\pi^0 K^-$	1.43 ± 0.50
$\Sigma^0\pi^0\eta^0$	1.53 ± 0.27	$pK^-\eta^0$	0.30 ± 0.13
$\Sigma^0\eta^0\eta^0$	$(3.00 \pm 1.24) \times 10^{-2}$	$p\pi^-\bar{K}^0$	5.78 ± 1.89
$\Sigma^0\pi^+\pi^-$	3.27 ± 0.66	$n\pi^0\bar{K}^0$	8.89 ± 1.73
$\Sigma^0 K^+K^-$	0.14 ± 0.08	$n\bar{K}^0\eta^0$	0.27 ± 0.10
$\Sigma^0 K^0\bar{K}^0$	$(7.02 \pm 1.32) \times 10^{-2}$	$n\pi^+K^-$	8.69 ± 1.65
$\Sigma^-\pi^+\pi^0$	1.68 ± 0.48	$\Sigma^+\pi^0\pi^-$	0.61 ± 0.20
$\Sigma^-\pi^+\eta^0$	6.16 ± 0.77	$\Sigma^+\pi^-\eta^0$	0.68 ± 0.30
$\Sigma^-K^+\bar{K}^0$	0.41 ± 0.07	$\Sigma^+K^0 K^-$	0.59 ± 0.11

 TABLE VI. Numerical results for $\mathcal{B}(\Xi_c^0 \rightarrow \mathbf{B}_n PP')$.

DCS mode	$10^5\mathcal{B}$	DCS mode	$10^5\mathcal{B}$
$\Lambda^0\pi^0 K^0$	8.84 ± 1.61	$\Xi^0 K^0 K^0$	$(8.53 \pm 3.72) \times 10^{-2}$
$\Lambda^0 K^0\eta^0$	0.67 ± 0.19	$p\pi^-\eta^0$	18.2 ± 5.8
$\Lambda^0\pi^-K^+$	13.7 ± 2.7	$pK^0 K^-$	6.79 ± 2.37
$\Sigma^0\pi^0 K^0$	5.47 ± 0.89	$n\pi^0\pi^0$	47.4 ± 4.0
$\Sigma^0 K^0\eta^0$	$(7.91 \pm 2.78) \times 10^{-2}$	$n\pi^0\eta^0$	9.14 ± 2.90
$\Sigma^0\pi^-K^+$	1.35 ± 0.57	$n\eta^0\eta^0$	0.13 ± 0.11
$\Sigma^-\pi^0 K^+$	7.23 ± 1.80	$n\pi^+\pi^-$	93.7 ± 8.0
$\Sigma^-K^+\eta^0$	0.30 ± 0.07	nK^+K^-	$0.48^{+0.65}_{-0.48}$
$\Sigma^-\pi^+K^0$	37.5 ± 4.6	$nK^0\bar{K}^0$	3.63 ± 0.61
$\Xi^-K^+\bar{K}^0$	0.81 ± 0.16	$\Sigma^+\pi^-K^0$	2.41 ± 0.72

FIG. 2. Topology diagram of parameter a_5 .

smaller than the one of $(2.0 \pm 0.4) \times 10^{-3}$ observed by BESIII [9] but aligns with the upper limit prediction of 8×10^{-4} from Belle [53]. This suggests that resonant contributions still exist. Working backward, it suggests that $\mathcal{B}(\Lambda_c^+ \rightarrow (\Xi(1690)^0 \rightarrow \Sigma^+ K^-) K^+)$ is about $(1.5 \pm 0.4) \times 10^{-3}$. Moreover, the predicted branching ratios for $\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^0 \pi^+ \pi^0)$ and $\mathcal{B}(\Lambda_c^+ \rightarrow \Xi^- \pi^+ K^+)$ are approximately half the magnitude of their experimental measurements, hinting the presence of other resonant contributions from excited state particles. We emphasize the need for further experimental investigations to confirm their existences and extract the branch fractions of these states.

TABLE VII. Numerical results for $\langle \alpha \rangle (\Lambda_c^+ \rightarrow \mathbf{B}_n P P')$.

CF mode	$\langle \alpha \rangle$	CS mode	$\langle \alpha \rangle$	DCS mode	$\langle \alpha \rangle$
$\Lambda^0 \pi^+ \eta^0$	0.1 ± 0.6	$\Lambda^0 \pi^0 K^+$	-0.4 ± 0.4	$\Lambda^0 K^+ K^0$	0.14 ± 0.30
$\Lambda^0 K^+ \bar{K}^0$	0.63 ± 0.30	$\Lambda^0 K^+ \eta^0$	0.17 ± 0.32	$\Sigma^0 K^+ K^0$	-0.3 ± 0.4
$\Sigma^0 \pi^+ \pi^0$	$0.73^{+0.27}_{-0.35}$	$\Lambda^0 \pi^+ K^0$	-0.96 ± 0.04	$\Sigma^- K^+ K^+$	-0.3 ± 0.4
$\Sigma^0 \pi^+ \eta^0$	$-0.86^{+0.28}_{-0.14}$	$\Sigma^0 \pi^0 K^+$	0.965 ± 0.021	$p \pi^0 K^0$	$-0.965^{+0.05}_{-0.035}$
$\Sigma^0 K^+ \bar{K}^0$	$-0.80^{+0.5}_{-0.20}$	$\Sigma^0 K^+ \eta^0$	0.15 ± 0.21	$p K^0 \eta^0$	$-0.74^{+0.4}_{-0.26}$
$\Sigma^- \pi^+ \pi^+$	$0.73^{+0.27}_{-0.35}$	$\Sigma^0 \pi^+ K^0$	0.3 ± 0.7	$p \pi^- K^+$	0.69 ± 0.27
$\Xi^- \pi^+ K^+$	$-0.85^{+0.4}_{-0.15}$	$\Sigma^- \pi^+ K^+$	0.6 ± 0.4	$n \pi^0 K^+$	$-0.93^{+0.10}_{-0.07}$
$\Xi^0 \pi^0 K^+$	-0.54 ± 0.33	$\Xi^- K^+ K^+$	$-0.75^{+0.5}_{-0.25}$	$n K^+ \eta^0$	$0.6^{+0.4}_{-0.7}$
$\Xi^0 K^+ \eta^0$	-0.29 ± 0.20	$\Xi^0 K^+ K^0$	$-0.72^{+0.5}_{-0.28}$	$n \pi^+ K^0$	0.60 ± 0.31
$\Xi^0 \pi^+ K^0$	-0.04 ± 0.35	$p \pi^0 \pi^0$	$0.86^{+0.14}_{-0.25}$	$\Sigma^+ K^0 K^0$	-0.3 ± 0.4
$p \pi^0 \bar{K}^0$	$0.6^{+0.4}_{-0.5}$	$p \pi^0 \eta^0$	$-0.93^{+0.16}_{-0.07}$		
$p \bar{K}^0 \eta^0$	0.0 ± 0.5	$p \eta^0 \eta^0$	-0.6 ± 0.4		
$p \pi^+ K^-$	$0.80^{+0.20}_{-0.4}$	$p \pi^+ \pi^-$	$-0.65^{+0.4}_{-0.35}$		
$n \pi^+ \bar{K}^0$	$-0.68^{+0.5}_{-0.32}$	$p K^+ K^-$	$-0.88^{+0.20}_{-0.12}$		
$\Sigma^+ \pi^0 \pi^0$	$-0.95^{+0.10}_{-0.05}$	$p K^0 \bar{K}^0$	-0.3 ± 0.7		
$\Sigma^+ \pi^0 \eta^0$	$-0.86^{+0.28}_{-0.14}$	$n \pi^+ \pi^0$	0.3 ± 0.7		
$\Sigma^+ \eta^0 \eta^0$	0.10 ± 0.07	$n \pi^+ \eta^0$	$0.6^{+0.4}_{-0.5}$		
$\Sigma^+ \pi^+ \pi^-$	-0.1 ± 0.5	$n K^+ \bar{K}^0$	$0.89^{+0.11}_{-0.27}$		
$\Sigma^+ K^+ K^-$	0.0 ± 0.9	$\Sigma^+ \pi^0 K^0$	0.3 ± 0.7		
$\Sigma^+ K^0 \bar{K}^0$	0.0 ± 0.4	$\Sigma^+ K^0 \eta^0$	0.15 ± 0.21		
		$\Sigma^+ \pi^- K^+$	$0.91^{+0.09}_{-0.21}$		

TABLE VIII. Numerical results for $\langle\alpha\rangle(\Xi_c^+ \rightarrow \mathbf{B}_n PP')$.

CF mode	$\langle\alpha\rangle$	CS mode	$\langle\alpha\rangle$	DCS mode	$\langle\alpha\rangle$
$\Lambda^0 \pi^+ \bar{K}^0$	$-0.5^{+0.8}_{-0.5}$	$\Lambda^0 \pi^+ \pi^0$	0.3 ± 0.6	$\Lambda^0 \pi^0 K^+$	$-0.90^{+0.24}_{-0.10}$
$\Sigma^0 \pi^+ \bar{K}^0$	0.0 ± 0.6	$\Lambda^0 \pi^+ \eta^0$	$0.5^{+0.5}_{-0.6}$	$\Lambda^0 K^+ \eta^0$	$0.78^{+0.22}_{-0.33}$
$\Xi^- \pi^+ \pi^+$	$-0.6^{+0.6}_{-0.4}$	$\Lambda^0 K^+ \bar{K}^0$	-0.2 ± 0.4	$\Lambda^0 \pi^+ K^0$	$-0.85^{+0.33}_{-0.15}$
$\Xi^0 \pi^+ \pi^0$	$-0.6^{+0.6}_{-0.4}$	$\Sigma^0 \pi^+ \pi^0$	$-0.94^{+0.18}_{-0.06}$	$\Sigma^0 \pi^0 K^+$	-0.1 ± 0.5
$\Xi^0 \pi^+ \eta^0$	$-0.96^{+0.04}_{-0.04}$	$\Sigma^0 \pi^+ \eta^0$	$0.77^{+0.23}_{-0.24}$	$\Sigma^0 K^+ \eta^0$	$-0.73^{+0.5}_{-0.27}$
$\Xi^0 K^+ \bar{K}^0$	$0.95^{+0.05}_{-0.10}$	$\Sigma^0 K^+ \bar{K}^0$	$-0.96^{+0.06}_{-0.04}$	$\Sigma^0 \pi^+ K^0$	$0.95^{+0.05}_{-0.08}$
$p \bar{K}^0 \bar{K}^0$	$-0.67^{+0.7}_{-0.33}$	$\Sigma^- \pi^+ \pi^+$	$-0.6^{+0.5}_{-0.4}$	$\Sigma^- \pi^+ K^+$	$-0.66^{+0.5}_{-0.34}$
$\Sigma^+ \pi^0 \bar{K}^0$	$-0.85^{+0.30}_{-0.15}$	$\Xi^- \pi^+ K^+$	0.6 ± 0.4	$\Xi^- K^+ K^+$	$0.96^{+0.04}_{-0.05}$
$\Sigma^+ \bar{K}^0 \eta^0$	$-0.95^{+0.11}_{-0.05}$	$\Xi^0 \pi^0 K^+$	0.5 ± 0.4	$\Xi^0 K^+ K^0$	-0.32 ± 0.29
$\Sigma^+ \pi^+ K^-$	$0.79^{+0.21}_{-0.26}$	$\Xi^0 K^+ \eta^0$	0.16 ± 0.33	$p \pi^0 \pi^0$	0.0 ± 0.4
		$\Xi^0 \pi^+ K^0$	$0.84^{+0.16}_{-0.33}$	$p \pi^0 \eta^0$	$-0.87^{+0.28}_{-0.13}$
		$p \pi^0 \bar{K}^0$	0.969 ± 0.023	$p \eta^0 \eta^0$	$-0.6^{+0.9}_{-0.4}$
		$p \bar{K}^0 \eta^0$	$-0.74^{+0.5}_{-0.26}$	$p \pi^+ \pi^-$	0.0 ± 0.4
		$p \pi^+ K^-$	$0.70^{+0.30}_{-0.32}$	$p K^+ K^-$	-0.1 ± 0.6
		$n \pi^+ \bar{K}^0$	-0.5 ± 0.4	$p K^0 \bar{K}^0$	0.1 ± 0.9
		$\Sigma^+ \pi^0 \pi^0$	$-0.94^{+0.16}_{-0.06}$	$n \pi^+ \eta^0$	$-0.87^{+0.28}_{-0.13}$
		$\Sigma^+ \pi^0 \eta^0$	$0.94^{+0.06}_{-0.13}$	$n K^+ \bar{K}^0$	0.0 ± 0.6
		$\Sigma^+ \eta^0 \eta^0$	0.3 ± 0.4	$\Sigma^+ \pi^0 K^0$	0.2 ± 0.6
		$\Sigma^+ \pi^+ \pi^-$	$-0.81^{+0.26}_{-0.19}$	$\Sigma^+ K^0 \eta^0$	0.64 ± 0.32
		$\Sigma^+ K^+ K^-$	$-0.92^{+0.25}_{-0.08}$	$\Sigma^+ \pi^- K^+$	$0.89^{+0.11}_{-0.29}$
		$\Sigma^+ K^0 \bar{K}^0$	-0.3 ± 0.5		

 TABLE IX. Numerical results for $\langle\alpha\rangle(\Xi_c^0 \rightarrow \mathbf{B}_n PP')$.

CF mode	$\langle\alpha\rangle$	CS mode	$\langle\alpha\rangle$	DCS mode	$\langle\alpha\rangle$
$\Lambda^0 \pi^0 \bar{K}^0$	0.1 ± 0.5	$\Lambda^0 \pi^0 \pi^0$	$0.965^{+0.035}_{-0.08}$	$\Lambda^0 \pi^0 K^0$	$-0.95^{+0.09}_{-0.05}$
$\Lambda^0 \bar{K}^0 \eta^0$	$0.6^{+0.4}_{-0.5}$	$\Lambda^0 \pi^0 \eta^0$	0.967 ± 0.016	$\Lambda^0 K^0 \eta^0$	0.0 ± 0.5
$\Lambda^0 \pi^+ K^-$	0.4 ± 0.5	$\Lambda^0 \eta^0 \eta^0$	0.31 ± 0.35	$\Lambda^0 \pi^- K^+$	$-0.71^{+0.31}_{-0.29}$
$\Sigma^0 \pi^0 \bar{K}^0$	-0.1 ± 0.5	$\Lambda^0 \pi^+ \pi^-$	$0.88^{+0.12}_{-0.34}$	$\Sigma^0 \pi^0 K^0$	0.965 ± 0.034
$\Sigma^0 \bar{K}^0 \eta^0$	-0.67 ± 0.29	$\Lambda^0 K^+ K^-$	-0.3 ± 0.7	$\Sigma^0 K^0 \eta^0$	0.64 ± 0.32
$\Sigma^0 \pi^+ K^-$	$-0.93^{+0.14}_{-0.07}$	$\Lambda^0 K^0 \bar{K}^0$	$-0.89^{+0.18}_{-0.11}$	$\Sigma^0 \pi^- K^+$	0.2 ± 0.7
$\Sigma^- \pi^+ \bar{K}^0$	$0.71^{+0.29}_{-0.33}$	$\Sigma^0 \pi^0 \pi^0$	0.0 ± 0.6	$\Sigma^- \pi^0 K^+$	0.4 ± 0.5
$\Xi^- \pi^+ \pi^0$	$-0.6^{+0.6}_{-0.4}$	$\Sigma^0 \pi^0 \eta^0$	$0.68^{+0.32}_{-0.4}$	$\Sigma^- K^+ \eta^0$	$-0.74^{+0.5}_{-0.26}$
$\Xi^- \pi^+ \eta^0$	0.1 ± 0.6	$\Sigma^0 \eta^0 \eta^0$	0.3 ± 0.4	$\Sigma^- \pi^+ K^0$	$0.80^{+0.20}_{-0.26}$
$\Xi^- K^+ \bar{K}^0$	0.961 ± 0.028	$\Sigma^0 \pi^+ \pi^-$	$-0.73^{+0.33}_{-0.27}$	$\Xi^- K^+ \bar{K}^0$	0.40 ± 0.25
$\Xi^0 \pi^0 \pi^0$	-0.3 ± 0.7	$\Sigma^0 K^+ K^-$	0.1 ± 0.5	$\Xi^0 K^0 K^0$	$0.96^{+0.04}_{-0.06}$
$\Xi^0 \pi^0 \eta^0$	$0.94^{+0.06}_{-0.14}$	$\Sigma^0 K^0 \bar{K}^0$	$-0.83^{+0.23}_{-0.17}$	$p \pi^- \eta^0$	$-0.87^{+0.28}_{-0.13}$
$\Xi^0 \eta^0 \eta^0$	$-0.86^{+0.27}_{-0.14}$	$\Sigma^- \pi^+ \pi^0$	0.97 ± 0.02	$p K^0 K^-$	0.0 ± 0.6
$\Xi^0 \pi^+ \pi^-$	0.0 ± 0.9	$\Sigma^- \pi^+ \eta^0$	$0.96^{+0.04}_{-0.07}$	$n \pi^0 \pi^0$	0.0 ± 0.4
$\Xi^0 K^+ K^-$	0.0 ± 0.9	$\Sigma^- K^+ \bar{K}^0$	-0.2 ± 0.7	$n \pi^0 \eta^0$	$-0.87^{+0.28}_{-0.13}$
$\Xi^0 K^0 \bar{K}^0$	-0.06 ± 0.24	$\Xi^- \pi^0 K^+$	0.96 ± 0.04	$n \eta^0 \eta^0$	$-0.6^{+0.9}_{-0.4}$

(Table continued)

TABLE IX. (Continued)

CF mode	$\langle\alpha\rangle$	CS mode	$\langle\alpha\rangle$	DCS mode	$\langle\alpha\rangle$
$pK^-\bar{K}^0$	$-0.6^{+0.5}_{-0.4}$	$\Xi^-K^+\eta^0$	0.71 ± 0.22	$n\pi^+\pi^-$	0.0 ± 0.4
$n\bar{K}^0\bar{K}^0$	$0.67^{+0.33}_{-0.35}$	$\Xi^-\pi^+K^0$	-0.2 ± 0.6	nK^+K^-	0.1 ± 0.9
$\Sigma^+\pi^0K^-$	$-0.74^{+0.35}_{-0.26}$	$\Xi^0\pi^0K^0$	-0.965 ± 0.025	$nK^0\bar{K}^0$	-0.1 ± 0.6
$\Sigma^+K^-\eta^0$	-0.54 ± 0.33	$\Xi^0K^0\eta^0$	$-0.2^{+1.1}_{-0.8}$	$\Sigma^+\pi^-K^0$	$-0.65^{+0.5}_{-0.35}$
$\Sigma^+\pi^-\bar{K}^0$	0.0 ± 0.5	$\Xi^0\pi^-K^+$	-0.5 ± 0.5		
		$p\pi^0K^-$	0.0 ± 0.7		
		$pK^-\eta^0$	$0.78^{+0.22}_{-0.4}$		
		$p\pi^-\bar{K}^0$	$-0.88^{+0.27}_{-0.12}$		
		$n\pi^0\bar{K}^0$	-0.4 ± 0.4		
		$n\bar{K}^0\eta^0$	-0.3 ± 0.7		
		$n\pi^+K^-$	-0.31 ± 0.35		
		$\Sigma^+\pi^0\pi^-$	$-0.6^{+0.5}_{-0.4}$		
		$\Sigma^+\pi^-\eta^0$	0.5 ± 0.5		
		$\Sigma^+K^0K^-$	-0.58 ± 0.35		

TABLE X. Decay branching ratios and averaged up-down asymmetries for CF and DCS mixed-mode processes involving K_S^0 and K_L^0 .

Channels	\mathcal{B}	$\langle\alpha\rangle$	Channels	\mathcal{B}	$\langle\alpha\rangle$
$\Lambda_c^+ \rightarrow \Lambda^0 K^+ K_S^0$	$(2.73 \pm 0.49) \times 10^{-3}$	0.65 ± 0.30	$\Xi_c^0 \rightarrow \Sigma^+\pi^-K_S^0$	$(1.35 \pm 0.45) \times 10^{-2}$	-0.10 ± 0.50
$\Lambda_c^+ \rightarrow \Lambda^0 K^+ K_L^0$	$(3.13 \pm 0.53) \times 10^{-3}$	0.62 ± 0.30	$\Xi_c^0 \rightarrow \Sigma^+\pi^-K_L^0$	$(1.29 \pm 0.46) \times 10^{-2}$	0.00 ± 0.50
$\Lambda_c^+ \rightarrow \Sigma^0 K^+ K_S^0$	$(1.27 \pm 0.48) \times 10^{-4}$	$-0.85^{+0.40}_{-0.15}$	$\Xi_c^0 \rightarrow \Sigma^0\pi^0K_S^0$	$(1.06 \pm 0.24) \times 10^{-2}$	-0.20 ± 0.50
$\Lambda_c^+ \rightarrow \Sigma^0 K^+ K_L^0$	$(1.08 \pm 0.35) \times 10^{-4}$	$-0.73^{+0.60}_{-0.27}$	$\Xi_c^0 \rightarrow \Sigma^0\pi^0K_L^0$	$(1.20 \pm 0.27) \times 10^{-2}$	0.00 ± 0.50
$\Lambda_c^+ \rightarrow p\pi^0 K_S^0$	$(1.88 \pm 0.14) \times 10^{-2}$	$0.60^{+0.40}_{-0.50}$	$\Xi_c^0 \rightarrow \Sigma^-\pi^+K_S^0$	$(1.81 \pm 0.34) \times 10^{-2}$	0.60 ± 0.40
$\Lambda_c^+ \rightarrow p\pi^0 K_L^0$	$(2.02 \pm 0.15) \times 10^{-2}$	0.50 ± 0.50	$\Xi_c^0 \rightarrow \Sigma^-\pi^+K_L^0$	$(2.43 \pm 0.41) \times 10^{-2}$	$0.78^{+0.22}_{-0.28}$
$\Lambda_c^+ \rightarrow p\eta^0 K_S^0$	$(3.41 \pm 0.49) \times 10^{-3}$	0.00 ± 0.50	$\Xi_c^0 \rightarrow \Xi^-K^+K_S^0$	$(1.97 \pm 0.65) \times 10^{-3}$	0.96 ± 0.00
$\Lambda_c^+ \rightarrow p\eta^0 K_L^0$	$(3.97 \pm 0.58) \times 10^{-3}$	0.00 ± 0.50	$\Xi_c^0 \rightarrow \Xi^-K^+K_L^0$	$(2.29 \pm 0.75) \times 10^{-3}$	$0.96^{+0.04}_{-0.05}$
$\Lambda_c^+ \rightarrow n\pi^+ K_S^0$	$(1.85 \pm 0.09) \times 10^{-2}$	$-0.76^{+0.40}_{-0.24}$	$\Xi_c^0 \rightarrow pK^-K_S^0$	$(5.37 \pm 1.70) \times 10^{-3}$	$-0.69^{+0.40}_{-0.31}$
$\Lambda_c^+ \rightarrow n\pi^+ K_L^0$	$(1.83 \pm 0.17) \times 10^{-2}$	$-0.60^{+0.50}_{-0.40}$	$\Xi_c^0 \rightarrow pK^-K_L^0$	$(4.79 \pm 1.67) \times 10^{-3}$	-0.50 ± 0.50
$\Lambda_c^+ \rightarrow \Sigma^+ K_S^0 K_S^0$	$(1.00^{+1.26}_{-1.00}) \times 10^{-4}$	0.00 ± 0.40	$\Xi_c^0 \rightarrow \Lambda^0\eta^0 K_S^0$	$(1.37 \pm 0.24) \times 10^{-3}$	$0.60^{+0.40}_{-0.50}$
$\Lambda_c^+ \rightarrow \Sigma^+ K_S^0 K_L^0$	$(5.46 \pm 2.09) \times 10^{-7}$	-0.30 ± 0.40	$\Xi_c^0 \rightarrow \Lambda^0\eta^0 K_L^0$	$(1.47 \pm 0.26) \times 10^{-3}$	$0.69^{+0.31}_{-0.40}$
$\Lambda_c^+ \rightarrow \Sigma^+ K_L^0 K_L^0$	$(0.73^{+1.10}_{-0.73}) \times 10^{-4}$	0.00 ± 0.50	$\Xi_c^0 \rightarrow \Xi^0 K_S^0 K_S^0$	$(1.10 \pm 0.19) \times 10^{-3}$	-0.09 ± 0.24
$\Xi_c^+ \rightarrow \Sigma^+\pi^0 K_S^0$	$(2.67 \pm 0.50) \times 10^{-2}$	$-0.89^{+0.25}_{-0.11}$	$\Xi_c^0 \rightarrow \Xi^0 K_S^0 K_L^0$	$(4.27 \pm 1.86) \times 10^{-7}$	$0.96^{+0.04}_{-0.06}$
$\Xi_c^+ \rightarrow \Sigma^+\pi^0 K_L^0$	$(3.32 \pm 0.59) \times 10^{-2}$	$-0.81^{+0.35}_{-0.19}$	$\Xi_c^0 \rightarrow \Xi^0 K_L^0 K_L^0$	$(1.15 \pm 0.20) \times 10^{-3}$	-0.03 ± 0.24
$\Xi_c^+ \rightarrow \Sigma^+\eta^0 K_S^0$	$(1.35 \pm 0.80) \times 10^{-4}$	$-0.96^{+0.07}_{-0.04}$	$\Xi_c^0 \rightarrow nK_S^0 K_S^0$	$(5.44 \pm 0.63) \times 10^{-3}$	0.60 ± 0.40
$\Xi_c^+ \rightarrow \Sigma^+\eta^0 K_L^0$	$(1.71 \pm 0.99) \times 10^{-4}$	$-0.90^{+0.24}_{-0.10}$	$\Xi_c^0 \rightarrow nK_S^0 K_L^0$	$(5.69 \pm 0.64) \times 10^{-3}$	$0.67^{+0.33}_{-0.35}$
$\Xi_c^+ \rightarrow \Sigma^0\pi^+ K_S^0$	$(3.33 \pm 0.55) \times 10^{-2}$	-0.10 ± 0.60	$\Xi_c^0 \rightarrow nK_L^0 K_L^0$	$(6.00 \pm 0.68) \times 10^{-3}$	$0.76^{+0.24}_{-0.31}$
$\Xi_c^+ \rightarrow \Sigma^0\pi^+ K_L^0$	$(4.04 \pm 0.67) \times 10^{-2}$	0.00 ± 0.60	$\Xi_c^0 \rightarrow \Lambda^0\pi^0 K_S^0$	$(6.65 \pm 1.64) \times 10^{-3}$	0.00 ± 0.50
$\Xi_c^+ \rightarrow \Xi^0 K^+ K_S^0$	$(8.32 \pm 3.76) \times 10^{-4}$	$0.92^{+0.08}_{-0.20}$	$\Xi_c^0 \rightarrow \Lambda^0\pi^0 K_L^0$	$(5.19 \pm 1.39) \times 10^{-3}$	0.20 ± 0.50
$\Xi_c^+ \rightarrow \Xi^0 K^+ K_L^0$	$(9.82 \pm 4.76) \times 10^{-4}$	0.96 ± 0.02	$\Xi_c^0 \rightarrow \Sigma^0\eta^0 K_S^0$	$(4.61 \pm 0.78) \times 10^{-4}$	-0.70 ± 0.28

(Table continued)

TABLE X. (Continued)

Channels	\mathcal{B}	$\langle\alpha\rangle$	Channels	\mathcal{B}	$\langle\alpha\rangle$
$\Xi_c^+ \rightarrow \Lambda^0 \pi^+ K_S^0$	$(4.02 \pm 3.32) \times 10^{-3}$	$-0.70_{-0.30}^{+0.70}$	$\Xi_c^0 \rightarrow \Sigma^0 \eta^0 K_L^0$	$(5.01 \pm 0.83) \times 10^{-4}$	-0.65 ± 0.29
$\Xi_c^+ \rightarrow \Lambda^0 \pi^+ K_L^0$	$(2.32_{-2.32}^{+2.71}) \times 10^{-3}$	$-0.30_{-0.70}^{+0.90}$			
$\Xi_c^+ \rightarrow p K_S^0 K_S^0$	$(1.31 \pm 0.35) \times 10^{-2}$	$-0.65_{-0.35}^{+0.70}$			
$\Xi_c^+ \rightarrow p K_S^0 K_L^0$	$(1.23 \pm 0.34) \times 10^{-2}$	$-0.67_{-0.33}^{+0.70}$			
$\Xi_c^+ \rightarrow p K_L^0 K_L^0$	$(1.16 \pm 0.35) \times 10^{-2}$	$-0.69_{-0.31}^{+0.60}$			

Notably, our prediction of $\mathcal{B}(\Xi_c^+ \rightarrow p\pi^+K^-) = (28.3 \pm 2.6) \times 10^{-3}$ significantly exceeds the results of $(11 \pm 4) \times 10^{-3}$ from LHCb [14] and $(4.5 \pm 2.2) \times 10^{-3}$ from Belle [1]. It is interesting to point out that $A(\Xi_c^+ \rightarrow p\pi^+K^-) = A(\Lambda_c^+ \rightarrow \Sigma^+\pi^-K^+)$ in the exact $SU(3)_F$ symmetry, but their released energies are in great difference of 0.9 GeV and 0.5 GeV for the former and latter, respectively,² leading to the hierarchy of $\Gamma(\Xi_c^+ \rightarrow p\pi^+K^-) \approx 10\Gamma(\Lambda_c^+ \rightarrow \Sigma^+\pi^-K^+)$. However, this analysis is in sharp contrast to the experimental data, indicating that their decay widths are approximately the same. This implies a large $SU(3)_F$ flavor symmetry breaking effect, which may come from resonant hadrons. We strongly suggest that future experiments revise these two channels.

We note that the χ^2 fit suffers a Z_2 ambiguity of $\mathcal{B}(A, B) = \mathcal{B}(A, -B)$. The ambiguity can be broken with an input of $\langle\alpha\rangle$ as $\langle\alpha\rangle(A, B) = \langle\alpha\rangle(A, -B)$. Here, we choose the $\langle\alpha\rangle$ of $\Lambda_c^+ \rightarrow \Xi^0\pi^0K^+$ and $\Lambda_c^+ \rightarrow \Xi^0K^+\eta^0$ to be negative as their amplitudes are contributed by Fig. 2 mainly, where x and y have a negative helicity in the chiral limit. Consequently, the helicity of \mathbf{B}_n is negative also, leading to a negative averaged up-down asymmetry. We list the predictions for the up-down asymmetries of $\langle\alpha\rangle(\Lambda_c^+, \Xi_c^+, \Xi_c^0 \rightarrow \mathbf{B}_n PP')$ in Tables VII–IX. Meanwhile, without considering CP violation of physical particles K_S^0 and K_L^0 , we can also give branching ratios and up-down asymmetries of three-body decay channels involving K_S^0 and K_L^0 of mixed-modes, which are presented in Table X. These results are acquired under the assumption of S-wave meson pairs in the final states, neglecting the contributions from pseudoscalar meson exchanges, which can be used to assess the dominance of S-wave meson pairs in nonleptonic three-body decays.

²From Eq. (6), we see that $\Gamma \propto \Delta E^4$ if A and B are held as constant, with ΔE the released energy.

IV. CONCLUSIONS

In our study of the antitriplet charmed baryon three-body weak decay of $\mathbf{B}_c \rightarrow \mathbf{B}_n PP'$ within the framework of the $SU(3)_F$ flavor symmetry, we have incorporated contributions from both $H(\bar{\mathbf{6}})$ and $H(\mathbf{15})$ to decompose the decays into 16 real amplitudes. Through a minimum χ^2 fit to 28 updated experimental data points, we have achieved a new fit with $\chi^2/\text{d.o.f} = 1.5$. This significantly improved the fitting results, affirming the validity of $SU(3)_F$ in charmed baryon three-body decays. Based on the branch ratio fitting results, we have analyzed potential sources of errors, including the remaining resonant and excited state contributions in the experimental data, the contributions of a_9 and a_{10} in $H(\mathbf{15})$, and the P-wave contribution in the final states. We have given one of those predictions of $\mathcal{B}(\Lambda_c^+ \rightarrow (\Xi(1690)^0 \rightarrow \Sigma^+K^-)K^+) = (1.5 \pm 0.4) \times 10^{-3}$, which needs to be further explored by experiments. Meanwhile, the possible $SU(3)_F$ symmetry breaking effect in processes, such as $\Xi_c^+ \rightarrow p\pi^+K^-$ and $\Lambda_c^+ \rightarrow \Sigma^+\pi^-K^+$, is notable in theoretical research. We have found that in the chiral limit, the $\langle\alpha\rangle$ of $\Lambda_c^+ \rightarrow \Xi^0\pi^0K^+$ and $\Lambda_c^+ \rightarrow \Xi^0K^+\eta^0$ is negative, which enables us to fix Z_2 ambiguity in the $SU(3)_F$ fit. We have also updated decays involving K_S^0 and K_L^0 . In particular, we find that three decay channels of $\Lambda_c^+ \rightarrow n\pi^+\pi^0$, $\Lambda_c^+ \rightarrow \Lambda^0K^+K^0$ and $\Xi_c^+ \rightarrow \Lambda^0\pi^+\pi^0$ are only contributed by $H(\mathbf{15})$, prompting the further theoretical analysis in classification. Lastly, measurements with large errors of channels $\Xi_c^+ \rightarrow \Sigma^+\pi^+\pi^-$, $\Xi_c^+ \rightarrow \Xi^0\pi^+\pi^0$, $\Xi_c^+ \rightarrow \Sigma^-\pi^+\pi^+$, and $\Xi_c^+ \rightarrow \Sigma^+K^+K^-$ should be remeasured for higher precision.

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APPENDIX: A-AMPLITUDE OF $B_c \rightarrow B_n PP'$ TABLE XI. A-amplitudes of $\Lambda_c^+ \rightarrow B_n PP'$.

CF mode	Ac_c^{-2}	CF mode	Ac_c^{-2}
$\Lambda^0 \pi^+ \eta^0$	$-\frac{2}{3}(a_2 - a_3 + a_5 + 3a_6 - a_7 + a_8)$	$p\pi^0 \bar{K}^0$	$-\frac{1}{\sqrt{2}}(2a_3 + 2a_4 - a_7 + a_8)$
$\Lambda^0 K^+ \bar{K}^0$	$-\sqrt{\frac{2}{3}}(a_2 - a_3 + a_5 - a_7 - a_8)$	$p\bar{K}^0 \eta^0$	$\frac{1}{\sqrt{6}}(-2a_3 + 2a_4 + a_7 - a_8)$
$\Sigma^0 \pi^+ \pi^0$	$-2a_4 - 2a_6$	$p\pi^+ K^-$	$2a_3 - 2a_6 - a_7 - a_8$
$\Sigma^0 \pi^+ \eta^0$	$\frac{2}{\sqrt{3}}(a_2 + a_3 + a_4 + a_5)$	$n\pi^+ \bar{K}^0$	$-2a_4 - 2a_6 - 2a_8$
$\Sigma^0 K^+ \bar{K}^0$	$\sqrt{2}(a_2 + a_3 + a_5)$	$\Sigma^+ \pi^0 \pi^0$	$4a_1 + 2a_2 + 2a_3 + 2a_4 - 2a_5$
$\Sigma^- \pi^+ \pi^+$	$-4a_4 - 4a_6$	$\Sigma^+ \pi^0 \eta^0$	$-\frac{2}{\sqrt{3}}(a_2 + a_3 + a_4 + a_5)$
$\Xi^- \pi^+ K^+$	$-2a_6$	$\Sigma^+ \eta^0 \eta^0$	$\frac{2}{3}(6a_1 + a_2 + a_3 + a_4 - a_5)$
$\Xi^0 \pi^0 K^+$	$-\sqrt{2}a_5$	$\Sigma^+ \pi^+ \pi^-$	$4a_1 + 2a_2 + 2a_3 - 2a_5 - 2a_6$
$\Xi^0 K^+ \eta^0$	$\sqrt{\frac{2}{3}}a_5$	$\Sigma^+ K^+ K^-$	$4a_1 - 2a_5$
$\Xi^0 \pi^+ K^0$	$-2a_5 - 2a_6$	$\Sigma^+ K^0 \bar{K}^0$	$4a_1 + 2a_2 + 2a_3$
CS mode	$Ac_c^{-1} s_c^{-1}$	CS mode	$Ac_c^{-1} s_c^{-1}$
$\Lambda^0 \pi^0 K^+$	$\frac{1}{\sqrt{3}}(-a_2 + a_3 + 2a_5 + a_7 + a_8)$	$p\eta^0 \eta^0$	$\frac{1}{3}(12a_1 + 2a_2 + 8a_3 - 4a_4 - 2a_5 - 3a_7 + 3a_8)$
$\Lambda^0 K^+ \eta^0$	$-\frac{1}{3}(-a_2 + a_3 + 2a_5 + 6a_6 + a_7 + 5a_8)$	$p\pi^+ \pi^-$	$4a_1 + 2a_2 - 2a_5 + a_7 + a_8$
$\Lambda^0 \pi^+ K^0$	$-\sqrt{\frac{2}{3}}(a_2 - a_3 - 2a_5 - a_7 + a_8)$	$pK^+ K^-$	$4a_1 + 2a_3 - 2a_5 - 2a_6 - a_7 - a_8$
$\Sigma^0 \pi^0 K^+$	$a_2 + a_3 - 2a_4 - 2a_6$	$pK^0 \bar{K}^0$	$4a_1 + 2a_2 + 2a_3 + 2a_4$
$\Sigma^0 K^+ \eta^0$	$\frac{1}{\sqrt{3}}(-a_2 - a_3 + 2a_4)$	$n\pi^+ \pi^0$	$-\sqrt{2}a_8$
$\Sigma^0 \pi^+ K^0$	$\sqrt{2}(a_2 + a_3 + a_4)$	$n\pi^+ \eta^0$	$\sqrt{\frac{2}{3}}(-2a_2 + 2a_4 - 2a_5 + a_7 + 2a_8)$
$\Sigma^- \pi^+ K^+$	$-4a_4 - 2a_6$	$nK^+ \bar{K}^0$	$-2a_2 - 2a_4 - 2a_5 - 2a_6 + a_7 - a_8$
$\Xi^- K^+ K^+$	$-4a_6$	$\Sigma^+ \pi^0 K^0$	$-\sqrt{2}(a_2 + a_3 + 2a_4)$
$\Xi^0 K^+ K^0$	$-2a_6$	$\Sigma^+ K^0 \eta^0$	$\sqrt{\frac{2}{3}}(-a_2 - a_3 + 2a_4)$
$p\pi^0 \pi^0$	$4a_1 + 2a_2 - 2a_5 + a_7 - a_8$	$\Sigma^+ \pi^- K^+$	$2a_2 + 2a_3 - 2a_6$
$p\pi^0 \eta^0$	$\frac{1}{\sqrt{3}}(-2a_2 + 2a_4 - 2a_5 - a_7 + a_8)$		
DCS mode	As_c^{-2}	DCS mode	As_c^{-2}
$\Lambda^0 K^+ K^0$	$-\sqrt{\frac{8}{3}}a_8$	$p\pi^- K^+$	$2a_2 + a_7 + a_8$
$\Sigma^0 K^+ K^0$	$2\sqrt{2}a_4$	$n\pi^0 K^+$	$-\frac{1}{\sqrt{2}}(2a_2 - a_7 + a_8)$
$\Sigma^- K^+ K^+$	$-4a_4$	$nK^+ \eta^0$	$\frac{1}{\sqrt{6}}(2a_2 + 4a_4 - a_7 + a_8)$
$p\pi^0 K^0$	$\frac{1}{\sqrt{2}}(-2a_2 - a_7 + a_8)$	$n\pi^+ K^0$	$-2a_2 + a_7 + a_8$
$pK^0 \eta^0$	$-\frac{1}{\sqrt{6}}(2a_2 + 4a_4 + a_7 - a_8)$	$\Sigma^+ K^0 K^0$	$4a_4$

TABLE XII. A-amplitudes of $\Xi_c^+ \rightarrow \mathbf{B}_n PP'$.

CF mode	Ac_c^{-2}	CF mode	Ac_c^{-2}
$\Lambda^0 \pi^+ \bar{K}^0$	$\sqrt{\frac{2}{3}}(3a_4 + a_8)$	$\Xi^0 K^+ \bar{K}^0$	$-2a_2 + a_7 + a_8$
$\Sigma^0 \pi^+ \bar{K}^0$	$\sqrt{2}(a_4 - a_8)$	$p \bar{K}^0 \bar{K}^0$	$4a_4$
$\Xi^- \pi^+ \pi^+$	$-4a_4$	$\Sigma^+ \pi^0 \bar{K}^0$	$\frac{1}{\sqrt{2}}(-2a_2 - 2a_4 - a_7 + a_8)$
$\Xi^0 \pi^+ \pi^0$	$\sqrt{2}a_4$	$\Sigma^+ \bar{K}^0 \eta^0$	$\frac{1}{\sqrt{6}}(-2a_2 + 2a_4 - a_7 + a_8)$
$\Xi^0 \pi^+ \eta^0$	$-\sqrt{\frac{2}{3}}(2a_2 + a_4 - a_7 + a_8)$	$\Sigma^+ \pi^+ K^-$	$2a_2 + a_7 + a_8$
CS mode	$Ac_c^{-1} s_c^{-1}$	CS mode	$Ac_c^{-1} s_c^{-1}$
$\Lambda^0 \pi^+ \pi^0$	$\frac{1}{\sqrt{3}}a_8$	$p \pi^0 \bar{K}^0$	$-\sqrt{2}(a_2 + a_3)$
$\Lambda^0 \pi^+ \eta^0$	$-\frac{1}{3}(-4a_2 - 2a_3 + 6a_4 + 2a_5 + 6a_6 + a_7 + 2a_8)$	$p \bar{K}^0 \eta^0$	$-\sqrt{\frac{2}{3}}(a_2 + a_3 + 4a_4)$
$\Lambda^0 K^+ \bar{K}^0$	$\frac{1}{\sqrt{6}}(4a_2 + 2a_3 + 6a_4 - 2a_5 - a_7 + a_8)$	$p \pi^+ K^-$	$2a_2 + 2a_3 - 2a_6$
$\Sigma^0 \pi^+ \pi^0$	$-2a_6 - a_8$	$n \pi^+ \bar{K}^0$	$-2a_6$
$\Sigma^0 \pi^+ \eta^0$	$\frac{1}{\sqrt{3}}(2a_3 - 2a_4 + 2a_5 + a_7 + 2a_8)$	$\Sigma^+ \pi^0 \pi^0$	$4a_1 + 2a_3 - 2a_5 - a_7 + a_8$
$\Sigma^0 K^+ \bar{K}^0$	$\frac{1}{\sqrt{2}}(2a_3 + 2a_4 + 2a_5 + a_7 - a_8)$	$\Sigma^+ \pi^0 \eta^0$	$\frac{1}{\sqrt{3}}(-2a_3 + 2a_4 - 2a_5 + a_7 - a_8)$
$\Sigma^- \pi^+ \pi^+$	$-4a_6$	$\Sigma^+ \eta^0 \eta^0$	$\frac{1}{3}(12a_1 + 8a_2 + 2a_3 - 4a_4 - 2a_5 + 3a_7 - 3a_8)$
$\Xi^- \pi^+ K^+$	$-4a_4 - 2a_6$	$\Sigma^+ \pi^+ \pi^-$	$4a_1 + 2a_3 - 2a_5 - 2a_6 - a_7 - a_8$
$\Xi^0 \pi^0 K^+$	$\frac{1}{\sqrt{2}}(-2a_2 + 2a_4 - 2a_5 + a_7 + a_8)$	$\Sigma^+ K^+ K^-$	$4a_1 + 2a_2 - 2a_5 + a_7 + a_8$
$\Xi^0 K^+ \eta^0$	$\frac{1}{\sqrt{6}}(2a_2 - 2a_4 + 2a_5 - a_7 - 5a_8)$	$\Sigma^+ K^0 \bar{K}^0$	$4a_1 + 2a_2 + 2a_3 + 2a_4$
$\Xi^0 \pi^+ K^0$	$-2a_2 - 2a_4 - 2a_5 - 2a_6 + a_7 - a_8$		
DCS mode	As_c^{-2}	DCS mode	As_c^{-2}
$\Lambda^0 \pi^0 K^+$	$\frac{1}{2\sqrt{3}}(4a_2 + 2a_3 + 4a_5 - a_7 + a_8)$	$p \pi^0 \eta^0$	$-\frac{2}{\sqrt{3}}a_5$
$\Lambda^0 K^+ \eta^0$	$-\frac{1}{6}(4a_2 + 2a_3 + 12a_4 + 4a_5 + 12a_6 - a_7 + a_8)$	$p \eta^0 \eta^0$	$\frac{2}{3}(6a_1 + 4a_2 + 4a_3 + 4a_4 - a_5)$
$\Lambda^0 \pi^+ K^0$	$\frac{1}{\sqrt{6}}(4a_2 + 2a_3 + 4a_5 - a_7 - a_8)$	$p \pi^+ \pi^-$	$4a_1 - 2a_5$
$\Sigma^0 \pi^0 K^+$	$\frac{1}{2}(2a_3 - 4a_6 + a_7 - a_8)$	$p K^+ K^-$	$4a_1 + 2a_2 + 2a_3 - 2a_5 - 2a_6$
$\Sigma^0 K^+ \eta^0$	$-\frac{1}{2\sqrt{3}}(2a_3 + 4a_4 + a_7 - a_8)$	$p K^0 \bar{K}^0$	$4a_1 + 2a_2 + 2a_3$
$\Sigma^0 \pi^+ K^0$	$\frac{1}{\sqrt{2}}(2a_3 + a_7 + a_8)$	$n \pi^+ \eta^0$	$-\sqrt{\frac{8}{3}}a_5$
$\Sigma^- \pi^+ K^+$	$-2a_6$	$n K^+ \bar{K}^0$	$-2a_5 - 2a_6$
$\Xi^- K^+ K^+$	$-4a_4 - 4a_6$	$\Sigma^+ \pi^0 K^0$	$\frac{1}{\sqrt{2}}(-2a_3 + a_7 - a_8)$
$\Xi^0 K^+ K^0$	$-2a_4 - 2a_6 - 2a_8$	$\Sigma^+ K^0 \eta^0$	$-\frac{1}{\sqrt{6}}(2a_3 + 4a_4 - a_7 + a_8)$
$p \pi^0 \pi^0$	$4a_1 - 2a_5$	$\Sigma^+ \pi^- K^+$	$2a_3 - 2a_6 - a_7 - a_8$

TABLE XIII. A-amplitudes of CF mode and CS mode $\Xi_c^0 \rightarrow \mathbf{B}_n PP'$.

CF mode	Ac_c^{-2}	CF mode	Ac_c^{-2}
$\Lambda^0 \pi^0 \bar{K}^0$	$-\frac{1}{2\sqrt{3}}(2a_2 + 4a_3 + 6a_4 + 2a_5 - a_7 + a_8)$	$\Xi^0 \pi^0 \eta^0$	$\frac{2}{\sqrt{3}}(a_2 + a_3 + a_4)$
$\Lambda^0 \bar{K}^0 \eta^0$	$-\frac{1}{6}(2a_2 + 4a_3 - 6a_4 + 2a_5 - 12a_6 - a_7 + a_8)$	$\Xi^0 \eta^0 \eta^0$	$-\frac{2}{3}(6a_1 + a_2 + a_3 + a_4 - 4a_5)$
$\Lambda^0 \pi^+ K^-$	$\frac{1}{\sqrt{6}}(2a_2 + 4a_3 + 2a_5 - a_7 - a_8)$	$\Xi^0 \pi^+ \pi^-$	$-4a_1 - 2a_2 - 2a_3$
$\Sigma^0 \pi^0 \bar{K}^0$	$\frac{1}{2}(2a_2 + 2a_4 + 2a_5 + 4a_6 + a_7 - a_8)$	$\Xi^0 K^+ K^-$	$-4a_1 + 2a_5$
$\Sigma^0 \bar{K}^0 \eta^0$	$\frac{1}{2\sqrt{3}}(2a_2 - 2a_4 + 2a_5 + a_7 - a_8)$	$\Xi^0 K^0 \bar{K}^0$	$-4a_1 - 2a_2 - 2a_3 + 2a_5 + 2a_6$
$\Sigma^0 \pi^+ K^-$	$-\frac{1}{\sqrt{2}}(2a_2 + 2a_5 + a_7 + a_8)$	$pK^- \bar{K}^0$	$2a_6$
$\Sigma^- \pi^+ \bar{K}^0$	$2a_4 + 2a_6 - 2a_8$	$n\bar{K}^0 \bar{K}^0$	$4a_4 + 4a_6$
$\Xi^- \pi^+ \pi^0$	$\sqrt{2}a_4$	$\Sigma^+ \pi^0 K^-$	$\sqrt{2}a_5$
$\Xi^- \pi^+ \eta^0$	$-\sqrt{\frac{2}{3}}(2a_3 + a_4 + a_7 - a_8)$	$\Sigma^+ K^- \eta^0$	$-\sqrt{\frac{2}{3}}a_5$
$\Xi^- K^+ \bar{K}^0$	$-2a_3 + 2a_6 - a_7 - a_8$	$\Sigma^+ \pi^- \bar{K}^0$	$2a_5 + 2a_6$
$\Xi^0 \pi^0 \pi^0$	$-4a_1 - 2a_2 - 2a_3 - 2a_4$		
CS mode	$Ac_c^{-1} s_c^{-1}$	CS mode	$Ac_c^{-1} s_c^{-1}$
$\Lambda^0 \pi^0 \pi^0$	$\frac{1}{\sqrt{6}}(12a_1 + 4a_2 + 2a_3 - 2a_5 + a_7 - a_8)$	$\Xi^- \pi^0 K^+$	$-\frac{1}{\sqrt{2}}(2a_3 - 2a_4 - 2a_6 + a_7 + a_8)$
$\Lambda^0 \pi^0 \eta^0$	$\frac{1}{3\sqrt{2}}(-4a_2 - 2a_3 + 6a_4 + 2a_5 + 6a_6 - a_7 + a_8)$	$\Xi^- K^+ \eta^0$	$\frac{1}{\sqrt{6}}(2a_3 - 2a_4 - 6a_6 + a_7 + 5a_8)$
$\Lambda^0 \eta^0 \eta^0$	$\frac{1}{\sqrt{6}}(12a_1 + 4a_2 + 6a_3 - 4a_4 - 6a_5 - 12a_6 - a_7 + a_8)$	$\Xi^- \pi^+ K^0$	$-2a_3 - 2a_4 - a_7 + a_8$
$\Lambda^0 \pi^+ \pi^-$	$\frac{1}{\sqrt{6}}(12a_1 + 4a_2 + 2a_3 - 2a_5 + a_7 + a_8)$	$\Xi^0 \pi^0 K^0$	$\sqrt{2}(a_2 + a_3 + 2a_4 + a_5 + a_6)$
$\Lambda^0 K^+ K^-$	$\frac{1}{\sqrt{6}}(12a_1 + 2a_2 + 4a_3 - 4a_5 - a_7 - a_8)$	$\Xi^0 K^0 \eta^0$	$\sqrt{\frac{2}{3}}(a_2 + a_3 - 2a_4 + a_5 - 3a_6)$
$\Lambda^0 K^0 \bar{K}^0$	$\sqrt{6}(2a_1 + a_2 + a_3 + a_4 - a_5)$	$\Xi^0 \pi^- K^+$	$-2a_2 - 2a_3 - 2a_5$
$\Sigma^0 \pi^0 \pi^0$	$-\frac{1}{\sqrt{2}}(4a_1 + 2a_3 - 2a_5 - 4a_6 - a_7 + a_8)$	$p\pi^0 K^-$	$\sqrt{2}(a_5 + a_6)$
$\Sigma^0 \pi^0 \eta^0$	$\frac{1}{\sqrt{6}}(2a_3 - 2a_4 - 2a_5 - 6a_6 - a_7 + a_8)$	$pK^- \eta^0$	$-\sqrt{\frac{2}{3}}(a_5 + 3a_6)$
$\Sigma^0 \eta^0 \eta^0$	$-\frac{1}{3\sqrt{2}}(12a_1 + 8a_2 + 2a_3 - 4a_4 - 2a_5 + 3a_7 - 3a_8)$	$p\pi^- \bar{K}^0$	$2a_5$
$\Sigma^0 \pi^+ \pi^-$	$\frac{1}{2}(-4a_1 - 2a_3 + 2a_5 + a_7 + a_8)$	$n\pi^0 \bar{K}^0$	$-\sqrt{2}(a_2 + a_3 + a_5 - a_6)$
$\Sigma^0 K^+ K^-$	$-\frac{1}{\sqrt{2}}(4a_1 + 2a_2 + a_7 + a_8)$	$n\bar{K}^0 \eta^0$	$-\sqrt{\frac{2}{3}}(a_2 + a_3 + 4a_4 + a_5 + 3a_6)$
$\Sigma^0 K^0 \bar{K}^0$	$-\sqrt{2}(2a_1 + a_2 + a_3 + a_4 - a_5)$	$n\pi^+ K^-$	$2a_2 + 2a_3 + 2a_5$
$\Sigma^- \pi^+ \pi^0$	$\sqrt{2}(a_6 - a_8)$	$\Sigma^+ \pi^0 \pi^-$	$\sqrt{2}a_6$
$\Sigma^- \pi^+ \eta^0$	$\sqrt{\frac{2}{3}}(2a_3 - 2a_4 - 3a_6 + a_7 + 2a_8)$	$\Sigma^+ \pi^- \eta^0$	$-\sqrt{\frac{2}{3}}(2a_5 + 3a_6)$
$\Sigma^- K^+ \bar{K}^0$	$2a_3 + 2a_4 + a_7 - a_8$	$\Sigma^+ K^0 K^-$	$-2a_5$

TABLE XIV. A-amplitudes of DCS mode $\Xi_c^0 \rightarrow \mathbf{B}_n PP'$.

DCS mode	As_c^{-2}	DCS mode	As_c^{-2}
$\Lambda^0 \pi^0 K^0$	$\frac{1}{2\sqrt{3}}(-4a_2 - 2a_3 - 4a_5 - a_7 + a_8)$	$\Xi^0 K^0 K^0$	$-4a_4 - 4a_6$
$\Lambda^0 K^0 \eta^0$	$-\frac{1}{6}(4a_2 + 2a_3 + 12a_4 + 4a_5 + 12a_6 + a_7 - a_8)$	$p\pi^-\eta^0$	$-\sqrt{\frac{8}{3}}a_5$
$\Lambda^0 \pi^- K^+$	$\frac{1}{\sqrt{6}}(4a_2 + 2a_3 + 4a_5 + a_7 + a_8)$	$pK^0 K^-$	$-2a_5 - 2a_6$
$\Sigma^0 \pi^0 K^0$	$\frac{1}{2}(2a_3 - 4a_6 - a_7 + a_8)$	$n\pi^0 \pi^0$	$4a_1 - 2a_5$
$\Sigma^0 K^0 \eta^0$	$\frac{1}{2\sqrt{3}}(2a_3 + 4a_4 - a_7 + a_8)$	$n\pi^0 \eta^0$	$\frac{2}{\sqrt{3}}a_5$
$\Sigma^0 \pi^- K^+$	$\frac{1}{\sqrt{2}}(-2a_3 + a_7 + a_8)$	$n\eta^0 \eta^0$	$\frac{2}{3}(6a_1 + 4a_2 + 4a_3 + 4a_4 - a_5)$
$\Sigma^- \pi^0 K^+$	$\frac{1}{\sqrt{2}}(2a_3 + a_7 - a_8)$	$n\pi^+ \pi^-$	$4a_1 - 2a_5$
$\Sigma^- K^+ \eta^0$	$-\frac{1}{\sqrt{6}}(2a_3 + 4a_4 + a_7 - a_8)$	$nK^+ K^-$	$4a_1 + 2a_2 + 2a_3$
$\Sigma^- \pi^+ K^0$	$2a_3 - 2a_6 + a_7 + a_8$	$nK^0 \bar{K}^0$	$4a_1 + 2a_2 + 2a_3 - 2a_5 - 2a_6$
$\Xi^- K^+ \bar{K}^0$	$-2a_4 - 2a_6 + 2a_8$	$\Sigma^+ \pi^- K^0$	$-2a_6$

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