Correlation function of flavored fermion in holographic QCD

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By using the gauge-gravity duality, we investigate the correlation function of flavored fermion in the $\mathrm{D}_p/\mathrm{D}_{p+4}$ model as top-down approaches of holographic QCD for $p = 4, 3$. The bulk spinor, as the source of the flavored fermion in QCD, is identified to the worldvolume fermion on the flavor D_{p+4}-branes and the standard form of its action can be therefore obtained by the T-duality rules in string theory. Keeping this in hand, we afterwards generalize the prescription for two-point correlation function in the AdS/CFT dictionary into general D-brane backgrounds and apply it to the case of $p = 4, 3$, i.e., the D4/D8 and D3/D7 approaches, respectively. Resultantly, our numerical calculation with the bubble background always displays discrete peaks in the correlation functions which imply the bound states created by the flavored fermions as the confinement in QCD. With the black brane background, the on-shell condition illustrated by the correlation function covers basically the dispersion curves of fermion obtained by the hard thermal loop approximation in the hot medium. Finally, we interpret the flavored fermions in the bubble background as baryons by taking into account a baryon vertex, then find the two-point correlation function is able to fit the lowest baryon spectrum. In this sense, we conclude remarkably that our top-down approach in this work could reveal the fundamental properties of QCD both in the confined and deconfined phases.

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I. INTRODUCTION

The gauge-gravity duality and AdS/CFT correspondence have become very powerful tools for investigating strongly coupled quantum field theory (QFT) by analyzing the corresponding classical gravity system [1–3] in holography. Particularly, the real-time two-point correlation functions of (composite) operators are significant to understand the QFT at finite temperature or with dense matter since the information about the collective behavior, e.g., the properties of transport, existence of (quasi-) particles, are encoded in them. The prescription to compute the two-point retarded Green functions in AdS/CFT has been justified in many different ways, e.g., in literature [4–15], which have instrumentally extracted many important insights about strongly coupled QFT systems from AdS/CFT. On the other hand, quantum chromodynamics (QCD) is the underlying theory to describe the property of strong interaction, however it is extremely complex to solve in the low-energy (strongly coupled) region due to its asymptotic freedom, especially at finite temperature with dense matter [15,16]. Therefore investigating QCD through gauge-gravity duality naturally becomes an interesting topic. Although there are several bottom-up models, e.g., [17–19] attempting to give a holographic version of QCD, the top-down approaches based on D4/D8 (the Witten-Sakai-Sugimoto model [20–22]) and D3/D7 [14] models are the most famous and successful achievements in holography since almost all the elementary features of QCD are included in these models in a very simple way [23–25].

Moreover, it is known that the self-energy of fermion has been studied with a very long history and attracted many interests since fermion is one of the fundamental elements in QFT as well as in nuclear physics. For example, in a weakly coupled system, the self-energy of fermion can be analyzed by the perturbation method of QFT or be characterized by the thermal mass in the hot medium [26,27] while the results may not hold in the strong coupling regime [28,29]. Hence there are many researches about fermionic correlations in order to investigate the properties of the strongly coupled fermionic system [11,12,30–33] in holography. As we can see, the presented works about holographic fermion are based on bottom-up approaches or with minimal coupled fermions, and the bulk fermionic field is identified to the fundamental representation of a $U(N)$ group when the chemical potential is taken into account especially. However, such a fermionic field in bulk is less clear in the top-down models, e.g., D4/D8 and D3/D7 approaches.

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Motivated by this issue, in this work we attempt to find a holographic correspondence to evaluate the two-point correlation function of the flavored fermion in $\mathrm{D}_p/\mathrm{D}_{p+4}$ model (for $p = 4, 3$, i.e., the D4/D8 and D3/D7 approach) as two top-down approaches to holographic QCD. Specifically, the $\mathrm{D}_p/\mathrm{D}_{p+4}$ model holographically dual to QCD consists of N_c D_p-branes as colors and N_f probe D_{p+4}-branes as flavors which are intersected to each other. In the large N_c limit, the dynamics of the bulk geometry is described approximately by the type II supergravity. Due to the supersymmetry on the N_f probe D_{p+4}-branes, we identify the bulk spinor (as the source of the flavored fermion) to the worldvolume fermionic field on the probe D_{p+4}-branes and this may be the only consistent correspondence by analyzing the bulk spectrum in this holographic system. In this sense, the action of the bulk fermion can be obtained by the T-duality rules in string theory [34,35] and it reveals the bulk fermion is not minimally coupled. Keeping this in hand, we further generalize the prescription for two-point correlation function in AdS/CFT dictionary into general D-brane backgrounds with respect to the flavored fermion in the dual theory. Afterwards, we apply our method to the D4/D8 and D3/D7 models as tests in order to evaluate the two-point retarded Green functions variously.

Our numerical calculation displays that the retarded Green functions behave similarly in the D4/D8 and D3/D7 approaches. That is, with bubble background, the Green functions always contain many discrete peaks representing various bound states in QCD and it is consistent with the QCD confinement since the bubble background corresponds to the confined phase of QCD in holography [14,20,21,23,25,36,37]. Remarkably, the bound energies obtained in the Green function also agree with the numerical evaluation in [36] for $p = 4$ quantitatively. With the black brane background, the particle on-shell condition obtained from the Green functions covers the dispersion curves obtained from the hard thermal loop approximation and the effective thermal mass of fermion can be also determined numerically while the chemical potential is not turned on in our current work. We notice the behaviors of confined Green functions in our top-down approaches are very different from those in the bottom-up approaches or with minimal coupled fermion [30–33], while the deconfined Green functions (obtained from the black brane background) behave similarly. Thus it illustrates seemingly that the property of QCD confinement is less clear in the previous works with bottom-up models or with minimally coupled fermion, e.g., [30–33]. Finally, we discuss how to interpret the fermionic bound states in the confined phase as baryon states by including a baryonic brane as baryon vertex [38] with the framework of $\mathrm{D}_p/\mathrm{D}_{p+4}$ model which may support that the open strings on the D-brane behave somehow as baryons [39,40].

The outline of this manuscript is as follows. In Sec. II, we review the $\mathrm{D}_p/\mathrm{D}_{p+4}$ model as holographic QCD briefly for the case of $p = 4, 3$. In Sec. III, we discuss how to identify the bulk spinor, its associated action, and the holographic prescription to compute the two-point correlation function in D-brane background. In Secs. 4 and 5, we apply our method to D4/D8 and D3/D7 models respectively in order to evaluate the retarded Green functions for flavored fermions then analyze the numerical results. Summary and discussion are given in Sec. VI.

II. THE $\mathrm{D}_p/\mathrm{D}_{p+4}$ MODEL AS HOLOGRAPHIC QCD

In this section, let us review the holographic QCD through gauge-gravity duality based on the type II string theories. In order to obtain a dual field theory close to QCD, we start with the D-brane system consisting of N_c coincident D_p-branes and N_f coincident D_{p+4}-branes.

Taking the near-horizon limit $\alpha' \to 0$ and the large N_c limit by keeping N_f finite, the bulk dynamic is described approximately by the type II supergravity (SUGRA) for N_c coincident D_p-branes whose action takes the following form as [37,41],

$$S_{\mathrm{II}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left[e^{-2\phi}(\mathcal{R} + 4\partial_M\phi\partial^M\phi) - \frac{g_s^2}{2}|F_{p+2}|^2\right], \tag{2.1}$$

where $\mathcal{R}, \phi, C_{p+1}$ is respectively the 10-dimensional (10d) scalar curvature, the dilaton and the Ramond-Ramond (R-R) $p+1$-form field with its strength $F_{p+2} = dC_{p+1}$. The constant κ_{10} is the gravity coupling constant related to the 10d Newtonian constant G_{10} as $2\kappa_{10}^2 = 16\pi G_{10} = (2\pi)^7 l_s^8 g_s^2$ where the string coupling constant and string length is denoted by g_s, l_s. Since we will discuss the D3/D7 and D4/D8 approaches based on IIB and IIA string theory respectively, the cases for $p = 3, 4$ are only considered here. Vary the action (2.1) with respect to the metric, the dilaton, and the R-R field, then the associated equations of motion can be obtained accordingly and they can be solved in general as the black brane solution [41],

$$\begin{aligned} ds^2 &= H_p^{-\frac{1}{2}}[f(r)dt^2 + d\vec{x}\cdot d\vec{x}] + H_p^{\frac{1}{2}}\left[\frac{dr^2}{f(r)} + r^2 d\Omega_{8-p}^2\right], \\ e^\phi &= H_p^{-\frac{p-3}{4}}, \qquad C_{01\ldots p} = g_s^{-1}H_p^{-1}, \\ F_{r01\ldots p} &= \frac{(7-p)g_s^{-1}h_p^{7-p}}{r^{8-p}H_p^2}, \end{aligned} \tag{2.2}$$

where the functions $f(r), H_p(r)$ are given respectively as,

$$f(r) = 1 - \frac{r_H^{7-p}}{r^{7-p}}, \qquad H_p(r) = 1 + \frac{r_p^{7-p}}{r^{7-p}}. \tag{2.3}$$

Here r is the radial coordinate of the bulk which corresponds usually to the holographic direction. r_H, r_p are two constants related respectively to the position of the horizon and the R-R charge carried by the D_p-branes.[1] We note that in the near-horizon limit, the harmonic function $H_p(r)$ becomes $H_p(r) \to r_p^{7-p}/r^{7-p}$.

The dual field theory associated with the bulk geometry (2.2) in the large N_c limit can be examined by taking into account a probe D_p-brane located at the holographic boundary $r \to \infty$. And as it is known, the resultantly dual field theory is respectively the $\mathcal{N} = 4$ super Yang-Mills (YM) theory [14] for $p = 3$ and the type $\mathcal{N} = (2, 0)$ super conformal field theory (SCFT) reduced on a circle [20,37] for $p = 4$, however both of them are less close to QCD due to the presence of supersymmetry. Thus Witten proposed a scheme [20] to construct the bulk geometry presented in (2.2) in order to obtain a dual field theory close to QCD with confinement. Specifically, the first step is to perform the double Wick rotation $\{x^0 \to -ix^p, x^p \to -ix^0\}$ on the D_p-brane where $x^0 = t, x^p$ refer to the time and the pth spacial direction of the D_p-brane. So the geometry (2.2) becomes a bubble configuration as

$$
\begin{aligned}
ds^2 &= H_p^{-\frac{1}{2}}[\eta_{\mu\nu}dx^\mu dx^\nu + f(r)(dx^p)^2] \\
&\quad + H_p^{\frac{1}{2}}\left[\frac{dr^2}{f(r)} + r^2 d\Omega_{8-p}^2\right], \\
f(r) &= 1 - \frac{r_{KK}^{7-p}}{r^{7-p}}, \qquad H_p(r) = 1 + \frac{r_p^{7-p}}{r^{7-p}}, \\
\mu, \nu &= 0, 1 \ldots p-1,
\end{aligned}
\tag{2.4}
$$

which is defined only for $r \in [r_{KK}, \infty)$. We have replaced r_H with r_{KK} in (2.4) because there is not a horizon in the bubble configuration. Notice that in the black brane solution (2.2), $x^0 = t$ is compactified on a circle which implies in the bubble case x^p becomes now periodic as

$$
x^p \sim x^p + \delta x^p, \qquad \delta x^p = \frac{2\pi}{M_{KK}}, \tag{2.5}
$$

where M_{KK} is the Klein-Kaluza (KK) energy scale. So the second step to finding a QCD-like theory is to get rid of all massless field other than the gauge fields by imposing the periodic and antiperiodic boundary condition to bosons and supersymmetric fermions respectively along the periodic direction x^p. Therefore the supersymmetric fermion acquires mass of order M_{KK} which would be decoupled to the low-energy theory, while the gauge boson remains to be massless. Accordingly, the dual field theory on the D_p-brane is p-dimensional pure $U(N_c)$ Yang-Mills theory in the large N_c limit below the energy scale M_{KK}. We note that the wrap factor $H_p(r)$ in (2.4) never goes to zero, hence the behavior of the Wilson loop in this geometry obeys the area law which implies the dual field theory will exhibit confinement. Altogether, these D_p-branes compactified on a circle correspond holographically to the color sector of QCD and its number N_c is identified to the color number. The $U(N_c)$ gauge field on the D_p-branes is therefore identified as the gluon field and in the large N_c limit, the bulk geometry is described by (2.4) with confinement. To further obtain a deconfined phase of QCD at finite temperature,[2] we can compactify the pth direction in the black brane solution (2.2) as (2.5) then follow the same discussion as it is in the confined geometry to construct the dual theory [14,20,37]. Afterwards the dual theory could be identified as deconfined QCD at finite temperature since there is a horizon in (2.2) introducing the Hawking temperature in the dual theory.

TABLE I. The D-brane configuration of the D_p/D_{p+4} system for $p = 3$, 4. “-” represents that the D-branes extend along this direction and x^p is the periodic direction.

	x^μ	x^p	$x^{p+1}(r)$	$x^{p+2}, \ldots x^{p+5}$	$x^{p+6}, \ldots x^9$
D_p	-	-			
D_{p+4}	-		-	-	

As QCD also includes fermions as flavors, the existing N_f coincident D_{p+4}-branes in this holographic model can just play the role of flavor. The D-brane configuration of the D_p/D_{p+4} system is given in Table I for the D3/D7 and D4/D8 approaches in this manuscript. Since N_f is set to be finite, the D_{p+4}-branes become the probes embedded into (2.2) or (2.4) and $N_c \to \infty$ implies D_p-brane dominates the bulk geometry. By analyzing the spectrum of the open string connecting the N_c D_p-branes and N_f D_{p+4}-branes in R-sector, the low-energy theory includes fermions in the fundamental representation of $U(N_c)$ and $U(N_f)$ which can be identified nicely as the fundamental quarks in this model [14,21,22]. In this sense, the dual theory contains both colors and flavors, thus it is expected to be the holographic version of QCD.

III. THE HOLOGRAPHIC CORRESPONDENCE WITH FLAVORED FERMION

As our goal is to work out the two-point correlation function of the flavored fermions through gauge-gravity

[1] In our solution, the constant h_p is defined as $(h_p^{7-p})^2 = (r_p^{7-p})^2 + r_p^{7-p} r_H^{7-p}$.

[2] Identifying the black brane solution (2.2) to the deconfined phase of QCD may contain some issues [42,43]. Nonetheless we will continue our discussion by this identification as most works with the D_p/D_{p+4} system. The reason is that the black brane solution corresponds to the dual theory at finite temperature which may include some properties of hot QCD, close to the deconfinement in QCD.

duality, in this section, let us attempt to find the associated holographic correspondence in the top-down approach first, then discuss how to generalize the prescription in the AdS/CFT dictionary into a nonconformal background to compute the Green function.

A. Identification of the bulk spinor

To begin with, let us recall the principle of the AdS/CFT which means in general the partition function of the dual field theory Z_{QCD} is equal to its gravitational partition function Z_{gravity} in the bulk. Since the dual theory living in the boundary $\partial\mathcal{M}$ of (2.2) or (2.4) would be expected to be QCD as it is reviewed in Sec. I, we use Z_{QCD} here to refer to its partition function. For a spinor ψ in the bulk $\mathcal{M}$ whose boundary value is $\psi_0 = \lim_{r\to\infty}\psi$, we can write down

$$Z_{\mathrm{QCD}}[\bar{\psi}_0,\psi_0] = Z_{\mathrm{gravity}}[\bar{\psi},\psi], \tag{3.1}$$

with

$$Z_{\mathrm{QCD}}[\bar{\psi}_0,\psi_0] = \left\langle \exp\left\{\int_{\partial\mathcal{M}}(\bar{\chi}\psi_0+\bar{\psi}_0\chi)d^Dx\right\}\right\rangle,$$
$$Z_{\mathrm{gravity}}[\bar{\psi},\psi] = \exp\left\{\int_{\mathcal{M}}\mathcal{L}^{\mathrm{ren}}_{\mathrm{gravity}}[\bar{\psi},\psi]d^{D+1}x\right\}. \tag{3.2}$$

And $\mathcal{L}^{\mathrm{ren}}_{\mathrm{gravity}}$ refers to the classical renormalized Lagrangian of the bulk field ψ. In our concern, χ refers to the flavored fermion in the dual field theory (QCD). Then the two-point retarded Green function G_R for the spinor χ can be evaluated by following [10,13,14] as

$$\langle\chi(\omega,\vec{k})\rangle = G_R(\omega,\vec{k})\psi_0(\omega,\vec{k}), \tag{3.3}$$

and

$$\langle\bar{\chi}(\omega,\vec{k})\rangle = -\frac{\delta S^{\mathrm{ren}}_{\mathrm{gravity}}}{\delta\psi_0} = \Pi_0(\omega,\vec{k}),$$
$$S^{\mathrm{ren}}_{\mathrm{gravity}} = \int_{\mathcal{M}}\mathcal{L}^{\mathrm{ren}}_{\mathrm{gravity}}[\bar{\psi}_0,\psi_0]d^{D+1}x. \tag{3.4}$$

Here $\omega,\vec{k}$ refers respectively to the frequency and 3-momentum of the associated Fourier modes. We note that the relation of the Euclidean Green function G_E and the real-time Green function G_R is given by $G_E(\omega_E,\vec{k}) = G_R(\omega,\vec{k})$ with $\omega_E = -i\omega$.

The next step is to identify the bulk spinor field ψ in order to investigate its action $S^{\mathrm{ren}}_{\mathrm{gravity}}[\psi]$ in holography. We notice that, in several bottom-up or phenomenological holographic approaches, e.g., [12,30–33], the bulk field ψ is usually identified to a fundamental fermion of the $U(N)$ group whose action is the minimally coupled Dirac action. However such a bulk field ψ is less clear in the $\mathrm{D}_p/\mathrm{D}_{p+4}$-brane system with a bosonic background (2.2) or (2.4) since in general, the bulk modes created by the strings in the type II string theory do not include a $U(N)$ fundamental fermion. To figure out this problem, it is worth retaking into account the interaction of the open strings in the $\mathrm{D}_p/\mathrm{D}_{p+4}$-brane system according to the string theory. For example, it is known that the AdS/CFT dictionary for the approach of vector with a source current

$$\left\langle \exp\left\{\int_{\partial\mathcal{M}} J^\mu A^{(0)}_\mu d^Dx\right\}\right\rangle = \exp\{-S_{\mathrm{bulk}}[A_\mu]\},$$
$$A^{(0)}_\mu = \lim_{r\to\infty}A_\mu \tag{3.5}$$

illustrates that the correlator of the flavored current operator J^μ living on a probe D_p-brane at the boundary could be evaluated by varying the action of the worldvolume vector A_μ on the D_{p+4}-branes as the bulk action $S_{\mathrm{bulk}}[A_\mu]$ [5,10,14], since the D_{p+4}-branes as flavors extend along the holographically radial direction of the bulk according to Table I. In this sense, the source term $J^\mu A^{(0)}_\mu$ presented in (3.5) implies the interaction of the (p,p) and $(p+4,p+4)$ strings.[3]

Keeping the above in mind and turning to our case, once the spinor χ in (3.1) and (3.2) is identified to the flavored fermion living on the D_p-brane at boundary, we must find its fermionic dual field ψ by analyzing the low-energy spectrum of type II string theories. Among the fields in the massless spectrum of the $\mathrm{D}_p/\mathrm{D}_{p+4}$ model, the only possible choice for ψ should be the fermionic field on the D_{p+4}-branes.[4] In this sense the source term $\bar{\chi}\psi_0+\bar{\psi}_0\chi$ presented in (3.2) corresponds to the interaction of $(p,p+4)$ and $(p+4,p+4)$ strings at boundary, which reveals the interaction to boundary theory involving the fermionic part to A_μ presented in (3.5), as it is displayed in Fig. 1. We note that, in the D-brane setup given in Table I on the bubble background (2.4), the $(p+4,p+4)$ string remains to be supersymmetric [37,40,44] since there is not any mechanism to break down the supersymmetry on the D_{p+4}-branes in principle while the D_p-brane is not supersymmetric due to the boundary condition on its compactified direction. Therefore the fermionic field ψ in the worldvolume of D_{p+4}-branes must exist in the low-energy theory, and in our $\mathrm{D}_p/\mathrm{D}_{p+4}$ model, it is seemingly the suitable candidate uniquely as the source of the flavored fermion χ living in the boundary which may also be the only consistent interpretation in this system. However in the black brane solution (2.2), the worldvolume fermion on the D_{p+4}-branes may acquire thermal mass due to the contribution from loop diagrams. This means the supersymmetry may break down so that the supersymmetric

[3] Here we use (p,q) to refer to an open string connecting D_p- and D_q-branes.

[4] Since we are working in the bosonic background given in (2.2) or (2.4), the fermions in the sector of close string can be neglected.

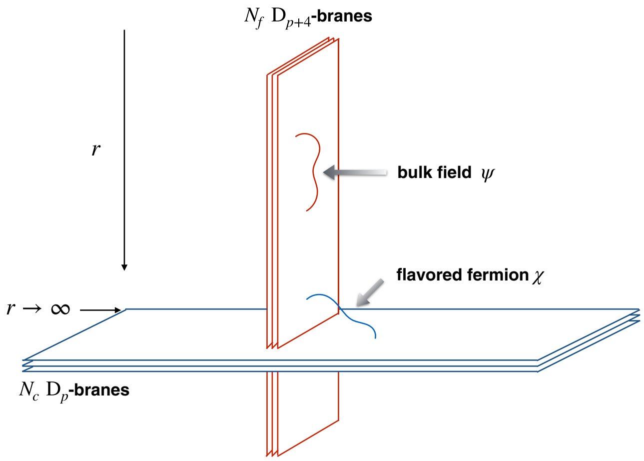

FIG. 1. The correspondence of the bulk field ψ and boundary operator χ in the $\mathrm{D}_p/\mathrm{D}_{p+4}$ system. Here r refers to the radial direction as the holographic direction. χ is a hadronic field as a gauge-invariant operator produced by multiple fundamental quark fields, or equivalently by multiple $(p, p+4)$ strings.

fermion could decouple to the low-energy theory. Nonetheless, it would be interesting to investigate the fermionic correlation in the black brane background (as the deconfined phase of QCD) as a comparison to several bottom-up approaches with deconfined fermions [30–33]. Therefore we will identify the bulk field ψ to the supersymmetric fermion on worldvolume of the D_{p+4}-brane in order to continue our discussion.

In addition, we also need an interpretation of the dual operator χ in terms of hadron physics because it is presented in the dual theory which is very close to QCD in holography. As the bulk field ψ and its dual operator χ are usually gauge-invariant operators and share same quantum numbers in gauge-gravity duality or AdS/CFT [14,37,41], it implies χ must be a color singlet in the language of QCD. On the other hand, the bulk field ψ is in the adjoint representation of the flavor group (just as its bosonic superpartner), which means dual operator χ is also the adjoint representation of the flavor group thus it must be a fermionic hadronic field. Mathematically, χ is not the fundamental representation of the color group; instead it must be the singlet irreducible representation obtained by decomposing the tensor product of the fundamental representation of the color group. So χ could be a mesino (the supersymmetric fermionic meson) [36,39,40] or baryon produced by multiple fundamental quark fields, or equivalently by multiple $(p, p+4)$ strings. This would be clear by recalling the construction of the hadronic state in QCD with $SU(3)$ color group. For example, the two-quark color singlet can be obtained by decomposing the tensor product of irreducible representation of $SU(3)$ in QCD as $\mathbf{3} \otimes \mathbf{3} = \mathbf{8} \oplus \mathbf{1}$, where $\mathbf{1}$ refers to the two-quark color singlet, i.e., meson. Similarly, for the three-quark color singlet, we have $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8}^* \oplus \mathbf{1}$ where $\mathbf{1}$ refers to the three-quark color singlet, i.e., baryon. Therefore χ refers to such a color singlet as a hadronic operator in our holographic approach with the $\mathrm{D}_p/\mathrm{D}_{p+4}$ model. However, interpreting χ as mesino is less close to the realistic hadron physics since mesino is always absent in QCD, thus we attempt to interpret χ as baryon by taking into account a baryon vertex in this work and we will discuss it in details in the end of this paper.

B. Action for the bulk spinor

In the last section, since the bulk field dual to the flavored fermion is identified to the worldvolume fermion on the N_f coincident D_{p+4}-branes, let us take a look at the action for such a fermionic field on the worldvolume of the D-brane.

As it is known that the action for the worldvolume fields on a D-brane is in principle obtained under the rule of T-duality [37,41] in string theory which includes a bosonic part consisting of the Dirac-Born-Infeld (DBI) term plus the Wess-Zumino (WZ) term, and a fermionic part. Since our concern is the worldvolume fermion on the D-brane, let us focus the fermionic part of the D_p-brane action which on the bosonic background is given by [34,35],

$$S_f^{\mathrm{D}_p} = \frac{iT_p}{2}\int d^{p+1}x e^{-\phi}\sqrt{-(g+\mathcal{F})}\bar{\Psi}(1-\Gamma_{\mathrm{D}_p}) \times \left(\Gamma^\alpha \hat{D}_\alpha - \Delta + \mathrm{L}_{\mathrm{D}_p}\right)\Psi, \tag{3.6}$$

where, for the type IIA string theory (p is even number),

$$\begin{aligned}
\hat{D}_\alpha &= \nabla_\alpha + \frac{1}{4\cdot 2!}H_{\alpha NK}\Gamma^{NK}\bar{\gamma} + \frac{1}{8}e^\phi\left(\frac{1}{2!}F_{NK}\Gamma^{NK}\Gamma_\alpha\bar{\gamma} + \frac{1}{4!}F_{KLNP}\Gamma^{KLNP}\Gamma_\alpha\right),\\
\Delta &= \frac{1}{2}\left(\Gamma^M\partial_M\phi + \frac{1}{2\cdot 3!}H_{MNK}\Gamma^{MNK}\bar{\gamma}\right) + \frac{1}{8}e^\phi\left(\frac{3}{2!}F_{MN}\Gamma^{MN}\bar{\gamma} + \frac{1}{4!}F_{KLNP}\Gamma^{KLNP}\right),\\
\Gamma_{\mathrm{D}_p} &= \frac{1}{\sqrt{-(g+\mathcal{F})}}\sum_q \frac{\epsilon^{\alpha_1\ldots\alpha_{2q}\beta_1\ldots\beta_{p-2q+1}}}{q!2^q(p-2q+1)!}\mathcal{F}_{\alpha_1\alpha_2}\ldots\mathcal{F}_{\alpha_{2q-1}\alpha_{2q}}\Gamma_{\beta_1\ldots\beta_{p-2q+1}}\bar{\gamma}^{\frac{p-2q+2}{2}},\\
\mathrm{L}_{\mathrm{D}_p} &= \sum_q \frac{\epsilon^{\alpha_1\ldots\alpha_{2q}\beta_1\ldots\beta_{p-2q+1}}}{q!2^q(p-2q+1)!}\frac{(-\bar{\gamma})^{\frac{p}{2}-q+1}}{\sqrt{-(g+\mathcal{F})}}\mathcal{F}_{\alpha_1\alpha_2}\ldots\mathcal{F}_{\alpha_{2q-1}\alpha_{2q}}\Gamma_{\beta_1\ldots\beta_{p-2q+1}}{}^\lambda\hat{D}_\lambda,
\end{aligned} \tag{3.7}$$

for the type IIB string theory (p is odd number),

$$\begin{aligned}
\hat{D}_\alpha &= \nabla_\alpha + \frac{1}{4\cdot 2!}H_{\alpha NK}\Gamma^{NK}\bar{\gamma} - \frac{1}{8}e^{\phi}\left(F_N\Gamma^N\Gamma_\alpha\bar{\gamma} + \frac{1}{3!}F_{KLN}\Gamma^{KLN}\Gamma_\alpha + \frac{1}{2\cdot 5!}F_{KLMNP}\Gamma^{KLMNP}\Gamma_\alpha\bar{\gamma}\right),\\
\Delta &= \frac{1}{2}\left(\Gamma^M\partial_M\phi + \frac{1}{2\cdot 3!}H_{MNK}\Gamma^{MNK}\bar{\gamma}\right) + \frac{1}{2}e^{\phi}\left(F_M\Gamma^M\bar{\gamma} + \frac{1}{2\cdot 3!}F_{KLN}\Gamma^{KLN}\right),\\
\Gamma_{\mathrm{D}_p} &= \frac{1}{\sqrt{-(g+\mathcal{F})}}\sum_q \frac{\epsilon^{\alpha_1\ldots\alpha_{2q}\beta_1\ldots\beta_{p-2q+1}}}{q!2^q(p-2q+1)!}\mathcal{F}_{\alpha_1\alpha_2}\ldots\mathcal{F}_{\alpha_{2q-1}\alpha_{2q}}\Gamma_{\beta_1\ldots\beta_{p-2q+1}}\bar{\gamma}^{\frac{p-2q+1}{2}},\\
\mathrm{L}_{\mathrm{D}_p} &= \sum_q \frac{\epsilon^{\alpha_1\ldots\alpha_{2q}\beta_1\ldots\beta_{p-2q+1}}}{q!2^q(p-2q+1)!}\frac{(-i\sigma_2)(\bar{\gamma})^{\frac{p-2q+1}{2}}}{\sqrt{-(g+\mathcal{F})}}\mathcal{F}_{\alpha_1\alpha_2}\ldots\mathcal{F}_{\alpha_{2q-1}\alpha_{2q}}\Gamma_{\beta_1\ldots\beta_{p-2q+1}}{}^{\lambda}\hat{D}_\lambda.
\end{aligned} \tag{3.8}$$

Since the two-point Green function in holography is our concern, only the quadratic term in fermionic action is given in (3.6). Then let us clarify the notation presented in (3.6)–(3.8). T_p refers to the tension of a D_p-brane given as $T_p = g_s^{-1}(2\pi)^{-p}l_s^{-(p+1)}$. The indices denoted by capital letters $K, L, M, N\ldots$ run over the 10d spacetime and indices denoted by lowercase letters $a, b, \ldots$ run over the tangent space of the 10d spacetime. We use Greek alphabet α, β, λ to denote the indices running over the worldvolume of the D_p-brane. The metric is written in terms of elfbein as $g_{MN} = e^a_M\eta_{ab}e^b_N$, so the gamma matrices are defined by

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \qquad \{\Gamma^M, \Gamma^N\} = 2g^{MN}, \tag{3.9}$$

with $e^a_M\Gamma^M = \gamma^a$. $\omega_{\alpha ab}$ refers to the spin connection and $\nabla_\alpha = \partial_\alpha + \frac{1}{4}\omega_{\alpha ab}\gamma^{ab}$ is the covariant derivative for fermion. The gamma matrix with multiple indices is defined as the alternate commutators or anticommutators, e.g.,

$$\gamma^{ab} = \frac{1}{2}[\gamma^a, \gamma^b], \qquad \gamma^{abc} = \frac{1}{2}\{\gamma^a, \gamma^{bc}\},$$
$$\gamma^{abcd} = \frac{1}{2}[\gamma^a, \gamma^{bcd}]\ldots \tag{3.10}$$

$\Gamma^{MNK\ldots}$ shares the same definition as $\gamma^{abc\ldots}$. $\bar{\gamma}$ is defined as $\bar{\gamma} = \gamma^{01\ldots9}$ and σ_2 refers to the associated Pauli matrix. The worldvolume field $\mathcal{F}$ is given as $\mathcal{F} = B + (2\pi\alpha')f$ where B is the NS-NS 2-form in type II string theory with $H = dB$ and f is the Yang-Mills field strength. $F_M, F_{MN}, F_{KLM}\ldots$ refer to the associated field strength of the massless R-R fields presented in IIA and IIB string theory. We note that, in our $\mathrm{D}_p/\mathrm{D}_{p+4}$ approach, p should be replaced by $p+4$ in the above formulas since the worldvolume fermion on the probe D_{p+4}-brane is our concern.

C. The prescription for two-point correlation function

In this section, let us attempt to generalize the prescription in the AdS/CFT dictionary for two-point correlation function into the background presented in (2.2) and (2.4). We first summarize the steps as follows then give some comments.

(1) Since one dimension of the D_p-brane is compactified on a circle, the dual theory is effectively p dimensional below the energy scale provided by the circle size. So we need to simplify the T-dualitized action (3.6) with respect to the background fields given in (2.2) and (2.4)[5] by integrating out the dependence on S^{8-p} in order to obtain an effective $p+1$ dimensional bulk action involving the bulk spinor $\psi(x,r)$ as

$$\begin{aligned}
S_{p+1} &\propto i\int d^Dxdr\bar{\psi}\gamma^r\partial_r\psi + \ldots\\
&= i\int d^Dx(\bar{\psi}\gamma^r\psi)|^{r=\infty}_{r=r_{KK,H}}\\
&\quad - i\int d^Dxdr\partial_r\bar{\psi}\gamma^r\psi + \ldots
\end{aligned} \tag{3.11}$$

(2) Derive the equation of motion for $\psi(x,r)$ by varying the $p+1$ dimensional action obtained in Step 1, then solve it by using the Fourier mode as the ansatz for the $\psi(x,r)$ as,

$$\psi(x,r) = e^{ik\cdot x}\beta(\omega,\vec{k},r), \qquad k_\mu = (-\omega,\vec{k}). \tag{3.12}$$

(3) Impose the solution for $\psi(x,r)$ obtained in Step 2 into the $p+1$ dimensional action obtained in Step 1, so the $p+1$ dimensional fermionic action becomes on-shell S^{cl}_{p+1}, then define the boundary value of ψ as ψ_0, hence the two-point correlation function is obtained as

[5]When we recall the background (2.2) or (2.4), it means we will use their near-horizon version for holography.

$$G_R = -\frac{1}{Z_{\text{gravity}}}\frac{\delta}{\delta\bar{\psi}_0}\frac{\delta}{\delta\psi_0}Z_{\text{gravity}}[\bar{\psi},\psi]|_{\bar{\psi}_0,\psi_0=0}$$
$$= -\frac{\delta}{\delta\bar{\psi}_0}\frac{\delta}{\delta\psi_0}S^{\text{cl}}_{p+1}[\bar{\psi},\psi], \tag{3.13}$$

leading to

$$\bar{\Pi}_0 = G_R\psi_0, \tag{3.14}$$

where Π_0 is defined in (3.4). Accordingly, one can obtain the correlation function by solving ψ_0 and Π_0 in gravity side. The above steps are based on the prescription in the AdS/CFT dictionary for two-point correlation function [10,14,37]. Note that the last term in (3.11) would vanish once the classical solution (3.12) is imposed since (as we will see) this term is nothing but the Dirac equation. Accordingly, in the actual calculation, we will introduce a ratio function defined as

$$\bar{\Pi} = \xi\psi, \tag{3.15}$$

where

$$\Pi = -\frac{\delta}{\delta\psi}S^{\text{cl}}_{p+1} = -i\bar{\psi}\gamma^r, \tag{3.16}$$

so that the boundary value of ξ is the correlation function. Therefore, our goal is to derive and solve the equations for ξ with the in-falling boundary conditions usually used in AdS/CFT.

Besides, in the AdS/CFT correspondence, the on-shell action (3.11) would always include a finite part due to the isometry of AdS or equivalently the conformal symmetry in the dual theory [14]. Hence the holographic renormalization could work consistently to remove the divergence presented in action (3.11). However, in a general background, the on-shell action (3.11) might not include any finite parts which could lead to a nonrenormalizable dual theory. Nevertheless, it is possible to define a finite Green function consistently by rescaling the boundary value ψ_0 with respect to ψ which, as we will see, is equivalent to exact the finite part of ξ given in (3.15). Therefore in the following sections, we will test our prescription in the top-down D4/D8 and D3/D7 approaches by holography.

IV. APPROACH TO THE D4/D8 MODEL

In this section, let us apply our prescription to the D4/D8 model (Witten-Salai-Sugimoto model) to compute the two-point Green function. We will first introduce the D4/D8 model briefly, then test our prescription and analyze the results numerically in the $1+3$ dimensional QCD. In Sec. IV A, we perform Step 1 in the prescription which is to obtain a 5d effective bulk action for fermion with respect to the bubble and black D4 background. In Sec. IV B, we follow Step 2 and Step 3, that is to solve the associated equations of motion for the bulk field, define its boundary value then derive its on-shell action to evaluate the correlation function.

A. The 5d action for the bulk spinor

The D4/D8 model is based on type IIA string theory and it corresponds to the case of $p=4$ as it is discussed in Sec. II. For the readers' convenience, let us give its supergravity solution which is to set $p=4$ in the background geometry (2.4) for the confined phase as [21]

$$ds_c^2 = \left(\frac{r}{R}\right)^{3/2}[\eta_{\mu\nu}dx^\mu dx^\nu + f(r)(dx^4)^2] + \left(\frac{R}{r}\right)^{3/2}\left[\frac{dr^2}{f(r)} + r^2 d\Omega_4^2\right],$$
$$e^\phi = \left(\frac{r}{R}\right)^{3/4}, \qquad F_4 = dC_3 = 3R^3 g_s^{-1}\epsilon_4,$$
$$f(r) = 1 - \frac{r_{KK}^3}{r^3}, \qquad \mu,\nu = 0,\ldots 3 \tag{4.1}$$

Here R relates to the radius of the spacetime, ϵ_4 is the volume form of a unit S^4, and we have taken the near-horizon limit. In the language of QCD, the parameters presented in (4.1) can be expressed as

$$R^3 = \frac{1}{2}\frac{g_{\text{YM}}^2 N_c l_s^2}{M_{KK}}, \qquad r_{KK} = \frac{2}{3}g_{\text{YM}}^2 N_c M_{KK} l_s^2,$$
$$g_s = \frac{1}{2\pi}\frac{g_{\text{YM}}^2}{M_{KK} l_s}, \tag{4.2}$$

where g_{YM} refers to the Yang-Mills coupling constant in the dual theory. For the deconfined phase, the bulk geometry in the near-horizon limit is obtained from (2.2) with $p=4$ as

$$ds_d^2 = \left(\frac{r}{R}\right)^{3/2}[-f_T(r)dt^2 + \delta_{ij}dx^i dx^j + (dx^4)^2] + \left(\frac{R}{r}\right)^{3/2}\left[\frac{dr^2}{f(r)} + r^2 d\Omega_4^2\right]$$
$$e^\phi = \left(\frac{r}{R}\right)^{3/4}, \qquad F_4 = dC_3 = 3R^3 g_s^{-1}\epsilon_4,$$
$$f_T(r) = 1 - \frac{r_H^3}{r^3}, \qquad i,j = 1,2,3. \tag{4.3}$$

As it is discussed in Sec. II, the bulk spinor is identified to the fermionic field on the worldvolume of the probe D8-branes; we need the induced metric on the worldvolume of the D8-branes which according to Table I is given as

$$
\begin{aligned}
ds^2_{\mathrm{D8},c} &= \left(\frac{r}{R}\right)^{3/2}\eta_{\mu\nu}dx^\mu dx^\nu + \left(\frac{R}{r}\right)^{3/2}\left[\frac{dr^2}{f(r)} + r^2 d\Omega_4^2\right],\\
ds^2_{\mathrm{D8},d} &= \left(\frac{r}{R}\right)^{3/2}[-f_T(r)dt^2 + \delta_{ij}dx^i dx^j]\\
&\quad + \left(\frac{R}{r}\right)^{3/2}\left[\frac{dr^2}{f_T(r)} + r^2 d\Omega_4^2\right] \qquad (4.4)
\end{aligned}
$$

for the confined and deconfined phases respectively. Considering the simplest configuration to reveal the chirality in the dual theory, we note that the D8- and anti-D8-branes are embedded at the antipodal position of x^4, i.e., $x^4 = \mathrm{const}$ in the bubble D4-brane background (2.4). For the black brane background (2.2), the D8- and anti-D8-branes are embedded parallel which also implies $x^4 = \mathrm{const}$ as it is illustrated in Fig. 2. Then the action for the worldvolume fermion on D8-branes can be collected from (3.6) and (3.7) as

$$
\begin{aligned}
S_f^{\mathrm{D8}} &= \frac{iT_8}{2}\int d^9x e^{-\phi}\sqrt{-g}\bar{\Psi}P_-\left[\Gamma^\alpha\nabla_\alpha + \frac{1}{8\cdot 4!}\right.\\
&\quad \left.\times e^{\phi}F_{KLNP}(\Gamma^\alpha\Gamma^{KLNP}\Gamma_\alpha - \Gamma^{KLNP}) - \frac{1}{2}\Gamma^M\partial_M\phi\right]\Psi. \qquad (4.5)
\end{aligned}
$$

Since our concern is the fermionic part, we have turned off the irrelative bosonic fields by setting $\mathcal{F} = 0$. Imposing the solution for the dilaton ϕ and the R-R form F_4 presented in (4.1) or (4.3), the action (4.5), after some algebraic calculations, can be simplified in the confined background as

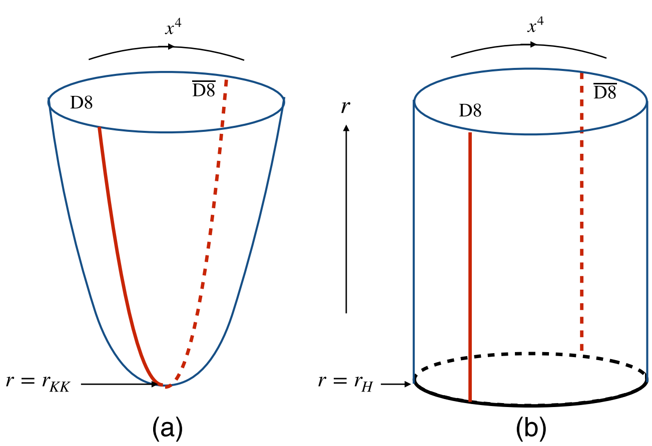

FIG. 2. The configuration of the D4/D8 model in the bubble (confined) background (a) and black brane (deconfined) background (b) on $r - x^4$ plane. Red line refers to the D8-branes (D8) and anti-D8-branes ($\overline{\mathrm{D8}}$).

$$
\begin{aligned}
S^{\mathrm{D8}}_{f,c} &= \frac{i\mathcal{T}_c}{(2\pi\alpha')^2\Omega_4}\int d^4x dZ d\Omega_4 \bar{\Psi}P_-\left(\frac{2}{3}M_{KK}K^{\frac{7}{12}}\gamma^m\nabla_m^{S^4}\right.\\
&\quad \left. + K^{\frac{5}{12}}\gamma^\mu\partial_\mu + M_{KK}K^{\frac{13}{12}}\gamma^Z\partial_Z, + \frac{13}{12}M_{KK}ZK^{\frac{1}{12}}\gamma^4\right)\Psi, \qquad (4.6)
\end{aligned}
$$

with

$$
\begin{aligned}
K(Z) &= 1 + Z^2,\\
\mathcal{T}_c &= \frac{1}{2}\left(\frac{2}{3}\right)^{13/2}T_8\Omega_4(2\pi\alpha')^2(M_{KK}R)^{11/2}R^5,\\
P_- &= \frac{1}{2}(1 - \Gamma_{\mathrm{D8}}). \qquad (4.7)
\end{aligned}
$$

In the deconfined background, the action (4.5) becomes

$$
\begin{aligned}
S^{\mathrm{D8}}_{f,d} &= \frac{i\mathcal{T}_d}{(2\pi\alpha')^2\Omega_4}\int d^4x dZ d\Omega_4 \bar{\Psi}P_-\left[\frac{4}{3}\pi TZK^{1/12}\gamma^m\nabla_m^{S^4}\right.\\
&\quad + K^{5/12}\gamma^0\partial_0 + ZK^{-1/12}\gamma^i\partial_i + 2\pi TZK^{7/12}\gamma^Z\partial_Z\\
&\quad \left. + \pi T\left(\frac{13}{6}Z^2K^{-5/12} + K^{7/12}\right)\gamma^Z\right]\Psi, \qquad (4.8)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{T}_d &= \frac{1}{2}\left(\frac{2}{3}\right)^{13/2}T_8\Omega_4(2\pi\alpha')^2(2\pi TR)^{11/2}R^5,\\
r_H &= \frac{16}{9}\pi^2R^3T^2, \qquad T = \beta_t^{-1}. \qquad (4.9)
\end{aligned}
$$

Here T, β_t refers to the Hawking temperature and the period of the time direction in (2.2). We have used Ψ to denote the worldvolume fermion whose boundary value relates to ψ_0. The index m runs over S^4 and the unit volume of S^4 is given as $\Omega_4 = 8\pi^2/3$. We recall that in the D4/D8 model, the flavor branes are introduced as N_f pairs of probe D8- and anti-D8-branes located at the antipodal points of x^4. Hence the Cartesian coordinate Z is usually used as

$$
K(Z) = 1 + Z^2 = \frac{r^3}{r^3_{KK,H}}, \qquad (4.10)
$$

so that the D8- and anti-D8-branes are located respectively at $Z \to \pm\infty$ representing approximately the chiral symmetry $U(N_f)_R \times U(N_f)_L$ at boundary. And the dual theory on the D4-branes at boundary takes the chirally symmetric action as QCD as [44]

$$
\begin{aligned}
S &= \int_{\mathrm{D4}} d^5x\sqrt{-g}\left[\delta(Z - Z_{+\infty})\chi_L^\dagger\bar{\sigma}^\mu(i\nabla_\mu + A_\mu)\chi_L\right.\\
&\quad \left. + \delta(Z - Z_{-\infty})\chi_R^\dagger\bar{\sigma}^\mu(i\nabla_\mu + A_\mu)\chi_R\right]\\
&\quad - \frac{1}{4g^2_{\mathrm{YM}}}\int_{\mathrm{D4}} d^5x\sqrt{-g}\,\mathrm{Tr}F_{\mu\nu}F^{\mu\nu}, \qquad \mu,\nu = 0, 1\ldots 3, \qquad (4.11)
\end{aligned}
$$

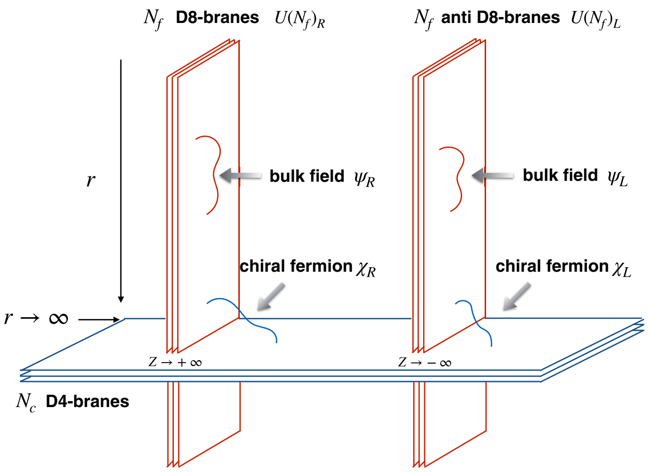

FIG. 3. The correspondence of the bulk fields $\psi_{R,L}$ and flavored fermions $\chi_{R,L}$ consisting of chiral quarks in the D4/D8 model. The N_f pairs of D8- and anti-D8-branes are located at $Z \to \pm\infty$ at boundary providing $U(N_f)_R \times U(N_f)_L$ chiral symmetry in the dual theory. The bulk field $\psi_{R,L}$ as source couples to the chiral quarks in $\chi_{R,L}$ respectively as it is in QCD.

where we use Weyl spinor $\chi_{R,L}$ to denote the fundamental chiral fermion and F_{MN} is the gauge field strength of the gluon A_M. Therefore the source term of spinor presented in (3.2) can be written in terms of Weyl spinors as

$$\left\langle \exp\left\{ \int_{\partial\mathcal{M}} (\chi_L^\dagger \psi_L + \chi_R^\dagger \psi_R + \mathrm{H.c.}) d^D x \right\} \right\rangle, \tag{4.12}$$

once we focus on the Green function of chiral fermion, as it is illustrated in Fig. 3.

In order to compute the two-point Green function, let us reduce the fermionic action (4.6) and (4.8) to a 5d form as most works about AdS/CFT. First, we decompose the 10d spinor into a $3+1$ dimensional part $\psi(x, Z)$ with holographic coordinate Z, an S^4 part φ, and a remaining 2d part β as[6]

$$\psi(x, Z) \otimes \varphi(S^4) \otimes \beta, \tag{4.13}$$

so that the worldvolume fermion on D8-brane should be $\Psi = \psi \otimes \varphi$. Then the associated 10d gamma matrices are chosen as

$$\begin{aligned}
\gamma^\mu &= \sigma_1 \otimes \boldsymbol{\gamma}^\mu \otimes \mathbf{1}, \qquad \mu = 0, 1, 2, 3\\
\gamma^Z &= \sigma_1 \otimes \boldsymbol{\gamma} \otimes \mathbf{1},\\
\gamma^4 &= \sigma_2 \otimes \mathbf{1} \otimes \tilde{\boldsymbol{\gamma}},\\
\gamma^m &= \sigma_2 \otimes \mathbf{1} \otimes \boldsymbol{\gamma}^m, \qquad m = 6, 7, 8, 9,\\
\boldsymbol{\gamma} &= i\boldsymbol{\gamma}^0\boldsymbol{\gamma}^1\boldsymbol{\gamma}^2\boldsymbol{\gamma}^3,\\
\tilde{\boldsymbol{\gamma}} &= i\boldsymbol{\gamma}^6\boldsymbol{\gamma}^7\boldsymbol{\gamma}^8\boldsymbol{\gamma}^9,
\end{aligned} \tag{4.14}$$

where we use bold to denote the 4×4 gamma matrices. $\boldsymbol{\gamma}^m, m = 6, 7, 8, 9$ refers to the gamma matrix on tangent space of S^4. We note that the 10d chirality matrix takes a very simple form as $\bar{\gamma} = \sigma_3 \otimes \mathbf{1} \otimes \mathbf{1}$ in this decomposition. Choosing the σ_3 representation, β can therefore be decomposed by the eigenstates of σ_3 with

$$\sigma_3\beta_\pm = \beta_\pm, \qquad \sigma_1\beta_\pm = \beta_\mp, \qquad \sigma_2\beta_\pm = \pm i\beta_\mp, \tag{4.15}$$

where we use $\beta_\pm$ to denote the two eigenstates of σ_3. Since the kappa symmetry fixes the condition $\bar{\gamma}\Psi = \Psi$,[7] we have to chose $\beta = \beta_+$ on the D8-brane. In addition, as φ must satisfy the Dirac equation on S^4, we can decompose it by using the spherical harmonic function with the eigenstates of $\Gamma^{\underline{m}}\nabla_m^{S^4}$ as [45]

$$\boldsymbol{\gamma}^m \nabla_m^{S^4} \varphi^{\pm l,s} = i\Lambda_l^\pm \varphi^{\pm l,s};\ \Lambda_l^\pm = \pm(2+l), \quad l = 0, 1\ldots \tag{4.16}$$

Here s, l represents the angular quantum numbers of the spherical harmonic function.

Putting (4.13) into (4.6) and (4.8) with the decomposition (4.14)–(4.16) for φ and β, we could obtain a 5d effective action for the fermion on the worldvolume of the D8-branes after integrating the S^4 part as

$$\begin{aligned}
S_{f,c}^{\mathrm{D8}} = \frac{i\mathcal{T}_c}{(2\pi\alpha')^2} \int d^4x dZ \bar{\psi} \bigg(&-\frac{2}{3} M_{KK} \Lambda_l K^{\frac{7}{12}} + K^{\frac{5}{12}} \boldsymbol{\gamma}^\mu \partial_\mu \\
&+ M_{KK} K^{\frac{13}{12}} \boldsymbol{\gamma} \partial_Z + \frac{13}{12} M_{KK} Z K^{\frac{1}{12}} \boldsymbol{\gamma} \bigg) \psi,
\end{aligned} \tag{4.17}$$

and

$$\begin{aligned}
S_{f,d}^{\mathrm{D8}} = \frac{i\mathcal{T}_d}{(2\pi\alpha')^2} \int d^4x dZ \bar{\psi} \bigg[&-\frac{4}{3}\pi T Z K^{1/12} \Lambda_l + K^{5/12} \boldsymbol{\gamma}^0 \partial_0 \\
&+ Z K^{-1/12} \boldsymbol{\gamma}^i \partial_i + 2\pi T Z K^{7/12} \boldsymbol{\gamma} \partial_Z \\
&+ \pi T \left(\frac{13}{6} Z^2 K^{-5/12} + K^{7/12} \right) \boldsymbol{\gamma} \bigg] \psi,
\end{aligned} \tag{4.18}$$

where $\Lambda_l = |\Lambda_l^\pm|$. As we are going to focus on the Green function in the dual theory according to the Sec. III C, the action (4.17) can be further simplified by rescaling $\psi \to 2\pi\alpha' K^{-13/24}\psi$ as

$$\begin{aligned}
S_{f,c}^{\mathrm{D8}} = i\mathcal{T}_c \int d^4x dZ \bar{\psi} \bigg(&-\frac{2}{3} M_{KK} \Lambda_l K^{-1/2} + K^{-2/3} \boldsymbol{\gamma}^\mu \partial_\mu \\
&+ M_{KK} \boldsymbol{\gamma} \partial_Z \bigg) \psi,
\end{aligned} \tag{4.19}$$

while the action (4.18) is rescaled by $\psi \to 2\pi\alpha' K^{-7/24} \times Z^{-1/2}\psi$ as

[6] We note that in D dimension, a spinor has $[D/2]$ components where $[D/2]$ refers to the integer part of $D/2$.

[7] This condition could be opposite on the anti-D8-branes.

$$S_{f,d}^{\mathrm{D8}} = i\mathcal{T}_d \int d^4xdZ\bar{\psi}\left(-\frac{4}{3}\pi TK^{-1/2}\Lambda_l + Z^{-1}K^{-1/6}\gamma^0\partial_0 + K^{-2/3}\gamma^i\partial_i + 2\pi T\gamma\partial_Z + \pi TZK^{-1}\gamma\right)\psi. \quad (4.20)$$

And we will use these 5d fermionic actions (4.19) and (4.20) to evaluate the two-point Green function.

B. Use the prescription

Since the 5d effective action is obtained in Sec. IV A, in this section, let us use Step 2 and Step 3 in the prescription to evaluate the two-point Green function with confined (2.4) and deconfined (2.2) geometry by using action (4.19) and (4.20) respectively.

1. Confined phase

Let us start with Step 2 in Sec. III C, i.e., solve the equation of motion associated to action (4.19) which is derived as

$$\left(-\frac{2}{3}M_{KK}\Lambda_l K^{-1/2} + K^{-2/3}\gamma^\mu\partial_\mu + M_{KK}\gamma\partial_Z\right)\psi = 0. \quad (4.21)$$

Picking up the ansatz in Weyl basis as

$$\psi = e^{ik\cdot x}\begin{pmatrix}\psi_R \\ \psi_L\end{pmatrix}, \qquad k\cdot x = k_\mu x^\mu = -\omega t + \vec{k}\cdot\vec{x}, \quad (4.22)$$

Eq. (4.21) becomes

$$\begin{aligned} &-\frac{2}{3}M_{KK}\Lambda_l K^{-1/2}\psi_R - K^{-2/3}(\sigma\cdot k)\psi_L + M_{KK}\partial_Z\psi_R = 0, \\ &-\frac{2}{3}M_{KK}\Lambda_l K^{-1/2}\psi_L - K^{-2/3}(\bar{\sigma}\cdot k)\psi_R - M_{KK}\partial_Z\psi_L = 0, \end{aligned} \quad (4.23)$$

where we have chosen the 5d gamma matrices as

$$\gamma^\mu = i\begin{pmatrix}0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0\end{pmatrix}, \qquad \gamma = \begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}, \quad (4.24)$$

with $\sigma^\mu = (1, -\sigma^i), \bar{\sigma}^\mu = (1, \sigma^i)$. Solving (4.23), we can further obtain the decoupled equations for $\psi_{R,L}$ as

$$\begin{aligned} &\partial_Z^2\psi_L + \frac{2K'}{3K}\partial_Z\psi_L + \left(\frac{\Lambda_l K'}{9K^{3/2}} - \frac{4}{9}\frac{\Lambda_l^2}{K} - \frac{k^2}{M_{KK}^2}\frac{1}{K^{4/3}}\right)\psi_L = 0, \\ &\partial_Z^2\psi_R + \frac{2K'}{3K}\partial_Z\psi_R - \left(\frac{\Lambda_l K'}{9K^{3/2}} + \frac{4}{9}\frac{\Lambda_l^2}{K} + \frac{k^2}{M_{KK}^2}\frac{1}{K^{4/3}}\right)\psi_R = 0, \end{aligned} \quad (4.25)$$

where "$'$" refers to the derivative with respect to Z. The above equations can be analytically solved at $Z \to +\infty$ as

$$\begin{aligned} \psi_R &= AZ^{\frac{2}{3}\Lambda_l} + BZ^{-\frac{1}{3}-\frac{2}{3}\Lambda_l}, \\ \psi_L &= CZ^{-\frac{1}{3}+\frac{2}{3}\Lambda_l} + DZ^{-\frac{2}{3}\Lambda_l}, \end{aligned} \quad (4.26)$$

where A, B, C, D are constant spinors dependent on $\omega, \vec{k}, \Lambda_l$ and furthermore one can find the following relations as

$$C = \frac{3}{(1-4\lambda)M_{KK}}(\bar{\sigma}\cdot k)A, \qquad B = \frac{3}{(1+4\lambda)M_{KK}}(\sigma\cdot k)D, \quad (4.27)$$

once the analytical solution (4.26) is plugged back into (4.23). Besides, we notice that the solution at $Z \to -\infty$ is equivalent to interchange $\psi_{R,L}$ in the solution (4.26). Λ_l is the eigenvalue of the S^4 part and it satisfies $\Lambda_l \geq 2$ according to (4.16). So we obtain the boundary behavior of ψ as

$$\lim_{Z\to+\infty}\psi \to \begin{pmatrix}\psi_R \\ 0\end{pmatrix}, \qquad \lim_{Z\to-\infty}\psi \to \begin{pmatrix}0 \\ \psi_L\end{pmatrix}. \quad (4.28)$$

This can be nicely interpreted as the source coupling to the chiral fermion in each boundary theory, as it is illustrated in Fig. 3. Hence we will focus on the calculation with respect to the N_f D8-branes since the discussion would be exactly same on the anti-D8-branes. Afterwards, let us take care of the boundary value ψ_0 of ψ at $Z \to +\infty$ by picking up its dominantly finite part in (4.26) as [13,14]

$$\psi_0 = \lim_{Z\to+\infty} Z^{-\frac{2}{3}\Lambda_l}\psi = \begin{pmatrix}A \\ 0\end{pmatrix}. \quad (4.29)$$

Next, the conjugate momentum Π_0 associated with ψ_0 can be evaluated by using (3.4). To this goal, let us take a look at the action (4.19),

$$\begin{aligned} S_{f,c}^{\mathrm{D8}} &= i\mathcal{T}_c \int d^4xdZ\bar{\psi}\left(-\frac{2}{3}M_{KK}\Lambda_l K^{-1/2} + K^{-2/3}\gamma^\mu\partial_\mu + M_{KK}\gamma\partial_Z\right)\psi, \\ &= i\mathcal{T}_c \int d^4x(\bar{\psi}M_{KK}\gamma\psi)|_{-\infty}^{+\infty} - i\mathcal{T}_c \int d^4xdZ\left(\partial_Z\bar{\psi}M_{KK}\gamma + \frac{2}{3}M_{KK}\Lambda_l K^{-1/2}\bar{\psi} - K^{-2/3}\partial_\mu\bar{\psi}\gamma^\mu\right)\psi, \\ &\equiv \int d^4x(\Pi\psi)|_{-\infty}^{+\infty} = \int d^4x(\Pi_R\psi_R + \Pi_L\psi_L)|_{-\infty}^{+\infty}, \end{aligned} \quad (4.30)$$

where

$$\Pi_R = -\mathcal{T}_c M_{KK} \psi_L^\dagger, \qquad \Pi_L = \mathcal{T}_c M_{KK} \psi_R^\dagger. \tag{4.31}$$

Notice that the term in the third line of (4.30) vanishes since it is nothing but the conjugate equation to (4.21). So once we impose the solution (4.26) to (4.30), it implies the action (4.30) includes divergence at boundary $Z \to \infty$ as[8]

$$S_{f,c}^{\mathrm{D8}} \supseteq -\mathcal{T}_c M_{KK} \int d^4x \left[C^\dagger A Z^{-\frac{1}{3}+\frac{4}{3}\Lambda_l} + D^\dagger A + \mathrm{H.c.}\right]\Big|_{Z\to+\infty} + \ldots \tag{4.32}$$

Therefore we can see it requests a holographic boundary counterterm S_{ct} as

$$S_{ct} = \mathcal{T}_c M_{KK} \int d^4x \left[C^\dagger A Z^{-\frac{1}{3}+\frac{4}{3}\Lambda_l} + \mathrm{H.c.}\right]\Big|_{Z\to+\infty}, \tag{4.33}$$

to remove the divergence in (4.30). We note that action (4.32) contains a finite part which will survive after holographic renormalization. And this should relate to the residual isometry of AdS_7 since the D4/D8 model is based on the holographic correspondence on $\mathrm{AdS}_7 \times S^4$ [20,37]. Then the conjugate momentum on the D8-branes to ψ_0 is evaluated as

$$\Pi_0 = -\frac{\delta S^{\mathrm{ren}}}{\delta \psi_0} = \mathcal{T}_c M_{KK}(0, D^\dagger), \qquad S^{\mathrm{ren}} = S_{f,c}^{\mathrm{D8}} + S_{ct}, \tag{4.34}$$

which implies the Green function $G_R(\omega, \vec{k})$ is obtained by

$$\mathcal{T}_c M_{KK} D = G_R(\omega, \vec{k}) A, \tag{4.35}$$

according to (3.3) and (3.4). Using the representation

$$\psi_R = \begin{pmatrix} \psi_R^{(1)} \\ \psi_R^{(2)} \end{pmatrix}, \qquad \psi_L = \begin{pmatrix} \psi_L^{(1)} \\ \psi_L^{(2)} \end{pmatrix}, \tag{4.36}$$

and introducing the ratio

$$\xi_1 = \frac{\psi_L^{(1)}}{\psi_R^{(1)}}, \qquad \xi_2 = \frac{\psi_L^{(2)}}{\psi_R^{(2)}}, \tag{4.37}$$

the Green function can therefore be written as

$$G_R^{(1,2)} = \mathcal{T}_c M_{KK} \lim_{Z\to\infty} Z^{\frac{4}{3}\Lambda_l} \xi_{1,2}(Z), \tag{4.38}$$

satisfying the equation of motion,

$$\xi_{1,2}' = -\frac{4}{3}\Lambda_l K^{-1/2} \xi_{1,2} - \frac{K^{-2/3}}{M_{KK}}(-\omega + h \cdot \mathrm{k}) - \frac{K^{-2/3}}{M_{KK}}(-\omega - h \cdot \mathrm{k}) \xi_{1,2}^2, \tag{4.39}$$

according to (4.23). Here we have set $k_\mu = (-\omega, \mathrm{k}, 0, 0)$ with

$$\frac{\psi_R^{(2)}}{\psi_R^{(1)}} = \frac{\psi_L^{(2)}}{\psi_L^{(1)}} = h. \tag{4.40}$$

On the other hand, since the dual theory is chirally symmetric, we obtain

$$\psi_R|_{Z\to+\infty} = \psi_L|_{Z\to-\infty}, \tag{4.41}$$

which implies [36]

$$\psi_R(0) = \pm \psi_L(0). \tag{4.42}$$

Therefore we can set $h = \pm 1$ in (4.40) for $\xi_{1,2}$ respectively. Altogether, (4.39) could be numerically solved with the incoming wave boundary condition $\xi_{1,2}(0) = \pm 1$.

2. Deconfined phase

In the deconfined phase, the equation of motion for ψ reads from (4.20) as

$$\left(-\frac{4}{3}\pi T K^{-1/2}\Lambda_l + Z^{-1}K^{-1/6}\boldsymbol{\gamma}^0\partial_0 + K^{-2/3}\boldsymbol{\gamma}^i\partial_i + 2\pi T \boldsymbol{\gamma}\partial_Z + \pi T Z K^{-1}\boldsymbol{\gamma}\right)\psi = 0. \tag{4.43}$$

Using the ansatz

$$\psi = e^{ik\cdot x}\begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}, \qquad \psi_{R,L} = \begin{pmatrix} \psi_{R,L}^{(1)} \\ \psi_{R,L}^{(2)} \end{pmatrix}, \tag{4.44}$$

with $k_\mu = (-\omega, \mathrm{k}, 0, 0)$, (4.43) reduces to

$$\begin{aligned} &\partial_Z \psi_R^{(1,2)} + \left(\frac{ZK^{-1}}{2} - \frac{2}{3}K^{-1/2}\Lambda_l\right)\psi_R^{(1,2)} + \left(\frac{\omega}{2\pi T}Z^{-1}K^{-1/6} + \frac{h\cdot \mathrm{k}}{2\pi T}K^{-2/3}\right)\psi_L^{(1,2)} = 0 \\ &\partial_Z \psi_L^{(1,2)} + \left(\frac{ZK^{-1}}{2} + \frac{2}{3}K^{-1/2}\Lambda_l\right)\psi_L^{(1,2)} + \left(\frac{h\cdot \mathrm{k}}{2\pi T}K^{-2/3} - \frac{\omega}{2\pi T}Z^{-1}K^{-1/6}\right)\psi_R^{(1,2)} = 0, \end{aligned} \tag{4.45}$$

[8] Since the solution at $Z \to -\infty$ can be obtained by interchanging the roles of $\psi_{R,L}$ in (4.26), the action (4.30) has same asymptotic behavior at $Z \to -\infty$ on the anti-D8-branes.

which can be solved analytically at $Z \to \infty$ as

$$\psi_R^{(1,2)} = AZ^{\frac{2}{3}\Lambda_l - \frac{1}{2}} + BZ^{-\frac{2}{3}\Lambda_l - \frac{5}{6}},$$
$$\psi_L^{(1,2)} = CZ^{-\frac{2}{3}\Lambda_l - \frac{1}{2}} + DZ^{\frac{2}{3}\Lambda_l - \frac{5}{6}}. \quad (4.46)$$

The asymptotics (4.46) lead to the on-shell action (4.20) taking form at $Z \to \infty$ as

$$S_{f,d}^{\mathrm{D8}} \supseteq 2\pi T \int d^4x dZ i\bar{\psi}\boldsymbol{\gamma}\partial_Z\psi = -2\pi T \int d^4x \psi_L^\dagger \psi_R + \cdots$$
$$= -2\pi T \int d^4x D^\dagger A Z^{\frac{4}{3}\Lambda_l - \frac{4}{3}}, \quad (4.47)$$

which however does not include any finite part for $\Lambda_l \geq 2$. Although this behavior might imply the deconfined hot QCD is probably nonrenormalizable in holography, it is possible to evaluate its correlation function with the prescription in Sec. III C if we take the boundary value ψ_0 of ψ carefully as

$$\psi_0 = \lim_{Z \to +\epsilon^{-1}} \epsilon^{\frac{1}{6}}\psi \to \epsilon^{\frac{1}{6}}\begin{pmatrix}\psi_R \\ 0\end{pmatrix} = \epsilon^{-\frac{2}{3}\Lambda_l + \frac{2}{3}}\begin{pmatrix}A \\ 0\end{pmatrix}, \epsilon \to 0. \quad (4.48)$$

Thus conjugate momentum for ψ_0 is

$$\Pi_0 = -\frac{\delta S_{f,d}^{\mathrm{D8}}}{\delta\psi_0} = \mathcal{T}_d(2\pi T)(0, D^\dagger)\epsilon^{-\frac{2}{3}\Lambda_l + \frac{2}{3}}, \quad (4.49)$$

which reduces to finite Green function as $G_R \sim D/A$. Then let us use the ratio given in (4.37) so that the Green function can be written as

$$G_R^{(1,2)} = \mathcal{T}_d(2\pi T)\lim_{Z \to +\infty} Z^{1/3}\xi_{1,2}(Z). \quad (4.50)$$

And the equation for $\xi_{1,2}$ can be obtained from (4.45) as

$$\xi_{1,2}' = -\frac{4}{3}K^{-1/2}\Lambda_l\xi_{1,2} - \frac{h \cdot \mathrm{k}}{2\pi T}K^{-2/3} + \frac{\omega}{2\pi T}Z^{-1}K^{-1/6}$$
$$+ \left(\frac{\omega}{2\pi T}Z^{-1}K^{-1/6} + \frac{h \cdot \mathrm{k}}{2\pi T}K^{-2/3}\right)\xi_{1,2}^2, \quad (4.51)$$

which can be solved numerically by using the incoming wave boundary condition $\xi_{1,2}(0) = \pm i$ on the horizon.

C. The numerical analysis

In this section, we evaluate numerically the retarded Green functions by solving (4.39) and (4.51) with incoming wave boundary conditions.

In confined phase, the results are collected in Figs. 4 and 5. The behavior of the peaks in the Green function displays discreteness as $\omega^2 - k^2 = M_n^2$ basically where M_n represents various constants. According to the general form of the fermionic propagator

$$G_R(\omega, \vec{k}) = \frac{1}{ik_\mu\gamma^\mu - M_n}, \qquad M_n = m_n - \Sigma(k), \quad (4.52)$$

here $m_n, \Sigma(k)$ refers respectively to the eigenvalue of the bare mass and self-energy, M_n corresponds to the on-shell energy of the various bound states consisting of the flavored fermions (quarks). And we believe this result is consistent with the property of the bubble background (4.1) since its dual theory is expected to be QCD with confinement states, e.g., meson, baryon states. We will further discuss these bound states at the end of this manuscript. Moreover, for $\omega > 0$, the location of the peak in the Green function can be read approximately once we set $\mathrm{k} = 0$, so that the peaks correspond to $\omega = m_n - \Sigma(k)$ as it is illustrated in Fig. 5. In this sense, we find quantitatively, for $\Lambda_l = 2, l = 0$, the bound energy (the position of the peak) is located at $\omega \simeq 1.6, 2.7, 3.7\ldots$ in $G_R^{(1)}$ and at $\omega \simeq 2.2, 3.1, 4.2\ldots$ in $G_R^{(2)}$. Furthermore, the location of the peaks also depends on the value of Λ_l as it is displayed in Fig. 6. Accordingly, if we identify $h = \mp 1$ as the parity of the bound states, the on-shell energy evaluated by the Green function agrees consistently with the spectrum of the worldvolume fermion on the D8-branes with various Λ_l in this model as it is calculated in [36].

Next, let us take a look at the Green function as the spectral function obtained in the deconfined phase illustrated in Figs. 7 and 8 where the peaks in the Green function have been fitted by green lines. Since the relation of $G_R^{(1,2)}$ is given as $G_R^{(2)}(\omega, \mathrm{k}) = -G_R^{(1)}(\omega, -\mathrm{k})$, let us focus on $G_R^{(2)}(\omega, \mathrm{k})$ for convenience. First, we can see the behaviors of the particle on shell (peaks in the Green function) are very different from those in the confined case which satisfies $\omega \simeq \mathrm{k}$ for $\mathrm{k} \gg 1$ and $\omega \simeq \mathrm{k}^2 + \ldots$ for $\mathrm{k} \ll 1$. While we discuss the positive frequency mode, it is same as the negative frequency mode. This behavior is in qualitative agreement with the dispersion curves of fermions obtained by the hard thermal loop (HTL) approximation in hot QCD, up to one-loop calculation, as [46]

$$\omega(\mathrm{k}) \simeq m_f \pm \frac{1}{3}\mathrm{k} + \frac{1}{3m_f}\mathrm{k}^2, \qquad \mathrm{k} \ll 1;$$
$$\omega(\mathrm{k}) \simeq \mathrm{k}, \qquad \mathrm{k} \gg 1, \quad (4.53)$$

where m_f is the effective mass of fermion generated by the medium effect given by

$$m_f = \sqrt{\frac{C_F}{8}}g_{\mathrm{YM}}T. \quad (4.54)$$

We note that chemical potential can also contribute to mass of fermion while it is not turned on in the current

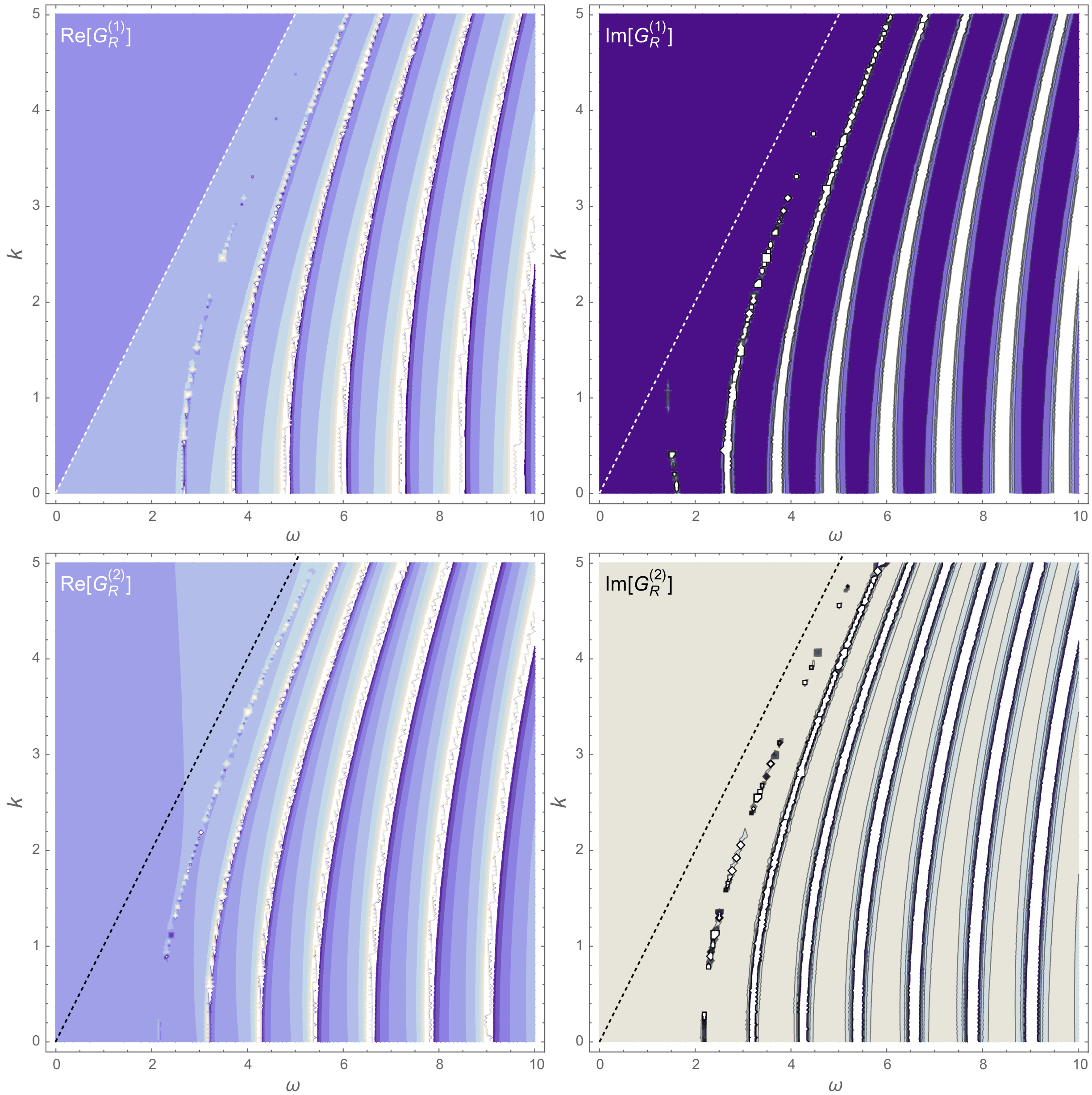

FIG. 4. Density plot of the confined retarded Green function $G_R^{(1,2)}$ as the spectral function of $1+3$ dimensional QCD from the D4/D8 model. The white regions refer to the peaks in the Green function and the dashed lines refer to $\omega = \mathrm{k}$ as the light cones. The parameters are chosen as $\Lambda_l = 2, l = 0, \mathcal{T}_c = 1$ in the unit of $M_{KK} = 1$.

holographic setup and C_F is suggested to be $C_F = 4/3$ for fundamental quarks. According to our numerical calculations presented in Fig. 8, the effective mass of fermion is evaluated to be $m_f = \omega(\mathrm{k}=0) \simeq 2.9 \times (2\pi T)$ which is expected to be the nonperturbative results predicted by holography. However, we must keep in mind that the holographic approach is valid in the strong coupling region, i.e., the ’t Hooft coupling constant $\lambda = g_{\mathrm{YM}}^2 N_c$ is finite and satisfies $\lambda \gg 1$ for $N_c \to \infty$. Hence the consistent interpretation here should be that, on the QFT side, the high order contribution of Yang-Mills coupling constant g_{YM} by HTL perturbation is suppressed in the large N_c limit since in this limit we have $g_{\mathrm{YM}} \ll 1$. Second, our current numerical evaluation of the dispersion curves implies the minus sign should be picked up in (4.53) for $\mathrm{k} \ll 1$ while there are two branches given by HTL approximation in (4.53). And following the discussion in [30,31], this branch could correspond to the dispersion curves of the fermionic plasmino at high temperature. Nonetheless, since the numerical calculation always displays a width of the peaks in the spectral function, the dispersion curves of quarks (which are expected to be $\omega(\mathrm{k}) \simeq m_f + \frac{1}{3}\mathrm{k} + \frac{1}{3m_f}\mathrm{k}^2$ for

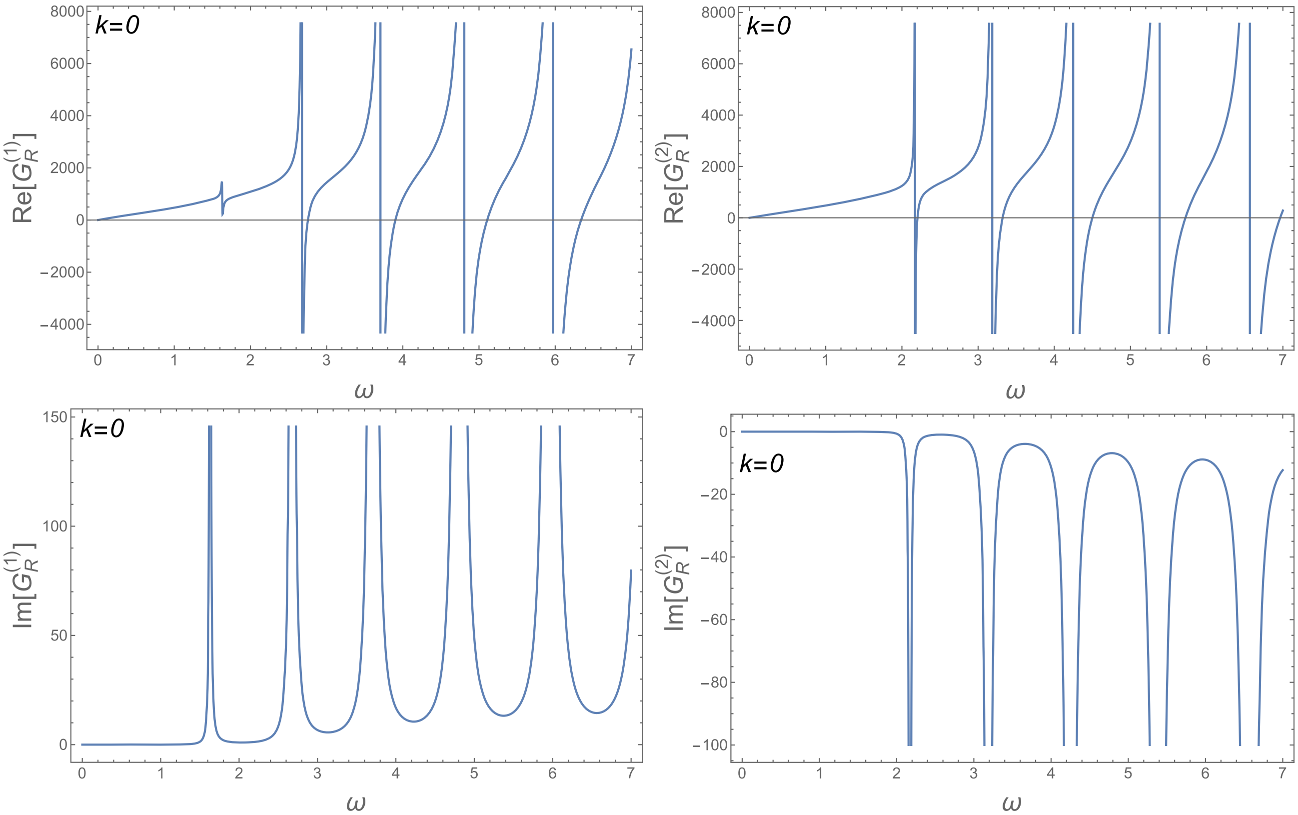

FIG. 5. The confined retarded Green function $G_R^{(1,2)}$ as the spectral function of $1+3$ dimensional QCD from the D4/D8 model. The parameters are chosen as $\Lambda_l = 2, \mathrm{k} = 0, \mathcal{T}_c = 1$ in the unit of $M_{KK} = 1$.

$\mathrm{k} \ll 1$) may also be included in the spectral function. However, distinguishing exactly the dispersion curves of fundamental quark from plasmino would very much depend on the trick of the numerical calculation we chose. In this sense, it would be less necessary to further compare our current numerical results with the HTL approximation. Overall, we believe our results, based on the holography with the principle in string theory, reveal mostly the fundamental properties of both confinement and deconfinement in QCD, while they are very different from some bottom-up approaches or phenomenological approaches in this model as [30,31] where the features of QCD confinement are less clear.

V. APPROACH TO THE D3/D7 MODEL

In order to further test the prescription in Sec. III C, let us attempt to use it in the D3/D7 approach in this section, which is a holographic version of $1+2$ dimensional QCD. Similarly as the case of the D4/D8 approach, in Sec. V A, we perform Step 1 in the prescription to obtain a 4d effective fermionic action with respect to the bubble and black D3 background. In Sec. V B, we perform Step 2 and Step 3 which is to solve the equations of motion for the bulk field, define its boundary value, and derive its on-shell action in order to evaluate the correlation function.

A. The 4d action for the bulk spinor

We start with the SUGRA solutions (2.4) and (2.2) in the case of $p = 3$ for the IIB SUGRA. So in the near horizon limit, the bubble D3-brane background is summarized as[9]

$$ds_c^2 = \frac{r^2}{R^2}\left[\eta_{\alpha\beta}dx^\alpha dx^\beta + f(r)(dx^3)^2\right] + \frac{R^2}{r^2}\left[\frac{dr^2}{f(r)} + r^2 d\Omega_5^2\right],$$
$$f(r) = 1 - \frac{r_{KK}^4}{r^4}, \qquad F_5 = dC_4 = 4R^4 g_s^{-1}\epsilon_5,$$
$$\alpha, \beta = 0, 1, 2, \tag{5.1}$$

which corresponds to the confined phase of the dual theory and the black D3-brane background is given as

$$ds_d^2 = \frac{r^2}{R^2}\left[-f_T(r)dt^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2\right] + \frac{R^2}{r^2}\left[\frac{dr^2}{f(r)} + r^2 d\Omega_5^2\right],$$
$$f_T(r) = 1 - \frac{r_H^4}{r^4}, \qquad F_5 = dC_4 = 4R^4 g_s^{-1}\epsilon_5, \tag{5.2}$$

[9]In the IIB SUGRA, the R-R 5-form is usually defined to be self-dual which is replaced as $F_5 \to \frac{1}{2}(F_5 + \star F_5)$.

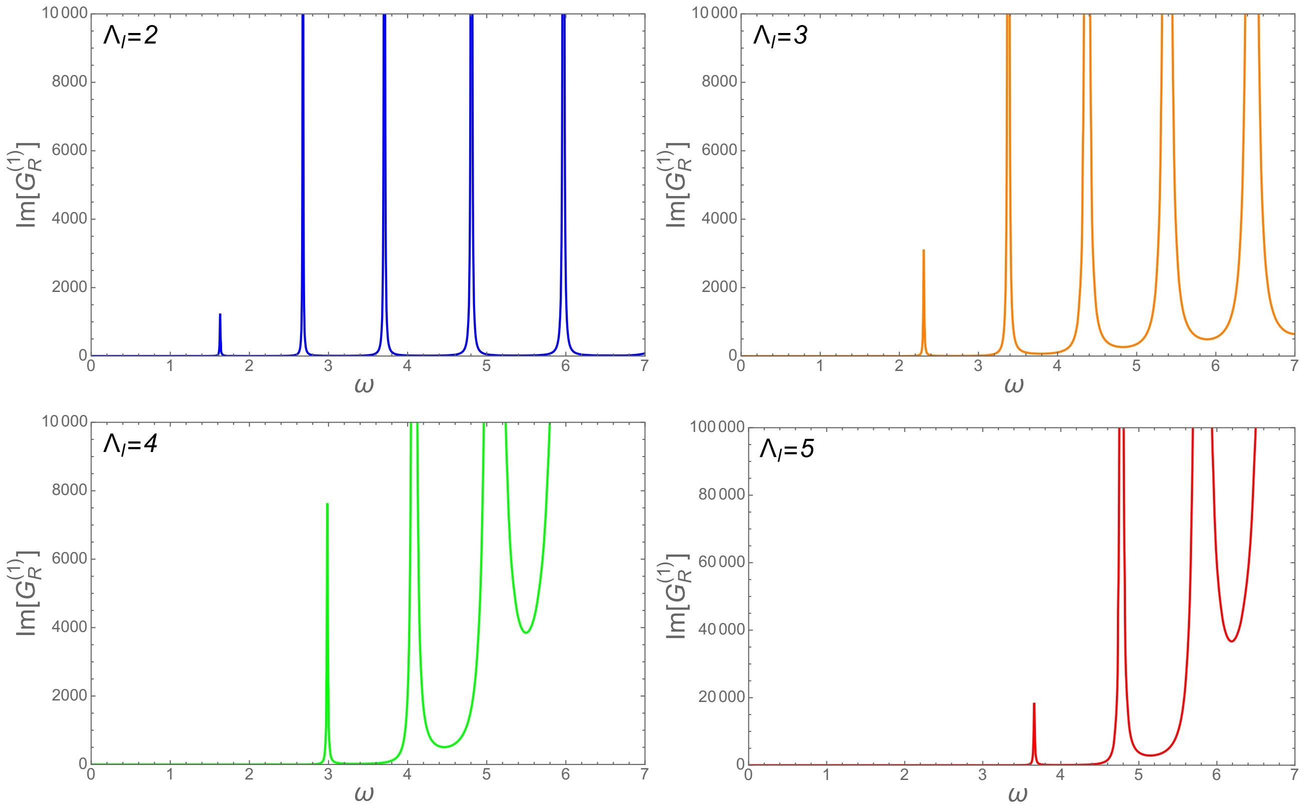

FIG. 6. The imaginary part of confined retarded Green function $G_R^{(1)}$ as the spectral function from the D4/D8 model with various Λ_l for $\mathrm{k}=0$.

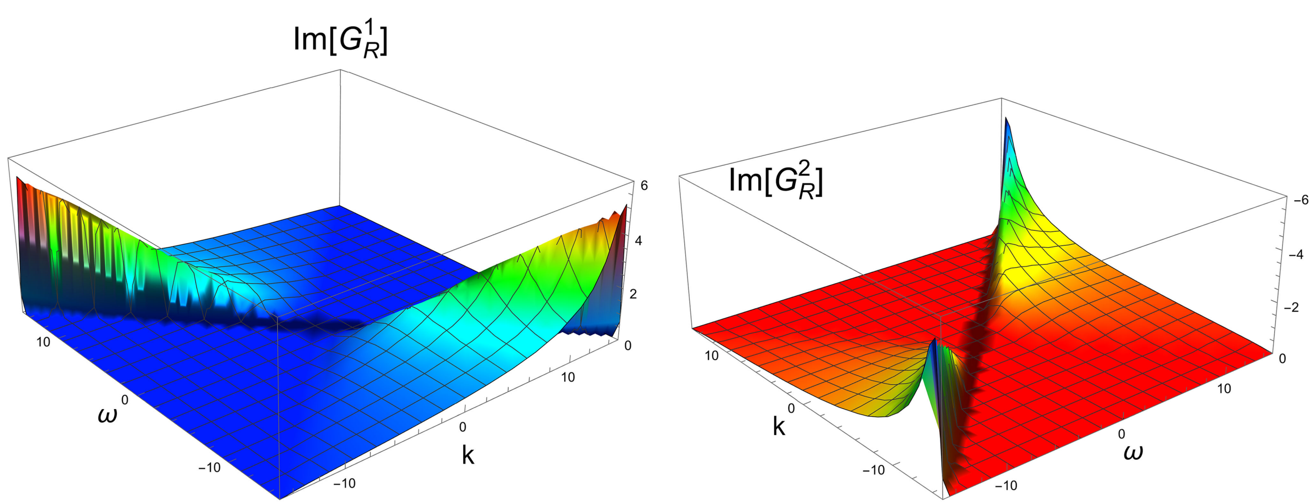

FIG. 7. The 3d plot of the imaginary part of the Green function $G_R^{(1,2)}$ from the black D4-brane background. The parameters are chosen as $\Lambda_l = 2, l = 0, \mathcal{T}_d = 1$ in the unit of $2\pi T = 1$.

which corresponds to the deconfined phase of the dual theory. Here we use ϵ_5 to refer to the volume form of a unit S^5 and $R^4 = 4\pi g_s N_c l_s^4$. Note that the dilaton field vanishes for $p = 3$ due to (2.2) and since x^3 is compactified on a circle, the dual field theory is effectively the $1+2$ dimensional Yang-Mills theory below the energy scale $M_{KK} = 2\pi\beta_3^{-1}$ where β_3 refers to the period of x^3 [14,37].

Then let us introduce the probe D7-branes as flavors embedded into the background (5.1) and (5.2) in order to investigate its worldvolume fermions. According to Table I,

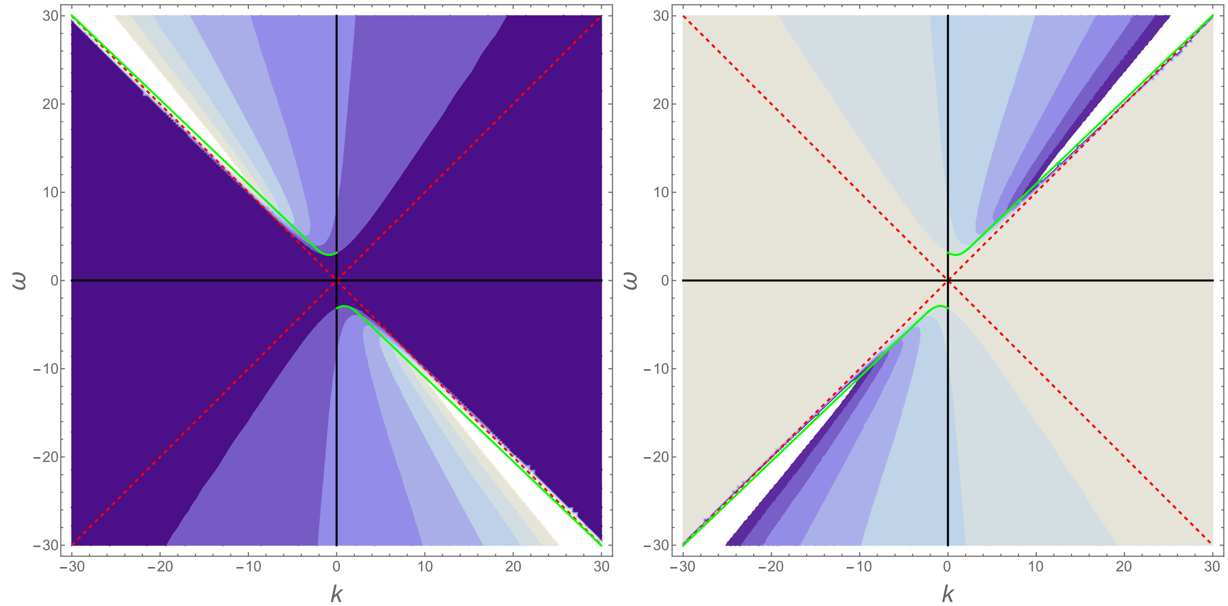

FIG. 8. Density plot of the imaginary part of the Green function $G_R^{(1,2)}$ from the black D4-brane background. The white regions represent the protrusion or pit, i.e., the peaks, in the Green functions which are fitted by green lines. The dashed lines refer to $\omega = \pm k$ as the light cone and the parameters are chosen as $\Lambda_l = 2, l = 0, \mathcal{T}_d = 1$ in the unit of $2\pi T = 1$.

the D7-branes are located at $x^{3,9} = \text{const}$, hence we impose the coordinate transformation,

$$r = \frac{r_{KK,H}}{\sqrt{2}}\left(\frac{1}{\zeta^2} + \zeta^2\right)^{1/2}, \zeta^2 = \rho^2 + u^2, u \equiv x^9, \quad (5.3)$$

to the radial and S^5 part in (5.1) and (5.2), then they can be rewritten as

$$\frac{R^2}{r^2}\left[\frac{dr^2}{f(r)} + r^2 d\Omega_5^2\right] = \frac{R^2}{\zeta^2}\left(d\rho^2 + \rho^2 d\Omega_4^2 + du^2\right), \quad (5.4)$$

where $\rho^2 = \sum_{m=4}^{8} x^m x_m$ and $d\Omega_4^2$ refers to the metric on S^4 whose solid angle is defined by parametrizing $x^{4,5...8}$. We note that due to $r \geq r_{KK,H}$, it reduces to two branches for ζ equivalently as $0 < \zeta \leq 1$ and $\zeta \geq 1$. In our setup, we chose the branch of $\zeta \geq 1$. Therefore the induced metric on the flavor D7-branes reads

$$\begin{aligned} ds^2_{\mathrm{D7},c} &= \frac{r^2}{R^2}\eta_{\alpha\beta}dx^\alpha dx^\beta + \frac{R^2}{\zeta^2}\left(d\rho^2 + \rho^2 d\Omega_4^2\right), \\ &\alpha, \beta = 0, 1, 2, \\ ds^2_{\mathrm{D7},d} &= \frac{r^2}{R^2}\left[-f_T(r)dt^2 + (dx^1)^2 + (dx^2)^2\right] \\ &\quad + \frac{R^2}{\zeta^2}\left(d\rho^2 + \rho^2 d\Omega_4^2\right). \end{aligned} \quad (5.5)$$

We note that $u = L$ as a constant represents the separation of the D3- and D7-branes and it is proportional to the bare mass of fundamental quarks in the D3/D7 approach [14] as the vacuum expectation value (VEV) of a (3,7) string. For chirally symmetric dual theory, we may simply set $L = 0$ which implies the fundamental quarks are massless. Keeping these in mind, the action for the worldvolume fermions on the D7-brane is given by (3.6) and (3.8) as

$$\begin{aligned} S_f^{\mathrm{D7}} = &\frac{iT_7}{2}\int d^8x\sqrt{-g}\bar{\Psi}P_- \\ &\times\left(\Gamma^\alpha\nabla_\alpha - \frac{1}{2\cdot 8\cdot 5!}F_{KLMNP}\Gamma^\alpha\Gamma^{KLMNP}\Gamma_\alpha\right)\Psi, \end{aligned} \quad (5.6)$$

where we have imposed $\bar{\gamma}\bar{\Psi} = \bar{\Psi}$ and F_{KLMNP} refers to the component of the R-R 5-form presented in (5.1) and (5.2). After some straightforward calculations, the action (5.6) can be written as

$$\begin{aligned} S_{f,c}^{\mathrm{D7}} = &\frac{i}{2}T_7R^2\int d^3x d\rho d\Omega_4 \bar{\Psi}P_-\left(\frac{\rho^4 r^2 R}{\zeta^5}\gamma^\alpha\partial_\alpha + \frac{3\rho^5 r^3}{2\zeta^6}\sqrt{f}\gamma^\rho\right. \\ &\left. + \frac{\rho^4 r^3}{\zeta^4 R}\gamma^\rho\partial_\rho + \frac{\rho^3 r^3}{\zeta^4 R}\gamma^m D_m + \frac{\rho^4 r^3}{\zeta^5 R}\right)\Psi, \end{aligned} \quad (5.7)$$

$$\begin{aligned} S_{f,d}^{\mathrm{D7}} = &\frac{i}{2}T_7R^2\int d^3x d\rho d\Omega_4 \bar{\Psi}P_-\left[\frac{\rho^4 r^2 R}{\zeta^5}\gamma^0\partial_0 + \frac{\rho^4\sqrt{f_T}r^3}{2\zeta^5 R}\right. \\ &\times\left(\frac{\zeta f'}{2f} + \frac{3\rho\sqrt{f_T}}{\zeta}\right)\gamma^4 + \frac{\rho^4\sqrt{f_T}r^2 R}{\zeta^5}\gamma^\alpha\partial_\alpha \\ &\left. + \frac{\rho^4\sqrt{f_T}r^3}{\zeta^4 R}\gamma^\rho\partial_\rho + \frac{\rho^3\sqrt{f_T}r^3}{\zeta^4 R}\gamma^m D_m + \frac{\rho^4\sqrt{f_T}r^3}{\zeta^5 R}\right]\Psi, \end{aligned} \quad (5.8)$$

where we have used $\gamma^{0123\rho}\Psi = \Psi$ and

$$P_- = \frac{1}{2}(1 - \Gamma_{\mathrm{D7}}), \tag{5.9}$$

with respect to the bubble (5.1) and black brane (5.2) background. Follow the decomposition presented in (4.13)–(4.15) with the eigen equation on S^4 (4.16) and integrate the S^4 part, and the action (5.7) can be further written as a 4d form as

$$S_{f,c}^{\mathrm{D7}} = i\mathcal{T}\int d^3x d\rho \bar{\psi}\left[\frac{1}{\zeta} - \frac{1}{\rho}\Lambda_l + \frac{R^2}{r\zeta}\boldsymbol{\gamma}^\alpha\partial_\alpha + \left(\frac{2\rho}{\zeta^2} - \frac{2}{\rho}\right)\boldsymbol{\gamma} + \boldsymbol{\gamma}\partial_\rho\right]\psi \tag{5.10}$$

by rescaling $\psi \to (2\pi\alpha')\psi\frac{\zeta^2}{\rho^2}\frac{1}{r^{3/2}}$ and action (5.8) can be written as

$$S_{f,d}^{\mathrm{D7}} = i\mathcal{T}\int d^3x d\rho \bar{\psi}\left[\frac{1}{\sqrt{f_T}}\frac{R}{r\zeta}\boldsymbol{\gamma}^0\partial_0 + \frac{R}{r\zeta}\boldsymbol{\gamma}^\alpha\partial_\alpha + \frac{1}{R}\boldsymbol{\gamma}\partial_\rho + \left(\frac{3\rho\sqrt{f_T}}{2R\zeta^2} + \frac{f_T'}{4f_T R}\right)\boldsymbol{\gamma} + \frac{1}{R}\left(\frac{1}{\zeta} - \frac{f_T'}{4f_T}\right) - \frac{1}{R\rho}(\Lambda_l + 2) + \frac{\rho}{R\zeta^2}\left(\frac{5}{2} - \frac{3}{2}\sqrt{f_T}\right)\right]\psi \tag{5.11}$$

by rescaling $\psi \to (2\pi\alpha')\psi\frac{\zeta^2}{\rho^2 r^{3/2} f_T^{1/4}}$ where

$$\mathcal{T} = \frac{1}{2}T_7 R^2 \Omega_4 (2\pi\alpha')^2. \tag{5.12}$$

We will use these 4d fermionic actions (5.10) and (5.11) to compute the two-point correlation function in the $2+1$ dimensional dual theory.

B. Use the prescription

Let us use the prescription in Sec. III C to study the two-point Green function for the $1+2$ dimensional QCD in the D3/D7 model.

1. Confined phase

For the confined phase, the equation of motion for the worldvolume ψ can be obtained by varying (5.10) as

$$\left[\frac{1}{\xi} - \frac{1}{\rho}\Lambda_l + \frac{R^2}{r\xi}\boldsymbol{\gamma}^\alpha\partial_\alpha + \left(\frac{2\rho}{\xi^2} - \frac{2}{\rho}\right)\boldsymbol{\gamma} + \boldsymbol{\gamma}\partial_\rho\right]\psi = 0. \tag{5.13}$$

Using the ansatz given in (4.22), we can obtain the coupled equations as

$$\begin{aligned}&\left(\frac{1}{\xi} - \frac{1}{\rho}\Lambda_l + \frac{2\rho}{\xi^2} - \frac{2}{\rho} + \partial_\rho\right)\psi_R + i\frac{R^2}{r\xi}\sigma^\alpha\partial_\alpha\psi_L = 0,\\ &i\frac{R^2}{r\xi}\bar{\sigma}^\alpha\partial_\alpha\psi_R + \left(\frac{1}{\xi} - \frac{1}{\rho}\Lambda_l - \frac{2\rho}{\xi^2} + \frac{2}{\rho} - \partial_\rho\right)\psi_L = 0.\end{aligned} \tag{5.14}$$

Setting $k_\mu = (-\omega, \mathrm{k}, 0, 0)$, the equations presented in (5.14) can be written as decoupled second order differential equations, which, at boundary $r \to \infty$ ($\rho \to \infty$), take the asymptotic form as

$$\begin{aligned}\psi_R'' + \frac{2}{\rho}\psi_R' - \frac{(\Lambda_l^2 + 3\Lambda_l + 2)}{\rho^2}\psi_R = 0,\\ \psi_L'' + \frac{2}{\rho}\psi_L' - \frac{(\Lambda_l^2 + \Lambda_l)}{\rho^2}\psi_L = 0.\end{aligned} \tag{5.15}$$

And they can be solved exactly as,

$$\begin{aligned}\psi_R &= A\rho^{1+\Lambda_l} + B\rho^{-2-\Lambda_l},\\ \psi_L &= C\rho^{\Lambda_l} + D\rho^{-1-\Lambda_l},\end{aligned} \tag{5.16}$$

where A, B, C, D are integration constant. As we have outlined $\Lambda_l \geq 2$, so the boundary value ψ_0 of ψ should be defined as

$$\psi_0 = \lim_{\rho\to\infty}\rho^{-1-\Lambda_l}\psi = \rho^{-1-\Lambda_l}\begin{pmatrix}\psi_R\\ 0\end{pmatrix} = \begin{pmatrix}A\\ 0\end{pmatrix}. \tag{5.17}$$

Then the on-shell action (5.10) will include a finite part as

$$\begin{aligned}S_{f,c}^{\mathrm{D7}} &\supseteq -\mathcal{T}\int d^3x\left[(\psi_L^\dagger\psi_R)|_{\rho\to\infty} + \mathrm{H.c.} + \cdots\right]\\ &= -\mathcal{T}\int d^3x\left[C^\dagger A\rho^{2\Lambda_l+1} + D^\dagger A + \mathrm{H.c.}\right]\Big|_{\rho\to\infty},\end{aligned} \tag{5.18}$$

thus it requests a holographic counterterm as

$$S_{ct} = \mathcal{T}\int d^3x\left[C^\dagger A\rho^{2\Lambda_l+1} + \mathrm{H.c.}\right]\Big|_{\rho\to\infty}. \tag{5.19}$$

The existing finite part in (5.18) is due to the residual isometry of AdS_5 since the D3-brane solution (5.1) and (5.2) as bulk geometry is $\mathrm{AdS}_5 \times S^5$ asymptotically. Following the discussion in Secs. 3 and 4, the retarded Green function is given by

$$\mathcal{T}D = G_R(\omega, \vec{k})A. \tag{5.20}$$

Using the representation (4.36) and introducing the ratio (4.37), the Green function can be written as

$$G_R^{(1,2)} = \lim_{\rho\to\infty}\rho^{2\Lambda_l+2}\xi_{1,2}, \tag{5.21}$$

where the equation for $\xi_{1,2}$ can be obtained from (5.14) as

$$\xi'_{1,2} = \frac{R^2}{r\zeta}(\omega - h\cdot \mathrm{k}) + 2\left(\frac{1}{\zeta} - \frac{1}{\rho}\Lambda_l\right)\xi_1 + \frac{R^2}{r\zeta}(\omega + h\cdot \mathrm{k})\xi_{1,2}^2, \tag{5.22}$$

which will be evaluated numerically with the incoming wave boundary condition and $h = \pm 1$ for $\xi_{1,2}$ respectively.

2. Deconfined phase

For the deconfined phase, the equation of motion for the worldvolume ψ is given by varying action (5.11) which is

$$\left[\frac{1}{\sqrt{f_T}}\frac{R}{r\zeta}\gamma^0\partial_0 + \frac{R}{r\zeta}\gamma^\alpha\partial_\alpha + \frac{1}{R}\gamma\partial_\rho + \left(\frac{3\rho\sqrt{f_T}}{2R\zeta^2} + \frac{f'_T}{4f_T R}\right)\gamma + \frac{1}{R}\left(\frac{1}{\zeta} - \frac{f'_T}{4f_T}\right) - \frac{1}{R\rho}(\Lambda_l + 2) + \frac{\rho}{R\zeta^2}\left(\frac{5}{2} - \frac{3}{2}\sqrt{f_T}\right)\right]\psi = 0. \tag{5.23}$$

Using the ansatz (4.22), the coupled equations for $\psi_{L,R}$ are

$$\begin{aligned}&\left[\frac{1}{R}\partial_\rho + \frac{3\rho\sqrt{f_T}}{2R\zeta^2} + \frac{f'_T}{4f_T R} + \frac{1}{R}\left(\frac{1}{\zeta} - \frac{f'_T}{4f_T}\right) - \frac{1}{R\rho}(\Lambda_l + 2) + \frac{\rho}{R\zeta^2}\left(\frac{5}{2} - \frac{3}{2}\sqrt{f_T}\right)\right]\psi_R + i\frac{R}{r\zeta}\left(\frac{1}{\sqrt{f_T}}\partial_0 + \sigma^\alpha\partial_\alpha\right)\psi_L = 0,\\ &\left[-\frac{1}{R}\partial_\rho - \frac{3\rho\sqrt{f_T}}{2R\zeta^2} - \frac{f'_T}{4f_T R} + \frac{1}{R}\left(\frac{1}{\zeta} - \frac{f'_T}{4f_T}\right) - \frac{1}{R\rho}(\Lambda_l + 2) + \frac{\rho}{R\zeta^2}\left(\frac{5}{2} - \frac{3}{2}\sqrt{f_T}\right)\right]\psi_L + i\frac{R}{r\zeta}\left(\frac{1}{\sqrt{f_T}}\partial_0 + \bar{\sigma}^\alpha\partial_\alpha\right)\psi_R = 0,\end{aligned} \tag{5.24}$$

which can reduce to two second-order differential equations. At boundary $r \to \infty$ ($\rho \to \infty$), they are

$$\begin{aligned}\psi''_R + \frac{5}{\rho}\psi'_R - \frac{(5 + 2\Lambda_l)(2\Lambda_l - 3)}{4\rho^2} &= 0,\\ \psi''_R + \frac{5}{\rho}\psi'_R - \frac{(2\Lambda_l - 5)(2\Lambda_l + 3)}{4\rho^2} &= 0,\end{aligned} \tag{5.25}$$

which can be solved as

$$\begin{aligned}\psi_R &= \rho^{\Lambda_l - \frac{3}{2}}A + \rho^{-\Lambda_l - \frac{3}{2}}B,\\ \psi_L &= \rho^{-\Lambda_l - \frac{3}{2}}C + \rho^{-\frac{5}{2} + \Lambda_l}D.\end{aligned} \tag{5.26}$$

While the boundary value of ψ remains to be ψ_R, the on-shell action (5.11) taking the form as

$$\begin{aligned}S^{\mathrm{D7}}_{f,c} &\supseteq -\mathcal{T}\int d^3x\left[(\psi_L^\dagger\psi_R)|_{\rho\to\infty} + \mathrm{H.c.} + \cdots\right]\\ &= -\mathcal{T}\int d^3x\left[\rho^{2\Lambda_l - 4}D^\dagger A + \mathrm{H.c.}\right]\big|_{\rho\to\infty}\end{aligned} \tag{5.27}$$

does not include any finite part which implies the deconfined phase of 3d QCD is also nonrenormalizable in holography. In order to obtain a finite Green function, we can follow the same discussion as it is in the D4/D8 model. Therefore, let us first take the boundary value ψ_0 of ψ carefully as

$$\psi_0 = \lim_{\rho\to\epsilon^{-1}}\epsilon^{\frac{1}{2}}\psi = \epsilon^{-\Lambda_l + 2}\begin{pmatrix}A\\0\end{pmatrix}, \epsilon \to 0. \tag{5.28}$$

Then the conjugate momentum Π_0 for ψ_0 is given as

$$\Pi_0 = -\frac{\delta S^{\mathrm{D8}}_{f,d}}{\delta\psi_0} = \mathcal{T}(0, D^\dagger)\epsilon^{-\Lambda_l + 2}. \tag{5.29}$$

Accordingly, it leads to a finite Green function as

$$\bar{\Pi}_0 = G_R(\omega, \vec{k})\psi_0, \qquad G_R^{(1,2)} = \lim_{\epsilon\to 0}\epsilon^{-1}\xi_{1,2}, \tag{5.30}$$

where the equation for $\xi_{1,2}$ defined in (4.37) can be obtained from (5.24) as

$$\xi'_{1,2} = \frac{R^2}{r\zeta}\left(\frac{\omega}{\sqrt{f_T}} - h\cdot \mathrm{k}\right)\xi_2 + \frac{R^2}{r\zeta}\left(\frac{\omega}{\sqrt{f_T}} + h\cdot \mathrm{k}\right)\xi_{1,2}^2 + 2\left[\left(\frac{1}{\zeta} - \frac{f'_T}{4f_T}\right) - \frac{1}{\rho}(\Lambda_l + 2) + \frac{\rho}{\zeta^2}\left(\frac{5}{2} - \frac{3}{2}\sqrt{f_T}\right)\right]\xi_{1,2}. \tag{5.31}$$

And we will evaluate the Green function with the incoming wave boundary condition on the horizon and $h = \pm 1$ for $\xi_{1,2}$ respectively.

C. The numerical analysis

In this section, let us summarize the numerical results by solving (5.22) and (5.31) in the D3/D7 approach. The behavior of two-point Green function in the confined phase is illustrated in Fig. 9 which is basically similar to the approach in the D4/D8 model. The peaks in the Green function display the discreteness representing the on-shell bound states as confinement in 3d QCD, as $\omega \simeq 1.6, 2.6, 4.1\ldots$ in $G_R^{(1)}$ and $\omega \simeq 1.9, 3.4, 4.8\ldots$ in $G_R^{(2)}$ which is only quantitatively different from the results in the D4/D8 model. To specify the position of the on-shell energy, we have plotted out the relation of the Green function G_R as a function of ω which is illustrated in Fig. 10. Note that, while we have set $L = 0$ for the chirally symmetric theory, the position of the peaks in the Green function depends on

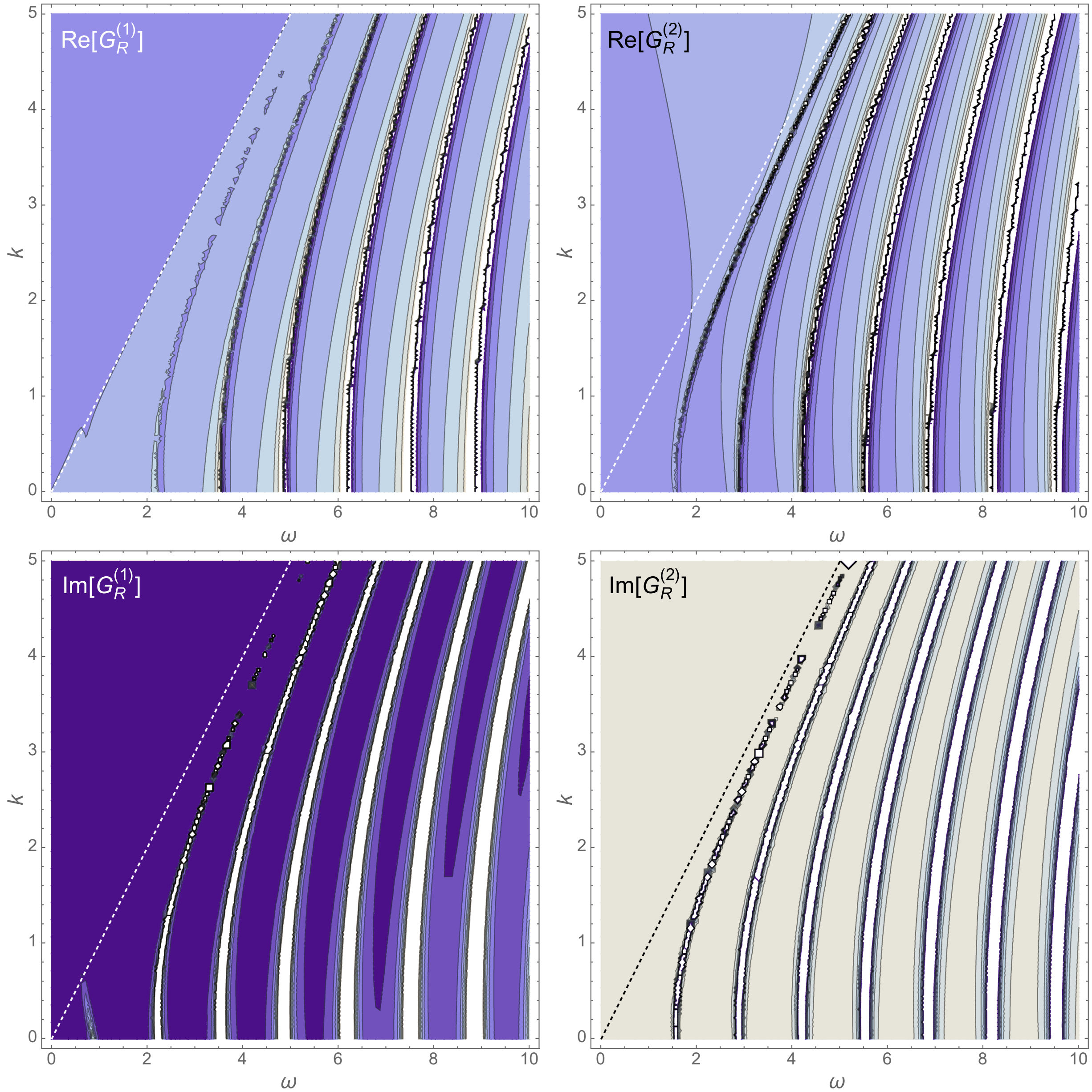

FIG. 9. Density plot of the confined retarded Green function $G_R^{(1,2)}$ as the spectral function of the $1+2$ dimensional QCD from the D3/D7 model. The parameters are chosen as $M_{KK}=1, \Lambda_l=2, L=0$. The white regions refer to the peaks in the Green function and dashed lines refer to the light cones.

the quark mass L as it is illustrated in Fig. 11. The dependence on L in Green function is different from the D4/D8 approach, since there is not a direction perpendicular to both D4- and D8-branes in the D4/D8 model [which means the VEV of a (4,8) string is vanished, i.e., the bare mass of fundamental quark is vanished]. Furthermore, our numerical calculation reveals that the on-shell mass of the bound fermion is suppressed by the quark bare mass due to the correction of the self-energy $\Sigma(k)$ in (4.52). In the deconfined phase, we plot out the imaginary part of the Green function as the spectral function in Figs. 12 and 13 in which the on-shell condition fitted by green lines is in qualitative agreement with the dispersion curves obtained by the hard thermal loop approximation presented in (4.53). The effective mass of fermion in hot medium is evaluated as $m_f \simeq 4.8(2\pi T)$ by the D3/D7 model which also implies the high order contribution in HTL approximation might be suppressed in the large N_c limit. Altogether, the D3/D7 approach also reveals mostly the fundamental properties of QCD as it is expected.

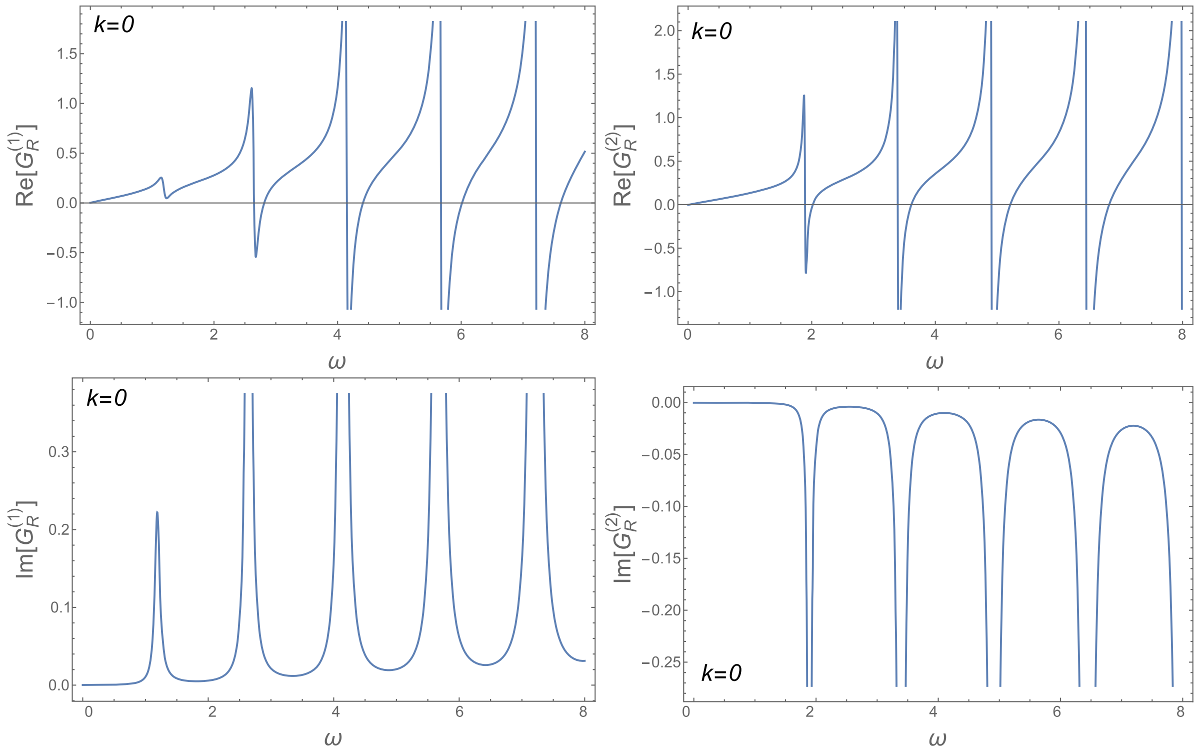

FIG. 10. The confined retarded Green function $G_R^{(1,2)}$ as the spectral function of the $1+2$ dimensional QCD from the D3/D7 approach. The parameters are set as $M_{KK}=1, \Lambda_l=2, L=0$.

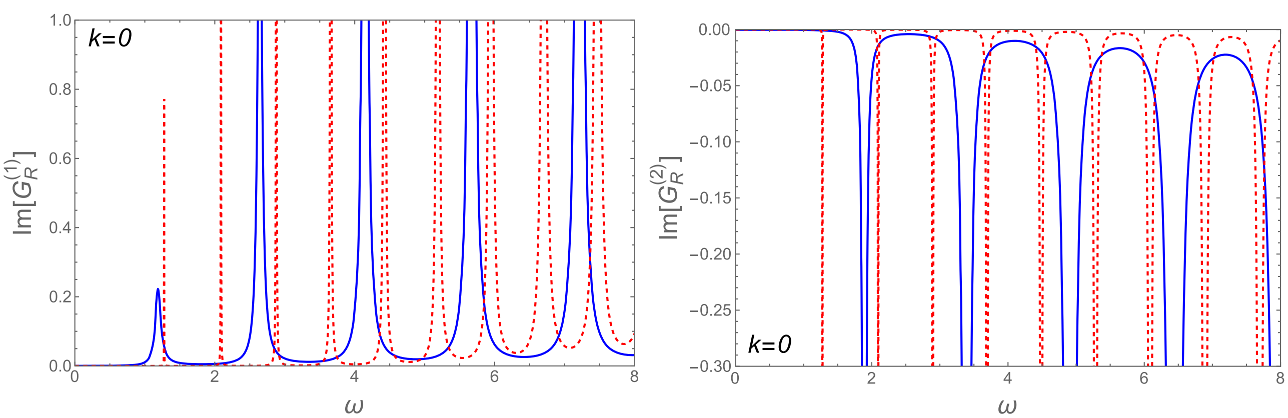

FIG. 11. Imaginary part of the confined retarded Green function $G_R^{(1,2)}$ with $L=0$ (solid blue line) and $L=1$ (dashed red line).

VI. SUMMARY AND DISCUSSION

In this work, we study the correlation function of the flavored fermion in the $\mathrm{D}_p/\mathrm{D}_{p+4}$ model ($p=4$, 3) as the holographic top-down approaches to QCD. Since our concern is the fermionic correlator, the bulk spinor is identified to the worldvolume fermion created by the $(p+4, p+4)$ string on the D_{p+4}-brane. Then we pick up the action for the worldvolume fermion on the D-brane which is obtained by T-duality in string theory, and generalize the prescription for two-point correlation function in the AdS/CFT dictionary into general D-brane background with respect to the flavored fermion in the dual theory. Afterwards we apply our viewpoint to the case of $p=4$, 3 respectively. The numerical calculation and result show that the Green functions in both D4/D8 and D3/D7 approaches reveal the discrete peaks with the bubble background which represents the bound states as confinement in QCD. In particular, the various bound energies agree basically with the numerical evaluation in [36] for $p=4$ quantitatively. With the black brane background, the on-shell condition given by the Green function agrees qualitatively with the dispersion curves obtained by the hard thermal loop

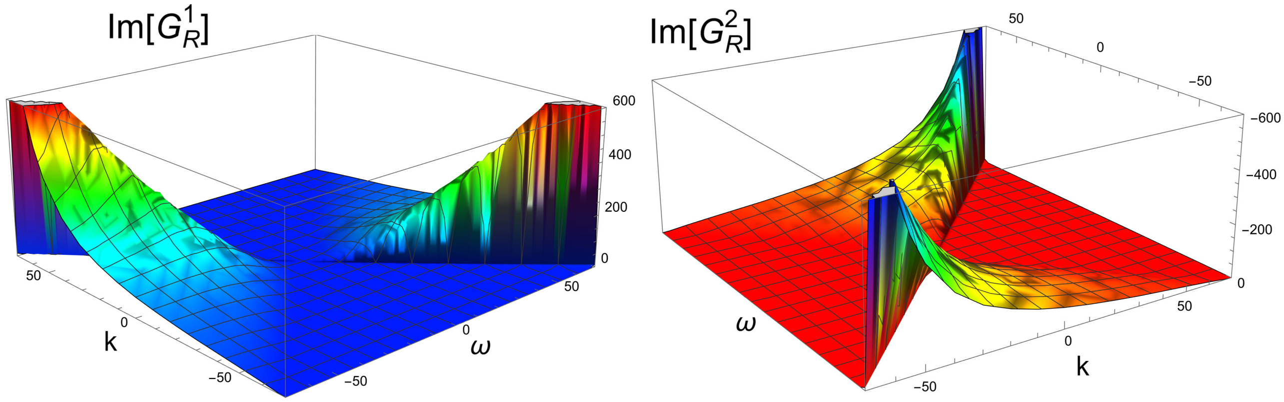

FIG. 12. The 3d plot of the imaginary part of the Green function $G_R^{(1,2)}$ from the black D3-brane background. The parameters are chosen as $\Lambda_l = 2, l = 0, \mathcal{T} = 1$ in the unit of $2\pi T = 1$.

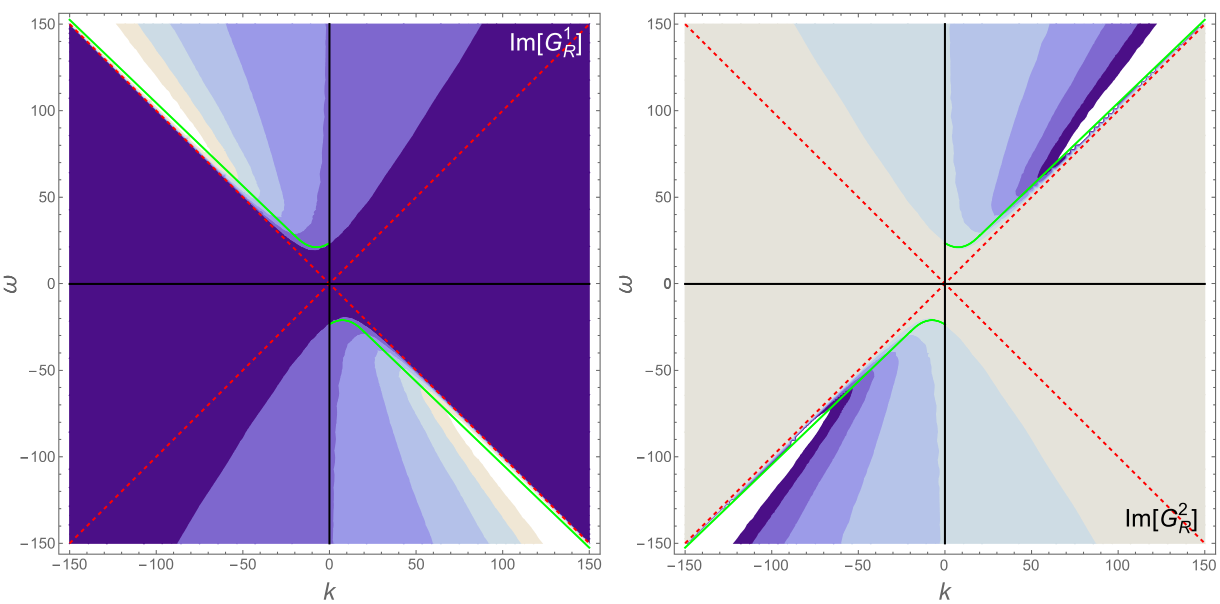

FIG. 13. Density plot of the imaginary part of the Green function $G_R^{(1,2)}$ from the black D3-brane background. The white regions represent the protrusion or pit, i.e., the peaks, in the Green functions which is fitted by green lines. The dashed lines refer to $\omega = \pm k$ as the light cone and the parameters are chosen as $\Lambda_l = 2, l = 0, \mathcal{T} = 1$ in the unit of $2\pi T = 1$.

approximation which displays the behavior of the deconfined fermion in hot medium. Overall our approaches to the D4/D8 and D3/D7 models illustrate the fundamental properties of QCD in holography which would therefore be remarkable.

Let us now give some comments about this work. First, although the on-shell condition in the deconfined phase is in qualitative agreement with several bottom-up approaches involving minimally coupled fermion, e.g., [30,31], the behavior of Green function in confined phase is totally different. The reasons are mostly that, as the dependence of the inner sphere S^4 is neglected and the probe brane is absent in the bottom-up approaches, the action for the bulk fermion would take a different form so that the asymptotic behavior of fermion, related to the Green function, is also different. And notice that the action (3.6) for worldvolume fermion on a D-brane is not minimally coupled in general, thus it is not surprising that the results in our top-down approach could be different from those in the bottom-up approaches. However, the property of confinement in QCD is less clear in the previous works with bottom-up models or with minimally coupled fermion.

Second, in the deconfined phase, while our analysis is consistent with the hard thermal loop approximation, we must keep in mind that the QFT approach is only valid in weak coupled field theory. Therefore our current result in the deconfined phase (which is valid in the strong coupling region) may imply the high order contribution from the

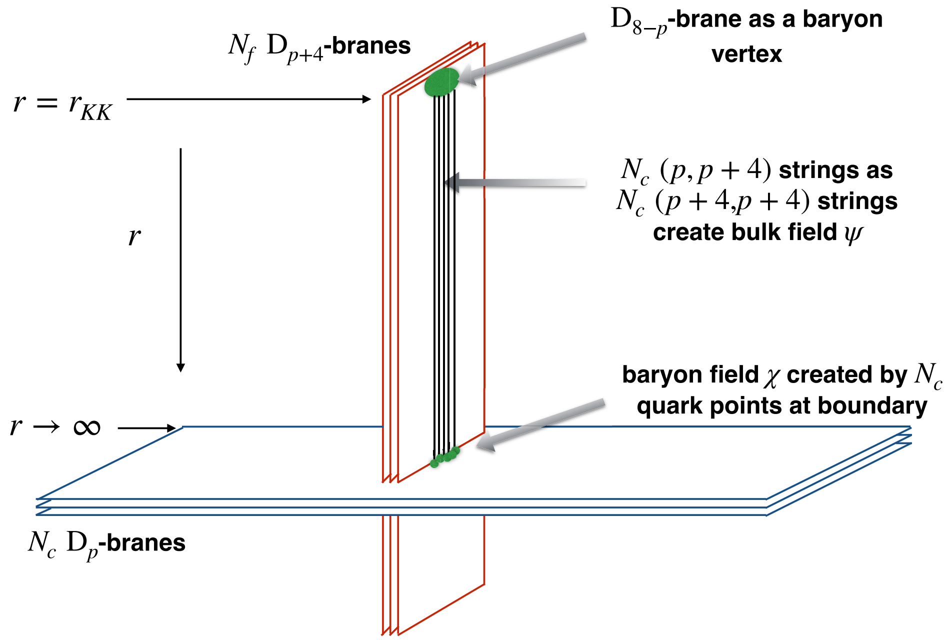

FIG. 14. The D-brane configuration of $\mathrm{D}_p/\mathrm{D}_{p+4}$ system with a baryon vertex. The bulk field ψ is created by N_c $(p, p+4)$ strings as quarks and the dual field χ at boundary is produced by N_c quark points as a baryon field.

hard thermal loop approximation at large N_c limit is suppressed. On the other hand, the hard thermal loop approximation implies the chemical potential may also contribute to the effective thermal mass of fermion while it is not included in this work. However, if the chemical potential (relating to the gauge field potential on the worldvolume of the D_{p+4}-brane) is taken into account in this setup, its equation of motion coupling to bulk fermion would be totally different from the bottom-up approaches with minimally coupled fermion according to action (3.6)–(3.8), because the bulk fermion is identified to be the fundamental representation of a $U(N)$ group in most bottom-up approaches while it is not in our top-down approach. Accordingly, we believe the Green function with nonvanished chemical potential in our top-down approach will display a very different behavior from it in the bottom-up approaches and we will leave this for a future work.

Last but not least, let us discuss the interpretation of the dual operator χ to the bulk fermion in terms of hadron physics and outline how to interpret it as a baryon field in the confined phase. As we have specified that to interpret the dual field χ as baryon is closer to the realistic QCD, so it is necessary to let χ produced by multiple fundamental quarks take baryon numbers on the side of string theory. Fortunately, this can be achieved by following Witten's [38], that is to introduce a D_{8-p}-brane wrapped on S^{8-p} [38,47,48] as a baryon vertex located at $r = r_{KK}$ in the bubble background. Since the N_c fundamental quarks created by the $(p, p+4)$ string must always take baryon numbers, there must be N_c $(p, p+4)$ strings as fundamental quarks connect the wrapped D_{8-p}-brane and stretch to the bulk boundary totally inside the D_{p+4}-branes thus they are flavored, colored, baryonic strings and are also $(p+4, p+4)$ strings. By taking into account a probe D_p-brane at the holographic boundary $r \to \infty$, the D-brane configuration with a baryon vertex is illustrated in Fig. 14. In this configuration, the $(p, p+4)$ strings as fundamental quarks are colored, flavored, and take baryon numbers, hence the dual field χ produced by multiple N_c $(p, p+4)$ string points is a baryonic hadron field, i.e., a baryon field. On the other hand, since the $(p, p+4)$ strings are also $(p+4, p+4)$ strings, the bulk field ψ dual to χ created by the multiple quarks is also baryonic. So it provides a nicely holographic correspondence with respect to the baryon field. In this sense, the bound states as the various peaks presented in the confined Green function refer to the baryon spectrum in holography. And it could be possible to fit the experimental data if we further identify the quantum number l, s in (4.16) to the isospin and spin of the baryon. In this sense, we can identify, for examples, the first three lowest states with same parity presented in Fig. 6 as proton, $N(1440)$ and $N(1710)$ for $l = 1$ as [49]. Thus our approach with the D4/D8 model gives the mass ratios of $N(1440)$ and proton, $N(1710)$ and proton as $M_{N(1440)}/M_{\mathrm{proton}} \simeq 1.51, M_{N(1710)}/M_{\mathrm{proton}} \simeq 1.95$, which are very close to the experimental data $M^{\mathrm{exp}}_{N(1440)}/M^{\mathrm{exp}}_{\mathrm{proton}} \simeq 1.53$, $M^{\mathrm{exp}}_{N(1440)}/M^{\mathrm{exp}}_{\mathrm{proton}} \simeq 1.82$ [50]. And this viewpoint may also support that the open strings on the D-brane behaves somehow as baryons in the D_p/D_{p+4} model as it is discussed in [39]. We further comment at last that to interpret χ as quark field, i.e., produced by a single $(p, p+4)$ string, may be possible in the deconfined background since in hadron physics a free quark as a free color charge could be observables theoretically in the deconfinement phase of QCD and baryon will be dissolved in the deconfinement phase.

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