Photon quantization in cosmological spaces

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Canonical quantization of the photon—a free massless vector field—is considered in cosmological spacetimes in a two-parameter family of linear gauges that treat all the vector potential components on equal footing. The goal is setting up a framework for computing photon two-point functions appropriate for loop computations in realistic inflationary spacetimes. The quantization is implemented without relying on spacetime symmetries, but rather it is based on the classical canonical structure. Special attention is paid to the quantization of the canonical first-class constraint structure that is implemented as the condition on the physical states. This condition gives rise to subsidiary conditions that the photon two-point functions must satisfy. Some of the de Sitter space photon propagators from the literature are found not to satisfy these subsidiary conditions, bringing into question their consistency.

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I. INTRODUCTION

The massless vector field—the photon—couples conformally to gravity in four spacetime dimensions, and effectively does not see the cosmological expansion. That is why its linear dynamics is not as interesting compared to nonconformally coupled fields that experience gravitational particle production [1–3] at the linear level. Nevertheless, coupling of photons to other fields might break conformality. Particularly interesting cases occur in inflation where the conformal coupling of the photon is broken by quantum loops generated from its interactions with light spectator scalars [4–29] or gravitons [30–36]. Scalars and gravitons are nonconformally coupled to gravity and experience a huge enhancement of their infrared sector due to the rapid expansion during inflation, the effects of which are communicated to the photon via loops. Most of the loop computations involving photons thus far have been worked out in a rigid de Sitter background, for which the two-point functions (propagators) comprising the loop expansion are known. It would be interesting to understand how the results, in particular the secular corrections to photons, are modulated in realistic slow-roll inflationary spacetimes. This work aims to advance this understanding by considering the photon and its two-point functions in a two-parameter family of linear gauges in general expanding cosmological spaces.

The specific goals of this article are threefold:

(i) To understand the photon quantization without relying on background symmetries or on covariant gauge fixing;

(ii) To set up a framework that will facilitate the computations of photon two-point functions in D-dimensional cosmological spaces, and to derive all the subsidiary conditions the two-point functions must satisfy, with the goal of performing dimensionally regulated loop computations in realistic inflationary spacetimes;

(iii) To demonstrate that cosmological evolution is not in conflict with gauge invariance, as suggested in [37], and that there is no contribution to the photon energy-momentum tensor coming from the gauge-fixing terms in any cosmological spacetime.

Given the aims listed above, in this work we consider the photon (electromagnetism) is spatially flat cosmological spacetimes in a two-parameter family of linear gauges preserving cosmological symmetries,

$$S_{\rm gf}[A_\mu] = \int d^Dx\sqrt{-g}\bigg[-\frac{1}{2\xi}(\nabla^\mu A_\mu - 2\zeta n^\mu A_\mu)^2\bigg], \qquad (1)$$

where ξ and ζ are arbitrary gauge-fixing parameters, and where $n_\mu = \delta_\mu^0\mathcal{H}$ is a nondynamical timelike vector, invariant under spatial rotations and translations. In the limit $\zeta = 0$ this gauge-fixing functional reduces to the general covariant gauge, which one might be tempted to consider as a natural choice. However, experience in de Sitter space suggests that noncovariant gauge choices can lead to simpler two-point functions [38] that can considerably simplify often quite involved computations. To this end we set up the framework that will facilitate identifying simple gauges in D-dimensional inflationary spacetimes, and streamline their computation.

In de Sitter space both the photon and the massive vector two-point functions have been worked out: in general covariant gauge [39–42], and in the simple gauge [38], while for more general spacetimes only a few results exist—the unitary gauge massive vector propagator [43] in power-law inflation, and photon propagator in arbitrary four-dimensional Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime in a conformal gauge [44] that takes the same form as in flat space on the account of conformal coupling. Exact gauges, such as the Coulomb one in which the two-point function takes the same form as in flat space in four-dimensional spacetime [45], are also legitimate choices. However, for practical applications the D-dimensional multiplier gauges such as (1) are usually preferable.

Because of the noncovariant gauge fixing (1), and because of the reduced symmetry of cosmological spaces compared to maximally symmetric ones, we have to understand how to compute photon two-point functions without relying on a large number of symmetries that usually simplify problems. To this end we consider the canonical quantization of the photon field similar to the Gupta-Bleuler quantization [46,47]. The approach taken here mainly differs by not relying on the on symmetries of the system. Rather it is based on the classical canonical structure in the *multiplier gauge* defined by the gauge-fixing functional (1). In that sense the quantization outlined here can be considered as the derivation of the Gupta-Bleuler quantization. This includes the derivation of the somewhat *ad hoc* Gupta-Bleuler subsidiary condition on the physical space of states that here arises as a natural consequence of the canonical first-class constraint structure and the correspondence principle.

Several works in recent years have considered the quantization of the photon field in cosmological spaces [37,42,48,49], choosing to preserve general covariance as much as possible. While there is nothing wrong with maintaining covariance, for practical purposes this is not always the most convenient choice, and covariant approaches offer little help when abandoning manifest covariance by gauge choices. This is why we opt to consider the canonical quantization formalism from the ground up, divorced from spacetime symmetries. We only consider spatially flat FLRW spacetimes, and not spatially closed ones where the issue of linearization instability arises [50–59].

Proper understanding of photon quantization has practical consequences. Computations of quantum loop corrections in inflation are notoriously difficult, and the choice of gauge can make a world of difference. It is sufficient to compare the computations of the one-graviton-loop vacuum polarization in de Sitter in noncovariant gauges [30] and covariant gauges [33] to realize that de Sitter symmetries do not play the same convenient organizational role as do Poincaré symmetries in flat space. Therefore, it is advantageous to consider different gauges in cosmological spacetimes. We ultimately express the photon two-point functions in terms of a few scalar mode functions that cosmologists are accustomed to working with. Identifying gauges admitting simple solutions for these scalar mode functions is left for future work.

The two-point functions of the linear theory serve to compute loop corrections to physical observables. It is in no way obvious for gauge theories in multiplier gauges, such as (1), how to properly define quantum observables. It was suggested in [37] that the expectation value of the photon energy-momentum tensor depends on whether the contribution from the gauge-fixing term (1) is included or not. For covariant gauges (with $\zeta = 0$) in de Sitter, the gauge-fixing term is supposed to contribute as a cosmological constant to the energy-momentum tensor. This is in conflict with the correspondence principle, and would essentially allow for quantum measurements of first-class constraints. However, when ordering of products of field operators comprising the energy-momentum tensor is considered carefully, as in Sec. VIII, the issue is fully resolved. The gauge-fixing functional (1) cannot contribute to the energy-momentum tensor for any admissible state, in any cosmological spacetime.

The results of this work also include the unexpected observation that the subsidiary conditions for the photon two-point functions derived here are not satisfied for some of the de Sitter space photon propagators reported in the literature. More details on this issue are given by the end of the concluding Sec. IX, while further investigations into the issue are left for future work [60,61].

The paper is organized in nine sections, with the current one laying out the background and the motivation. Section II gives some of the properties of scalar field modes and two-point functions that we make use of in subsequent sections. Section III presents the details of implementing multiplier/average gauges in the canonical formulation of the classical photon. This structure lends itself to canonical quantization that is discussed in Sec. IV. In Sec. V the dynamics of the field operators is translated into equations of motion for the mode functions, while Sec. VI discusses the construction of the space of states and the conditions the physical states must respect, as well as the role that spacetime symmetries play in this construction. Section VII concerns the main goal of this paper of constructing photon two-point functions from the photon mode functions, and demonstrates how such construction satisfies all the subsidiary conditions. Section VIII discusses two simple observables and the issue of proper operator ordering of observables. Section IX contains the discussion of the construction presented in the paper, and provides the check of the photon propagators in the literature versus the conditions presented here, not all of which are found to be satisfied.

II. PRELIMINARIES

The mode functions of linear higher spin fields in cosmological spaces can often be expressed in terms of the scalar mode functions. Consequently, the same is often true for two-point functions as well. This indeed is the case for the photon mode functions and two-point functions that we consider in this work. This section serves to introduce the background cosmological space, to define the notation, and to summarize some of the basic results for scalar fields that will be used in subsequent sections.

A. FLRW spacetime

The geometry of homogeneous, isotropic, and spatially flat expanding spacetimes is described by the FLRW invariant line element,

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\eta)(-d\eta^2 + d\vec{x}^2). \quad (2)$$

The flat spatial sections are covered by $(D-1)$ Cartesian coordinates, $x_i \in (-\infty, \infty)$, while the evolution is expressed in terms of either the physical time t or the conformal time η. The dynamics of the expansion is encoded in the scale factor a that also provides the connection between the two time coordinates, $dt = a d\eta$. For our purposes the conformal time coordinate is preferable, since then the FLRW metric, $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$, is conformally flat, where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$ is the D-dimensional Minkowski metric.

The relevant information about the dynamics of the expansion is usually captured by the first few derivatives of the scale factor. The conformal Hubble rate $\mathcal{H}$ and the principal slow-roll parameter ϵ capture the first two derivatives of the scale factor,

$$\mathcal{H}(\eta) = \frac{1}{a}\frac{da}{d\eta}, \qquad \epsilon(\eta) = 1 - \frac{1}{\mathcal{H}^2}\frac{d\mathcal{H}}{d\eta}. \quad (3)$$

The more commonly used physical Hubble rate H is related to the conformal Hubble rate as $H = \mathcal{H}/a$, while the principal slow-roll parameter is related to the often used deceleration parameter, $q = \epsilon - 1$. Accelerating FLRW spacetimes, such as primordial inflation, are characterized by $0 < \epsilon(\eta) \ll 1$. Even though the results of this work apply to arbitrary accelerating FLRW spacetimes, it is nonetheless helpful at times to have a concrete spacetime in mind. One such tractable example is that of power-law inflation [62,63] for which the Hubble rate and the scale factor depend on time as

$$\epsilon = \mathrm{const} \Rightarrow \mathcal{H}(\eta) = \frac{H_0}{1-(1-\epsilon)H_0(\eta-\eta_0)},$$
$$a(\eta) = \left(\frac{\mathcal{H}}{H_0}\right)^{\frac{1}{1-\epsilon}}, \quad (4)$$

where η_0 is the initial time at which $a(\eta_0) = 1$ and $\mathcal{H}(\eta_0) = H_0$.

B. Scalar mode functions

The equation of motion for the conformally rescaled scalar field modes of comoving momentum $\vec{k}$ in FLRW generically takes the form

$$\left[\partial_0^2 + k^2 - \left(\lambda^2 - \frac{1}{4}\right)(1-\epsilon)^2\mathcal{H}^2\right]\mathscr{U}_\lambda(\eta, \vec{k}) = 0, \qquad k = \|\vec{k}\|, \quad (5)$$

where λ descends from the mass term and the nonminimal coupling term. Both ϵ and λ are time-dependent in general, though there are notable cases where they are constant: in power-law inflation ϵ is constant, while a massless, nonminimally coupled scalar has a constant λ. The equation of motion (5) admits two linearly independent solutions, which we take to be complex conjugates of each other,

$$\mathscr{U}_\lambda(\eta, \vec{k}) = \alpha(\vec{k})\mathcal{U}_\lambda(\eta, k) + \beta(\vec{k})\mathcal{U}_\lambda^*(\eta, k), \quad (6)$$

where $\alpha(\vec{k})$ and $\beta(\vec{k})$ are arbitrary constants of integration. In practice it is convenient to choose the independent solutions, $\mathcal{U}_\lambda$ and $\mathcal{U}_\lambda^*$, that are some appropriate generalization of positive- and negative-frequency modes (Chernikov-Tagirov-Bunch-Davies modes [64,65]), at least in the ultraviolet. As a concrete example one can keep in mind power-law inflation (4) where $0 < \epsilon = \mathrm{const} < 1$, for which the positive-frequency mode function is[1]

$$\epsilon = \mathrm{const} \Rightarrow \mathcal{U}_\lambda(\eta, k) = e^{\frac{i\pi}{4}(2\lambda+1)}\sqrt{\frac{\pi}{4(1-\epsilon)\mathcal{H}}}H_\lambda^{(1)}\left(\frac{k}{(1-\epsilon)\mathcal{H}}\right), \quad (7)$$

where $H_\lambda^{(1)}$ is the Hankel function of the first kind. Note, however, that no assumptions on ϵ are made throughout the paper. Equation (5) also implies a nonvanishing Wronskian for the two independent solutions,

$$\mathcal{U}_\lambda(\eta, k)\partial_0\mathcal{U}_\lambda^*(\eta, k) - \mathcal{U}_\lambda^*(\eta, k)\partial_0\mathcal{U}_\lambda(\eta, k) = i, \quad (8)$$

where the normalization is chosen for convenience, as appropriate for mode functions of scalar field operators.

[1] The phase factor in the definition (7) ensures that the Wronskian in (8) is correct for both real and imaginary values of λ.

The free coefficients in (6) then have an interpretation of Bogolyubov coefficients that have to satisfy

$$|\alpha(\vec{k})|^2 - |\beta(\vec{k})|^2 = 1. \tag{9}$$

When solving for the photon field mode functions in Sec. V C we will encounter scalar mode equations (5) with only special combinations of parameters for which

$$\partial_0\left[\left(\lambda+\frac{1}{2}\right)(1-\epsilon)\right] = 0. \tag{10}$$

That allows us to introduce convenient recurrence relations for contiguous mode functions without solving equations of motion explicitly,

$$\begin{aligned}\left[\partial_0 + \left(\lambda+\frac{1}{2}\right)(1-\epsilon)\mathcal{H}\right]\mathcal{U}_\lambda &= -ik\mathcal{U}_{\lambda+1},\\ \left[\partial_0 - \left(\lambda+\frac{1}{2}\right)(1-\epsilon)\mathcal{H}\right]\mathcal{U}_{\lambda+1} &= -ik\mathcal{U}_\lambda,\end{aligned} \tag{11}$$

where $\mathcal{U}_{\lambda+1}$ satisfies equation (5) with $\lambda\to\lambda+1$, and where the proportionality constant was chosen for convenience. The Wronskian (8) can consequently be expressed as,

$$\mathrm{Re}[\mathcal{U}_\lambda(\eta,k)\mathcal{U}^*_{\lambda+1}(\eta,k)] = \frac{1}{2k}. \tag{12}$$

Confirming the recurrence relations (11) is accomplished by plugging them into the equation of motion (5) and applying condition (10) when commuting time derivatives. In more special cases, such as power-law inflation (4), the recurrence relations (11) can be inferred from the properties of the solutions for the mode functions, such as recurrence relations for Hankel functions that appear as solutions for the mode functions (7) (cf. Sec. 2.2.1 from [60]). Recurrence relations (11) help keep the expressions compact, reducing the clutter of the computation. They will be valid only for time-independent ζ form the gauge-fixing term (1), but generalization to time-dependent values should be straightforward.

C. Scalar two-point functions

Nonequilibrium loop computations in quantum field theory require the use of several different two-point functions [66–73]. The positive-frequency Wightman function can perhaps be considered the fundamental one, as the remaining ones can all be expressed in terms of it. The Wightman function for scalar fields in FLRW can be expressed in terms of conformally rescaled scalar mode functions introduced in the preceding subsection as a sum-over-modes,[2]

$$i[{}^-\Delta^+]_\lambda(x;x') = (aa')^{-\frac{D-2}{2}} \times \int \frac{d^{D-1}k}{(2\pi)^{D-1}}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\mathscr{U}_\lambda(\eta,\vec{k})\mathscr{U}^*_\lambda(\eta',\vec{k}). \tag{13}$$

It satisfies the equation of motion inherited from the mode equation (5),

$$[\Box - (\lambda_0^2-\lambda^2)(1-\epsilon)^2H^2]i[{}^-\Delta^+]_\lambda(x;x') = 0, \tag{14}$$

where $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$ is the d'Alembertian, ∇_μ is the D-dimensional covariant derivative, and

$$\lambda_0 = \frac{D-1-\epsilon}{2(1-\epsilon)}. \tag{15}$$

The negative-frequency Wightman function is a complex conjugate of the positive-frequency one, $i[{}^+\Delta^-]_\lambda(x;x') = \{i[{}^-\Delta^+]_\lambda(x;x')\}^*$. The two Wightman functions can serve to define the Feynman propagator,

$$\begin{aligned}i[{}^+\Delta^+]_\lambda(x;x') &= \theta(\eta-\eta')i[{}^-\Delta^+]_\lambda(x;x')\\ &\quad + \theta(\eta'-\eta)i[{}^+\Delta^-]_\lambda(x;x'),\end{aligned} \tag{16}$$

which satisfies a sourced equation of motion,

$$[\Box-(\lambda_0^2-\lambda^2)(1-\epsilon)^2H^2]i[{}^+\Delta^+]_\lambda(x;x') = \frac{i\delta^D(x-x')}{\sqrt{-g}}, \tag{17}$$

because of the step function θ in its definition. Finally, the Dyson propagator is the complex conjugate of (16), $i[{}^-\Delta^-]_\lambda(x;x') = \{i[{}^+\Delta^+]_\lambda(x;x')\}^*$, and it satisfies the equation of motion that is a conjugate of (17). The sum-over-modes representations for different two-point functions are inferred from the one for the Wightman function (13) and their respective definitions.

III. CLASSICAL PHOTON IN FLRW

The massless vector field—the photon—in general D-dimensional curved spacetimes is given by the covariantized action for electromagnetism,

[2] The sum-over-modes representation needs to be regulated for it to be valid on the entire range of coordinates. This is accomplished by appending an infinitesimal imaginary part to time coordinates: $\eta\to\eta-i\delta/2$, $\eta'\to\eta'-i\delta/2$. Note that this substitution is first performed on the argument of the mode functions, and only then is the complex conjugation in (13) taken. In this sense, the two-point function is defined as the distributional $\delta\to 0_+$ limit of an analytic function.

$$S[A_\mu] = \int d^Dx\sqrt{-g}\left[-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\right], \qquad (18)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor for the vector potential A_μ. This action is invariant under $U(1)$ gauge transformations, $A_\mu \to A_\mu + \partial_\mu \Lambda$, where $\Lambda(x)$ is an arbitrary scalar function. Consequently the covariant Maxwell's equations inherit this property, meaning their solutions are not fully determined by specifying initial conditions for the vector potential components. This implies that the number of physical propagating degrees of freedom is smaller than the number of dynamical fields. Just as in flat space, the propagating degrees of freedom of the free photon are the $(D-2)$ transverse polarizations. Their dynamics is well understood, since in $D=4$ they couple conformally to gravity and effectively do not see the expansion. However, formulating interacting gauge theories in terms of physical propagating degrees of freedom only is rather impractical at best. It is often advantageous to consider the questions of the dynamics and of observables separately. One first fixes the gauge to remove the ambiguities in the dynamical equations, and solves for the gauge-fixed dynamics. Then one obtains observables by projecting out the physical information from the gauge-fixed solutions.

Particularly convenient gauges for loop computations in quantum field theory are the so-called *average gauges*, known as *multiplier gauges* in classical theory [74]. These are not imposed by following the Dirac-Bergmann algorithm [75] and imposing gauge conditions that eliminate some of the vector potential components. Instead, multiplier gauges treat all the components of the vector potential on equal footing, and fix the ambiguities in the dynamics by fixing directly the Lagrange multipliers. This procedure is ultimately equivalent to adding a gauge-fixing term to the gauge-invariant action (18). A specific choice for the multiplier will lead to the gauge-fixing term in (1) we consider here. This sort of gauge fixing is not often encountered in classical theories, perhaps giving the impression that there is something innately quantum about gauge-fixing terms added to gauge-invariant actions. This is far from true, as the rationale behind gauge-fixing terms is essentially the classical canonical structure in multiplier gauges. That is why this section is devoted to presenting the details of implementing multiplier gauges, in particular for the photon in (18). Canonical formulation in multiplier gauges subsequently lends itself readily to canonical quantization that is considered in Sec. IV.

A. Gauge-invariant canonical formulation

Our starting point of implementing a multiplier gauge is the canonical formulation of the action (18) that we derive in this subsection. We start by decomposing the indices in (18) into spatial and temporal ones,

$$S[A_\mu] = \int d^Dx\, a^{D-4}\left[\frac{1}{2}F_{0i}F_{0i} - \frac{1}{4}F_{ij}F_{ij}\right], \qquad (19)$$

where henceforth all such decomposed indices are written as lowered and the convention that repeated spatial indices are summed over is implied. We follow [76] in reformulating (19) as a first order canonical action. This requires first promoting all the time derivatives to independent velocity fields,

$$\partial_0 A_0 \to V_0, \qquad F_{0i} = \partial_0 A_i - \partial_i A_0 \to V_i, \qquad (20)$$

and introducing accompanying Lagrange multipliers Π_0 and Π_i that ensure on-shell equivalence of the intermediate extended action,

$$\begin{aligned}\mathcal{S}[A_0,V_0,\Pi_0,A_i,V_i,\Pi_i] = \int d^Dx\bigg\{ & a^{D-4}\left[\frac{1}{2}V_iV_i - \frac{1}{4}F_{ij}F_{ij}\right] \\ & + \Pi_0(\partial_0 A_0 - V_0) \\ & + \Pi_i(\partial_0 A_i - \partial_i A_0 - V_i)\bigg\}, \qquad (21)\end{aligned}$$

to the original action (18). Solving for the velocity fields on-shell, which here is possible only for V_i,

$$\frac{\delta\mathcal{S}}{\delta V_i} \approx 0, \qquad \Rightarrow V_i \approx \bar{V}_i = a^{4-D}\Pi_i, \qquad (22)$$

and plugging this into the extended action above produces the canonical action

$$\begin{aligned}\mathscr{S}[A_0,\Pi_0,A_i,\Pi_i,\ell] &\equiv \mathcal{S}[A_0,V_0\to\ell,\Pi_0,A_i,\bar{V}_i,\Pi_i] \\ &= \int d^Dx[\Pi_0\partial_0 A_0 + \Pi_i\partial_0 A_i \\ &\quad - \mathscr{H} - \ell\Psi_1], \qquad (23)\end{aligned}$$

where

$$\mathscr{H} = \frac{a^{4-D}}{2}\Pi_i\Pi_i + \Pi_i\partial_i A_0 + \frac{a^{D-4}}{4}F_{ij}F_{ij} \qquad (24)$$

is the canonical Hamiltonian density and $\Psi_1 = \Pi_0$ is the primary constraint generated by the Lagrange multiplier ℓ (which is just a relabeled field V_0). Variation of (23) with respect to the canonical variables produces canonical equations of motion

$$\partial_0 A_0 \approx \ell, \qquad \partial_0\Pi_0 \approx \partial_i\Pi_i, \qquad \partial_0 A_i \approx a^{4-D}\Pi_i + \partial_i A_0,$$
$$\partial_0\Pi_i \approx a^{D-4}\partial_j F_{ji}, \qquad (25)$$

while variation with respect to the Lagrange multiplier ℓ gives the primary constraint

$$\Psi_1 \approx 0. \tag{26}$$

Note that in this section we employ Dirac's notation where $=$ stands for an off-shell (strong) equality, while $\approx$ stands for an on-shell (weak) equality. The equations above are equivalent to Hamilton's equations descending from the total Hamiltonian $\mathscr{H}_{\rm tot} = \mathscr{H} + \ell \Psi_1$, where the Poisson brackets are determined by the symplectic part of the canonical action (23), with the nonvanishing ones being

$$\{A_0(\eta,\vec{x}), \Pi_0(\eta,\vec{x}')\} = \delta^{D-1}(\vec{x}-\vec{x}'),$$
$$\{A_i(\eta,\vec{x}), \Pi_j(\eta,\vec{x}')\} = \delta_{ij}\delta^{D-1}(\vec{x}-\vec{x}'), \tag{27}$$

and the Lagrange multiplier ℓ has vanishing brackets with all the canonical variables. Note that the constraint equation (26) does not follow from the total Hamiltonian, but needs to be considered in addition to Hamilton's equations.

The consistency of the primary constraint (26) requires it to be conserved, which in turn generates a secondary constraint,

$$0 \approx \partial_0 \Psi_1 \approx \partial_i \Pi_i \equiv \Psi_2, \tag{28}$$

the conservation of which generates no further constraints,

$$\partial_0 \Psi_2 \approx 0. \tag{29}$$

The primary and secondary constraints form a complete set of first-class constraints,

$$\{\Psi_1(\eta,\vec{x}), \Psi_2(\eta,\vec{x}')\} = 0, \tag{30}$$

implying that the Lagrange multiplier ℓ is left undetermined by the equations of motion, which is how gauge symmetries manifest themselves in the canonical formulation. Fixing this ambiguity is what the next section is devoted to.

While any solution to the dynamical equations (25) and the accompanying constraint equations (26) and (29) describes the same physical system, observables in gauge theories cannot depend on the arbitrary Lagrange multipliers such as ℓ. This is guaranteed by requiring observables to have on-shell vanishing brackets with all the first-class constraints. For the case at hand, this means that for some $\mathscr{O}(\eta,\vec{x})$ to be an observable it has to satisfy

$$\{\Psi_1(\eta,\vec{x}), \mathscr{O}(\eta,\vec{x}')\} \approx 0, \qquad \{\Psi_2(\eta,\vec{x}), \mathscr{O}(\eta,\vec{x}')\} \approx 0, \tag{31}$$

which guarantees that it does not depend on the arbitrary Lagrange multiplier ℓ.

B. Gauge-fixed canonical formulation

There are multiple ways of fixing the ambiguity of the dynamical equations (25) of the preceding section. The Dirac-Bergman algorithm requires the specification of gauge conditions in the form of off-shell equalities that eliminate part of the dynamical canonical fields. Such gauge choices are known as *exact gauges*, and the Coulomb gauge is one classic example. However, exact gauges are often impractical to use in quantized theories. Preferred choices are the so-called multiplier gauges (also called average gauges) that do not eliminate any of the dynamical fields, but rather treat all of them on an equal footing. Implementing a multiplier gauge for the problem at hand is accomplished by fixing by hand the multiplier ℓ to be a function of canonical pairs, without explicitly specifying any gauge conditions,

$$\ell \to \overline{\ell}(A_0, \Pi_0, A_i, \Pi_i). \tag{32}$$

Employing this choice at the level of equations of motion (25) produces a set of gauge-fixed equations of motion,

$$\partial_0 A_0 \approx \overline{\ell}(A_0, \Pi_0, A_i, \Pi_i), \tag{33}$$

$$\partial_0 \Pi_0 \approx \partial_i \Pi_i, \tag{34}$$

$$\partial_0 A_i \approx a^{4-D}\Pi_i + \partial_i A_0, \tag{35}$$

$$\partial_0 \Pi_i \approx a^{D-4}\partial_j F_{ji}, \tag{36}$$

in addition to the two first-class constraints that remain unchanged,

$$\Psi_1 = \Pi_0 \approx 0, \qquad \Psi_2 = \partial_i \Pi_i \approx 0. \tag{37}$$

This system of equations now forms a well-defined initial value problem (provided that $\overline{\ell}$ is not chosen in some pathological manner). A useful way of viewing these equations is to consider the two first-class constraints as conditions on the initial value surface. The evolution will ensure they are preserved for all times. This way we split the problem into four dynamical equations (33)–(36) describing evolution and two kinematic equations (37) constraining the initial conditions. The latter cut defines a subspace of the space of solutions of the former. This shows that choosing $\ell \to \overline{\ell}$ at the level of the equations of motion leads to a well-defined dynamical problem and in that sense it fixes the gauge. However, it is more convenient to implement this gauge at the level of the action.

Instead of fixing the multiplier at the level of equations of motion, we can fix it in the canonical action (23) directly. This defines the *gauge-fixed canonical action*,

$$\mathscr{S}_\star[A_0, \Pi_0, A_i, \Pi_i] \equiv \mathscr{S}[A_0, \Pi_0, A_i, \Pi_i, \ell \to \overline{\ell}]$$
$$= \int d^D x [\Pi_0 \partial_0 A_0 + \Pi_i \partial_0 A_i - \mathscr{H}_\star], \tag{38}$$

where the gauge-fixed Hamiltonian density is

$$\mathscr{H}_\star = \mathscr{H} + \bar{\ell}\Psi_1. \tag{39}$$

Note that this gauge-fixed action is not equivalent to canonical action (23) as it no longer encodes a variational principle with respect to the multiplier ℓ. The equations of motion generated by the gauge-fixed canonical action are[3]

$$\partial_0 A_0 \approx \bar{\ell} + \frac{\partial\bar{\ell}}{\partial\Pi_0}\Pi_0, \tag{40}$$

$$\partial_0\Pi_0 \approx \partial_i\Pi_i - \frac{\partial\bar{\ell}}{\partial A_0}\Pi_0, \tag{41}$$

$$\partial_0 A_i \approx a^{4-D}\Pi_i + \partial_i A_0 + \frac{\partial\bar{\ell}}{\partial\Pi_i}\Pi_0, \tag{42}$$

$$\partial_0\Pi_i \approx a^{D-4}\partial_j F_{ji} - \frac{\partial\bar{\ell}}{\partial A_i}\Pi_0. \tag{43}$$

This is now a fully determined set of coupled partial differential equations in the sense that specifying initial conditions fully fixes the evolution. However, note that these are not immediately equivalent to Eqs. (33)–(37). First, the four dynamical equations are all modified by additional terms, and more importantly the constraint equations are absent. The remedy is to consider the gauge-fixed action to encode the dynamics only, and to require the first-class constraints (37) to be satisfied *in addition* to the dynamical equations by considering them to be *subsidiary conditions* on the initial value surface,

$$\Psi_1(\eta_0,\vec{x}) = \Pi_0(\eta_0,\vec{x}) \approx 0, \qquad \Psi_2(\eta_0,\vec{x}) = \partial_i\Pi_i(\eta_0,\vec{x}) \approx 0. \tag{44}$$

Equations of motion (40)–(43) guarantee that imposing constraints (44) on the initial value surface is sufficient to guarantee their conservation. The system of Eqs. (40)–(44) is now obviously equivalent to the original system (33)–(37).

The choice for the multiplier in (32) is not dictated by physical principles, but is rather a matter of convenience, as in fact any gauge choice is. In this work we consider only linear gauges natural for free theories. We require them (i) to respect homogeneity and isotropy of the FLRW spacetime, (ii) not to introduce additional dimensionful scales, (iii) to be composed of commensurate terms, and (iv) to respect Lorentz invariance in the Minkowski limit. This essentially restricts the choice of the multiplier to a two-parameter family,

$$\bar{\ell} = -\frac{\xi}{2}a^{4-D}\Pi_0 + \partial_i A_i - (D-2-2\zeta)\mathcal{H}A_0, \tag{45}$$

where ξ and ζ are two arbitrary dimensionless gauge-fixing parameters.[4] This particular gauge choice then produces the equations of motion

$$\partial_0 A_0 \approx -\xi a^{4-D}\Pi_0 + \partial_i A_i - (D-2-2\zeta)\mathcal{H}A_0, \tag{46}$$

$$\partial_0\Pi_0 \approx \partial_i\Pi_i + (D-2-2\zeta)\mathcal{H}\Pi_0, \tag{47}$$

$$\partial_0 A_i \approx a^{4-D}\Pi_i + \partial_i A_0, \tag{48}$$

$$\partial_0\Pi_i \approx \partial_i\Pi_0 + a^{D-4}\partial_j F_{ji} \tag{49}$$

that are generated by the gauge-fixed Hamiltonian

$$\mathscr{H}_\star = \frac{a^{4-D}}{2}(\Pi_i\Pi_i - \xi\Pi_0\Pi_0) + \Pi_i\partial_i A_0 + \Pi_0\partial_i A_i - (D-2-2\zeta)\mathcal{H}\Pi_0 A_0 + \frac{a^{D-4}}{4}F_{ij}F_{ij}. \tag{50}$$

The two first-class constraints (44) satisfy closed equations of motion,

$$\partial_0\Psi_1 \approx \Psi_2 + (D-2-2\zeta)\mathcal{H}\Psi_1, \qquad \partial_0\Psi_2 \approx \nabla^2\Psi_1, \tag{51}$$

where $\nabla^2 \equiv \partial_i\partial_i$ is the Laplace operator, exemplifying the fact that the dynamics preserves (44) if imposed on the initial value surface.

We have defined the gauge-fixed system in terms of the gauge-fixed canonical action (38) with the choice for the multiplier in (45) describing the dynamics, and the two subsidiary conditions (44) accounting for first-class constraints. Despite being the superior formulation for analyzing the structure of gauge theories, the canonical formulation is often less intuitive than the configurations space formulation. There is an associated gauge-fixed configuration space action associated with the canonical one. By solving for the canonical momenta on-shell,

$$\frac{\delta\mathscr{S}_\star}{\delta\Pi_0} \approx 0 \Rightarrow \Pi_0 \approx \bar{\Pi}_0 = -\frac{a^{D-4}}{\xi}(\partial_0 A_0 + (D-2-2\zeta)\mathcal{H}A_0 - \partial_i A_i), \tag{52}$$

$$\frac{\delta\mathscr{S}_\star}{\delta\Pi_i} \approx 0 \Rightarrow \Pi_i \approx \bar{\Pi}_i = a^{D-4}(\partial_0 A_i - \partial_i A_0), \tag{53}$$

[3]Partial derivatives of $\bar{\ell}$ are generalized to functional derivatives in an obvious way whenever necessary. This detail is omitted for notational simplicity.

[4]Having ξ and ζ be time-dependent functions would be just as easy, but we do not consider it for simplicity; the generalization is straightforward.

and inserting the solutions into the gauge-fixed canonical action produces precisely the associated gauge-fixed configuration space action

$$S_\star[A_\mu] = \int d^Dx\sqrt{-g}\bigg[-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2\xi}(\nabla^\mu A_\mu - 2\zeta n^\mu A_\mu)^2\bigg] \tag{54}$$

with the gauge-fixing term (1). Thus we have derived how the gauge-fixing terms arise in classical gauge theories. The subsidiary conditions (44) in the configuration space formulation take the form

$$\xi a^{2-D}\Psi_1 \approx (\nabla^\mu - 2\zeta n^\mu)A_\mu \approx 0, \qquad a^{4-D}\Psi_2 \approx \partial_i F_{0i} \approx 0. \tag{55}$$

IV. QUANTIZED PHOTON IN FLRW

The classical theory in the multiplier gauge is formulated so that the dynamics is represented by a gauge-fixed action without constraints, and the first-class constraints are imposed as subsidiary conditions on the initial value surface. The canonical quantization of such a gauge-fixed theory is most naturally implemented in the Heisenberg picture, where field operators account for the dynamics, and the state vector accounts for the initial conditions.

A. Dynamics

The usual rules of canonical quantization are sufficient to quantize the gauge-fixed dynamics of the classical theory. Canonical fields are promoted to Hermitian field operators,

$$A_0(x) \to \hat{A}_0(x), \qquad \Pi_0(x) \to \hat{\Pi}_0(x),$$
$$A_i(x) \to \hat{A}_i(x), \qquad \Pi_i(x) \to \hat{\Pi}_i(x), \tag{56}$$

and their Poisson brackets (27) to commutators,

$$[\hat{A}_0(\eta,\vec{x}), \hat{\Pi}_0(\eta,\vec{x}')] = i\delta^{D-1}(\vec{x}-\vec{x}'),$$
$$[\hat{A}_i(\eta,\vec{x}), \hat{\Pi}_j(\eta,\vec{x}')] = \delta_{ij} i\delta^{D-1}(\vec{x}-\vec{x}'). \tag{57}$$

The gauge-fixed equations of motion (46)–(49) remain unchanged,

$$\partial_0\hat{A}_0 = -\xi a^{4-D}\hat{\Pi}_0 + \partial_i\hat{A}_i - (D-2-2\zeta)\mathcal{H}\hat{A}_0, \tag{58}$$

$$\partial_0\hat{\Pi}_0 = \partial_i\hat{\Pi}_i + (D-2-2\zeta)\mathcal{H}\hat{\Pi}_0, \tag{59}$$

$$\partial_0\hat{A}_i = a^{4-D}\hat{\Pi}_i + \partial_i\hat{A}_0, \tag{60}$$

$$\partial_0\hat{\Pi}_i = \partial_i\hat{\Pi}_0 + a^{D-4}\partial_j\hat{F}_{ji}, \tag{61}$$

and are generated by the Hamiltonian (50) with fields promoted to field operators.

B. Subsidiary condition

Implementing the first-class constraints in the quantized theory is less straightforward. It definitely must involve Hermitian constraint operators,

$$\hat{\Psi}_1(x) = \hat{\Pi}_0(x), \qquad \hat{\Psi}_2(x) = \partial_i\hat{\Pi}_i(x). \tag{62}$$

However, the constraints cannot be imposed as operator equalities as that would contradict canonical commutation relations (57). To understand how to quantize the first-class constraints we better first consider the correspondence principle, which tells us that matrix elements of Hermitian first-class constraints have to vanish at initial time,

$$\langle\Omega_1|\hat{\Psi}_1(\eta_0,\vec{x})|\Omega_2\rangle = 0, \qquad \langle\Omega_1|\hat{\Psi}_2(\eta_0,\vec{x})|\Omega_2\rangle = 0. \tag{63}$$

This cannot be satisfied by requiring that the Hermitian constraints themselves annihilate the state vector (as required in [77,78]), as that would again contradict the canonical commutation relations (57). However, it is consistent to require that the state ket-vector is annihilated by an invertible non-Hermitian linear combination of the two constraints (62),

$$\hat{K}(\vec{x}) = \int d^{D-1}x'[f_1(\eta_0,\vec{x}-\vec{x}')\hat{\Psi}_1(\eta_0,\vec{x}') + f_2(\eta_0,\vec{x}-\vec{x}')\hat{\Psi}_2(\eta_0,\vec{x}')], \tag{64}$$

that we refer to as a *subsidiary non-Hermitian constraint operator*, and that its conjugate annihilates the state bra-vector,

$$\hat{K}(\vec{x})|\Omega\rangle = 0, \qquad \langle\Omega|\hat{K}^\dagger(\vec{x}) = 0, \qquad \forall\,\vec{x}. \tag{65}$$

This condition is a generalization of the Gupta-Bleuler subsidiary condition for covariant photon gauges in flat space. It is divorced from spacetime symmetries and symmetries of the gauge-fixing term, and is rather based solely on the canonical structure.

The conditions in (63) are preserved in time,

$$\langle\Omega_1|\hat{\Psi}_1(\eta,\vec{x})|\Omega_2\rangle = 0, \qquad \langle\Omega_1|\hat{\Psi}_2(\eta,\vec{x})|\Omega_2\rangle = 0, \tag{66}$$

due to the equations of motion the Hermitian constraints satisfy,

$$\partial_0\hat{\Psi}_1 = \hat{\Psi}_2 + (D-2-2\zeta)\mathcal{H}\hat{\Psi}_1, \qquad \partial_0\hat{\Psi}_2 = \nabla^2\hat{\Psi}_1. \tag{67}$$

This implies that the time-independent non-Hermitian constraint can be expressed in terms of Hermitian ones at any point in time, since there are f_1 and f_2 at any time η such that

$$\hat{K}(\vec{x}) = \int d^{D-1}x' [f_1(\eta, \vec{x} - \vec{x}')\hat{\Psi}_1(\eta, \vec{x}') + f_2(\eta, \vec{x} - \vec{x}')\hat{\Psi}_2(\eta, \vec{x}')]. \tag{68}$$

Given the equations of motion (67), the coefficient functions have to satisfy

$$\partial_0 f_1 = -\nabla^2 f_2 - (D - 2 - 2\zeta)\mathcal{H} f_1, \qquad \partial_0 f_2 = -f_1. \tag{69}$$

The subsidiary constraint operator (64) commutes with its conjugate

$$[\hat{K}(\vec{x}), \hat{K}^\dagger(\vec{x}')] = 0, \tag{70}$$

and consequently a matrix element of any polynomial functional of Hermitian constraints vanishes,

$$\langle \Omega_1 | \mathscr{P}[\hat{\Psi}_1(\eta, \vec{x}), \hat{\Psi}_2(\eta, \vec{x})] | \Omega_2 \rangle = 0, \tag{71}$$

as required by the correspondence principle. In particular, the two-point functions of Hermitian constraints must vanish,

$$\langle \Omega | \hat{\Psi}_1(\eta, \vec{x}) \hat{\Psi}_1(\eta', \vec{x}') | \Omega \rangle = 0, \tag{72a}$$

$$\langle \Omega | \hat{\Psi}_1(\eta, \vec{x}) \hat{\Psi}_2(\eta', \vec{x}') | \Omega \rangle = 0, \tag{72b}$$

$$\langle \Omega | \hat{\Psi}_2(\eta, \vec{x}) \hat{\Psi}_2(\eta', \vec{x}') | \Omega \rangle = 0. \tag{72c}$$

For later sections it is useful to define some shorthand notation. Namely, we can invert relation (68) and express the Hermitian constraints in terms of the non-Hermitian ones,

$$\hat{\Psi}_1(x) = \hat{K}_1^\dagger(x) + \hat{K}_1(x), \qquad \hat{\Psi}_2(x) = \hat{K}_2^\dagger(x) + \hat{K}_2(x), \tag{73}$$

where pieces $\hat{K}_1$ and $\hat{K}_2$ contain just $\hat{K}$, and their conjugates contain just $\hat{K}^\dagger$, such that for any physical state we have

$$\hat{K}_1(x)|\Omega\rangle = \hat{K}_2(x)|\Omega\rangle = 0, \qquad \langle \Omega | \hat{K}_1^\dagger(x) = \langle \Omega | \hat{K}_2^\dagger(x) = 0. \tag{74}$$

C. Quantum observables

A classical observable $\mathscr{O}$ is a quantity composed out of canonical fields that has vanishing Poisson brackets with all the first-class constraints (31). When promoting such a classical observable to a quantum observable we have to promote $\mathscr{O}$ to an operator $\hat{\mathscr{O}}$, by promoting the canonical fields it is composed of to canonical field operators. This process requires that we address the question of operator ordering in $\hat{\mathscr{O}}$.

In quantum gauge theories there is an additional operator ordering issue compared to quantum theories without constraints. To understand this ordering issue consider a trivial classical "observable",[5]

$$\mathscr{O}(\eta, \vec{x}, \vec{x}') = A_0(\eta, \vec{x}) \Psi_2(\eta, \vec{x}') \approx 0, \tag{75}$$

that vanishes on account of being proportional to one of the first-class constraints (31). When promoting this quantity to an operator we may first consider Weyl ordering (denoted by subscript W henceforth). In this particular case there is no need for explicit Weyl ordering as the involved field operators commute,

$$[\hat{\mathscr{O}}(\eta, \vec{x}, \vec{x}')]_{\rm W} \equiv \frac{1}{2}(\hat{A}_0(\eta, \vec{x})\hat{\Psi}_2(\eta, \vec{x}') + \hat{\Psi}_2(\eta, \vec{x}')\hat{A}_0(\eta, \vec{x})) = \hat{A}_0(\eta, \vec{x})\hat{\Psi}_2(\eta, \vec{x}'). \tag{76}$$

However, taking the expectation value produces a nonvanishing result,

$$\langle \Omega | [\hat{\mathscr{O}}(\eta, \vec{x}, \vec{x}')]_{\rm W} | \Omega \rangle = {\rm Re}([\hat{K}_2(\eta, \vec{x}'), \hat{A}_0(\eta, \vec{x})]), \tag{77}$$

where we use the shorthand notation of the decomposition in (73). This is independent of any physical state satisfying (65), as it is only the commutator that appears on the right-hand side of (77). This nonvanishing expectation value violates the correspondence principle; even though the right-hand side does not depend on the quantum state, the choice of $\hat{K}$ is still largely arbitrary, and consequently so is $\hat{K}_2$. The proper way to order the operators (henceforth denoted by subscript g for "gauge") in observable (75) is to (i) decompose the Hermitian constraint operator $\hat{\Psi}_2$ into the non-Hermitian subsidiary constraint operator $\hat{K}$ and its conjugate $\hat{K}^\dagger$, and (ii) put all $\hat{K}$ operators on the right of the product and all $\hat{K}^\dagger$ operators on the left of the product,

$$[\hat{\mathscr{O}}(\eta, \vec{x}, \vec{x}')]_{\rm g} = \hat{K}_2^\dagger(\eta, \vec{x}')\hat{A}_0(\eta, \vec{x}) + \hat{A}_0(\eta, \vec{x})\hat{K}_2(\eta, \vec{x}'). \tag{78}$$

Such ordering guarantees that the expectation value vanishes due to (74) for any physical state,

$$\langle \Omega | [\hat{\mathscr{O}}(\eta, \vec{x}, \vec{x}')]_{\rm g} | \Omega \rangle = 0. \tag{79}$$

It is useful to note that the properly ordered observable can be written in terms of the Weyl-ordered observable plus the commutator accounting for the difference,

[5] We consider for simplicity a product of two fields evaluated at different spatial points, to avoid having to multiply distributions in the quantized theory.

$$[\hat{\mathscr{O}}(\eta,\vec{x},\vec{x}')]_{\rm g} = [\hat{\mathscr{O}}(\eta,\vec{x},\vec{x}')]_{\rm W} - {\rm Re}([\hat{K}_2(\eta,\vec{x}'),\hat{A}_0(\eta,\vec{x})]). \tag{80}$$

We comment more on the significance of this in Sec. VIII B. The difference between two orderings is ultimately the Faddeev-Popov ghost contribution.

Having addressed the operator ordering in (78), we need to consider further quantum properties of this trivial observable, which are usually encoded by correlators (n-point functions). Since it contains a Hermitian constraint and vanishes trivially classically, its n-point functions ought to vanish as well. However, while the operator ordering in (79) guarantees this is satisfied for the expectation values (one-point function), the two-point function of this operator does not vanish,

$$\begin{aligned}&\langle\Omega|[\hat{\mathscr{O}}(\eta,\vec{x},\vec{x}')]_{\rm g}[\hat{\mathscr{O}}(\eta,\vec{y},\vec{y}')]_{\rm g}|\Omega\rangle\\ &\quad= [\hat{A}_0(\eta,\vec{x}),\hat{K}_2^\dagger(\eta,\vec{y}')][\hat{K}_2(\eta,\vec{x}'),\hat{A}_0(\eta,\vec{y})] \neq 0,\end{aligned} \tag{81}$$

which is problematic. It would not be reasonable to conclude that quantum mechanics prevents this object from being a trivial observable, on account of its correlators not vanishing. That would imply a significant reduction in the number of observables in the quantized theory, with respect to the classical theory, and would provide a way to measure a quantum violation of classical first-class constraints. The resolution is to require that the product of operators associated with the observable has to first be properly ordered in the gauge sense,

$$\begin{aligned}&[\hat{\mathscr{O}}(\eta,\vec{x},\vec{x}')\hat{\mathscr{O}}(\eta,\vec{y},\vec{y}')]_{\rm g}\\ &= \hat{K}_2^\dagger(\eta,\vec{x}')\hat{K}_2^\dagger(\eta,\vec{y}')\hat{A}_0(\eta,\vec{x})\hat{A}_0(\eta,\vec{y})\\ &\quad+\hat{K}_2^\dagger(\eta,\vec{x}')\hat{A}_0(\eta,\vec{x})\hat{A}_0(\eta,\vec{y})\hat{K}_2(\eta,\vec{y}')\\ &\quad+\hat{K}_2^\dagger(\eta,\vec{y}')\hat{A}_0(\eta,\vec{x})\hat{A}_0(\eta,\vec{y})\hat{K}_2(\eta,\vec{x}')\\ &\quad+\hat{A}_0(\eta,\vec{x})\hat{A}_0(\eta,\vec{y})\hat{K}_2(\eta,\vec{x}')\hat{K}_2(\eta,\vec{y}'),\end{aligned} \tag{82}$$

and only then should the expectation value be taken. This way the two-point function also vanishes,

$$\langle\Omega|[\hat{\mathscr{O}}(\eta,\vec{x},\vec{x}')\hat{\mathscr{O}}(\eta,\vec{y},\vec{y}')]_{\rm g}|\Omega\rangle = 0. \tag{83}$$

The operator in (82) can also be expressed in terms of the Weyl-ordered products and compensating commutators, analogous to (80). The extension of this prescription to higher n-point functions should be straightforward.

V. FIELD OPERATOR DYNAMICS

The dynamics of the linear quantized theory is completely accounted for by the field operators. In this section we consider the dynamics of photon field operators in comoving momentum space, and we express the solutions in terms of a few scalar mode functions introduced in Sec. II B. Canonical commutation relations fix the normalization of these scalar mode functions and imply the commutation relations for time-independent momentum space operators. The section concludes by computing the non-Hermitian subsidiary constraint operator and discussing the freedom in how it is chosen.

A. Field operators in momentum space

The FLRW spacetime is homogeneous and isotropic, and the analysis of dynamics considerably simplifies by working in comoving momentum space. It is first advantageous to decompose the spatial components of the vector potential,

$$\hat{A}_i = \hat{A}_i^T + \hat{A}_i^L, \qquad \hat{\Pi}_i = \hat{\Pi}_i^T + \hat{\Pi}_i^L, \tag{84}$$

into its transverse and longitudinal parts,

$$\begin{aligned}\hat{A}_i^T &= \mathbb{P}_{ij}^T\hat{A}_j, &\quad \hat{\Pi}_i^T &= \mathbb{P}_{ij}^T\hat{\Pi}_j,\\ \hat{A}_i^L &= \mathbb{P}_{ij}^L\hat{A}_j, &\quad \hat{\Pi}_i^L &= \mathbb{P}_{ij}^L\hat{\Pi}_j,\end{aligned} \tag{85}$$

defined in terms of the transverse and longitudinal projection operators,

$$\mathbb{P}_{ij}^T = \delta_{ij} - \frac{\partial_i\partial_j}{\nabla^2}, \qquad \mathbb{P}_{ij}^L = \frac{\partial_i\partial_j}{\nabla^2}, \tag{86}$$

that are orthogonal, $\mathbb{P}_{ij}^T\mathbb{P}_{jk}^L = \mathbb{P}_{ij}^L\mathbb{P}_{jk}^T = 0$, and idempotent, $\mathbb{P}_{ij}^T\mathbb{P}_{jk}^T = \mathbb{P}_{ik}^T$, $\mathbb{P}_{ij}^L\mathbb{P}_{jk}^L = \mathbb{P}_{ik}^L$. The conveniently rescaled spatial Fourier transforms of such decomposed field operators are

$$\hat{A}_0(\eta,\vec{x}) = a^{-\frac{D-2-2\zeta}{2}}\int\frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}e^{i\vec{k}\cdot\vec{x}}\hat{\mathcal{A}}_0(\eta,\vec{k}), \tag{87a}$$

$$\hat{\Pi}_0(\eta,\vec{x}) = a^{\frac{D-2-2\zeta}{2}}\int\frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}e^{i\vec{k}\cdot\vec{x}}\hat{\pi}_0(\eta,\vec{k}), \tag{87b}$$

$$\hat{A}_i^L(\eta,\vec{x}) = a^{-\frac{D-2-2\zeta}{2}}\int\frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}e^{i\vec{k}\cdot\vec{x}}\frac{(-i)k_i}{k}\hat{\mathcal{A}}_L(\eta,\vec{k}), \tag{87c}$$

$$\hat{\Pi}_i^L(\eta,\vec{x}) = a^{\frac{D-2-2\zeta}{2}}\int\frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}e^{i\vec{k}\cdot\vec{x}}\frac{(-i)k_i}{k}\hat{\pi}_L(\eta,\vec{k}), \tag{87d}$$

$$\hat{A}_i^T(\eta,\vec{x}) = a^{-\frac{D-4}{2}}\int\frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}e^{i\vec{k}\cdot\vec{x}}\sum_{\sigma=1}^{D-2}\varepsilon_i(\sigma,\vec{k})\hat{\mathcal{A}}_{T,\sigma}(\eta,\vec{k}), \tag{87e}$$

$$\hat{\Pi}_i^T(\eta,\vec{x}) = a^{\frac{D-4}{2}}\int\frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}e^{i\vec{k}\cdot\vec{x}}\sum_{\sigma=1}^{D-2}\varepsilon_i(\sigma,\vec{k})\hat{\pi}_{T,\sigma}(\eta,\vec{k}), \tag{87f}$$

where the momentum space Hermitian operators behave under conjugation as $\hat{\mathcal{O}}^\dagger(\vec{k}) = \hat{\mathcal{O}}(-\vec{k})$. Here we introduced transverse polarization tensors with the following properties:

$$k_i\varepsilon_i(\sigma,\vec{k}) = 0, \quad \varepsilon_i^*(\sigma,\vec{k}) = \varepsilon_i(\sigma,-\vec{k}), \varepsilon_i^*(\sigma,\vec{k})\varepsilon_i(\sigma',\vec{k}) = \delta_{\sigma\sigma'},$$
$$\sum_{\sigma=1}^{D-2}\varepsilon_i^*(\sigma,\vec{k})\varepsilon_j(\sigma,\vec{k}) = \delta_{ij} - \frac{k_ik_j}{k^2}. \tag{88}$$

The canonical commutators of the momentum space field operators are now

$$[\hat{\mathcal{A}}_0(\eta,\vec{k}),\hat{\pi}_0(\eta,\vec{k}')] = [\hat{\mathcal{A}}_L(\eta,\vec{k}),\hat{\pi}_L(\eta,\vec{k}')] = i\delta^{D-1}(\vec{k}+\vec{k}'), \tag{89a}$$

$$[\hat{\mathcal{A}}_{T,\sigma}(\eta,\vec{k}),\hat{\pi}_{T,\sigma'}(\eta,\vec{k}')] = \delta_{\sigma\sigma'}\, i\,\delta^{D-1}(\vec{k}+\vec{k}'), \tag{89b}$$

while the momentum space equations of motion for the transverse sector are

$$\partial_0\hat{\mathcal{A}}_{T,\sigma} = \hat{\pi}_{T,\sigma} + \frac{1}{2}(D-4)\mathcal{H}\hat{\mathcal{A}}_{T,\sigma}, \tag{90}$$

$$\partial_0\hat{\pi}_{T,\sigma} = -k^2\hat{\mathcal{A}}_{T,\sigma} - \frac{1}{2}(D-4)\mathcal{H}\hat{\pi}_{T,\sigma}, \tag{91}$$

and the ones for the scalar sector read

$$\partial_0\hat{\mathcal{A}}_0 = -\xi a^{2-2\zeta}\hat{\pi}_0 + k\hat{\mathcal{A}}_L - \frac{1}{2}(D-2-2\zeta)\mathcal{H}\hat{\mathcal{A}}_0, \tag{92}$$

$$\partial_0\hat{\pi}_0 = k\hat{\pi}_L + \frac{1}{2}(D-2-2\zeta)\mathcal{H}\hat{\pi}_0, \tag{93}$$

$$\partial_0\hat{\mathcal{A}}_L = a^{2-2\zeta}\hat{\pi}_L - k\hat{\mathcal{A}}_0 + \frac{1}{2}(D-2-2\zeta)\mathcal{H}\hat{\mathcal{A}}_L, \tag{94}$$

$$\partial_0\hat{\pi}_L = -k\hat{\pi}_0 - \frac{1}{2}(D-2-2\zeta)\mathcal{H}\hat{\pi}_L. \tag{95}$$

Note that the Fourier transforms in (87) represent a time-dependent canonical transformation, so that the momentum space Hamiltonian generating the dynamics is

$$\begin{aligned}\hat{\mathrm{H}}_\star(\eta) = &\int d^{D-1}k \sum_{\sigma=1}^{D-2}\left[\frac{1}{2}\hat{\pi}_{T,\sigma}^\dagger\hat{\pi}_{T,\sigma} + \frac{k^2}{2}\hat{\mathcal{A}}_{T,\sigma}^\dagger\hat{\mathcal{A}}_{T,\sigma} + \frac{(D-4)}{4}\mathcal{H}(\hat{\pi}_{T,\sigma}^\dagger\hat{\mathcal{A}}_{T,\sigma} + \hat{\mathcal{A}}_{T,\sigma}^\dagger\hat{\pi}_{T,\sigma})\right] \\ &+ \int d^{D-1}k\left[\frac{a^{2-2\zeta}}{2}\hat{\pi}_L^\dagger\hat{\pi}_L - \frac{\xi a^{2-2\zeta}}{2}\hat{\pi}_0^\dagger\hat{\pi}_0 + \frac{k}{2}(\hat{\pi}_0^\dagger\hat{\mathcal{A}}_L + \hat{\mathcal{A}}_L^\dagger\hat{\pi}_0 - \hat{\pi}_L^\dagger\hat{\mathcal{A}}_0 - \hat{\mathcal{A}}_0^\dagger\hat{\pi}_L)\right. \\ &\left. + \frac{(D-2-2\zeta)}{4}\mathcal{H}(\hat{\pi}_L^\dagger\hat{\mathcal{A}}_L + \hat{\mathcal{A}}_L^\dagger\hat{\pi}_L - \hat{\pi}_0^\dagger\hat{\mathcal{A}}_0 - \hat{\mathcal{A}}_0^\dagger\hat{\pi}_0)\right],\end{aligned} \tag{96}$$

where the arguments of all the fields are $(\eta,\vec{k})$. Note that because the system is linear, the operator ordering of the Hamiltonian does not matter when generating the field operator equations of motion. It does matter, however, when observables are concerned, as discussed in Sec. IV C.

B. Subsidiary conditions in momentum space

The subsidiary condition introduced in Sec. IV B also takes a considerably simpler form in comoving momentum space. For the Hermitian constraints we define Fourier transforms

$$\hat{\Psi}_1(\eta,\vec{x}) = a^{\frac{D-2-2\zeta}{2}}\int\frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}e^{i\vec{k}\cdot\vec{x}}\hat{\psi}_1(\eta,\vec{k}), \tag{97a}$$

$$\hat{\Psi}_2(\eta,\vec{x}) = a^{\frac{D-2-2\zeta}{2}}\int\frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}e^{i\vec{k}\cdot\vec{x}}k\hat{\psi}_2(\eta,\vec{k}), \tag{97b}$$

such that the momentum space Hermitian constraints have the same dimensions,

$$\hat{\psi}_1(\eta,\vec{k}) = \hat{\pi}_0(\eta,\vec{k}), \qquad \hat{\psi}_2(\eta,\vec{k}) = \hat{\pi}_L(\eta,\vec{k}), \tag{98}$$

and satisfy closed momentum space equations of motion,

$$\begin{aligned}\partial_0\hat{\psi}_1 &= k\hat{\psi}_2 + \frac{1}{2}(D-2-2\zeta)\mathcal{H}\hat{\psi}_1, \\ \partial_0\hat{\psi}_2 &= -k\hat{\psi}_1 - \frac{1}{2}(D-2-2\zeta)\mathcal{H}\hat{\psi}_2.\end{aligned} \tag{99}$$

The momentum space non-Hermitian constraint operator is introduced in the same manner,

$$\hat{K}(\vec{x}) = a^{\frac{D-2-2\zeta}{2}}\int\frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}}e^{i\vec{k}\cdot\vec{x}}\hat{\mathcal{K}}(\vec{k}), \tag{100}$$

which translates into a simple linear combination of the two momentum space Hermitian constraint operators,

$$\hat{\mathcal{K}}(\vec{k}) = c_1(\eta,\vec{k})\hat{\psi}_1(\eta,\vec{k}) + c_2(\eta,\vec{k})\hat{\psi}_2(\eta,\vec{k}). \tag{101}$$

The momentum space equivalent of the position space subsidiary condition (65) on the space of states now reads

$$\hat{\mathcal{K}}(\vec{k})|\Omega\rangle = 0, \qquad \langle\Omega|\hat{\mathcal{K}}^\dagger(\vec{k}) = 0, \qquad \forall \vec{k}. \tag{102}$$

The condition of non-Hermiticity of the subsidiary constraint operator, necessary for consistency with canonical commutation relations, in momentum space translates into

$$\hat{\mathcal{K}}^\dagger(\vec{k}) \neq e^{i\gamma(\vec{k})}\hat{\mathcal{K}}(-\vec{k}), \tag{103}$$

where $\gamma(\vec{k})$ is an arbitrary real function reflecting the fact that subsidiary conditions (102) are defined up to an arbitrary phase.

The conservation of (101), together with the equations of motion (99) for the Hermitian constraints, implies equations of motion for the coefficient functions,

$$\partial_0 c_1 = k c_2 - \frac{1}{2}(D-2-2\zeta)\mathcal{H}c_1, \tag{104a}$$

$$\partial_0 c_2 = -k c_1 + \frac{1}{2}(D-2-2\zeta)\mathcal{H}c_2. \tag{104b}$$

The decomposition of the Hermitian constraints in terms of the non-Hermitian ones (73) in momentum space now reads

$$\hat{\psi}_1(\eta,\vec{k}) = \hat{\mathcal{K}}_1^\dagger(\eta,-\vec{k}) + \hat{\mathcal{K}}_1(\eta,\vec{k}), \tag{105a}$$

$$\hat{\psi}_2(\eta,\vec{k}) = \hat{\mathcal{K}}_2^\dagger(\eta,-\vec{k}) + \hat{\mathcal{K}}_2(\eta,\vec{k}), \tag{105b}$$

where

$$\hat{K}_1(\eta,\vec{x}) = a^{\frac{D-2-2\zeta}{2}} \int \frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}} e^{i\vec{k}\cdot\vec{x}} \hat{\mathcal{K}}_1(\eta,\vec{k}), \tag{106a}$$

$$\hat{K}_2(\eta,\vec{x}) = a^{\frac{D-2-2\zeta}{2}} \int \frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}} e^{i\vec{k}\cdot\vec{x}} k\hat{\mathcal{K}}_2(\eta,\vec{k}). \tag{106b}$$

C. Solving for dynamics

Solving for dynamics means expressing the time-dependent field operators in the Heisenberg picture in terms of initial conditions given at some η_0. This is what the usual solving for operators in terms of creation/annihilation operators is, which can also be seen as expressing field operators in the Heisenberg picture in terms of ones in the Schrödinger picture. In this section we express the solutions of the field operators in terms of the scalar mode functions satisfying mode equations, with solutions dependent on the specific FLRW background.

1. Transverse sector

The two transverse sector equations of motion (90) and (91) combine into a single second order one,

$$\left[\partial_0^2 + k^2 - \left(\lambda_T^2 - \frac{1}{4}\right)(1-\epsilon)^2\mathcal{H}^2\right]\hat{\pi}_{T,\sigma} = 0, \tag{107}$$

$$\hat{\mathcal{A}}_{T,\sigma} = -\frac{1}{k^2}\left[\partial_0 + \left(\lambda_T + \frac{1}{2}\right)(1-\epsilon)\mathcal{H}\right]\hat{\pi}_{T,\sigma}, \tag{108}$$

where we introduce

$$\lambda_T = \frac{D-5+\epsilon}{2(1-\epsilon)}. \tag{109}$$

The second order equation (107) is just the scalar mode equation (5) with $\lambda \to \lambda_T$. Furthermore, we have that $(\lambda_T + \frac{1}{2})(1-\epsilon) = (D-4)/2$ is time-independent, so that recurrence relations (11) are applicable. Therefore, we can write the solutions as

$$\hat{\pi}_{T,\sigma}(\eta,\vec{k}) = -ik\mathcal{U}_{\lambda_T}(\eta,k)\hat{b}_T(\sigma,\vec{k}) + ik\mathcal{U}_{\lambda_T}^*(\eta,k)\hat{b}_T^\dagger(\sigma,-\vec{k}), \tag{110}$$

$$\hat{\mathcal{A}}_{T,\sigma}(\eta,\vec{k}) = \mathcal{U}_{\lambda_T+1}(\eta,k)\hat{b}_T(\sigma,\vec{k}) + \mathcal{U}_{\lambda_T+1}^*(\eta,k)\hat{b}_T^\dagger(\sigma,-\vec{k}). \tag{111}$$

It follows now from the canonical commutation relations (89b) and the Wronskian (12) that the initial condition operators satisfy creation/annihilation commutation relations,

$$[\hat{b}_T(\sigma,\vec{k}), \hat{b}_T^\dagger(\sigma',\vec{k}')] = \delta_{\sigma\sigma'}\delta^{D-1}(\vec{k}-\vec{k}'). \tag{112}$$

Explicit solutions for the transverse sector mode functions depend on the particular FLRW background only, and not on the gauge-fixing parameters. This reflects the fact that the transverse polarizations are the physical propagating degrees of freedom of the photon in spatially flat cosmological spaces.

2. Scalar sector

In the scalar sector the two equations for canonical momenta (93) and (95) decouple from the rest. They combine into a second order equation,

$$\left[\partial_0^2 + k^2 - \left(\lambda^2 - \frac{1}{4}\right)(1-\epsilon)^2\mathcal{H}^2\right]\hat{\pi}_L = 0, \tag{113}$$

$$\hat{\pi}_0 = -\frac{1}{k}\left[\partial_0 + \left(\lambda + \frac{1}{2}\right)(1-\epsilon)\mathcal{H}\right]\hat{\pi}_L, \tag{114}$$

taking the form of the scalar mode equation, where the parameter

$$\lambda = \frac{D-3+\epsilon-2\zeta}{2(1-\epsilon)} \tag{115}$$

satisfies the relation (10). Therefore, according to (5), (6), and (11), the solutions are given in terms of scalar mode functions

$$\hat{\pi}_L(\eta,\vec{k}) = k\mathcal{U}_\lambda(\eta,k)\hat{b}_P(\vec{k}) + k\mathcal{U}_\lambda^*(\eta,k)\hat{b}_P^\dagger(-\vec{k}), \quad (116)$$

$$\hat{\pi}_0(\eta,\vec{k}) = ik\mathcal{U}_{\lambda+1}(\eta,k)\hat{b}_P(\vec{k}) - ik\mathcal{U}_{\lambda+1}^*(\eta,k)\hat{b}_P^\dagger(-\vec{k}). \quad (117)$$

These solutions now source the two remaining scalar sector equations (92) and (94), which again combine into a single second order one,

$$\left[\partial_0^2 + k^2 - \left(\lambda^2 - \frac{1}{4}\right)(1-\epsilon)^2\mathcal{H}^2\right]\hat{\mathcal{A}}_0 = a^{2-2\zeta}[-2\xi(1-\zeta)\mathcal{H}\hat{\pi}_0 + (1-\xi)k\hat{\pi}_L], \quad (118)$$

$$\hat{\mathcal{A}}_L = \frac{1}{k}\left[\partial_0 + \left(\lambda + \frac{1}{2}\right)(1-\epsilon)\mathcal{H}\right]\hat{\mathcal{A}}_0 + \frac{\xi a^{2-2\zeta}}{k}\hat{\pi}_0. \quad (119)$$

The source for the second order equation above already suggests what is likely the simplest choice of gauge-fixing parameters—$\xi = 1$ and $\zeta = 1$—which turns it into a homogeneous one. In the de Sitter space limit $\epsilon = 0$, this corresponds to the simple noncovariant gauge due to Woodard [38]. Equations (118) and (119) are solved by

$$\hat{\mathcal{A}}_0(\eta,\vec{k}) = \mathcal{U}_\lambda(\eta,k)\hat{b}_H(\vec{k}) + \mathcal{U}_\lambda^*(\eta,k)\hat{b}_H^\dagger(-\vec{k}) + v_0(\eta,k)\hat{b}_P(\vec{k}) + v_0^*(\eta,k)\hat{b}_P^\dagger(-\vec{k}), \quad (120)$$

$$\hat{\mathcal{A}}_L(\eta,\vec{k}) = -i\mathcal{U}_{\lambda+1}(\eta,k)\hat{b}_H(\vec{k}) + i\mathcal{U}_{\lambda+1}^*(\eta,k)\hat{b}_H^\dagger(-\vec{k}) - iv_L(\eta,k)\hat{b}_P(\vec{k}) + iv_L^*(\eta,k)\hat{b}_P^\dagger(-\vec{k}), \quad (121)$$

where the homogeneous parts solve the scalar mode equation (5), while the particular mode functions v_0 and v_L satisfy sourced mode equations,

$$\left[\partial_0^2 + k^2 - \left(\lambda^2 - \frac{1}{4}\right)(1-\epsilon)^2\mathcal{H}^2\right]v_0 = a^{2-2\zeta}[-2i\xi(1-\zeta)k\mathcal{H}\mathcal{U}_{\lambda+1} + (1-\xi)k^2\mathcal{U}_\lambda], \quad (122)$$

$$v_L = \frac{i}{k}\left[\partial_0 + \left(\lambda + \frac{1}{2}\right)(1-\epsilon)\mathcal{H}\right]v_0 - \xi a^{2-2\zeta}\mathcal{U}_{\lambda+1}, \quad (123)$$

and we conveniently normalize them to

$$\mathrm{Re}[v_0(\eta,k)\mathcal{U}_{\lambda+1}^*(\eta,k) + v_L(\eta,k)\mathcal{U}_\lambda^*(\eta,k)] = 0. \quad (124)$$

This fixes the commutation relations between the time-independent operators, the only nonvanishing ones being

$$[\hat{b}_P(\vec{k}), \hat{b}_H^\dagger(\vec{k}')] = [\hat{b}_H(\vec{k}), \hat{b}_P^\dagger(\vec{k}')] = -\delta^{D-1}(\vec{k} - \vec{k}'). \quad (125)$$

These are not the canonical commutation relations for the creation and annihilation operators that one is accustomed to working with. Nonetheless, they are perfectly valid solutions. In fact, a simple non-Bogolyubov transformation,

$$\hat{b}_1(\vec{k}) = \frac{1}{\sqrt{2}}(\hat{b}_H^\dagger(-\vec{k}) + \hat{b}_P^\dagger(-\vec{k})), \quad (126a)$$

$$\hat{b}_2(\vec{k}) = \frac{1}{\sqrt{2}}(\hat{b}_H(\vec{k}) - \hat{b}_P(\vec{k})), \quad (126b)$$

leads to more familiar creation/annihilation operators with nonvanishing commutators,

$$[\hat{b}_1(\vec{k}), \hat{b}_1^\dagger(\vec{k}')] = [\hat{b}_2(\vec{k}), \hat{b}_2^\dagger(\vec{k}')] = \delta^{D-1}(\vec{k} - \vec{k}'). \quad (127)$$

However, for our purposes it is far more convenient to work with $\hat{b}_P$ and $\hat{b}_H$ directly, as they readily translate to the subsidiary condition on physical states.

3. Non-Hermitian subsidiary constraint

The conservation of the non-Hermitian subsidiary constraint operator (101) implies two Eqs. (104) that combine into a single second order one,

$$\left[\partial_0^2 + k^2 - \left(\lambda^2 - \frac{1}{4}\right)(1-\epsilon)^2\mathcal{H}^2\right]c_1 = 0, \quad (128)$$

$$c_2 = \frac{1}{k}\left[\partial_0 + \left(\lambda + \frac{1}{2}\right)(1-\epsilon)\mathcal{H}\right]c_1, \quad (129)$$

with λ defined in (115). Again we recognize the scalar mode equation (5) and apply the recurrence relation (11) to write the general solutions in a convenient form:

$$c_1(\eta,\vec{k}) = i\beta(-\vec{k})\mathcal{U}_\lambda(\eta,k) - i\alpha(\vec{k})\mathcal{U}_\lambda^*(\eta,k), \quad (130)$$

$$c_2(\eta,\vec{k}) = \beta(-\vec{k})\mathcal{U}_{\lambda+1}(\eta,k) + \alpha(\vec{k})\mathcal{U}_{\lambda+1}^*(\eta,k), \quad (131)$$

where $\alpha(\vec{k})$ and $\beta(\vec{k})$ are free coefficients. Upon using the Wronskian (12) the non-Hermitian constraint evaluates to

$$\hat{\mathcal{K}}(\vec{k}) = \alpha(\vec{k})\hat{b}_P(\vec{k}) + \beta(-\vec{k})\hat{b}_P^\dagger(-\vec{k}), \quad (132)$$

and the condition of non-Hermiticity (103) now translates into the condition on the coefficients,

$$\left|\frac{\alpha(\vec{k})}{\beta(\vec{k})}\right| \neq 1. \quad (133)$$

The way that the free coefficients appear in (132) is reminiscent of Bogolyubov coefficients, and ultimately they have such an interpretation. Since the overall normalization of $\hat{\mathcal{K}}$ is immaterial we may parametrize it conveniently as[6]

$$\hat{\mathcal{K}}(\vec{k}) = \mathcal{N}(\vec{k}) e^{i\theta(\vec{k})} (e^{-i\varphi(\vec{k})} \mathrm{ch}[\rho(\vec{k})] \hat{b}_P(\vec{k}) + e^{i\varphi(-\vec{k})} \mathrm{sh}[\rho(-\vec{k})] \hat{b}_P^\dagger(-\vec{k})), \tag{134}$$

where we introduced a normalization coefficient,

$$\mathcal{N}(\vec{k}) = (\mathrm{ch}[\rho(\vec{k})]\mathrm{ch}[\rho(-\vec{k})] - \mathrm{sh}[\rho(\vec{k})]\mathrm{sh}[\rho(-\vec{k})])^{-\frac{1}{2}} = (\mathrm{ch}[\rho(\vec{k}) - \rho(-\vec{k})])^{-\frac{1}{2}}, \tag{135}$$

and where $\theta(\vec{k})$, $\varphi(\vec{k})$, and $\rho(\vec{k})$ are arbitrary real functions. Fixing these functions is a matter of convenience, whether it is respecting some symmetry or some other requirement. Since $\hat{\mathcal{K}}(\vec{k})$ will annihilate the ket state, it is convenient to employ it in computations, instead of using $\hat{b}_P(\vec{k})$. It is likewise advantageous to introduce another non-Hermitian operator associated with $\hat{b}_H(\vec{k})$,

$$\hat{\mathcal{B}}(\vec{k}) = \mathcal{N}(\vec{k}) e^{i\theta(\vec{k})} (e^{-i\varphi(\vec{k})} \mathrm{ch}[\rho(-\vec{k})] \hat{b}_H(\vec{k}) + e^{i\varphi(-\vec{k})} \mathrm{sh}[\rho(\vec{k})] \hat{b}_H^\dagger(-\vec{k})), \tag{136}$$

that preserves the form of nonvanishing commutators (125),

$$[\hat{\mathcal{K}}(\vec{k}), \hat{\mathcal{B}}^\dagger(\vec{k}')] = [\hat{\mathcal{B}}(\vec{k}), \hat{\mathcal{K}}^\dagger(\vec{k}')] = -\delta^{D-1}(\vec{k} - \vec{k}'). \tag{137}$$

In this sense (134)–(136) can be seen as a Bogolyubov transformation preserving the noncanonical commutation relations (125).

We can now also evaluate the parts of the non-Hermitian decomposition (105) in terms of the scalar mode functions that will prove useful later,

$$\hat{\mathcal{K}}_1(\eta, \vec{k}) = ik\mathcal{N}(\vec{k}) e^{-i\theta(\vec{k})} [e^{i\varphi(\vec{k})} \mathrm{ch}[\rho(-\vec{k})] \mathcal{U}_{\lambda+1}(\eta, k) + e^{-i\varphi(-\vec{k})} \mathrm{sh}[\rho(\vec{k})] \mathcal{U}_{\lambda+1}^*(\eta, k)] \hat{\mathcal{K}}(\vec{k}), \tag{138a}$$

$$\hat{\mathcal{K}}_2(\eta, \vec{k}) = k\mathcal{N}(\vec{k}) e^{-i\theta(\vec{k})} [e^{i\varphi(\vec{k})} \mathrm{ch}[\rho(-\vec{k})] \mathcal{U}_{\lambda}(\eta, k) - e^{-i\varphi(-\vec{k})} \mathrm{sh}[\rho(\vec{k})] \mathcal{U}_{\lambda}^*(\eta, k)] \hat{\mathcal{K}}(\vec{k}). \tag{138b}$$

VI. CONSTRUCTING THE SPACE OF STATES

The preceding section considered the quantization of the dynamics of field operators and of the subsidiary non-Hermitian constraint operator. To complete the quantization we need to construct a space of states on which the field operators act. This cannot be the usual Fock space due to the subsidiary condition (102) that forces upon us an indefinite inner product space. The construction of the space of states in quantized theories is typically intricately connected to the symmetries of the system. Here we discuss two concepts of symmetries arising in multiplier gauges: *physical symmetries* that are symmetries of the gauge-invariant action (18), and *gauge-fixed symmetries* that are symmetries of the gauge-fixed action (54). The former are actual symmetries of the system and characterize physical properties of the state, while the latter are symmetries of the gauge-fixed dynamics and are a matter of choice. Even though in the case at hand the gauge-fixed symmetries coincide with the physical symmetries, as both actions (18) and (54) are invariant under spatial Euclidean transformations, in general this need not be the case. For example, in the de Sitter space limit ($\epsilon = 0$) the gauge-invariant action would be invariant under the maximal number of isometries, while the gauge-fixed action for $\zeta \neq 0$ would be invariant under Euclidean spatial transformations only. Thus we would be able to define a state respecting physical de Sitter symmetries, but the gauge-fixed dynamics could not be made de Sitter invariant. For such a state correlators of gauge-independent operators would exhibit physical symmetries, despite the fact that the correlators of gauge-dependent quantities would not. This is why understanding the distinction between the two is important. The physical symmetries will influence the construction of the transverse sector of the space of states, while the gauge-fixed symmetries will dictate the construction of the scalar sector.

A. FLRW symmetries

Flat FLRW spacetimes have $\frac{1}{2}D(D-1)$ isometries of the $(D-1)$-dimensional Euclidean spaces that make the equal-time spatial slices. They consist of $(D-1)$ spatial translations,

$$\eta \to \eta, \qquad x_i \to x_i + \alpha_i, \tag{139}$$

and of $\frac{1}{2}(D-1)(D-2)$ spatial rotations, whose infinitesimal form is

$$\eta \to \eta, \qquad x_i \to x_i + 2\omega_{ij}x_j, \qquad (\omega_{ij} = -\omega_{ji}). \tag{140}$$

Both the gauge-invariant and the gauge-fixed photon actions, (18) and (54), are invariant under infinitesimal active transformations of the vector potential, associated with spatial translations,

$$A_\mu(x) \to A_\mu(x) - \alpha_i \partial_i A_\mu(x), \tag{141}$$

[6]This parametrization technically covers just half of the parameter space. The other half is covered by interchanging the roles of $\hat{b}_P(\vec{k})$ and $\hat{b}_P^\dagger(-\vec{k})$. Even though there should be no obstructions to this choice, it turns out to be inconsistent with manifest Poincaré symmetries in flat space, and we do not consider it.

and with spatial rotations,

$$A_\mu(x) \to A_\mu(x) + 2\omega_{ij}x_i\partial_j A_\mu(x) + 2\delta^i_\mu\omega_{ij}A_j(x). \tag{142}$$

B. Physical symmetries

Even though the active transformations (141) and (142) are symmetry transformations of the gauge-invariant action (18), they are ambiguous on the account of gauge transformations that carry no physical meaning. This means we can combine (141) and (142) with a gauge transformation and change their form without affecting the physical content. The most convenient choice fixing the ambiguity is requiring that the generators of these transformations take a gauge-invariant form themselves. This is accomplished by modifying (141) and (142) by a gauge transformation to read, respectively,

$$\begin{aligned} A_\mu(x) &\to A_\mu(x) - \alpha_i F_{i\mu}(x), \\ A_\mu(x) &\to A_\mu(x) + 2\omega_{ij}x_i F_{j\mu}(x). \end{aligned} \tag{143}$$

Thus, the conserved Noether charges associated with the two symmetry transformations of the gauge-invariant action (18) are, respectively, the total linear momentum and total angular momentum,

$$P_i = \int d^{D-1}x(-F_{ij}\Pi_j), \qquad M_{ij} = \int d^{D-1}x(2x_{[i}F_{j]k}\Pi_k). \tag{144}$$

They satisfy $E(D-1)$ algebra on-shell,

$$\begin{aligned} &\{P_i, P_j\} \approx 0, \qquad \{M_{ij}, P_k\} \approx 2P_{[i}\delta_{j]k}, \\ &\{M_{ij}, M_{kl}\} \approx 4\delta_{i][k}M_{l][j}, \end{aligned} \tag{145}$$

and serve as generators of corresponding symmetry transformations. Their structure is more transparent if we write them out in terms of longitudinal and transverse components of the canonical variables (84)–(86), and recognize the constraints (55),

$$P_i = \int d^{D-1}x(-\Pi^T_j\partial_i A^T_j - A^T_i\Psi_2), \tag{146}$$

$$M_{ij} = \int d^{D-1}x(2x_{[i}F^T_{j]k}\Pi^T_k + 2x_{[i}A^T_{j]}\Psi_2). \tag{147}$$

Quantizing these symmetry generators implies promoting fields to field operators, which necessitates proper operator ordering in order for them to be observables. First, the parts of (146) and (147) containing constraints should be ordered according to the prescription outlined in Sec. IV C,

$$\hat{P}_i = \hat{P}^T_i - \int d^{D-1}x(\hat{K}^\dagger_2\hat{A}^T_i + \hat{A}^T_i\hat{K}_2), \tag{148}$$

$$\hat{M}_{ij} = \hat{M}^T_{ij} + \int d^{D-1}x(2\hat{K}^\dagger_2 x_{[i}\hat{A}^T_{j]} + 2x_{[i}\hat{A}^T_{j]}\hat{K}_2). \tag{149}$$

Second, the purely transverse parts should be normal-ordered according to the standard prescription that is best implemented in momentum space where all the annihilation operators of the transverse sector are put to the right of all the creation operators. Using Fourier transforms of field operators (87), the solutions of the transverse field operators (111) and (110), and the Wronskian (12) of the mode function, the normal-ordered purely transverse parts of the operators evaluate to

$$\hat{P}^T_i = \int d^{D-1}k k_i \hat{\mathcal{E}}^\dagger_j(\vec{k})\hat{\mathcal{E}}_j(\vec{k}), \tag{150}$$

$$\begin{aligned} \hat{M}^T_{ij} = \int d^{D-1}k \bigg[&\hat{\mathcal{E}}^\dagger_k(\vec{k})\left(ik_i\frac{\partial}{\partial k_j} - ik_j\frac{\partial}{\partial k_i}\right)\hat{\mathcal{E}}_k(\vec{k}) \\ &+ 2\hat{\mathcal{E}}^\dagger_{[i}(\vec{k})\hat{\mathcal{E}}_{j]}(\vec{k})\bigg], \end{aligned} \tag{151}$$

with the expressions written compactly using a shorthand notation,

$$\hat{\mathcal{E}}_i(\vec{k}) = \sum_{\sigma=1}^{D-2}\varepsilon_i(\sigma,\vec{k})\hat{b}_T(\sigma,\vec{k}). \tag{152}$$

These generators of physical symmetries commute with the non-Hermitian constraint,

$$[\hat{\mathcal{K}}(\vec{k}), \hat{P}_i] = 0, \qquad [\hat{\mathcal{K}}(\vec{k}), \hat{M}_{ij}] = 0, \tag{153}$$

and preserve the algebra (145) at the level of matrix elements,

$$\begin{aligned} &\langle\psi|[\hat{P}_i,\hat{P}_j]|\psi'\rangle = 0, \qquad \langle\psi|[\hat{M}_{ij},\hat{P}_k]|\psi'\rangle = i\langle\psi|2\hat{P}_{[i}\delta_{j]k}|\psi'\rangle, \\ &\langle\psi|[\hat{M}_{ij},\hat{M}_{kl}]|\psi'\rangle = i\langle\psi|4\delta_{i][k}M_{l][j}|\psi'\rangle. \end{aligned} \tag{154}$$

In fact, it is only the purely transverse parts of (148) and (151) that contribute to the matrix elements of the algebra.

The dynamics of field operators is given by the gauge-fixed action. As a consequence the physical symmetry generators (148) and (149) are not time-independent. That is why it is meaningless to require there exists a state that is an eigenstate of these generators in the usual sense. However, the matrix elements of physical symmetry generators are conserved in time, since only the purely transverse part provides a nonvanishing contribution to them. The fact that this part is time-independent can be seen from the solutions given in (150) and (151). This implies that we can still define a notion of an eigenstate $|\Omega\rangle$ of generators by the property that an expectation value of any polynomial of generators equals the polynomial of expectation values,

$$\langle\Omega|\mathscr{P}(\hat{P}_i,\hat{M}_{ij})|\Omega\rangle=\mathscr{P}(\bar{P}_i,\bar{M}_{ij}),\qquad\langle\Omega|\hat{P}_i|\Omega\rangle=\bar{P}_i,\qquad\langle\Omega|\hat{M}_{ij}|\Omega\rangle=\bar{M}_{ij}.\tag{155}$$

This condition, together with the algebra (154), implies there is only one such state that is a simultaneous eigenstate of both, and that it has to have vanishing expectation values,

$$\bar{P}_i=0,\qquad\bar{M}_{ij}=0.\tag{156}$$

Another thing becomes evident upon closer examination of (155)—it is only the transverse sector that is affected by these conditions, and the full state will be the tensor product between the transverse sector and the scalar sector, $|\Omega\rangle=|\Omega_T\rangle\otimes|\Omega_0\rangle$. The only scalar sector operators appearing in the generators are constraints, which are ordered such that terms containing them drop out from any expectation values:

$$\langle\Omega|\mathscr{P}(\hat{P}_i,\hat{M}_{ij})|\Omega\rangle=\langle\Omega_T|\mathscr{P}(\hat{P}_i^T,\hat{M}_{ij}^T)|\Omega_T\rangle.\tag{157}$$

Since the transverse sector is unconstrained, its space of states can be constructed as the usual Fock space. There must be some annihilation operator $\hat{c}_T(\sigma,\vec{k})$ that annihilates the vacuum,

$$\hat{c}_T(\sigma,\vec{k})|\Omega_T\rangle=0,\qquad\forall\ \sigma,\vec{k},\tag{158}$$

so that the rest of the Fock space is generated by acting with the associated creation operators $\hat{c}_T^\dagger(\sigma,\vec{k})$ on that vacuum. The most general choice respecting isotropy and homogeneity is given by the Bogolyubov transformation,

$$\hat{c}_T(\sigma,\vec{k})=e^{-i\varphi_T(k)}\mathrm{ch}[\rho_T(k)]\hat{b}_T(\sigma,\vec{k})+e^{i\varphi_T(k)}\mathrm{sh}[\rho_T(k)]\hat{b}_T^\dagger(\sigma,-\vec{k}),\tag{159}$$

where $\varphi_T(k)$ and $\rho_T(k)$ are arbitrary real functions. The vacuum defined in (158) is now an eigenstate of the purely transverse parts of generators (150) and (151),

$$\hat{P}_i^T|\Omega_T\rangle=0,\qquad\hat{M}_{ij}^T|\Omega_T\rangle=0,\tag{160}$$

with vanishing eigenvalues, and thus corresponds to the state respecting physical cosmological symmetries. Note that the procedure of this section has fixed only the transverse sector of the state, while leaving the scalar sector unfixed. The scalar sector has to be fixed from different considerations that the following section is devoted to.

C. Gauge-fixed symmetries

The conserved Noether charges associated with spatial translations and spatial rotations that follow from the gauge-fixed action (54) are, respectively,

$$P_i^\star=\int d^{D-1}x\Big(-\Pi_0\partial_iA_0-\Pi_j\partial_iA_j\Big),\tag{161}$$

$$M_{ij}^\star=\int d^{D-1}x\Big(2\Pi_0x_{[i}\partial_{j]}A_0+2\Pi_kx_{[i}\partial_{j]}A_k+2\Pi_{[i}A_{j]}\Big).\tag{162}$$

They are generators of the corresponding symmetry transformations of the gauge-fixed dynamics, and they satisfy the $E(D-1)$ algebra off-shell,

$$\{P_i^\star,P_j^\star\}=0,\qquad\{M_{ij}^\star,P_k^\star\}=2P_{[i}^\star\delta_{j]k},\qquad\{M_{ij}^\star,M_{kl}^\star\}=4\delta_{i][k}M_{l][j}^\star.\tag{163}$$

The structure of these charges and their quantization is more transparent when written in terms of transverse and longitudinal components of the canonical fields,

$$P_i^\star=\int d^{D-1}x\Big[-\Pi_j^T\partial_iA_j^T+(\Psi_2A_i^L-\Psi_1\partial_iA_0)\Big],\tag{164}$$

$$M_{ij}^\star=\int d^{D-1}x\Big[2x_{[i}F_{j]k}^T\Pi_k^T+2(\Psi_1x_{[i}\partial_{j]}A_0-\Psi_2x_{[i}A_{j]}^L)\Big].\tag{165}$$

Classically these symmetry generators are observables, differing from the gauge-invariant ones (146) and (147) only off-shell. Quantizing the gauge-fixed generators and requiring they remain observables produces the following operator ordering, according to Sec. IV C:

$$\hat{P}_i^\star=\hat{P}_i^T+\int d^{D-1}x(\hat{K}_2^\dagger\hat{A}_i^L+\hat{A}_i^L\hat{K}_2-\hat{K}_1^\dagger\partial_i\hat{A}_0-\partial_i\hat{A}_0\hat{K}_1),\tag{166}$$

$$\hat{M}_{ij}^\star=\hat{M}_{ij}^T+\int d^{D-1}x(2\hat{K}_1^\dagger x_{[i}\partial_{j]}\hat{A}_0+2x_{[i}\partial_{j]}\hat{A}_0\hat{K}_1-2\hat{K}_2^\dagger x_{[i}\hat{A}_{j]}^L-2x_{[i}\hat{A}_{j]}^L\hat{K}_2),\tag{167}$$

where the normal-ordered purely transverse parts were already given in (150) and (151). These quantum generators respect the $E(D-1)$ algebra at the operator level,

$$[\hat{P}_i^\star,\hat{P}_j^\star]=0,\qquad[\hat{M}_{ij}^\star,\hat{P}_k^\star]=2i\hat{P}_{[i}^\star\delta_{j]k},\qquad[\hat{M}_{ij}^\star,\hat{M}_{kl}^\star]=4i\delta_{i][k}\hat{M}_{l][j}^\star.\tag{168}$$

In order for the translationally and rotationally invariant physical state to exist, it must be annihilated by both symmetry generators (166) and (167), and by the non-Hermitian constraint (134). This implies that the non-Hermitian constraint $\mathcal{K}(\vec{k})$ must commute with the gauge-fixed generators, modulo $\hat{\mathcal{K}}$ itself. That is already satisfied for translations,

$$[\hat{\mathcal{K}}(\vec{k}), \hat{P}_i^\star] = k_i \hat{\mathcal{K}}(\vec{k}), \tag{169}$$

but only after requiring

$$\rho(\vec{k}) = \rho(k), \qquad \varphi(\vec{k}) = \varphi(k) \tag{170}$$

is it satisfied for rotations,

$$[\hat{\mathcal{K}}(\vec{k}), \hat{M}_{ij}^\star] = ie^{i\theta(\vec{k})}\left(k_i \frac{\partial}{\partial k_j} - k_j \frac{\partial}{\partial k_i}\right)(e^{-i\theta(\vec{k})}\hat{\mathcal{K}}(\vec{k})). \tag{171}$$

With the restrictions (170) implemented in (166) and (167), the gauge-fixed generators take the form

$$\hat{P}_i^\star = \hat{P}_i^T + \int d^{D-1}k k_i \left[\hat{\mathcal{K}}^\dagger(\vec{k})\hat{\mathcal{B}}(\vec{k}) + \hat{\mathcal{B}}^\dagger(\vec{k})\hat{\mathcal{K}}(\vec{k})\right], \tag{172}$$

$$\begin{aligned}\hat{M}_{ij}^\star = \hat{M}_{ij}^T + \int d^{D-1}k(-i) \\ \times \Bigg[(e^{i\theta(\vec{k})}\hat{\mathcal{B}}^\dagger(\vec{k}))\left(k_i \frac{\partial}{\partial k_j} - k_j \frac{\partial}{\partial k_i}\right)(e^{-i\theta(\vec{k})}\hat{\mathcal{K}}(\vec{k})) \\ + (e^{i\theta(\vec{k})}\hat{\mathcal{K}}^\dagger(\vec{k}))\left(k_i \frac{\partial}{\partial k_j} - k_j \frac{\partial}{\partial k_i}\right)(e^{-i\theta(\vec{k})}\hat{\mathcal{B}}(\vec{k}))\Bigg].\end{aligned} \tag{173}$$

Next we turn to finding an eigenstate of gauge-fixed generators (172) and (173) with vanishing eigenvalues. This is now a condition on the scalar sector, since the transverse sector has already been fixed in Sec. VI B. Given that the subsidiary condition (102) acts on the scalar sector only, $\hat{\mathcal{K}}(\vec{k})|\Omega_0\rangle = 0$, by simple inspection of generators (172) and (173) it is clear that the sought-for state vector has to be annihilated by $\hat{\mathcal{B}}(\vec{k})$,

$$\hat{\mathcal{B}}(\vec{k})|\Omega_0\rangle = 0, \tag{174}$$

in addition to being annihilated by $\hat{\mathcal{K}}(\vec{k})$. This guarantees that the state is annihilated by the gauge-fixed symmetry generators,

$$\hat{P}_i^\star|\Omega\rangle = 0, \qquad \hat{M}_{ij}^\star|\Omega\rangle = 0. \tag{175}$$

Having defined the homogeneous and isotropic state, next we construct the scalar sector space of states. Since the operators acting on this vector space are $\hat{\mathcal{K}}(\vec{k})$ and $\hat{\mathcal{B}}(\vec{k})$, the rest of the basis vectors are generated by acting with operators $\hat{\mathcal{K}}^\dagger(\vec{k})$ and $\hat{\mathcal{B}}^\dagger(\vec{k})$. However, this will not be a Fock space, since these operators are not the standard creation/annihilation operators due to their algebra. There are several features to notice.

1. Indefinite metric (inner product) space

In the scalar sector the space of states is spanned by $\hat{\mathcal{K}}^\dagger(\vec{k})$ and $\hat{\mathcal{B}}^\dagger(\vec{k})$ acting on $|\Omega_0\rangle$. In such a space there are states of vanishing and negative norm (in addition to positive norm). It is not difficult to construct examples. The two state vectors,

$$\begin{aligned}|\psi_1\rangle = \int d^{D-1}k f(\vec{k})\hat{\mathcal{K}}^\dagger(\vec{k})|\Omega_0\rangle, \\ |\psi_2\rangle = \int d^{D-1}k f(\vec{k})\hat{\mathcal{B}}^\dagger(\vec{k})|\Omega_0\rangle,\end{aligned} \tag{176}$$

are not orthogonal to each other, $\langle\psi|\psi\rangle \neq 0$, but also both have a vanishing norm,

$$\langle\psi_1|\psi_1\rangle = 0, \qquad \langle\psi_2|\psi_2\rangle = 0. \tag{177}$$

This is a consequence of commutation relations (137). It is also straightforward to demonstrate the existence of negative norm states, e.g.

$$|\psi_3\rangle = |\psi_1\rangle + |\psi_2\rangle \Rightarrow \langle\psi_3|\psi_3\rangle = -2\int d^{D-1}k|f(\vec{k})|^2 < 0. \tag{178}$$

Even though this might seem disconcerting at first, it is not really an issue, as it does not affect the physical states defined by (102).

2. Physical subspace is positive-definite

The physical subspace of the entire space of states is defined by a subsidiary condition on the scalar sector space of states, $\hat{\mathcal{K}}(\vec{k})|\Omega_0^{\rm phys}\rangle = 0$. If the "vacuum" state of that subspace is defined by condition (174) consistent with manifest homogeneity and isotropy, then it can be shown that the remaining members of the physical subspace take the form

$$|\Omega_0^{\rm phys}\rangle = |\Omega_0\rangle + \sum_{n=1}^{\infty}\int d^{D-1}k_1 \cdots d^{D-1}k_n f_n(\vec{k}_1, \ldots, \vec{k}_n)\hat{\mathcal{K}}^\dagger(\vec{k}_1)\cdots\hat{\mathcal{K}}^\dagger(\vec{k}_n)|\Omega_0\rangle \tag{179}$$

that all have a unit norm,

$$\langle\Omega_0^{\rm phys}|\Omega_0^{\rm phys}\rangle = \langle\Omega_0|\Omega_0\rangle = 1. \tag{180}$$

This form is dictated by the conditions (102) and (174), and the algebra of operators (137) spanning the scalar sector space of states. Physically there is no difference whatsoever which of the representatives in (179) we choose to represent the state. Therefore, the choice is delegated to a matter of convenience, which is obviously the physical and homogeneous state.

VII. TWO-POINT FUNCTIONS

The two-point functions fully characterize Gaussian quantum states in free theories. Moreover, they are basic ingredients for nonequilibrium perturbative computations in field theory. In this section we first discuss state-independent properties of the two-point functions—the equations of motion they satisfy and the various subsidiary conditions they have to respect. By the end of the section we express the photon two-point functions in terms of a few scalar mode functions introduced in Sec. V. Thus, we reduce the future tasks of computing photon two-point functions in FLRW spaces to computing several scalar mode functions and the corresponding sum-over-modes.

A. General properties

The positive-frequency Wightman function for the photon is defined as an expectation value of a product of two vector potential field operators,

$$i[{}_\mu^-\Delta_\nu^+](x;x') = \langle\Omega|\hat{A}_\mu(x)\hat{A}_\nu(x')|\Omega\rangle, \tag{181}$$

while the negative-frequency Wightman function, $i[{}_\mu^+\Delta_\nu^-](x;x') = \{i[{}_\mu^-\Delta_\nu^+](x;x')\}^*$, is a complex conjugate that reverses the order of operators in the product in (181). These two can be used to define the Feynman propagator,

$$\begin{aligned} i[{}_\mu^+\Delta_\nu^+](x;x') &= \langle\Omega|\mathcal{T}(\hat{A}_\mu(x)\hat{A}_\nu(x'))|\Omega\rangle \\ &= \theta(\eta-\eta')i[{}_\mu^-\Delta_\nu^+](x;x') \\ &\quad + \theta(\eta'-\eta)i[{}_\mu^+\Delta_\nu^-](x;x'), \end{aligned} \tag{182}$$

and its conjugate, $i[{}_\mu^-\Delta_\nu^-](x;x') = \{i[{}_\mu^+\Delta_\nu^+](x;x')\}^*$, called the Dyson propagator. The four two-point functions are completely determined by specifying the quantum state. Nonetheless, there are general properties that they have to satisfy for any allowed states. These properties are useful as checks of the consistency of two-point functions. We derive and discuss them here.

First, the field operator equations of motion (58)–(61) can be written in a more familiar covariant form,

$$\begin{aligned} &\mathcal{D}_\mu{}^\nu\hat{A}_\nu = 0, \\ &\mathcal{D}_{\mu\nu} = g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu \\ &\qquad + \frac{1}{\xi}(\nabla_\mu + 2\zeta n_\mu)(\nabla_\nu - 2\zeta n_\nu) - R_{\mu\nu}. \end{aligned} \tag{183}$$

As a consequence, the Wightman function satisfies the same homogeneous equation of motion on both external points,

$$\mathcal{D}_\mu{}^\rho i[{}_\rho^-\Delta_\nu^+](x;x') = 0, \qquad \mathcal{D}_\nu'^{\sigma} i[{}_\mu^-\Delta_\sigma^+](x;x') = 0, \tag{184}$$

and the canonical commutation relations (57) guarantee that the Feynman propagator satisfies inhomogeneous equations,

$$\mathcal{D}_\mu{}^\rho i[{}_\rho^+\Delta_\nu^+](x;x') = g_{\mu\nu}\frac{i\delta^D(x-x')}{\sqrt{-g}}, \tag{185a}$$

$$\mathcal{D}_\nu'^{\sigma} i[{}_\mu^+\Delta_\sigma^+](x;x') = g_{\mu\nu}\frac{i\delta^D(x-x')}{\sqrt{-g}}. \tag{185b}$$

These are not the only state-independent equations that the photon two-point functions satisfy. The quantization in Sec. IV required the two-point functions of Hermitian constraints (72) to vanish according to the correspondence principle. By expressing the Hermitian constraints in terms of the derivatives of vector potential field operators,

$$(\nabla^\mu - 2\zeta n^\mu)\hat{A}_\mu = \xi a^{2-D}\hat{\Pi}_0 = \xi a^{2-D}\hat{\Psi}_1, \tag{186a}$$

$$(2g^{ij}\delta^\mu_{[i}\partial_{0]}\partial_j)\hat{A}_\mu = a^{2-D}\partial_i\hat{\Pi}_i = a^{2-D}\hat{\Psi}_2, \tag{186b}$$

we can translate this quantization requirement into subsidiary conditions for the Wightman function,

$$(\nabla^\mu - 2\zeta n^\mu)(\nabla'^\nu - 2\zeta n'^\nu)i[{}_\mu^-\Delta_\nu^+](x;x') = 0, \tag{187a}$$

$$(\nabla^\mu - 2\zeta n^\mu)(2g'^{kl}\delta^\nu_{[k}\partial'_{0]}\partial'_l)i[{}_\mu^-\Delta_\nu^+](x;x') = 0, \tag{187b}$$

$$(2g^{ij}\delta^\mu_{[i}\partial_{0]}\partial_j)(\nabla'^\nu - 2\zeta n'^\nu)i[{}_\mu^-\Delta_\nu^+](x;x') = 0, \tag{187c}$$

$$(2g^{ij}\delta^\mu_{[i}\partial_{0]}\partial_j)(2g'^{kl}\delta^\nu_{[k}\partial'_{0]}\partial'_l)i[{}_\mu^-\Delta_\nu^+](x;x') = 0, \tag{187d}$$

and for the Feynman propagator,

$$(\nabla^\mu - 2\zeta n^\mu)(\nabla'^\nu - 2\zeta n'^\nu)i[{}_\mu^+\Delta_\nu^+](x;x') = -\xi\frac{i\delta^D(x-x')}{\sqrt{-g}}, \tag{188a}$$

$$(\nabla^\mu - 2\zeta n^\mu)(2g'^{kl}\delta^\nu_{[k}\partial'_{0]}\partial'_l)i[{}_\mu^+\Delta_\nu^+](x;x') = 0, \tag{188b}$$

$$(2g^{ij}\delta^\mu_{[i}\partial_{0]}\partial_j)(\nabla'^\nu - 2\zeta n'^\nu)i[{}_\mu^+\Delta_\nu^+](x;x') = 0, \tag{188c}$$

$$(2g^{ij}\delta^{\mu}_{[i}\partial_{0]}\partial_j)(2g'^{kl}\delta^{\nu}_{[k}\partial'_{0]}\partial'_l)i[^{+}_{\mu}\Delta^{+}_{\nu}](x;x') = \partial_i\partial'_i \frac{i\delta^D(x-x')}{\sqrt{-g}}. \tag{188d}$$

These are useful as consistency checks of two-point functions. Failure to satisfy them signals inconsistencies of photon two-point functions.

The two-derivative subsidiary conditions (187) and (188) are independent of the choice of state, and in particular of the choice of the pure gauge sector. There is a different way of expressing (187) and (188), in terms of single-derivative subsidiary conditions. These derive from considering correlators between Hermitian constraints and vector field operators,

$$(\nabla^\mu - 2\zeta n^\mu)i[^{-}_{\mu}\Delta^{+}_{\nu}](x;x') = \xi a^{2-D}\langle\Omega|\hat{\Psi}_1(x)\hat{A}_\nu(x')|\Omega\rangle, \tag{189}$$

$$(2g^{ij}\delta^{\mu}_{[i}\partial_{0]}\partial_j)i[^{-}_{\mu}\Delta^{+}_{\nu}](x;x') = a^{2-D}\langle\Omega|\hat{\Psi}_2(x)\hat{A}_\nu(x')|\Omega\rangle. \tag{190}$$

Given the decomposition (73) of Hermitian constraints into non-Hermitian ones, and the subsidiary condition in the state (65), the right-hand sides above reduce to

$$(\nabla^\mu - 2\zeta n^\mu)i[^{-}_{\mu}\Delta^{+}_{\nu}](x;x') = \xi a^{2-D}\langle\Omega|[\hat{K}_1(x),\hat{A}_\nu(x')]|\Omega\rangle, \tag{191}$$

$$(2g^{ij}\delta^{\mu}_{[i}\partial_{0]}\partial_j)i[^{-}_{\mu}\Delta^{+}_{\nu}](x;x') = a^{2-D}\langle\Omega|[\hat{K}_2(x),\hat{A}_\nu(x')]|\Omega\rangle. \tag{192}$$

Evaluating the position space commutators on the right-hand side above is simpler if we first compute the momentum space commutators,

$$[\hat{\mathcal{K}}(\vec{k}),\hat{\mathcal{A}}_0(\eta,\vec{k}')] = -e^{i\theta(\vec{k})}\mathscr{U}^*_\lambda(\eta,k)\delta^{D-1}(\vec{k}+\vec{k}'), \tag{193}$$

$$[\hat{\mathcal{K}}(\vec{k}),\hat{\mathcal{A}}_L(\eta,\vec{k}')] = -ie^{i\theta(\vec{k})}\mathscr{U}^*_{\lambda+1}(\eta,k)\delta^{D-1}(\vec{k}+\vec{k}'), \tag{194}$$

where we defined the two scalar mode functions,

$$\mathscr{U}_\lambda(\eta,k) = e^{i\varphi(k)}\mathrm{ch}[\rho(k)]\mathcal{U}_\lambda(\eta,k) - e^{-i\varphi(k)}\mathrm{sh}[\rho(k)]\mathcal{U}^*_\lambda(\eta,k), \tag{195}$$

$$\mathscr{U}_{\lambda+1}(\eta,k) = e^{i\varphi(k)}\mathrm{ch}[\rho(k)]\mathcal{U}_{\lambda+1}(\eta,k) + e^{-i\varphi(k)}\mathrm{sh}[\rho(k)]\mathcal{U}^*_{\lambda+1}(\eta,k), \tag{196}$$

that also satisfy the recurrence relations (11),

$$\left[\partial_0 + \left(\lambda+\frac{1}{2}\right)(1-\epsilon)\mathcal{H}\right]\mathscr{U}_\lambda = -ik\mathscr{U}_{\lambda+1}, \tag{197a}$$

$$\left[\partial_0 - \left(\lambda+\frac{1}{2}\right)(1-\epsilon)\mathcal{H}\right]\mathscr{U}_{\lambda+1} = -ik\mathscr{U}_\lambda. \tag{197b}$$

Using these recurrence relations, the commutators (193) and (194), Fourier transforms (87) and (106), and the operators (138), it is straightforward to show that the right-hand sides of (191) and (192) are

$$(\nabla^\mu - 2\zeta n^\mu)i[^{-}_{\mu}\Delta^{+}_{\nu}](x;x') = -\xi\partial'_\nu\left\{\left(\frac{a'}{a}\right)^\zeta i[^{-}\Delta^{+}]_{\lambda+1}(x;x')\right\}, \tag{198}$$

$$\begin{aligned}(2g^{ij}\delta^{\mu}_{[i}\partial_{0]}\partial_j)&i[^{-}_{\mu}\Delta^{+}_{\nu}](x;x') \\ &= -(\partial_0 + 2\zeta\mathcal{H})\partial'_\nu\left\{\left(\frac{a'}{a}\right)^\zeta i[^{-}\Delta^{+}]_{\lambda+1}(x;x')\right\},\end{aligned} \tag{199}$$

where we recognized the scalar two-point function (13),

$$i[^{-}\Delta^{+}]_{\lambda+1}(x;x') = (aa')^{-\frac{D-2}{2}}\int\frac{d^{D-1}k}{(2\pi)^{D-1}}e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \times \mathscr{U}_{\lambda+1}(\eta,k)\mathscr{U}^*_{\lambda+1}(\eta',k). \tag{200}$$

It satisfies the scalar two-point function equation of motion (14), which in turn guarantees that conditions (187) are satisfied. Analogous results for the Feynman propagator are derived in the same manner,

$$(\nabla^\mu - 2\zeta n^\mu)i[^{+}_{\mu}\Delta^{+}_{\nu}](x;x') = -\xi\partial'_\nu\left\{\left(\frac{a'}{a}\right)^\zeta i[^{+}\Delta^{+}]_{\lambda+1}(x;x')\right\}, \tag{201}$$

$$\begin{aligned}(2g^{ij}\delta^{\mu}_{[i}\partial_{0]}\partial_j)&i[^{+}_{\mu}\Delta^{+}_{\nu}](x;x') \\ &= -(\partial_0 + 2\zeta\mathcal{H})\partial'_\nu\left\{\left(\frac{a'}{a}\right)^\zeta i[^{+}\Delta^{+}]_{\lambda+1}(x;x')\right\} \\ &\quad + \delta^0_\nu\frac{i\delta^D(x-x')}{a^{D-2}},\end{aligned} \tag{202}$$

and the scalar Feynman propagator satisfies (17), which guarantees conditions (188) hold. Expressions (198) and (199) for the Wightman function and (201) and (202) for the Feynman propagator are the promised single-derivative subsidiary conditions.

This is the point where the formalism outlined in this work makes connection with the Becchi-Rouet-Stora-Tyutin (BRST) quantization and Faddeev-Popov (FP) ghosts. On the right-hand sides of the expressions for the single derivative conditions above we can recognize the FP ghost two-point function

$$\langle\bar{c}(x)c(x')\rangle = \left(\frac{a'}{a}\right)^{\zeta} i[^{-}\Delta^{+}]_{\lambda+1}(x;x') \tag{203}$$

that satisfies the equation of motion

$$(\nabla'^{\mu} - 2\zeta n'^{\mu})\nabla'_{\mu}\langle\bar{c}(x)c(x')\rangle = 0, \tag{204}$$

such that the single derivative subsidiary condition (198) takes the form of the Ward-Takahashi identity,

$$(\nabla^{\mu} - 2\zeta n^{\mu}) i[^{-}_{\mu}\Delta^{+}_{\nu}](x;x') = -\xi\partial'_{\nu}\langle\bar{c}(x)c(x')\rangle. \tag{205}$$

This is precisely the condition that descends from the BRST quantization. In addition to the gauge-fixing action functional (1), there one introduces the accompanying FP ghost action for Grassmanian fields c and $\bar{c}$,

$$S_{\rm gh}[\bar{c},c] = \int d^D x\sqrt{-g}(\nabla^{\mu}\bar{c} + 2\zeta n^{\mu}\bar{c})(\nabla_{\mu}c), \tag{206}$$

such that the total action is invariant under global BRST transformations,

$$A_\mu \to A_\mu + \theta\xi\partial_\mu c, \qquad \bar{c} \to \bar{c} - \theta(\nabla^{\mu} - 2\zeta n^{\mu})A_\mu, \\ c \to c, \tag{207}$$

parametrized by an infinitesimal Grassmanian parameter θ. This implies a conserved BRST charge generating the transformation. The physical states in BRST quantization are required to be invariant under the action of this charge, which yields the Ward-Takahashi identity (205) as a consequence. It is Eq. (204) that guarantees the single-derivative subsidiary conditions (198) and (199) are consistent with the vanishing of the double-derivative subsidiary conditions (187a)–(187d). Thus the correlators of Hermitian constraints vanish, and the correspondence principle is respected.

B. Mode sum representation

The components of the photon two-point functions can be expressed as integrals over modes of products of mode functions. This is accomplished by using conditions imposed on the state both in the physical sector (158) and in the gauge sector in (102) and (174), and the momentum space representation of the field operators (87),

$$i[^{-}_{0}\Delta^{+}_{0}](x;x') = (aa')^{-\frac{D-2-2\zeta}{2}} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} [-\mathscr{U}_{\lambda}(\eta,k)\mathscr{V}^{*}_{0}(\eta',k) - \mathscr{V}_{0}(\eta,k)\mathscr{U}^{*}_{\lambda}(\eta',k)], \tag{208}$$

$$i[^{-}_{0}\Delta^{+}_{i}](x;x') = (aa')^{-\frac{D-2-2\zeta}{2}} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \frac{k_i}{k} [\mathscr{U}_{\lambda}(\eta,k)\mathscr{V}^{*}_{L}(\eta',k) + \mathscr{V}_{0}(\eta,k)\mathscr{U}^{*}_{\lambda+1}(\eta',k)], \tag{209}$$

$$\begin{aligned} i[^{-}_{i}\Delta^{+}_{j}](x;x') = {} & (aa')^{-\frac{D-4}{2}} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \mathscr{U}_{\lambda_T+1}(\eta,k)\mathscr{U}^{*}_{\lambda_T+1}(\eta',k) \\ & - (aa')^{-\frac{D-2-2\zeta}{2}} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')} \frac{k_i k_j}{k^2} [\mathscr{U}_{\lambda+1}(\eta,k)\mathscr{V}^{*}_{L}(\eta',k) + \mathscr{V}_{L}(\eta,k)\mathscr{U}^{*}_{\lambda+1}(\eta',k)], \end{aligned} \tag{210}$$

where we introduced the following shorthand notation for scalar mode functions,

$$\mathscr{U}_{\lambda_T+1}(\eta,k) = e^{i\varphi_T(k)}{\rm ch}[\rho_T(k)]\mathcal{U}_{\lambda_T+1}(\eta,k) - e^{-i\varphi_T(k)}{\rm sh}[\rho_T(k)]\mathcal{U}^{*}_{\lambda_T+1}(\eta,k), \tag{211}$$

$$\mathscr{V}_{0}(\eta,k) = e^{i\varphi(k)}{\rm ch}[\rho(k)]v_0(\eta,k) - e^{-i\varphi(k)}{\rm sh}[\rho(k)]v^{*}_{0}(\eta,k), \tag{212}$$

$$\mathscr{V}_{L}(\eta,k) = e^{i\varphi(k)}{\rm ch}[\rho(k)]v_L(\eta,k) + e^{-i\varphi(k)}{\rm sh}[\rho(k)]v^{*}_{L}(\eta,k), \tag{213}$$

in addition to the ones already defined in (195) and (196). The Wightman function constructed this way is guaranteed to satisfy both the equations of motion (184) *and* the appropriate subsidiary conditions (187). The Feynman propagator follows from the Wightman function simply from the definition (182) and satisfies the equations of motion (185) and subsidiary conditions (188). This guarantees that the perturbation theory based on these two-point functions will yield correct results.

VIII. SIMPLE OBSERVABLES

The two-point functions computed according to the preceding section can be used to compute quantum loop corrections to various observables in spatially flat cosmological spaces. However, in addition to working out the two-point functions, we also have to address the question of ordering products of field operators comprising the observables. This is very much related to the fact that observables have to be independent of the gauge-fixing parameter ξ. To elucidate this point, in this section we consider two simple observables: the tree-level field strength correlator and the one-loop energy-momentum tensor.

A. Field strength correlators

The simplest observable one can think of is the tree-level off-coincident field strength correlator,

$$\langle\Omega|\hat{F}_{\mu\nu}(x)\hat{F}_{\rho\sigma}(x')|\Omega\rangle = 4(\delta^{\alpha}_{[\mu}\partial_{\nu]})(\delta^{\beta}_{[\rho}\partial'_{\sigma]})i[{}^{-}_{\alpha}\Delta^{+}_{\beta}](x;x'), \tag{214}$$

expressed in terms of derivatives acting on the vector potential two-point function. The field strength tensor is an observable, as it fully commutes with the Hermitian constraints, and thus also with the non-Hermitian constraint. This is obvious if we write it in terms of canonical variables of the transverse and scalar sectors,

$$\hat{F}_{0i} = a^{4-D}\hat{\Pi}^{T}_{i} + a^{4-D}\hat{\Pi}^{L}_{i}, \qquad \hat{F}_{ij} = 2\partial_{[i}\hat{A}^{T}_{j]}. \tag{215}$$

This also makes it clear that the correlator (214) receives contributions from the transverse sector only, and that any gauge-dependence drops out,

$$\langle\Omega|\hat{F}_{0i}(x)\hat{F}_{0j}(x')|\Omega\rangle = \partial_0\partial'_0\langle\Omega|\hat{A}^{T}_{i}(x)\hat{A}^{T}_{j}(x')|\Omega\rangle, \tag{216}$$

$$\langle\Omega|\hat{F}_{0i}(x)\hat{F}_{kl}(x')|\Omega\rangle = 2\partial_0(\delta^{n}_{[l}\partial'_{k]})\langle\Omega|\hat{A}^{T}_{i}(x)\hat{A}^{T}_{n}(x')|\Omega\rangle, \tag{217}$$

$$\langle\Omega|\hat{F}_{ij}(x)\hat{F}_{kl}(x')|\Omega\rangle = 4(\delta^{m}_{[j}\partial_{i]})(\delta^{n}_{[l}\partial'_{k]})\langle\Omega|\hat{A}^{T}_{m}(x)\hat{A}^{T}_{n}(x')|\Omega\rangle. \tag{218}$$

The specific forms that these correlators take depend on the particular FLRW spacetime, and on the state of the transverse modes that is chosen, i.e. on the free coefficients chosen in (159). In the four-dimensional limit the transverse photons are conformally coupled, and there exists a conformal vacuum state that has to reproduce the flat space correlators,

$$\langle\Omega|\hat{F}_{\mu\nu}(x)\hat{F}_{\rho\sigma}(x')|\Omega\rangle \overset{D\to4}{\longrightarrow} \frac{2}{\pi^2(\Delta x^2)^2} \times\left[\eta_{\mu[\rho}\eta_{\sigma]\nu} - 4\eta_{\alpha[\mu}\eta_{\nu][\sigma}\eta_{\rho]\beta}\frac{\Delta x^{\alpha}\Delta x^{\beta}}{\Delta x^2}\right]. \tag{219}$$

This is a simple check that any photon two-point function of a physically conformal vacuum state in FLRW space must satisfy.

B. Energy-momentum tensor

The energy-momentum tensor is perhaps the simplest one-loop observable, composed of a single photon two-point function. There is an ambiguity, even at the classical level, in how we define even the observable, which consequently appears in the quantized theory as well. This ambiguity, however, vanishes on-shell both in the classical and in the quantum cases. Most of this subsection is devoted to the discussion of how to properly define the quantum energy-momentum tensor.

1. Classical energy-momentum tensor

Two sensible definitions for the energy-momentum tensor of the photon field are possible. It can be defined either as a variation of the gauge-invariant action (18),

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \left(\delta^{\rho}_{\mu}\delta^{\sigma}_{\nu} - \frac{1}{4}g_{\mu\nu}g^{\rho\sigma}\right)g^{\alpha\beta}F_{\rho\alpha}F_{\sigma\beta}, \tag{220}$$

or as a variation of the gauge-fixed action (54),

$$T^{\star}_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S_{\star}}{\delta g^{\mu\nu}} = T_{\mu\nu} + T^{\rm gf}_{\mu\nu}, \tag{221}$$

which, in addition to the gauge-invariant part, contains an extra contribution from the gauge-fixing term (1),

$$\begin{aligned} T^{\rm gf}_{\mu\nu} &= \frac{-2}{\sqrt{-g}}\frac{\delta S_{\rm gf}}{\delta g^{\mu\nu}} \\ &= -\frac{2}{\xi}(A_{(\mu}\nabla_{\nu)} + 2\zeta n_{(\mu}A_{\nu)})(\nabla^{\rho}A_{\rho} - 2\zeta n^{\rho}A_{\rho}) \\ &\quad + \frac{g_{\mu\nu}}{\xi}\bigg[(A_{\rho}\nabla^{\rho} + 2\zeta n^{\rho}A_{\rho})(\nabla^{\sigma}A_{\sigma} - 2\zeta n^{\sigma}A_{\sigma}) \\ &\quad + \frac{1}{2}(\nabla^{\rho}A_{\rho} - 2\zeta n^{\rho}A_{\rho})^2\bigg]. \end{aligned} \tag{222}$$

Both definitions give energy-momentum tensors conserved on-shell, but for the first definition (220) we have to use the constraint equations (44), while the conservation for the second definition (222) relies on the gauge-fixed dynamical equations (46)–(49) only. At the classical level the two definitions give the same answer on-shell. This is best seen by expressing the gauge-fixing contribution in terms of canonical variables,

$$T^{\rm gf}_{00} = -a^{2-D}\left[A_0\Psi_2 + A_k\partial_k\Psi_1 + \frac{\xi}{2}a^{4-D}\Psi_1^2\right], \tag{223a}$$

$$T^{\rm gf}_{0i} = -a^{2-D}[A_0\partial_i\Psi_1 + A_i\Psi_2], \tag{223b}$$

$$T^{\rm gf}_{ij} = -a^{2-D}\left[2A_{(i}\partial_{j)}\Psi_1 + \delta_{ij}\left(A_0\Psi_2 - A_k\partial_k\Psi_1 - \frac{\xi}{2}a^{4-D}\Psi_1^2\right)\right], \tag{223c}$$

and noting that every term contains at least one of the first-class constraints (37). In fact, it is only the transverse modes that contribute to the energy-momentum tensor on-shell. This is clear since the only contributing part is the gauge-invariant one (220) that is composed out of field strength tensors only, which contain only transverse fields and constraints, as discussed in Sec. VIII A. The same properties of the energy-momentum tensor are maintained in the quantized theory if attention is paid to operator ordering.

2. Quantum energy-momentum tensor

When defining operators associated with quantum observables attention needs to be paid to the ordering of products of field operators. Usually Weyl ordering of field operators is employed. However, this is not fully satisfactory in gauge theories as, in general, it does not respect the correspondence principle. For the energy-momentum tensor this is the question of the contribution of the gauge-fixing part (222). In the classical theory this contribution vanishes, and it is sensible to demand the same property in the quantized theory. This is accomplished by correct operator ordering. Here we discuss the quantization of the two parts of the definition (220) and (222) separately.

Gauge-invariant part. For the gauge-invariant part (220) defining an operator is straightforward, since when expressed in terms of the canonical fields all the terms are composed either solely of transverse fields or solely of constraints. Therefore, we may define the operator to be Weyl-ordered, and the expectation value essentially reduces to the coincident limit of the field strength correlator,

$$\langle\Omega|\hat{T}_{\mu\nu}(x)|\Omega\rangle = \left(\delta^\rho_\mu\delta^\sigma_\nu - \frac{1}{4}g_{\mu\nu}g^{\rho\sigma}\right)g^{\alpha\beta} \times\frac{1}{2}\langle\Omega|\{\hat{F}_{\rho\alpha}(x),\hat{F}_{\sigma\beta}(x)\}|\Omega\rangle. \tag{224}$$

If the off-coincident field strength correlator (214) is computed in D dimensions we can take a dimensionally regulated coincidence limit required above. The precise value of (224) depends on the transverse photon state and the particular FLRW background.

Gauge-fixing part. The gauge-fixing contribution (222) to the energy-momentum tensor contains constraints, as evident from (223). Therefore, the operator associated with the gauge-fixing part of the energy-momentum tensor has to be ordered properly, as explained in Sec. IV C. The Hermitian constraints have to be split into parts containing the non-Hermitian subsidiary constraint $\hat{K}$ and parts containing its conjugate $\hat{K}^\dagger$, and the former has to be put to the right of the product and the latter to the left. This is accomplished by making use of decompositions in (73),

$$\hat{T}^{\rm gf}_{00} \equiv [\hat{T}^{\rm gf}_{00}]_g = -a^{2-D}\left[(\hat{K}_2^\dagger\hat{A}_0 + \partial_k\hat{K}_1^\dagger\hat{A}_k) + (\hat{A}_0\hat{K}_2 + \hat{A}_k\partial_k\hat{K}_1) + \frac{\xi}{2}a^{4-D}\hat{\Psi}_1^2\right], \tag{225a}$$

$$\hat{T}^{\rm gf}_{0i} \equiv [\hat{T}^{\rm gf}_{0i}]_g = -a^{2-D}\left[(\partial_i\hat{K}_1^\dagger\hat{A}_0 + \hat{K}_2^\dagger\hat{A}_i) + (\hat{A}_0\partial_i\hat{K}_1 + \hat{A}_i\hat{K}_2)\right], \tag{225b}$$

$$\hat{T}^{\rm gf}_{ij} \equiv [\hat{T}^{\rm gf}_{ij}]_g = -a^{2-D}\left[2\partial_{(i}\hat{K}_1^\dagger\hat{A}_{j)} + \delta_{ij}(\hat{K}_2^\dagger\hat{A}_0 - \partial_k\hat{K}_1^\dagger\hat{A}_k) + 2\hat{A}_{(i}\partial_{j)}\hat{K}_1 + \delta_{ij}(\hat{A}_0\hat{K}_2 - \hat{A}_k\partial_k\hat{K}_1) - \frac{\xi}{2}\delta_{ij}a^{4-D}\hat{\Psi}_1^2\right]. \tag{225c}$$

Note that for terms composed solely of constraints this ordering is immaterial since the non-Hermitian subsidiary constraint and its conjugate commute. Defined this way it is manifest that the expectation value always vanishes,

$$\langle\Omega|\hat{T}^{\rm gf}_{\mu\nu}(x)|\Omega\rangle = 0, \tag{226}$$

as it should, without any additional arguments.

Even though the operator ordering in (225) leads to correct and consistent results, it is rather unwieldy to use it in practice. In general Weyl-ordered products are far more convenient to use. We can indeed commute the operators in products of (222) to the Weyl-ordered form, but this leaves nonvanishing commutators accounting for the difference between two ordering prescriptions,

$$[\hat{T}^{\rm gf}_{00}]_g = [\hat{T}^{\rm gf}_{00}]_{\rm W} + a^{2-D}\,{\rm Re}([\hat{K}_2,\hat{A}_0] + [\partial_k\hat{K}_1,\hat{A}_k]), \tag{227a}$$

$$[\hat{T}^{\rm gf}_{0i}]_g = [\hat{T}^{\rm gf}_{0i}]_{\rm W} + a^{2-D}\,{\rm Re}([\partial_i\hat{K}_1,\hat{A}_0] + [\hat{K}_2,\hat{A}_i]), \tag{227b}$$

$$[\hat{T}^{\rm gf}_{ij}]_g = [\hat{T}^{\rm gf}_{ij}]_{\rm W} + a^{2-D}\,{\rm Re}(2[\partial_{(i}\hat{K}_1,\hat{A}_{j)}] + \delta_{ij}[\hat{K}_2,\hat{A}_0] - \delta_{ij}[\partial_k\hat{K}_1,\hat{A}_k]). \tag{227c}$$

The Weyl-ordered parts above can easily be defined from the covariant expression (222) directly, by simply symmetrizing the products of Hermitian operators,

$$[T^{\rm gf}_{\mu\nu}]_{\rm W} = -\frac{1}{\xi}\{\hat{A}_{(\mu},(\nabla_{\nu)} + 2\zeta n_{\nu)})(\nabla^\rho\hat{A}_\rho - 2\zeta n^\rho\hat{A}_\rho)\} + \frac{g_{\mu\nu}}{2\xi}\left[\{\hat{A}_\rho,(\nabla^\rho + 2\zeta n^\rho)(\nabla^\sigma\hat{A}_\sigma - 2\zeta n^\sigma\hat{A}_\sigma)\} + (\nabla^\rho\hat{A}_\rho - 2\zeta n^\rho\hat{A}_\rho)^2\right]. \tag{228}$$

The nonvanishing commutators in (227) that are just c-numbers can be evaluated as coincidence limits of position space commutators using (191)–(199). This leads to the following relation between the properly gauge-ordered gauge-fixing contribution to the energy-momentum tensor and the Weyl-ordered contribution,

$$[\hat{T}^{\rm gf}_{\mu\nu}]_g = [\hat{T}^{\rm gf}_{\mu\nu}]_{\rm W} - {\rm Re}\bigg\{\Big[2(\nabla_{(\mu}\nabla'_{\nu)} + 2\zeta n_{(\mu}\nabla'_{\nu)}) - g_{\mu\nu}(\nabla^\rho\nabla'_\rho + 2\zeta n^\rho\nabla'_\rho)\Big] \times \bigg[\left(\frac{a'}{a}\right)^\zeta i[{}^-\Delta^+](x;x')\bigg]_{x'\to x}\bigg\}. \tag{229}$$

The result in (226) is guaranteed only if we take the left-hand side of the expression above as the definition, or if we take the full right-hand side. This shows how the Weyl-ordered product is incomplete when defining the observable, as it needs to be supplemented by the contribution in the second line of (229). This c-number contribution can, in fact, be recognized as the FP ghost contribution to the energy-momentum tensor that descends from the FP ghost action (206). Thus, we see that even in the linear Abelian gauge theory, and even without being explicitly introduced, the FP ghosts naturally arise as commutators accounting for the difference between proper operator ordering of operators and Weyl ordering of operators. Either form of the definition we adopt, (225) or (229), leads to the vanishing expectation value (226). Therefore, one has two options: either gauge-order operators containing constraints or Weyl-order all the products *and* add the FP ghost contributions.

Had we ignored the question of operator ordering and defined (228) as the observable, its expectation value would no longer vanish identically. This is what is behind the conclusion in [37,79,80] that the gauge-fixing contribution to the energy-momentum tensor of the photon engenders a nonvanishing cosmological constant contribution. This conclusion, however, does not hold up. Reference [49] attempted to address the question of the vanishing gauge-fixing contribution to the energy-momentum tensor in de Sitter space by considering Weyl ordering of operators, but without introducing FP ghosts and instead employing adiabatic subtraction to obtain (226). That approach cannot be correct, as it suggests that gauge-independence has something to do with the divergent UV structure of the theory, and it leaves the option for the gauge-fixing part of the energy-momentum tensor to produce a physical contribution in some spacetime other than de Sitter. Moreover, their conclusion that FP ghosts are not necessary when operators are Weyl ordered contradicts the results of this section, as well as contradicting consistent analogs for Stueckelberg vector fields [81,82].

IX. DISCUSSION

In this work we considered the canonical quantization of the photon (massless vector field) in spatially flat FLRW spacetimes in the two-parameter family of linear gauges (1). We used this framework to demonstrate that observables with appropriate operator ordering respect expectations set by the correspondence principle. In particular, we had demonstrated how the gauge-fixing term (1) does not contribute to the energy-momentum tensor expectation value for any physical state.

Along the way we had elucidated how the canonical formulation is the appropriate framework for the Gupta-Bleuler quantization, and we had derived the subsidiary condition on the physical state from the first-class constraint structure of the classical theory. This subsidiary condition translates into subsidiary conditions on two-point functions derived in Sec. VII A. The form of these subsidiary conditions depends on the two gauge-fixing parameters ξ and ζ from (1), as in fact does the two-point function itself. Two-point functions with free gauge-fixing parameters are particularly useful for computations, as they allow for explicit checks of gauge independence of observables that should not depend on the gauge-fixing parameters. The construction of inflationary gauge-independent quantum observables is still in its early stages [83].

Different two-point functions considered in Sec. VII are basic building blocks of nonequilibrium loop computations in the Schwinger-Keldysh formalism appropriate for early universe cosmology. We believe that the framework set up in this paper will facilitate the construction of photon propagators in realistic inflationary spacetimes that will in turn allow the investigation of slow-roll corrections to large infrared effects found when photons interact with spectator scalars and gravitons in de Sitter [13–35]. We had expressed the photon two-point function in Sec. VII B in terms of several scalar mode functions. The equations of motion that these scalar mode functions satisfy are collected in Sec. V C. In that form it should be considerably easier to identify convenient choices for gauge-fixing parameters that lead to simpler propagators for given inflationary backgrounds. Constructing such two-point functions will be the subject of future work [84]. Furthermore, collecting subsidiary conditions for two-point functions in Sec. VII A, in both the double derivative and the single derivative forms, will facilitate consistency checks of photon two-point functions constructed in future studies.

Before embarking to compute the photon propagators in power-law and slow-roll inflation it is instructive to check the existing two-point functions in de Sitter space ($\epsilon = 0$) from the literature versus subsidiary conditions for the Wightman function (187) and for the Feynman propagator (188). The checks are summarized in Table I. Unexpectedly, they reveal that the general covariant gauge two-point functions satisfy all the required subsidiary conditions only

TABLE I. Checks of subsidiary conditions for massless photon two-point functions in de Sitter reported in the literature. The checks refer to both the Wightman functions and Feynman propagators. The first four entries with $\zeta = 0$ correspond to covariant gauges, while the fifth entry with $\zeta = 1$ corresponds to a conformal gauge in $D = 4$.

de Sitter propagator	$\langle \hat{\Pi}_0 \hat{\Pi}_0 \rangle$	$\langle \hat{\Pi}_0 \partial_i \hat{\Pi}_i \rangle$	$\langle \partial_i \hat{\Pi}_i \partial_j \hat{\Pi}_j \rangle$
Allen-Jacobson [39] ($\xi = 1, \zeta = 0, D$)	✗	✓	✓
Tsamis-Woodard [40] ($\xi = 0, \zeta = 0, D$)	✓	✓	✓
Youssef [41] ($\xi, \zeta = 0, D = 4$)	✗	✓	✓
Fröb-Higuchi [42] ($\xi, \zeta = 0, D$)	✗	✓	✓
Woodard [38] ($\xi = 1, \zeta = 1, D$)	✓	✓	✓

in the exact transverse gauge limit $\xi \to 0$. For nonvanishing ξ the double divergence of the covariant gauge two-point functions fails to vanish off-coincidence,

$$\nabla^\mu \nabla'^\nu i[{}_\mu^- \Delta_\nu^+](x;x') = -\frac{\xi H^D \Gamma(D)}{(4\pi)^{\frac{D}{2}} \Gamma(\frac{D}{2})} = \nabla^\mu \nabla'^\nu i[{}_\mu^+ \Delta_\nu^+](x;x'), \quad x \neq x', \tag{230}$$

violating subsidiary conditions (187) and (188). This points to inconsistencies of known results even for the relatively simple case of the maximally symmetric de Sitter space. These inconsistencies will be examined in more detail and addressed elsewhere [60,61].

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