Implications of the weak gravity conjecture for de Sitter space decay by flux discharge

Nemanja Kaloper[1,*] and Alexander Westphal[2,†]

[1]QMAP, Department of Physics and Astronomy, University of California Davis, California 95616, USA
[2]Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, 22607 Hamburg, Germany

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We examine the implications of the weak gravity conjecture for the mechanisms for discharging cosmological constant via membrane nucleations. Once screening fluxes and membranes which source them enter, and weak gravity bounds are enforced, a generic de Sitter space must be unstable. We show that when all the flux terms which screen and discharge the cosmological constant are dominated by quadratic and higher order terms, the bounds from weak gravity conjecture and naturalness lead toward anthropic outcomes. In contrast, when the flux sectors are dominated by linear flux terms, anthropics may be avoided, and the cosmological constant may naturally decay toward smallest possible values.

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I. INTRODUCTION

In the recent sequence of papers [1–5], we have initiated a novel program for addressing the cosmological constant problem and de Sitter space decay by using the discharge of 4-form fluxes which screen and then relax the total cosmological constant. As a result de Sitter space in this approach is intrinsically unstable. It decays into a fractal-like patchwork of regions of ever smaller constant curvature. In its simplest incarnation, our approach is a generalization of the idea pioneered in [6,7] about discharging a cosmological constant by nonperturbative membrane nucleation processes, and it also benefits from results of [8–12].

Let us briefly summarize the main features of the "mechanics" of flux discharge here. Regardless of the details of the flux sector, when the cosmological constant is very large, larger than a certain scale Λ_* set by the membrane quantum numbers, the discharge proceeds by the nucleation of membranes whose size is comparable to the environment curvature radius—i.e., the horizon size. During this "boiling" stage, the discharge is essentially unsuppressed. One could try a quick estimate $S_{\mathrm{bounce}} \simeq -\frac{12\pi^2 M_{\mathrm{Pl}}^4 \Delta\Lambda}{\Lambda_{\mathrm{out}}\Lambda_{\mathrm{in}}}$, which is $\mathcal{O}(1)$ at the cutoff [1–3] and resembles the Hawking-Moss instanton action [13], which supports that nucleation rates are not suppressed. A more incisive analysis of S_{bounce} actually shows that $\Lambda \to \infty$ and $\Lambda = 0$ are branch points of S_{bounce} as opposed to poles, and so the limits are more delicate. We will show here that when $\Lambda \to \infty$, $S_{\mathrm{bounce}} \to 0$, and so in this regime, the decay rate is

$$\Gamma \to 1, \tag{1}$$

which indicates barrierless tunneling in the Euclidean theory. This could happen in the limits of the well-known analyses of [14,15], for very fast bubble nucleations [16–18]. In the thin wall limit of tunneling between a false and a true vacuum for a scalar field, the wall tension measures the barrier area, which controls the tunneling rate. Hence, when the cosmological constant dominates over the tension terms, the barrier is negligible, which yields an unsuppressed nucleation rate.

Even more accurately, the rapid decay during this stage is moderated by a small bounce action, which eventually gradually increases toward $S_{\mathrm{bounce}} \simeq +\frac{24\pi^2 M_{\mathrm{Pl}}^4}{\Lambda_{\mathrm{out}}}$ as the initial cosmological constant decreases. The resulting decay rate disfavors the largest possible values of the cosmological constant and favors the smallest ones as the terminal outcome, because the more curved backgrounds are more unstable. To be relevant, this regime must involve the cosmological constant values below the cutoff; otherwise, it is excluded from the effective theory description. When this holds, the cosmological constant will discharge to the smallest values achievable, for as long as the rate remains nonzero. Due to the increase of the bounce action with the decrease of Λ, the resulting distribution of cosmological constant values is skewed toward the smallest possible

*kaloper@physics.ucdavis.edu
†alexander.westphal@desy.de

values that can be reached given a set of membrane charges. This stage is important for populating the de Sitter landscape in ways that reach the smallest values of the terminal cosmological constant.

Once Λ decreases to below the critical scale, the discharge rate dramatically slows down. There are important quantitative differences between the dynamics depending on whether discharge processes are controlled by linear 4-form flux terms, or by quadratic and higher power ones. If linear flux terms dominate discharge processes, as in [1–5], the discharge channels shut off faster, since the rates strongly depend on the initial cosmological constant. As noted above, for these processes the decay rate asymptotes toward an essential singularity as $\Lambda \to 0^+$,

$$\Gamma \to \exp\left(-\frac{24\pi^2 M_{\rm Pl}^4}{\Lambda_{\rm out}}\right), \tag{2}$$

where $M_{\rm Pl}^2 = 1/8\pi G_N$. This "braking" stage protects the tiniest values of cosmological constant by making those geometries most stable. Together with the faster discharge, the "boiling" regime preceding it, the overall dynamics favors de Sitter spaces with the smallest attainable cosmological constant [1–5]. This occurs for all generic, natural values of the initial cosmological constant near and below the cutoff of the theory, at a scale $\mathcal{M}_{\rm UV} \le M_{\rm Pl}$.

This is in contrast to previous works [6,7,9–12], where the flux sector does not include linear 4-form terms but starts with F^2. Then, whenever the initial cosmological constant is not fine tuned, but starts near the cutoff value $\Lambda \sim \mathcal{M}_{\rm UV}^4 \lesssim M_{\rm Pl}^4$, the terms that control the transitions select decay channels which have a broad regime with rates without accumulation points, which asymptote to a constant value that depends on the *difference* of the cosmological constants in the parent and descendant bubbles, and not the individual values, so

$$\Gamma \simeq \exp\left(-\frac{27\pi^2}{2}\frac{\mathcal{T}^4}{(\Delta\Lambda)^3}\right) \to \exp\left(-\frac{27\pi^2}{2}\frac{\mathcal{T}^4}{(2\Lambda_{\rm QFT})^{3/2}\mathcal{Q}^3}\right). \tag{3}$$

Here, $\Lambda_{\rm QFT}$ is the cosmological constant from the field theory sector, which is being neutralized, and $\mathcal{T}$ and $\mathcal{Q}$ membrane tension and charge, respectively. In such cases, one finds that the terminal distribution of cosmological constant values can be uniform if the phase space of possible values were equiprobable, and if the preceding "boiling" stage was not long (or completely excluded). In this case, one can select the final value by resorting to the anthropic principle.

The "boundary" between the "boiling" and "braking" stages is controlled by the ratio of the cosmological constant before a transition and, in general, the tension and charge of the membrane, in the units of Planck scale. The precise value of the critical value Λ_*, where the transition happens, is detail dependent (and actually can be quite broad), and we will outline the possibilities below. We stress this presupposes that both the "boiling" and "braking" stages are below the cutoff, within the realm of effective theory. This is not automatic for the "boiling" stage.

This argument indicates that together, naturalness in the QFT sense and any constraints on membrane charges and tensions relative to the cutoff impose restrictions on the cosmological constant cancellation via screening and membrane discharge. Staying below the cutoff is not only an issue of reliability but of caution born of necessity: above the cutoff lurks the wormhole regime, which is notoriously unreliable [19–22]. So the "brute force" tuning and retuning of parameters in the equations which arise in the semiclassical limit may not be arbitrarily done to evade those restrictions. In this paper, we explore these restrictions and the model-building requirements they impose. Specifically, we deploy the bounds on the membrane dynamics that arise from the weak gravity conjecture (WGC) [23] and apply them to the generic natural boroughs of the landscape, where the field theory contribution to the vacuum energy is technically natural, of an order $\Lambda_{\rm QFT} \lesssim \mathcal{M}_{\rm UV}^4$.

It should be immediately clear that the WGC bounds affect the nature of de Sitter space fundamentally once we introduce the flux screening and discharge mechanisms. Once the cosmological constant receives the contributions from fluxes, and charged tensional membranes are included, so that fluxes can change by membrane nucleation, the WGC bounds immediately imply that there is no absolutely stable de Sitter space. The reason is simple: to stop the quantum mechanically induced discharges, membranes must be decoupled, which means, the tension of all the membranes that can change the cosmological constant must go to, formally, infinity—or in practice, above the cutoff. This is precisely the limit prohibited by WGC [23]. Even if we do not violate WGC, this limit indicates that the WGC bounds, which constrain the ratios of charges, tensions, and the cutoff, will affect the specific details of discharge dynamics.

We find that when the 4-form sector is dominated by quadratic and higher powers of fluxes, the natural discharges by membranes that obey the WGC are mediated by the same types of instantons as in [6,7], which admit a broad regime with the asymptotic decay rate given by (3). The "boiling" regime is pushed out of the range of the effective theory, and the initial vacuum energy to be canceled, near the QFT cutoff, is already at the edge of the "braking" regime. Such setups can therefore be naturally used to provide a framework for anthropic selection of the terminal cosmological constant, as in [11].

If we chose to violate WGC for *all* charged membranes in the theory, the low scale attractor (2) will appear, and the "boiling" stage may reappear below the cutoff. However, the WGC violations required to make the setup natural are quite severe because the charges of affected membranes become very small, and restoring WGC by adding membranes that satisfy the bounds can also restore channels for dominantly uniform discharge, that bring back anthropics, or force more fine tuning.

In contrast, when the linear flux terms are present, and when they dominate over higher powers in the effective action, the dynamics change significantly [1–5]. When the fluxes are natural, of the order of the cutoff-scale QFT vacuum energy $\Lambda_{\rm QFT} \simeq \mathcal{M}_{\rm UV}^4$, and they satisfy WGC, both the "boiling" and "braking" stages are below the cutoff, and they, together, favor the nucleation of a sequence of bubbles ending with the smallest possible terminal cosmological constant. Hence, in this case, the universes with tiny cosmological constant could arise naturally, without invoking anthropic reasoning. We also note, that even if some of the membranes involved are discharged by processes dominated by quadratic flux terms, the outcome may remain unaffected as long as there are many decay channels dominated by linear flux terms. A more detailed investigation of the evolution, which involves a variety of decay channels, of both types mentioned here and with a range of charges and tensions, is therefore warranted.

The paper is organized as follows: in the next section, we review WGC, starting with point particles and then provide a general set of inequalities for charged membranes in four dimensions. Next, in Sec. III. we discuss the effective action for fluxes and charged tensional membranes using magnetic duals of 4-forms, and revisit the mechanics of flux discharges of [1–5]. In Sec. IV, we turn to the implications of WGC and naturalness for flux discharge processes and explain the limitations, which arise for effective theory description of de Sitter decay. We give a summary of the results and discuss open questions in the last section.

II. WGC IN A NUTSHELL: A LIGHTNING REVIEW

If objects supporting event horizons were really forever, the retrieval of information about the material that went into them may be impossible. In quantum physics, this suggests that event horizons may catalyze unitarity loss, and hence, endanger and obstruct quantum mechanics itself [24]. Preempting this implies subtle consistency conditions on the models of matter coupled to quantum gravity. A specific application concerns charged black holes. As is well known, generic black holes are actually not black since they radiate like black bodies at Hawking temperature. To ensure that they radiate out the charge that went in with the material that formed a black hole, it is necessary that there are sufficiently light charged particles that can stream outside. This imposes a condition on charge per unit mass [23], which is now called the electric WGC: for each conserved gauge charge, there must be a sufficiently light charge carrier, such that

$$\frac{e}{m} \geq \sqrt{G_N}, \tag{4}$$

where e and m are the carrier charge and mass. This can be deduced very simply from conservation laws [25,26]: consider a black hole of mass M with charge Q, where by conservation of mass and charge $M \geq \sum_i m_i$ (as we allow for energy contribution from neutral sources) and $Q = \sum_i e_i$. Thus,

$$\frac{M}{Q} = \frac{1}{Q}\sum m_i = \frac{1}{Q}\sum \frac{m_i}{e_i} e_i \geq \frac{1}{Q}\left(\frac{m}{e}\right)_{\rm min} \sum e_i = \left(\frac{m}{e}\right)_{\rm min}. \tag{5}$$

Applying this to extremal black holes $M = Q/\sqrt{G_N}$, which have the largest ratio Q/M due to the horizon regularity constraints yields the strongest bound: Eq. (4), precisely.

However, many charged black holes can become ultracold in the extremal limit and cease to emit Hawking radiation. If they were to last forever, they would cause problems behaving as troublesome remnants [27]. Yet even if Hawking radiation ceases, there are nonperturbative, nonthermal processes, which lend to black hole discharge [28]. Essentially, these are variants of Schwinger charged particle production in background electric fields [29]. This decay channel arises thanks to the Heisenberg uncertainty principle, whereby a particle-antiparticle pair emerges in an external electric field E. The field accelerates virtual particles in the pair away from each other and transfers enough energy to them that they can get on shell instead of annihilating away.

A very nice intuitive argument is given in [30], building on the work of [31], which we revisit here. We will model the pair creation and their separation due to the work of the background field as the process of initially accelerating a negative energy "virtual" particle, which gains enough energy due to the acceleration to become a positive energy particle that propagates away, leaving behind a "hole"—a positive energy antiparticle after charge conjugation—that propagates away in the opposite direction. Working in the rest frame of one of the pair, which is also initially the rest frame of the pair, the dispersion relation after a small displacement δz is $(\varepsilon + eE\delta z)^2 - \vec{p}^2 = m^2$. Solving for p_z, with the initial condition $\varepsilon = -m$ at $\delta z = 0$ (recall that $c = \hbar = k_B = 1$),

$$p_z = \sqrt{(-m + eE\delta z)^2 - m^2} = \sqrt{eE\delta z(eE\delta z - 2m)}. \tag{6}$$

Clearly, the square root vanishes at $\delta z = 0$ and $\delta z = 2m/eE$. In between these two locations, the square root is imaginary, and so the Euclidean momentum $\pi_z = -ip_z$ is real, $\pi_z = \sqrt{eE\delta z(2m - eE\delta z)}$. In this regime, the particle is "virtual", with imaginary momentum, being accelerated by the electric field E toward positive energies.

Since the mass shells are tilted by the electric field potential energy, the particle can tunnel from the negative mass shell to the positive one, and subsequently, propagate out to infinity [30]. This occurs when the Euclidean momentum vanishes at $\delta z = 2m/eE$, which can be understood as the instant where the particle gains enough energy through the work of the electric field. Indeed, since $\delta W \simeq eE\delta z$, $\delta W \simeq 2m$ implies that enough energy is transferred at $\delta z \simeq 2m/eE$.

We can estimate the particle production rate to the leading order by employing thr WKB approximation and computing the Euclidean action by integrating over the region where p_z is imaginary (the "barrier"),

$$S_{\mathrm{E}} = \int_0^{2m/eE} dz \pi_z = \frac{\pi}{2}\frac{m^2}{eE}, \tag{7}$$

to get the tunneling wave function $\Psi = e^{-S_{\mathrm{E}}}$. The rate is given by $\Gamma \sim |\Psi|^2$; hence,

$$\Gamma \simeq e^{-\pi m^2/eE}. \tag{8}$$

In weak electric fields $E \to 0$, the rate goes to zero, while for strong fields $E > \pi m^2/e$, the exponential suppression disappears, and the rate is polynomially fast.

Applying this formula to a charged black hole, and taking the strongest electric field available just outside of the outer event horizon, yields $E = Q/r_+^2$, where Q is the black hole charge and $r_+ = G_N M + \sqrt{G_N^2 M^2 - G_N Q^2}$, where M is the black hole mass, Q its charge, and G_N Newton's constant. Therefore,

$$\Gamma \simeq e^{-\pi m^2 r_+^2/eQ}. \tag{9}$$

It turns out that, despite technical subtleties, this equation gives the correct leading order decay rate describing black hole discharge due to nonperturbative quantum effects [28,30]. Note, that these processes do not cease in the extremal limit, and that discharge continues even when $M = Q/\sqrt{G_N}$. Further note, that while these processes are slow for large black holes, they speed up as the mass decreases. They can also be augmented by spurts of Hawking radiation that can restart the charge loss by light particle emission, and go faster when charge carriers are light, $m \ll M_{\mathrm{Pl}}$. But at least in principle, as long as the Euclidean action S_{E} can continuously decrease to ≤ 1, the discharge can proceed—and speed up near the end—with the black hole disappearing. As shown in [25], reaching $m^2 r_+^2/eQ < 1$ is inevitable as long as Eq. (4) holds. To see this, substitute $r_+ = G_N M + \sqrt{G_N^2 M^2 - G_N Q^2}$ into $m^2 r_+^2/eQ < 1$. The resulting inequality after squaring it up and manipulating terms as in [25] becomes

$$Q^2 + \frac{eQ}{G_N m^2} \ge 2\frac{M}{m}\sqrt{eQ}. \tag{10}$$

Next, demand $M \ge Qm/e$ to ensure that kinematical constraints can be met, for simplicity define $Q = \zeta^2$ and divide everything by ζ^2 [25]; this maps (10) to

$$f(\zeta) \ge 0, \qquad f(\zeta) = 1 + \frac{e}{G_N m^2 \zeta^2} - \frac{2}{\sqrt{e}\zeta}. \tag{11}$$

The function $f(\zeta)$ has two zeros, one at $\zeta = 0$ and the other away, at a location approximately determined by $\zeta_0 \simeq e^{3/2}/2G_N m^2$. In between these two values, f is negative, and so the inequality cannot be satisfied there. To satisfy the inequality, ζ must exceed ζ_0. However, as the black hole charge $\zeta = Q^2$ decreases from some large initial value, the root ζ_0 must approach the origin, which means that at fixed e, the mass m must be dialed up to satisfy $f \ge 0$—eventually running afoul of Eq. (4). The fastest way for this to occur is along the parameter space curve extremizing f in the ζ direction, which implies $\zeta_{\mathtt{max}} = e^{3/2} G_N m^2$. Hence, the strongest bound comes from imposing (10) at this value of ζ. Substituting in (10), we find that $f \ge 0$ implies

$$f(\zeta_{\mathtt{max}}) = 1 - \frac{G_N m^2}{e^2} \ge 0 \Rightarrow \frac{\sqrt{G_N} m}{e} \le 1, \tag{12}$$

which is precisely the same as the bound of Eqs. (4), (5). This implies that as long as there are light particles carrying charge e which obey (4), charged black holes cannot linger forever. Both perturbative and nonperturbative particle production processes can discharge them. Conversely, if (4) does not hold for any charged species, neither discharge channel will be generally accessible, and remnants, and perhaps other problems, would seem to be difficult to avoid [23,25]. Equation (4) provides protection from such problems.

There is also a magnetic variant of WGC, which deals with the interplay of magnetic solitons with gravity [23] (see also [32]). An issue here is that in the weak coupling regime of gauge theory magnetic monopoles are very heavy, with the mass $m_{\mathtt{monopole}} \sim \mathcal{M}_{\mathrm{UV}}/e^2$, where $\mathcal{M}_{\mathrm{UV}}$ is the UV cutoff of the theory. Combining this with the WGC bound $m_{\mathtt{monopole}} \le e_{\mathtt{magnetic}}\sqrt{G_N} \simeq \sqrt{G_N}/e$ yields

$$\mathcal{M}_{\mathrm{UV}} \le e\sqrt{G_N}, \tag{13}$$

which means that the cutoff of a weakly coupled gauge theory must be below the Planck scale. This ensures that the monopole is not a black hole: combining $m_{\mathtt{monopole}} \sim \mathcal{M}_{\mathrm{UV}}/e^2$ with the size of the monopole core $R_{\mathtt{monopole}} \sim 1/\mathcal{M}_{\mathrm{UV}}$ yields immediately $m_{\mathtt{monopole}} \le G_N R_{\mathtt{monopole}}$ [23], violating the hoop conjecture, which black holes satisfy.

The WGC bounds discussed here for point particles can be generalized for extended objects. Specifically, we will be interested in the implications of charged tensional membranes in four dimensions. For them, the electric weak gravity bound generalizes to

$$\frac{\mathcal{Q}}{\mathcal{T}} \geq \sqrt{G_N}. \tag{14}$$

The statement of WGC then is that in the spectrum of the theory, which includes charged membranes, for each type of charge there must be at least one membrane, which satisfies the inequality (14). The magnetic form of the bound is a bit more subtle, having been deduced [32] to be the bound on the cutoff of the theory,

$$\mathcal{M}_{\mathrm{UV}} \leq \frac{\mathcal{Q}^{1/3}}{G_N^{1/6}}, \tag{15}$$

found by estimating the membrane tension in more than $4D$ by the energy stored in the field sourced by $\mathcal{Q}$, using the expected scaling of the gravitational radius with the higher-dimensional gravitational constant, requiring that there exists a magnetic membrane without the horizon, and then dimensionally reducing the result to $4D$. The inequalities (14), (15) will play important role in our arguments in what follows.

III. DISCHARGES WITH LINEAR AND QUADRATIC FLUX TERMS

General theories of 4-forms coupled to gravity and sourced by charged tensional membranes were examined in [4,5]. They split into two qualitatively different classes, depending on whether the linear 4-form flux terms are present and dominant in the action, or not. For this reason, we will focus here on the more special limiting forms, comprised of only linear and quadratic terms, which simplifies the discussion without any loss of generality. Further, the technical analysis simplifies by replacing the 4-form with its magnetic dual, $\mathcal{F} \leftrightarrow {}^*\lambda$, and replacing the 4-form Lagrangian $\mathcal{L}(\mathcal{F})$ with its Legendre transform $\mathcal{L}(\lambda)$ [33,34]. As explained in [4], this amounts to the Routhian transformation of the theory. Concretely, we start with

$$S = \int d^4x \left\{ \sqrt{g}\left(\frac{M_{\mathrm{Pl}}^2}{2}R - \mathcal{L}_{\mathrm{QFT}} - \frac{1}{48}\mathcal{F}_{\mu\nu\lambda\sigma}^2\right) - \frac{\alpha}{24}\epsilon^{\mu\nu\lambda\sigma}\mathcal{F}_{\mu\nu\lambda\sigma}\right\}, \tag{16}$$

motivated by, e.g., [35], where α is a fixed 4-form theory coupling parameter induced by nontrivial CP-breaking effects [35]. We then add the boundary term $\int d^4x \frac{1}{3}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu(\lambda\mathcal{A}_{\nu\lambda\sigma})$ to (16), define the new variable $\tilde{\mathcal{F}}_{\mu\nu\lambda\sigma} = \mathcal{F}_{\mu\nu\lambda\sigma} - (2\lambda - \alpha)\sqrt{g}\epsilon_{\mu\nu\lambda\sigma}$, and integrate $\tilde{\mathcal{F}}$ out. The resulting "bulk" action is [4]

$$S = \int d^4x \left\{ \sqrt{g}\left(\frac{M_{\mathrm{Pl}}^2}{2}R - \mathcal{L}_{\mathrm{QFT}} - 2\left(\lambda - \frac{\alpha}{2}\right)^2\right) + \frac{1}{3}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu(\lambda)\mathcal{A}_{\nu\lambda\sigma}\right\}. \tag{17}$$

We next expand $2(\lambda - \frac{\alpha}{2})^2 = 2\lambda^2 - 2\alpha\lambda + \alpha^2/2$ and absorb the flux-independent term $\alpha^2/2$ into the QFT vacuum energy, $\mathcal{L}_{\mathrm{QFT}} + \alpha^2/2 \to \mathcal{L}_{\mathrm{QFT}}$. Further, since 4-form should be viewed as a higher rank gauge theory, we add to (17) the gauge field charges—the charged tensional membranes—and boundary terms required to properly provide junction conditions across the membrane walls. This is motivated by the general lore that quantum gravity does not coexist with global symmetries [36,37], and without charges, the 4-form theory would in fact admit generalized higher-form symmetries. When charges are present, those are broken by gauge currents [38,39].

Finally, we parametrize $2\alpha = c_1\mathcal{M}_{\mathrm{UV}}^2$ and note that the value of c_1 controls how close this term is to the cutoff. Such terms arise naturally in axion physics, when the CP-violating effects in some nontrivial gauge theory max out [35]. We could even imagine that such a theory has an axion with a very large decay constant $f \gtrsim M_{\mathrm{Pl}}$ and the quantum gravity effects break shift symmetry very strongly. In any case, the final effective action for a single gauge sector coupled to gravity, which we will use in what follows, is

$$S = \int d^4x \left\{ \sqrt{g}\left(\frac{M_{\mathrm{Pl}}^2}{2}R - \mathcal{L}_{\mathrm{QFT}} + c_1\mathcal{M}_{\mathrm{UV}}^2\lambda - 2\lambda^2\right) + \frac{1}{3}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu(\lambda)\mathcal{A}_{\nu\lambda\sigma}\right\} - \int d^3\xi\sqrt{\gamma}M_{\mathrm{Pl}}^2[K] - \mathcal{T}_{\mathcal{A}}\int d^3\xi\sqrt{\gamma}_{\mathcal{A}} - \mathcal{Q}_{\mathcal{A}}\int\mathcal{A}, \tag{18}$$

where $\mathcal{T}_{\mathcal{A}}$ and $\mathcal{Q}_{\mathcal{A}}$ are the membrane tension and charge, respectively, the term $\propto K$ is the Israel-Gibbons-Hawking term for gravity which encodes boundary conditions across membrane walls, and $[\ldots]$ is the jump across a membrane. The coordinates ξ are coordinates along a membrane world volume, embedding it in spacetime. The charge terms are

$$\int\mathcal{A} = \frac{1}{6}\int d^3\xi\mathcal{A}_{\mu\nu\lambda}\frac{\partial x^\mu}{\partial\xi^\alpha}\frac{\partial x^\nu}{\partial\xi^\beta}\frac{\partial x^\lambda}{\partial\xi^\gamma}\epsilon^{\alpha\beta\gamma}. \tag{19}$$

We take $\mathcal{T}_{\mathcal{A}} > 0$ to avoid problems with ghosts and negative energies. This is a special case of actions discussed in [4], which suffices for our purposes here.

To study the discharge processes, we Wick rotate the action (18) to Euclidean time. This Euclidean action controls the nucleation rates $\Gamma \sim e^{-S_E}$ [14]. The analysis is given in detail in [1–4], and we just summarize it here. To transition to a Euclidean picture, we replace $t = -ix_E^0$, which gives $-i\int d^4x\sqrt{g}\mathcal{L}_{\mathrm{QFT}} = -\int d^4x_E\sqrt{g}\mathcal{L}^E_{\mathrm{QFT}}$; with the standard conventions, $\mathcal{A}_{0jk}=\mathcal{A}^E_{0jk}$, $\mathcal{A}_{jkl}=\mathcal{A}^E_{jkl}$ we get $\mathcal{F}_{\mu\nu\lambda\sigma}=\mathcal{F}^E_{\mu\nu\lambda\sigma}$, and $\epsilon_{0ijk}=\epsilon^E_{0ijk}$, $\epsilon^{0ijk}=-\epsilon_E^{0ijk}$. The membrane source terms transform to $-i\mathcal{T}_{\mathcal{A}}\int d^3\xi\sqrt{\gamma} = -\mathcal{T}_{\mathcal{A}}\int d^3\xi_E\sqrt{\gamma}$ and $i\mathcal{Q}_{\mathcal{A}}\int\mathcal{A}_i = -\mathcal{Q}_{\mathcal{A}}\int\mathcal{A}_i$. The Euclidean action by $iS=-S_E$ is (below we drop the subscript E)

$$S_E = \int d^4x_E\left\{\sqrt{g}\left(-\frac{M_{\mathrm{Pl}}^2}{2}R_E + \mathcal{L}(\lambda) + \Lambda_{\mathrm{QFT}} - c_1\mathcal{M}_{\mathrm{UV}}^2\lambda + 2\lambda^2\right) + \frac{1}{3}\epsilon_E^{\mu\nu\lambda\sigma}\partial_\mu(\lambda)\mathcal{A}^E_{\nu\lambda\sigma}\right\} \\ + \int d^3\xi\sqrt{\gamma}M_{\mathrm{Pl}}^2[K_E] + \mathcal{T}_{\mathcal{A}}\int d^3\xi_E\sqrt{\gamma}_{\mathcal{A}} - \frac{\mathcal{Q}_{\mathcal{A}}}{6}\int d^3\xi_E\mathcal{A}^E_{\mu\nu\lambda}\frac{\partial x^\mu}{\partial\xi^\alpha}\frac{\partial x^\nu}{\partial\xi^\beta}\frac{\partial x^\lambda}{\partial\xi^\gamma}\epsilon_E^{\alpha\beta\gamma}. \tag{20}$$

In the action, we set $\langle\mathcal{L}^E_{\mathrm{QFT}}\rangle = \Lambda_{\mathrm{QFT}}$, with Λ_{QFT} the regulated matter sector vacuum energy to an arbitrary loop order, since we consider transitions on backgrounds with local $O(4)$ symmetry that have minimal Euclidean action and dominate the evolution [14,15]. When the QFT vacuum energy is natural, QFT/gravity couplings imply $\Lambda_{\mathrm{QFT}} = \mathcal{M}_{\mathrm{UV}}^4 + \ldots \equiv M_{\mathrm{Pl}}^2\lambda_{\mathrm{QFT}}$, where $\mathcal{M}_{\mathrm{UV}}^4$ is the QFT UV cutoff and ellipsis denote subleading terms [40,41]. Hence, the total cosmological constant in any bulk patch is

$$\Lambda_{\mathtt{total}} = \Lambda_{\mathrm{QFT}} - c_1\mathcal{M}_{\mathrm{UV}}^2\lambda + 2\lambda^2, \tag{21}$$

where λ can vary from patch to patch across membrane walls.

A nucleation of a membrane changes the flux of λ inside it, and hence, the total cosmological constant in the interior. The resulting geometry comprises of two de Sitter patches glued along the membrane, with tension and charge controlling the membrane-sourced discontinuity. Away from the membrane, de Sitter patches are described with the metrics,

$$ds_E^2 = dr^2 + a^2(r)d\Omega_3, \tag{22}$$

where $d\Omega_3$ is the line element on a unit S^3. The warp factor a is the solution of the Euclidean "Friedmann equation",

$$3M_{\mathrm{Pl}}^2\left(\left(\frac{a'}{a}\right)^2 - \frac{1}{a^2}\right) = -\Lambda_{\mathtt{total}}. \tag{23}$$

The prime designates an r derivative. From here on, we will drop the subscript "$\mathtt{total}$". The boundary conditions induced on a membrane for gauge fields and gravity are [1–3]

$$a_{\mathrm{out}} = a_{\mathrm{in}}, \qquad \lambda_{\mathrm{out}} - \lambda_{\mathrm{in}} = \frac{\mathcal{Q}_{\mathcal{A}}}{2}, \\ M_{\mathrm{Pl}}^2\left(\frac{a'_{\mathrm{out}}}{a} - \frac{a'_{\mathrm{in}}}{a}\right) = -\frac{\mathcal{T}_{\mathcal{A}}}{2}, \tag{24}$$

in the coordinate system, where the outward membrane normal vector is oriented in the direction of the radial coordinate; r measures the distance in this direction. Subscripts "out" and "in" refer to the membrane's exterior ("parent de Sitter") and interior ("descendant de Sitter"), respectively. The discontinuities in λ and a' follow since a membrane is a Dirac δ-function source of charge and tension.

We proceed by solving (23) for $a' = \zeta_j\sqrt{1-\frac{\Lambda a^2}{3M_{\mathrm{Pl}}^2}}$, where $\zeta_j = \pm 1$ designate the two possible branches of the square root. Using this and the junction conditions for the magnetic fluxes (24), the value of Λ_{QFT} cancels out, and we obtain [1–5]

$$\zeta_{\mathrm{out}}\sqrt{1-\frac{\Lambda_{\mathrm{out}}a^2}{3M_{\mathrm{Pl}}^2}} = -\frac{\mathcal{T}_{\mathcal{A}}}{4M_{\mathrm{Pl}}^2}\left(1+\frac{2M_{\mathrm{Pl}}^2\mathcal{M}_{\mathrm{UV}}^2\mathcal{Q}_{\mathcal{A}}}{3\mathcal{T}_{\mathcal{A}}^2}\left(c_1 - 4\frac{\lambda}{\mathcal{M}_{\mathrm{UV}}^2}\right)\right)a, \\ \zeta_{\mathrm{in}}\sqrt{1-\frac{\Lambda_{\mathrm{in}}a^2}{3M_{\mathrm{Pl}}^2}} = \frac{\mathcal{T}_{\mathcal{A}}}{4M_{\mathrm{Pl}}^2}\left(1-\frac{2M_{\mathrm{Pl}}^2\mathcal{M}_{\mathrm{UV}}^2\mathcal{Q}_{\mathcal{A}}}{3\mathcal{T}_{\mathcal{A}}^2}\left(c_1 - 4\frac{\lambda}{\mathcal{M}_{\mathrm{UV}}^2}\right)\right)a, \tag{25}$$

where we take the flux to be made up of a large number of charge units, $\lambda \gg \mathcal{Q}_{\mathcal{A}}$. If this were not so, we would replace $\lambda \to \lambda_{\mathrm{out}} - \mathcal{Q}_{\mathcal{A}}/4$ in (25) (in the large flux case, the distinction between the "in" and "out" fluxes in these equations is irrelevant).

The Eqs. (25) play a crucial role, since they select the membrane discharge channel, which relaxes the vacuum energy, and control relaxation dynamics. The point is, that the right-hand side (rhs) of (25) can be written as $\mp\frac{\mathcal{T}_{\mathcal{A}}}{4M_{\mathrm{Pl}}^2}(1\pm q)$, where

$$q = \frac{2M_{\mathrm{Pl}}^2\mathcal{M}_{\mathrm{UV}}^2\mathcal{Q}_{\mathcal{A}}}{3\mathcal{T}_{\mathcal{A}}^2}\left(c_1 - 4\frac{\lambda}{\mathcal{M}_{\mathrm{UV}}^2}\right). \tag{26}$$

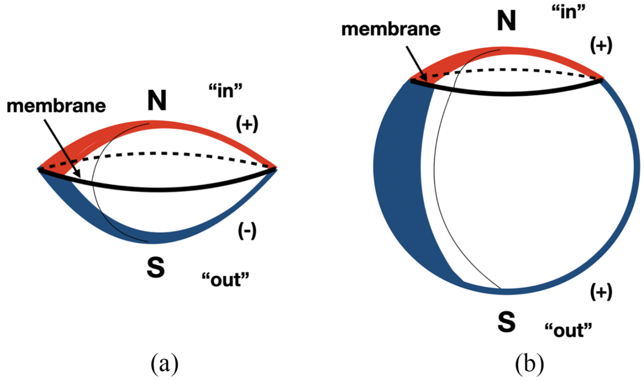

FIG. 1. (a) a $|q| < 1$ instanton mediating $dS \to dS$ with $(\zeta_{\rm out}, \zeta_{\rm in}) = (-,+)$. (b) a large flux, $|q| > 1$ instanton mediating $dS \to dS$ with $(\zeta_{\rm out}, \zeta_{\rm in}) = (+,+)$.

Thus, when $|q| < 1$, the terms in the parenthesis on the rhs of (25) keep the same sign, and Eqs. (25) only have solutions when $(\zeta_{\rm out}, \zeta_{\rm in}) = (-,+)$. Conversely, when $|q| > 1$, the solutions describing $dS \to dS$ transitions can only be found when $(\zeta_{\rm out}, \zeta_{\rm in}) = (+,+)$ or $(-,-)$. Of the latter two cases, $(\zeta_{\rm out}, \zeta_{\rm in}) = (+,+)$ dominates over $(\zeta_{\rm out}, \zeta_{\rm in}) = (-,-)$ because it has a smaller Euclidean action. The instantons mediating $(-,+)$ and $(+,+)$ processes are given in Fig. 1.

Which of the two channels is selected by the boundary conditions has a dramatic effect on the discharge dynamics. The bounce action is [1,2,7]

$$S_{\rm bounce} = S_{\rm out} - S_{\rm in} - \pi^2 a^3 \mathcal{T}_A, \tag{27}$$

with (where $k \in \{\text{out}, \text{in}\}$)

$$S_k = 18\pi^2 \frac{M_{\rm Pl}^4}{\Lambda_k}\left(\frac{2}{3} - \zeta_k\left(1 - \frac{\Lambda_k a^2}{3M_{\rm Pl}^2}\right)^{1/2} + \frac{\zeta_k}{3}\left(1 - \frac{\Lambda_k a^2}{3M_{\rm Pl}^2}\right)^{3/2}\right). \tag{28}$$

Its value depends on the membrane radius at nucleation a, which in turn depends on the microscopic parameters and Λ according to

$$\frac{1}{a^2} = \frac{\Lambda_{\rm out}}{3M_{\rm Pl}^2} + \left(\frac{\mathcal{T}_A}{4M_{\rm Pl}^2}\right)^2 (1+q)^2 = \frac{\Lambda_{\rm in}}{3M_{\rm Pl}^2} + \left(\frac{\mathcal{T}_A}{4M_{\rm Pl}^2}\right)^2 (1-q)^2. \tag{29}$$

From this formula, we deduce there are two regimes of bubble nucleation for a fixed set of parameters, depending on which term on the rhs of (29) the dominant contribution to the membrane radius comes from. The boundary between the two regimes is controlled by the critical value of the cosmological constant, roughly set by $\Lambda_* \simeq 3(\frac{\mathcal{T}_A}{4M_{\rm Pl}})^2(1+q)^2$.

To infer a more precise description, we can rewrite the bounce action (27) in terms of the *out* cosmological constant, membrane charge and tension, and the cutoff scale $\mathcal{M}_{\rm UV}$. First, we can evaluate (28) by eliminating the square root terms on the rhs using the junction conditions (25). Next, we express $\Lambda_{\rm in}$ in terms of $\Lambda_{\rm out}$ and membrane charge $\mathcal{Q}$ using (21) and the second of (24). Finally, we eliminate powers of the membrane radius at nucleation a using Eq. (29). Then we can consider specific limits of this action, e.g., fixing the tension $\mathcal{T}$ and varying $\mathcal{Q}$ and $\Lambda_{\rm out}$ relative to it to explore the possible tunneling regimes mediated by the instantons of Fig. 1.

It is tempting to take a shortcut and merely focus on the leading order terms in this panoply of pieces in the limits $\Lambda \to \infty$ and $\Lambda \to 0$ to get the essential behavior of the bounce action (27) while skipping the algebraic tedium. This actually works in the limit $\Lambda \to 0$. However, the limit $\Lambda \to \infty$ is more delicate. The reason is that the $a(\Lambda)$ dependence in (29) and the terms $\propto \mathcal{T}_A a$, which the bounce action (27) depends on after the square roots in (28) are evaluated using (25), show that $\Lambda \to \infty$ and $\Lambda = 0$ are branch points of the bounce action viewed as a function of Λ (this can also be seen in scalar field tunneling in, e.g., [15]). In particular, although $a(\Lambda)$ vanishes as $\Lambda \to \infty$ in (29), the terms $\propto \mathcal{T}_A a$ in the expression for the bounce action get multiplied by positive powers of Λ and hence, may not be negligible. Thus, it is prudent to determine the exact form of $S_{\rm bounce}$ before taking the limits.

The calculation is straightforward albeit tedious; a simplifying step is to write the terms $\frac{\zeta_k}{3}(1 - \frac{\Lambda_k a^2}{3M_{\rm Pl}^2})^{3/2} = \mp \frac{\mathcal{T}_A}{4M_{\rm Pl}^2}(1 \pm q)(1 - \frac{\Lambda_k a^2}{3M_{\rm Pl}^2})$, where the upper sign on the rhs corresponds to $k = out$ and the lower sign to $k = in$, respectively, given our conventions and definitions here. This gives, after straightforward steps,

$$S_{\rm bounce} = 12\pi^2 \frac{M_{\rm Pl}^4}{\Lambda_{\rm out}}\left(1 + \frac{\mathcal{T}_A a}{4M_{\rm Pl}^2}(1+q)\right) - 12\pi^2 \frac{M_{\rm Pl}^4}{\Lambda_{\rm in}}\left(1 - \frac{\mathcal{T}_A a}{4M_{\rm Pl}^2}(1-q)\right),$$
$$\frac{\mathcal{T}_A a}{4M_{\rm Pl}^2} = \left(\frac{\frac{3}{\Lambda_{\rm out}}\left(\frac{\mathcal{T}_A}{4M_{\rm Pl}}\right)^2}{1 + \frac{3}{\Lambda_{\rm out}}\left(\frac{\mathcal{T}_A}{4M_{\rm Pl}}\right)^2 (1+q)^2}\right)^{1/2},$$
$$\Lambda_{\rm in} = \Lambda_{\rm out} + \frac{3\mathcal{T}_A^2}{4M_{\rm Pl}^2} q. \tag{30}$$

Here, of course, $q < 0$ since we are focusing on transitions which reduce $\Lambda_{\rm out}$. It is now clear that $\Lambda_{\rm out} \to \infty$ and $\Lambda_{\rm out} = 0$ are branch points. To take the limits, it is further convenient to factorize this equation as a product of poles and functions which only include the branch points.

A shortcut is to bring (30) under a common denominator, substitute $\Lambda_{\rm out} = \Lambda_{\rm out}(a)$, which turns the numerator into a polynomial in a, and factorize the polynomial. We obtain

$$S_{\tt bounce} = \frac{18\pi^2 M_{\rm Pl}^4 \mathcal{T}_{\mathcal{A}}}{\Lambda_{\rm out}\Lambda_{\rm in} a}\left(1+\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2}(1+q)\right)\left(1-\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2}(1-q)\right),$$
$$\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2} = \left(\frac{\frac{3}{\Lambda_{\rm out}}\left(\frac{\mathcal{T}_{\mathcal{A}}}{4M_{\rm Pl}}\right)^2}{1+\frac{3}{\Lambda_{\rm out}}\left(\frac{\mathcal{T}_{\mathcal{A}}}{4M_{\rm Pl}}\right)^2(1+q)^2}\right)^{1/2}, \qquad \Lambda_{\rm in} = \Lambda_{\rm out} + \frac{3\mathcal{T}_{\mathcal{A}}^2}{4M_{\rm Pl}^2} q. \tag{31}$$

Lastly, we can eliminate $\Lambda_{\rm out}, \Lambda_{\rm in}$ using Eq. (29), where we obtain

$$\Lambda_{\rm out} = \frac{3M_{\rm Pl}^2}{a^2}\left(1-\left(\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2}\right)(1+q)\right)\left(1+\left(\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2}\right)(1+q)\right),$$
$$\Lambda_{\rm in} = \frac{3M_{\rm Pl}^2}{a^2}\left(1-\left(\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2}\right)(1-q)\right)\left(1+\left(\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2}\right)(1-q)\right), \tag{32}$$

after factorizing the squares. Substituting these into (31), we finally find

$$S_{\tt bounce} = \frac{2\pi^2 a^3 \mathcal{T}_{\mathcal{A}}}{(1-\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2}(1+q))(1+\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2}(1-q))},$$
$$\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2} = \left(\frac{\frac{3}{\Lambda_{\rm out}}\left(\frac{\mathcal{T}_{\mathcal{A}}}{4M_{\rm Pl}}\right)^2}{1+\frac{3}{\Lambda_{\rm out}}\left(\frac{\mathcal{T}_{\mathcal{A}}}{4M_{\rm Pl}}\right)^2(1+q)^2}\right)^{1/2}. \tag{33}$$

It is now clear that for the $dS \to dS$ transitions, the bounce action (31) remains nonnegative. Given the positivity of the cosmological constants and the tension, and $q < 0$, the only way it could ever be negative is if the last factor is negative, or alternatively, if $\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2}(1+q) > 1$. But given the definition of $\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2}$ in (33), we see that this can not happen,

$$\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2}(1+q) = \left(\frac{\frac{3}{\Lambda_{\rm out}}\left(\frac{\mathcal{T}_{\mathcal{A}}}{4M_{\rm Pl}}\right)^2}{\frac{1}{(1+q)^2}+\frac{3}{\Lambda_{\rm out}}\left(\frac{\mathcal{T}_{\mathcal{A}}}{4M_{\rm Pl}}\right)^2}\right)^{1/2} \le 1. \tag{34}$$

Using these equations we also see that if the cosmological constant dependent term dominates the rhs of (29)—i.e., in the limit $\Lambda_{\rm out} \to \infty$—the membrane's radius at nucleation is $a \simeq \frac{\sqrt{3}M_{\rm Pl}}{\sqrt{\Lambda_{\rm out}}}$ and $\frac{\mathcal{T}_{\mathcal{A}} a}{4M_{\rm Pl}^2}(1+q) \simeq \frac{\mathcal{T}_{\mathcal{A}}}{4M_{\rm Pl}^2}\frac{\sqrt{3}M_{\rm Pl}}{\sqrt{\Lambda_{\rm out}}}(1+q) < 1$. Thus,

$$S_{\tt bounce} \simeq 2\pi^2 a^3 \mathcal{T}_{\mathcal{A}} \simeq \frac{6\sqrt{3}\pi^2 M_{\rm Pl}^3 \mathcal{T}_{\mathcal{A}}}{(\Lambda_{\rm out})^{3/2}}. \tag{35}$$

When $\mathcal{T}_{\mathcal{A}} < \mathcal{M}_{\rm UV}^3$ and $\Lambda_{\tt out} \simeq \mathcal{M}_{\rm UV}^4 \simeq M_{\rm Pl}^4$, this gives[1] $S_{\tt bounce} \simeq \frac{\mathcal{T}_{\mathcal{A}}}{M_{\rm Pl}^3} < 1$. This is the initial regime which we model build to be in,[2] since in this regime an initial de Sitter space with a large cosmological constant "boils" the bubbles of the smaller cosmological constant that can start populating the landscape. As we noted in the Introduction, this is the regime of barrierless tunneling, where $dS \to dS$ decays are unsuppressed. Also, (35) grows bigger as $\Lambda_{\tt out}$ decreases, which means that de Sitter spaces with the largest Λ decay faster than those with a smaller Λ. This shows the trend of evolution: fast decay of the large Λ's and increased stability of subsequent lower Λ spaces. Clearly, if for any reason the "boiling" stage is pushed above the cutoff, the theory would not be under control in that regime, and this regime could not be invoked to set an initial population of Λ's. If this were realized, the landscape can turn into a desert.

This regime ends when the total cosmological constant discharges enough so that the second term in (29) dominates. For $|q| < 1$, that occurs when $\Lambda < \Lambda_* = 3(\frac{\mathcal{T}_{\mathcal{A}}}{4M_{\rm Pl}})^2$. The discharge proceeds by the $|q| < 1$ instanton in Fig. (1), for which $(\zeta_{\rm out}, \zeta_{\rm in}) = (-, +)$. The action (27) gradually asymptotes to

$$S_{\tt bounce} \simeq \frac{24\pi^2 M_{\rm Pl}^4}{\Lambda_{\rm out}}\left(1-\frac{8}{3}\frac{M_{\rm Pl}^2\Lambda_{\rm out}}{\mathcal{T}_{\mathcal{A}}^2}\right), \tag{36}$$

[1]The factor of $6\sqrt{3}\pi^2$ can be easily compensated initially by picking the scale $\mu = \mathcal{T}_{\mathcal{A}}^{1/3} < \mathcal{M}_{\rm UV}/5$. This may require appropriately reducing charges to keep $|q| < 1$; we will return to the precise details in later work.

[2]Or to never be in this regime, if we prefer to eventually rely on anthropics, see the discussion later on.

in this limit, with $S_{\mathrm{bounce}} > 0$ because $\Lambda < 3M_{\mathrm{Pl}}^2(\frac{\mathcal{T}_{\mathcal{A}}}{4M_{\mathrm{Pl}}^2})^2$. This action asymptotes to a pole at $\Lambda_{\mathrm{out}} = 0$, and leads to the decay rate of Eq. (2), $\Gamma \simeq \exp(-24\pi^2 M_{\mathrm{Pl}}^4/\Lambda_{\mathrm{out}})$. Just like the discharges are rapid when $\Lambda > 3(\frac{\mathcal{T}_{\mathcal{A}}}{4M_{\mathrm{Pl}}})^2$, they are very slow when $\Lambda < 3(\frac{\mathcal{T}_{\mathcal{A}}}{4M_{\mathrm{Pl}}})^2$. As a result, the total evolution, which results from combining the stages controlled by (35) and (36), if both can be fit below the cutoff in the same effective theory, would favor the smallest values of $\Lambda_{\mathtt{terminal}}$. The "boiling" stage is setting up the distribution and the "braking" stage preserving it, and therefore, in tandem, they provide a framework for naturally solving the cosmological constant problem. On the other hand, if the "boiling" regime were completely excised, being pushed above the cutoff, the resulting landscape could be very desolate with a naturally large initial Λ. In this case, the decay of the cosmological constant may be very slow, and if the membrane charges are small, it would need to go through many steps until reaching the terminal Λ values near zero. At practical times, the distribution of Λ would be biased toward larger values, and the prospect of an ultimately empty universe [42] would loom large.

In the other case, when $|q| > 1$, the "braking" regime starts when $\Lambda < \frac{M_{\mathrm{Pl}}^6 \mathcal{Q}_{\mathcal{A}}^2}{12\mathcal{T}_{\mathcal{A}}^2}$, or when $\Lambda < \frac{4M_{\mathrm{Pl}}^2\lambda^2\mathcal{Q}_{\mathcal{A}}^2}{3\mathcal{T}_{\mathcal{A}}^2}$, depending on whether linear or quadratic terms dominate q. The discharges are mediated by the $|q| > 1$ instanton in Fig. 1, for which $(\zeta_{\mathrm{out}}, \zeta_{\mathrm{in}}) = (+,+)$. For this instanton, in the bounce action (27), (28), the leading terms in (28) cancel out completely for both "in" and "out" contributions, and the subleading terms converge to (see, e.g., [1,2,6,7,14])

$$S_{\mathrm{bounce}} \simeq \frac{27\pi^2}{2}\frac{\mathcal{T}_{\mathcal{A}}^4}{(\Delta\Lambda)^3}. \tag{37}$$

In this regime, the decay rate saturates for a broad range of Λ as the cosmological constant decreases. When the quadratic flux terms dominate in q, the relative stability of de Sitter spaces with small cosmological constant is set by the ratio[3] $\mathcal{T}_{\mathcal{A}}^4/(\Delta\Lambda)^3 \simeq \mathcal{T}_{\mathcal{A}}^4/(2\Lambda_{\mathtt{QFT}})^{3/2}\mathcal{Q}_{\mathcal{A}}^3$, with the decay rate approaching (3). For a natural value of the screened initial vacuum energy $\Lambda_{\mathtt{QFT}} \simeq \mathcal{M}_{\mathrm{UV}}^4$, this immediately shows that we need, somewhat loosely,

$$\mathcal{T}_{\mathcal{A}}^4 > (2\Lambda_{\mathtt{QFT}})^{3/2}\mathcal{Q}_{\mathcal{A}}^3, \tag{38}$$

to have a chance for sufficient longevity of de Sitter regions with small cosmological constant, necessary to fit a realistic late universe cosmology. If the tension were too low, the small curvature de Sitter spaces could decay too fast. However, since the rate is approximately constant, when (38) holds, and if the "boiling" regime (35) is not too long, the discharges can produce a multiverse with all values of $\Lambda_{\mathtt{terminal}}$ approximately equally likely, and long-lived. If this happens, then invoking anthropics can be used to address the observed smallness of the cosmological constant. As we will see below, this can naturally occur when all flux discharge processes are dominated by quadratic or higher order flux terms.

IV. WGC VERSUS DISCHARGES

We now impose the WGC bounds of Sec. II on the discharge dynamics of the previous section. We will normalize the inequalities (14), (15) using the Planck scale instead of Newton's constant, ignoring the numerical factor of $\sqrt{8\pi} \simeq 5$, thus working with the original normalizations introduced in [23]. The $\mathcal{O}(1)$ numerical factors will be of little consequence in this work, although in general one should be careful with their accounting since they can affect normalization of some physical parameters, as for example the overall normalization of the bounce action, the duration of slow roll inflation, and so on [43,44]. In any case, the electric and magnetic weak gravity bounds that we will use are

$$M_{\mathrm{Pl}}\frac{\mathcal{Q}}{\mathcal{T}} \geq 1, \tag{39}$$

and

$$\mathcal{Q}M_{\mathrm{Pl}} \geq \mathcal{M}_{\mathrm{UV}}^3. \tag{40}$$

In addition, following the approach of [5], we will impose a bound on the flux variation for each type of flux involved in screening and discharge. The reason for this is that in hindsight, when the 4-forms are generalized by adding a dynamical longitudinal mode and a mass term, which arises naturally whenever the 4-forms realize monodromy field theories in $4D$, as in [45–56], in the axial gauge the longitudinal modes are monodromy-spanning "axions", whose total range must be limited by at least the requirement that their energy density does not exceed the Planckian energy, so the effective theory with gravity remains under control. Depending on the specifics of the theory, the bounds could be even tighter. Here, imagining that the effective theory enjoys protection from the gauge symmetries of the 4-form sectors, both continuous and discrete, we will require that it remains below the cutoff scale,

$$||T^\mu{}_\nu(4-\mathrm{form})|| \lesssim \mathcal{M}_{\mathrm{UV}}^4 \lesssim M_{\mathrm{Pl}}^4, \tag{41}$$

where by $||T^\mu{}_\nu(4-\mathrm{form})||$, we mean the operatorial norm of the stress energy tensor of the 4-form sector, i.e., its largest eigenvalue. With this in place, we are ready to find

[3]The appearance of $\Lambda_{\mathtt{QFT}}$ follows from requiring natural screening of vacuum energy by fluxes [5,11]. If the linear flux terms dominate, we would find $\mathcal{T}_{\mathcal{A}}^4 > M_{\mathrm{Pl}}^6\mathcal{Q}_{\mathcal{A}}^3$ instead of (38).

the implications of these bounds on the dynamics of screening and discharge. Conveniently, the technical aspects of this analysis are simplified by separately considering the purely quadratic flux case, as an avatar of frameworks where the linear flux term is subleading, and purely linear term, without any loss of generality.

A. Quadratic flux dominance

We will explicitly work with a single species of membranes for the most part, since the nucleations proceed one bubble at a time. However, we bear in mind that, to be able to approach the observably allowed values of the terminal cosmological constant, we need a multiplicity of different membranes once we impose the field theory cutoff on the flux range [5,11]. This means that in the formulas below we should really replace expressions like $N_{\mathcal{A}}^2 \mathcal{Q}_{\mathcal{A}}^2$ by $\sum_i N_i^2 \mathcal{Q}_i^2$ etc. This in turn means, that in our comparison of the cosmological constant to be canceled and the cutoff, there is an in-principle multiplicative species factor, counting each flux that contributes to $\Lambda_{\mathtt{total}}$. Since it is $\lesssim \mathcal{O}(100)$, we will ignore it in what follows. When the screening terms in $\Lambda_{\mathtt{total}}$ are dominated by the quadratic flux contributions, such that

$$\Lambda_{\mathtt{total}} = \Lambda_{\mathtt{QFT}} + 2\sum \lambda_i^2 + \ldots \simeq -|\Lambda_{\mathtt{QFT}}| + 2\sum_i \lambda_i^2, \tag{42}$$

we must take $\Lambda_{\mathtt{QFT}} < 0$ to have a chance to cancel it [6,7,11]. Then, for a natural value of $\Lambda_{\mathtt{QFT}} \sim \mathcal{M}_{\mathrm{UV}}^4 \sim M_{\mathrm{Pl}}^4$, the dynamics of discharge produces a nested system of bubbles bounded by membranes. Nucleation processes are controlled by Eqs. (25)–(29), and, crucially, by the value of q. In the limit when quadratic terms dominate, q is given by

$$q_i = \frac{8}{3}\frac{M_{\mathrm{Pl}}^2 \lambda_i \mathcal{Q}_i}{\mathcal{T}_i^2}, \tag{43}$$

for each individual flux λ_i. The parameter q_i is proportional to the slope of the tangent to the "spectral parabola" as depicted in Fig. 2.

Since fluxes are quantized, $\lambda_i = \frac{1}{2} N_i \mathcal{Q}_i$ (1/2 comes from our normalization of λ). Then, plugging this into the formula for q_i yields $q_i = \frac{4}{3} N_i \frac{M_{\mathrm{Pl}}^2 \mathcal{Q}_i^2}{\mathcal{T}_i^2}$, or, using $\gamma_{\mathrm{WGC}} = M_{\mathrm{Pl}} \mathcal{Q}_i / \mathcal{T}_i$,

$$q_i = \frac{4}{3} N_i \gamma_{\mathrm{WGC}}^2, \tag{44}$$

where γ_{WGC} is precisely the ratio of charge to tension in Planck units, which is subject to the electric weak gravity bound (39). Thus, if WGC is obeyed by a membrane "i", $\gamma_{\mathrm{WGC}} > 1$, and since we must screen a natural vacuum energy $\Lambda_{\mathtt{QFT}}$ with multiple units of flux, $N_i > 1$. Therefore,

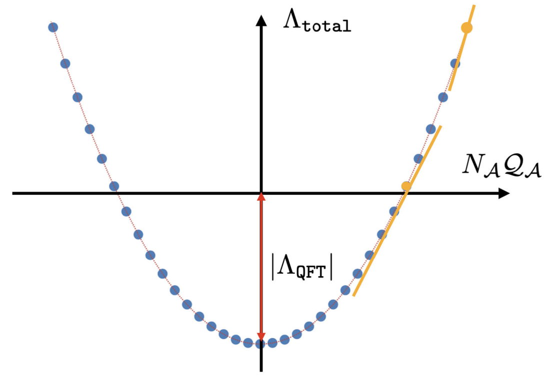

FIG. 2. Λ-parabola, depicting the spectrum of Λ as a function of the screening flux. In the full multidimensional flux space, Λ is a paraboloid, and here, we depict its projection to a single coordinate plane. The gold lines are tangents to the parabola whose slope is q, which controls the discharge process. The discrete points are the actual values of the quantized fluxes and the corresponding cosmological constant of the Λ-discretuum.

$q > 1$ for any type of membrane obeying WGC, for all transitions which occur until Λ reaches its smallest positive value. As a result, the discharge processes of the natural vacuum energy by emission of these membranes can only proceed by the instanton with $|q| > 1$ of Fig. 1. This means the bounce action for these processes generically asymptotically approaches Eq. (37) as the cosmological constant diminishes, which remains a good approximation for much of the discharge sequence.

The relevant inequalities to check further are (38) and (40). In fact, we can rewrite all three of these inequalities in terms of dimensionless ratios, as follows (for $\Lambda_{\mathtt{QFT}} \sim \mathcal{M}_{\mathrm{UV}}^4$):

$$\begin{aligned} &\left(\frac{\mathcal{T}_i}{M_{\mathrm{Pl}}^3}\right)^4 > \left(\frac{\mathcal{M}_{\mathrm{UV}}}{M_{\mathrm{Pl}}}\right)^6 \left(\frac{\mathcal{Q}_i}{M_{\mathrm{Pl}}^2}\right)^3, \quad \text{stability;}\\ &\left(\frac{\mathcal{Q}_i}{M_{\mathrm{Pl}}^2}\right) \geq \left(\frac{\mathcal{T}_i}{M_{\mathrm{Pl}}^3}\right), \quad \text{electric WGC;}\\ &\left(\frac{\mathcal{Q}_i}{M_{\mathrm{Pl}}^2}\right) \geq \left(\frac{\mathcal{M}_{\mathrm{UV}}}{M_{\mathrm{Pl}}}\right)^3, \quad \text{magnetic WGC.} \end{aligned} \tag{45}$$

Furthermore, since the membrane charge and tension are distributed quantities, we should require that they are below the cutoff scale, $\mathcal{Q}_i < \mathcal{M}_{\mathrm{UV}}^2$ and $\mathcal{T}_i < \mathcal{M}_{\mathrm{UV}}^3$, so that they can be reliably included in the sub-cutoff effective description based on the low energy actions which we deploy here. In fact, these bounds are redundant: the electric WGC bound in (45) follows from the magnetic WGC bound and $\mathcal{T}_i < \mathcal{M}_{\mathrm{UV}}^3$. However, we will retain the electric WGC bound for convenience with calculations below. We note that all of these inequalities can be satisfied simultaneously for some $\mathcal{M}_{\mathrm{UV}} \lesssim M_{\mathrm{Pl}}$. On the other hand, model building "economics" suggests that $\mathcal{M}_{\mathrm{UV}}$ is to be looked for not too far below M_{Pl} in order to be able to use as few fluxes as possible [11]. Add to this the argument of the previous section about the existence of the "boiling" stage, which further reaffirms this expectation.

Finally, we note that, for as long as $|q| > 1$, the scale that separates the "boiling" from the "braking" stage for quadratic flux is given by

$$\Lambda_* \simeq \frac{4}{3}\frac{\lambda^2 M_{\rm Pl}^2 \mathcal{Q}_i^2}{\mathcal{T}_i^2} \simeq \frac{2}{3}\gamma_{\rm WGC}^2 \mathcal{M}_{\rm UV}^4. \tag{46}$$

When $\gamma_{\rm WGC} > 1$, the critical value Λ_* is above the cutoff, and the "boiling" stage is effectively excised out of the quadratic flux effective theory, and so essentially for values of $\Lambda \lesssim \mathcal{M}_{\rm UV}^4 \sim M_{\rm Pl}^4$, the discharges occur near the edge or during the "braking" stage (with the decay rate reduction being progressively more efficient as Λ decreases), with the bounce action asymptotically approaching the formula given in Eq. (37), $S_{\rm bounce} \simeq \frac{27\pi^2}{2}\frac{\mathcal{T}_i^4}{(\Delta\Lambda)^3}$. Since $\Delta\Lambda = 2\lambda\Delta\lambda$, and initially, $\lambda \simeq \sqrt{\Lambda} \gtrsim \sqrt{\Lambda_{\rm QFT}}$, the initial bounce action will start smaller than the asymptotic value $S_{\rm bounce} \simeq \frac{27\pi^2}{2}\frac{\mathcal{T}_i^4}{(2\Lambda_{\rm QFT})^{3/2}\mathcal{Q}_i^3}$, which may permit discharges at an approximately uniform rate, independent of initial and final cosmological constant values. The discharges will cease once the flux becomes small enough to obey the stability bound (38). Thus, the theory where flux contributions to the cosmological constant are dominated by quadratic terms may provide an arena where invoking the anthropic principle can be used to address the observed smallness of the cosmological constant, as in [11].

Higher powers of flux do not affect this conclusion much. If, e.g., $2\lambda^2$ is replaced by $\mathcal{L}(\lambda) = 2\lambda^2(1 + c_3\lambda/\mathcal{M}_{\rm UV}^2 + \ldots)$, as in the examples of [4], and higher order terms are suppressed by the cutoff, or by $M_{\rm Pl}$, these terms will be subleading in the effective theory. If, on the other hand, the suppression is weaker for any single one of them, that term might compete with the quadratic flux term at large flux, and perhaps even produce novel regimes with tiny total cosmological constant, rearranging the effective theory near them but behaving similarly to when the quadratic dominates [4].

Thus, the bottom line is that these processes can discharge the cosmological constant at a nearly constant rate from one value to another before the discharges stop completely, setting essentially an approximately uniform distribution of values at late times, without automatically favoring any particular value of Λ, including the smallest ones. This sets the stage for invoking anthropic principle.

In contrast, in [1–5], we have been pursuing a framework where the dominant flux terms are linear, and the instantons, which discharge the cosmological constant, have $|q| < 1$, so that their bounce action asymptotes to a pole at a tiny value of Λ, which, as we argued, can favor the smallest Λ without anthropics. As we noted in the

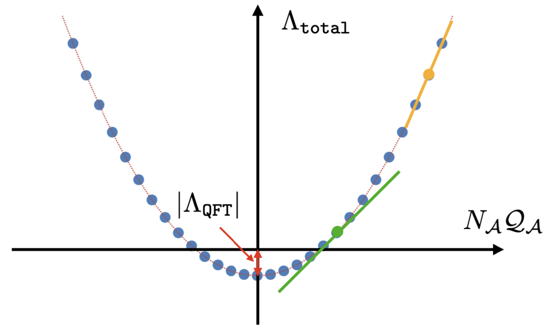

FIG. 3. Λ-parabola, depicting the spectrum of Λ as a function of the screening flux, but for smaller values of $\mathcal{Q}_i$ than in Fig. 2. As a result, the slope of the tangent as measured by q becomes smaller, and if $|q| < 1$, discharges would be mediated by the $|q| < 1$ instanton of Fig. 1. We show in the text that this is unfounded when quadratic flux terms dominate.

Introduction, one might try to adopt similar processes to the cases when higher powers of the flux dominate, and the linear term is absent. One might think that by arbitrarily reducing the membrane charge, this might make q smaller than unity, so that the $|q| < 1$ instantons of Fig. 1 take over the discharges. If this had been possible, the spectrum of Λ as a function of the fluxes would have been altered, looking like Fig. 3, where for small parent Λ the slope of the tangent to the parabola would have been below unity. However, there are problems with this approach.

Seeing the problem is straightforward. To get $|q| < 1$, so that the corresponding instantons are activated, formula (44) shows that we must violate the electric weak gravity bound *considerably*: solving (44) for N_i,

$$N_i = \frac{3}{4}\frac{q_i}{\gamma_{\rm WGC}^2}, \tag{47}$$

and so if $|q_i| < 1$ and $\gamma_{\rm WGC} > 1$, we find $N_i < 0.75$—which completely excludes the possibility of screening *any* value of field theory vacuum energy by quantized fluxes. Indeed, to have a chance to screen a natural field theory vacuum energy $\lambda_{\rm QFT} \simeq \mathcal{M}_{\rm UV}^4$ and then relax the total by subsequent membrane nucleations, we need multiple units of flux: $N_i \gg 1$. Hence, we need a serious violation of the electric weak gravity bound: $|q_i| < 1$ implies

$$\gamma_{\rm WGC} < \sqrt{\frac{3}{4N_i}}, \tag{48}$$

and so if $N_i \gg 1$, we find $\gamma_{\rm WGC} \ll 1$. Next, we can ignore the stability bound of Eqs. (45), since the bounce action in the "braking" stage for this case is not (37), but (36), and so the stability is enforced by the $\Lambda \to 0$ pole.

However, if we rewrite Λ_* for the case $|q| < 1$ as

$$\Lambda_* = \frac{3}{16}\frac{\mathcal{T}_i^2}{M_{\rm Pl}^2} = \frac{3}{16}\frac{\mathcal{Q}_i^2}{\gamma_{\rm WGC}^2}, \qquad (49)$$

we see that $\gamma_{\rm WGC} \ll 1$ implies that charges are very small: $\mathcal{Q}_i \simeq \frac{4}{\sqrt{3}}\gamma_{\rm WGC}\sqrt{\Lambda_*} \ll \frac{4}{\sqrt{3}}\sqrt{\Lambda_*}$. From Eqs. (21) and the second of (24), it then follows that

$$\Delta\Lambda \simeq 2\lambda\Delta\lambda = \lambda\mathcal{Q}_i \simeq \frac{4}{\sqrt{3}}\gamma_{\rm WGC}\sqrt{\Lambda_{\tt QFT}}\sqrt{\Lambda_*} \ll \frac{4}{\sqrt{3}}\mathcal{M}_{\rm UV}^2\sqrt{\Lambda_*} \simeq \frac{\mathcal{M}_{\rm UV}^2}{M_{\rm Pl}}\mathcal{T}_i. \qquad (50)$$

Thus, since $\mathcal{M}_{\rm UV} \sim M_{\rm Pl}$ and $\mathcal{T}_i < \mathcal{M}_{\rm UV}^3$, in this regime, the individual discharges change the cosmological constant by a tiny amount, $\Delta\Lambda/\Lambda \ll \mathcal{T}_i/\mathcal{M}_{\rm UV}^3$. Thus, to relax it to nearly zero, we need many subsequent transitions during the "braking" stage, which will be ever more slow due to the attractor behavior of (36). Such a slow discharge sequence practically stabilizes the de Sitter background, and it could bring back the specter of the empty universe problem [42], since many small slow steps could result in difficult reheating.[4]

The empty universe problem could be averted by adding a single membrane with tension and charge that satisfy WGC bounds. This would maintain the option of UV completing the theory, and it would mediate faster nucleations in the "braking" stage, by using the $|q| > 1$ nucleation processes with the bounce action (37). Even if those satisfy the stability bound (38), the transitions would be generically faster than the ones mediating (50). Discharges mediated by such a membrane (or membranes) can overtake the processes which have the attractor behavior and avert the empty universe. However, since the decay rate in this case may be uniform, this channel could usher the anthropics back. This is the obstacle to some of the proposals in [58], in using $|q| < 1$ instantons when quadratic (and higher power) fluxes dominate. The point is not that those small values of Λ are not populated but *how* they are populated. Indeed, on general grounds the whole landscape will be populated [59], but the details of the landscape "demographics" must be looked at on a case to case basis.

B. Linear flux dominance

We have already extensively discussed aspects of Λ discharge when linear flux terms dominate in [1–5]. Here, we will revisit some of those results with particular attention given to the role of WGC bounds. First off,

[4]Dynamics of stable bubbles and domain walls with small tensions and charges can also be constrained by cosmology [57], although such bounds are not very practical in an empty universe.

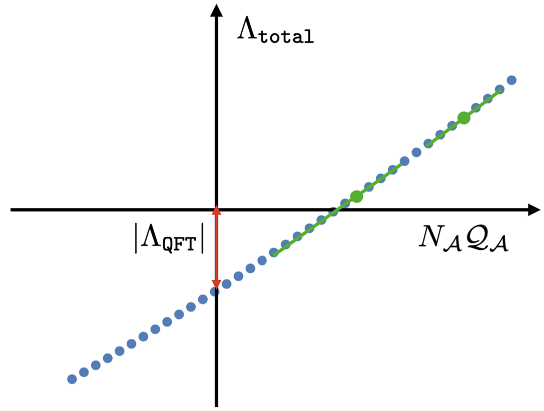

FIG. 4. Almost-linear spectrum of Λ as a function of the screening flux, projected onto a single coordinate plane. The slope of the tangent is practically a constant, and when $|q| < 1$ the membrane discharges are mediated by the $|q| < 1$ instantons of Fig. 1.

the screened total cosmological constant, as, e.g., examined in [5], is

$$\Lambda_{\tt total} = \Lambda_{\tt QFT} - c_1\mathcal{M}_{\rm UV}^2\lambda + 2\lambda^2 = \Lambda_{\tt QFT} - \frac{c_1^2}{2}\mathcal{M}_{\rm UV}^4 + 2\left(\lambda - \frac{c_1}{4}\mathcal{M}_{\rm UV}^2\right)^2, \qquad (51)$$

where we implicitly take the linear term to dominate over the quadratic one and complete the squares in the second line for the sake of convenience. In the case of multiple fluxes, we can rewrite this as

$$\Lambda_{\tt eff} = 2\sum_i\left(\lambda_i - \frac{c_{1i}}{4}\mathcal{M}_{\rm UV}^2\right)^2,$$
$$\Lambda_{\tt eff} = \Lambda_{\tt total} - \Lambda_{\tt QFT} + \sum_i \frac{c_{1i}^2}{2}\mathcal{M}_{\rm UV}^4. \qquad (52)$$

The spectrum of values of Λ is depicted in Fig. 4. Note that in this case it does not matter if $\Lambda_{\tt QFT}$ is positive or negative (it had to be negative when quadratic fluxes dominate for screening to work). Due to the fact that linear fluxes dominate, they can screen $\Lambda_{\tt QFT}$ of either sign.

In this limit, the slope parameter is

$$q_i = \frac{2c_{1i}}{3}\frac{M_{\rm Pl}^2\mathcal{M}_{\rm UV}^2\mathcal{Q}_i}{\mathcal{T}_i^2} = \frac{2c_{1i}}{3}\frac{M_{\rm Pl}\mathcal{M}_{\rm UV}^2}{\mathcal{T}_i}\gamma_{\rm WGC} = \frac{2c_{1i}}{3}\frac{\mathcal{M}_{\rm UV}^2}{\mathcal{Q}_i}\gamma_{\rm WGC}^2. \qquad (53)$$

The equation for Λ_* is still given by the expression (49), $\Lambda_* = \frac{3}{16}\mathcal{T}_i^2/M_{\rm Pl}^2 = \frac{3}{16}\mathcal{Q}_i^2/\gamma_{\rm WGC}^2$. The important point is that now the parameter q_i is completely independent of the units of flux N_i—which can be as large as one wishes while q_i is still fixed. We can keep the charge near the cutoff, $\mathcal{Q}_i \sim \mathcal{M}_{\rm UV}^2$, and $\gamma_{\rm WGC} \lesssim 1$, but close to the bound, and ensure that $|q_i| < 1$ by choosing $c_{1i} < 1$. When $\gamma_{\rm WGC} \lesssim 1$, Λ_* might thus be mere few units of charge above zero, but faster discharges during the "boiling" regime could discharge it to near zero.

Then, to satisfy WGC, we in principle need to have only one charge per gauge group, which satisfies the electric bound. Saturating the bound is acceptable, when the charge is light enough. So for each gauge group, we need one of the membranes to obey $\gamma_{\rm WGC} \simeq 1$. From Eq. (53), for membranes with $\gamma_{\rm WGC} \simeq 1$, we need $2c_{1i}\mathcal{M}_{\rm UV}^2 < 3\mathcal{Q}_i$ to get $|q_i| < 1$. This together with $\mathcal{Q}_i \leq \mathcal{M}_{\rm UV}^2$ constrains the charge to $2c_{1i}\mathcal{M}_{\rm UV}^2/3 < \mathcal{Q}_i \leq \mathcal{M}_{\rm UV}^2$, which can be met by $c_{1i} \leq 1$. Meeting these bounds may be easiest done near the Planckian cutoff, $c_{1i}M_{\rm Pl} \sim \mathcal{M}_{\rm UV}$, which can also keep $\mathcal{T}_i < \mathcal{M}_{\rm UV}^3$ and so retain an episode of "boiling" in the theory below the cutoff. So a membrane with a charge of order of the cutoff and tension somewhat smaller could also yield $|q_i| \lesssim 1$, while marginally satisfying WGC bounds. For other membranes which carry the gauge charge of the same group, we can then violate the electric weak gravity bound, that will make achieving $|q_i| < 1$ much easier. This can happen if, for example, those membranes are domain walls separating multiple vacua after a symmetry breaking at low energies, which could be viewed analogously to particles with fractional charges in QFT.

Alternatively, we may even allow a membrane whose charge to tension ratio obeys the electric weak gravity bound to participate in the discharge of Λ even if it has $|q_i| \gtrsim 1$, as long as there are other species of membranes with $|q_i| < 1$ (which may violate the WGC bounds). The reason is that although the processes for this one specific discharge channel are mediated by the $|q| \gtrsim 1$ instanton, which uniformizes the distribution of the Λ values which are linked by these transitions, there are many more channels that proceed via the $|q| < 1$ instantons. Those can still bias the overall distributions of Λ towards the smallest possible values and have larger charges that require fewer steps to get the cosmological constant close to zero. Once near zero, those small values remain extremely stable. If a $|q| \gtrsim 1$ channel is present, those values might not be absolutely stable: they could decay, for example, to regions $\Lambda < 0$ *eventually*. But as we have seen, once a region of the universe ends up in the "braking" regime of either $|q| < 1$ or $|q| \gtrsim 1$ instanton discharge, it is very stable and very long-lived. Yet, when the terminal distribution of Λ is biased towards smallest possible values, we may still avoid invoking the anthropic arguments to explain why the cosmological constant is not huge. Exactly how close to zero it can be is controlled by the model building aspects of the theory, which we described in some detail in [1–5]. We direct an interested reader to those references for additional information.

V. SUMMARY

In this paper, we have examined in detail implications of the weak gravity conjecture for the mechanisms for discharging cosmological constant via membrane nucleations. This is a natural and interesting question, given the role which the WGC bounds play in blocking the existence of eternal event horizons in gravity theories in order to protect unitarity. In the frameworks where the cosmological constant is screened by 4-form fluxes, and then the total background value of Λ is discharged away by the nucleation of membranes, stable eternal de Sitter spaces do not even exist. In fact, starting with a theory which has fluxes and membranes, the only way to recover an eternal de Sitter is to decouple all of the membranes in the theory by, e.g., sending their tensions to infinity. But this would violate the WGC bounds completely; compliance with the conjecture rules out eternal de Sitter just like the WGC bounds applied to charged particles rules out eternal extremal black holes [23]. Conversely, in the example where quadratic flux terms control the discharge processes, we saw that if the WGC bounds are violated, de Sitter space will not reach near-Minkowski limit unless the theory is fine tuned. From this point of view, an eternal de Sitter geometry is really analogous to a remnant, with regions forever removed from a dweller in the space.

The details of the WGC bounds, when combined with naturalness of the initial, maximal cosmological constant, place limits on the decay processes of de Sitter space and the cosmological constant that sources it. The possible outcomes fall in two different classes. When the flux terms which control the screening and discharge of the cosmological constant are dominated by quadratic and higher order terms, the bounds from weak gravity conjecture and naturalness point toward anthropic scenarios. Interestingly, even if the WGC bounds are deliberately violated, the discharge rates still do not easily pick a small Λ. On the other hand, if the theory involves linear flux terms, which dominate below the cutoff, anthropics could be avoided, because a large cosmological constant naturally decays toward smaller values, with nonuniform decay rates.

This is because the evolution is comprised of two discharge regimes: the "boiling" stage, followed by a "braking" stage. For the cases when the linear fluxes are present and dominant, with WGC-compliant branes, the narrower "boiling" stage processes have fast discharge rates, which generate the descendant regions with curvatures biased towards the smallest possible values of $\Lambda > 0$. The subsequent "braking" stage in turn slows down the most the decay rates of regions with smallest positive Λ after "boiling" has ended. Together, these stages produce a distribution of Λ, which is biased towards the smallest values.

Conversely, if the controlling fluxes are dominated by quadratic or higher order terms, purposefully violating the WGC bounds may reproduce the enhanced "braking" at small values of Λ, making de Sitter patches with small Λ more stable than the strongly curved ones. However, the price to pay is that charges must be very small if the screening and discharge adjustment are to be natural. This affects the "boiling" stage, which could resurrect the empty universe problem. Further, by current lore, UV completing the theory needs the means to maintain WGC. This can be done by adding a membrane which satisfies the electric WGC bound, charged under the same gauge group as the membranes that are adjusting Λ, one per gauge group. If gravity is universal, these membranes should also partake in the cosmological constant adjustment. If they obey WGC bounds, their vacuum energy would be dominated by quadratic flux terms, and so they would yield discharge processes which would be uniform, since it would be mediated by the $(+,+)$ instanton. These processes can be faster than the discharges using WGC-violating membranes and flatten the distribution of terminal Λ at the small Λ end. This would usher back the anthropics.

Note that our investigation of the discharges was consistently carried out below the cutoff of the effective theory, which avoids the direct confrontation with quandary that is the wormhole regime [19–22]. In this sense, the WGC limits are useful, since they "regulate" the boundary conditions which quantum gravity imposes on the low energy effective theory, with some confidence that the phenomena retained because they obey the WGC bounds are meaningfully accounted for.

Regarding the final numerical values of Λ, precisely how small those can be depends on the model building details, as noted in previous work [1–5]. More precise model building is required to give a more specific answer. Finally, note that even if the various regimes occur concurrently, i.e., if the discharge processes are more diversified, involving both processes mediated by $|q| > 1$ and $|q| < 1$ instantons, as long as at least some channels are dominated by linear fluxes, the spectrum of Λ will be skewed toward the smallest values. A more detailed investigation to explore phenomenologically viable scenarios is therefore warranted.

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