Magnetogenesis from an anisotropic universe

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The existence of large-scale anisotropy cannot be ruled out by the cosmic microwave background (CMB) radiation. Over the years, several models have been proposed in the context of anisotropic inflation to account for the CMB's cold spot and hemispheric asymmetry. However, any small-scale anisotropy, if it exists during inflation, is not constrained, due to its nonlinear evolution in the subsequent phase. This small-scale anisotropy during inflation can play a nontrivial role in giving rise to the cosmic magnetic field, which is the subject of our present study. Assuming a particular phenomenological form of an anisotropic inflationary universe, we have shown that it can generate a large-scale magnetic field at the 1 Mpc scale with a magnitude $\sim 4 \times 10^{-20}$ G, within the observed bound. Because of the anisotropy, the conformal flatness property is lost, and the Maxwell field is generated even without explicit coupling. This immediately resolves the strong coupling problem in the standard magnetogenesis scenario. In addition, assuming very low conductivity during the reheating era, we can further observe the evolution of the electromagnetic field with the equation of state ω_{eff} and its effects on the present-day magnetic field.

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I. INTRODUCTION

It is well known that our Universe is magnetized on all observational scales, from planets and stars to large-scale galaxies and galaxy clusters. In particular, the magnetic field strength has been observed in the range from μG for galaxies and galaxy clusters to a few G for planets and 10^{12} G for neutron stars. From gamma-ray observations and Faraday rotation measurements, the magnetic field in the intergalactic medium (IGM) has also been shown to be bounded, with the strength ranging from 10^{-10} to 10^{-22} G [1–5]. It is possible that the primordial magnetic fields on a large scale ($\sim$1 Mpc) were generated during the big bang or later and survived until today as a relic. The origin of the magnetic field in galaxies and galaxy clusters can be explained through classical magnetohydrodynamic processes magnifying the tiny seed magnetic field. It is important to identify the origin of the primordial magnetic fields. There have been some proposed mechanisms for generating large-scale primordial magnetic fields, which can be found in the interesting review papers listed in Refs. [6–21]. Among them, the Ratra model [21] is the most accepted one, where the electromagnetic fields are generated during the inflationary era by breaking the conformal invariance of the Maxwell term (i.e., $F_{\mu\nu}F^{\mu\nu}$) through nonminimal coupling(s) with other fields such as scalar fields.

Inspired by the mechanism in [16,18,21–24], we propose another mechanism for generating the primordial magnetic fields through anisotropic spacetime. In particular, the Maxwell field experiences the existence of the anisotropy of spacetime during the inflationary era, leading to the genesis of the primordial magnetic field. In cosmology, there exists a nice classification of homogeneous but anisotropic spacetimes called the Bianchi universe [25–29]. It turns out that the Bianchi type-I metric is the simplest one and can be regarded as a straightforward extension of the Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime. Hence, we chose the Bianchi type-I metric for our study.

Remarkably, it has long been argued that the very early Universe, which is close to the initial singularity, should be strongly anisotropic [30–32]. During an inflationary phase of the early Universe, all spatial anisotropies, which could happen in a preinflationary phase, should decrease very quickly, such that the Universe speedily approaches a locally isotropic state, as pointed out in Refs. [33,34]. It should be noted that this scenario is consistent with the so-called cosmic no-hair conjecture, which claims that all initial anisotropies and inhomogeneities should disappear in a late-time universe [35,36]. Very interestingly, some recent unavoidable anomalies in the cosmic microwave background (CMB) radiation confirmed by Planck [37,38], such as the cold spot and hemispheric asymmetry, have challenged the standard inflationary universe models,

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which are based on a cosmological principle stating that the Universe should be homogeneous and isotropic on large scales. In addition, other interesting observational evidences, which have called the validity of cosmological principle into question, have been listed in a recent interesting review [39]. These remarkable points lead us to a possible scenario of an anisotropic inflationary Universe in early times. Many papers have been working on anisotropic inflation—e.g., see Refs. [25–27], as well as Refs. [40–45].

This paper does not discuss the origin and evolution of anisotropy in spacetime. Instead, we treat it as a perturbation over the FLRW background. As the spatial anisotropy breaks the conformal flatness of the background in the electromagnetic (EM) field, we do not need any explicit coupling with the scalar field as proposed in the literature [16–20] for gauge field production during inflation. In this context, it is important to mention the challenges in inflationary magnetogenesis—namely, the strong coupling problem and the backreaction problem [10,16,46,47]. In the literature, several mechanisms and different types of coupling [46] have been introduced to overcome these problems. However, in our formalism, the strong coupling problem is readily solved in this paper, since no explicit coupling is involved with the gauge field. However, there might still be a possibility of backreaction, which we will study in detail as we proceed.

The paper is organized as follows: An introduction of the present paper has been written in Sec. I. In Sec. II, we describe the basic formalism and the quantization of the gauge field in an anisotropic background. In Sec. III, we show the evolution of the gauge field during the inflationary era and the strength of the present-day magnetic field under an instant reheating scenario. However, a scenario might occur when the Universe undergoes a prolonged reheating era, affecting the magnetic field. We discuss the evolution in such a scenario in Sec. IV. Finally, we discuss the findings and implications of this proposal in Sec. V.

II. THE SETUP

We introduce the spatial anisotropy in the background through the homogeneous but anisotropic Bianchi type-I metric. In general, this type of metric can be written as

$$ds^2 = a^2(\eta)[-d\eta^2 + b^2(\eta)dx^2 + dy^2 + dz^2], \tag{1}$$

where η is the conformal time, $a(\eta)$ is the overall scale factor, and $b(\eta)$ is the anisotropic factor along the x direction. In our phenomenological model, we impose the condition that the anisotropy in the spacetime exists only in the inflationary era, although the spacetime remains continuous. This ansatz guarantees that the conformal flatness is restored after inflation. These conditions can be satisfied by various models of the anisotropic factor $b(\eta)$. However, we take a particular model that satisfies all the necessary conditions:

$$b(\eta) = 1 + \alpha e^{-(\frac{\eta}{\eta_m})^2}. \tag{2}$$

In the above Eq. (2), α is a dimensionless parameter that determines the strength of the anisotropy. Furthermore, η_m is a parameter that dictates the overall behavior of the anisotropic background. An example of the anisotropic background is shown in Fig. 1. Here, α and η_m are free parameters of the anisotropic model. In this paper, we do not discuss the origin of such anisotropy. However, the anisotropy, particularly near the end of inflation, may be a combined effect of quantum field theory and the sudden breakdown of slow-roll conditions. We will come back to this issue in the future.

The action of Einstein scalar-vector theory can be given by

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right], \tag{3}$$

where G is the gravitational constant, ϕ is a scalar field, and $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the vector field $A_\mu(t, \mathbf{x})$ describing the electromagnetic field. In this paper, the dynamics of the scalar field and the metric itself due to the anisotropy present are beyond our scope. Therefore, we will mainly discuss the dynamics of the EM field during the inflationary era due to the spatially anisotropic background. Therefore, the Lagrangian of interest here is the Lagrangian corresponding to the electromagnetic field, which, according to Eq. (3), is given by

$$\mathcal{L}_{\rm em} = \frac{1}{2a^2}g^{jn}A'_jA'_n - \frac{1}{4}g^{im}g^{jn}F_{ij}F_{mn}, \tag{4}$$

where the prime denotes a derivative with respect to conformal time. In this paper, all the physical quantities are denoted with the lower index—e.g., the physical

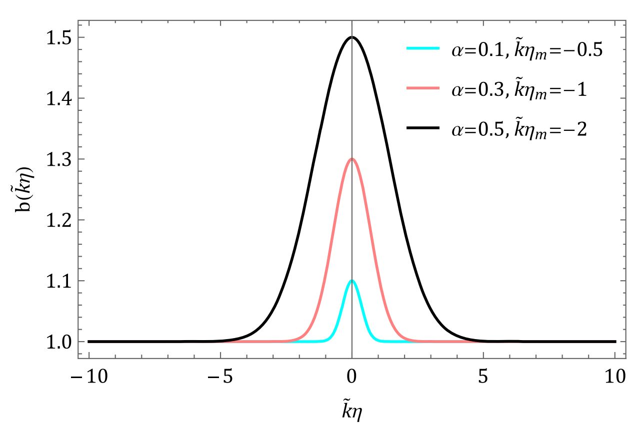

FIG. 1. Behavior of the anisotropic factor $b(\tilde{k}\eta)$ with $\tilde{k}\eta$ for anisotropic free parameters α and $\tilde{k}\eta_m$.

momentum is denoted by k_i, and the vector potential is given by A_i. We set $A_0 = 0$ as the choice of gauge, and unlike the case of conformally flat spacetime, the $\nu = 0$ component satisfies the modified constraint equation,

$$\tilde{g}^{im}\partial_i A'_m = 0, \tag{5}$$

where we have defined $\tilde{g}^{ij} = a^2 g^{ij}$ for simplicity of calculation. It can be further shown that the above equation boils down to the usual Coulomb condition for the conformally flat case. However, the raising (and lowering) of the indices is accomplished through the metric component $g^{ij}(g_{ij})$. Similarly, the dynamical equation of motion for the magnetic vector potential A_i can be calculated from the $\nu = j$ component, which boils down to

$$A''_n + \frac{b'}{b}A'_n + \tilde{g}_{jn}\tilde{g}'^{jk}A'_k - \tilde{g}^{im}\partial_i F_{mn} = 0. \tag{6}$$

It is important to note that the metric components play a crucial role in the dynamics of the field. In the case of a standard conformally flat background, the term with the metric component's derivative vanishes, giving us the regular plane wave solutions. Now, we will promote the fields and their conjugates as operators. The conjugate momentum operator corresponding to the field operator A_i turns out as

$$\Pi^i = \frac{\partial \mathcal{L}_{\rm em}}{\partial A'_i} = \frac{1}{a^2} g^{im} A'_m. \tag{7}$$

To quantize the field, we decompose the magnetic vector potential A_i as

$$A_i(\eta, \mathbf{x}) = \sum_p \int \frac{d^3k}{(2\pi)^3}\left(a_{\mathbf{k}}^{(p)} u_i^{(p)}(\eta) e^{ik_n x^n} + a_{\mathbf{k}}^{\dagger(p)} u_i^{*(p)}(\eta) e^{-ik_n x^n}\right). \tag{8}$$

In the above Eq. (8), (p) is the polarization index, and $a_{\mathbf{k}}^{(p)}$ and $a_{\mathbf{k}}^{\dagger(p)}$ are the annihilation and creation operators corresponding to the polarization mode (p). They follow the general commutation relation

$$[a_{\mathbf{k}}^{(p)}, a_{\mathbf{k}'}^{(q)}] = \delta^{pq}(2\pi)^3\delta^3(\mathbf{k} - \mathbf{k}'). \tag{9}$$

In this article, the boldface letters represent vector quantities.

In this context, it is important to discuss the commutation relation of the magnetic vector potential A_i and its conjugate momentum Π^i. We impose the commutation relation, such that the constraint of Eq. (5) on vector potential A_i is satisfied:

$$[A_i(\eta,\mathbf{x}), A_j(\eta,\mathbf{y})] = 0; \quad [\Pi_i(\eta,\mathbf{x}), \Pi_j(\eta,\mathbf{y})] = 0, \tag{10}$$

$$[A_i(\eta,\mathbf{x}), \Pi^j(\eta,\mathbf{y})] = \frac{i}{\sqrt{-g}} \int \frac{d^3k}{(2\pi)^3} e^{ik_n(x^n - y^n)} \times \left(\delta_i^j - \frac{k_i k^j}{k_n k^n}\right). \tag{11}$$

With the quantization of the field, we now get the mode function equations using Eq. (6). The mode functions satisfy the relation

$$u''_n + \frac{b'}{b}u'_n + \tilde{g}'^{jl}\tilde{g}_{jn}u'_l + \tilde{g}^{im}(k_m k_i u_n - k_n k_i u_m) = 0. \tag{12}$$

The polarization index is omitted here, as all the polarization modes follow the same equation of motion. Similarly, the constraint Eq. (5) in terms of the mode function becomes

$$\tilde{g}^{\rm in} k_i u'_n = 0. \tag{13}$$

Interestingly, Eq. (12) contains the derivative of the metric coefficients, which works as the source of particle production during the inflationary era. As the mode function equation contains the derivative of the metric components, by substituting the metric components, we get the modified mode function equations as

$$\begin{aligned}
&u''_1 - \frac{b'}{b}u'_1 + k_2^2 u_1 + k_3^2 u_1 - k_1 k_2 u_2 - k_1 k_3 u_3 = 0,\\
&u''_2 + \frac{b'}{b}u'_2 + \frac{k_1^2}{b^2}u_1 + k_3^2 u_2 - \frac{k_1 k_2}{b^2}u_1 - k_2 k_3 u_3 = 0,\\
&u''_3 + \frac{b'}{b}u'_3 + \frac{k_1^2}{b^2}u_3 + k_2^2 u_3 - \frac{k_1 k_3}{b^2}u_1 - k_2 k_3 u_2 = 0.
\end{aligned} \tag{14}$$

Moreover, all the mode functions u_1, u_2, and u_3 satisfy the constraint in Eq. (13), which explicitly boils down to

$$\frac{k_1}{b^2}u'_1 + k_2 u'_2 + k_3 u'_3 = 0. \tag{15}$$

The mode functions follow the normalization condition,

$$(u_i^{(p)}\tilde{g}^{jm}u_m^{*\prime(p)} - u_i^{*(p)}\tilde{g}^{jm}u_m^{\prime(p)}) = \frac{i}{2b}\left(\delta_i^j - \frac{k_i k^j}{k_n k^n}\right). \tag{16}$$

Utilizing the above formalism of quantization along with the constraint relation, we evolve the mode function in different phases of the Universe until the present epoch.

III. EVOLUTION OF ELECTROMAGNETIC FIELD DURING INFLATIONARY ERA

According to our model in Eq. (1), the anisotropy in the spacetime exists only towards the end of inflation. After the end of inflation, within a short period, the spacetime essentially becomes FLRW again, as seen from Fig. 1.

However, it is important to mention that the large-scale production of the EM field is not affected due to this short presence of anisotropy after inflation. It is also evident that $b \to 1$ toward past infinity ensures that the Bunch-Davis vacuum condition is satisfied in the infinite past. Furthermore, we assume the background spacetime is de Sitter in nature—i.e., $a = -1/(H\eta)$—where H is the Hubble parameter during inflation and remains constant throughout the entire inflation. Following these initial conditions, we numerically solve the mode function equations shown in Eq. (14). We consider $k_1 = k_2 = k_3 = \tilde{k} \sim 1\,\mathrm{Mpc}^{-1}$ for simplification, and the Hubble parameter $H = 10^{-5} M_{\rm pl}$ remains constant throughout the inflationary era. Here, $M_{\rm pl} = \sqrt{1/(8\pi G)}$ is the reduced Planck mass. We redefine the conformal time η as a dimensionless parameter $x = \tilde{k}\eta$. In terms of this new variable, Eq. (2) can be rewritten as

$$b(x) = 1 + \alpha e^{-(\frac{x}{x_m})^2}, \tag{17}$$

where the parameters α and $\tilde{k}\eta_m$ are chosen accordingly to avoid the backreaction from anisotropy, which essentially means that the anisotropy acts as a perturbation over the FLRW universe. A detailed discussion of the anisotropic backreaction is done in a later section. In terms of the redefined variables $x = \tilde{k}\eta, \tilde{u}_i = \sqrt{\tilde{k}} u_i$, and $\tilde{k} = k_1 = k_2 = k_3$, the mode function equations can be written as

$$\begin{aligned}
&\frac{d^2\tilde{u}_1}{dx^2} - \frac{1}{b}\frac{db}{dx}\frac{d\tilde{u}_1}{dx} + 2\tilde{u}_1 - \tilde{u}_2 - \tilde{u}_3 = 0, \\
&\frac{d^2\tilde{u}_2}{dx^2} + \frac{1}{b}\frac{db}{dx}\frac{d\tilde{u}_2}{dx} + \frac{\tilde{u}_2 - \tilde{u}_1}{b^2} - \tilde{u}_3 = 0, \\
&\frac{d^2\tilde{u}_3}{dx^2} + \frac{1}{b}\frac{db}{dx}\frac{d\tilde{u}_3}{dx} + \frac{\tilde{u}_3 - \tilde{u}_1}{b^2} - \tilde{u}_2 = 0,
\end{aligned} \tag{18}$$

along with the constraint equation

$$\frac{1}{b^2}\frac{d\tilde{u}_1}{dx} + \frac{d\tilde{u}_2}{dx} + \frac{d\tilde{u}_3}{dx} = 0. \tag{19}$$

By solving Eq. (18), we can obtain the mode function solution for different choices of the parameters α and x_m, as shown in Fig. 2. The above figure shows that the mode function grows in time due to anisotropy, particularly near the end of inflation. For values $\alpha < 0.03$, field production stops altogether. Hence, we get a lower bound on the anisotropic parameter $\alpha \geq 0.03$. The upper bound on α is discussed in later sections.

A. Power spectrum of the electromagnetic field during the inflationary era

The stress energy-momentum tensor corresponding to the produced EM field is given by

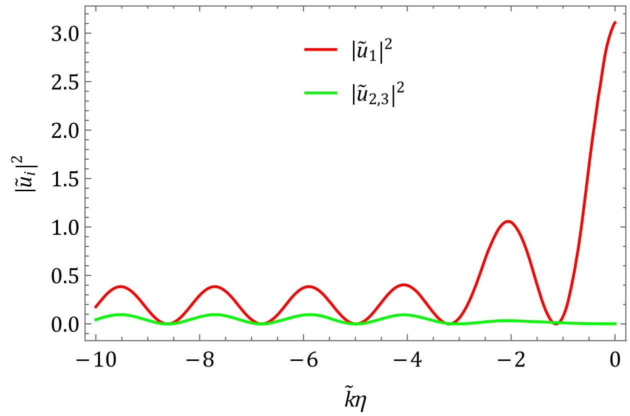

FIG. 2. Evolution of the mode functions $\tilde{u}_1$, $\tilde{u}_2$, and $\tilde{u}_3$ with $x = \tilde{k}\eta$ for the value of the anisotropic parameter $\alpha = 3$ and $x_m = -2$. As the anisotropy exists only along the x direction, the mode function equation corresponding to the x direction ($\tilde{u}_1$) behaves differently than the other two ($\tilde{u}_2, \tilde{u}_3$).

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta[\sqrt{-g}\mathcal{L}]}{\delta g^{\mu\nu}}. \tag{20}$$

As a result, the energy-momentum tensor corresponding to the electromagnetic part of the Lagrangian boils down to

$$T_{mn} = -\frac{1}{4} g_{mn} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} + g^{\mu\nu} F_{m\mu} F_{n\nu}. \tag{21}$$

The total energy density of the system is given by the T_{tt} component of the energy-momentum tensor. Therefore, the total electromagnetic energy density of the system is

$$\rho = -\langle T^0_0 \rangle = \frac{1}{2a^2} g^{ij} \langle A'_i A'_j \rangle + \frac{1}{4} g^{ij} g^{ab} \langle F_{ia} F_{jb} \rangle. \tag{22}$$

Thus, we have the electric field and magnetic field energy densities as

$$\begin{aligned}
\rho_E(x,\eta) &= \frac{1}{2a^2} g^{ij} \langle A'_i A'_j \rangle, \\
\rho_B(x,\eta) &= \frac{1}{4} g^{ij} g^{mn} \langle F_{ij} F_{mn} \rangle,
\end{aligned} \tag{23}$$

respectively, where the expectation values are taken with respect to the initial Bunch-Davies (BD) vacuum. In the momentum space, these energy densities can be written as (See Appendix A)

$$\begin{aligned}
\rho_E(k,\eta) &= \frac{1}{2a^4} \sum_p \int \frac{d^3k}{(2\pi)^3} u_i^{(p)} \tilde{g}^{ij} u_j^{*\prime(p)}, \\
\rho_B(k,\eta) &= \frac{1}{4a^4} \sum_p \int \frac{d^3k}{(2\pi)^3} \tilde{g}^{ij} \tilde{g}^{mn} \\
&\quad \times [(k_i k_n u_m^{(p)} u_j^{*(p)} - k_i k_j u_m^{(p)} u_n^{*(p)}) \\
&\quad + (k_m k_j u_i^{(p)} u_n^{*(p)} - k_m k_n u_i u_j^{*(p)})].
\end{aligned} \tag{24}$$

In order to determine the strength of the magnetic field in the present era, we first define the power spectrum of the electromagnetic field as

$$\mathcal{P}_{E/B}(k,\eta)=\frac{\partial\rho_{E/B}}{\partial\ln k}; \tag{25}$$

as already stated earlier, each polarization mode follows the same equation of motion. Therefore, all the polarization modes have equal contributions. Summing over all the polarization modes and using the assumption that the amplitudes of all the momenta $k_1=k_2=k_3=\tilde{k}$ are the same, we calculate the power spectra of the electric and magnetic fields as

$$\mathcal{P}_E(\eta,\tilde{k})=\frac{\tilde{k}^3}{2\pi^2a^4}\left(\frac{|u_1'(\eta)|^2}{b^2}+|u_2'(\eta)|^2+|u_3'(\eta)|^2\right), \tag{26}$$

$$\begin{aligned}\mathcal{P}_B(\eta,\tilde{k})=\frac{\tilde{k}^5}{2\pi^2a^4}\Bigg[&\frac{1}{b^2}(2|u_1|^2+|u_2|^2+|u_3|^2\\&-2\Re(u_1u_2^*)-2\Re(u_1u_3^*))\\&+(|u_2|^2+|u_3|^2-2\Re(u_2u_3^*))\Bigg].\end{aligned} \tag{27}$$

With these forms of the power spectrum, our goal would be to calculate its strength at present. However, before that, we will calculate the condition for which the produced electromagnetic field should not backreact to the background during inflation.

B. Backreaction of anisotropic background and generated EM field

In the previous section, we have briefly discussed the backreaction and strong coupling problem of inflationary magnetogenesis. In a general large-scale gauge field production scenario, a scalar field is coupled to the EM field to break the conformal invariance. Depending on the choice of the coupling function, it is possible to have a strong coupling problem, and different scenarios have been discussed in the literature [10,16,46,47]. For the sake of completeness, we discuss it here briefly. In order to have a sustainable production of the electromagnetic field during inflation, the coupling function is often chosen to be an increasing function of time. However, it needs to revert to unity to restore the regular Maxwellian electromagnetism at the end of inflation. Hence, it needs to be very small at the start of the inflationary era, so the effective charge of electrons will be very large, and we cannot treat the gauge field as a free field during the inflationary era. In this proposal, there is no such direct coupling between the inflaton field and the EM field. Therefore, we do not need to worry about the strong coupling issue in this scenario. However, we must ensure that the anisotropy energy density or the generated EM field does not jeopardize the inflation. To this extent, we calculate the energy density produced by the anisotropic background and get a lower bound on the anisotropic parameter $\tilde{k}\eta_m$ and α introduced in Eq. (17). The energy-momentum tensor of the background $T_{\mu\nu}$ is dictated by the Einstein equation in terms of the Einstein tensor $G_{\mu\nu}$ as

$$G_{\mu\nu}=8\pi GT_{\mu\nu}. \tag{28}$$

In the case of a Bianchi type-I background as introduced in Eq. (1), the 00 component of the Einstein tensor can be calculated as

$$G_{00}=\frac{a'(3a'b+2ab')}{a^2b}. \tag{29}$$

Thanks to this result, we can calculate the energy density corresponding to the anisotropic background. It turns out as

$$\begin{aligned}\rho_{\mathrm{total}}&=-T_0^0=-\frac{1}{8\pi G}G_0^0=\frac{1}{8\pi G}\left(3\frac{a'^2}{a^4}+2\frac{a'}{a^3}\frac{b'}{b}\right)\\&=3H^2M_{\mathrm{pl}}^2+2HM_{\mathrm{pl}}^2\frac{b'}{ab},\end{aligned} \tag{30}$$

where $H\equiv a'/a^2$ is the Hubble parameter in conformal time during the inflation, and a is the scale factor in de Sitter spacetime ($a=-1/(H\eta)$). From the dynamics of the inflaton field during inflation, we already know that the total energy of the inflaton field is given by $\rho_{\mathrm{inf}}=3H^2M_{\mathrm{pl}}^2$. Therefore, the total background energy density in Eq. (30) consists of two parts. The first part we call inflationary energy density, and the second part is the energy density due to anisotropy in the background,

$$\rho_{\mathrm{inf}}=3H^2M_{\mathrm{pl}}^2, \qquad \rho_{\mathrm{anis}}=2HM_{\mathrm{pl}}^2\frac{b'}{ab}. \tag{31}$$

In our proposition, we have mentioned earlier that the anisotropy should act as a perturbation. Therefore, we must ensure that the anisotropic energy density is much lower than the inflaton energy density. Furthermore, the electromagnetic energy density has to be lower than the anisotropic and inflaton energy densities. From the Planck data [38], we know that the temperature anisotropy in CMB is $\frac{\Delta T}{T}\sim10^{-5}$. If the anisotropic energy is closer to the perturbative limit toward the end of the inflationary era, it will not affect the CMB map, as observed by Planck. We define the e-folding number during the inflationary era as $N=\ln(\frac{a}{a_{\mathrm{end}}})$, where a_{end} is the scale factor at the end of inflation. By this definition, the e-folding number at the end of inflation $N_{\mathrm{end}}=0$. Moreover, the total e-folding number during the inflation is $N_{\mathrm{tot}}\simeq60$. The anisotropic factor b in terms of the e-folding number can be written as

$$b(N) = 1 + \alpha \exp\left[-e^{2(N_m - N)}\right], \tag{32}$$

where N_m is the e-folding number corresponding to the conformal time η_m. We can calculate the ratio of the anisotropic energy density and the inflationary energy density in terms of the e-folding number N as follows:

$$\left|\frac{\rho_{\rm anis}}{\rho_{\rm inf}}\right| = \left|\frac{2}{3H}\frac{b'}{ab}\right| = \left|\frac{2}{3}\frac{db}{dN}\frac{1}{b}\right|. \tag{33}$$

In order to have sustainable inflation, such that the anisotropic energy density does not affect the inflation energy density, we need to have $|\frac{\rho_{\rm anis}}{\rho_{\rm inf}}| < 1$ throughout the entirety of the inflation. Thus, the ratio gives us an upper bound on α, which dictates the strength of the anisotropy. In Fig. 3, we can see that the ratio of the energy densities reaches its maximum toward the end of inflation. Thus, we can choose our parameters such that the ratio is up to the perturbative level ($\sim$0.5). It gives us the upper bound $\alpha \leq 1.48$. Still, the CMB remains unaffected due to the presence of spatial anisotropy. However, it is worth mentioning here that we do not consider the dynamics of the anisotropy in the background. In order to make sure that the anisotropic background comes in toward the end of inflation, we take the upper limit on the parameter $\tilde{k}\eta_m \geq -2$. Furthermore, during inflation, the EM field also gets produced. It is also necessary to ensure that the generated gauge field energy density does not violate the inflationary energy density. We can see that the maximum production occurs toward the end of inflation from the nature of the coupling function introduced in Eq. (17). Thus, to avoid the backreaction problem, it is sufficient to satisfy

$$\rho_E + \rho_B \leq \rho_{\rm inf}.$$

We can obtain the values of the energy densities of the electric and magnetic fields from Eq. (23) and integrate over all the modes inside the horizon during the inflationary era,

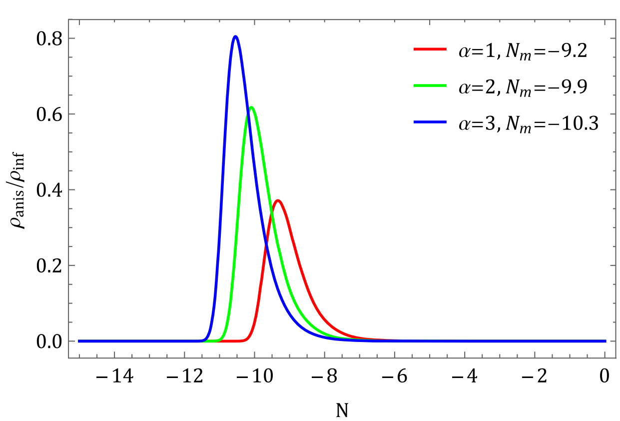

FIG. 3. Evolution of the ratio of anisotropic energy density to inflationary energy density with the e-folding number N with different anisotropic parameters α and N_m.

which finally boils down to (See Appendix B for full calculation)

$$\rho_E = \frac{H^4}{2\pi^2}\int_{x_i}^{x_f} dx x^3 \left[\frac{1}{b^2}\left|\frac{d\tilde{u}_1}{dx}\right|^2 + \left|\frac{d\tilde{u}_2}{dx}\right|^2 + \left|\frac{d\tilde{u}_3}{dx}\right|^2\right], \tag{34}$$

$$\begin{aligned}\rho_B = \frac{H^4}{\pi^2}\int_{x_i}^{x_f} dx x^3 \bigg[&\frac{1}{b^2}(2|\tilde{u}_1|^2 + |\tilde{u}_2|^2 + |\tilde{u}_3|^2 \\ &- 2\Re(\tilde{u}_1\tilde{u}_2^*) - 2\Re(\tilde{u}_1\tilde{u}_3^*)) \\ &+ (|\tilde{u}_2|^2 + |\tilde{u}_3|^2 - 2\Re(\tilde{u}_2\tilde{u}_3^*))\bigg],\end{aligned} \tag{35}$$

where we recall the variables $x = \tilde{k}\eta$, $\tilde{u}_i = \sqrt{\tilde{k}}u_i$. Evaluating the integrations numerically, the ratio of the energy densities turns out to be $\frac{\rho_{\rm E}+\rho_{\rm B}}{\rho_{\rm inf}} \sim 10^{-9}$ for the anisotropic parameter $\tilde{k}\eta_m = -1$ and $\alpha = 1.45$. As the generated electromagnetic energy density is very low compared to the background inflaton energy density, the backreaction problem is also avoided. Therefore, with this formalism, we can sustainably produce the EM field during inflation without worrying about the strong coupling or backreaction problem. On the other hand, ensuring that the generated EM field does not surpass the energy density of the inflationary background is also necessary. Equation (17) shows that the maximum energy density occurs at $\eta = 0$. However, we have stated that the inflation ends at η_f, so there is no production of the large-scale magnetic field in the postinflationary era. If the electromagnetic energy density is less than the anisotropic energy density at the end of inflation, all the sufficient conditions for no backreaction are satisfied. To this end, we reiterate that the anisotropic parameter α is so chosen that the anisotropic energy density remains subdominant compared to the inflaton energy density. We further show that the produced energy density of the electromagnetic field is less than the anisotropic energy density. In conclusion, the produced electromagnetic field affects neither the inflationary nor the anisotropic background. Therefore, this formalism effectively produces a magnetic field without special coupling to avoid the backreaction effect.

IV. POSTINFLATIONARY EVOLUTION

The anisotropic factor b goes to unity after the end of inflation, and the spacetime becomes conformally flat. The EM field evolves as a usual Maxwellian field subsequently. However, depending on the evolution of the Universe, we can have two different scenarios of field evolution: (i) In the first scenario, it is assumed that the Universe instantly goes into radiation domination—i.e., the inflaton field instantly decays and produces radiation. (ii) In the second scenario, the inflaton decays within a finite time, and therefore it goes through a brief reheating era, having a nonzero e-folding number and very low conductivity. The dynamics of the subsequent evolution of the Universe dictate the present

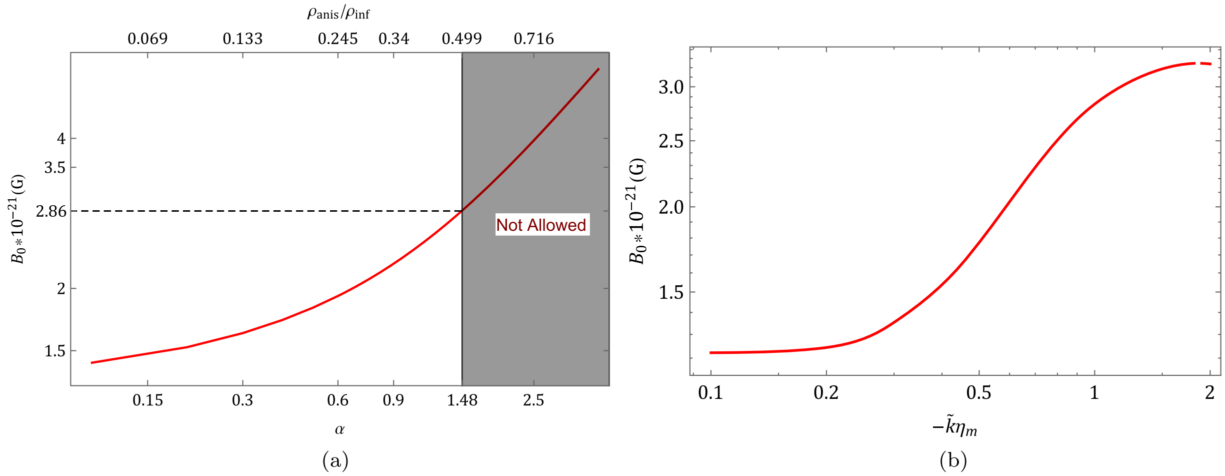

FIG. 4. (a) Variation of the present strength of the magnetic field B_0 with α for a fixed value of the anisotropic parameter $\tilde{k}\eta_m = -1$. (b) Variation of the present strength of the magnetic field B_0 with $\tilde{k}\eta_m$ for a fixed value of the anisotropic parameter $\alpha = 1.45$; the ratio of the anisotropic energy density in this case remains constant at $\rho_{\rm anis}/\rho_{\rm inf} = 0.492$.

strength of the observed magnetic field. We will discuss both scenarios in the next subsections.

A. The case of instantaneous reheating

Here in this section, we will find the strength of the magnetic field in the present time, considering an instantaneous reheating scenario. In this case, after the end of the inflationary era, the Universe instantly thermalizes and goes to the radiation-dominated era. As the conductivity of the Universe becomes very large, the electric field dies out instantly. However, the magnetic field produced during the inflationary era decays as a radiation density $\mathcal{P}_B \propto a^{-4}$. Therefore, incorporating the conservation of entropy, we can compute the strength of the magnetic field, relating it to the field strength at the end of inflation. We have already calculated the power spectra of the magnetic field during the inflationary era in Eq. (27). In terms of the mode functions, the explicit expression for the present-day magnetic field turns out as

$$B_0 = \left(\frac{-\tilde{k}\eta_f a_f H}{\pi^2 a_0}\right)^2 \sqrt{|\tilde{u}_1(x_f)|^2 + |\tilde{u}_2(x_f)|^2 + |\tilde{u}_3(x_f)|^2}. \tag{36}$$

The above expression is in units of GeV^2. Here, we recall that the variable $\tilde{u}_i = \sqrt{\tilde{k}}u_i$ (with $i = 1, 2, 3$), H is the Hubble parameter during inflation, and $\tilde{k}$ is the scale under consideration in which we will estimate the strength of the magnetic field. Furthermore, in order to calculate the value of B_0 from Eq. (36), we first need to evaluate the value of the ratio $\frac{a_f}{a_0}$. We evaluate the value to be $\frac{a_0}{a_f} \approx 10^{30}(H/10^{-5}M_{\rm pl})^{1/2}$. Here, in particular, we have taken the value of the Hubble parameter to be $H = 10^{-5}M_{\rm pl}$. With the numerical solution of the mode functions from Fig. 2 at the end of inflation $\tilde{k}\eta_f = -0.0001$ and Eq. (36), we can evaluate the strength of the magnetic field at present day using the conversion $1\mathrm{G} = 1.95 \times 10^{-20}\ \mathrm{GeV}^2$ for different values of the anisotropic parameters α and $\tilde{k}\eta_m$.

In Fig. 4, we can see the variation of the present-day magnetic field B_0 with α for a fixed value of $\tilde{k}\eta_m$, as well as the variation of magnetic field strength with $\tilde{k}\eta_m$. The maximum value of B_0 obtained in the instant reheating scenario for a fixed value of $\tilde{k}\eta_m$ is $B_0 = 2.86 \times 10^{-21}$ G by varying α. Similarly, for a fixed value of α, the maximum value of B_0 obtained is $B_0 = 3.24 \times 10^{-21}$ G. Experiments like Faraday rotation and gamma-ray observation impose a bound on the present strength of the primordial magnetic field: $10^{-10}\ \mathrm{G} \lesssim B_0 \lesssim 10^{-22}$ G [5]. Therefore, this proposal can generate a large-scale magnetic field within the experimental bound of the present-day intergalactic magnetic field. With the choice of the anisotropic parameter in the range $0.03 \leq \alpha \leq 1.48$, the backreaction or the strong coupling problem is also avoided.

B. The case of prolonged reheating with constant equation of state

In the last section, we saw that we can generate the required strength of the magnetic field in the instant reheating scenario. However, if we consider a reheating phase with a nonzero e-folding number, then the conductivity of the Universe does not reach infinity instantly. Instead, during this period, the conductivity of the Universe may remain very low. As a result, the electric field does not go to zero immediately and induces a magnetic field during this period. This conversion of the electric field into the magnetic field during the reheating phase occurs through Faraday induction [48]. This conversion of an electric field to a magnetic field makes it diluted slowly during the reheating era compared to the previous case of $\mathcal{P}_B \propto a^{-4}$. Thus, a finite reheating era further strengthens the magnetic field on a large scale and gives us bounds on the EOS during the reheating era. After the inflation ends, the anisotropic factor b goes to unity after a very short period, and the EM field evolves in the usual manner. Following the regular Maxwellian evolution, the equation of motion of the mode functions $u_i(\tilde{k},\eta)$ during the reheating becomes

$$u_i''^{(\mathrm{re})}(\tilde{k},\eta) + 3\tilde{k}^2 u_i^{(\mathrm{re})}(\tilde{k},\eta) = 0, \tag{37}$$

where $i = 1, 2, 3$ are the indices corresponding to the three spatial components of the gauge field, and $u_i^{(\mathrm{re})}(\tilde{k},\eta)$ are the mode functions during reheating. Furthermore, we consider the universe a poor conductor during this period. To be precise, we take the conductivity to be zero. The solution of the mode function from Eq. (37), along with the proper normalization condition, gives us

$$\begin{aligned} u_1^{(\mathrm{re})}(\tilde{k},\eta) &= \frac{1}{\sqrt{6\sqrt{3}\tilde{k}}}[\alpha_1(\tilde{k})e^{-i\sqrt{3}\tilde{k}(\eta-\eta_f)} \\ &\quad + \beta_1(\tilde{k})e^{i\sqrt{3}\tilde{k}(\eta-\eta_f)}], \\ u_{2,3}^{(\mathrm{re})}(\tilde{k},\eta) &= -\frac{1}{2\sqrt{6\sqrt{3}\tilde{k}}}[\alpha_{2,3}(\tilde{k})e^{-i\sqrt{3}\tilde{k}(\eta-\eta_f)} \\ &\quad + \beta_{2,3}(\tilde{k})e^{i\sqrt{3}\tilde{k}(\eta-\eta_f)}], \end{aligned} \tag{38}$$

where α_i and β_i are the integration constants, and η_f denotes the end of inflation. The integration constants are evaluated at the end of inflation η_f by equating the junction conditions of the inflationary and reheating eras:

$$u_i^{(\mathrm{re})}(\tilde{k},\eta_f) = u_i(\tilde{k},\eta_f) \quad \text{and} \quad u_i'^{(\mathrm{re})}(\tilde{k},\eta_f) = u_i'(\tilde{k},\eta_f). \tag{39}$$

In the above Eq. (39), $u_i(\tilde{k},\eta)$ are mode functions during the inflationary era which follow Eq. (14), and $u_i^{(\mathrm{re})}$ are mode functions during the reheating era following Eq. (37). This immediately leads to the integration constants

$$\begin{aligned} \alpha_1(\tilde{k}) &= \sqrt{\frac{3\sqrt{3}\tilde{k}}{2}}u_1(\tilde{k},\eta_f) + i\sqrt{\frac{\sqrt{3}}{2\tilde{k}}}u_1'(\tilde{k},\eta_f), \\ \beta_1(\tilde{k}) &= \sqrt{\frac{3\sqrt{3}\tilde{k}}{2}}\tilde{u}_1(\tilde{k},x_f) - i\sqrt{\frac{\sqrt{3}}{2\tilde{k}}}u_1'(\tilde{k},\eta_f), \\ \alpha_{2,3}(\tilde{k}) &= -\sqrt{6\sqrt{3}\tilde{k}}u_{2,3}(\tilde{k},\eta_f) - i\sqrt{\frac{2\sqrt{3}}{\tilde{k}}}u_{2,3}'(\tilde{k},\eta_f), \\ \beta_{2,3}(\tilde{k}) &= -\sqrt{6\sqrt{3}\tilde{k}}u_{2,3}(\tilde{k},\eta_f) + i\sqrt{\frac{2\sqrt{3}}{\tilde{k}}}u_{2,3}'(\tilde{k},\eta_f). \end{aligned} \tag{40}$$

With all these, we can now compute the time-evolving power spectrum during reheating as (refer to Appendix D for elaborate discussions)

$$\begin{aligned} \mathcal{P}_B(\eta,\tilde{k}) &= \frac{\tilde{k}^5}{\pi^2 a^4}(|u_1^{(\mathrm{re})}|^2 + |u_2^{(\mathrm{re})}|^2 + |u_3^{(\mathrm{re})}|^2) \\ &= \sum_i \frac{\tilde{k}^4}{\pi^2 a^4}|\tilde{u}_i^{(\mathrm{re})}|^2 \\ &= \frac{\tilde{k}^4}{\pi^2 a^4}\left[\frac{1}{6\sqrt{3}}(|\alpha_1|^2 + |\beta_1|^2 + 2|\alpha_1||\beta_1|\cos[\mathrm{Arg}(\alpha_1\beta_1^*) - 2\sqrt{3}\tilde{k}(\eta-\eta_f)])\right. \\ &\quad + \frac{1}{24\sqrt{3}}(|\alpha_2|^2 + |\beta_2|^2 + 2|\alpha_2||\beta_2|\cos[\mathrm{Arg}(\alpha_2\beta_2^*) - 2\sqrt{3}\tilde{k}(\eta-\eta_f)]) \\ &\quad \left. + \frac{1}{24\sqrt{3}}(|\alpha_3|^2 + |\beta_3|^2 + 2|\alpha_3||\beta_3|\cos[\mathrm{Arg}(\alpha_3\beta_3^*) - 2\sqrt{3}\tilde{k}(\eta-\eta_f)])\right]. \end{aligned} \tag{41}$$

In order to estimate the strength of the magnetic field during the reheating era, we first need to evaluate the term $\eta - \eta_f$. Following Ref. [48], the term is calculated as

$$\eta - \eta_f = \int_{a_f}^{a} \frac{da}{a^2 H}. \tag{42}$$

As the Hubble constant H is present in the above equation, it is evident that the quantity $\eta - \eta_f$ depends on the background's evolution during the inflationary era—in particular, how the inflaton energy density is converted into radiation energy density. In general, there are two scenarios:

(1) Evolution through a time-independent, effective equation of state.
(2) Perturbative decay of inflaton into radiation (perturbative reheating scenario).

Here in this paper, we will only discuss evolution through an independent constant effective EOS. In this context, we follow the methodology proposed by Kamionkowski *et al.*

in Ref. [49]. Here, the evolution of the background is parametrized by a constant effective EOS $\omega_{\rm eff}$. Therefore, the Hubble parameter during the reheating evolves as $H \propto a^{-\frac{3}{2}(1+\omega_{\rm eff})}$. The physical parameters of reheating, like the e-folding number of the reheating era N_{re} and the reheating temperature $T_{\rm re}$, can be expressed in terms of the inflationary parameters and effective EOS $\omega_{\rm eff}$ as [50]

$$N_{\rm re} = \frac{1}{3\omega_{\rm eff}-1}\left[\ln(\rho_f) - \ln\left(\frac{\pi^2 g_{\rm re}}{30}\right) - \frac{1}{3}\ln\left(\frac{43}{11 g_{s,\rm re}}\right) - 4\ln\left(\frac{a_0 T_0}{k}\right) + 4\ln(H_k) + 4N_k\right], \tag{43}$$

$$T_{\rm re} = \left(\frac{43}{11 g_{s,\rm re}}\right)^{1/3}\left(\frac{a_0 T_0}{k} H_k e^{-N_k} e^{-N_{\rm re}}\right), \tag{44}$$

where H_k denotes the Hubble parameter at the time of horizon crossing, $k/a_0 = 0.05\ \mathrm{Mpc}^{-1}$ is the pivot scale, $g_{\rm re}$ is the degrees of freedom during reheating, and N_k is the total e-folding number from the end of inflation till horizon crossing. As we have not considered any particular inflation potential in this paper, we develop a model-independent way to determine N_k following Ref. [51]. In the calculation of N_k (see Appendix C), we have taken the central values of the scalar spectral index $n_s = 0.9649$ and scalar perturbation amplitude $\ln[10^{10}\mathcal{A}_s] = 3.044$, considering the constraints provided by the Planck data [38], and as an input parameter we have chosen $N_k = 50$. With this choice of n_s, N_k, we get an upper bound on the effective EOS $\omega_{\rm eff} < 0.164$ from the BBN bound of reheating temperature $T_{\rm re} \sim 10^{-2}$ GeV. Now, in order to connect the reheating parameters $N_{\rm re}, T_{\rm re}$ to the strength of the primordial magnetic field, we need to evaluate the quantity $\eta - \eta_f$ in Eq. (42). It is evaluated following the evolution of the Hubble parameter during the reheating era. As the EOS is constant $\omega_{\rm eff}$, the variation of the Hubble parameter during the reheating era $(H_{\rm re})$ is related to the Hubble parameter at the end of inflation (H_f) as

$$H_{\rm re} = H_f\left(\frac{a_{\rm re}}{a_f}\right)^{-\frac{3}{2}(1+\omega_{\rm eff})}, \tag{45}$$

where the subscript "re" represents the end of reheating. Thus, $a_{\rm re}$ and $H_{\rm re}$ are the scale factor and Hubble parameter at the end of reheating, respectively. Following the above relation, the term in Eq. (42) boils down to

$$\eta - \eta_f = \frac{2}{1+3\omega_{\rm eff}}\left(\frac{1}{aH} - \frac{1}{a_f H_f}\right). \tag{46}$$

Substituting the value of the extra reheating term $\eta - \eta_f$, we can calculate the present strength of the magnetic field as a function of the effective EOS $\omega_{\rm eff}$. After the end of reheating, the conductivity of the Universe goes to infinity. Therefore, the electric field goes to zero, and the Faraday conversion of the electric field into the magnetic field stops at the end of reheating, and the magnetic field decays as radiation (a^{-4}) until now. From the conservation of magnetic energy density, the present strength of the magnetic field can be calculated from the relation as follows:

$$\left.\frac{\partial \rho_B}{\partial \ln k}\right|_0 = \left(\frac{a_{re}}{a_0}\right)^4 \left.\frac{\partial \rho_B}{\partial \ln k}\right|_{\rm re}. \tag{47}$$

Evolving through the reheating era, the strength of the magnetic field in the present era turns out to be

$$B_0 = \frac{\sqrt{2}}{6\pi\sqrt{3}}\left(\frac{\tilde{k}}{a_0}\right)^2\left[\mathcal{I}_1 + \frac{1}{4}(\mathcal{I}_2 + \mathcal{I}_3)\right]^{1/2}, \tag{48}$$

where

$$\mathcal{I}_i = |\alpha_i|^2 + |\beta_i|^2 + 2|\alpha_i||\beta_i|\cos(\mathrm{Arg}(\alpha_i\beta_i^*) - \Phi),$$
$$\Phi = \frac{4\tilde{k}\sqrt{3}}{(1+3\omega_{\rm eff})a_f H_f}\left(\left(\frac{H_f}{H_{\rm re}}\right)^{\delta} - 1\right). \tag{49}$$

In the above Eq. (48), $\delta = (3\omega_{\rm eff}+1)/(3\omega_{\rm eff}+3)$. Varying the EOS $\omega_{\rm eff}$, we get the present-day strength of the magnetic field of $B_0 \sim 4\times10^{-20}$ G, which is 1 order higher than what was predicted for the instantaneous reheating case, which is $\sim 3\times10^{-21}$ G. Furthermore, from the observed strength of the magnetic field, we also get a lower bound of the EOS $\omega_{\rm eff} > 0.132$. From Fig. 5, we see that the present strength of the magnetic field increases due to the Faraday conversion of the electric field into the magnetic field; such an increment is quite insensitive to the reheating EOS. This increment is small, since the strength of the electric field compared to the magnetic field at the end of inflation is not significantly higher.

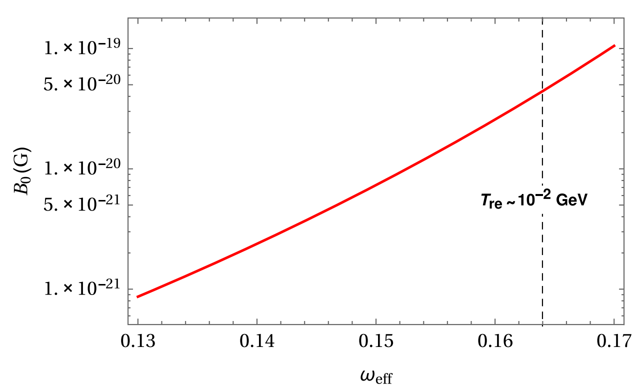

FIG. 5. Variation of present magnetic strength with effective equation of state $\omega_{\rm eff}$ for the choice of anisotropic parameters $\alpha = 1.45$ and $\tilde{k}\eta_m = -1$. The model-independent formalism input parameter $N_k = 50$, and the inflationary parameter $n_s = 0.9649$.

V. SUMMARY AND CONCLUSIONS

This paper proposes a new formalism to generate large-scale magnetic fields during the inflationary era. The novelty of the work lies in the generation of fields during inflation. Several works have been produced in the context of inflationary magnetogenesis. However, all the previous works rely on the conformal breaking coupling of the EM field with some scalar field or gravity. In the present case, we have taken the underlying background to be an anisotropic one (Bianchi type I), keeping the conformal property intact. Due to this, our model does not suffer from the usual strong coupling problem. In the process, we have introduced two parameters α and η_m to characterize the behavior of the anisotropic scale factor $b(\eta)$. By appropriately tuning those anisotropic parameters, we have further addressed the backreaction problem. If we take the ratio of the anisotropic energy density $\rho_{\rm anis}$ and the inflaton energy density $\rho_{\rm inf}$ to be $\rho_{\rm anis}/\rho_{\rm inf} \leq 0.5$, we get an upper bound of $\alpha \leq 1.48$. Furthermore, to ensure that electromagnetic field gets produced during the inflationary era, we get a lower bound on the parameter of $\alpha \geq 0.03$. The parameter η_m is so chosen that the anisotropy appears toward the end of inflation. For this, we have taken $\tilde{k}\eta_m \geq -2$, ensuring that the anisotropy is localized and short-lived. With this choice of parameters, we find that the ratio of the energy density of the generated EM field to the total inflaton energy density is $\sim 10^{-9}$, which implies that the electromagnetic energy density is also lower than the anisotropic energy density. Therefore, the generated electromagnetic field backreacts neither on the inflaton field nor on the anisotropic background. Finally, this set of parameters gives us a present strength of the magnetic field $B_0 \sim 3 \times 10^{-21}$ G, for $\alpha = 1.45$ and $\tilde{k}\eta_m = -2$, which is well within the latest bounds of present-day magnetic field strength. However, if we consider an elongated reheating period followed by inflation, the magnetic field strength further increases. This increase in strength occurs due to Faraday's conversion of the electric field to the magnetic field. By this prolonged reheating era, we get the present strength of the magnetic field $B_0 \sim 4 \times 10^{-20}$ G. Through the introduction of the reheating era, we also get a tight constraint on the range of the equation of state $0.132 < \omega_{\rm eff} < 0.164$ for the particular choice of inflationary parameters n_s and N_k. Due to the presence of anisotropy, there might be interesting signatures of the anisotropy on gravitational waves at small scales. Further, it would be most interesting to investigate the origin of such anisotropy, particularly near the end of inflation. All those questions we leave for our future study.

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APPENDIX A: POWER SPECTRUM OF THE ELECTROMAGNETIC FIELD DURING THE INFLATIONARY ERA

We have the energy-momentum tensor corresponding to the free Maxwellian Lagrangian,

$$T_{mn} = -\frac{1}{4} g_{mn} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} + g^{\mu\nu} F_{m\mu} F_{n\nu}. \quad \text{(A1)}$$

The energy density of the electromagnetic field is obtained from the "00" component of the energy-momentum tensor, which boils down to

$$T_{00} = \frac{1}{2} g^{ij} A_i' A_j' + \frac{a^2}{4} g^{im} g^{jn} F_{ij} F_{mn}. \quad \text{(A2)}$$

Upon trading the EM field into the quantum operator and referring to Eq. (23), we have the expressions for the electric and magnetic field energy densities as

$$\begin{aligned} \rho_E(x,\eta) &= \frac{1}{2a^2} g^{ij} \langle A_i' A_j' \rangle, \\ \rho_B(x,\eta) &= \frac{1}{4} g^{ij} g^{mn} \langle F_{ij} F_{mn} \rangle, \end{aligned} \quad \text{(A3)}$$

where the expectation value is obtained over the BD vacuum. The expectation value of $g^{im} g^{jn} F_{ij} F_{mn}$ in the BD vacuum in terms of mode functions boils down to

$$\begin{aligned} \langle g^{im} g^{jn} F_{ij} F_{mn} \rangle &= \sum_p \int \frac{d^3\tilde{k}}{(2\pi)^3} \left[\frac{2\tilde{k}^2}{a^4 b^2} (2|u_1|^2 + |u_2|^2 + |u_3|^2 - u_1 u_2^* - u_2 u_1^* - u_1 u_3^* - u_3 u_1^*) + \frac{2\tilde{k}^2}{a^4} (|u_2|^2 + |u_3|^2 - u_2 u_3^* - u_3 u_2^*) \right] \\ &= \sum_p \int \frac{d^3\tilde{k}}{(2\pi)^3} \left[\frac{2\tilde{k}^2}{a^4 b^2} (2|u_1|^2 + |u_2|^2 + |u_3|^2 - 2\Re(u_1 u_2^*) - 2\Re(u_1 u_3^*)) + \frac{2\tilde{k}^2}{a^4} (|u_2|^2 + |u_3|^2 - 2\Re(u_2 u_3^*)) \right]. \end{aligned} \quad \text{(A4)}$$

Substituting the expectation value of the term $g^{im}g^{jn}F_{ij}F_{mn}$ into Eq. (A3), we get the energy density of the magnetic field. With the polarization index, the expression for the magnetic field energy density turns out as

$$\rho_B(\tilde{k},\eta) = \sum_p \int \frac{d^3\tilde{k}}{(2\pi)^3}\frac{\tilde{k}^2}{2a^4}\left[\frac{1}{b^2}(2|u_1^{(p)}|^2 + |u_2^{(p)}|^2 + |u_3^{(p)}|^2 - 2\Re(u_1^{(p)}u_2^{*(p)}) - 2\Re(u_1^{(p)}u_3^{*(p)})) + (|u_2^{(p)}|^2 + |u_3^{(p)}|^2 - 2\Re(u_2^{(p)}u_3^{*(p)}))\right]. \quad \text{(A5)}$$

As all the polarization modes behave the same way, summing over all the polarization, we finally get the energy density of the magnetic field:

$$\rho_B(\tilde{k},\eta) = \int \frac{d^3\tilde{k}}{(2\pi)^3}\frac{\tilde{k}^2}{a^4}\left[\frac{1}{b^2}(2|u_1|^2 + |u_2|^2 + |u_3|^2 - 2\Re(u_1u_2^*) - 2\Re(u_1u_3^*)) + (|u_2|^2 + |u_3|^2 - 2\Re(u_2u_3^*))\right]. \quad \text{(A6)}$$

Similarly, evaluating the expectation value of the term $g^{ij}A_i'A_j'$ and substituting it back into Eq. (A3), we get the energy density of the electric field in terms of mode functions as

$$\rho_E(\tilde{k},\eta) = \int \frac{d^3\tilde{k}}{(2\pi)^3}\frac{1}{a^4}\left(\frac{|u_1'|^2}{b^2} + |u_2'|^2 + |u_3'|^2\right). \quad \text{(A7)}$$

APPENDIX B: BACKREACTION OF THE ANISOTROPIC BACKGROUND AND GENERATED ELECTROMAGNETIC FIELD

The energy-momentum tensor of the background $T_{\mu\nu}$ is dictated by the Einstein equation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad \text{(B1)}$$

where $G_{\mu\nu}$ is the Einstein tensor and G is the gravitational constant. The Einstein tensor can be calculated in terms of the Riemann tensor $(R_{\mu\nu})$ and Ricci scalar (R),

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}.$$

In the case of a Bianchi type-I background as introduced in Eq. (1), the "00" component of the Einstein tensor turns out as

$$G_{00} = \frac{a'(3a'b + 2ab')}{a^2b}. \quad \text{(B2)}$$

This essentially gives us the background energy density,

$$\rho_{\text{total}} = -T_0^0 = \frac{1}{8\pi G}\left(3\frac{a'^2}{a^4} + 2\frac{a'}{a^3}\frac{b'}{b}\right) = 3H^2M_{pl}^2 + 2HM_{pl}^2\frac{b'}{ab}. \quad \text{(B3)}$$

The ratio of anisotropic energy density to the inflaton energy density is given in terms of the e-folding number N:

$$\left|\frac{\rho_{\text{anis}}}{\rho_{\text{inf}}}\right| = \frac{2HM_{pl}^2\frac{b'}{ab}}{3H^2M_{pl}^2} = \frac{2}{3}\frac{db}{dN}\frac{1}{b},$$

with the e-folding number defined as $dN = d\ln a$, where a is the scale factor. From the above equation, we can get the ratio of the anisotropic and the inflationary energy densities.

The backreaction problem can be evaded if the total energy of the generated EM field is less than the energy density of the inflaton field—that is,

$$\rho_E + \rho_B \le \rho_{\text{inf}}. \quad \text{(B4)}$$

The total energy densities of the EM field are given by Eq. (24). Integrating all the modes, we get the total energy density. The modes involved are given by $\tilde{k}_i = a_iH$, which crosses the horizon at the beginning of inflation, and $k_f = a_fH$ are the modes that cross the horizon at the end of inflation. We have the total energy density of the electric field expressed as

$$\begin{aligned}
\rho_E &= \int_{\tilde{k}_i}^{\tilde{k}_f}\frac{d\tilde{k}}{\tilde{k}}\frac{\tilde{k}^3}{2\pi^2a^4}\left(\frac{|u_1'(\eta)|^2}{b(\eta)^2} + |u_2'(\eta)|^2 + |u_3'(\eta)|^2\right)\\
&= \frac{1}{2\pi^2}\int_{\tilde{k}_i}^{\tilde{k}_f}d\tilde{k}\frac{\tilde{k}^3}{a^4}\left(\frac{1}{b^2}\left|\frac{d\tilde{u}_1}{dx}\right|^2 + \left|\frac{d\tilde{u}_2}{dx}\right|^2 + \left|\frac{d\tilde{u}_3}{dx}\right|^2\right)\\
&= \frac{H^4}{2\pi^2}\int_{\tilde{k}_i}^{\tilde{k}_f}d(\tilde{k}\eta)(\tilde{k}^3\eta^3)\left(\frac{1}{b^2}\left|\frac{d\tilde{u}_1}{dx}\right|^2 + \left|\frac{d\tilde{u}_2}{dx}\right|^2 + \left|\frac{d\tilde{u}_3}{dx}\right|^2\right)\\
&\quad (\text{Substituted } a = -1/H\eta)\\
&= \frac{H^4}{2\pi^2}\int_{x_i}^{x_f}dx x^3\left(\frac{1}{b^2}\left|\frac{d\tilde{u}_1}{dx}\right|^2 + \left|\frac{d\tilde{u}_2}{dx}\right|^2 + \left|\frac{d\tilde{u}_3}{dx}\right|^2\right). \quad \text{(B5)}
\end{aligned}$$

Similarly, the total energy density of the magnetic field is calculated to be

$$\rho_B = \frac{H^4}{2\pi^2}\int_{x_i}^{x_f}dx x^3\left[\frac{1}{b^2}(2|\tilde{u}_1|^2 + |\tilde{u}_2|^2 + |\tilde{u}_3|^2 - 2\Re(\tilde{u}_1\tilde{u}_2^*) - 2\Re(\tilde{u}_1\tilde{u}_3^*)) + (|\tilde{u}_2|^2 + |\tilde{u}_3|^2 - 2\Re(\tilde{u}_2\tilde{u}_3^*))\right], \quad \text{(B6)}$$

where $x = \tilde{k}\eta$ and $\tilde{u}_i = \sqrt{\tilde{k}}u_i$. Integrating over the limits numerically with the solutions of the mode functions, we get the total energy density of the generated EM field.

Finally, comparing the energy density of the EM field to the inflaton field ($\rho_{\rm inf} = 3H^2M_{pl}^2$), we get

$$\frac{\rho_E + \rho_B}{\rho_{\rm inf}} \sim 10^{-9}. \tag{B7}$$

After inflation, the production of the electromagnetic field stops altogether. If we consider an instant reheating scenario, it essentially behaves as a radiation field. Thus, by conservation of entropy, we have

$$\begin{aligned} a_0^4 \frac{\partial \rho_B}{\partial \ln k}\bigg|_0 &= a_f^4 \frac{\partial \rho_B}{\partial \ln k}\bigg|_{\eta_f} \\ \Rightarrow \frac{\partial \rho_B}{\partial \ln k}\bigg|_0 &= \left(\frac{a_f}{a_0}\right)^4 \frac{\partial \rho_B}{\partial \ln k}\bigg|_{\eta_f}, \end{aligned} \tag{B8}$$

where "0" denotes the present epoch, a_0 represents the scale factor at present, η_f is the conformal time at the end of inflation, and a_f is the scale factor corresponding to η_f. Implementing Eq. (B8), we can evaluate the present-day magnetic field strength.

APPENDIX C: CALCULATION OF TOTAL e-FOLDING NUMBER OF INFLATION

We have the expression of the total e-folding number of inflation N_k from [51] as

$$N_k = \int_{t_k}^{t_f} H(t)dt. \tag{C1}$$

The Hubble parameter explicitly depends on the background evolution. Therefore, to calculate the actual Hubble parameter, we will Taylor-expand around the conformal time of horizon crossing t_k to incorporate the background effects,

$$H(t) = H_k + \dot{H}_k(t - t_k) + \frac{1}{2}\ddot{H}_k(t - t_k)^2. \tag{C2}$$

In the above Eq. (C2), we consider only terms up to $\mathcal{O}(\ddot{H}_k)$. The total duration of the inflation is represented as $\Delta t = t - t_k$. Then, by Eq. (C2), the Hubble parameter at the end of inflation can be written as

$$H_f = H_k + \dot{H}_k\Delta t + \ddot{H}_k(\Delta t)^2. \tag{C3}$$

Consequently, the duration of inflation (Δt) can be expressed in terms of the Hubble parameter and the derivative of it as

$$\Delta t = \frac{|\dot{H}_k|}{\ddot{H}_k}\left(1 - \sqrt{1 - \frac{2\ddot{H}_k}{|\dot{H}_k|^2}(H_k - H_f)}\right). \tag{C4}$$

Finally, the total e-folding number during inflation turns out to be

$$\begin{aligned} N_k &= \int_{t_k}^{t_f} H(t)dt \\ &= \frac{H_k|\dot{H}_k|}{\ddot{H}_k}\left(1 - \sqrt{1 - \frac{2\ddot{H}_k}{|\dot{H}_k|^2}(H_k - H_f)}\right) \\ &\quad - \frac{|\dot{H}_k|^3}{2\ddot{H}_k^2}\left(1 - \sqrt{1 - \frac{2\ddot{H}_k}{|\dot{H}_k|^2}(H_k - H_f)}\right)^2 \\ &\quad + \frac{|\dot{H}_k|^3}{6\ddot{H}_k^2}\left(1 - \sqrt{1 - \frac{2\ddot{H}_k}{|\dot{H}_k|^2}(H_k - H_f)}\right)^3. \end{aligned} \tag{C5}$$

The slow-roll parameters are also connected through the Hubble parameter and its derivatives. In terms of the inflationary Hubble parameter, the scalar perturbation amplitude $\mathcal{A}_s$ and the scalar spectral index n_s are related as

$$\begin{aligned} |\dot{H}_k| &= \frac{H_k^4}{4\mathcal{A}_s M_{\rm Pl}^2}, \\ \ddot{H}_k &= \frac{H_k^5}{4\mathcal{A}_s M_{\rm Pl}^2}\left(\frac{H_k^2}{\mathcal{A}_s M_{\rm Pl}^2} - (1 - n_s)\right). \end{aligned} \tag{C6}$$

Equation (C5) can also be inverted to take N_k as an input parameter, and correspondingly, we can calculate the quantity H_f. For this study, we have taken $N_k = 50$ and get $H_f \sim 10^{13}$ GeV.

APPENDIX D: MAGNETIC FIELD POWER SPECTRA DURING THE REHEATING ERA

The power spectrum of the magnetic field in the post-inflationary era becomes

$$\begin{aligned} \mathcal{P}_B(\eta, \tilde{k}) &= \frac{\tilde{k}^5}{\pi^2 a^4}(|u_1^{(\rm re)}|^2 + |u_2^{(\rm re)}|^2 + |u_3^{(\rm re)}|^2) = \frac{\tilde{k}^4}{\pi^2 a^4}\sum_i |\tilde{u}_i^{(\rm re)}|^2 \\ &= \frac{\tilde{k}^4}{\pi^2 a^4}\bigg[\frac{1}{6\sqrt{3}}(1 + 2|\beta_1|^2 + 2\sqrt{1 + |\beta_1|^2}|\beta_1| \cos[\mathrm{Arg}(\alpha_1\beta_1^*) - 2\sqrt{3}\tilde{k}(\eta - \eta_f)]) \\ &\quad + \frac{1}{24\sqrt{3}}(1 + 2|\beta_2|^2 + 2\sqrt{1 + |\beta_2|^2}|\beta_2| \cos[\mathrm{Arg}(\alpha_2\beta_2^*) - 2\sqrt{3}\tilde{k}(\eta - \eta_f)]) \\ &\quad + \frac{1}{24\sqrt{3}}(1 + 2|\beta_3|^2 + 2\sqrt{1 + |\beta_3|^2}|\beta_3| \cos[\mathrm{Arg}(\alpha_3\beta_3^*) - 2\sqrt{3}\tilde{k}(\eta - \eta_f)])\bigg], \end{aligned} \tag{D1}$$

where we have substituted the mode function solutions during the reheating era in terms of the Bogoliubov coefficients. The term $\eta - \eta_f$ is calculated as

$$\eta - \eta_f = \int \frac{da}{a^2 H}. \tag{D2}$$

Using the proper relations and substituting the value, we get the power spectrum of the magnetic field at the end of reheating as

$$\begin{aligned} \mathcal{P}_B(\tilde{k},\eta)\Big|_{\mathrm{re}} = \frac{\tilde{k}^4}{\pi^2 a_{\mathrm{re}}^4}\Bigg[\frac{1}{6\sqrt{3}}\left(1+2|\beta_1|^2+2\sqrt{1+|\beta_1|^2}|\beta_1|\cos\left[\mathrm{Arg}(\alpha_1\beta_1^*)-\frac{4\sqrt{3}\tilde{k}}{(1+3\omega_{\mathrm{eff}})a_f H_f}\left(\left(\frac{H_f}{H_{\mathrm{re}}}\right)^{\delta}-1\right)\right]\right) \\ +\frac{1}{24\sqrt{3}}\left(1+2|\beta_2|^2+2\sqrt{1+|\beta_2|^2}|\beta_2|\cos\left[\mathrm{Arg}(\alpha_2\beta_2^*)-\frac{4\sqrt{3}\tilde{k}}{(1+3\omega_{\mathrm{eff}})a_f H_f}\left(\left(\frac{H_f}{H_{\mathrm{re}}}\right)^{\delta}-1\right)\right]\right) \\ +\frac{1}{24\sqrt{3}}\left(1+2|\beta_3|^2+2\sqrt{1+|\beta_3|^2}|\beta_3|\cos\left[\mathrm{Arg}(\alpha_3\beta_3^*)-\frac{4\sqrt{3}\tilde{k}}{(1+3\omega_{\mathrm{eff}})a_f H_f}\left(\left(\frac{H_f}{H_{\mathrm{re}}}\right)^{\delta}-1\right)\right]\right)\Bigg]. \end{aligned} \tag{D3}$$

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