

Electromagnetic properties of the T_{cc}^+ molecular states

Ya-Ding Lei^{1,*} and Hao-Song Li^{1,2,3,4,†}

¹*School of Physics, Northwest University, Xian 710127, China*

²*Institute of Modern Physics, Northwest University, Xian 710127, China*

³*Shaanxi Key Laboratory for Theoretical Physics Frontiers, Xian 710127, China*

⁴*Peng Huanwu Center for Fundamental Theory, Xian 710127, China*



(Received 14 December 2023; accepted 26 March 2024; published 15 April 2024)

In this work, we discuss the electromagnetic properties of the S -wave and D -wave T_{cc}^+ molecular states, which include the magnetic moments, transition magnetic moments, and radiative decay widths. According to our results, the magnetic moment of T_{cc}^+ state observed experimentally is $-0.09\mu_N$. Meanwhile, we also discuss the relations between the transition magnetic moments of the S -wave T_{cc}^+ molecular states and the radiative decay widths, and we analyze the proportionality between the magnetic moments of the T_{cc}^+ molecular states. These results provide further information on the inner structure of T_{cc}^+ molecular states and deepen the understanding of electromagnetic properties of doubly charmed tetraquarks.

DOI: 10.1103/PhysRevD.109.076014

I. INTRODUCTION

In 2003, the Belle collaboration observed $X(3872)$, which proved the existence of exotic hadronic states [1]. The mass of $X(3872)$ is very close to the mass threshold of $D\bar{D}^*$, which promoted the theoretical studies of $D^{(*)}D^{(*)}$ states. Quantum chromodynamics (QCD) allows for the existence of multi-quark states, such as tetraquark states and pentaquark states. In 2007, the Belle collaboration reported the first nonzero charge $Z_c(4430)^+$ with quark configuration of $cu\bar{c}\bar{d}$ [2]. Subsequently, a series of tetraquark states were observed, such as $Z_c(3900)^+$ [3,4] and $Z_c(4020)^+$ [5]. These works aroused the interest of theorists in doubly heavy tetraquarks, and as experimental precision improved, more and more exotic tetraquark states were observed and studied [6–17].

In 2021, the LHCb collaboration observed the doubly charmed tetraquark state T_{cc}^+ composed of four quarks $cc\bar{u}\bar{d}$ and quantum number $J^P = 1^+$ [18,19]. This exotic state shows a narrow peak in the mass spectrum of the $D^0D^0\pi^+$ meson. The mass of the T_{cc}^+ state with respect to the $D^{*+}D^0$ mass threshold and the width are

$$\delta_{m_{\text{BW}}} = -273 \pm 61 \pm 5_{-14}^{+11} \text{ keV},$$

$$\Gamma_{m_{\text{BW}}} = 410 \pm 165 \pm 43_{-38}^{+18} \text{ keV}.$$

For the doubly charmed tetraquarks, their properties have been studied extensively in the past decades, and there exists abundant experimental and theoretical works [20–39].

Whether T_{cc}^+ is compact tetraquark states or a molecular states remains debated. For example, in Ref. [40], the author argue that T_{cc}^+ state is more likely to be a molecular state. In Ref. [41], the authors argue that T_{cc}^+ state is highly likely to be in a compact configuration. There are many theoretical studies of the interpretation of compact tetraquark states [42,43]. In addition, the interpretation of molecular state is reasonable [44–48], since the mass of the T_{cc}^+ state is very close to the $D^{*+}D^0$ mass threshold. In Ref. [49], the authors studied the DD^* hadronic molecule interpretation of T_{cc}^+ and calculated the mass and decay width of T_{cc}^+ using the one-meson exchange potential model. In Ref. [50], the authors assumed that T_{cc}^+ is an isoscalar DD^* molecule and used the effective Lagrangian method to study the partial decay width of $T_{cc}^+ \rightarrow DD\pi$, which resulted in smaller than the central experimental value of the Breit-Wigner fit. In Ref. [51], the authors studied the decay of T_{cc}^+ in the molecular interpretation using effective field theory, and calculated differential distributions in the invariant mass.

For exotic hadronic state, the magnetic moments and the transition magnetic moments are crucial observable physical quantities, which can be used to distinguish between different configurations, and the electromagnetic

*yadinglei@stumail.nwu.edu.cn

†haosongli@nwu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

properties of exotic hadronic states can also help us obtain information about their geometry.

There are a number of theoretical works around the nature of tetraquark states using different approaches, including lattice QCD [52–55], constituent quark model [56–59], and QCD sum rules [60–62]. The constituent quark model has been widely used to describe the magnetic moments of exotic hadronic state quantitatively [63–66]. In Ref. [67], the authors studied the ground state of the doubly heavy tetraquarks in the constituent quark model and estimated the magnetic moments of the predicted tetraquark states. In Ref. [68], the authors studied all possible configurations of the ground fully heavy tetraquark states in the constituent quark model and discussed the spectroscopy behaviors for the fully heavy tetraquark system.

In this work, we adopt the constituent quark model to study the electromagnetic properties of the T_{cc}^+ molecular states, including the magnetic moments, the transition magnetic moments, and the radiative decay widths, and we discuss the effect of orbital excitation on the magnetic moments.

The structure of this paper is as follows. In Sec. II, we construct the flavor-spin wave functions of the T_{cc}^+ molecular states. In Sec. III, we calculate the magnetic moments of the S -wave channel and D -wave channel T_{cc}^+ molecular states. In Sec. IV, we calculate the transition magnetic moments and the radiative decay widths of the S -wave T_{cc}^+ molecular states. Finally, we summarize our work and compare our results with those of other studies in Sec. V.

II. WAVE FUNCTION

The wave function of the hadronic state ψ can be expressed as

$$\psi = \phi_{\text{color}} \otimes \eta_{\text{flavor}} \otimes \xi_{\text{spin}} \otimes R_{\text{space}}, \quad (1)$$

where ϕ_{color} , η_{flavor} , ξ_{spin} , and R_{space} represent the color wave function, the flavor wave function, the spin wave function, and the spatial wave function, respectively.

There are four quarks, $cc\bar{q}_1\bar{q}_2$, in the T_{cc}^+ states. With the SU(2) flavor symmetry, we construct the wave functions of T_{cc}^+ states, we can obtain the flavor wave functions of the T_{cc}^+ states by adding the heavy quark c into the flavor wave functions of the two light quarks according to the structures $(Q_1\bar{q}_1)(Q_2\bar{q}_2)$ in the molecular model. Here, Q and q denote the heavy quark and the light quark, respectively. Meanwhile, not all T_{cc}^+ molecular states with combinations of spin and isospin exist. Due to Bose-Einstein symmetry, for the DD and D^*D^* states, their quantum numbers $(I + J)$ need to be odd [69].

The T_{cc}^+ molecular states are composed of $D^{(*)}D^{(*)}$. For example, it is composed of $D^{(*)0}D^{(*)+}$ with $(I, I_3) = (0, 0)$, their flavor wave functions are

$$\begin{aligned} \eta_{\text{flavor}} &= \frac{1}{\sqrt{2}}(c\bar{u})(c\bar{d}) - \frac{1}{\sqrt{2}}(c\bar{d})(c\bar{u}), \\ &= \frac{1}{\sqrt{2}}|D^{(*)0}D^{(*)+}\rangle - \frac{1}{\sqrt{2}}|D^{(*)+}D^{(*)0}\rangle. \end{aligned} \quad (2)$$

Considering the spin wave functions of the T_{cc}^+ molecular states, we can obtain the flavor-spin wave functions. For example, for the T_{cc}^+ molecular composed of D^*D^* with $J^P = 1^+$, its flavor-spin wave functions can be written as

$$\begin{aligned} |\psi\rangle &= \left\{ \frac{1}{\sqrt{2}}|D^{*0}D^{*+}\rangle - \frac{1}{\sqrt{2}}|D^{*+}D^{*0}\rangle \right\} \\ &\otimes \left\{ \frac{1}{\sqrt{2}}|1, 1\rangle|1, 0\rangle - \frac{1}{\sqrt{2}}|1, 0\rangle|1, 1\rangle \right\}. \end{aligned} \quad (3)$$

The wave functions of the T_{cc}^+ states with different isospins are similar. In Table I, we collect the flavor wave functions η_{flavor} and spin wave function ξ_{spin} of S -wave T_{cc}^+ molecular states.

TABLE I. The flavor wave functions η_{flavor} and spin wave functions ξ_{spin} of S -wave $D^{(*)}D^{(*)}$ molecular states, where I and I_3 represent the isospin and its third components of the $D^{(*)}D^{(*)}$ system, respectively, and S and S_3 represent the spin and its third components of the $D^{(*)}D^{(*)}$ system, respectively.

Systems	$ I, I_3\rangle$	η_{flavor}
$D^{(*)}D^{(*)}$	$ 0, 0\rangle$	$\frac{1}{\sqrt{2}} D^{(*)0}D^{(*)+}\rangle - \frac{1}{\sqrt{2}} D^{(*)+}D^{(*)0}\rangle$
	$ 1, 1\rangle$	$ D^{(*)+}D^{(*)+}\rangle$
	$ 1, 0\rangle$	$\frac{1}{\sqrt{2}} D^{(*)0}D^{(*)+}\rangle + \frac{1}{\sqrt{2}} D^{(*)+}D^{(*)0}\rangle$
	$ 1, -1\rangle$	$ D^{(*)0}D^{(*)0}\rangle$
Systems	$ S, S_3\rangle$	ξ_{spin}
DD	$ 0, 0\rangle$	$ 0, 0\rangle 0, 0\rangle$
DD^*	$ 1, 1\rangle$	$ 0, 0\rangle 1, 1\rangle$
	$ 1, 0\rangle$	$ 0, 0\rangle 1, 0\rangle$
	$ 1, -1\rangle$	$ 0, 0\rangle 1, -1\rangle$
D^*D^*	$ 2, 2\rangle$	$ 1, 1\rangle 1, 1\rangle$
	$ 2, 1\rangle$	$\frac{1}{\sqrt{2}} 1, 1\rangle 1, 0\rangle + \frac{1}{\sqrt{2}} 1, 0\rangle 1, 1\rangle$
	$ 2, 0\rangle$	$\frac{1}{\sqrt{6}} 1, 1\rangle 1, -1\rangle + \frac{\sqrt{2}}{\sqrt{3}} 1, 0\rangle 1, 0\rangle + \frac{1}{\sqrt{6}} 1, -1\rangle 1, 1\rangle$
	$ 2, -1\rangle$	$\frac{1}{\sqrt{2}} 1, 0\rangle 1, -1\rangle + \frac{1}{\sqrt{2}} 1, -1\rangle 1, 0\rangle$
	$ 2, -2\rangle$	$ 1, -1\rangle 1, -1\rangle$
	$ 1, 1\rangle$	$\frac{1}{\sqrt{2}} 1, 1\rangle 1, 0\rangle - \frac{1}{\sqrt{2}} 1, 0\rangle 1, 1\rangle$
	$ 1, 0\rangle$	$\frac{1}{\sqrt{2}} 1, 1\rangle 1, -1\rangle - \frac{1}{\sqrt{2}} 1, -1\rangle 1, 1\rangle$
	$ 1, -1\rangle$	$\frac{1}{\sqrt{2}} 1, 0\rangle 1, -1\rangle - \frac{1}{\sqrt{2}} 1, -1\rangle 1, 0\rangle$
	$ 0, 0\rangle$	$\frac{1}{\sqrt{3}} 1, 1\rangle 1, -1\rangle - \frac{1}{\sqrt{3}} 1, 0\rangle 1, 0\rangle + \frac{1}{\sqrt{3}} 1, -1\rangle 1, 1\rangle$

III. MAGNETIC MOMENT PROPERTIES

In constituent quark model, the magnetic moment of the hadronic state μ can be written as the sum of the spin magnetic moments μ_{spin} and orbital magnetic moments μ_{orbital} of its corresponding constituents. The total magnetic moment can be expressed as

$$\mu = \mu_{\text{spin}} + \mu_{\text{orbital}}, \quad (4)$$

where the spin magnetic moment μ_{spin} is the sum of the spins of the each constituent and orbital magnetic moment μ_{orbital} is related to the orbital angular momentum between the constituents.

In the numerical analysis, the magnetic moment μ_H can be calculated by inserting the z component of the magnetic moment operator into the corresponding wave function of the hadron, and the magnetic moment can be expressed as

$$\mu_H = \langle \psi_H | \hat{\mu}_z | \psi_H \rangle, \quad (5)$$

where $|\psi_H\rangle$ is the wave function corresponding to the hadronic state.

When only considering the S -wave channel, there only exists the spin magnetic moment, and the spin operator can be expressed as

$$\hat{\mu}_{\text{spin}} = \sum_i \frac{e_i}{2m_i} \hat{\sigma}_i, \quad (6)$$

where e_i , m_i , and σ_i represent the charge, mass and Pauli's spin operator of the i th constituent of the hadron H , respectively.

In our calculations, we consider the following S -wave and D -wave channel T_{cc}^+ molecular states

$$\begin{aligned} D^* D^* [J^P = 0^+] &: |^1 S_0\rangle, |^5 D_0\rangle, \\ D^* D^* [J^P = 1^+] &: |^3 S_1\rangle, |^3 D_1\rangle, |^5 D_1\rangle, \\ D^* D^* [J^P = 2^+] &: |^5 S_2\rangle, |^1 D_2\rangle, |^3 D_2\rangle, |^5 D_2\rangle. \end{aligned} \quad (7)$$

Here, we take $|^{2S+1} L_J\rangle$ to denote the quantum numbers of the corresponding channel, where S , L , and J represent the quantum numbers of spin angular momentum, orbital angular momentum, and total angular momentum, respectively.

According to Eq. (5), the magnetic moments of the S -wave T_{cc}^+ molecular states can be obtained. Because there are different flavor wave functions with different isospins, the magnetic moments of the S -wave T_{cc}^+ molecular states with different isospins are different. We take DD^* with $|S, S_3\rangle = |1, 1\rangle$, $(I, I_3) = (0, 0)$ as an example to obtain the magnetic moments of the S -wave $D^* D^*$ molecular state:

$$\mu_{DD^*}(1^+) = \frac{1}{2}(\mu_{D^{*0}} + \mu_{D^{*+}}). \quad (8)$$

In the above results, the magnetic moment of the S -wave T_{cc}^+ molecular state is a linear combination of the magnetic moments of the charmed mesons $D^{(*)}$. Then, we take the charmed meson D^{*+} as an example to obtain the magnetic moment of the charmed meson. In the numerical analysis, we use the constituent quark masses $m_u = 0.336$ GeV, $m_d = 0.336$ GeV, $m_c = 1.660$ GeV [70] to calculate the magnetic moments of $D^{(*)}$ mesons, the flavor-spin wave function can be written as

$$\chi_{D^{*+}}^{(1,1)} = |c\bar{d}\rangle \otimes |\uparrow\uparrow\rangle, \quad (9)$$

where χ_f^s is the flavor-spin wave function of the calculated hadron, the superscript s and the subscript f represent the spin and flavor wave functions, respectively, $\uparrow$ means the third component of the quark spin is $1/2$. According to Eq. (5), inserting the spin magnetic moment operator $\hat{\mu}_{\text{spin}}$ into the corresponding flavor-spin wave function, we can calculate the magnetic moment of the D^{*+} meson as follows:

$$\begin{aligned} \mu_{D^{*+}} &= \langle \chi_{D^{*+}}^{(1,1)} | \hat{\mu}_z | \chi_{D^{*+}}^{(1,1)} \rangle, \\ &= \langle c\bar{d}\uparrow\uparrow | \hat{\mu}_z | c\bar{d}\uparrow\uparrow \rangle, \\ &= \mu_c + \mu_{\bar{d}}. \end{aligned} \quad (10)$$

The magnetic moment of D^{*+} meson is $\mu_c + \mu_{\bar{d}}$, we can obtain the magnetic moment of D^{*0} by the same method, and we list their results in Table II.

We can obtain the magnetic moment result of the DD^* state by a linear combination of the D^* meson magnetic moments. In the same way, we calculate the magnetic moments of the T_{cc}^+ molecular states with different isospins, then the magnetic moments of the S -wave T_{cc}^+ molecular states are listed in Table III.

On the basis of the magnetic moment results obtained in above work, we summarized the following key points.

- (1) The magnetic moments of the S -wave channel T_{cc}^+ molecular states range from $-2.97\mu_N$ to $2.62\mu_N$, $D^* D^*(2^+)$ with $(I, I_3) = (1, -1)$ and $D^* D^*(2^+)$ with $(I, I_3) = (1, 1)$ have minimum and maximum magnetic moment, respectively.
- (2) The S -wave T_{cc}^+ molecular states $DD^*(1^+)$ and $D^* D^*(1^+)$ with $(I, I_3) = (0, 0)$ have different quark constituents, but their magnetic moments are the same since they have the same total angular momentum.

TABLE II. The magnetic moment of S -wave meson D^* , Here, the magnetic moment is in units of the nuclear magnetic moment μ_N .

Mesons	Expressions	Results
D^{*0}	$\mu_c + \mu_{\bar{u}}$	-1.49
D^{*+}	$\mu_c + \mu_{\bar{d}}$	1.31

TABLE III. The S -wave channel magnetic moments of the T_{cc}^+ molecular states with different flavor representations. The unit is the nuclear magnetic moment μ_N .

$\frac{1}{\sqrt{2}} D^{(*)0}D^{(*)+}\rangle - \frac{1}{\sqrt{2}} D^{(*)+}D^{(*)0}\rangle$			
Systems	J^P	Expressions	Magnetic moments
DD^*	1^+	$\frac{1}{2}(\mu_{D^{*0}} + \mu_{D^{*+}})$	-0.09
D^*D^*	1^+	$\frac{1}{2}(\mu_{D^{*0}} + \mu_{D^{*+}})$	-0.09
$ D^{(*)+}D^{(*)+}\rangle$			
Systems	J^P	Expressions	Magnetic moments
DD	0^+	0	0
DD^*	1^+	$\mu_{D^{*+}}$	1.31
D^*D^*	0^+	0	0
	2^+	$2\mu_{D^{*+}}$	2.62
$\frac{1}{\sqrt{2}} D^{(*)0}D^{(*)+}\rangle + \frac{1}{\sqrt{2}} D^{(*)+}D^{(*)0}\rangle$			
Systems	J^P	Expressions	Magnetic moments
DD	0^+	0	0
DD^*	1^+	$\frac{1}{2}(\mu_{D^{*0}} + \mu_{D^{*+}})$	-0.09
D^*D^*	0^+	0	0
	2^+	$\mu_{D^{*0}} + \mu_{D^{*+}}$	-0.18
$ D^{(*)0}D^{(*)0}\rangle$			
Systems	J^P	Expressions	Magnetic moments
DD	0^+	0	0
DD^*	1^+	$\mu_{D^{*0}}$	-1.49
D^*D^*	0^+	0	0
	2^+	$2\mu_{D^{*0}}$	-2.97

- (3) The S -wave channel T_{cc}^+ molecular states with same $I(J^P)$ and different I_3 quantum numbers have different magnetic moments, since they have different flavor wave functions as listed in Table I.
- (4) The magnetic moments of the S -wave channel DD states with $(I, I_3) = (1, 1)$, $(I, I_3) = (1, 0)$, $(I, I_3) = (1, -1)$ are all zero. At the same isospin, the magnetic moments of S -wave channel $D^*D^*(2^+)$ state and $DD^*(1^+)$ state satisfy the relation $\frac{\mu_{D^*D^*(2^+)}}{\mu_{DD^*(1^+)}} = 2$, since their magnetic moments are composed of spin magnetic moments and their spin angular momenta are proportional.

Analyzing the numerical results in Table III, since the mass of the T_{cc}^+ ground state with spin-parity quantum numbers $J^P = 1^+$ and isospin $I = 0$ is very close to the $D^{*+}D^0$ mass threshold, the magnetic moment of the T_{cc}^+ state observed experimentally is $-0.09\mu_N$.

In previous work, there exists the study of the magnetic moments of compact T_{cc}^+ tetraquark states. In Ref. [36], the

authors have already discussed the magnetic moments of the compact T_{cc}^+ tetraquark with $J^P = 1^+$ in diquark-antidiquark picture using the light cone QCD sum rules. We compare the magnetic moments of the compact T_{cc}^+ tetraquark and the T_{cc}^+ molecular states with $I(J^P) = 0(1^+)$. The value of the magnetic moment of the compact T_{cc}^+ tetraquark state with $I(J^P) = 0(1^+)$ is greater than $0.6\mu_N$, and the value of the magnetic moment of the T_{cc}^+ molecular state with $I(J^P) = 0(1^+)$ is near 0. Thus the different inner structures of doubly charmed tetraquark states lead to different magnetic moments.

Then, when we consider the magnetic moments of D -wave channel T_{cc}^+ molecular states, the total magnetic moment consists of the spin magnetic moment and the orbital magnetic moment, and the operator of orbital magnetic moment can be expressed as

$$\hat{\mu}_{\text{orbital}} = \mu_{\alpha\beta}^L \hat{L}_z = \left(\frac{m_\alpha \mu_\beta}{m_\alpha + m_\beta} + \frac{m_\beta \mu_\alpha}{m_\alpha + m_\beta} \right) \hat{L}_z, \quad (11)$$

where the subscripts α and β are constituent mesons, and we adopt the constituent mesons masses $m_{D^0} = 1864.84$ MeV, $m_{D^+} = 1869.65$ MeV, $m_{D^{*0}} = 2006.85$ MeV, and $m_{D^{*+}} = 2010.26$ MeV [71].

The spin-orbital wave function $|^{2S+1}L_J\rangle$ can be expanded using the orbital wave function Y_{L,m_L} and the spin wave function χ_{S,m_S} as follows:

$$|^{2S+1}L_J\rangle = \sum_{m_L, m_S} C_{Lm_L, Sm_S}^{JM} Y_{L,m_L} \chi_{S,m_S}, \quad (12)$$

where C_{Lm_L, Sm_S}^{JM} is the Clebsch-Gordan coefficient, then the D -wave channel spin-orbital wave function can be expressed as follows:

$$\begin{aligned}
|^3D_1\rangle &= \frac{\sqrt{3}}{\sqrt{5}} Y_{2,2} \chi_{1,-1} - \frac{\sqrt{3}}{\sqrt{10}} Y_{2,1} \chi_{1,0} + \frac{1}{\sqrt{10}} Y_{2,0} \chi_{1,1}, \\
|^5D_1\rangle &= \frac{1}{\sqrt{5}} Y_{2,2} \chi_{2,-1} - \frac{\sqrt{3}}{\sqrt{10}} Y_{2,1} \chi_{2,0} + \frac{\sqrt{3}}{\sqrt{10}} Y_{2,0} \chi_{2,1} \\
&\quad - \frac{1}{\sqrt{5}} Y_{2,-1} \chi_{2,2}, \\
|^3D_2\rangle &= \frac{\sqrt{2}}{\sqrt{3}} Y_{2,2} \chi_{1,0} - \frac{1}{\sqrt{3}} Y_{2,1} \chi_{1,1}, \\
|^5D_2\rangle &= \frac{\sqrt{2}}{\sqrt{7}} Y_{2,2} \chi_{2,0} - \frac{\sqrt{3}}{\sqrt{7}} Y_{2,1} \chi_{2,1} + \frac{\sqrt{2}}{\sqrt{7}} Y_{2,0} \chi_{2,2}, \\
|^5D_0\rangle &= \frac{1}{\sqrt{5}} Y_{2,2} \chi_{2,-2} - \frac{1}{\sqrt{5}} Y_{2,1} \chi_{2,-1} + \frac{1}{\sqrt{5}} Y_{2,0} \chi_{2,0} \\
&\quad - \frac{1}{\sqrt{5}} Y_{2,-1} \chi_{2,1} + \frac{1}{\sqrt{5}} Y_{2,-2} \chi_{2,2}. \quad (13)
\end{aligned}$$

Combining Eq. (13) and the obtained results of the magnetic moments of S -wave channel as listed in Table II,

the magnetic moments of D -wave channel can be calculated. We take the magnetic moment $\mu_{|{}^3D_1\rangle}$ with $(I, I_3) = (0, 0)$ as an example, which can be expressed as

$$\begin{aligned}\mu_{|{}^3D_1\rangle} &= \frac{3}{5} \left(-\frac{1}{2}\mu_{D^{*0}} - \frac{1}{2}\mu_{D^{*+}} + 2\mu_{D^{*0}D^{*+}}^L \right) \\ &\quad + \frac{3}{10}\mu_{D^{*0}D^{*+}}^L + \frac{1}{10} \left(\frac{1}{2}\mu_{D^{*0}} + \frac{1}{2}\mu_{D^{*+}} \right), \\ &= -\frac{1}{4}\mu_{D^{*0}} - \frac{1}{4}\mu_{D^{*+}} + \frac{3}{2}\mu_{D^{*0}D^{*+}}^L,\end{aligned}\quad (14)$$

then we list the magnetic moments of the D -wave channel with different isospins in Table IV.

Comparing the magnetic moments of the S -wave channel and D -wave channel T_{cc}^+ states, we can find that the magnetic moments of the S -wave channel and D -wave channel T_{cc}^+ states are not the same. Due to the orbital excitation of the D -wave channel, the magnetic moments of $|{}^3D_1\rangle$, $|{}^5D_1\rangle$, $|{}^3D_2\rangle$, and $|{}^5D_2\rangle$ state have a variant of 2% with respect to the S -wave magnetic moments. In the S -wave channel, the value of the $|{}^1S_2\rangle$ state is 0, but the value of the $|{}^1D_2\rangle$ state is $2\mu_{D^{*+}}^L$ in the D -wave channel. The magnetic moment of $DD^*|{}^3D_2\rangle$ state is approximately five times the magnetic moment of $DD^*|{}^3D_1\rangle$ state. Since the values of the orbital magnetic moment and the magnetic moment of D meson satisfy the proportionality relation. For DD^* state with $(I, I_3) = (1, 1)$, the value of the orbital magnetic moment $\mu_{D^{*+}D^{*+}}^L$ is close to one-half the value of the magnetic moment $\mu_{D^{*+}}$. In addition, some different states have the same value. For example, for the T_{cc}^+ states with $(I, I_3) = (1, 1)$ and $J^P = 2^+$, there exist $|{}^1D_2\rangle$, $|{}^3D_2\rangle$, $|{}^5D_2\rangle$ states. They have different expressions but the results of these D -wave magnetic moments are same, since the value of the orbital magnetic moment $\mu_{D^{*+}D^{*+}}^L$ is approximately equal to the magnetic moments $\mu_{D^{*+}}$. Therefore, the magnetic moments of these states have the same value.

We discuss the $S-D$ wave coupling of T_{cc} molecular states. Taking the DD state with $I(J^P) = 0(1^+)$ as an example, we study the $S-D$ wave coupling. The wave function can be written as

$$\begin{aligned}|\psi\rangle &\sim |DD^*, I=0, J=1\rangle \otimes Y_{0,0}R_{S1}(r) \\ &\quad + |D^*D^*, I=0, J=1\rangle \otimes Y_{0,0}R_{S2}(r) \\ &\quad + D\text{-wave contribution},\end{aligned}\quad (15)$$

where $R_{Si}(r)$ is the radial wave function of the S -wave DD state. When the cutoff parameter is 1.05 GeV, the binding energy relative to the DD^* threshold is 1.24 MeV and the corresponding root-mean-square radius is 3.11 fm which is comparable to the size of the deuteron. The dominant channel is $DD^*|{}^3S_1\rangle$, with a probability 96.39%, the contribution of the $D^*D^*|{}^3S_1\rangle$ state is 2.79%, the probability of the D wave is around 1% [45]. Since the contribution of the D -wave channel is quite small, the

TABLE IV. The D -wave magnetic moments of the T_{cc}^+ states in the molecular model with different flavor representations. The unit is the nuclear magnetic moment μ_N .

$\frac{1}{\sqrt{2}} D^{*0}D^{*+}\rangle - \frac{1}{\sqrt{2}} D^{*+}D^{*0}\rangle$				
States	J^P	Quantities	Expressions	Results
DD^*	1^+	$\mu_{ ^3D_1\rangle}$	$-\frac{1}{4}\mu_{D^{*0}} - \frac{1}{4}\mu_{D^{*+}} + \frac{3}{2}\mu_{DD^*}^L$	-0.02
	2^+	$\mu_{ ^3D_2\rangle}$	$\frac{1}{6}\mu_{D^{*0}} + \frac{1}{6}\mu_{D^{*+}} + \frac{5}{3}\mu_{DD^*}^L$	-0.10
D^*D^*	1^+	$\mu_{ ^3D_1\rangle}$	$-\frac{1}{4}\mu_{D^{*0}} - \frac{1}{4}\mu_{D^{*+}} + \frac{3}{2}\mu_{D^{*0}D^{*+}}^L$	-0.09
		$\mu_{ ^5D_1\rangle}$	$\frac{1}{4}\mu_{D^{*0}} + \frac{1}{4}\mu_{D^{*+}} + \frac{1}{2}\mu_{D^{*0}D^{*+}}^L$	-0.09
$ D^{*+}D^{*+}\rangle$				
States	J^P	Quantities	Expressions	Results
DD^*	1^+	$\mu_{ ^3D_1\rangle}$	$-\frac{1}{2}\mu_{D^{*+}} + \frac{3}{2}\mu_{D^{*+}D^{*+}}^L$	0.29
	2^+	$\mu_{ ^3D_2\rangle}$	$\frac{1}{3}\mu_{D^{*+}} + \frac{5}{3}\mu_{D^{*+}D^{*+}}^L$	1.49
D^*D^*	0^+	$\mu_{ ^5D_0\rangle}$	0	0
	2^+	$\mu_{ ^1D_2\rangle}$	$2\mu_{D^{*+}D^{*+}}^L$	2.62
		$\mu_{ ^3D_2\rangle}$	$\frac{1}{3}\mu_{D^{*+}} + \frac{5}{3}\mu_{D^{*+}D^{*+}}^L$	2.62
		$\mu_{ ^5D_2\rangle}$	$\mu_{D^{*+}} + \mu_{D^{*+}D^{*+}}^L$	2.62
$\frac{1}{\sqrt{2}} D^{*0}D^{*+}\rangle + \frac{1}{\sqrt{2}} D^{*+}D^{*0}\rangle$				
States	J^P	Quantities	Expressions	Results
DD^*	1^+	$\mu_{ ^3D_1\rangle}$	$-\frac{1}{4}\mu_{D^{*0}} - \frac{1}{4}\mu_{D^{*+}} + \frac{3}{2}\mu_{DD^*}^L$	-0.02
	2^+	$\mu_{ ^3D_2\rangle}$	$\frac{1}{6}\mu_{D^{*0}} + \frac{1}{6}\mu_{D^{*+}} + \frac{5}{3}\mu_{DD^*}^L$	-0.10
D^*D^*	0^+	$\mu_{ ^5D_0\rangle}$	0	0
	2^+	$\mu_{ ^1D_2\rangle}$	$2\mu_{D^{*0}D^{*+}}^L$	-0.18
		$\mu_{ ^3D_2\rangle}$	$\frac{1}{6}\mu_{D^{*0}} + \frac{1}{6}\mu_{D^{*+}} + \frac{5}{3}\mu_{D^{*0}D^{*+}}^L$	-0.18
		$\mu_{ ^5D_2\rangle}$	$\frac{1}{2}\mu_{D^{*0}} + \frac{1}{2}\mu_{D^{*+}} + \mu_{D^{*0}D^{*+}}^L$	-0.18
$ D^{*0}D^{*0}\rangle$				
States	J^P	Quantities	Expressions	Results
DD^*	1^+	$\mu_{ ^3D_1\rangle}$	$-\frac{1}{2}\mu_{D^{*0}} + \frac{3}{2}\mu_{D^{*0}D^{*0}}^L$	-0.34
	2^+	$\mu_{ ^3D_2\rangle}$	$\frac{1}{3}\mu_{D^{*0}} + \frac{5}{3}\mu_{D^{*0}D^{*0}}^L$	-1.70
D^*D^*	0^+	$\mu_{ ^5D_0\rangle}$	0	0
	2^+	$\mu_{ ^1D_2\rangle}$	$2\mu_{D^{*0}D^{*0}}^L$	-2.97
		$\mu_{ ^3D_2\rangle}$	$\frac{1}{3}\mu_{D^{*0}} + \frac{5}{3}\mu_{D^{*0}D^{*0}}^L$	-2.97
		$\mu_{ ^5D_2\rangle}$	$\mu_{D^{*0}} + \mu_{D^{*0}D^{*0}}^L$	-2.97

dominant channels are S -wave channels [$\int dr r^2(|R_{S1}|^2 + |R_{S2}|^2) = 99\%$]. The contribution of the D -wave channel can be neglected. Thus the influence of $S-D$ wave coupling is quite minor.

IV. TRANSITION MAGNETIC MOMENT AND RADIATIVE DECAY WIDTH

In this section, we discuss the transition magnetic moments of the S -wave T_{cc}^+ molecular states, which can provide an important reference for the study of the radiative

decay behavior of T_{cc}^+ states. For the transition magnetic moments between these discussed D^*D^* states, the calculation is similar to those used to obtain the magnetic moments of S -wave T_{cc}^+ states, except that the flavor-spin wave functions of the initial and final states are different. The transition magnetic moment of S -wave T_{cc}^+ molecular states are obtained by the following equation:

$$\mu_{H \rightarrow H'} = \langle \psi_{H'} | \hat{\mu}_z e^{-i\mathbf{k} \cdot \mathbf{r}} | \psi_H \rangle, \quad (16)$$

where $\mathbf{k}$ is the momentum of the emitted photon. When the momentum of the emitted photon is rather small, the factor $\langle R_i' | e^{-i\mathbf{k} \cdot \mathbf{r}} | R_i \rangle$ is approximately equal to 1, the spatial wave functions of the initial and final states do not affect the results of the transition magnetic moment. Thus the above equation can be approximated as

$$\mu_{H \rightarrow H'} = \langle \psi_{H'} | \hat{\mu}_z | \psi_H \rangle. \quad (17)$$

We take $\mu_{D^*D^*[2^+] \rightarrow DD^*[1^+]}$ with $(I, I_3) = (1, 0)$ as an example to illustrate the procedure of getting the transition magnetic moments between the S -wave $D^{(*)}D^{(*)}$ molecular states. According to Table I, the flavor-spin wave functions of the $D^*D^*[2^+]$ and $DD^*[1^+]$ states with $(I, I_3) = (1, 0)$ can be constructed as

$$\begin{aligned} \chi_{D^*D^*[2^+]} &= \left[\frac{1}{\sqrt{2}} |D^0 D^{*+}\rangle + \frac{1}{\sqrt{2}} |D^+ D^{*0}\rangle \right] \\ &\quad \otimes \left[\frac{1}{\sqrt{2}} |1, 1\rangle |1, 0\rangle + \frac{1}{\sqrt{2}} |1, 0\rangle |1, 1\rangle \right], \\ \chi_{DD^*[1^+]} &= \left[\frac{1}{\sqrt{2}} |D^0 D^{*+}\rangle + \frac{1}{\sqrt{2}} |D^+ D^{*0}\rangle \right] \\ &\quad \otimes |0, 0\rangle |1, 1\rangle. \end{aligned} \quad (18)$$

According to Eq. (17), inserting the magnetic moment operator $\hat{\mu}_z$ into the flavor-spin wave functions of the corresponding initial and final states, we can calculate the transition magnetic moment of the $D^*D^*[2^+] \rightarrow DD^*[1^+]\gamma$ process

$$\begin{aligned} \mu_{D^*D^*[2^+] \rightarrow DD^*[1^+]} &= \langle \chi_{DD^*[1^+]} | \hat{\mu}_z | \chi_{D^*D^*[2^+]} \rangle, \\ &= \left\langle \chi_{\frac{1}{\sqrt{2}}|D^0 D^{*+}\rangle + \frac{1}{\sqrt{2}}|D^+ D^{*0}\rangle} \left| \hat{\mu}_z \right| \chi_{\frac{1}{\sqrt{2}}|1, 1\rangle |1, 0\rangle + \frac{1}{\sqrt{2}}|1, 0\rangle |1, 1\rangle} \right\rangle, \\ &= \frac{1}{2\sqrt{2}} \mu_{D^{*+} \rightarrow D^+} + \frac{1}{2\sqrt{2}} \mu_{D^{*0} \rightarrow D^0}. \end{aligned} \quad (19)$$

In the above expression, transition magnetic moment of the S -wave T_{cc}^+ molecular states is linear combinations of the transition magnetic moments between its constituent $D^{(*)}$ mesons. We take the $D^{*+} \rightarrow D^+\gamma$ process as an example to illustrate the transition magnetic moment

TABLE V. Transition magnetic moments of S -wave charmed meson D^* . Here, the transition magnetic moment is in units of the nuclear magnetic moment μ_N .

Processes	Expressions	Results
$D^{*0} \rightarrow D^0\gamma$	$\mu_c - \mu_{\bar{u}}$	2.24
$D^{*+} \rightarrow D^+\gamma$	$\mu_c - \mu_{\bar{d}}$	-0.55

between $D^{(*)}$ mesons, we construct their flavor-spin wave functions as follows:

$$\begin{aligned} \chi_{D^{*+}}^{[1,0]} &= \frac{1}{\sqrt{2}} |c\bar{d}\rangle \otimes |\uparrow\downarrow + \downarrow\uparrow\rangle, \\ \chi_{D^{*0}}^{[0,0]} &= \frac{1}{\sqrt{2}} |c\bar{d}\rangle \otimes |\uparrow\downarrow - \downarrow\uparrow\rangle. \end{aligned} \quad (20)$$

Then, the transition magnetic moment of the $D^{*+} \rightarrow D^+\gamma$ process is

$$\begin{aligned} \mu_{D^{*+} \rightarrow D^+\gamma} &= \langle \chi_{D^+}^{[0,0]} | \hat{\mu}_z | \chi_{D^{*+}}^{[1,0]} \rangle, \\ &= \left\langle \frac{c\bar{d}\uparrow\downarrow - c\bar{d}\downarrow\uparrow}{\sqrt{2}} \left| \hat{\mu}_z \right| \frac{c\bar{d}\uparrow\downarrow + c\bar{d}\downarrow\uparrow}{\sqrt{2}} \right\rangle, \\ &= \mu_c - \mu_{\bar{d}}. \end{aligned} \quad (21)$$

As a result, the magnetic moment of the $D^{*+} \rightarrow D^+\gamma$ process is $\mu_c - \mu_{\bar{d}}$. Using the same method, we can obtain the transition magnetic moments of the $D^{*0} \rightarrow D^0\gamma$, and we collect their results in Table V.

Combining the transition magnetic moments of the $D^{*+} \rightarrow D^+\gamma$ and $D^{*0} \rightarrow D^0\gamma$ processes linearly, we obtain the transition magnetic moments of the S -wave T_{cc}^+ molecular states, which are collected in Table VI.

Analyzing the above results for the transition magnetic moments of S -wave T_{cc}^+ molecular states, we summarize several key points:

- (1) The transition magnetic moments of the T_{cc}^+ molecular states can be expressed as linear combinations of the transition magnetic moments of the corresponding constituents, for example the transition magnetic moments of the process $D^*D^*[1^+] \rightarrow DD^*[1^+]\gamma$ consists of transition magnetic moments of $D^{*+} \rightarrow D^+\gamma$ process and $D^{*0} \rightarrow D^0\gamma$ process.
- (2) The maximum transition magnetic moment is $2.24\mu_N$, corresponding to the process of $DD^*[1^+] \rightarrow DD[0^+]\gamma$ with $(I, I_3) = (1, -1)$, and the minimum transition magnetic moment is $-0.91\mu_N$, corresponding to the processes of $D^*D^*[0^+] \rightarrow DD^*[1^+]\gamma$ with $(I, I_3) = (1, -1)$.
- (3) For radiative decay processes with $I = 1$, their transition magnetic moments satisfy some proportional relations, for example

TABLE VI. The transition magnetic moments between the S -wave T_{cc}^+ molecular states with different flavor representations. Here, the magnetic moment is in unit of the nuclear magnetic moment μ_N .

$\frac{1}{\sqrt{2}} D^{(*)0}D^{(*)+}\rangle - \frac{1}{\sqrt{2}} D^{(*)+}D^{(*)0}\rangle$		
Process	Expression	Result
$D^*D^* 1^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	$-\frac{1}{2\sqrt{2}}\mu_{D^{*+}\rightarrow D^+} - \frac{1}{2\sqrt{2}}\mu_{D^{*0}\rightarrow D^0}$	-0.60
$ D^{(*)+}D^{(*)+}\rangle$		
Processes	Expressions	Results
$DD^* 1^+\rangle \rightarrow DD 0^+\rangle\gamma$	$\mu_{D^{*+}\rightarrow D^+}$	-0.55
$D^*D^* 0^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	$-\frac{1}{\sqrt{6}}\mu_{D^{*+}\rightarrow D^+}$	0.23
$D^*D^* 2^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	$\frac{1}{\sqrt{2}}\mu_{D^{*+}\rightarrow D^+}$	-0.39
$\frac{1}{\sqrt{2}} D^{(*)0}D^{(*)+}\rangle + \frac{1}{\sqrt{2}} D^{(*)+}D^{(*)0}\rangle$		
Processes	Expressions	Results
$DD^* 1^+\rangle \rightarrow DD 0^+\rangle\gamma$	$\frac{1}{2}\mu_{D^{*+}\rightarrow D^+} + \frac{1}{2}\mu_{D^{*0}\rightarrow D^0}$	0.84
$D^*D^* 0^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	$-\frac{1}{2\sqrt{6}}\mu_{D^{*+}\rightarrow D^+} - \frac{1}{2\sqrt{6}}\mu_{D^{*0}\rightarrow D^0}$	-0.34
$D^*D^* 2^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	$\frac{1}{2\sqrt{2}}\mu_{D^{*+}\rightarrow D^+} + \frac{1}{2\sqrt{2}}\mu_{D^{*0}\rightarrow D^0}$	0.60
$ D^{(*)0}D^{(*)0}\rangle$		
Processes	Expressions	Results
$DD^* 1^+\rangle \rightarrow DD 0^+\rangle\gamma$	$\mu_{D^{*0}\rightarrow D^0}$	2.24
$D^*D^* 0^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	$-\frac{1}{\sqrt{6}}\mu_{D^{*0}\rightarrow D^0}$	-0.91
$D^*D^* 2^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	$\frac{1}{\sqrt{2}}\mu_{D^{*0}\rightarrow D^0}$	1.56

$$\begin{aligned} \frac{\mu_{D^*D^*|2^+\rangle \rightarrow DD^*|1^+\rangle\gamma}}{\mu_{D^*D^*|0^+\rangle \rightarrow DD^*|1^+\rangle\gamma}} &= -\sqrt{3}, \\ \frac{\mu_{DD^*|1^+\rangle \rightarrow DD|0^+\rangle\gamma}}{\mu_{D^*D^*|2^+\rangle \rightarrow DD^*|1^+\rangle\gamma}} &= \sqrt{2}. \end{aligned}$$

The radiative widths of the S -wave T_{cc}^+ molecular states can provide important information for exploring its inner structure, so the radiative width is also a crucial physical quantity, and the radiative decay width is closely related to the transition magnetic moment. Based on the transition magnetic moments obtained above, we can calculate the radiative decay width of the S -wave T_{cc}^+ molecular state. The relationship between the radiative decay width $\Gamma_{H \rightarrow H'\gamma}$ and the transition magnetic moment can be expressed as

$$\Gamma_{H \rightarrow H'\gamma} = \begin{cases} \alpha_{\text{EM}} \frac{E_\gamma^3}{3m_p^2} \frac{J_H+1}{J_H} \frac{|\mu_{H \rightarrow H'}|^2}{\mu_N^2}, & J_H = J_{H'} \\ \alpha_{\text{EM}} \frac{E_\gamma^3}{3m_p^2} J_H \frac{|\mu_{H \rightarrow H'}|^2}{\mu_N^2}, & J_H = J_{H'} + 1 \\ \alpha_{\text{EM}} \frac{E_\gamma^3}{3m_p^2} \frac{J_{H'}(2J_{H'}+1)}{2J_H+1} \frac{|\mu_{H \rightarrow H'}|^2}{\mu_N^2}, & J_H = J_{H'} - 1 \end{cases} \quad (22)$$

In the above equation, the electromagnetic fine structure constant α_{EM} is $1/137$, and m_p is the mass of proton with

TABLE VII. The radiative decay widths between the S -wave T_{cc}^+ molecular states. Here, the radiative decay width is in units of keV.

$\frac{1}{\sqrt{2}} D^{(*)0}D^{(*)+}\rangle - \frac{1}{\sqrt{2}} D^{(*)+}D^{(*)0}\rangle$	
Process	Radiative decay width
$D^*D^* 1^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	5.24
$ D^{(*)+}D^{(*)+}\rangle$	
Processes	Radiative decay widths
$DD^* 1^+\rangle \rightarrow DD 0^+\rangle\gamma$	2.23
$D^*D^* 0^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	1.18
$D^*D^* 2^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	2.18
$\frac{1}{\sqrt{2}} D^{(*)0}D^{(*)+}\rangle + \frac{1}{\sqrt{2}} D^{(*)+}D^{(*)0}\rangle$	
Processes	Radiative decay widths
$DD^* 1^+\rangle \rightarrow DD 0^+\rangle\gamma$	5.23
$D^*D^* 0^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	2.63
$D^*D^* 2^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	5.24
$ D^{(*)0}D^{(*)0}\rangle$	
Processes	Radiative decay widths
$DD^* 1^+\rangle \rightarrow DD 0^+\rangle\gamma$	37.52
$D^*D^* 0^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	18.81
$D^*D^* 2^+\rangle \rightarrow DD^* 1^+\rangle\gamma$	36.68

$m_p = 0.938$ GeV [61]. For the $H \rightarrow H'\gamma$ process, E_γ is the momentum of the emitted photon, which can be written as

$$E_\gamma = \frac{m_H^2 - m_{H'}^2}{2m_H}, \quad (23)$$

where H and H' represent the tetraquark molecular states we discuss, and m_H and $m_{H'}$ represent the masses of the corresponding hadrons, respectively. In Table VII, the expressions and results of the S -wave T_{cc}^+ molecular states are collected.

As presented in Table VII, the effect of the transition magnetic moments on the radiative decay width of the S -wave T_{cc}^+ molecular states is significant. For example, the $D^*D^*|2^+\rangle \rightarrow DD^*|1^+\rangle\gamma$ process has a larger radiative decay width than the $D^*D^*|0^+\rangle \rightarrow DD^*|1^+\rangle\gamma$ process, since the $D^*D^*|2^+\rangle \rightarrow DD^*|1^+\rangle\gamma$ process has a larger transition magnetic moment than the $D^*D^*|0^+\rangle \rightarrow DD^*|1^+\rangle\gamma$ process. The radiative decay width of the $DD^*|1^+\rangle \rightarrow DD|0^+\rangle\gamma$ with $(I, I_3) = (1, -1)$ is much larger than that of $DD^*|1^+\rangle \rightarrow DD|0^+\rangle\gamma$ with $(I, I_3) = (1, 1)$, which is similar to the radiative decay behavior of the $D^{*0} \rightarrow D^0\gamma$ and $D^{*+} \rightarrow D^+\gamma$ processes. In addition, most of the radiative decay widths with $(I, I_3) = (1, 0)$ and $(I, I_3) = (0, 0)$ are around 5.0 keV, while the decay width of $D^*D^*|0^+\rangle \rightarrow DD^*|1^+\rangle\gamma$ process is the less than 3.0 keV, and the radiative decay widths of the processes $DD^*|1^+\rangle \rightarrow DD|0^+\rangle\gamma$,

$D^*D^*[0^+] \rightarrow DD^*[1^+]\gamma$, $D^*D^*[2^+] \rightarrow DD^*[1^+]\gamma$ with $(I, I_3) = (1, -1)$ are greater than 10.0 keV. Analyzing the numerical results, it can be noted that the values of the radiative decay widths with different isospins vary considerably.

V. SUMMARY

The observation of the T_{cc}^+ state provides a new platform to study the doubly charmed tetraquark state, theorists have done extensive research around the nature of the doubly charmed tetraquark state. The electromagnetic properties of hadronic molecular states can also help us to further understand their inner structure, but there is no more progress in related research.

In this work, we construct the flavor-spin wave functions of T_{cc}^+ molecular states using the constituent quark model. We systematically study their electromagnetic properties, including magnetic moments, transition magnetic moments, and radiative decay widths. According to our results, the magnetic moment of the T_{cc}^+ state observed experimentally is $-0.09\mu_N$. In addition, we discuss the effect of orbital excitations on the magnetic moments, and we find the D -wave magnetic moment have a variant of 2% with respect to the S -wave magnetic moments.

Meanwhile, we also discuss the radiative decay widths: the results show that the radiative decay widths are closely related to the transition magnetic moments, and there are several radiative decay processes of T_{cc}^+ molecular states with significant widths. We compare our results with other studies on the natural properties of T_{cc}^+ in Table VIII. We hope that the present study will inspire more research on the

TABLE VIII. Comparison of the quantities of the tetraquark T_{cc}^+ state in the article with the results of other studies, including nonrelativistic quark model (NQM) [23], MIT bag model [72], diffusion Monte Carlo (DMC) model [73], constituent quark model (CQM) [67], one-boson-exchange (OBE) [50], effective field theory (EFT) [51], coupled-channel effective field theory (CCEFT) [24]. Here, the magnetic moment is in units of μ_N and the radiative decay width is in units of keV.

Quantities	$\mu_{T_{cc}^+}$	$\Gamma[T_{cc}^+ \rightarrow D^+ D^0 \gamma]$
NQM [23]	0.13	...
MIT bag model [72]	0.88	...
DMC [73]	0.28	...
CQM [67]	0.732	...
OBE [50]	...	10.0
EFT [51]	...	2.8
CCEFT [24]	...	5.0
This work	-0.09	5.23

electromagnetic properties of T_{cc}^+ molecular states, which can enrich our understanding of the inner structure of hadronic molecular states.

ACKNOWLEDGMENTS

This project is supported by the National Natural Science Foundation of China under Grant No. 11905171. This work is also supported by the Natural Science Basic Research Plan in Shaanxi Province of China (Grant No. 2022JQ-025) and Shaanxi Fundamental Science Research Project for Mathematics and Physics (Grant No. 22JSQ016).

-
- [1] S. K. Choi *et al.* (Belle Collaboration), Observation of a narrow charmonium-like state in exclusive $B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi$ decays, *Phys. Rev. Lett.* **91**, 262001 (2003).
 - [2] S. K. Choi *et al.* (Belle Collaboration), Observation of a resonance-like structure in the $\pi^\pm \psi'$ mass distribution in exclusive $B \rightarrow K \pi^\pm \psi'$ decays, *Phys. Rev. Lett.* **100**, 142001 (2008).
 - [3] M. Ablikim *et al.* (BESIII Collaboration), Observation of a charged charmoniumlike structure in $e^+e^- \rightarrow \pi^+ \pi^- J/\psi$ at $\sqrt{s} = 4.26$ GeV, *Phys. Rev. Lett.* **110**, 252001 (2013).
 - [4] Z. Q. Liu *et al.* (Belle Collaboration), Study of $e^+e^- \rightarrow \pi^+ \pi^- J/\psi$ and observation of a charged charmoniumlike state at Belle, *Phys. Rev. Lett.* **110**, 252002 (2013); **111**, 019901(E) (2013).
 - [5] M. Ablikim *et al.* (BESIII Collaboration), Observation of a charged charmoniumlike structure in $e^+e^- \rightarrow (D^* \bar{D}^*)^\pm \pi^\mp$ at $\sqrt{s} = 4.26$ GeV, *Phys. Rev. Lett.* **112**, 132001 (2014).
 - [6] A. Ali, A. Y. Parkhomenko, Q. Qin, and W. Wang, Prospects of discovering stable double-heavy tetraquarks at a Tera-Z factory, *Phys. Lett. B* **782**, 412 (2018).
 - [7] Y. Xing and R. Zhu, Weak decays of stable doubly heavy tetraquark states, *Phys. Rev. D* **98**, 053005 (2018).
 - [8] A. Ali, Q. Qin, and W. Wang, Discovery potential of stable and near-threshold doubly heavy tetraquarks at the LHC, *Phys. Lett. B* **785**, 605 (2018).
 - [9] A. Francis, R. J. Hudspith, R. Lewis, and K. Maltman, Evidence for charm-bottom tetraquarks and the mass dependence of heavy-light tetraquark states from lattice QCD, *Phys. Rev. D* **99**, 054505 (2019).
 - [10] L. Maiani, A. D. Polosa, and V. Riquer, Hydrogen bond of QCD, *Phys. Rev. D* **100**, 014002 (2019).
 - [11] C. E. Fontoura, G. Krein, A. Valcarce, and J. Vijande, Production of exotic tetraquarks $QQ\bar{q}\bar{q}$ in heavy-ion collisions at the LHC, *Phys. Rev. D* **99**, 094037 (2019).
 - [12] L. Leskovec, S. Meinel, M. Pflaumer, and M. Wagner, Lattice QCD investigation of a doubly-bottom $\bar{b}\bar{b}ud$

- tetraquark with quantum numbers $I(J^P) = 0(1^+)$, *Phys. Rev. D* **100**, 014503 (2019).
- [13] P. Junnarkar, N. Mathur, and M. Padmanath, Study of doubly heavy tetraquarks in Lattice QCD, *Phys. Rev. D* **99**, 034507 (2019).
- [14] S. S. Agaev, K. Azizi, B. Barsbay, and H. Sundu, Weak decays of the axial-vector tetraquark $T_{bb,\bar{u}\bar{d}}^-$, *Phys. Rev. D* **99**, 033002 (2019).
- [15] G. Yang, J. Ping, and J. Segovia, Doubly-heavy tetraquarks, *Phys. Rev. D* **101**, 014001 (2020).
- [16] L. Tang, B. D. Wan, K. Maltman, and C. F. Qiao, Doubly heavy tetraquarks in QCD sum rules, *Phys. Rev. D* **101**, 094032 (2020).
- [17] E. Hernández, J. Vijande, A. Valcarce, and J. M. Richard, Spectroscopy, lifetime and decay modes of the T_{bb}^- tetraquark, *Phys. Lett. B* **800**, 135073 (2020).
- [18] R. Aaij *et al.* (LHCb Collaboration), Observation of an exotic narrow doubly charmed tetraquark, *Nat. Phys.* **18**, 751 (2022).
- [19] R. Aaij *et al.* (LHCb Collaboration), Study of the doubly charmed tetraquark T_{cc}^+ , *Nat. Commun.* **13**, 3351 (2022).
- [20] A. Feijoo, W. H. Liang, and E. Oset, $D^0 D^0 \pi^+$ mass distribution in the production of the T_{cc} exotic state, *Phys. Rev. D* **104**, 114015 (2021).
- [21] R. Chen, Q. Huang, X. Liu, and S. L. Zhu, Predicting another doubly charmed molecular resonance $T_{cc}^+(3876)$, *Phys. Rev. D* **104**, 114042 (2021).
- [22] L. Qiu, C. Gong, and Q. Zhao, Coupled-channel description of charmed heavy hadronic molecules within the meson-exchange model and its implication, *arXiv:2311.10067*.
- [23] C. Deng and S. L. Zhu, T_{cc}^+ and its partners, *Phys. Rev. D* **105**, 054015 (2022).
- [24] L. Meng, G. J. Wang, B. Wang, and S. L. Zhu, Probing the long-range structure of the T_{cc}^+ with the strong and electromagnetic decays, *Phys. Rev. D* **104**, 051502 (2021).
- [25] J. Z. Wang, Z. Y. Lin, and S. L. Zhu, Cut structures and an observable singularity in the three-body threshold dynamics: The T_{cc}^+ case, *arXiv:2309.09861*.
- [26] S. Noh, W. Park, and S. H. Lee, The doubly-heavy tetraquarks ($qq'\bar{Q}\bar{Q}'$) in a constituent quark model with a complete set of harmonic oscillator bases, *Phys. Rev. D* **103**, 114009 (2021).
- [27] S. S. Agaev, K. Azizi, and H. Sundu, Newly observed exotic doubly charmed meson T_{cc}^+ , *Nucl. Phys. B* **975**, 115650 (2022).
- [28] N. N. Achasov and G. N. Shestakov, Triangle singularities in the $T_{cc}^+ \rightarrow D^{*+} D^0 \rightarrow \pi^+ D^0 D^0$ decay width, *Phys. Rev. D* **105**, 096038 (2022).
- [29] U. Özdem, Magnetic moments of the doubly charged axial-vector T_{cc}^{++} states, *Phys. Rev. D* **105**, 054019 (2022).
- [30] M. Albaladejo, T_{cc}^+ coupled channel analysis and predictions, *Phys. Lett. B* **829**, 137052 (2022).
- [31] S. S. Agaev, K. Azizi, and H. Sundu, Hadronic molecule model for the doubly charmed state T_{cc}^+ , *J. High Energy Phys.* **06** (2022) 057.
- [32] E. Braaten, L. P. He, K. Ingles, and J. Jiang, Triangle singularity in the production of $T_{cc}^+(3875)$ and a soft pion, *Phys. Rev. D* **106**, 034033 (2022).
- [33] L. M. Abreu, H. P. L. Vieira, and F. S. Navarra, Multiplicity of the doubly charmed state T_{cc}^+ in heavy-ion collisions, *Phys. Rev. D* **105**, 116029 (2022).
- [34] Y. Hu, J. Liao, E. Wang, Q. Wang, H. Xing, and H. Zhang, Production of doubly charmed exotic hadrons in heavy ion collisions, *Phys. Rev. D* **104**, L111502 (2021).
- [35] Y. Huang, H. Q. Zhu, L. S. Geng, and R. Wang, Production of T_{cc}^+ exotic state in the $\gamma p \rightarrow D^+ \bar{T}_{cc}^- \Lambda_c^+$ reaction, *Phys. Rev. D* **104**, 116008 (2021).
- [36] K. Azizi and U. Özdem, Magnetic dipole moments of the T_{cc}^+ and $Z_{\bar{V}}^{++}$ tetraquark states, *Phys. Rev. D* **104**, 114002 (2021).
- [37] Q. Xin and Z. G. Wang, Analysis of the doubly-charmed tetraquark molecular states with the QCD sum rules, *Eur. Phys. J. A* **58**, 110 (2022).
- [38] K. Azizi and U. Özdem, Exploring the magnetic dipole moments of $T_{QQ\bar{q}\bar{s}}$ and $T_{QQ\bar{s}\bar{s}}$ states in the framework of QCD light-cone sum rules, *J. High Energy Phys.* **03** (2023) 166.
- [39] Z. G. Wang, Analysis of the axialvector doubly heavy tetraquark states with QCD sum rules, *Acta Phys. Pol. B* **49**, 1781 (2018).
- [40] H. Yun, D. Park, S. Noh, A. Park, W. Park, S. Cho, J. Hong, Y. Kim, S. Lim, and S. H. Lee, $X(3872)$ and T_{cc} : Structures and productions in heavy ion collisions, *Phys. Rev. C* **107**, 014906 (2023).
- [41] S. Noh and W. Park, Nonrelativistic quark model analysis of T_{cc} , *Phys. Rev. D* **108**, 014004 (2023).
- [42] A. Esposito, C. A. Manzarì, A. Pilloni, and A. D. Polosa, Hunting for tetraquarks in ultraperipheral heavy ion collisions, *Phys. Rev. D* **104**, 114029 (2021).
- [43] A. Esposito, L. Maiani, A. Pilloni, A. D. Polosa, and V. Riquer, From the line shape of the $X(3872)$ to its structure, *Phys. Rev. D* **105**, L031503 (2022).
- [44] Z. S. Jia, M. J. Yan, Z. H. Zhang, P. P. Shi, G. Li, and F. K. Guo, Hadronic decays of the heavy-quark-spin molecular partner of T_{cc}^+ , *Phys. Rev. D* **107**, 074029 (2023).
- [45] N. Li, Z. F. Sun, X. Liu, and S. L. Zhu, Coupled-channel analysis of the possible $D^{(*)} D^{(*)}$, $\bar{B}^{(*)} \bar{B}^{(*)}$ and $D^{(*)} \bar{B}^{(*)}$ molecular states, *Phys. Rev. D* **88**, 114008 (2013).
- [46] M. J. Yan and M. P. Valderrama, Subleading contributions to the decay width of the T_{cc}^+ tetraquark, *Phys. Rev. D* **105**, 014007 (2022).
- [47] Z. M. Ding, J. He, and X. Liu, New reaction approach to reflect exotic structure of hadronic molecular state, *Phys. Rev. D* **107**, 094012 (2023).
- [48] P. G. Ortega, J. Segovia, D. R. Entem, and F. Fernandez, Nature of the doubly-charmed tetraquark T_{cc}^+ in a constituent quark model, *Phys. Lett. B* **841**, 137918 (2023); **847**, 138308(E) (2023).
- [49] H. Ren, F. Wu, and R. Zhu, Hadronic molecule interpretation of T_{cc}^+ and its beauty partners, *Adv. High Energy Phys.* **2022**, 9103031 (2022).
- [50] X. Z. Ling, M. Z. Liu, L. S. Geng, E. Wang, and J. J. Xie, Can we understand the decay width of the T_{cc}^+ state?, *Phys. Lett. B* **826**, 136897 (2022).
- [51] S. Fleming, R. Hodges, and T. Mehen, T_{cc}^+ decays: Differential spectra and two-body final states, *Phys. Rev. D* **104**, 116010 (2021).

- [52] P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, and M. Wagner, $ud\bar{b}\bar{b}$ tetraquark resonances with lattice QCD potentials and the Born-Oppenheimer approximation, *Phys. Rev. D* **96**, 054510 (2017).
- [53] G. Yang, J. Ping, and J. Segovia, Exotic resonances of fully-heavy tetraquarks in a lattice-QCD inspired quark model, *Phys. Rev. D* **104**, 014006 (2021).
- [54] M. Sadl and S. Prelovsek, Tetraquark systems $\bar{b}b\bar{d}u$ in the static limit and lattice QCD, *Phys. Rev. D* **104**, 114503 (2021).
- [55] M. Padmanath and S. Prelovsek, Signature of a doubly charm tetraquark pole in DD^* scattering on the lattice, *Phys. Rev. Lett.* **129**, 032002 (2022).
- [56] E. J. Eichten and C. Quigg, Heavy-quark symmetry implies stable heavy tetraquark mesons $Q_i Q_j \bar{q}_k \bar{q}_l$, *Phys. Rev. Lett.* **119**, 202002 (2017).
- [57] Y. Tan and J. Ping, $Y(4626)$ in a chiral constituent quark model, *Phys. Rev. D* **101**, 054010 (2020).
- [58] C. Deng, H. Chen, and J. Ping, Towards the understanding of fully-heavy tetraquark states from various models, *Phys. Rev. D* **103**, 014001 (2021).
- [59] Z. Zhao, K. Xu, A. Kaewsnod, X. Liu, A. Limphirat, and Y. Yan, Study of charmoniumlike and fully-charm tetraquark spectroscopy, *Phys. Rev. D* **103**, 116027 (2021).
- [60] M. L. Du, W. Chen, X. L. Chen, and S. L. Zhu, Exotic $QQ\bar{q}\bar{q}$, $QQ\bar{q}\bar{s}$ and $QQ\bar{s}\bar{s}$ states, *Phys. Rev. D* **87**, 014003 (2013).
- [61] W. Chen, T. G. Steele, and S. L. Zhu, Exotic open-flavor $bc\bar{q}\bar{q}$, $bc\bar{s}\bar{s}$ and $qc\bar{q}\bar{b}$, $sc\bar{s}\bar{b}$ tetraquark states, *Phys. Rev. D* **89**, 054037 (2014).
- [62] N. Su and H. X. Chen, S- and P-wave fully strange tetraquark states from QCD sum rules, *Phys. Rev. D* **106**, 014023 (2022).
- [63] H. Y. Zhou, F. L. Wang, Z. W. Liu, and X. Liu, Probing the electromagnetic properties of the $\Sigma_c^{(*)}D^{(*)}$ -type doubly charmed molecular pentaquarks, *Phys. Rev. D* **106**, 034034 (2022).
- [64] F. Gao and H. S. Li, Magnetic moments of hidden-charm strange pentaquark states, *Chin. Phys. C* **46**, 123111 (2022).
- [65] F. L. Wang, H. Y. Zhou, Z. W. Liu, and X. Liu, What can we learn from the electromagnetic properties of hidden-charm molecular pentaquarks with single strangeness?, *Phys. Rev. D* **106**, 054020 (2022).
- [66] F. L. Wang, S. Q. Luo, H. Y. Zhou, Z. W. Liu, and X. Liu, Exploring the electromagnetic properties of the $\Xi_c^{(\prime,*)}\bar{D}_s^*$ and $\Omega_c^{(*)}\bar{D}_s^*$ molecular states, *Phys. Rev. D* **108**, 034006 (2023).
- [67] T. W. Wu and Y. L. Ma, Doubly heavy tetraquark multiplets as heavy antiquark-diquark symmetry partners of heavy baryons, *Phys. Rev. D* **107**, L071501 (2023).
- [68] H. T. An, S. Q. Luo, Z. W. Liu, and X. Liu, Spectroscopic behavior of fully heavy tetraquarks, *Eur. Phys. J. C* **83**, 740 (2023).
- [69] F. Z. Peng, M. J. Yan, and M. Pavon Valderrama, Heavy- and light-flavor symmetry partners of the $T_{cc}^+(3875)$, the $X(3872)$, and the $X(3960)$ from light-meson exchange saturation, *Phys. Rev. D* **108**, 114001 (2023).
- [70] G. J. Wang, L. Meng, H. S. Li, Z. W. Liu, and S. L. Zhu, Magnetic moments of the spin- $\frac{1}{2}$ singly charmed baryons in chiral perturbation theory, *Phys. Rev. D* **98**, 054026 (2018).
- [71] R. L. Workman *et al.* (Particle Data Group), Review of particle physics, *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).
- [72] W. X. Zhang, H. Xu, and D. Jia, Masses and magnetic moments of hadrons with one and two open heavy quarks: Heavy baryons and tetraquarks, *Phys. Rev. D* **104**, 114011 (2021).
- [73] H. Mutuk, Masses and magnetic moments of doubly heavy tetraquarks via diffusion Monte Carlo method, [arXiv:2312.13383](https://arxiv.org/abs/2312.13383).