Investigation of the mass spectra of singly heavy baryons Σ_Q, Ξ'_Q, and $\Omega_Q(Q=c,b)$ in the Regge trajectory model

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Very recently, LHCb Collaboration observed that two new Ω_c^0 states decay into $\Xi_c^+ K^-$ with masses of about 3185 MeV and 3327 MeV. However, their spin-parity quantum numbers J^P have not been determined. In this paper, we exploit the quark-diquark model, the linear Regge trajectory and the perturbation treatment method to analyze the mass spectra of the discovered experimental data for the singly heavy baryons Σ_c/Σ_b, Ξ'_c/Ξ'_b, and Ω_c/Ω_b. In addition, we further predict the mass spectra of several unobserved Σ_c/Σ_b, Ξ'_c/Ξ'_b, and Ω_c/Ω_b baryons. In the case of the $\Omega_c(3185)^0$ and $\Omega_c(3327)^0$ states, we determine $\Omega_c(3185)^0$ as $2S$ state and $\Omega_c(3327)^0$ as $1D$ state with $J^P = 1/2^+$ and $J^P = 3/2^+$, respectively. An overall good agreement of the obtained predictions with available experimental data are found.

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I. INTRODUCTION

With the discovery of more and more highly excited strongly interacting particles in experiments, such as LHCb, Belle, *BABAR*, and CLEO, a deeper understanding of the singly heavy baryons has been gained. In the quark-diquark picture, singly heavy baryons are composed of an anticolor triplet ($\bar{3}_c$) diquark with spin-1 ($S_d = 1$), formed by two light quarks, and a heavy quark ($S_Q = 1/2$). The latest review of particle physics by PDG can shed new light on the singly heavy baryons Σ_c/Σ_b, Ξ'_c/Ξ'_b, and Ω_c/Ω_b.

From PDG [1] in 2022, the establishment of S-, P- and D-wave excited states are gradually improved providing valuable insights into the fundamental structure and behavior for the Σ_c/Σ_b, Ξ'_c/Ξ'_b, and Ω_c/Ω_b baryons. In the Σ_c/Σ_b baryons, the $\Sigma_c(2455)^{0,+,++}$ and $\Sigma_c(2520)^{0,+,++}$ states can be well-interpreted as S-wave charmed baryons with $J^P = 1/2^+$, and $J^P = 3/2^+$, respectively. The triplet of the excited $\Sigma_c(2800)^{0,+,++}$ states decaying to $\Lambda_c^+\pi$ were observed by Belle Collaboration in 2005 [2]. The four ground states $\Sigma_b(5815)^{-+}$ and $\Sigma_b^*(5835)^{-+}$ of Σ_b have been observed by the CDF Collaboration in Ref. [3] with $J^P = 1/2^+$ and $J^P = 3/2^+$, respectively. In the Ξ'_c/Ξ'_b baryons, the neutral state Ξ_c^0 and its charged partner $\Xi_c(2645)^+$ were reported by CLEO in the decay channels $\Xi_c^+\pi^-$ [4] and $\Xi_c^0\pi^+$ [5] as S-wave states with $J^P = 1/2^+$, and $J^P = 3/2^+$, respectively. However, the J^P of $\Xi_c(2923)$ [6] and $\Xi_c(2930)^+$ [7], which are good candidates for P-wave states, are yet to be determined. Similarly, LHCb observed two new charged $\Xi'_b(5935)^-$ and $\Xi_b^*(5955)^-$ states of the Ξ'_b baryons in Ref. [8]. They were proposed to be the $J^P = 1/2^+$, and $J^P = 3/2^+$ ground states. Note that the Ξ'_b baryon has only one neutral state $\Xi_b(5945)^0$ with $J^P = 3/2^+$ based on quark-model expectations. Therefore, the discovery of these singly heavy baryons have great significance for research.

In the study of Ω_c, only two ground states Ω_c^0 and $\Omega_c(2770)^0$ were discovered experimentally with $J^P = 1/2^+$, and $J^P = 3/2^+$, respectively. In 2017, the LHCb Collaboration reported five new narrow excited states of Ω_c in the decay channel $\Xi_c^+ K^-$ [9] which were later confirmed by Belle [10] with interesting spin-parity properties and inner structures. For a discussion of the excited Ω_c states we refer to Refs. [11–22] or recent explorations given in Refs. [23–27]. In 2020, the LHCb Collaboration reported the discovery of four narrow excited Ω_b states in the decay channel $\Xi_b^0 K^-$ [28]. In Refs. [29,30], the authors used the constituent quark model to obtain masses compatible with the experiment. Very recently, LHCb observed two new narrow Ω_c states decaying into $\Xi_c^+ K^-$ [31] with masses of $\Omega_c(3185)^0$ and $\Omega_c(3327)^0$ about 3185 MeV and 3327 MeV. The value of J^P for the newly discovered states remains unclear.

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In this paper, we study the mass spectra of the singly heavy baryons Σ_c/Σ_b, Ξ'_c/Ξ'_b, and Ω_c/Ω_b from the Regge trajectory and the spin-dependent potential. By analyzing the Regge trajectory formula, we get the spin-average masses of the baryons. In addition, to obtain the mass shifts, we exploit new scaling relations to calculate the spin-coupling parameters. In the end, the properties of the charmed baryons and the bottom baryons will be discussed.

This paper is organized as follows. We analyze the Regge trajectory formula to give the spin-average mass $\bar{M}$ of excited states of the Σ_c/Σ_b, Ξ'_c/Ξ'_b, and Ω_c/Ω_b baryons in Sec. II. In Sec. III, we review about the spin-dependent Hamiltonian and the scaling relations. We calculate the mass spectra of the Ω_c/Ω_b baryons in Sec. IV. In Sec. V, we discuss the mass spectra of the Σ_c/Σ_b baryons. In Sec. VI, a similar mass analysis is given for the Ξ'_c/Ξ'_b baryons. Finally, we outline our conclusion in Sec. VII.

II. THE REGGE TRAJECTORY AND THE SPIN-AVERAGE MASSES

In the QCD rotating-string model [32,33], the strong interaction binds the heavy and light quarks inside the hadron, where one end of the string is a heavy quark and the other is a light antiquark or light diquark moving around the heavy quark. Based on this model, it is interesting to investigate the Regge trajectory behavior of the hadronic system.

For the orbital excitations of the baryons, we obtain the spin-average mass $\bar{M}$ and angular momentum L following the equations given by Refs. [34,35]:

$$\bar{M} = \frac{m_{\mathrm{cur}Q}}{\sqrt{1-v_Q^2}} + \frac{\alpha}{\omega}\int_0^{v_Q}\frac{du}{\sqrt{1-u^2}} + \frac{m_{\mathrm{cur}d}}{\sqrt{1-v_d^2}} + \frac{\alpha}{\omega}\int_0^{v_d}\frac{du}{\sqrt{1-u^2}}, \quad (1)$$

$$L = \frac{m_{\mathrm{cur}Q}v_Q^2}{\sqrt{1-v_Q^2}} + \frac{\alpha}{\omega^2}\int_0^{v_Q}\frac{u^2du}{\sqrt{1-u^2}} + \frac{m_{\mathrm{cur}d}v_d^2}{\sqrt{1-v_d^2}} + \frac{\alpha}{\omega^2}\int_0^{v_d}\frac{u^2du}{\sqrt{1-u^2}}, \quad (2)$$

where α is the QCD string tension coefficient, and v_Q, v_d the velocity of the string end tied to between the heavy quark Q and light diquark d. We define the velocity $v_i = \omega r_i$ ($i = Q, d$), where ω and r_i are the angular velocity and the position from the center-of-mass, respectively. For simplicity, we have chosen the velocity of light c = 1. The light diquark is ultrarelativistic, we take the velocity of light diquark $v_d \approx 1$ for approximation. Then $m_{\mathrm{cur}Q}$ and $m_{\mathrm{cur}d}$ can be regarded as current mass of the heavy quark and light diquark, respectively. Including relativistic effects, one can obtain the constituent quark masses

$$M_Q = \frac{m_{\mathrm{cur}Q}}{\sqrt{1-v_Q^2}}, \qquad m_d = \frac{m_{\mathrm{cur}d}}{\sqrt{1-v_d^2}}. \quad (3)$$

Equations (1) and (2) can be integrated to give

$$\bar{M} = M_Q + m_d + M_Q v_Q^2 + \frac{\pi\alpha}{2\omega}, \quad (4)$$

$$L = \frac{1}{\omega}\left(m_d + M_Q v_Q^2 + \frac{\pi\alpha}{4\omega}\right), \quad (5)$$

where for the string ending at the heavy quark we use the boundary condition,

$$\frac{\alpha}{\omega} = \frac{m_{\mathrm{cur}Q}v_Q}{1-v_Q^2} \approx M_Q v_Q. \quad (6)$$

Substituting Eq. (6) into Eqs. (4) and (5) eliminating the angular velocity ω gives the spin-averaged mass formula [36–38] for the orbital excited states,

$$(\bar{M} - M_Q)^2 = \pi\alpha L + a_0, \quad (7)$$

where the intercept factor $a_0 = (m_d + M_Q v_Q^2)^2$ depends on the diquark mass m_d and the nonrelativistic 3-kinematic energy $M_Q v_Q^2 = P_Q^2/M_Q$ for the heavy quark. Note that the nonrelativistic kinematic 3-momentum P_Q is conserved in the heavy quark limit, which has been associated with both M_Q and v_Q. Using a variant of Eq. (3), the velocity v_Q is

$$v_Q = \left(1 - \frac{m_{\mathrm{cur}Q}^2}{M_Q^2}\right)^{\frac{1}{2}}, \quad (8)$$

and the spin-averaged mass formula (7) becomes

$$\bar{M} = M_Q + \sqrt{\alpha\pi L + \left(m_d + M_Q\left(1 - \frac{m_{\mathrm{cur}Q}^2}{M_Q^2}\right)\right)^2}. \quad (9)$$

Here, M_Q and m_d are the constituent masses of the heavy quark and the diquark, respectively. L is the orbital angular momentum of the baryon systems ($L = 0, 1, 2, \cdots$). Accordingly, the current masses, the constituent quark masses and the string tension are applied in Eq. (9) as listed in Table I, which were previously determined in Refs. [30,38] via matching the measured mass spectra of the singly heavy baryons.

To obtain the spin-average masses of the orbital and radial excited states Σ_c/Σ_b, Ξ'_c/Ξ'_b, and Ω_c/Ω_b, we reexamine the Regge-like mass relation Eq. (9). By an analysis of the experimental data given by PDG [1] we suggest that the slope ratio of the Regge trajectory between the radial and angular momentum is 1.37: 1. Accordingly, $\pi\alpha L$ in Eq. (9) is replaced by $\pi\alpha(L + 1.37n)$,

TABLE I. The current masses and the constituent quark masses (in GeV) of the quark and the string tensions α (in GeV^2) of the singly heavy baryons.

Parameters	M_c	M_b	$m_{\mathrm{cur}c}$	$m_{\mathrm{cur}b}$	m_{nn}	m_{ns}	m_{ss}	$\alpha(cnn)$	$\alpha(cns)$	$\alpha(css)$	$\alpha(bnn)$	$\alpha(bns)$	$\alpha(bss)$
Input	1.44	4.48	1.275	4.18	0.745	0.872	0.991	0.212	0.255	0.316	0.246	0.307	0.318

$$\bar{M} = M_Q + \sqrt{\alpha\pi(L + 1.37n) + \left(m_d + M_Q\left(1 - \frac{m_{\mathrm{cur}Q}^2}{M_Q^2}\right)\right)^2}, \tag{10}$$

where n is a radial quantum number ($n = 0, 1, 2, \cdots$). We use Eq. (10) to calculate the spin-average masses of the Σ_c/Σ_b, Ξ_c'/Ξ_b', and Ω_c/Ω_b baryons. The results are listed in Table II.

Accordingly, the squared mass difference $(M - \bar{M})^2$ of the heavy-light hadronic system is related to L and n by

$$(M - \bar{M})^2 = \alpha\pi(L + 1.37n) + \left(m_d + M_Q\left(1 - \frac{m_{\mathrm{cur}Q}^2}{M_Q^2}\right)\right)^2. \tag{11}$$

The squared mass difference $(M - \bar{M})^2$ for the charm baryons is calculated and plotted against L in Figs. 1, 3, and 5 with $n = 0$, 1, 2, 3, and 4. Similarly, the results of the bottom baryons are shown in Figs. 2, 4, and 6. The (red) solid circles correspond to the observed (mean) masses and the empty circles indicate the predicted value in Figs. 1–6. It can be seen that $(M - \bar{M})^2$ increases with both L and n.

TABLE II. Spin-average masses (MeV) of the Σ_Q, Ξ_Q' and Ω_Q ($Q = c$, b) baryons predicted by Eq. (10).

State (MeV)	$\bar{M}(L=0)$	$\bar{M}(L=1)$	$\bar{M}(L=2)$	$\bar{M}(L=3)$	$\bar{M}(L=4)$
Σ_c ($n=0$)	2496.09	2774.67	3004.41	3204.48	3384.07
Σ_c ($n=1$)	2864.00	3081.28	3272.98	3446.45	3606.07
Σ_c ($n=2$)	3154.71	3339.01	3506.94	3662.22	3807.34
Σ_c ($n=3$)	3402.82	3565.72	3716.99	3858.83	3992.79
Σ_c ($n=4$)	3622.91	3770.48	3909.24	4040.61	4165.65
Σ_b ($n=0$)	5819.26	6082.84	6308.71	6509.53	6692.16
Σ_b ($n=1$)	6169.94	6385.49	6578.96	6756.02	6920.23
Σ_b ($n=2$)	6459.29	6646.17	6818.12	6978.25	7128.70
Σ_b ($n=3$)	6711.34	6878.62	7034.95	7182.24	7321.88
Σ_b ($n=4$)	6937.62	7090.42	7234.73	7371.84	7502.73
Ξ_c' ($n=0$)	2623.09	2923.52	3172.61	3390.14	3585.72
Ξ_c' ($n=1$)	3020.26	3256.13	3464.71	3653.72	3827.81
Ξ_c' ($n=2$)	3335.98	3536.63	3719.68	3889.09	4047.52
Ξ_c' ($n=3$)	3606.16	3783.79	3948.88	4103.75	4250.10
Ξ_c' ($n=4$)	3846.19	4007.27	4158.82	4302.36	4439.03
Ξ_b' ($n=0$)	5945.40	6244.84	6500.28	6726.81	6932.46
Ξ_b' ($n=1$)	6343.45	6586.94	6805.02	7004.30	7188.93
Ξ_b' ($n=2$)	6670.17	6880.69	7074.14	7254.12	7423.09
Ξ_b' ($n=3$)	6954.03	7142.16	7317.82	7483.20	7639.93
Ξ_b' ($n=4$)	7208.47	7380.11	7542.13	7695.98	7842.80
Ω_c ($n=0$)	2742.09	3081.49	3361.85	3606.22	3825.70
Ω_c ($n=1$)	3190.46	3455.72	3689.93	3901.95	4097.11
Ω_c ($n=2$)	3545.42	3770.62	3975.91	4165.78	4343.25
Ω_c ($n=3$)	3848.62	4047.77	4232.76	4406.23	4570.10
Ω_c ($n=4$)	4117.71	4298.17	4467.90	4628.60	4781.59
Ω_b ($n=0$)	6053.78	6344.51	6595.64	6819.96	7024.57
Ω_b ($n=1$)	6441.18	6681.30	6897.68	7096.22	7280.72
Ω_b ($n=2$)	6763.76	6972.99	7165.96	7345.97	7515.32
Ω_b ($n=3$)	7046.08	7233.94	7409.77	7575.63	7733.04
Ω_b ($n=4$)	7300.27	7472.21	7634.78	7789.38	7937.07

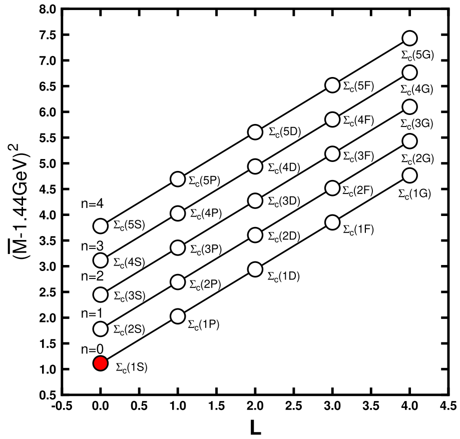

FIG. 1. Spin-average mass of Σ_c baryons.

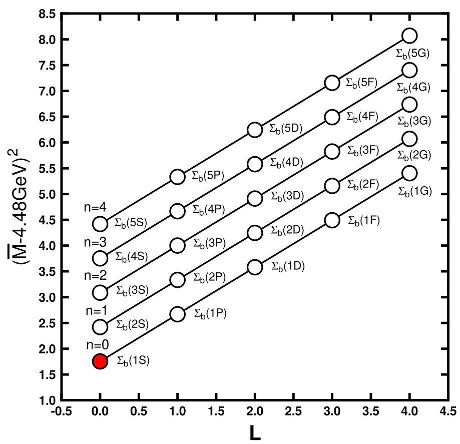

FIG. 2. Spin-average mass of Σ_b baryons.

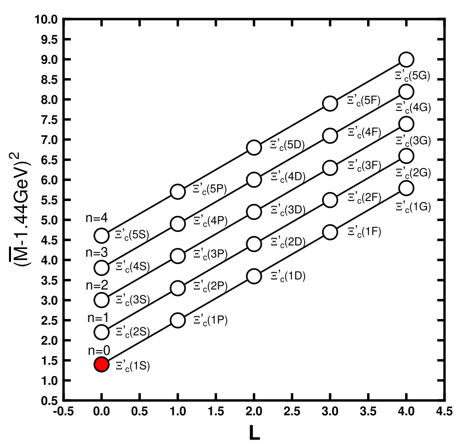

FIG. 3. Spin-average mass of Ξ'_c baryons.

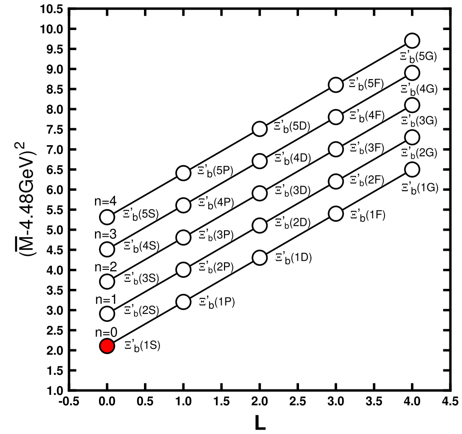

FIG. 4. Spin-average mass of Ξ'_b baryons.

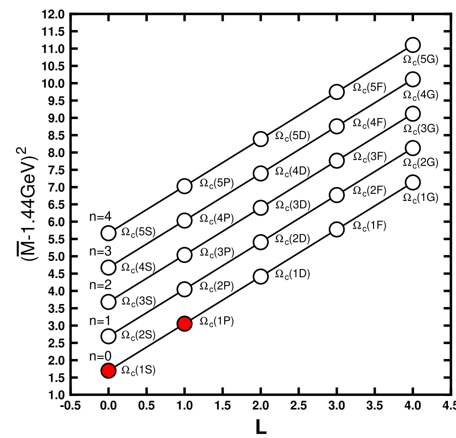

FIG. 5. Spin-average mass of Ω_c baryons.

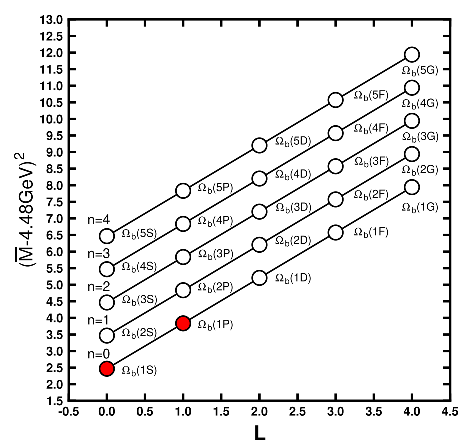

FIG. 6. Spin-average mass of Ω_b baryons.

III. THE SPIN-DEPENDENT POTENTIAL AND THE SCALING RELATIONS

Even though the baryon is a three-body system under the strong interaction, it is helpful to understand the measured mass data of the excited baryons using a simple heavy quark-diquark picture. To estimate the mass splitting for the singly heavy baryons, we consider the spin-dependent Hamiltonian H^{SD} [11,39] between the heavy quark (Q) and the spin-1 diquark (d) as

$$H^{\mathrm{SD}} = a_1 \mathbf{L}\cdot\mathbf{S}_d + a_2 \mathbf{L}\cdot\mathbf{S}_Q + b_1 S_{12} + c_1 \mathbf{S}_d\cdot\mathbf{S}_Q, \qquad (12)$$

where a_1, a_2, b_1, and c_1 are the spin-coupling parameters. The first two terms are spin-orbit interactions, the third is the tensor energy, and the last is the contact interaction between the heavy quark spin $\mathbf{S}_Q$ and the diquark spin $\mathbf{S}_d$. For the particular choice $L = 0$ for the S-wave baryons in Appendix A, the first three terms of Eq. (12) can be eliminated and only the last term survives, see Eq. (A1). Here, $S_{12} = 3(\mathbf{S}_d\cdot\hat{\mathbf{r}})(\mathbf{S}_Q\cdot\hat{\mathbf{r}})/r^2 - \mathbf{S}_d\cdot\mathbf{S}_Q$ in Ref. [27] with $L = 1$ and $L = 2$ can be given by

$$L = 1\colon\ S_{12} = -\frac{3}{5}\left[(\mathbf{L}\cdot\mathbf{S}_d)(\mathbf{L}\cdot\mathbf{S}_Q) + (\mathbf{L}\cdot\mathbf{S}_Q)(\mathbf{L}\cdot\mathbf{S}_d) - \frac{4}{3}(\mathbf{S}_d\cdot\mathbf{S}_Q)\right], \qquad (13)$$

$$L = 2\colon\ S_{12} = -\frac{1}{7}\left[(\mathbf{L}\cdot\mathbf{S}_d)(\mathbf{L}\cdot\mathbf{S}_Q) + (\mathbf{L}\cdot\mathbf{S}_Q)(\mathbf{L}\cdot\mathbf{S}_d) - 4(\mathbf{S}_d\cdot\mathbf{S}_Q)\right]. \qquad (14)$$

Combined with the experimental data [9] of the $\Omega_c(css)$, we used the Regge trajectory Eq. (9) to fit the constituent quark masses of the charm quark (c) and two strange quarks (ss) in Ref. [30], the results are $M_c = 1.44$ GeV and $m_{ss} = 0.991$ GeV. In the case of doubly strange Qss baryons with the mass of the diquark ss comparable with the mass of the heavy quark Q ($m_{ss} \approx M_c$), the finite-mass effect of the heavy quark may become important and makes it appropriate to go beyond the jj coupling. Therefore, in contrast to the scheme used in Ref. [39], we proposed a new scheme of state classification named the JLS coupling [30]. The first three terms are treated as operators H_1^{SD} defining representations and the last term $H_2^{\mathrm{SD}} = c_1 S_d\cdot S_Q$ in Eq. (12) as a perturbation. The operator H_1^{SD} is given by

$$H_1^{\mathrm{SD}} = a_1(\mathbf{L}\cdot\mathbf{S}_d) + a_2(\mathbf{L}\cdot\mathbf{S}_Q) + b_1 S_{12}. \qquad (15)$$

Using the bases $|J, j\rangle$ in terms of eigenvalues J, j of the total angular momentum $\mathbf{J}$ and total light-quark angular momentum $\mathbf{j}$, respectively, in order to diagonalize the mass operators H_1^{SD} and H_2^{SD}, we can obtain the mass shifts ΔM of P-wave in Eq. (B6) and D-wave in Eq. (C4) for the singly heavy baryons, see Appendix B and C. In this scheme, the P-wave states of the baryons may be classified as ${}^{2S+1}P_J = {}^2P_{1/2}$, ${}^4P_{1/2}$, ${}^2P_{3/2}$, ${}^4P_{3/2}$, ${}^4P_{5/2}$ and the D-wave states as ${}^{2S+1}D_J = {}^4D_{1/2}$, ${}^2D_{3/2}$, ${}^4D_{3/2}$, ${}^2D_{5/2}$, ${}^4D_{5/2}$, ${}^4D_{7/2}$.

Next, it is necessary to estimate the four spin-coupling parameters a_1, a_2, b_1, and c_1 in the heavy-light quark system. If Eq. (12) is taken as a spin-relevant relativistic correction, the parameters a_1, a_2, b_1, and c_1 are related to the magnetic moment $\mathbf{S}_Q/M_Q$ of the heavy quark. Therefore, these parameters can be considered roughly inversely proportional to the heavy quark mass (M_Q). In Ref. [39], the authors calculated the parameters of the partner in baryons using the scaling relations

$$\begin{aligned} a_1(b) &= a_1(c), \\ a_2(b) &= \frac{M_c}{M_b} a_2(c), \\ b_1(b) &= \frac{M_c}{M_b} b_1(c), \end{aligned} \qquad (16)$$

with the constituent quark masses (M_c, M_b) of the heavy quark in baryons. The parameter c_1 is expected to be negligible, because it should be very small in the P-wave states of the baryons.

In order to calculate the mass splitting of all excited states, we utilize the scaling relations based on the similarity between a baryon and its the partner baryons in the color configurations to study the spin-coupling parameters. In this subsection, we need to generalize Eq. (16) and consider the parameter c_1 which should include the effect of the principal quantum number N together with the radial quantum number n and orbital quantum number L [11,40–43]. The parameters a_1, a_2, and b_1 are obtained by following the scaling rules:

(i) The parameter a_1 is proportional to $\frac{1}{M_Q m_d}\langle\frac{1}{r}\rangle$;

(ii) The parameter a_2 is proportional to $\frac{1}{M_Q m_d}\langle\frac{1}{r}\rangle$;

(iii) The tensor parameter b_1 is proportional to $\frac{1}{M_Q m_d}\langle\frac{1}{r^3}\rangle$.

Here, $\langle 1/r\rangle = 1/((n+L+1)^2 a_B)$, $\langle 1/r^3\rangle = 1/(L(L+1/2)(L+1)(n+L+1)^3 a_B^3)$ and a_B is the Bohr radius. According to the scaling rules, a_2 can be of the same order as a_1 with the same n, L in the excited states, while the parameter b_1 should be smaller than the a_1, a_2, as b_1 scales with $\langle\frac{1}{r^3}\rangle$.

In order to obtain the parameter c_1 in Eq. (12), we need a scaling rule similar to (i)–(iii). Considering that c_1 becomes dominant in determining the mass splitting Eq. (A5), we can estimate c_1 based on the hyperfine structure term given by [44,45]

$$H^{hp} = \frac{8}{9M_Q m_d}\nabla^2 V \mathbf{S}_d\cdot\mathbf{S}_Q = \frac{32\pi\alpha_s}{9M_Q m_d}\mathbf{S}_d\cdot\mathbf{S}_Q \delta^3(\mathbf{r}), \qquad (17)$$

where ∇^2 is the Laplacian operator and $\delta^3(\mathbf{r})$ is the three-dimensional delta distribution. The derivative of the Coulomb potential V gives $\nabla^2 V = 4\pi\alpha_s\delta^3(\mathbf{r})$ with the strong coupling α_s. By taking the average $\langle\delta^3(\mathbf{r})\rangle = |\psi(0)|^2$ established for the hydrogenlike atoms wave function $\psi(\mathbf{r})$ of S-wave ($L = 0$) [46], Eq. (17) becomes

$$\langle H^{hp}\rangle = \frac{32\pi\alpha_s}{9M_Q m_d}\frac{1}{N^3 a_B^3}\langle \mathbf{S}_d\cdot\mathbf{S}_Q\rangle, \tag{18}$$

with $N = n + L + 1$. To extend Eq. (18) further to the excited states of the baryons, we introduce a parameter λ as follows:

$$\langle H^{hp}\rangle = \frac{32\pi\alpha_s}{9M_Q m_d}\frac{1}{(L+\lambda)N^3 a_B^3}\langle \mathbf{S}_d\cdot\mathbf{S}_Q\rangle. \tag{19}$$

Based on the systematic analysis of experimental values, we find the parameter $\lambda = 3.3$. Analyzing the coefficient in Eq. (19), the parameter c_1 is inversely proportional to M_Q, m_d, and $(L+\lambda)N^3$. Thus, the scaling rule of c_1 can be determined as follows:

(iv) The parameter c_1 is proportional to $\frac{1}{M_Q m_d}\frac{1}{(L+\lambda)N^3}$.

Eventually, the scaling relations of the spin-coupling parameters in Eq. (12) for the baryon system are

$$\begin{cases} a_1(B_a,(n+1)L) = \frac{M'_Q m'_d}{M_Q m_d}\frac{N'_{a_1}}{N_{a_1}} a_1(B'_a,(n'+1)L'), \\ a_2(B_a,(n+1)L) = \frac{M'_Q m'_d}{M_Q m_d}\frac{N'_{a_2}}{N_{a_2}} a_2(B'_a,(n'+1)L'), \\ b_1(B_a,(n+1)L) = \frac{M'_Q m'_d}{M_Q m_d}\frac{N'_{b_1}}{N_{b_1}} b_1(B'_a,(n'+1)L'), \\ c_1(B_a,(n+1)L) = \frac{M'_Q m'_d}{M_Q m_d}\frac{N'_{c_1}}{N_{c_1}} c_1(B'_a,(n'+1)L'), \end{cases} \tag{20}$$

where $n, n' = 0, 1, 2, \ldots, L, L' = S, P, D, F, \cdots$, and B_a, B'_a are baryons with $N_{a_1} = (n+L+1)^2 = N_{a_2}$, $N_{b_1} = L(L+1/2)(L+1)(n+L+1)^3$, $N_{c_1} = (L+\lambda)(n+L+1)^3$ corresponding to the similar form of N'_{a_1}, N'_{a_2}, N'_{b_1}, N'_{c_1} with L' and n', respectively. The prime denotes the quantities of the baryon B'_a obtained from experiments, distinguishing them from that of an unobserved baryon B_a.

IV. THE BARYONS Ω_c AND Ω_b

For Ω_c baryon family, it was a pleasant surprise that the LHCb Collaboration recently discovered five new narrow Ω_c states observed in decay channel $\Xi_c^+ K$ [9]; $\Omega_c(3000)^0$, $\Omega_c(3050)^0$, $\Omega_c(3065)^0$, $\Omega_c(3090)^0$, $\Omega_c(3120)^0$, the measured masses are

$$\begin{aligned} &\Omega_c(3000)^0: M = 3000.4 \pm 0.2 \pm 0.1 \text{ MeV}, \\ &\Omega_c(3050)^0: M = 3050.2 \pm 0.1 \pm 0.1 \text{ MeV}, \\ &\Omega_c(3065)^0: M = 3065.6 \pm 0.1 \pm 0.3 \text{ MeV}, \\ &\Omega_c(3090)^0: M = 3090.2 \pm 0.3 \pm 0.5 \text{ MeV}, \\ &\Omega_c(3120)^0: M = 3119.1 \pm 0.3 \pm 0.9 \text{ MeV}. \end{aligned}$$

Later, the Belle Collaboration confirmed the existence of these states [47]. In Ref. [30], the authors employ the quark model to analyze the narrow Ω_c states, and suggested that the parity was negative for all of five states. These can be interpreted as $1P$-wave charmed baryon candidates. Correspondingly, the bases $|J, j\rangle$ of the five excited Ω_c states are

$$\begin{aligned} &\text{State}|J,j\rangle: |1/2,0\rangle\, |1/2,1\rangle\, |3/2,1\rangle\, |3/2,2\rangle\, |5/2,2\rangle \\ &M(\Omega_c,1P): 3000.4 \text{ MeV}, 3050.2 \text{ MeV}, 3065.6 \text{ MeV}, \\ &\qquad 3090.2 \text{ MeV}, 3119.1 \text{ MeV}, \end{aligned} \tag{21}$$

At the same time, by fitting, the spin coupling parameters a_1, a_2, b_1, and c_1 are also obtained in Ref. [30],

$$\begin{aligned} &a_1(\Omega_c,1P) = 26.96 \text{ MeV}, \quad a_2(\Omega_c,1P) = 25.76 \text{ MeV}, \\ &b_1(\Omega_c,1P) = 13.51 \text{ MeV}, \quad c_1(\Omega_c,1P) = 4.04 \text{ MeV}. \end{aligned} \tag{22}$$

These results are the same as those in both of Refs. [27,48]. For more information of the Ω_c baryons, we recommend interested readers to see Refs. [11–22,24,49].

To elaborate on the mass shifts $\Delta M(J, j)$ for the entire baryon systems, we utilize the parameters (22) of the $1P$-wave Ω_c states as the object of the scaling relations in Eq. (20) to calculate the parameters of the other states. Adding the spin-average mass $\bar{M}$, the baryon mass becomes $M(J, j) = \bar{M} + \Delta M(J, j)$, where details of calculating $\Delta M(J, j)$ and $M(J, j)$ are presented in the Appendix. Therefore, the mass spectra of the singly heavy baryons can be predicted.

In earlier times, the observed $1S$-wave states Ω_c^0 and $\Omega_c(2770)^0$ with $J^P = 1/2^+$ and $J^P = 3/2^+$, corresponding to the masses $M(\Omega_c, 1/2^+) = 2695.2$ MeV and $M(\Omega_c, 3/2^+) = 2765.9$ MeV, respectively, had already been established. As seen in our model calculations, by using Eqs. (10), (20) and the parameters (22) with $L' = 1$, $n' = 0$ for the $1P$-wave Ω_c states, the calculation of the spin-averaged mass for the $1S$-wave Ω_c states with $L = 0$, $n = 0$ gives

$$\begin{aligned} \bar{M}(\Omega_c,1S) &= M_c + \left(m_{ss} + M_c\left(1 - \frac{m_{\text{curc}}^2}{M_c^2}\right)\right) \\ &= 2742.09 \text{ MeV}, \end{aligned} \tag{23}$$

and the parameter is

$$\begin{aligned} c_1(\Omega_c, 1S) &= \frac{N_c'}{N_c} c_1(\Omega_c, 1P) \\ &= \frac{(L'+3.3)(n'+L'+1)^3}{(L+3.3)(n+L+1)^3} c_1(\Omega_c, 1P) \\ &= \frac{(1+3.3)(0+1+1)^3}{(0+3.3)(0+0+1)^3} 4.04\ \text{MeV} \\ &= 42.11\ \text{MeV}. \end{aligned} \tag{24}$$

Substituting Eq. (23) and Eq. (24) into Eq. (A5), we obtain the masses $M(\Omega_c, 1/2^+) = 2699.98$ MeV and $M(\Omega_c, 3/2^+) = 2763.15$ MeV as shown in Table V of two ground states for Ω_c^0, which are in good agreement with the experimental values.

The mixed state $\Omega_c(3327)^0$ has been speculated as a $2S$ state in Ref. [50] and as a $1D$ state in Refs. [51–53]. However, we still need more observable objects to get clarity about the internal structure. In addition, in Ref. [24] the authors suggested that $\Omega_c(3185)^0$ may be regarded as a $2S$ state with $J^P = 1/2^+$ or $J^P = 3/2^+$, or their overlapping structure, and $\Omega_c(3185)^0$ is interpreted as a P-wave state in Ref. [54]. Very recently, the $\Omega_c(3185)^0$ and $\Omega_c(3327)^0$ states of Ω_c baryons were observed by LHCb Collaboration [31] with masses 3185.1 MeV and 3327.1 MeV, respectively. The quantum numbers of these states remain to be determined. According to our model, the calculation of the spin-averaged mass and the parameter for the Ω_c states in $2S$-wave ($L = 0$, $n = 1$) are obtained by using Eqs. (10), (20), and (22),

$$\bar{M}(\Omega_c, 2S) = M_c + \sqrt{\pi\alpha(\Omega_c) \times 1.37 + \left(m_{ss} + M_c\left(1 - \frac{m_{\text{curc}}^2}{M_c^2}\right)\right)^2} = 3190.46\ \text{MeV}, \tag{25}$$

$$c_1(\Omega_c, 2S) = \frac{(L'+3.3)(n'+L'+1)^3}{(L+3.3)(n+L+1)^3} c_1(\Omega_c, 1P) = \frac{(1+3.3)(0+1+1)^3}{(0+3.3)(1+0+1)^3} 4.04\ \text{MeV} = 5.26\ \text{MeV}. \tag{26}$$

Hence, the masses of the $2S$-wave Ω_c states are 3185.20 MeV and 3193.09 MeV as listed in Table V with $J^P = 1/2^+$ and $J^P = 3/2^+$, respectively. The $\Omega_c(3185)^0$ can be grouped into the $2S$ state. We assign $J^P = 1/2^+$ for $\Omega_c(3185)^0$. On the other hand, we also have calculated the spin-average of the $1D$-wave ($L = 2$, $n = 0$) for Ω_c states,

$$\bar{M}(\Omega_c, 1D) = M_c + \sqrt{2\pi\alpha(\Omega_c) + \left(m_{ss} + M_c\left(1 - \frac{m_{\text{curc}}^2}{M_c^2}\right)\right)^2} = 3361.85\ \text{MeV}, \tag{27}$$

the parameters are

$$\begin{aligned} a_1(\Omega_c, 1D) &= \frac{(n'+L'+1)^2}{(n+L+1)^2} a_1(\Omega_c, 1P) \\ &= \frac{(0+1+1)^2}{(0+2+1)^2} 26.96\ \text{MeV} \\ &= 11.98\ \text{MeV}, \end{aligned} \tag{28}$$

$$\begin{aligned} a_2(\Omega_c, 1D) &= \frac{(n'+L'+1)^2}{(n+L+1)^2} a_2(\Omega_c, 1P) \\ &= \frac{(0+1+1)^2}{(0+2+1)^2} 25.76\ \text{MeV} \\ &= 11.45\ \text{MeV}, \end{aligned} \tag{29}$$

$$\begin{aligned} b_1(\Omega_c, 1D) &= \frac{L'(L'+\frac{1}{2})(L'+1)(n'+L'+1)^3}{L(L+\frac{1}{2})(L+1)(n+L+1)^3} b_1(\Omega_c, 1P) \\ &= \frac{(1+\frac{1}{2})(1+1)(0+1+1)^3}{2(2+\frac{1}{2})(2+1)(0+2+1)^3} 13.51\ \text{MeV} \\ &= 0.80\ \text{MeV}, \end{aligned} \tag{30}$$

$$\begin{aligned} c_1(\Omega_c, 1D) &= \frac{(L'+3.3)(n'+L'+1)^3}{(L+3.3)(n+L+1)^3} c_1(\Omega_c, 1P) \\ &= \frac{(1+3.3)(0+1+1)^3}{(2+3.3)(0+2+1)^3} 4.04\ \text{MeV} \\ &= 0.97\ \text{MeV}, \end{aligned} \tag{31}$$

and the masses are

$$M(\Omega_c, 1D): \ 3308.41\ \mathrm{MeV}, 3326.92\ \mathrm{MeV}, 3342.64\ \mathrm{MeV}, 3356.96\ \mathrm{MeV}, 3373.08\ \mathrm{MeV}, 3397.52\ \mathrm{MeV}. \quad (32)$$

Using experimental values in Ref. [31], $\Omega_c(3327)^0$ is assigned by us as a $1D$ state with $J^P = 3/2^+$ rather than a $2S$ state, as the hyperfine splitting $3327.1\ \mathrm{MeV} - 3185.1\ \mathrm{MeV} = 142.0\ \mathrm{MeV}$ between $\Omega_c(3185)^0$ and $\Omega_c(3327)^0$ is much larger than the result 5.26 MeV of our model calculation. In this work, by using the scaling relations Eq. (20) we get the spin coupling parameters a_1, a_2, b_1, and c_1 as shown in Table III. We use the mass splitting Eqs. (A5), (B8), and (C6) to calculate the mass spectra of the Ω_c states. The results are given in Table V for the Ω_c baryons and compared with other models.

For Ω_b baryon family, in the quark model Ω_b^- is the ground state of Ω_b, where the system (bss) consists of a bottom quark (b) and a spin-1 diquark (ss). The mass of the Ω_b^- state is $M(\Omega_b^-) = 6045.2$ MeV with $J^P = 1/2^+$ identified in Ref. [1]. As a result, our calculation agrees very well with PDG for the Ω_b^- state. Recently, the LHCb experiment reported four extremely narrow Ω_b states in the decay channel $\Xi_b^0 K$ [28]. According to our discussion with the observations, the four states $\Omega_b(6316)^-$, $\Omega_b(6330)^-$, $\Omega_b(6340)^-$, $\Omega_b(6350)^-$ may be assigned as $1P$-wave excitations around the spin-average mass $\bar{M} = 6344.51$ MeV with $J^P = 1/2^-$, $1/2^-$, $3/2^-$ and $3/2^-$, respectively. Based on the masses of the $1P$-wave Ω_b states, we predict that there exists another excited Ω_b state with $J^P = 5/2^?$ in addition to the four Ω_b states observed by the LHCb Collaboration. The spin coupling parameters a_1, a_2, b_1, c_1 and the masses $M(1/2,0)$, $M(1/2,1)$, $M(3/2,1)$, $M(3/2,2)$, $M(5/2,2)$ are given by

$$\bar{M} = 6344.51\ \mathrm{MeV}, \quad a_1 = 8.67\ \mathrm{MeV}, \quad a_2 = 8.28\ \mathrm{MeV}, \quad b_1 = 4.34\ \mathrm{MeV}, \quad c_1 = 1.30\ \mathrm{MeV}, \quad (33)$$

$$M(\Omega_b, 1P): \ 6318.95\ \mathrm{MeV}, 6334.95\ \mathrm{MeV}, 6339.90\ \mathrm{MeV}, 6347.80\ \mathrm{MeV}, 6357.09\ \mathrm{MeV}. \quad (34)$$

Therefore, the mass of the excited Ω_b state with $J^P = 5/2^?$ is about 6357 MeV. This can be compared with values from Ref. [29]. For the Ω_b baryons, we calculate the parameters a_1, a_2, b_1, and c_1 as shown in Table IV, while our mass results are compared to results of other models in Table VI. These mass predictions presented in Table V and Table VI for the Ω_Q ($Q = c$, b) baryons will be helpful for future experimental searches.

TABLE III. The spin coupling parameters (MeV) of the Ω_c baryons.

State:	a_1	a_2	b_1	c_1
$1S$				42.11
$2S$				5.26
$3S$				1.56
$4S$				0.66
$5S$				0.34
$1P$	26.96	25.76	13.51	4.04
$2P$	11.98	11.45	4.00	1.20
$3P$	6.74	6.44	1.69	0.51
$4P$	4.31	4.12	0.86	0.26
$5P$	3.00	2.86	0.50	0.15
$1D$	11.98	11.45	0.80	0.97
$2D$	6.74	6.44	0.34	0.41
$3D$	4.31	4.12	0.17	0.21
$4D$	3.00	2.86	0.10	0.12
$5D$	2.20	2.10	0.06	0.08

TABLE IV. The spin coupling parameters (MeV) of the Ω_b baryons.

State:	a_1	a_2	b_1	c_1
$1S$				13.54
$2S$				1.69
$3S$				0.50
$4S$				0.21
$5S$				0.11
$1P$	8.67	8.28	4.34	1.30
$2P$	3.85	3.68	1.29	0.38
$3P$	2.17	2.07	0.54	0.16
$4P$	1.39	1.32	0.28	0.08
$5P$	0.96	0.92	0.16	0.05
$1D$	3.85	3.68	0.26	0.31
$2D$	2.17	2.07	0.11	0.13
$3D$	1.39	1.32	0.06	0.07
$4D$	0.96	0.92	0.03	0.04
$5D$	0.71	0.68	0.02	0.03

V. THE BARYONS Σ_c AND Σ_b

By analyzing the existing experimental data in PDG [1], we explore some patterns of the odd-parity Σ_Q ($Q = c$, b) baryons consisting of a light isospin-1 nonstrange diquark $(nn = uu, ud, dd)$ in a state of L with respect to the spin-1/2 heavy quark Q. So far, the Σ_Q baryons have been observed in experiments, and the data are available from the Particle Data Group, which provides us with more information to study the mass spectra of the Σ_Q states.

Reference [1] cites the two masses $M(\Sigma_c, 1/2^+) = 2452.65$ MeV, $M(\Sigma_c, 3/2^+) = 2517.4$ MeV for $\Sigma_c(2455)^+$, $\Sigma_c(2520)^+$ with $J^P = 1/2^+$ and $3/2^+$, respectively, which was discovered and identified as $1S$-wave states by the LHCb experiment. Accordingly, by using Particle Data Group masses, the spin-weighted average mass is obtained by [58]

TABLE V. The mass spectrum (MeV) of Ω_c baryons are given and compared with different quark models.

State	J^P	Baryon	Mass	Ours	EFG [11]	Reference [20]	Reference [55]
$1^1S_{1/2}$	$1/2^+$	Ω_c^0	2695.2	2699.98	2698	2695	2702
$1^3S_{3/2}$	$3/2^+$	$\Omega_c(2770)^0$	2765.9	2763.15	2768	2767	2772
$2^1S_{1/2}$	$1/2^+$	$\Omega_c(3185)^0$	3185.1	3185.20	3088	3100	3164
$2^3S_{3/2}$	$3/2^+$			3193.09	3123	3126	3197
$3^1S_{1/2}$	$1/2^+$			3543.86	3489	3436	3566
$3^3S_{3/2}$	$3/2^+$			3548.95	3510	3450	3571
$4^1S_{1/2}$	$1/2^+$			3847.96	3814	3737	3928
$4^3S_{3/2}$	$3/2^+$			3848.95	3830	3745	3910
$5^1S_{1/2}$	$1/2^+$			4117.37	4102	4015	4259
$5^3S_{3/2}$	$3/2^+$			4117.88	4114	4021	4222
$1^2P_{1/2}$	$1/2^-$	$\Omega_c(3000)^0$	3000.41	3001.93	2966	3011	
$1^4P_{1/2}$	$1/2^-$	$\Omega_c(3050)^0$	3050.19	3051.74	3055	2976	
$1^2P_{3/2}$	$3/2^-$	$\Omega_c(3065)^0$	3065.54	3067.14	3029	3028	3049
$1^4P_{3/2}$	$3/2^-$	$\Omega_c(3090)^0$	3090.10	3091.72	3054	2993	
$1^4P_{5/2}$	$5/2^-$	$\Omega_c(3120)^0$	3119.10	3120.64	3051	2947	3055
$2^2P_{1/2}$	$1/2^-$			3422.48	3384	3345	
$2^4P_{1/2}$	$1/2^-$			3442.70	3435	3315	
$2^2P_{3/2}$	$3/2^-$			3447.59	3415	3359	3408
$2^4P_{3/2}$	$3/2^-$			3460.73	3433	3330	
$2^4P_{5/2}$	$5/2^-$			3473.23	3427	3290	3393
$3^2P_{1/2}$	$1/2^-$			3752.49	3717	3644	
$3^4P_{1/2}$	$1/2^-$			3763.38	3754	3620	
$3^2P_{3/2}$	$3/2^-$			3765.54	3737	3656	3732
$3^4P_{3/2}$	$3/2^-$			3773.58	3752	3632	
$3^4P_{5/2}$	$5/2^-$			3780.50	3744	3601	3700
$4^2P_{1/2}$	$1/2^-$			4036.37	4009	3926	
$4^4P_{1/2}$	$1/2^-$			4043.18	4037	3903	
$4^2P_{3/2}$	$3/2^-$			4044.32	4023	3938	4031
$4^4P_{3/2}$	$3/2^-$			4049.72	4036	3915	
$4^4P_{5/2}$	$5/2^-$			4054.10	4028	3884	3983
$5^2P_{1/2}$	$1/2^-$			4290.35			
$5^4P_{1/2}$	$1/2^-$			4295.00			
$5^2P_{3/2}$	$3/2^-$			4295.68			4309
$5^4P_{3/2}$	$3/2^-$			4299.55			
$5^4P_{5/2}$	$5/2^-$			4302.57			4248
$1^4D_{1/2}$	$1/2^+$			3308.41	3287	3215	
$1^2D_{3/2}$	$3/2^+$	$\Omega_c(3327)^0$	3327.1	3326.92	3282	3231	
$1^4D_{3/2}$	$3/2^+$			3342.64	3298	3262	
$1^2D_{5/2}$	$5/2^+$			3356.96	3286	3188	3360
$1^4D_{5/2}$	$5/2^+$			3373.08	3297	3173	
$1^4D_{7/2}$	$7/2^+$			3397.52	3283	3136	3314
$2^4D_{1/2}$	$1/2^+$			3659.91	3623	3524	
$2^2D_{3/2}$	$3/2^+$			3670.21	3613	3538	
$2^4D_{3/2}$	$3/2^+$			3679.26	3627	3565	
$2^2D_{5/2}$	$5/2^+$			3687.03	3614	3502	3680
$2^4D_{5/2}$	$5/2^+$			3696.38	3626	3488	
$2^4D_{7/2}$	$7/2^+$			3709.95	3611	3456	3656

(Table continued)

TABLE V. *(Continued)*

State	J^P	Baryon	Mass	Ours	EFG [11]	Reference [20]	Reference [55]
$3^4D_{1/2}$	$1/2^+$			3956.72			
$3^2D_{3/2}$	$3/2^+$			3963.26			
$3^4D_{3/2}$	$3/2^+$			3969.13			
$3^2D_{5/2}$	$5/2^+$			3974.00			3974
$3^4D_{5/2}$	$5/2^+$			3980.09			
$3^4D_{7/2}$	$7/2^+$			3988.71			3968
$4^4D_{1/2}$	$1/2^+$			4219.44			
$4^2D_{3/2}$	$3/2^+$			4223.96			
$4^4D_{3/2}$	$3/2^+$			4228.08			
$4^2D_{5/2}$	$5/2^+$			4231.41			4248
$4^4D_{5/2}$	$5/2^+$			4235.69			
$4^4D_{7/2}$	$7/2^+$			4241.64			4258
$5^4D_{1/2}$	$1/2^+$			4458.12			
$5^2D_{3/2}$	$3/2^+$			4461.43			
$5^4D_{3/2}$	$3/2^+$			4464.47			
$5^2D_{5/2}$	$5/2^+$			4466.89			4505
$5^4D_{5/2}$	$5/2^+$			4470.06			
$5^4D_{7/2}$	$7/2^+$			4474.42			4529

TABLE VI. The mass spectrum (MeV) of Ω_b baryons are given and compared with different quark models.

State	J^P	Baryon	Mass	Ours	EFG [11]	Reference [56]	Reference [57]
$1^1S_{1/2}$	$1/2^+$	Ω_b^-	6045.2	6040.25	6064	6046	6054
$1^3S_{3/2}$	$3/2^+$			6060.55	6088	6082	6074
$2^1S_{1/2}$	$1/2^+$			6439.49	6450	6438	6455
$2^3S_{3/2}$	$3/2^+$			6442.03	6461	6462	6481
$3^1S_{1/2}$	$1/2^+$			6763.25	6804	6740	6832
$3^3S_{3/2}$	$3/2^+$			6764.01	6811	6753	6864
$4^1S_{1/2}$	$1/2^+$			7045.87	7091	7022	7190
$4^3S_{3/2}$	$3/2^+$			7046.19	7096	7030	7226
$5^1S_{1/2}$	$1/2^+$			7300.17	7338	7290	7531
$5^3S_{3/2}$	$3/2^+$			7300.33	7343	7296	7572
$1^2P_{1/2}$	$1/2^-$	$\Omega_b(6316)^-$	6315.6	6318.95	6330	4344	
$1^4P_{1/2}$	$1/2^-$	$\Omega_b(6330)^-$	6333.3	6334.95	6339	4345	
$1^2P_{3/2}$	$3/2^-$	$\Omega_b(6340)^-$	6339.7	6339.90	6331	4341	6348
$1^4P_{3/2}$	$3/2^-$	$\Omega_b(6350)^-$	6349.8	6347.80	6340	4343	
$1^4P_{5/2}$	$5/2^-$			6357.09	6334	4339	6362
$2^2P_{1/2}$	$1/2^-$			6670.62	6706	6596	
$2^4P_{1/2}$	$1/2^-$			6677.12	6710	6597	
$2^2P_{3/2}$	$3/2^-$			6678.69	6699	6594	6662
$2^4P_{3/2}$	$3/2^-$			6682.91	6705	6595	
$2^4P_{5/2}$	$5/2^-$			6686.93	6700	6592	6653
$3^2P_{1/2}$	$1/2^-$			6967.16	7003	6829	
$3^4P_{1/2}$	$1/2^-$			6970.66	7009	6830	
$3^2P_{3/2}$	$3/2^-$			6971.35	6998	6827	6962
$3^4P_{3/2}$	$3/2^-$			6973.94	7002	6828	
$3^4P_{5/2}$	$5/2^-$			6976.16	6996	6826	6689

(Table continued)

TABLE VI. *(Continued)*

State	J^P	Baryon	Mass	Ours	EFG [11]	Reference [56]	Reference [57]
$4^2P_{1/2}$	$1/2^-$			7230.27	7257	7044	
$4^4P_{1/2}$	$1/2^-$			7232.46	7265	7043	
$4^2P_{3/2}$	$3/2^-$			7232.83	7250	7043	7249
$4^4P_{3/2}$	$3/2^-$			7234.56	7258	7043	
$4^4P_{5/2}$	$5/2^-$			7235.97	7251	7042	7200
$5^2P_{1/2}$	$1/2^-$			7469.70			
$5^4P_{1/2}$	$1/2^-$			7471.19			
$5^2P_{3/2}$	$3/2^-$			7471.41			7526
$5^4P_{3/2}$	$3/2^-$			7472.65			
$5^4P_{5/2}$	$5/2^-$			7473.62			7458
$1^4D_{1/2}$	$1/2^+$			6578.47	6540	6485	
$1^2D_{3/2}$	$3/2^+$			6584.41	6530	6480	
$1^4D_{3/2}$	$3/2^+$			6589.46	6549	6482	
$1^2D_{5/2}$	$5/2^+$			6594.07	6520	6476	6629
$1^4D_{5/2}$	$5/2^+$			6599.25	6529	6478	
$1^4D_{7/2}$	$7/2^+$			6607.10	6517	6472	6638
$2^4D_{1/2}$	$1/2^+$			6888.04	6857	6730	
$2^2D_{3/2}$	$3/2^+$			6891.35	6846	6726	
$2^4D_{3/2}$	$3/2^+$			6894.26	6863	6727	
$2^2D_{5/2}$	$5/2^+$			6896.75	6837	6723	6659
$2^4D_{5/2}$	$5/2^+$			6899.76	6846	6724	
$2^4D_{7/2}$	$7/2^+$			6904.12	6834	6720	6643
$3^4D_{1/2}$	$1/2^+$			7159.80		6956	
$3^2D_{3/2}$	$3/2^+$			7161.90		6953	
$3^4D_{3/2}$	$3/2^+$			7163.79		6954	
$3^2D_{5/2}$	$5/2^+$			7165.35		6951	6689
$3^4D_{5/2}$	$5/2^+$			7167.31		6951	
$3^4D_{7/2}$	$7/2^+$			7170.08		6948	6648
$4^4D_{1/2}$	$1/2^+$			7405.49		7166	
$4^2D_{3/2}$	$3/2^+$			7406.94		7164	
$4^4D_{3/2}$	$3/2^+$			7408.27		7164	
$4^2D_{5/2}$	$5/2^+$			7409.34		7162	6719
$4^4D_{5/2}$	$5/2^+$			7410.71		7162	
$4^4D_{7/2}$	$7/2^+$			7412.63		7160	6653
$5^4D_{1/2}$	$1/2^+$			7631.64			
$5^2D_{3/2}$	$3/2^+$			7632.71			
$5^4D_{3/2}$	$3/2^+$			7633.68			
$5^2D_{5/2}$	$5/2^+$			7634.46			7458
$5^4D_{5/2}$	$5/2^+$			7635.48			
$5^4D_{7/2}$	$7/2^+$			7636.88			6658

$$\bar{M}^{\text{spin-weighted}} = \frac{\Sigma(2J+1)M(J)}{\Sigma(2J+1)} = \frac{(2\times 2453.75\ \text{MeV} + 4\times 2517.5\ \text{MeV})}{6} = 2496.25\ \text{MeV}. \tag{35}$$

As can be seen from Table II, the spin-average mass $\bar{M}(\Sigma_c, 1S) = 2496.09$ MeV is very close to the experimental value in Eq. (35). As the hyperfine splitting 2517.4 MeV − 2452.65 MeV = 64.75 MeV between $\Sigma_c(2455)^+$ and $\Sigma_c(2520)^+$ is regarded as a good reference for comparing the results of our model. For the Σ_c baryons, comparing the

TABLE VII. The spin coupling parameters (MeV) of the Σ_c baryons.

State:	a_1	a_2	b_1	c_1
$1S$				56.02
$2S$				7.00
$3S$				2.07
$4S$				0.88
$5S$				0.45
$1P$	35.86	34.27	17.97	5.37
$2P$	15.94	15.23	5.32	1.59
$3P$	8.97	8.57	2.25	0.67
$4P$	5.74	5.48	1.15	0.34
$5P$	3.98	3.81	0.67	0.20
$1D$	15.94	15.23	1.06	1.29
$2D$	8.97	8.57	0.45	0.55
$3D$	5.74	5.48	0.23	0.28
$4D$	3.98	3.81	0.13	0.16
$5D$	2.93	2.80	0.08	0.10

measured masses presented in Table IX with our prediction masses, and the parameters as shown in Table VII, it is seen that the masses of all these states are compatible with the experimental values (within few MeV). We employ Eq. (10) to calculate the spin-averaged mass $\bar{M}$ of $1S$-wave with $L = 0$, $n = 0$,

$$\bar{M}(\Sigma_c, 1S) = M_c + \left(m_{nn} + M_c\left(1 - \frac{m^2_{\mathrm{curc}}}{M^2_c}\right)\right) = 2496.09\ \mathrm{MeV}, \tag{36}$$

as well as the following rough estimate for the parameter c_1 by Eq. (20):

$$c_1(\Sigma_c, 1S) = \frac{M_c m_{ss}}{M_c m_{nn}} \frac{N'_c}{N_c} c_1(\Omega_c, 1P) = \frac{0.991}{0.745} \frac{(1+3.3)(0+1+1)^3}{(0+3.3)(0+0+1)^3} 4.04\ \mathrm{MeV} = 56.02\ \mathrm{MeV}, \tag{37}$$

with $c_1(\Omega_c, 1P) = 4.04$ MeV given in Eq. (22). Note that the heavy quark mass M_c cancels out for charmed baryons.

The $\Sigma_c(2800)$ observed by the Belle Collaboration [2] might be a good candidate for a $1P$-wave state (cf. e.g., Ref. [39]). For comparison with the experiment values, we also compute the parameters and the masses of the Σ_c in $1P$-wave states,

$$\bar{M} = 2774.67\ \mathrm{MeV}, \quad a_1 = 35.86\ \mathrm{MeV}, \quad a_2 = 34.27\ \mathrm{MeV}, \quad b_1 = 17.97\ \mathrm{MeV}, \quad c_1 = 5.37\ \mathrm{MeV}, \tag{38}$$

$$M(\Sigma_c, 1P){:}\ 2668.86\ \mathrm{MeV}, 2735.11\ \mathrm{MeV}, 2755.59\ \mathrm{MeV}, 2788.31\ \mathrm{MeV}, 2826.76\ \mathrm{MeV}. \tag{39}$$

Although the Belle Collaboration observed the excited $\Sigma_c(2800)$ state in the decay channel $\Lambda_c^+\pi$ [60] which mass at $M(\Sigma_c) = 2792$ MeV, the J^P has not been determined, making it difficult to determine its properties. The $\Sigma_c(2800)$ state is calculated by our model to own the mass 2788.31 MeV, which is in agreement with the experiment as show in Table IX. Hence, for $\Sigma_c(2800)$ we should advocate the fourth state $|^4P_{3/2}, 3/2^-\rangle$ of $1P$-wave. The nature of these states is discussed in Refs. [11,61].

In the Σ_b baryon family, there are four states with masses $M(\Sigma_b^+, 1/2^+) = 5810.56$ MeV and $M(\Sigma_b^{*+}, 3/2^+) = 5830.32$ MeV in PDG [1] for the Σ_b^+ and Σ_b^{*+} states, and $M(\Sigma_b^-, 1/2^+) = 5815.64$ MeV and $M(\Sigma_b^{*-}, 3/2^+) = 5834.74$ MeV for the Σ_b^- and Σ_b^{*-} states, respectively. It should be pointed out that the neutral $1S$-wave Σ_b^0, Σ_b^{*0} states are still missing. In addition, $\Sigma_b(6097)$ has been measured using fully reconstructed $\Lambda_b^0 \to \Lambda_c^+\pi^-$ and $\Lambda_c^+ \to \rho\kappa_c^+\pi^+$ decays in Ref. [3]. In our calculations, $\Sigma_b(6097)$ can be a good candidate of $1P$-wave excitations. Therefore, we assign $J^P = 5/2^-$ to the $\Sigma_b(6097)$ state. Finally, the spin-averaged mass, the parameters and the mass splitting are given by Eq. (10) and Eq. (20) in $1P$-wave ($L = 1$, $n = 0$),

$$\bar{M}(\Sigma_b, 1P) = M_b + \sqrt{\pi\alpha(\Sigma_b) + \left(m_{nn} + M_b\left(1 - \frac{m^2_{\mathrm{curb}}}{M^2_b}\right)\right)^2} = 6082.84\ \mathrm{MeV}, \tag{40}$$

$$a_1(\Sigma_b, 1P) = \frac{M_c m_{ss}}{M_b m_{nn}} \frac{(n' + L' + 1)^2}{(n + L + 1)^2} a_1(\Omega_c, 1P) = \frac{1.44 \times 0.991}{4.48 \times 0.745} \frac{(0+1+1)^2}{(0+1+1)^2} 26.96\ \mathrm{MeV} = 11.53\ \mathrm{MeV}, \tag{41}$$

$$a_2(\Sigma_b, 1P) = \frac{M_c m_{ss}}{M_b m_{nn}} \frac{(n' + L' + 1)^2}{(n + L + 1)^2} a_2(\Omega_c, 1P) = \frac{1.44 \times 0.991}{4.48 \times 0.745} \frac{(0+1+1)^2}{(0+1+1)^2} 25.76\ \mathrm{MeV} = 11.01\ \mathrm{MeV}, \tag{42}$$

$$
\begin{aligned}
b_1(\Sigma_b, 1P) &= \frac{M_c m_{ss}}{M_b m_{nn}} \frac{L'(L'+\frac{1}{2})(L'+1)(n'+L'+1)^3}{L(L+\frac{1}{2})(L+1)(n+L+1)^3} b_1(\Omega_c, 1P) \\
&= \frac{1.44 \times 0.991}{4.48 \times 0.745} \frac{(1+\frac{1}{2})(1+1)(0+1+1)^3}{(1+\frac{1}{2})(1+1)(0+1+1)^3} 13.51 \text{ MeV} \\
&= 5.78 \text{ MeV},
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
c_1(\Sigma_b, 1P) &= \frac{M_c m_{ss}}{M_b m_{nn}} \frac{(L'+3.3)(n'+L'+1)^3}{(L+3.3)(n+L+1)^3} c_1(\Omega_c, 1P) \\
&= \frac{1.44 \times 0.991}{4.48 \times 0.745} \frac{(1+3.3)(0+1+1)^3}{(1+3.3)(0+1+1)^3} 4.04 \text{ MeV} \\
&= 1.73 \text{ MeV},
\end{aligned} \tag{44}
$$

$$
M(\Sigma_b, 1P)\colon\ 6048.86 \text{ MeV}, 6070.13 \text{ MeV}, 6076.71 \text{ MeV}, 6087.21 \text{ MeV}, 6099.56 \text{ MeV}. \tag{45}
$$

Evidently, a_1, a_2, and b_1 reasonably fulfill (i)–(iii), and c_1 in (iv) becomes a nonvanishing but small value for the highly excited states. We exploit Eq. (B8) to calculate the mass splitting for the Σ_b states. The results of the parameters are listed in Table VIII and the masses in Table X. Under the analysis of the model, these results are consistent with the experimental values.

VI. THE BARYONS Ξ_c' AND Ξ_b'

In this section, based on our scheme, a similar method can be applied to the excited Ξ_Q' (csn or bsn) baryon systems in order to analyze their masses and parameters. For the Ξ_c' baryon system, the (S-wave) ground states with the spin-parity $J^P = 1/2^+$ and $J^P = 3/2^+$ correspond to $\Xi_c'^0$ and $\Xi_c(2645)^0$, as the masses at $M(\Xi_c'^0, 1/2^+) = 2578.7$ MeV and $M(\Xi_c^0, 3/2^+) = 2646.16$ MeV listed by the PDG [1] have been established. In this work, we use the scaling relations to calculate the spin-coupling parameters a_1, a_2, b_1, and c_1 as shown in Table XI for the Ξ_c' baryons. The mass results are listed in Table XIII and compared with other models.

As the classification of the P-wave states ($L = 1$) is similar to the other charm baryons, we use Eq. (B8) to calculate the mass splitting for the Ξ_c' states. By analyzing the model results in Table XIII, we find that $\Xi_c(2923)^0$ and $\Xi_c(2930)^0$ with the spin-parity $J^P = 3/2^-$ might be good candidates for P states of the Ξ_c' baryons. The masse $M(\Xi_c(2923)^0) = 2907.21$ MeV is only 15.83 MeV lower than the mass of the state $\Xi_c(2923)^0$, and lower than $M(\Xi_c(2930)^0) = 2935.17$ MeV, compared with experimental values within a reasonable range. For a more detailed analysis the Ξ_c' baryons see also Refs. [59,63].

In addition, the state $\Xi_c(3123)$ was also confirmed by the *BABAR* Collaboration [64], with a mass $M(\Xi_c{}^+) = 3122.9$ MeV listed in PDG [1]. From the analysis of our data in Table XIII we infer that the mass shifts of about 22 MeV in the $1D$-wave are relatively small. In the past, the quantum number of $\Xi_c(3123)$ was not determined. In our frame, it is possible to determine $\Xi_c(3123)$ as the second state with $J^P = 3/2^+$ or mixed with the first state, which can be a good candidate for a $1D$ state of the Ξ_c' baryons.

For the Ξ_b' baryon system, in 2015 the LHCb Collaboration observed two new charged states $\Xi_b'(5935)^-$ and $\Xi_b^*(5955)^-$ in the decay channel $\Xi_b'^0\pi^-$ [8]. The masses $M(\Xi_b'^-, 1/2^+) = 5935.02$ MeV and $M(\Xi_b^{*-}, 1/2^+) = 5955.33$ MeV were proposed to be the ground states $\Xi_b'^-$ and Ξ_b^{*-} with the $J^P = 1/2^+$ and $J^P = 3/2^+$, respectively. In our work, the ground states $\Xi_b'(5935)^-$ and $\Xi_b^*(5955)^-$ in Table XIV are in good agreement with other theoretical predictions as well as experimental measurements (see Ref. [8]).

TABLE VIII. The spin coupling parameters (MeV) of the Σ_b baryons.

State:	a_1	a_2	b_1	c_1
$1S$				18.00
$2S$				2.25
$3S$				0.67
$4S$				0.28
$5S$				0.14
$1P$	11.53	11.01	5.78	1.73
$2P$	5.12	4.90	1.71	0.51
$3P$	2.88	2.75	0.72	0.22
$4P$	1.84	1.76	0.37	0.11
$5P$	1.28	1.22	0.21	0.06
$1D$	5.12	4.90	0.34	0.42
$2D$	2.88	2.75	0.14	0.18
$3D$	1.84	1.76	0.07	0.09
$4D$	1.28	1.22	0.04	0.05
$5D$	0.94	0.90	0.03	0.03

TABLE IX. The mass spectrum (MeV) of Σ_c baryons are given and compared with different quark models.

State	J^P	Baryon	Mass	Ours	EFG [11]	Reference [59]	Reference [20]
$1^1S_{1/2}$	$1/2^+$	$\Sigma_c(2455)^+$	2452.65	2440.07	2443	2456	2452
$1^3S_{3/2}$	$3/2^+$	$\Sigma_c(2520)^+$	2517.4	2524.10	2519	2515	2518
$2^1S_{1/2}$	$1/2^+$			2857.00	2901	2850	2891
$2^3S_{3/2}$	$3/2^+$			2867.50	2936	2876	2917
$3^1S_{1/2}$	$1/2^+$			3152.63	3271	3091	3261
$3^3S_{3/2}$	$3/2^+$			3155.75	3293	3109	3274
$4^1S_{1/2}$	$1/2^+$			3401.95	3581		3593
$4^3S_{3/2}$	$3/2^+$			3403.26	3598		3601
$5^1S_{1/2}$	$1/2^+$			3622.47	3861		3900
$5^3S_{3/2}$	$3/2^+$			3623.14	3873		3906
$1^2P_{1/2}$	$1/2^-$			2668.86	2713	2702	2809
$1^4P_{1/2}$	$1/2^-$			2735.11	2799	2765	2755
$1^2P_{3/2}$	$3/2^-$			2755.59	2773	2785	2835
$1^4P_{3/2}$	$3/2^-$	$\Sigma_c(2800)^+$	2792	2788.31	2798	2798	2782
$1^4P_{5/2}$	$5/2^-$			2826.76	2789	2790	2710
$2^2P_{1/2}$	$1/2^-$			3037.05	3125	2971	3174
$2^4P_{1/2}$	$1/2^-$			3063.95	3172	3018	3128
$2^2P_{3/2}$	$3/2^-$			3070.46	3151	3036	3196
$2^4P_{3/2}$	$3/2^-$			3087.94	3172	3044	3151
$2^4P_{5/2}$	$5/2^-$			3104.56	3161	3040	3090
$3^2P_{1/2}$	$1/2^-$			3314.88	3455		3505
$3^4P_{1/2}$	$1/2^-$			3329.38	3488		3465
$3^2P_{3/2}$	$3/2^-$			3332.25	3469		3525
$3^4P_{3/2}$	$3/2^-$			3342.95	3486		3485
$3^4P_{5/2}$	$5/2^-$			3352.15	3475		3433
$4^2P_{1/2}$	$1/2^-$			3550.56	3743		3814
$4^4P_{1/2}$	$1/2^-$			3559.61	3770		3777
$4^2P_{3/2}$	$3/2^-$			3561.13	3753		3832
$4^4P_{3/2}$	$3/2^-$			3568.32	3768		2796
$4^4P_{5/2}$	$5/2^-$			3574.14	3757		3747
$5^2P_{1/2}$	$1/2^-$			3760.07			
$5^4P_{1/2}$	$1/2^-$			3766.26			
$5^2P_{3/2}$	$3/2^-$			3767.17			
$5^4P_{3/2}$	$3/2^-$			3772.32			
$5^4P_{5/2}$	$5/2^-$			3776.33			
$1^4D_{1/2}$	$1/2^+$			2933.33	3041	2949	3036
$1^2D_{3/2}$	$3/2^+$			2957.94	3040	2952	3112
$1^4D_{3/2}$	$3/2^+$			2978.85	3043	2964	3061
$1^2D_{5/2}$	$5/2^+$			2997.90	3023	2942	2993
$1^4D_{5/2}$	$5/2^+$			3019.35	3038	2963	2968
$1^4D_{7/2}$	$7/2^+$			3051.86	3013	2943	2909
$2^4D_{1/2}$	$1/2^+$			3233.06	3370		3376
$2^2D_{3/2}$	$3/2^+$			3246.75	3364		3398
$2^4D_{3/2}$	$3/2^+$			3258.79	3366		3442
$2^2D_{5/2}$	$5/2^+$			3269.13	3349		3316
$2^4D_{5/2}$	$5/2^+$			3281.56	3365		3339
$2^4D_{7/2}$	$7/2^+$			3299.62	3342		3265

(Table continued)

TABLE IX. *(Continued)*

State	J^P	Baryon	Mass	Ours	EFG [11]	Reference [59]	Reference [20]
$3^4D_{1/2}$	$1/2^+$			3481.42			
$3^2D_{3/2}$	$3/2^+$			3490.12			
$3^4D_{3/2}$	$3/2^+$			3497.93			
$3^2D_{5/2}$	$5/2^+$			3504.40			
$3^4D_{5/2}$	$5/2^+$			3512.51			
$3^4D_{7/2}$	$7/2^+$			3523.98			
$4^4D_{1/2}$	$1/2^+$			3699.28			
$4^2D_{3/2}$	$3/2^+$			3705.30			
$4^4D_{3/2}$	$3/2^+$			3710.77			
$4^2D_{5/2}$	$5/2^+$			3715.20			
$4^4D_{5/2}$	$5/2^+$			3720.89			
$4^4D_{7/2}$	$7/2^+$			3728.81			
$5^4D_{1/2}$	$1/2^+$			3896.23			
$5^2D_{3/2}$	$3/2^+$			3900.64			
$5^4D_{3/2}$	$3/2^+$			3904.68			
$5^2D_{5/2}$	$5/2^+$			3907.90			
$5^4D_{5/2}$	$5/2^+$			3912.12			
$5^4D_{7/2}$	$7/2^+$			3917.92			

TABLE X. The mass spectrum (MeV) of Σ_b baryons are given and compared with different quark models.

State	J^P	Baryon	Mass	Ours	EFG [11]	Reference [62]	Reference [57]
$1^1S_{1/2}$	$1/2^+$	Σ_b^+	5810.56	5801.27	5808	5811	5811
$1^3S_{3/2}$	$3/2^+$	$\Sigma_b{}^{*+}$	5830.32	5828.25	5834	5832	5830
$2^1S_{1/2}$	$1/2^+$			6167.69	6213	6262	6275
$2^3S_{3/2}$	$3/2^+$			6171.06	6226	6278	6291
$3^1S_{1/2}$	$1/2^+$			6458.62	6575	6605	6707
$3^3S_{3/2}$	$3/2^+$			6459.62	6583	6614	6720
$4^1S_{1/2}$	$1/2^+$			6711.06	6869	6927	7113
$4^3S_{3/2}$	$3/2^+$			6711.48	6876	6933	7124
$5^1S_{1/2}$	$1/2^+$			6937.48	7124	7231	7497
$5^3S_{3/2}$	$3/2^+$			6937.70	7129	7235	7506
$1^2P_{1/2}$	$1/2^-$			6048.86	6095	6104	
$1^4P_{1/2}$	$1/2^-$			6070.13	6101	6106	
$1^2P_{3/2}$	$3/2^-$			6076.71	6087	6100	6105
$1^4P_{3/2}$	$3/2^-$			6087.21	6096	6102	
$1^4P_{5/2}$	$5/2^-$	$\Sigma_b(6097)^-$	6098.0	6099.56	6084	6097	6118
$2^2P_{1/2}$	$1/2^-$			6371.29	6430	6355	
$2^4P_{1/2}$	$1/2^-$			6379.93	6440	6356	
$2^2P_{3/2}$	$3/2^-$			6382.02	6424	6353	6506
$2^4P_{3/2}$	$3/2^-$			6387.63	6430	6354	
$2^4P_{5/2}$	$5/2^-$			6392.96	6421	6351	6489
$3^2P_{1/2}$	$1/2^-$			6638.42	6742	6578	
$3^4P_{1/2}$	$1/2^-$			6643.07	6756	6579	
$3^2P_{3/2}$	$3/2^-$			6644.00	6736	6577	6884
$3^4P_{3/2}$	$3/2^-$			6647.43	6742	6577	
$3^4P_{5/2}$	$5/2^-$			6650.38	6732	6575	6840

(Table continued)

TABLE X. *(Continued)*

State	J^P	Baryon	Mass	Ours	EFG [11]	Reference [62]	Reference [57]
$4^2P_{1/2}$	$1/2^-$			6873.75	7008	6778	
$4^4P_{1/2}$	$1/2^-$			6876.66	7024	6779	
$4^2P_{3/2}$	$3/2^-$			6877.15	7003	6777	7242
$4^4P_{3/2}$	$3/2^-$			6879.45	7009	6778	
$4^4P_{5/2}$	$5/2^-$			6881.31	6999	6776	7174
$5^2P_{1/2}$	$1/2^-$			7087.08			
$5^4P_{1/2}$	$1/2^-$			7089.06			
$5^2P_{3/2}$	$3/2^-$			7089.35			7583
$5^4P_{3/2}$	$3/2^-$			7091.01			
$5^4P_{5/2}$	$5/2^-$			7092.30			7493
$1^4D_{1/2}$	$1/2^+$			6285.89	6311	6303	
$1^2D_{3/2}$	$3/2^+$			6293.79	6285	6298	
$1^4D_{3/2}$	$3/2^+$			6300.50	6326	6300	
$1^2D_{5/2}$	$5/2^+$			6306.62	6270	6294	6386
$1^4D_{5/2}$	$5/2^+$			6313.51	6284	6295	
$1^4D_{7/2}$	$7/2^+$			6323.94	6260	6290	6393
$2^4D_{1/2}$	$1/2^+$			6566.15	6636	6533	
$2^2D_{3/2}$	$3/2^+$			6570.54	6612	6529	
$2^4D_{3/2}$	$3/2^+$			6574.41	6647	6530	
$2^2D_{5/2}$	$5/2^+$			6577.73	6598	6526	6778
$2^4D_{5/2}$	$5/2^+$			6581.72	6612	6527	
$2^4D_{7/2}$	$7/2^+$			6587.52	6590	6524	6751
$3^4D_{1/2}$	$1/2^+$			6809.93		6738	
$3^2D_{3/2}$	$3/2^+$			6812.72		6736	
$3^4D_{3/2}$	$3/2^+$			6815.23		6736	
$3^2D_{5/2}$	$5/2^+$			6817.31		6734	7148
$3^4D_{5/2}$	$5/2^+$			6819.91		6735	
$3^4D_{7/2}$	$7/2^+$			6823.59		6733	7091
$4^4D_{1/2}$	$1/2^+$			7029.26		6923	
$4^2D_{3/2}$	$3/2^+$			7031.19		6922	
$4^4D_{3/2}$	$3/2^+$			7032.95		6922	
$4^2D_{5/2}$	$5/2^+$			7034.37		6921	7501
$4^4D_{5/2}$	$5/2^+$			7036.20		6921	
$4^4D_{7/2}$	$7/2^+$			7038.75		6920	7415
$5^4D_{1/2}$	$1/2^+$			7230.55			
$5^2D_{3/2}$	$3/2^+$			7231.97			
$5^4D_{3/2}$	$3/2^+$			7233.27			
$5^2D_{5/2}$	$5/2^+$			7234.30			7837
$5^4D_{5/2}$	$5/2^+$			7235.66			
$5^4D_{7/2}$	$7/2^+$			7237.52			7526

TABLE XI. The spin coupling parameters (MeV) of the Ξ_c' baryons.

State:	a_1	a_2	b_1	c_1
$1S$				47.86
$2S$				5.98
$3S$				1.77
$4S$				0.75
$5S$				0.38
$1P$	30.64	29.28	15.35	4.59
$2P$	13.62	13.01	4.55	1.36
$3P$	7.66	7.32	1.92	0.57
$4P$	4.90	4.68	0.98	0.29
$5P$	3.40	3.25	0.57	0.17
$1D$	13.62	13.01	0.91	1.10
$2D$	7.66	7.32	0.38	0.47
$3D$	4.90	4.68	0.20	0.24
$4D$	3.40	3.25	0.11	0.14
$5D$	2.50	2.39	0.07	0.09

TABLE XII. The spin coupling parameters (MeV) of the Ξ_b' baryons.

State:	a_1	a_2	b_1	c_1
$1S$				15.38
$2S$				1.92
$3S$				0.57
$4S$				0.24
$5S$				0.12
$1P$	9.85	9.41	4.94	1.48
$2P$	4.38	4.18	1.46	0.44
$3P$	2.46	2.35	0.62	0.18
$4P$	1.58	1.51	0.32	0.09
$5P$	1.09	1.05	0.18	0.05
$1D$	4.38	4.18	0.29	0.35
$2D$	2.46	2.35	0.12	0.15
$3D$	1.58	1.51	0.06	0.08
$4D$	1.09	1.05	0.04	0.04
$5D$	0.80	0.77	0.02	0.03

TABLE XIII. The mass spectrum (MeV) of Ξ_c' baryons are given and compared with different quark models.

State	J^P	Baryon	Mass	Ours	EFG [11]	Reference [59]	Reference [20]
$1^1S_{1/2}$	$1/2^+$	$\Xi_c'^0$	2578.70	2575.23	2579	2579	2471
$1^3S_{3/2}$	$3/2^+$	$\Xi_c(2645)^0$	2646.16	2647.02	2649	2649	2647
$2^1S_{1/2}$	$1/2^+$			3014.28	2983	2977	2937
$2^3S_{3/2}$	$3/2^+$			3023.25	3026	3007	3004
$3^1S_{1/2}$	$1/2^+$			3334.21	3377	3215	3303
$3^3S_{3/2}$	$3/2^+$			3336.87	3396	3236	3338
$4^1S_{1/2}$	$1/2^+$			3605.41	3695		3626
$4^3S_{3/2}$	$3/2^+$			3606.54	3709		3646
$5^1S_{1/2}$	$1/2^+$			3845.81	3978		3921
$5^3S_{3/2}$	$3/2^+$			3846.39	3989		3934
$1^2P_{1/2}$	$1/2^-$			2833.11	2854	2839	2877
$1^4P_{1/2}$	$1/2^-$			2889.71	2936	2900	2834
$1^2P_{3/2}$	$3/2^-$	$\Xi_c(2923)^0$	2923.04	2907.21	2912	2921	2899
$1^4P_{3/2}$	$3/2^-$	$\Xi_c(2930)^0$	2938.55	2935.17	2935	2932	2856
$1^4P_{5/2}$	$5/2^-$			2968.02	2929	2927	2798
$2^2P_{1/2}$	$1/2^-$			3218.35	3267	3094	3222
$2^4P_{1/2}$	$1/2^-$			3241.33	3313	3144	3189
$2^2P_{3/2}$	$3/2^-$			3246.89	3293	3172	3239
$2^4P_{3/2}$	$3/2^-$			3261.82	3311	3165	3206
$2^4P_{5/2}$	$5/2^-$			3276.02	3303	3170	3162
$3^2P_{1/2}$	$1/2^-$			3516.01	3598		3544
$3^4P_{1/2}$	$1/2^-$			3528.40	3630		3512
$3^2P_{3/2}$	$3/2^-$			3530.85	3613		3561
$3^4P_{3/2}$	$3/2^-$			3539.99	3628		3528
$3^4P_{5/2}$	$5/2^-$			3547.85	3619		3484

(Table continued)

TABLE XIII. *(Continued)*

State	J^P	Baryon	Mass	Ours	EFG [11]	Reference [59]	Reference [20]
$4^2P_{1/2}$	$1/2^-$			3770.84	3887		3837
$4^4P_{1/2}$	$1/2^-$			3778.57	3912		3808
$4^2P_{3/2}$	$3/2^-$			3779.87	3898		3851
$4^4P_{3/2}$	$3/2^-$			3786.01	3911		3823
$4^4P_{5/2}$	$5/2^-$			3790.99	3902		3784
$5^2P_{1/2}$	$1/2^-$			3998.38			
$5^4P_{1/2}$	$1/2^-$			4003.67			
$5^2P_{3/2}$	$3/2^-$			4004.44			
$5^4P_{3/2}$	$3/2^-$			4008.84			
$5^4P_{5/2}$	$5/2^-$			4012.27			
$1^4D_{1/2}$	$1/2^+$			3111.88	3163	3075	3147
$1^2D_{3/2}$	$3/2^+$	$\Xi_c(3123)^+$	3122.9	3132.91	3160	3089	3109
$1^4D_{3/2}$	$3/2^+$			3150.77	3167	3081	3090
$1^2D_{5/2}$	$5/2^+$			3167.05	3153	3091	3058
$1^4D_{5/2}$	$5/2^+$			3185.38	3166	3077	3039
$1^4D_{7/2}$	$7/2^+$			3213.14	3147	3078	2995
$2^4D_{1/2}$	$1/2^+$			3430.60	3505		3470
$2^2D_{3/2}$	$3/2^+$			3442.30	3497		3417
$2^4D_{3/2}$	$3/2^+$			3452.58	3506		3434
$2^2D_{5/2}$	$5/2^+$			3461.41	3493		3701
$2^4D_{5/2}$	$5/2^+$			3472.04	3504		3388
$2^4D_{7/2}$	$7/2^+$			3487.47	3486		3330
$3^4D_{1/2}$	$1/2^+$			3697.87			
$3^2D_{3/2}$	$3/2^+$			3705.31			
$3^4D_{3/2}$	$3/2^+$			3711.98			
$3^2D_{5/2}$	$5/2^+$			3717.51			
$3^4D_{5/2}$	$5/2^+$			3724.43			
$3^4D_{7/2}$	$7/2^+$			3734.23			
$4^4D_{1/2}$	$1/2^+$			3933.74			
$4^2D_{3/2}$	$3/2^+$			3938.88			
$4^4D_{3/2}$	$3/2^+$			3943.56			
$4^2D_{5/2}$	$5/2^+$			3947.34			
$4^4D_{5/2}$	$5/2^+$			3952.20			
$4^4D_{7/2}$	$7/2^+$			3958.98			
$5^4D_{1/2}$	$1/2^+$			4147.70			
$5^2D_{3/2}$	$3/2^+$			4151.47			
$5^4D_{3/2}$	$3/2^+$			4154.93			
$5^2D_{5/2}$	$5/2^+$			4157.68			
$5^4D_{5/2}$	$5/2^+$			4161.28			
$5^4D_{7/2}$	$7/2^+$			4166.24			

$\Xi_b(6227)$ which was found in both $\Lambda_b^0 K^-$ and $\Xi_b^0\pi^-$ channels [65], is identified in our model with the second excitation of the Ξ_b' baryons corresponding to $L = 1$, $n = 0$ and $J^P = 1/2^-$,

$$\bar{M} = 6244.84\ \text{MeV}, \qquad a_1 = 9.85\ \text{MeV}, \qquad a_2 = 9.41\ \text{MeV}, \qquad b_1 = 4.94\ \text{MeV}, \qquad c_1 = 1.48\ \text{MeV}, \tag{46}$$

$$M(\Xi_b', 1P):\ 6215.82\ \text{MeV}, 6233.90\ \text{MeV}, 6239.61\ \text{MeV}, 6248.59\ \text{MeV}, 6259.13\ \text{MeV}. \tag{47}$$

TABLE XIV. The mass spectrum (MeV) of Ξ_b' baryons are given and compared with different quark models.

State	J^P	Baryon	Mass	Ours	EFG [11]	Reference [56]	Reference [57]
$1^1S_{1/2}$	$1/2^+$	$\Xi_b'(5935)^-$	5935.02	5930.03	5936	5935	5935
$1^3S_{3/2}$	$3/2^+$	$\Xi_b^*(5955)^-$	5955.33	5953.08	5963	5958	
$2^1S_{1/2}$	$1/2^+$			6341.53	6329	6328	6329
$2^3S_{3/2}$	$3/2^+$			6344.41	6342	6343	
$3^1S_{1/2}$	$1/2^+$			6669.60	6687	6625	6700
$3^3S_{3/2}$	$3/2^+$			6670.46	6695	6634	
$4^1S_{1/2}$	$1/2^+$			6953.79	6978	6902	7051
$4^3S_{3/2}$	$3/2^+$			6954.15	6984	6907	
$5^1S_{1/2}$	$1/2^+$			7208.35	7229	7161	7386
$5^3S_{3/2}$	$3/2^+$			7208.53	7234	7165	
$1^2P_{1/2}$	$1/2^-$			6215.82	6227	6235	
$1^4P_{1/2}$	$1/2^-$	$\Xi_b(6227)^-$	6227.9	6233.90	6233	6237	
$1^2P_{3/2}$	$3/2^-$			6239.61	6224	6232	6229
$1^4P_{3/2}$	$3/2^-$			6248.59	6234	6234	
$1^4P_{5/2}$	$5/2^-$			6259.13	6226	6229	
$2^2P_{1/2}$	$1/2^-$			6574.81	6604	6494	
$2^4P_{1/2}$	$1/2^-$			6582.19	6611	6495	
$2^2P_{3/2}$	$3/2^-$			6583.98	6598	6492	6605
$2^4P_{3/2}$	$3/2^-$			6588.77	6605	6493	
$2^4P_{5/2}$	$5/2^-$			6593.33	6596	6490	
$3^2P_{1/2}$	$1/2^-$			6874.07	6905	6731	
$3^4P_{1/2}$	$1/2^-$			6878.04	6906	6732	
$3^2P_{3/2}$	$3/2^-$			6878.83	6897	6729	6961
$3^4P_{3/2}$	$3/2^-$			6881.77	6900	6730	
$3^4P_{5/2}$	$5/2^-$			6884.27	6897	6728	
$4^2P_{1/2}$	$1/2^-$			7138.00	7164	6949	
$4^4P_{1/2}$	$1/2^-$			7140.48	7174	6950	
$4^2P_{3/2}$	$3/2^-$			7140.90	7159	6948	7299
$4^4P_{3/2}$	$3/2^-$			7142.87	7163	6949	
$4^4P_{5/2}$	$5/2^-$			7144.47	7156	6947	
$5^2P_{1/2}$	$1/2^-$			7377.26			
$5^4P_{1/2}$	$1/2^-$			7378.96			
$5^2P_{3/2}$	$3/2^-$			7379.20			7622
$5^4P_{3/2}$	$3/2^-$			7380.61			
$5^4P_{5/2}$	$5/2^-$			7381.72			
$1^4D_{1/2}$	$1/2^+$			6480.78	6447	6380	
$1^2D_{3/2}$	$3/2^+$			6487.54	6431	6375	
$1^4D_{3/2}$	$3/2^+$			6493.27	6459	6377	
$1^2D_{5/2}$	$5/2^+$			6498.50	6420	6371	6510
$1^4D_{5/2}$	$5/2^+$			6504.38	6432	6373	
$1^4D_{7/2}$	$7/2^+$			6513.3	6414	6368	
$2^4D_{1/2}$	$1/2^+$			6794.07	6767	6632	
$2^2D_{3/2}$	$3/2^+$			6797.83	6751	6628	
$2^4D_{3/2}$	$3/2^+$			6801.13	6775	6630	
$2^2D_{5/2}$	$5/2^+$			6803.97	6740	6625	6751
$2^4D_{5/2}$	$5/2^+$			6807.38	6751	6626	
$2^4D_{7/2}$	$7/2^+$			6812.33	6736	6621	

(Table continued)

TABLE XIV. *(Continued)*

State	J^P	Baryon	Mass	Ours	EFG [11]	Reference [56]	Reference [57]
$3^4D_{1/2}$	$1/2^+$			7067.14		6861	
$3^2D_{3/2}$	$3/2^+$			7069.53		6859	
$3^4D_{3/2}$	$3/2^+$			7071.67		6860	
$3^2D_{5/2}$	$5/2^+$			7073.45		6856	6984
$3^4D_{5/2}$	$5/2^+$			7075.67		6857	
$3^4D_{7/2}$	$7/2^+$			7078.82		6854	
$4^4D_{1/2}$	$1/2^+$			7312.96		7072	
$4^2D_{3/2}$	$3/2^+$			7314.61		7070	
$4^4D_{3/2}$	$3/2^+$			7316.11		7071	
$4^2D_{5/2}$	$5/2^+$			7317.32		7069	7209
$4^4D_{5/2}$	$5/2^+$			7318.88		7069	
$4^4D_{7/2}$	$7/2^+$			7321.06		7067	
$5^4D_{1/2}$	$1/2^+$			7538.65			
$5^2D_{3/2}$	$3/2^+$			7539.77			
$5^4D_{3/2}$	$3/2^+$			7540.88			
$5^2D_{5/2}$	$5/2^+$			7541.76			7427
$5^4D_{5/2}$	$5/2^+$			7542.92			
$5^4D_{7/2}$	$7/2^+$			7544.51			

The predicted masses are compatible with the experimental values, closer to the second state or mixed with the first state. The same conclusion holds for the masses of the Ξ_b' baryons as shown in Table XII and Table XIV. The latter can be inquired also for a discussion of $\Xi_b(6227)$ in different models [56,57] and the well-matching with the experiment.

VII. SUMMARY

Stimulated by new excited states found by LHCb, in this paper we study the mass spectra of the heavy baryons and the internal structure. Comparing with the experimental data of discovered singly heavy baryons and with predictions of existing theoretical models, the internal interaction of hadrons and the structure of the Σ_Q, Ξ_Q', and Ω_Q ($Q = c$, b) baryons are being explored.

In this work, we use the JLS mixing scheme to study the S-, P-, and D-wave states of the baryons. To calculate the mass splitting of the singly heavy baryons, we discuss the Regge trajectory and the spin-dependent potential in the quark-diquark picture. In our model, we establish new scaling relations to determine the spin coupling parameters a_1, a_2, b_1, and c_1. The parameters for $1P$-wave states of the Ω_c baryons are treated as the object of the scaling relations. By analyzing the mass spectra of the discovered experimental data in PDG, we predict the mass spectra of several unobserved baryons. In addition, our analysis indicates the two new excited Ω_c states as $2^1S_{1/2}$ and $1^2D_{3/2}$ for $\Omega_c(3185)^0$ and $\Omega_c(3327)^0$, respectively. These predictions provide important references for future experimental exploration.

APPENDIX A: *S*-WAVE

Analyzing S-wave mass splitting with the orbital angular momentum $L = 0$, the singly heavy baryon is considered in the approximation of a system of a single heavy quark and a light diquark, with the heavy quark spin $S_Q = 1/2$ and diquark spin $S_d = 1$, respectively. Therefore, there are two possibilities for the total spin $\mathbf{S}$; one is $1/2$ and the other is $3/2$. In the scheme of LS coupling, the spin of the diquark $\mathbf{S}_d$ and the spin of the heavy quark $\mathbf{S}_Q$ couple to give $\mathbf{S}$ ($\mathbf{S} = \mathbf{S}_d + \mathbf{S}_Q$), before $\mathbf{S}$ is combined with $\mathbf{L}$ to generate the total angular momentum $\mathbf{J}$ ($\mathbf{J} = \mathbf{S} + \mathbf{L}$). We consider S-wave ($L = 0$) states in baryons Qqq, where the coupling of $L = 0$ with the spin $S = 1/2$ gives states with $J = 1/2$, while coupling with $S = 3/2$ leads to $J = 3/2$. In this case, the first three terms in Eq. (12) are eliminated, only the last term survives,

$$H_2^{\rm SD} = c_1 \mathbf{S}_d \cdot \mathbf{S}_Q. \tag{A1}$$

It is very convenient to analyze the influence of spin-spin interaction on the nontrivial terms for the mass splitting. The matrix elements of $\mathbf{S}_d \cdot \mathbf{S}_Q$ may be evaluated by explicit construction of states with the third component S_3 of the total spin given as linear combinations of the states $|S_{d3}, S_{Q3}\rangle$ and calculate the expectation value $\langle \mathbf{S}_d \cdot \mathbf{S}_Q \rangle = [S(S+1) - S_Q(S_Q+1) - S_d(S_d+1)]/2$ of $\mathbf{S}_d \cdot \mathbf{S}_Q$ as the square of the total spin $\mathbf{S} = \mathbf{S}_Q + \mathbf{S}_d$,

$$\mathbf{S}_d \cdot \mathbf{S}_Q = (\mathbf{S}^2 - \mathbf{S}_d^2 - \mathbf{S}_Q^2)/2. \tag{A2}$$

The two basis states are

$$\begin{aligned} |^2S_{1/2}, S_3 = 1/2\rangle &= \sqrt{\frac{2}{3}}\left|1, -\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|0, \frac{1}{2}\right\rangle, \\ |^4S_{3/2}, S_3 = 3/2\rangle &= \left|1, \frac{1}{2}\right\rangle. \end{aligned} \tag{A3}$$

The eigenvalues (two diagonal elements) of $\langle \mathbf{S}_d \cdot \mathbf{S}_Q \rangle$ in the basis $[^2S_{1/2}, {}^4S_{3/2}]$ can be obtained as

$$\langle \mathbf{S}_d \cdot \mathbf{S}_Q \rangle = \begin{bmatrix} \langle ^2S_{1/2}, S_3 = 1/2|\mathbf{S}_d \cdot \mathbf{S}_Q|^2S_{1/2}, S_3 = 1/2\rangle & \langle ^2S_{1/2}, S_3 = 1/2|\mathbf{S}_d \cdot \mathbf{S}_Q|^4S_{3/2}, S_3 = 3/2\rangle \\ \langle ^4S_{3/2}, S_3 = 3/2|\mathbf{S}_d \cdot \mathbf{S}_Q|^2S_{1/2}, S_3 = 1/2\rangle & \langle ^4S_{3/2}, S_3 = 3/2|\mathbf{S}_d \cdot \mathbf{S}_Q|^4S_{3/2}, S_3 = 3/2\rangle \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}. \tag{A4}$$

Combining with Eqs. (10) and (A4), the S-wave masses of the singly heavy baryons are

$$M = \bar{M} + c_1 \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}. \tag{A5}$$

APPENDIX B: *P*-WAVE

Let us consider the P-wave system with the orbital angular momentum $L = 1$. The spin of the diquark $S_d = 1$ can be coupled with the heavy quark spin $S_Q = 1/2$ and $L = 1$ to the total angular momentum $J = 1/2, 3/2$ or $1/2, 3/2, 5/2$ with negative parity $P = -1$. The expectation value of $\mathbf{L} \cdot \mathbf{S}$ in any coupling scheme is

$$\langle \mathbf{L} \cdot \mathbf{S} \rangle = [J(J+1) - L(L+1) - S(S+1)]/2, \tag{B1}$$

and the calculation of the operator $\mathbf{L} \cdot \mathbf{S}_i$ ($i = Q$, d) results in

$$\mathbf{L} \cdot \mathbf{S}_i = L_3 S_{i3} + (L_+ S_{i-} + L_- S_{i+})/2, \tag{B2}$$

with raising and lowering operator $L_\pm$, $S_{i\pm}$. The expectation values of $\mathbf{L} \cdot \mathbf{S}_d$, $\mathbf{L} \cdot \mathbf{S}_Q$, S_{12}, and $\mathbf{S}_d \cdot \mathbf{S}_Q$ in Eq. (12) in the $L - S$ basis can be constructed as linear combinations of the states $|S_{d3}, S_{Q3}, L_3\rangle$ of the third components of the respective angular momenta,

$$\begin{aligned} |^2P_{1/2}, J_3 = 1/2\rangle &= \frac{\sqrt{2}}{3}\left|1, -\frac{1}{2}, 0\right\rangle - \frac{1}{3}\left|0, \frac{1}{2}, 0\right\rangle - \frac{\sqrt{2}}{3}\left|0, -\frac{1}{2}, 1\right\rangle + \frac{2}{3}\left|-1, \frac{1}{2}, 1\right\rangle, \\ |^4P_{1/2}, J_3 = 1/2\rangle &= \frac{1}{\sqrt{2}}\left|1, \frac{1}{2}, -1\right\rangle - \frac{1}{3}\left|1, -\frac{1}{2}, 0\right\rangle - \frac{\sqrt{2}}{3}\left|0, \frac{1}{2}, 0\right\rangle + \frac{1}{3}\left|0, -\frac{1}{2}, 1\right\rangle + \frac{1}{3\sqrt{2}}\left|-1, \frac{1}{2}, 1\right\rangle, \\ |^2P_{3/2}, J_3 = 3/2\rangle &= \sqrt{\frac{2}{3}}\left|1, -\frac{1}{2}, 1\right\rangle - \sqrt{\frac{1}{3}}\left|0, \frac{1}{2}, 1\right\rangle, \\ |^4P_{3/2}, J_3 = 3/2\rangle &= \sqrt{\frac{3}{5}}\left|1, \frac{1}{2}, 0\right\rangle - \sqrt{\frac{2}{15}}\left|1, -\frac{1}{2}, 1\right\rangle - \frac{2}{\sqrt{15}}\left|0, \frac{1}{2}, 1\right\rangle, \\ |^4P_{5/2}, J_3 = 5/2\rangle &= \left|1, \frac{1}{2}, 1\right\rangle. \end{aligned} \tag{B3}$$

The expectation values of $\langle \mathbf{L} \cdot \mathbf{S}_i \rangle$, $\langle S_{12} \rangle$ and $\langle \mathbf{S}_d \cdot \mathbf{S}_Q \rangle$ are given by

$$\langle \mathbf{L}\cdot\mathbf{S}_d\rangle_{J=\frac{1}{2}} = \begin{bmatrix} -\frac{4}{3} & -\frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & -\frac{5}{3} \end{bmatrix}, \qquad \langle \mathbf{L}\cdot\mathbf{S}_Q\rangle_{J=\frac{1}{2}} = \begin{bmatrix} \frac{1}{3} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & -\frac{5}{6} \end{bmatrix}, \qquad \langle S_{12}\rangle_{J=\frac{1}{2}} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -1 \end{bmatrix},$$
$$\langle \mathbf{S}_d\cdot\mathbf{S}_Q\rangle_{J=\frac{1}{2}} = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix},$$
$$\langle \mathbf{L}\cdot\mathbf{S}_d\rangle_{J=\frac{3}{2}} = \begin{bmatrix} \frac{2}{3} & \frac{\sqrt{5}}{3} \\ -\frac{\sqrt{5}}{3} & \frac{2}{3} \end{bmatrix}, \qquad \langle \mathbf{L}\cdot\mathbf{S}_Q\rangle_{J=\frac{3}{2}} = \begin{bmatrix} -\frac{1}{6} & \frac{\sqrt{5}}{3} \\ \frac{\sqrt{5}}{3} & -\frac{1}{3} \end{bmatrix}, \qquad \langle S_{12}\rangle_{J=\frac{3}{2}} = \begin{bmatrix} 0 & -\frac{\sqrt{5}}{10} \\ -\frac{\sqrt{5}}{10} & \frac{4}{5} \end{bmatrix},$$
$$\langle \mathbf{S}_d\cdot\mathbf{S}_Q\rangle_{J=\frac{3}{2}} = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix},$$
$$\langle \mathbf{L}\cdot\mathbf{S}_d\rangle_{J=\frac{5}{2}} = 1, \qquad \langle \mathbf{L}\cdot\mathbf{S}_Q\rangle_{J=\frac{5}{2}} = \frac{1}{2}, \qquad \langle S_{12}\rangle_{J=\frac{5}{2}} = -\frac{1}{5}, \qquad \langle \mathbf{S}_d\cdot\mathbf{S}_Q\rangle_{J=\frac{5}{2}} = \frac{1}{2}. \tag{B4}$$

The matrix forms of these mass shifts are

$$\Delta\mathcal{M}_{J=1/2} = \begin{bmatrix} \frac{1}{3}(a_2-4a_1) & \frac{\sqrt{2}}{3}(a_2-a_1)+\frac{b_1}{\sqrt{2}} \\ \frac{\sqrt{2}}{3}(a_2-a_1)+\frac{b_1}{\sqrt{2}} & -\frac{5}{3}(a_1+\frac{1}{2}a_2)-b_1 \end{bmatrix} + \begin{bmatrix} -c_1 & 0 \\ 0 & \frac{1}{2}c_1 \end{bmatrix},$$
$$\Delta\mathcal{M}_{J=3/2} = \begin{bmatrix} \frac{2}{3}a_1-\frac{1}{6}a_2 & \frac{\sqrt{5}}{3}(a_2-a_1)-\frac{b_1}{2\sqrt{5}} \\ \frac{\sqrt{5}}{3}(a_2-a_1)-\frac{b_1}{2\sqrt{5}} & -\frac{1}{3}(2a_1+a_2)+\frac{4b_1}{5} \end{bmatrix} + \begin{bmatrix} -c_1 & 0 \\ 0 & \frac{1}{2}c_1 \end{bmatrix},$$
$$\Delta\mathcal{M}_{J=5/2} = a_1+\frac{1}{2}a_2-\frac{b_1}{5}+\frac{c_1}{2}. \tag{B5}$$

Diagonalizing the matrices Eq. (B5), one can compute the mass shifts $\Delta M(J,j)$ with the total angular momentum $\mathbf{J}$ and the total light-quark angular momentum $\mathbf{j}=\mathbf{L}+\mathbf{S}_d$, where $S_d=1$ is the spin of the diquark, so $j=0$, 1, 2,

$$\Delta M(1/2,0) = \frac{1}{4}\left(-6a_1-a_2-2b_1-\sqrt{\Delta_1(a_1,a_2,b_1)}\right)+c_1\Delta_3^+(a_1,a_2,b_1),$$
$$\Delta M(1/2,1) = \frac{1}{4}\left(-6a_1-a_2-2b_1+\sqrt{\Delta_1(a_1,a_2,b_1)}\right)+c_1\Delta_3^+(a_1,a_2,b_1),$$
$$\Delta M(3/2,1) = \frac{1}{20}\left(-5a_2+8b_1-\sqrt{\Delta_2(a_1,a_2,b_1)}\right)+c_1\Delta_4^+(a_1,a_2,b_1),$$
$$\Delta M(3/2,2) = \frac{1}{20}\left(-5a_2+8b_1+\sqrt{\Delta_2(a_1,a_2,b_1)}\right)+c_1\Delta_4^+(a_1,a_2,b_1),$$
$$\Delta M(5/2,2) = a_1+\frac{a_2}{2}-\frac{b_1}{5}+\frac{c_1}{2}, \tag{B6}$$

where six functions $\Delta_{1,2}(a_1,a_2,b_1)$, $\Delta_3^\pm(a_1,a_2,b_1)$ and $\Delta_4^\pm(a_1,a_2,b_1)$ are defined by

$$\Delta_1(a_1,a_2,b_1) = 4(a_1)^2-8a_1b_1+12(b_1)^2-4a_1a_2+20b_1a_1+9(a_2)^2,$$
$$\Delta_2(a_1,a_2,b_1) = 400(a_1)^2-80a_1b_1+84(b_1)^2-400a_1a_2-160b_1a_1+225(a_2)^2,$$
$$\Delta_3^+(a_1,a_2,b_1) = \frac{4-\left(-2-\frac{7a_2}{a_1}-\frac{6b_1}{a_1}+\frac{3}{a_1}\sqrt{\Delta_1(a_1,a_2,b_1)}\right)^2/\left(-2+\frac{2a_2}{a_1}+\frac{3b_1}{a_1}\right)^2}{8+\left(-2-\frac{7a_2}{a_1}-\frac{6b_1}{a_1}+\frac{3}{a_1}\sqrt{\Delta_1(a_1,a_2,b_1)}\right)^2/\left(-2+\frac{2a_2}{a_1}+\frac{3b_1}{a_1}\right)^2},$$
$$\Delta_3^-(a_1,a_2,b_1) = \Delta_3^+\left(\sqrt{\Delta_1}\to-\sqrt{\Delta_1}\right),$$
$$\Delta_4^+(a_1,a_2,b_1) = \frac{10-\left(40+\frac{5a_2}{a_1}-\frac{24b_1}{a_1}-\frac{3}{a_1}\sqrt{\Delta_2(a_1,a_2,b_1)}\right)^2/\left(10-\frac{10a_2}{a_1}+\frac{3b_1}{a_1}\right)^2}{20+\left(40+\frac{5a_2}{a_1}-\frac{24b_1}{a_1}-\frac{3}{a_1}\sqrt{\Delta_2(a_1,a_2,b_1)}\right)^2/\left(10-\frac{10a_2}{a_1}+\frac{3b_1}{a_1}\right)^2},$$
$$\Delta_4^-(a_1,a_2,b_1) = \Delta_4^+\left(\sqrt{\Delta_2}\to-\sqrt{\Delta_2}\right), \tag{B7}$$

with $\Delta^-_{3,4}(a_1, a_2, b_1)$ obtained from $\Delta^+_{3,4}(a_1, a_2, b_1)$ by merely replacing $\sqrt{\Delta_{1,2}} \to -\sqrt{\Delta_{1,2}}$. The mass spectra of the P-wave states for the baryons are

$$\begin{aligned} M(1/2,0) &= \bar{M} + \Delta M(1/2,0), \\ M(1/2,1) &= \bar{M} + \Delta M(1/2,1), \\ M(3/2,1) &= \bar{M} + \Delta M(3/2,1), \\ M(3/2,2) &= \bar{M} + \Delta M(3/2,2), \\ M(5/2,2) &= \bar{M} + \Delta M(5/2,2). \end{aligned} \tag{B8}$$

APPENDIX C: *D*-WAVE

For analyzing the D-wave system, the diquark spin $S_d = 1$ can be coupled with the heavy quark spin $S_Q = 1/2$ to determine the total spin $S = 1/2, 3/2$. Coupling of the orbital angular momentum $L = 2$ give six states with the total spin $J = 1/2, 3/2, 5/2$ or $3/2, 5/2, 7/2$ with positive parity $P = +1$. The relevant linear combinations of six basis states are

$$\begin{aligned} |{}^4D_{1/2}, J_3 = 1/2\rangle &= \frac{1}{\sqrt{10}}\left|1,\frac{1}{2},-1\right\rangle - \frac{1}{\sqrt{15}}\left|1,-\frac{1}{2},0\right\rangle - \sqrt{\frac{2}{15}}\left|0,\frac{1}{2},0\right\rangle + \frac{1}{\sqrt{5}}\left|0,-\frac{1}{2},1\right\rangle + \frac{1}{\sqrt{10}}\left|-1,\frac{1}{2},1\right\rangle \\ &\quad - \sqrt{\frac{2}{5}}\left|-1,-\frac{1}{2},2\right\rangle, \\ |{}^2D_{3/2}, J_3 = 3/2\rangle &= \sqrt{\frac{2}{15}}\left|1,-\frac{1}{2},1\right\rangle - \frac{1}{\sqrt{15}}\left|0,\frac{1}{2},1\right\rangle - \frac{2}{\sqrt{15}}\left|0,-\frac{1}{2},2\right\rangle + \sqrt{\frac{8}{15}}\left|-1,\frac{1}{2},2\right\rangle, \\ |{}^4D_{3/2}, J_3 = 3/2\rangle &= \frac{1}{\sqrt{5}}\left|1,\frac{1}{2},0\right\rangle - \sqrt{\frac{2}{15}}\left|1,\frac{1}{2},1\right\rangle - \frac{2}{\sqrt{15}}\left|0,\frac{1}{2},1\right\rangle + \frac{2}{\sqrt{15}}\left|0,-\frac{1}{2},2\right\rangle + \sqrt{\frac{2}{15}}\left|-1,\frac{1}{2},2\right\rangle, \\ |{}^2D_{5/2}, J_3 = 5/2\rangle &= \sqrt{\frac{2}{3}}\left|1,-\frac{1}{2},2\right\rangle - \sqrt{\frac{1}{3}}\left|0,\frac{1}{2},2\right\rangle, \\ |{}^4D_{5/2}, J_3 = 5/2\rangle &= \frac{3}{\sqrt{21}}\left|1,\frac{1}{2},1\right\rangle - \frac{2}{\sqrt{21}}\left|1,-\frac{1}{2},2\right\rangle - \frac{2\sqrt{2}}{\sqrt{21}}\left|0,\frac{1}{2},2\right\rangle, \\ |{}^4D_{7/2}, J_3 = 7/2\rangle &= \left|1,\frac{1}{2},2\right\rangle. \end{aligned} \tag{C1}$$

The expectation values of $\langle \mathbf{L}\cdot\mathbf{S}_i\rangle$ ($i = Q$, d), $\langle S_{12}\rangle$ and $\langle \mathbf{S}_d\cdot\mathbf{S}_Q\rangle$ are

$$\begin{aligned} &\langle \mathbf{L}\cdot\mathbf{S}_d\rangle_{J=\frac{1}{2}} = -3, \qquad \langle \mathbf{L}\cdot\mathbf{S}_Q\rangle_{J=\frac{1}{2}} = -\frac{3}{2}, \qquad \langle S_{12}\rangle_{J=\frac{1}{2}} = -1, \qquad \langle \mathbf{S}_d\cdot\mathbf{S}_Q\rangle_{J=\frac{1}{2}} = \frac{1}{2}, \\ &\langle \mathbf{L}\cdot\mathbf{S}_d\rangle_{J=\frac{3}{2}} = \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}, \qquad \langle \mathbf{L}\cdot\mathbf{S}_Q\rangle_{J=\frac{3}{2}} = \begin{bmatrix} \frac{1}{2} & 1 \\ 1 & -1 \end{bmatrix}, \qquad \langle S_{12}\rangle_{J=\frac{3}{2}} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}, \\ &\langle \mathbf{S}_d\cdot\mathbf{S}_Q\rangle_{J=\frac{3}{2}} = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \\ &\langle \mathbf{L}\cdot\mathbf{S}_d\rangle_{J=\frac{5}{2}} = \begin{bmatrix} \frac{4}{3} & -\frac{\sqrt{14}}{3} \\ -\frac{\sqrt{14}}{3} & -\frac{1}{3} \end{bmatrix}, \qquad \langle \mathbf{L}\cdot\mathbf{S}_Q\rangle_{J=\frac{5}{2}} = \begin{bmatrix} -\frac{1}{3} & \frac{\sqrt{14}}{3} \\ \frac{\sqrt{14}}{3} & -\frac{1}{6} \end{bmatrix}, \qquad \langle S_{12}\rangle_{J=\frac{5}{2}} = \begin{bmatrix} 0 & -\frac{\sqrt{14}}{14} \\ -\frac{\sqrt{14}}{14} & \frac{5}{7} \end{bmatrix}, \\ &\langle \mathbf{S}_d\cdot\mathbf{S}_Q\rangle_{J=\frac{5}{2}} = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \\ &\langle \mathbf{L}\cdot\mathbf{S}_d\rangle_{J=\frac{7}{2}} = 2, \qquad \langle \mathbf{L}\cdot\mathbf{S}_Q\rangle_{J=\frac{7}{2}} = 1, \qquad \langle S_{12}\rangle_{J=\frac{7}{2}} = -\frac{2}{7}, \qquad \langle \mathbf{S}_d\cdot\mathbf{S}_Q\rangle_{J=\frac{7}{2}} = \frac{1}{2}. \end{aligned} \tag{C2}$$

The matrix forms of these mass shifts are

$$\Delta\mathcal{M}_{J=1/2} = -3a_1 - \frac{3a_2}{2} - b_1 + \frac{c_1}{2},$$
$$\Delta\mathcal{M}_{J=3/2} = \begin{bmatrix} -2a_1 + \frac{1}{2}a_2 & -a_1 + a_2 + \frac{1}{2}b_1 \\ -a_1 + a_2 + \frac{1}{2}b_1 & -2a_1 - a_2 \end{bmatrix} + \begin{bmatrix} -c_1 & 0 \\ 0 & \frac{1}{2}c_1 \end{bmatrix},$$
$$\Delta\mathcal{M}_{J=5/2} = \begin{bmatrix} \frac{4}{3}a_1 - \frac{1}{3}a_2 & -\frac{\sqrt{14}}{3}a_1 + \frac{\sqrt{14}}{3}a_2 - \frac{\sqrt{14}}{14}b_1 \\ -\frac{\sqrt{14}}{3}a_1 + \frac{\sqrt{14}}{3}a_2 - \frac{\sqrt{14}}{14}b_1 & -\frac{1}{3}a_1 - \frac{1}{6}a_2 + \frac{5}{7}b_1 \end{bmatrix} + \begin{bmatrix} -c_1 & 0 \\ 0 & \frac{1}{2}c_1 \end{bmatrix},$$
$$\Delta\mathcal{M}_{J=7/2} = 2a_1 + a_2 - \frac{2}{7}b_1 + \frac{1}{2}c_1. \tag{C3}$$

Diagonalizing the matrices Eq. (C3), one can compute six mass shifts $\Delta M(J, j)$, where $S_d = 1$ is the spin of the diquark, so $j = 1, 2, 3$,

$$\Delta M(1/2, 1) = -3a_1 - \frac{3a_2}{2} - b_1 + \frac{c_1}{2},$$
$$\Delta M(3/2, 1) = \frac{1}{4}\left(-8a_1 - a_2 - \sqrt{\Theta_1(a_1, a_2, b_1)}\right) + c_1\Theta_3^+(a_1, a_2, b_1),$$
$$\Delta M(3/2, 2) = \frac{1}{4}\left(-8a_1 - a_2 + \sqrt{\Theta_1(a_1, a_2, b_1)}\right) + c_1\Theta_3^-(a_1, a_2, b_1),$$
$$\Delta M(5/2, 2) = \frac{1}{28}\left(14a_1 - 7a_2 + 10b_1 - \sqrt{\Theta_2(a_1, a_2, b_1)}\right) + c_1\Theta_4^+(a_1, a_2, b_1),$$
$$\Delta M(5/2, 3) = \frac{1}{28}\left(14a_1 - 7a_2 + 10b_1 + \sqrt{\Theta_2(a_1, a_2, b_1)}\right) + c_1\Theta_4^-(a_1, a_2, b_1),$$
$$\Delta M(7/2, 3) = 2a_1 + a_2 - \frac{2}{7}b_1 + \frac{c_1}{2}, \tag{C4}$$

where six functions $\Theta_{1,2}(a_1, a_2, b_1)$, $\Theta_3^{\pm}(a_1, a_2, b_1)$ and $\Theta_4^{\pm}(a_1, a_2, b_1)$ are defined by

$$\Theta_1(a_1, a_2, b_1) = 16(a_1)^2 - 32a_1a_2 + 25(a_2)^2 - 16a_1b_1 + 16a_2b_1 + 4(b_1)^2,$$
$$\Theta_2(a_1, a_2, b_1) = 1764(a_1)^2 - 2548a_1a_2 + 1225(a_2)^2 + 56a_1b_1 - 476a_2b_1 + 156(b_1)^2,$$
$$\Theta_3^+(a_1, a_2, b_1) = \frac{2 - \left(\frac{3a_2}{a_1} - \frac{1}{a_1}\sqrt{\Theta_1(a_1, a_2, b_1)}\right)^2 / \left(2 - \frac{2a_2}{a_1} - \frac{b_1}{a_1}\right)^2}{4 + \left(\frac{3a_2}{a_1} - \frac{1}{a_1}\sqrt{\Theta_1(a_1, a_2, b_1)}\right)^2 / \left(2 - \frac{2a_2}{a_1} - \frac{b_1}{a_1}\right)^2},$$
$$\Theta_3^-(a_1, a_2, b_1) = \Theta_3^+\left(\sqrt{\Theta_1} \to -\sqrt{\Theta_1}\right),$$
$$\Theta_4^+(a_1, a_2, b_1) = \frac{28 - \left(70 - \frac{7a_2}{a_1} - \frac{30b_1}{a_1} - \frac{3}{a_1}\sqrt{\Theta_2(a_1, a_2, b_1)}\right)^2 / \left(2 - \frac{2a_2}{a_1} - \frac{b_1}{a_1}\right)^2}{56 + \left(70 - \frac{7a_2}{a_1} - \frac{30b_1}{a_1} - \frac{3}{a_1}\sqrt{\Theta_2(a_1, a_2, b_1)}\right)^2 / \left(2 - \frac{2a_2}{a_1} - \frac{b_1}{a_1}\right)^2},$$
$$\Theta_4^-(a_1, a_2, b_1) = \Theta_4^+\left(\sqrt{\Theta_2} \to -\sqrt{\Theta_2}\right), \tag{C5}$$

with $\Theta_{3,4}^-(a_1, a_2, b_1)$ obtained from $\Theta_{3,4}^+(a_1, a_2, b_1)$ by merely replacing $\sqrt{\Theta_{1,2}} \to -\sqrt{\Theta_{1,2}}$. The mass spectra of the D-wave states for the baryons are

$$M(1/2, 1) = \bar{M} + \Delta M(1/2, 1),$$
$$M(3/2, 1) = \bar{M} + \Delta M(3/2, 1),$$
$$M(3/2, 2) = \bar{M} + \Delta M(3/2, 2),$$
$$M(5/2, 2) = \bar{M} + \Delta M(5/2, 2),$$
$$M(5/2, 3) = \bar{M} + \Delta M(5/2, 3),$$
$$M(7/2, 3) = \bar{M} + \Delta M(7/2, 3). \tag{C6}$$

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