Electron $g-2$ foreshadowing discoveries at FCC-ee

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A future e^+e^- circular collider (FCC-ee) may provide a unique probe of the electron Yukawa coupling through Higgs boson production on resonance. Motivated by this exciting possibility, we examine a simple model which can result in $\mathcal{O}(10)$ modifications of the Higgs coupling to electrons. The model can also lead to deviations in the electron anomalous magnetic moment, g_e-2, which at present shows a $+2.2\sigma$ or -3.7σ deviation, implied by differing precision determinations of the electromagnetic fine structure constant. The electron g_e-2 can be a forerunner for FCC-ee discoveries which, as we elucidate, may not be accessible to the high-luminosity LHC measurements. A simple extension of our model can also account for the current deviation in the muon $g_\mu-2$.

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I. INTRODUCTION

Open questions that are not answered within the Standard Model (SM), such as the identity of dark matter and the origin of the cosmic baryon asymmetry, highlight the need for new fundamental physics to describe the observed phenomena in nature. It is often assumed that the new physics may manifest itself through its interactions with the Higgs field. As such, deviations in the properties of the third generation fermions are deemed most likely, due to their stronger coupling with the Higgs and hence the fundamental new physics underlying its potential.

Despite the above expectations, one may entertain the possibility that new physics may show up in the Higgs couplings of the first generation fermions. In particular, the electron can provide clean signals, unlike the top or the tau. In the SM, the Yukawa coupling of the electron to the Higgs is extremely small, $\mathcal{O}(10^{-6})$, and very challenging to measure directly. However, this can provide an opportunity to find new effects unambiguously, far from the SM expectations.

In this work, we propose a simple model which can lead to $\mathcal{O}(10)$ enhancement of the electron Yukawa coupling. The model, in its basic form, only requires a new vector-like lepton and a weak scale singlet scalar. The SM Higgs coupling to electrons can be potentially probed at a future e^+e^- circular collider (FCC-ee) [1], down to $\sim$1.6 times the SM value [2], which would be an impressive improvement over the current limit at $\sim$260 times the SM expectation [3,4]. This bound is projected to be improved to $\sim$120 times the SM value by the end of the high-luminosity LHC (HL-LHC) running [5], far from the potential projection for FCC-ee, using resonant production. Hence, the model we propose can be tested at the FCC-ee. It is interesting that the type of physics we postulate may not be detectable by the HL-LHC, with $\sim$3 ab^{-1} of integrated luminosity. However, quite generally, we expect that the model can lead to deviations in the anomalous magnetic moment of the electron g_e-2, at levels that could be accessible to experiment in the coming years. With a modest extension, the model can also address the deviation in the muon $g_\mu-2$ [6–9], which is still under investigation by theory and experiment.

Ideas for measuring the Yukawa couplings of first generation fermions, using atomic clocks, have been considered in Ref. [10]. Enhanced electron Yukawa coupling in the context of two-Higgs doublet models has been discussed in Ref. [11]. A model of charged lepton masses which can lead to deviations in the electron and muon Yukawa couplings and their values of $g-2$ is presented in Ref. [12]. See Refs. [13–17], for other works where connections between lepton dipole moments and their Yukawa couplings have been examined.

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II. ELECTRON $g-2$

Precision measurements of the electron and muon $g-2$ can provide stringent tests of the SM. Currently, the status of $a_e \equiv (g_e-2)/2$ is not clear, since the most precise measured values of the fine-structure constant α do not agree. The most recent experimental value obtained using rubidium (Rb) atoms is [18]

$$\alpha^{-1}(\mathrm{Rb}) = 137.035999206(11), \tag{1}$$

which disagrees with that obtained earlier by another group using cesium (Cs) atoms [19],

$$\alpha^{-1}(\mathrm{Cs}) = 137.035999046(27), \tag{2}$$

leading to a discrepancy at the level of 5.5σ.

The SM prediction for a_e^{SM} from Ref. [20], when compared to the latest experimental determination a_e^{exp} [21], results in a deviation $\Delta a_e \equiv a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}}$, which depends on the value of α used as input. For $\alpha(\mathrm{Rb})$, we find

$$\begin{aligned}\Delta a_e(\mathrm{Rb}) &\equiv a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}}(\mathrm{Rb}) \\ &= [34 \pm 13(\mathrm{exp}) \pm 9(\alpha) \pm 2(\mathrm{th})] \times 10^{-14}.\end{aligned} \tag{3}$$

Summing errors in quadrature, we get

$$\Delta a_e(\mathrm{Rb}) = (34 \pm 16) \times 10^{-14}, \tag{4}$$

which leads to a positive deviation of 2.2σ. However, using the value $\alpha(\mathrm{Cs})$ yields

$$\begin{aligned}\Delta a_e(\mathrm{Cs}) &\equiv a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}}(\mathrm{Cs}) \\ &= [-101 \pm 13(\mathrm{exp}) \pm 23(\alpha) \pm 2(\mathrm{th})] \times 10^{-14},\end{aligned} \tag{5}$$

which gives

$$\Delta a_e(\mathrm{Cs}) = (-101 \pm 27) \times 10^{-14} \tag{6}$$

and thus leads to a negative discrepancy of 3.7σ.[1]

We note that the discrepancy between theory and experiment has grown since the experimental determinations of α in 2018 [19] and 2020 [18], for either value used as input. This is due to the new experimental result for a_e^{exp} [21], which is smaller than the previous determination [22] by 14×10^{-14}, but has less than half the uncertainty of the earlier measurement.

III. THE MODEL

We consider a theory where we add to the usual SM field content a singlet scalar ϕ and a family of heavy vectorlike fermions S_l. The fermion S_l, having the gauge charges of the SM right-handed electron, carries a lepton flavor number. Therefore, we have three distinct fermions S_e, S_μ, S_τ. The Lagrangian of this model is given by

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_\phi + \mathcal{L}_{S_e} + \mathcal{L}_{S_\mu} + \mathcal{L}_{S_\tau} + \mathrm{H.c.}, \tag{7}$$

$$\begin{aligned}\mathcal{L}_\phi &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 \\ &\quad - \frac{\mu}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4 - \kappa v\phi H^\dagger H,\end{aligned} \tag{8}$$

$$\begin{aligned}\mathcal{L}_{S_e} &= i\bar{S}_e\not{D}S_e - M_{S_e}\bar{S}_eS_e - M_{e,S_e}\bar{S}_{e,L}e_R - y_{S_e}\bar{L}_eHS_{e,R} \\ &\quad - \xi_e\phi\bar{S}_{e,L}e_R - g_{S_e}\phi\bar{S}_eS_e + \mathrm{H.c.},\end{aligned} \tag{9}$$

$$\mathcal{L}_{S_\ell} = \mathcal{L}_{S_e}(e \to \ell); \qquad \ell = \mu, \tau, \tag{10}$$

where H is the usual Higgs doublet, and v is the SM Higgs vacuum expectation value: $\langle H\rangle = v/\sqrt{2} \approx 174$ GeV. As explained in the Introduction, in this paper we are interested particularly in the modification of the physics of the electron, therefore for now we are going to focus only on the first three terms of the Lagrangian in Eq. (7). We will come back to the other terms at the end of the paper.

It has to be noted that the above setup, as it stands could lead to charged lepton-flavor violation. The form of the interactions in Eq. (9) implicitly assumes that such effects are absent or else sufficiently small, which is a phenomenologically motivated choice. However, without a good symmetry, one can in principle couple S_e to all three SM lepton generations and end up with flavor off-diagonal interactions. These could provide additional signals for our model, but one has to make sure they are not at unacceptable levels. This is a generic model building problem, whenever new vectorlike leptons are introduced. We will address this issue in the Appendix.

Note that while at the level of the unbroken symmetry ϕ cannot mix directly with the doublet field H, once the electroweak symmetry is broken ϕ can mix with the Higgs boson h (corresponding to the observed scalar at $\sim$125 GeV) and new mass mixing terms between S_e and the electron appear. Consequently, both the mass of the electron m_e and the effective coupling $y_e^h/\sqrt{2}$ between the Higgs boson h and the electron are modified. However, since the modifications to these parameters depend in different ways on the Lagrangian parameters, the SM relation between m_e and y_e^h is not preserved. This is the central mechanism which allowed us to obtain a large y_e^h, while keeping m_e at its measured value. We will illustrate this point using an effective field theory (EFT) approach.

IV. EFFECTIVE FIELD THEORY ANALYSIS

In order to elucidate the above mechanism, we now provide a simplified EFT analysis. To do so, let us assume parameters such that the cubic and the quartic terms in the ϕ

[1] A deviation of $|\Delta a_e| \sim 8 \times 10^{-14}$, within $\sim 1.6\sigma$ from $\Delta a_e(\mathrm{Rb})$, can be achieved using only SM fields by increasing the Yukawa coupling between the Higgs and the electron to $\sim$250 times its SM value, which is very close to the current experimental bounds. The deviation $\Delta a_e(\mathrm{Cs})$ can be addressed in this way only within $\sim 3.4\sigma$.

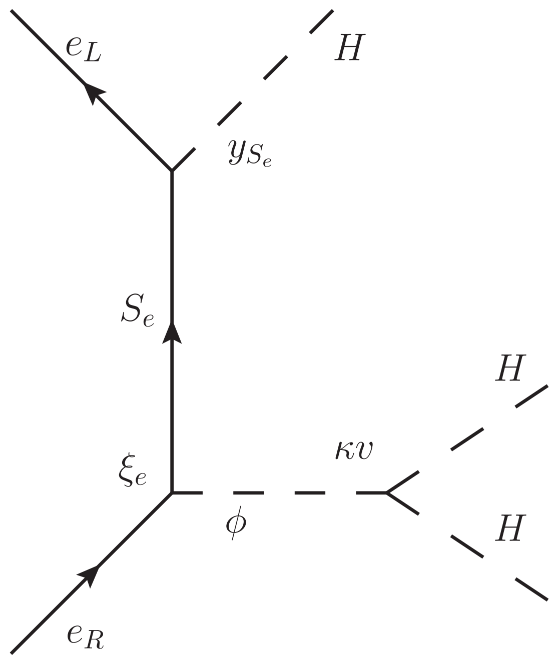

FIG. 1. Feynman diagram that can generate the operator in Eq. (11) if we assume S_e and ϕ to be heavy. Notice that we have ignored the numerical factors.

potential can be ignored, focusing only on the mass term and the linear term.[2] Treating both the S_e fermion and ϕ as heavy fields (Fig. 1), we may integrate them out and obtain the following dimension-6 operator:

$$O_6 = \kappa \frac{(H^\dagger H)\bar{L}He_R}{\Lambda^2}, \tag{11}$$

where

$$\Lambda^2 \equiv y_{S_e}^{-1}\xi_e^{-1}v^{-1}M_{S_e}m_\phi^2, \tag{12}$$

using the notation from Eq. (9).

After electroweak symmetry breaking, the contribution from the usual dimension-4 Higgs Yukawa coupling to the electron, $y_e H\bar{L}e_R$, and that from the O_6 of the EFT yield the following for the electron mass:

$$m_e = \frac{y_e v}{\sqrt{2}} + \frac{\kappa v^3}{\sqrt{8}\Lambda^2}. \tag{13}$$

[2]The last term in Eq. (8) generates a tadpole term for ϕ, and therefore a vacuum expectation value (v_ϕ). In our analysis, we have implicitly assumed the value of v_ϕ to be small compared to m_ϕ. We find that, in the decoupling limit $\kappa \to 0$, the potential of ϕ has only one minimum around the origin if $\mu < \sqrt{3\lambda}m_\phi$. Under this condition, which remains approximately true in the full theory as long as $\kappa \ll 1$, the resulting v_ϕ is small and mostly independent of the exact values of λ and μ.

The corresponding effective coupling $(y_e^h/\sqrt{2})h\bar{e}e$ of the electron to h will then be given by

$$y_e^h = y_e + \frac{3\kappa v^2}{2\Lambda^2}. \tag{14}$$

From Eq. (13), we see that one can choose y_e to be much larger than the SM value, as long as there is a sufficient degree of cancellation between the two terms that contribute to the electron mass, resulting in the measured value $m_e \approx 0.511$ MeV. However, such a cancellation will no longer be maintained for y_e^h, leading to a value $y_e^h \sim \mathcal{O}(y_e)$, which can be much larger than $y_e^{h,\mathrm{SM}} = \sqrt{2}m_e/v \approx 3 \times 10^{-6}$ predicted in the SM.

V. LEPTON MAGNETIC DIPOLE MOMENTS

The interactions between leptons and ϕ described in Eq. (7) induce new contributions to their a_ℓ. The largest contributions to Δa_ℓ for our choice of parameters are represented in Figs. 2 and 3. We are not going to discuss here other contributions (for example, those mediated by S_ℓ at one-loop) since they are subdominant; however, they have been included in the final result. The one-loop contribution of ϕ to a_ℓ in Fig. 2 is given by [23–25]

$$\Delta a_\ell = \frac{\lambda_\ell^2}{8\pi^2}x^2 \int_0^1 dz \frac{(1+z)(1-z)^2}{x^2(1-z)^2 + z}, \tag{15}$$

for a lepton ℓ of mass m_ℓ and $x \equiv m_\ell/m_\phi$. The coupling of ℓ to ϕ is denoted by λ_ℓ, corresponding to the interaction $\lambda_\ell \phi\bar{\ell}\ell$; this coupling results from the interactions in Eq. (7), after electroweak symmetry breaking, and in general corresponds to a complicated function of all the parameters of the Lagrangian.

The "Barr-Zee" diagram contribution to a_ℓ (Fig. 3), for a heavy fermion f loop, is given by [26–28]

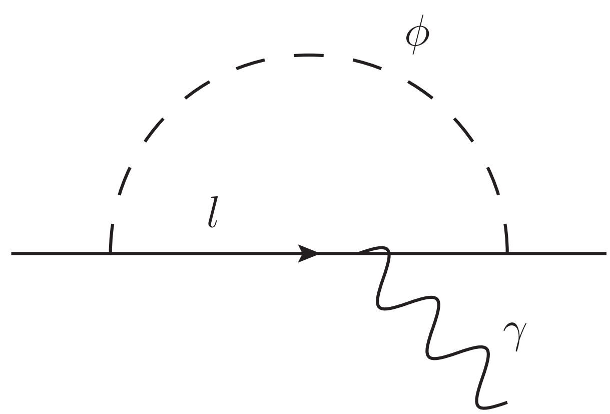

FIG. 2. Contribution to the $g-2$ of a lepton l induced by its coupling to the scalar ϕ.

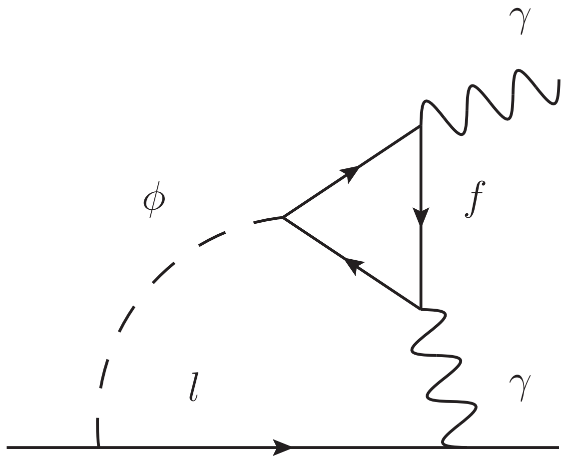

FIG. 3. The "Barr-Zee" contribution to the $g-2$ of a lepton l.

$$\Delta a_\ell^{\rm BZ}(f) = -\frac{\alpha}{6\pi}\frac{m_\ell}{m_f}\frac{\lambda_\ell\lambda_f}{\pi^2}Q_f^2 N_c^f I(y), \qquad (16)$$

where

$$I(y) = \frac{3}{4}y^2\int_0^1 dz\frac{1-2z(1-z)}{z(1-z)-y^2}\ln\frac{z(1-z)}{y^2}, \qquad (17)$$

with $y \equiv m_f/m_\phi$; m_f is the mass of f and λ_f is defined as the coupling $\lambda_f\phi\bar{f}f$. As for the case of λ_l, λ_f is obtained after the mass diagonalization of the Lagrangian and in general involves the parameters in Eqs. (8) and (9). However, in the case of $f = S_e$, $\lambda_f \simeq g_{S_e}$, since the corrections coming from the rotation of the mass matrix are subleading. Here, Q_f and N_c^f are the electric charge and the number of colors of f, respectively, with $N_c^f = 1(3)$ for SM leptons (quarks). For multiple heavy fermions f, one sums over them.

For fermions f much heavier than the scalar, $y^2 \gg 1$, the expression for $\Delta a_\ell^{\rm BZ}$ in Eq. (16) is approximated by [28,29]

$$\Delta a_\ell^{\rm BZ} \approx \frac{\lambda_\ell\kappa_\gamma m_\ell}{4\pi^2}(13/12 + \ln y), \qquad (18)$$

after integrating out f in the two-loop Barr-Zee diagram. In the above, κ_γ is given by (see, e.g., Ref. [30])

$$\kappa_\gamma \approx -\frac{2\alpha}{3\pi}\sum_f\frac{\lambda_f Q_f^2 N_c^f}{m_f}, \qquad (19)$$

where the sum is over fermions with similar values of $\ln(m_f/m_\phi)$. In general, the terms in Eq. (19) should be weighted by the corresponding values of the function $I(m_f/m_\phi)$. The above formula for κ_γ is obtained in the limit that $y^2 \gg 1$. Heavy fermions contribute to κ_γ significantly only if they have sizable couplings to ϕ.

VI. EXPERIMENTAL SIGNATURES

The main experimental signatures of this model are an increase of the value of the effective Yukawa coupling between the electron and Higgs and a deviation of the electron anomalous magnetic moment.[3] We will consider two scenarios depending on the value of the mass of the new scalar ϕ. In the first scenario, we fix $m_\phi = 150$ GeV and let the parameters y_e and g_{S_e} vary in the intervals[4] $0 < y_e < 2\times10^{-4}$ and $-1 < g_{S_e} < 1$, respectively. In the second scenario, we fix $m_\phi = 250$ GeV and make a scan over $0 < y_e < 5\times10^{-5}$ and $-1 < g_{S_e} < 1$. For both scenarios the other parameters are fixed to the following values; $\mu = m_\phi$, $\lambda = 1$, $\kappa = 10^{-3}$, $M_{S_e} = 1.5$ TeV, $M_{e,S_e} = 0$, $y_{S_e} = 1$, while ξ_e is determined by the condition that the mass of the electron stays at its measured value. We checked that $|\xi_e| < 1$ for all the parameter space in which we are interested.

In Figs. 4 and 5 we show the results of the aforementioned scans for the two scenarios. The blue and red bands show the points in the parameter space that allow to generate $\Delta a_e(\mathrm{Rb})$ or $\Delta a_e(\mathrm{Cs})$ respectively, within 1σ from their central values. For both scenarios the largest contribution to a_e comes from the Barr-Zee diagram in Eq. (18). The horizontal lines represent different values of

$$K_e \equiv |y_e^h/y_e^{h,\rm SM}|, \qquad (20)$$

which parametrizes the enhancement of the Higgs boson coupling to the electron, compared to its SM value. In particular, we highlight the present and future bounds on K_e coming from the LHC, HL-LHC, and the FCC-ee. It is interesting to notice that, in our model, a modification of the electron a_e implies a larger effective Yukawa. Moreover, the explanation of the deviation in Eq. (6) (Cs) also requires a much larger K_e than the deviation in Eq. (4) (Rb). Lastly, we note that, as shown in Figs. 4 and 5, increasing g_{S_e} leads to smaller values of y_e (and hence K_e) being compatible with our 1σ range for Δa_e. This is because, with our choice of parameters, the cancellation needed to get the correct electron mass, implied in Eq. (13), requires larger values of ξ_e, for larger values of y_e. However, making ξ_e larger will increase the effective coupling of ϕ to electrons, which will in turn lead to Δa_e being too large. Hence, the range of y_e for large g_{S_e} gets limited, as illustrated in the figures.

Regarding signatures at collider, we notice that ϕ could be produced through the same production mechanisms of a single Higgs. Therefore, it is possible to estimate the

[3]We note that, using the expressions from Ref. [31], one can check that corrections to electroweak oblique parameters are negligible.

[4]Solutions for a negative y_e are also possible with suitable choice of the other parameters.

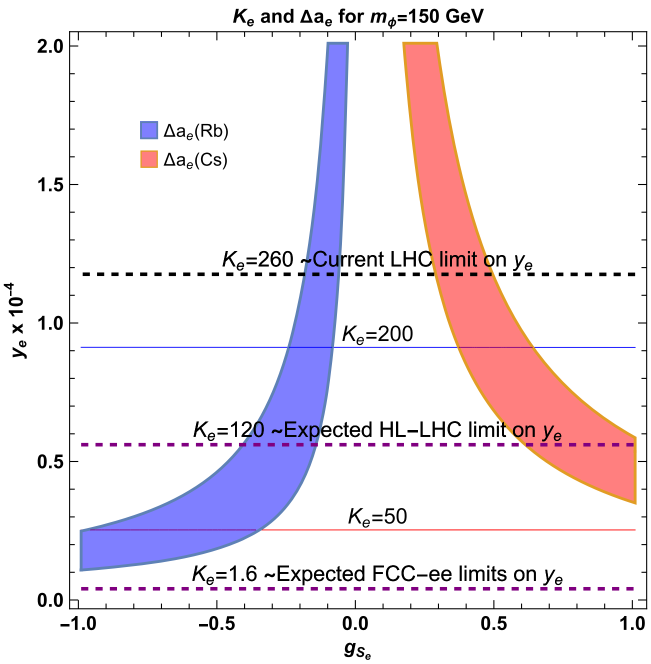

FIG. 4. Enhancement of the electron Yukawa K_e [see Eq. (20)] and Δa_e in terms of the Lagrangian paramenters y_e and g_{S_e}, for $m_\phi = 150$ GeV.

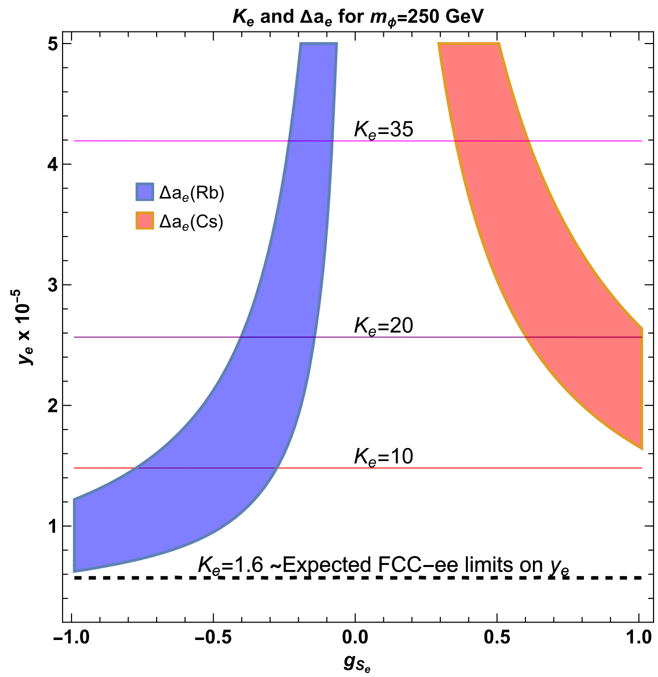

FIG. 5. Enhancement of the electron Yukawa K_e [see Eq. (20)] and Δa_e in terms of the Lagrangian paramenters y_e and g_{S_e}, for $m_\phi = 250$ GeV.

production cross section for ϕ using those of the SM Higgs. We find that the production cross sections are $\sigma(pp \to \phi) \lesssim 3$ fb and $\sigma(pp \to \phi) \ll 0.1$ fb, for $m_\phi = 150$ GeV and $m_\phi = 250$ GeV, respectively. Since ϕ decays mostly into electrons, with branching ratio ~93% and ~82% for $m_\phi = 150$ GeV and $m_\phi = 250$ GeV, respectively,[5] we can easily reinterpret the limits for Z' production [32,33]. We see that the direct discovery of ϕ seems to be beyond the capabilities of LHC, even once 3 ab^{-1} of integrated luminosity is reached. On the other hand, future colliders, and particularly an e^+e^- machine would probably be able to detect ϕ, particularly considering the current limits placed by LEP on production of new particles with large couplings to leptons [34]. We have made sure that these bounds are applied in the parameter space that we have considered. Finally, regarding S_e, studies based on models with similar properties to ours [35] seem to suggest that the detection of S_e is also currently beyond the capabilities of the LHC.

VII. MUON $g-2$

There is a longstanding discrepancy between the theoretical prediction for the moun $g_\mu - 2$ and its measured value. The most recent measurements at Fermilab [9,36], combined with a prior measurement at Brookhaven National Laboratory [6], yield the experimental value,

$$a_\mu^{\mathrm{exp}} = 116592059(22) \times 10^{-11}, \tag{21}$$

where $a_\mu \equiv (g_\mu - 2)/2$. Assuming the SM prediction given in Ref. [7],

$$a_\mu^{\mathrm{SM}} = 116591810(43) \times 10^{-11}, \tag{22}$$

the discrepancy

$$\Delta a_\mu \equiv a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = (249 \pm 48) \times 10^{-11} \tag{23}$$

would have a significance of 5.2σ. We note that the status of this discrepancy is currently under scrutiny and convergence of theory around another prediction presented in Ref. [8] would reduce its significance.

By introducing ϕ interactions with the muon, akin to those assumed for the electron, our setup can also provide an explanation of the deviation in Eq. (23). For our benchmark scenario we take $m_\phi = 150$ GeV, $M_{S_e} = M_{S_\mu} = 1.5$ TeV, $y_{S_\mu} = 1$, and $\xi_\mu = 1$. With this setup, we need to impose $M_{\mu,S_\mu} \sim 0.3$ GeV in order to guarantee that the muon mass and Yukawa coupling stay at their measured values after diagonalization. A large contribution to the muon $g-2$ comes from the Barr-Zee diagram in Eq. (18), which will include a sum over both S_e and S_μ. As a result of

[5] For $m_\phi = 150$ GeV, the total width of ϕ is ~2 MeV, where other relevant branching ratios are ~3% into $Wf'\bar{f}$ and ~2% into $b\bar{b}$. For $m_\phi = 250$ GeV, the total width ϕ is ~28 MeV and the other relevant branching ratios are WW at ~13% and ZZ at ~5%.

this, and the fact that we chose the masses of the heavy states to be the same, the Barr-Zee contribution will be proportional to $g_{S_e} + g_{S_\mu}$. We find that $g_{S_e} + g_{S_\mu} = 1$ implies,

$$\Delta a_\mu = 220 \times 10^{-11}, \tag{24}$$

which is enough to explain the deviation in Eq. (23), within one σ.

It is interesting to notice that Figs. 4 and 5 imply that, if $g_{S_e} + g_{S_\mu} \sim 1$, only $\Delta a_e(\mathrm{Cs})$ can be explained and either $K_e \gtrsim 80$ for $m_\phi = 150$ GeV or $K_e \gtrsim 12$ for $m_\phi = 250$ GeV. Another possible scenario is obtained by imposing $y_{S_\mu} = -1$ and $g_{S_e} + g_{S_\mu} = -1$, while keeping the other parameters the same, which yields

$$\Delta a_\mu = 221 \times 10^{-11}. \tag{25}$$

In this case, Figs. 4 and 5 imply that only $\Delta a_e(\mathrm{Rb})$ can be accounted for and either $K_e \gtrsim 15$ for $m_\phi = 150$ GeV or $K_e \gtrsim 2$ for $m_\phi = 250$ GeV.

Digital data related to this work are submitted on the arXiv repository as ancillary files [37].

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APPENDIX: FLAVOR SYMMETRIES

In order to avoid bounds from flavor-changing processes in our model, one could in principle enforce lepton flavor symmetries that only allow diagonal couplings. While the charged lepton mass matrix can be completely diagonal, this is not phenomenologically allowed for neutrinos, given the well-established observations of neutrino oscillations. Hence, one needs to be able to break such a symmetry. One possibility is to assume that neutrinos have Dirac masses and that the right-handed neutrinos ν_R^i, with $i = 1, 2, 3$, are singlets of the flavor symmetry. One can assign a separate $\mathbb{Z}_2$, for example, to each flavor and break them with a scalar χ_a, with $a = e, \mu, \tau$. This allows dimension-5 neutrino mass terms of the form

$$\frac{\chi_a H \bar{\nu}_R^i L_a}{M} + \mathrm{H.c.}, \tag{A1}$$

up to $\mathcal{O}(1)$ Wilson coefficients, suppressed by a scale M. We take M to be large, say $\sim 10^{18}$ GeV, near Planck mass. The vev of χ_a then needs to be $\sim 10^6$ GeV for a reasonable neutrino mass matrix.

Note that the size of charged lepton-number violating mixing-mass scale allowed by this setup would be at most at the level of $\langle \chi_a \rangle \langle \chi_b \rangle / M \sim$ keV, leading to a corresponding mixing angle $\theta \sim \mathrm{keV}/M_S \sim 10^{-9}$. One can show that with parameters near those assumed in our preceding discussions, one could achieve sufficient suppression of flavor changing processes to avoid conflict with the data. A detailed analysis requires more specific model building and is beyond the scope of this work.

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