# Interplay between the muon $g-2$ anomaly and the PTA nHZ gravitational waves from domain walls in the next-to-minimal supersymmetric standard model

Ming Xia Huang, Fei Wang,* and Ying Kai Zhang

*School of Physics and Laboratory of Zhongyuan Light, Zhengzhou University, Zhengzhou 450000, People's Republic of China*

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With some explicitly $Z_3$ breaking terms in the NMSSM (next-to-minimal supersymmetric standard model) effective superpotential and scalar potential, domain walls (DWs) from spontaneously breaking of the discrete symmetry in approximate $Z_3$-invariant NMSSM can collapse and lead to observable stochastic gravitational wave (GW) background signals. In the presence of a hidden sector, such terms may originate from the geometric superconformal breaking with holomorphic quadratic correction to frame function when the global scale-invariant superpotential is naturally embedded into the canonical superconformal supergravity models. The smallness of such mass parameters in the NMSSM may be traced back to the original superconformal invariance. Naive estimations indicate that a SUSY explanation to muon $g-2$ anomaly can have tension with the constraints on SUSY by pulsar timing arrays data, because large SUSY contributions to $\Delta a_\mu$ in general needs relatively light superpartners while present $\Omega_{gw}^0$ can set the lower bounds for $m_{\rm soft}$. We calculate numerically the signatures of GWs produced from the collapse of DWs and find that the observed nHZ stochastic GW background by NANOGrav, etc., can indeed be explained with proper tiny values of $\chi m_{3/2} \sim 10^{-14}$ eV for $\chi S^2$ case (and $\chi m_{3/2} \sim 10^{-10}$ eV for $\chi H_u H_d$ case), respectively. Besides, there are still some parameter points, whose GW spectra intersect with the NANOGrav signal region, that can explain the muon $g-2$ anomaly to $1\sigma$ range.

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## I. INTRODUCTION

Gravitational waves (GWs) can be produced in the early Universe, forming a stochastic GW background that can possibly be detected via radio telescopes using the pulsar timing arrays (PTAs) when the frequency lies around $10^{-8}$ Hz. Recently, various PTA collaborations, including the NANOGrav [1–3], EPTA [4], CPTA [5], and PPTA [6–8], have reported evidences for a stochastic GW background in the nHz frequency band. Many models have been proposed to explain this signal and potential sources for such a stochastic GW background involve supermassive black hole binaries [9–18], primordial black hole [19–24], primary perturbation [25–30], inflation [31–45], scalar induced gravitational waves [46–55], cosmic string [56–62], domian walls [63–73], first-order phase transitions [74–90], and other astrophysical/cosmological GWs or constraints [91–104]. Among all beyond the standard model (SM) fits to NANOGrav results, the domain walls (DWs), and phase transition sources perform better. So, it is well motivated to concentrate on the possibility that DWs from new physics beyond the SM act as possible cosmological sources of nHZ stochastic GWs. DWs are sheetlike topological defects generated from the spontaneous breaking of discrete symmetry, which can carry useful information related to the underlying theory. We would like to see if the recent nHz GW background data can shed new lights on the relevant new physics models that generate the DWs, for example, the popular $Z_3$-invariant next-to-minimal supersymmetric standard model (NMSSM) [105,106] of low-energy supersymmetry (SUSY).

Low-energy SUSY is the most promising new physics beyond the SM of particle physics. The SUSY framework can solve almost all the theoretical and aesthetic problems that bother the SM. In particular, the discovered 125 GeV Higgs scalar lies miraculously in the small '115–135' GeV window predicted by the low-energy SUSY, which can be seen as a strong hint of the existence of low-energy SUSY. NMSSM can elegantly solve the $\mu$ problem of minimal supersymmetric standard model (MSSM) with an additional singlet sector. In addition, with additional tree-level contributions or through

*Correspondence author: feiwang@zzu.edu.cn

doublet-singlet mixing, NMSSM can accommodate easily the discovered 125 GeV Higgs boson mass with low electroweak fine tuning. The $Z_3$-invariant NMSSM is the most predictive version of NMSSM realizations with less new input parameters. However, the VEVs of Higgs fields $S, H_u, H_d$ at the electroweak phase transition era will also trigger the breaking of the $Z_3$ discrete symmetry, potentially generating cosmologically problematic DWs because the DWs energy density decreases slower than radiation and matter, and would soon dominate the total energy density of the universe. As long as the discrete symmetry is exact, such topological configurations are stable. However, it was argued that DWs with a tension as large as $\sigma > \mathcal{O}(\mathrm{MeV}^3)$ must not exist in the present universe, which is known as Zel'dovich-Kobzarev-Okun bound [107]. Fortunately, the degeneracy of the minima from the discrete symmetry can be lifted by an energy bias in the potential, causing the collapse of DWs at the early time of the Universe and produce large amounts of GWs. Such GWs can form a stochastic background that persist to the present Universe, potentially being probed by recent PTA experiments.

There are several terms in the general NMSSM superpotential that can explicitly violate the $Z_3$ discrete symmetry to trigger the decay of DWs [108,109]. We argue that the smallness of the quadratic terms may be traced back to the original superconformal invariance for the matter part of the supergravity action when the global scale-free NMSSM is embedded to supergravity. Unlike the pure supergravity part, which breaks superconformal symmetry after the gauge fixing, the matter part alone remains superconformal. Additional real parts of the holomorphic functions in the frame function are adopted to break the superconformal symmetry, which result in the scale-invariance violation quadratic terms (such as $H_uH_d$ or $S^2$ terms) in the superpotential in the presence of a hidden sector.

It is well known that the theoretical prediction of the muon anomalous magnetic moment $a_\mu \equiv (g-2)_\mu/2$ for SM has subtle deviations from the experimental values. In fact, combining the recent reported FNAL muon $g-2$ measurement with the previous BNL + FNAL results [110–112], the updated world average experimental value of $a_\mu$ [113] has a $5.1\sigma$ deviation from the SM predictions [114] with

$$\Delta a_\mu^{\mathrm{FNAL+BNL}} = (24.9 \pm 4.8) \times 10^{-10}, \tag{1}$$

imposing very stringent constraints on various new physics models, including the $Z_3$-invariant NMSSM. So, it is fairly interesting to see if the approximate $Z_3$-invariant NMSSM (with tiny explicit discrete symmetry breaking terms) can consistently lead to the signals of GWs by the collapse of DWs in the parameter regions allowed by the muon $g-2$ explanation and collider data. Such a survey is fairly nontrivial, as large SUSY contributions to $\Delta a_\mu$ always prefer $m_{\mathrm{soft}} \sim \mathcal{O}(10^2)$ GeV while recent NANOGrav data can lead to approximate low bounds $m_{\mathrm{soft}} \gtrsim \mathcal{O}(1)$ TeV.

This paper is organized as follows. In Sec. II, we discuss the formation of DWs from the spontaneous breaking of $Z_3$ discrete symmetry. In Sec. III, the collapse of DWs from explicitly $Z_3$-violation terms are discussed. Such small $Z_3$ violation terms can be the consequence of superconformal violation in the matter part of supergravity in the presence of a hidden sector. In Sec. IV, the GWs signals related to the collapse of DWs are discussed. The interplay between the muon $g-2$ anomaly and the PTA nHZ GWs signals from DWs are also discussed. Sec. V contains our conclusions.

## II. DWs FROM $Z_3$-INVARIANT NMSSM

As noted previously, NMSSM is well motivated theoretically to solve the $\mu$ problem. A bare $\mu$ term $\mu H_u H_d$ in the NMSSM superpotential is forbidden if we impose a discrete $Z_3$ symmetry, under which the Higgs superfields $S, H_u, H_d$ transform nontrivially. The $Z_3$ invariant superpotential couplings are given by [105,106]

$$W_{Z_3\mathrm{NMSSM}} = W_{\mathrm{MSSM}}|_{\mu=0} + \lambda S H_u H_d + \frac{\kappa}{3} S^3, \tag{2}$$

with

$$\begin{aligned} W_{\mathrm{MSSM}}|_{\mu=0} = &\, y_{ij}^u Q_{L,i} H_u U_{L,j}^c - y_{ij}^d Q_{L,i} H_d D_{L,j}^c \\ &- y_{ij}^e L_{L,i} H_d E_{L,j}^c. \end{aligned} \tag{3}$$

The soft SUSY breaking parameters can be given as

$$\begin{aligned} \mathcal{L}_{Z_3\mathrm{NMSSM}}^{\mathrm{soft}} = &\, \mathcal{L}_{\mathrm{MSSM}}^{\mathrm{soft}}|_{B=0} - \left( A_\lambda \lambda S H_u H_d + A_\kappa \frac{\kappa}{3} S^3 \right) \\ &- m_S^2 |S|^2, \end{aligned} \tag{4}$$

by proper SUSY breaking mechanisms, such as AMSB [115] and Yukawa mediation mechanisms [116,117]. From the superpotential and the soft SUSY breaking terms, one obtains the Higgs potential

$$\begin{aligned} V = &\, |\lambda(H_u^+ H_d^- - H_u^0 H_d^0) + \kappa S^2|^2 \\ &+ (m_{H_u}^2 + |\lambda S|^2)(|H_u^0|^2 + |H_u^+|^2) \\ &+ (m_{H_d}^2 + |\lambda S|^2)(|H_d^0|^2 + |H_d^-|^2) \\ &+ \frac{g_1^2 + g_2^2}{8} (|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 \\ &+ \frac{g_2^2}{2} |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 + m_S^2 |S|^2 \\ &+ \left( \lambda A_\lambda (H_u^+ H_d^- - H_u^0 H_d^0) S + \frac{1}{3} \kappa A_\kappa S^3 + \mathrm{H.c.} \right), \end{aligned} \tag{5}$$

where $g_1$ and $g_2$ denote the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$ gauge couplings, respectively. Electroweak symmetry breaking (EWSB) are triggered by the VEVs $v_u \equiv \langle H_u \rangle$, $v_d \equiv \langle H_d \rangle$ while the effective $\mu_{\mathrm{eff}}$ parameter,

$$\mu_{\text{eff}} \equiv \lambda s, \tag{6}$$

is generated by the VEV $s \equiv \langle S \rangle$. Such Higgs VEVs also break spontaneously the $Z_3$ discrete symmetry and lead to the formation of DWs.

As the $Z_3$-NMSSM scalar potential is invariant under the $Z_3$ transformation, the scalar potential has three degenerate minima that can be parametrized by $(|s|\omega^i, |v_u|\omega^i, |v_d|\omega^i)$ for $(i = 0, 1, 2)$ with $\omega^3 = 1$. We use the conclusion in [118] that the true minimum of the potential does not spontaneously break CP as the VEVs can always be made real by an appropriate field redefinition, up to the existence of three degenerate vacua related to each other by $Z_3$ transformations.

DWs are located around boundaries of these three vacua. A planar domain wall solution perpendicular to the $z$-axis can be found by solving numerically the equation of motions for $S, H_u, H_d$ fields with the boundary conditions

$$\lim_{z\to-\infty}(S(z), H_u(z), H_d(z)) \to (|s|\omega^i, |v_u|\omega^i, |v_d|\omega^i),$$
$$\lim_{z\to\infty}(S(z), H_u(z), H_d(z)) \to (|s|\omega^{i+1}, |v_u|\omega^{i+1}, |v_d|\omega^{i+1}), \tag{7}$$

such that the configuration interpolates smoothly between two vacua at $z \to \pm\infty$.

We adopt linearized interpolation curves for the two asymptotic domains as our initial guess of the solution and iterate these procedures until the solution converges. From the configuration of the DWs, the spatial distribution of the energy density can be calculated by

$$\rho_w(z) = \sum_{\phi_i} |\nabla\phi_i|^2 + V_F + V_D + V_{\text{soft}} - V(s, v_u, v_d),$$
$$= \sum_{\phi_i = S, H_u, H_d} \left|\frac{\partial\phi_i(z)}{\partial z}\right|^2 + V(\phi_i) - V(s, v_u, v_d), \tag{8}$$

in which the boundary energy constant is subtracted such that $\rho_w \to 0$ is satisfied for $z \to \pm\infty$. The tension of the DWs can be calculated from the energy-momentum tensor of the static solution, which reduces to the integration of the energy density along the $z$-axis

$$\sigma = \int_{-\infty}^{\infty} dz \rho_w(z). \tag{9}$$

We show in Table I the tension and energy density of DWs from $Z_3$-NMSSM for some benchmark points.

TABLE I. The tension, energy density of DWs from $Z_3$-invariant NMSSM, and the SUSY contributions to $\Delta a_\mu$ for some benchmark points that can also satisfy all the low-energy collider constraints.

| $\tan\beta$ | $\lambda$ | $\kappa$ | $A_\lambda$ | $A_\kappa$ | $\mu_{\text{eff}}$ | $\chi m_{\frac{3}{2}}$ |
|---|---|---|---|---|---|---|
| 45.9079 | 0.0166 | 0.0031 | 697.4122 | −291.0022 | 882.0991 | $6.6132 \times 10^{-19}$ eV |

| Particle | Mass (GeV) | Particle | Mass (GeV) |
|---|---|---|---|
| $\tilde{g}$ | 10585.1482 | $\tilde{e}_L$ | 245.428730 |
| $h_1$ | 125.451953 | $\tilde{e}_R$ | 346.932050 |
| $h_2$ | 252.599429 | $\tilde{\nu}_{e_L}$ | 233.298454 |
| $h_3$ | 5918.80624 | $\tilde{\mu}_L$ | 245.428730 |
| $H^+$ | 5919.72521 | $\tilde{\mu}_R$ | 346.932050 |
| $a$ | 382.247468 | $\tilde{t}_1$ | 10072.4644 |
| $A$ | 5918.80456 | $\tilde{t}_2$ | 10188.7595 |
| $\tilde{d}_L$ | 10303.7859 | $\tilde{\nu}_{\mu_L}$ | 233.298454 |
| $\tilde{d}_R$ | 10303.6597 | $\tilde{\tau}_1$ | 2573.50938 |
| $\tilde{u}_L$ | 10303.5041 | $\tilde{\tau}_2$ | 2978.15121 |
| $\tilde{u}_R$ | 10303.5716 | $\tilde{\nu}_{\tau_L}$ | 2572.83541 |
| $\tilde{s}_L$ | 10303.7859 | $\tilde{\chi}_1$ | 116.573893 |
| $\tilde{s}_R$ | 10303.6597 | $\tilde{\chi}_2$ | −290.383594 |
| $\tilde{c}_L$ | 10303.5041 | $\tilde{\chi}_3$ | 344.081650 |
| $\tilde{c}_R$ | 10303.5716 | $\tilde{\chi}_4$ | −911.211934 |
| $\tilde{b}_1$ | 10184.2229 | $\tilde{\chi}_5$ | 911.837634 |
| $\tilde{b}_2$ | 10189.2163 | $\tilde{\chi}_1^\pm$ | 116.576768 |
| $M_W$ | 80.4200000 | $\tilde{\chi}_2^\pm$ | 914.030425 |
| $M_B$ | 4.18000000 | $M_t$ | 173.400000 |
| $M_Z$ | 91.1870000 | $M_\tau$ | 1.77699995 |
| $\sigma$ | $7.3394 \times 10^{11}$ GeV$^3$ | $t_{\text{dec}}$ | $3.6981 \times 10^{21}$ GeV$^{-1}$ |
| $\Omega_{\text{GW}} h^2$ | $2.1249 \times 10^{-9}$ | $f$ | 1.9025 nHz |

We know that the validity of perturbation theory up to the GUT scale implies $\lambda < 0.7 \sim 0.8$ [105] at the weak scale. Besides, the EWSB condition requires that

$$\frac{m_Z^2}{2} = |\lambda s|^2 + m_{H_u}^2 + \frac{2}{\tan^2\beta}(m_{H_d}^2 - m_{H_u}^2) + \mathcal{O}(1/\tan^4\beta). \tag{10}$$

So, the singlet VEV $s$ always takes values of order the soft SUSY breaking scale, which is always much larger than the electroweak (EW) scale VEV $v_u$ and $v_d$ and act as the dominant spontaneously $Z_3$ discrete symmetry breaking source. In this case, as noted in [108], the dominant contribution to the energy density and the tension of DWs comes from the singlet relevant terms in the scalar potential. From the equation of motion for the corresponding field, we can estimate the gradient of the field as

$$([\phi'(z)]^2)' = \frac{\partial V(\phi)}{\partial \phi}\phi' \Rightarrow \phi'(z) \sim \sqrt{V(\phi(z))}. \tag{11}$$

So, the characteristic length (thickness) for varying $\phi(z)$ from two adjacent minima $-v/2$ to $v/2$ in NMSSM can be estimated to be

$$\delta^{-1} \sim \frac{\sqrt{V}}{v} \sim \frac{\sqrt{\kappa A_\kappa s^3}}{s} \sim A_\kappa, \tag{12}$$

while the energy density of the DWs is

$$\rho = [\phi'(z)]^2 \sim V(z) \sim \kappa A_k s^3. \tag{13}$$

The tension of DW can be estimated in terms of the height of the potential energy $V_0$ separating the degenerate minima and the thickness $\delta$ by the relation $\sigma \sim \delta V_0$, giving $\sigma \sim (\kappa A_k s^3)/A_\kappa \sim A_\kappa^3/\kappa^2$. Such an estimation can be helpful to understand some of the subsequent numerical results.

When the exponentially damped friction force of DWs becomes irrelevant after the temperature of the background radiations becomes less than the mass of particles that interact with DWs, the dynamics of DWs is dominated by the tension force, which stretches them up to the horizon size. The evolution of DWs in this regime can be described by the scaling solution, which states that their energy density evolves according to the simple scaling law and there is about one DW per Hubble horizon. The energy density of DWs in the scaling regime decreases much more slowly than that of cold matters and radiations so that they gradually dominate the energy density of the Universe, drastically altering the subsequent evolution of the Universe.

## III. COLLAPSE OF DWs AND GWs

The formation of DWs from spontaneously discrete $Z_3$ breaking can be problematic in cosmology if they persist till now. They need to collapse before primordial Big Bang nucleosynthesis (BBN). If the discrete symmetry is broken explicitly by $1/M_P$ suppressed interactions, such operators can lead to divergent two- or three-loop diagrams that contribute a term linear in $S$, destabilizing the Planck/weak hierarchy unless the coupling is tiny. However, such tiny couplings conflict with the previous constraints from nucleosynthesis [119]. Therefore, explicitly $Z_3$ breaking model should have nonvanishing $\mu_0 \neq 0$ or $\mu' \neq 0$ with $\kappa \neq 0$ in the low-energy effective superpotential

$$W_{\not{Z}_3} \supseteq \mu_0 H_u H_d + \frac{1}{2}\mu' S^2, \tag{14}$$

and, at the same time, the absence of the $1/M_P$ suppressed $S^4/M_P$ operator [120]. Although the superpotential will not introduce new terms by the nonrenormalization theorem, such terms and soft SUSY breaking terms

$$\begin{aligned} -\mathcal{L}_{\text{soft;NMSSM}} \supseteq & -\mathcal{L}_{\text{soft;MSSM}} + m_S^2|S|^2 + \lambda A_\lambda H_u \cdot H_d S \\ & + \left(B\mu_0 H_u \cdot H_d + \frac{1}{2}B'\mu' S^2 + \text{H.c.}\right) \\ & + \frac{1}{3}\kappa A_\kappa S^3 \end{aligned} \tag{15}$$

always appear in the low-energy effective action for the light superfields [122] after integrating out the messenger/sequestered sector for the SUSY breaking. However, such $\mu_0$ and $\mu'$ terms can again lead to the existence of tadpole diagrams for $S$, possibly causing the shift in the potential for $S$ to slide to large values far above the weak scale. In fact, a more general conclusion in [119] states that, to ensure a model with singlets will be natural, typical symmetry (such as the gauged R-symmetry or modular symmetry for target space duality [123] instead of ordinary gauge symmetry) is needed to forbid odd-dimension terms in the Kahler potential $K$ and even-dimension terms in the superpotential $\hat{W}$, while any extra odd-dimension operators in $\hat{W}$ or even-dimension operators in $K$ are not harmful to the gauge hierarchy. So, the low-energy effective $\mu' S^2$ and $\mu_0 H_u H_d$ operators should be absent in the original superpotential $\hat{W}$ for supergravity to avoid problematic singlet tadpole problems and they can only come from the Kahler potential. Such small $\mu_0$ and $\mu'$ terms can be the natural consequence of small geometrical breaking of larger local superconformal symmetry from the bilinear dimensionless holomorphic terms in the frame function for the canonical superconformal supergravity (CSS) model [124], when the scale-invariant superpotential is embedded into it.

### A. Explicitly $Z_3$ violation term from geometrical superconformal breaking

The form of Poincare supergravity can be obtained from the underlying superconformal theory, which in addition to local supersymmetry of a Poincare supergravity has also extra local symmetries: Weyl symmetry, $\mathrm{U}(1)_R$ symmetry, special conformal symmetry, and special supersymmetry. General theory of supergravity in an arbitrary Jordan frame was derived in [125] from the $\mathrm{SU}(2,2|1)$ superconformal theory after gauge fixing. It was noted in [124] that the embedding of the globally superconformal theory into supergravity in the Jordan frame can be realized by simply adding the action of the global SUSY, interacting with gravity with conformal scalar-curvature coupling, to the action of supergravity. The scalar-gravity part of the $\mathcal{N}=1$, $d=4$ supergravity in a generic Jordan frame with frame function $\Phi(z,\bar{z})$ and superpotential $W(z)$ is given by [124]

$$\mathcal{L}_J^{\mathrm{s-g}} = \sqrt{-g_J}\left[\Phi\left(-\frac{1}{6}R(g_J)+\mathcal{A}_\mu^2(z,\bar{z})\right) + \left(\frac{1}{3}\Phi g_{\alpha\bar{\beta}} - \frac{\Phi_\alpha\Phi_{\bar{\beta}}}{\Phi}\right)\hat{\partial}_\mu z^\alpha \hat{\partial}^\mu \bar{z}^{\bar{\beta}} - V_J\right], \tag{16}$$

with

$$\Phi_\alpha \equiv \frac{\partial}{\partial z^\alpha}\Phi(z,\bar{z}), \qquad \Phi_{\bar{\beta}} \equiv \frac{\partial}{\partial \bar{z}^{\bar{\beta}}}\Phi(z,\bar{z}),$$
$$g_{\alpha\bar{\beta}} = \frac{\partial^2\mathcal{K}(z,\bar{z})}{\partial z^\alpha \partial \bar{z}^{\bar{\beta}}} \equiv \mathcal{K}_{\alpha\bar{\beta}}(z,\bar{z}), \tag{17}$$

and $\mathcal{A}_\mu$ depends on scalar fields as follows:

$$\mathcal{A}_\mu(z,\bar{z}) \equiv -\frac{i}{2\Phi}(\hat{\partial}_\mu z^\alpha \partial_\alpha \Phi - \hat{\partial}_\mu \bar{z}^{\bar{\alpha}} \partial_{\bar{\alpha}}\Phi). \tag{18}$$

For a particular choice of the frame function, the kinetic scalar terms are canonical when the on-shell auxiliary axial-vector field $\mathcal{A}_\mu$ vanishes. This requires that

$$\Phi(z,\bar{z}) = -3e^{-(1/3)\mathcal{K}(z,\bar{z})} = -3 + \delta_{\alpha\bar{\beta}} z^\alpha \bar{z}^{\bar{\beta}} + J(z) + \bar{J}(\bar{z}). \tag{19}$$

These $J(z)+\bar{J}(\bar{z})$ terms in the frame function not only break the continuous $R$ symmetry, but also break the discrete $Z_3$ symmetry.

In addition to the pure supergravity part in the total supergravity action, which breaks superconformal symmetry, the part of the action describing chiral and vector multiplets coupled to supergravity has a much larger local superconformal symmetry. Such a superconformal invariance needs to be broken to allow that most of the particles could be massive. However, this symmetry is broken down to the local Poincare supersymmetry only by the part of the action describing the self-interacting supergravity multiplet, sometimes requiring additional superconformal symmetry breaking sources in the matter sector. It was proposed in [126] that the superconformal symmetry of the matter multiplets in the supergravity action can be broken geometrically without introducing dimensional parameters into the underlying superconformal action, in which the moduli space of chiral fields including the compensator field is no longer flat.

We adopt the setting of Jordan frame NMSSM with the following real function

$$\mathcal{N} = -|X^0|^2 + |H_u|^2 + |H_d|^2 + |S|^2 + \chi\left(\frac{\bar{X}_0}{X_0}S^2 + \mathrm{H.c.}\right), \tag{20}$$

that describes the Kahler manifold of the embedding space, which leads to the superpotential in Jordan frame

$$V_J = G^{\alpha\bar{\beta}} W_\alpha \bar{W}_{\bar{\beta}} + \frac{1}{2}(\mathrm{Re}f)^{-1AB}\mathcal{P}_A\mathcal{P}_B, \tag{21}$$

with $G^{\alpha\bar{\beta}}$ the matter part of the inverse metric $G^{I\bar{J}}$ of the enlarged space including the compensator. The general expression of $G^{\alpha\bar{\beta}}$ can be calculated to be [126]

$$G^{\alpha\bar{\beta}} = \delta^{\alpha\bar{\beta}} - \frac{4\chi^2\delta^{\alpha\bar{\lambda}}\delta^{\sigma\bar{\beta}} a_{\sigma\zeta}\bar{a}_{\bar{\lambda}\bar{\xi}} z^\zeta \bar{z}^{\bar{\xi}}}{[3M_P^2 - \chi(a_{\gamma\eta}z^\gamma z^\eta + \bar{a}_{\bar{\gamma}\bar{\eta}}\bar{z}^{\bar{\gamma}}\bar{z}^{\bar{\eta}}) + 4\chi^2\delta^{\gamma\bar{\eta}} a_{\gamma\theta}\bar{a}_{\bar{\eta}\bar{\rho}} z^\theta \bar{z}^{\bar{\rho}}]} \tag{22}$$

for the real function

$$\mathcal{N}(X,\bar{X}) = -|X^0|^2 + |X^\alpha|^2 - \chi\left(a_{\alpha\beta}\frac{X^\alpha X^\beta \bar{X}^{\bar{0}}}{X^0} + \bar{a}_{\bar{\alpha}\bar{\beta}}\frac{\bar{X}^{\bar{\alpha}}\bar{X}^{\bar{\beta}}X^0}{\bar{X}^{\bar{0}}}\right). \tag{23}$$

So, for our case, we have

$$G^{S\bar{S}} = \delta^{S\bar{S}} - \frac{4\chi^2\bar{S}S}{[3M_P^2 - \chi(S^2+\bar{S}^2) + 4\chi^2(\bar{S}S)]}$$
$$\approx 1 - \frac{4\chi^2|S|^2}{3M_P^2} - \frac{\chi(S^2+S^{*2})}{9M_P^4} + \frac{16\chi^4|S|^4}{9M_P^4} + \cdots. \tag{24}$$

The SUSY preserving part of the scalar potential receives an additional term

$$\Delta V_J \supseteq -\left[\frac{4\chi^2|S|^2}{3M_P^2} + \frac{4\chi^3|S|^2(S^2+S^{*2})}{9M_P^4} - \frac{16\chi^4|S|^4}{9M_P^4} + \cdots\right] \times |\lambda(H_u^+H_d^- - H_u^0H_d^0) + \kappa S^2|^2. \tag{25}$$

We can see that the leading $Z_3$ breaking term in this scalar potential that is proportional to $(S^2+S^{*2})$ is suppressed by $1/M_P^4$. Similarly, for real function $\mathcal{N}(X,\bar{X})$ of the form

$$\mathcal{N}(X,\bar{X}) = -|X^0|^2 + |H_u|^2 + |H_d|^2 + |S|^2 + \chi\left(\frac{\bar{X}_0}{X_0}H_uH_d + \mathrm{H.c.}\right), \tag{26}$$

in the Jordan frame for NMSSM, we have

$$\begin{aligned}
G^{H_u\bar{H}_u} &= \delta^{H_u\bar{H}_u} - \frac{\chi^2\bar{H}_uH_u}{[3M_P^2 - \chi(H_uH_d + \bar{H}_u\bar{H}_d) + \chi^2(\bar{H}_uH_u + \bar{H}_dH_d)]},\\
G^{H_d\bar{H}_d} &= \delta^{H_d\bar{H}_d} - \frac{\chi^2\bar{H}_dH_d}{[3M_P^2 - \chi(H_uH_d + \bar{H}_u\bar{H}_d) + \chi^2(\bar{H}_uH_u + \bar{H}_dH_d)]},\\
G^{H_u\bar{H}_d} &= -\frac{\chi^2\bar{H}_dH_u}{[3M_P^2 - \chi(H_uH_d + \bar{H}_u\bar{H}_d) + \chi^2(\bar{H}_uH_u + \bar{H}_dH_d)]},\\
G^{H_d\bar{H}_u} &= -\frac{\chi^2\bar{H}_uH_d}{[3M_P^2 - \chi(H_uH_d + \bar{H}_u\bar{H}_d) + \chi^2(\bar{H}_uH_u + \bar{H}_dH_d)]}.
\end{aligned} \tag{27}$$

So, the scalar potential for NMSSM is given by

$$\begin{aligned}
V_J &= \lambda^2|S|^2|H_d|^2G^{H_u\bar{H}_u} + \lambda^2|S|^2|H_u|^2G^{H_d\bar{H}_d} + \lambda^2|S|^2G^{H_d\bar{H}_u}\bar{H}_dH_u + \lambda^2|S|^2G^{H_d\bar{H}_u}\bar{H}_dH_u\\
&\quad + |\lambda H_uH_d + \kappa S^2|^2 + \frac{1}{8}(g_1^2 + g_2^2)(|H_d|^2 - |H_u|^2)^2 + \frac{1}{2}g_2^2|H_uH_d^\dagger|^2,
\end{aligned} \tag{28}$$

with the correction

$$\Delta V_J \supseteq -4\left[\frac{\chi^2}{3M_P^2} + \frac{\chi^3(H_uH_d + \bar{H}_u\bar{H}_d)}{9M_P^4} + \frac{\chi^4(|H_u|^2 + |H_d|^2)}{9M_P^4}\right]\lambda^2|S|^2|H_d|^2|H_u|^2. \tag{29}$$

Therefore, the leading $Z_3$ breaking term that is proportional to $(H_uH_d + \bar{H}_u\bar{H}_d)$ in the scalar potential can be seen to be suppressed by $1/M_P^4$.

The frame function $\Phi(z^a,\bar{z}^a)$ and consequently the Kahler potential can relate to $\mathcal{N}(X,\bar{X})$ by the gauge fixing

$$X^0 = \bar{X}^{\bar{0}} = \sqrt{3}M_P, \qquad X^a = z^a, \tag{30}$$

with $\mathcal{N}(X,\bar{X}) = \Phi(z^a,\bar{z}^a) = -3e^{-K(z,\bar{z})/3}$.

In many popular SUSY breaking mechanisms, the total superpotential can be divided into the hidden sector and visible sector with $W = W_{\mathrm{vis}} + W_{\mathrm{hid}}$ for $\langle W_{\mathrm{vis}}\rangle \ll \langle W_{\mathrm{hid}}\rangle$, as the VEV is obviously dominated by the fields responsible for SUSY breaking even though all fields in the theory are summed over. So, under Kahler transformation to render the new Kahler potential canonical

$$\begin{aligned}
K(z,\bar{z}) &\to K(z,\bar{z}) + J(z) + \bar{J}(\bar{z}),\\
W(z) &\to e^{-J(z)/M_P^2}W(z),
\end{aligned} \tag{31}$$

the superpotential will receive a correction of the from

$$\delta W \simeq \frac{J(z)}{M_P^2}W \simeq \frac{J(z)}{M_P^2}\langle W_{\mathrm{hid}}\rangle \simeq m_{3/2}J(z). \tag{32}$$

Here the gravitino mass $m_{3/2}$ is

$$m_{3/2} = e^{\frac{K}{2M_P^2}}\frac{\langle W\rangle}{M_P^2} \simeq \frac{\langle W\rangle}{M_P^2}. \tag{33}$$

For $J(z) = \chi S^2$ (or $\chi H_uH_d$), the superpotential will receive (non-Planck scale suppressed) $Z_3$ breaking terms $\chi m_{3/2}S^2$ (or $\chi m_{3/2}H_uH_d$), respectively. Note that unpreferable tadpole term $\xi_F S$ will not be present in such $Z_3$ explicitly breaking superpotential because no dimensional parameters are introduced in $\mathcal{N}(X,\bar{X})$, which can be seen as an advantage of this CSS approach.

After the Kahler transformation with new frame function $\Phi'(z,\bar{z})$, the action in the Jordan frame is given by

$$\frac{\mathcal{L}_J}{\sqrt{-g}} = -\frac{1}{6}\Phi'(z,\bar{z})R(g_J) - \delta_{\alpha\bar{\beta}}\partial_\mu z^\alpha\partial^\mu z^{\bar{\beta}} - V_J'(z,\bar{z}), \tag{34}$$

with $V_J'$ the scalar potential for the superpotential $e^{-J}W$. It is obvious that the matter part of supergravity action contains nonminimal gravitational couplings of scalars to the Ricci scalar curvature in additional to the bosonic part of the supergravity action. To absorb the nonminimal gravitational couplings, we need to rescale the metric. Under the transformation

$$\tilde{g}_{\mu\nu} = e^{2\sigma(x)} g_{\mu\nu}, \tag{35}$$

the Ricci scalar changes as

$$\tilde{R} = e^{-2\sigma(x)}[R - 2(d-1)\nabla^2(\sigma) - (d-1)(d-2)(\nabla\sigma)^2], \tag{36}$$

with $\nabla^2\sigma = g^{\mu\nu}\nabla_\mu\nabla_\nu\sigma$ and $(\nabla\sigma)^2 = g^{\mu\nu}(\nabla_\mu\sigma)(\nabla_\nu\sigma)$. So, the metric can be rescaled as followings

$$e^{2\sigma} \equiv \Omega^2 = 1 - \frac{1}{3M_P^2}\{|S|^2 + |H_u|^2 + |H_d|^2\}, \tag{37}$$

to change the Jordan frame into the Einstein frame [127]. After metric rescaling, the scalar potential in the Einstein frame is given by

$$\begin{aligned} V_E &= \frac{V_J'}{\Omega^4} \\ &\simeq \left[1 + \frac{2}{3M_P^2}\{|S|^2 + |H_u|^2 + |H_d|^2\}\right] V_J'(e^{-J}W), \end{aligned} \tag{38}$$

with $V_J'(e^{-J}W)$ the SUSY scalar potential for the superpotential $e^{-J}W$. General discussions for nonminimal gravitational couplings involving the complex scalars can be found in our previous work [128], in which the polar variables $R$, $\Theta$ can be used to simplify the transformations. As anticipated, in small field values $\phi \ll M_P$, the new variables in the Einstein frame can reduce to the original field variables. The $m_{3/2}J(z)$ term in the superpotential can give the dominant (non-Planck scale suppressed) $Z_3$ breaking contribution in the Einstein frame scalar potential in comparison to the $Z_3$ breaking terms in (25) and (29).

### B. Additional explicitly $Z_3$ breaking terms after SUSY breaking

In the CSS approach with geometrical superconformal breaking in the matter sector, no tadpole terms in the superpotential and scalar potential will be generated after the Kahler transformation with SUSY breaking. The mediation mechanism of SUSY breaking that is responsible for the soft SUSY breaking parameters of NMSSM fields should also not reintroduce the tadpole terms. Unlike some gravity mediation mechanism, which can potentially trigger the generation of tadpole terms again, the tadpole terms in gauge mediated SUSY breaking (GMSB) mechanism can be forbidden by some discrete symmetry for the messenger sector. For example, the economical realization [129,130] (without the introduction of additional gauge fields) can be adopted

$$W \supseteq \sum_{i=0}^{n} \lambda_{X;i} X(\bar{\Phi}_{2i}\Phi_{2i} + \bar{\Phi}_{2i+1}\Phi_{2i+1} + \sum_{i=0}^{n} \lambda_{S;i} S\bar{\Phi}_{2i}\Phi_{2i+1} \tag{39}$$

to avoid the tadpole problem, which can preserve the $Z_3$ discrete symmetry in the messenger sector by introducing double families of messengers (in $\mathbf{5} \oplus \bar{\mathbf{5}}$ representation of SU(5)). In this case, the general soft SUSY breaking parameters in the NMSSM, such as $B$ and $B'$, can be estimated to leading order as

$$B' \simeq \frac{1}{16\pi^2}\frac{F_X}{M_m} 5 \sum_{j=1}^{N} \lambda_{S;j}^2, \qquad B = 0 \tag{40}$$

for the VEV of spurion $\langle X \rangle = M_m + \theta^2 F_X$. For a general messenger sector with proper messenger-matter interactions involving the $H_u$ or $H_d$ fields, the $B$ parameter will receive a similar contribution of order the soft SUSY breaking masses $m_{\text{soft}} \sim \frac{1}{16\pi^2}\frac{F_X}{M_m} \sim \mathcal{O}(\text{TeV})$.

In our subsequent studies, we will not specify concretely which SUSY breaking mechanism is used. The values $B \simeq B' \sim m_{\text{soft}}$ with $m_{\text{soft}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ are adopted in our numerical results for such explicitly $Z_3$ breaking soft SUSY parameters.

## IV. COLLAPSE OF DWs FROM EXPLICIT $Z_3$ BREAKING

The $\chi m_{3/2} S^2$ related terms in the scalar potential break explicitly the $Z_3$ symmetry. The scalar potential relevant to such a term is given by

$$\begin{aligned} \Delta\mathcal{L}_{S^2} \supseteq & \left[|\mu' S|^2 + \left\{\mu' S^*(\lambda H_u H_d + \kappa S^2) + \frac{B'\mu'}{2} S^2 + \text{H.c.}\right\}\right]\left[1 + \frac{2\{|S|^2 + |H_u|^2 + |H_d|^2\}}{3M_P^2}\right], \\ & - \left[\frac{4\chi^2|S|^2}{3M_P^2} + \frac{4\chi^3|S|^2(S^2 + S^{*2})}{9M_P^4} - \frac{16\chi^4|S|^4}{9M_P^4} + \cdots\right]|\lambda H_u H_d + \kappa S^2|^2, \\ & - \frac{8\chi^2|S|^2}{9M_P^4}\{|S|^2 + |H_u|^2 + |H_d|^2\} + \mathcal{O}(1/M_P^6), \end{aligned} \tag{41}$$

with $\mu' = 2\chi m_{3/2}$. We will neglect the subleading $M_P$ suppressed $Z_3$ breaking terms in our subsequent studies for the collapsing of DWs. The vacuum energy differences between $v_i$ and $v_j$ are defined as

$$(V_{\rm bias})_{i,j} = V_{v_i} - V_{v_j}, \quad \text{for } (i,j = 1, \omega, \omega^2) \tag{42}$$

which satisfies $(V_{\rm bias})_{1,\omega^2} - (V_{\rm bias})_{1,\omega} = (V_{\rm bias})_{\omega,\omega^2}$.

We can calculate the energy difference between adjacent vacuum

$$\begin{aligned}\Delta_{1,\omega} V_{\not{Z}_3;S^2} &= \Delta_{1,\omega^2} V_{\not{Z}_3;S^2} \\ &= [2\mu' s(\kappa s^2 - \lambda v_u v_d) + \mu' m_{\rm soft} s^2]\left(1 - \cos\frac{2\pi}{3}\right) \\ &= 3\mu' s(\kappa s^2 - \lambda v_u v_d) + \frac{3}{2}\mu' m_{\rm soft} s^2, \\ \Delta_{\omega,\omega^2} V_{\not{Z}_3;S^2} &= 0. \end{aligned} \tag{43}$$

Requiring the $v_1$ vacua to be the true minimum leads to the constraints $\Delta_{1,\omega} V_{\not{Z}_3;S^2} < 0$ and $\Delta_{1,\omega^2} V_{\not{Z}_3;S^2} < 0$, which can be satisfied with

$$\mu' < 0, \quad \text{or} \quad (\kappa s^2 - \lambda v_u v_d) + \frac{1}{2} m_{\rm soft} s < 0. \tag{44}$$

The degeneracy of the two minimum is the consequence of $CP$ conjugation of the $\omega$ and $\omega^2$ vacua. When the conditions in Eq. (44) are satisfied, we have the order of the vacuum energies $V_{v_{\omega^2}} = V_{v_\omega} > V_{v_1}$ with $v_1$ the true vacua and

$$|(V_{\rm bias})_{1,\omega^2}| = |(V_{\rm bias})_{1,\omega}|, \qquad |(V_{\rm bias})_{\omega,\omega^2}| = 0,$$

for the biases.

With bias terms, the energy difference between the neighboring vacua acts as a volume pressure $p_V \sim V_{\rm bias}$ on the wall, which tends to shrink the higher-energy domains until the whole space is filled by the only true vacuum, eventually collapsing the wall network. The collapse of DWs happens when this pressure force becomes greater than the tension force $p_T \sim \sigma / R_{\rm wall}$. In our case with two degenerate false vacua, the volume pressures from $(V_{\rm bias})_{1,\omega^2}$ and $(V_{\rm bias})_{1,\omega}$ are of the same strength, and they lead the true $v_1$ vacuum to expand against the false $v_\omega$ and $v_{\omega^2}$ vacua regions, reducing at the same rate the occupancy of such false vacuum states and leading to the fragmentation of the wall network, which will collapse.

If the conditions in Eq. (44) are not satisfied, the potential has two degenerate true minima $v_\omega$ and $v_{\omega^2}$. To spoil the problematic degeneracy between the $\omega$ and $\omega^2$ vacua, one can introduce additional messengers $\psi'$, $\bar{\psi}'$, which are charged under some hidden gauge group $SU(N)_h$ and $Z_3$. The $Z_3$ discrete symmetry can be anomalous under $SU(N)_h$, leading to an additional contribution to the bias of order $\Delta_H V \sim \lambda_H^4$ [131] so as that the degeneracy between the $\omega$ and $\omega^2$ vacua is spoiled (assuming that $V_{v_{\omega^2}} \gtrsim V_{v_\omega}$). Then, the vacuum energies and the biases satisfy

$$\begin{gathered} V_{v_1} > V_{v_{\omega^2}} \gtrsim V_{v_\omega}, \\ |(V_{\rm bias})_{1,\omega}| \gtrsim |(V_{\rm bias})_{1,\omega^2}| \gg |(V_{\rm bias})_{\omega,\omega^2}|. \end{gathered} \tag{45}$$

In such a scenario, due to the fact that the true vacuum $v_\omega$ is nearly degenerate with the false vacuum $v_{\omega^2}$, the annihilations driven by $(V_{\rm bias})_{1,\omega^2}$ and $(V_{\rm bias})_{1,\omega}$ are of similar strengths, increasing almost identically the $v_\omega$ and $v_{\omega^2}$ regions by volume pressure to diminish the original false vacuum $v_1$ region. As $v_{\omega^2}$ is still a false vacuum, the tiny $(V_{\rm bias})_{\omega,\omega^2}$ driven volume pressure on $v_{\omega^2}$ can slowly push the walls to increase the true vacuum $v_\omega$ region. So, after the period with a quick increment of the $v_{\omega^2}$ region driven by the $(V_{\rm bias})_{1,\omega^2}$ from original $v_1$ region, the $v_{\omega^2}$ region tends to diminish slowly by the tiny $(V_{\rm bias})_{\omega,\omega^2}$ driven volume pressure. In this case the decay time for DWs is dominated by the slow $(V_{\rm bias})_{\omega,\omega^2}$ driven collapse. Collapse of DWs in such a scenario with vacuum structure satisfying (45) is sensitive to the small contributions $(V_{\rm bias})_{\omega,\omega^2}$ that lift the degeneracy between $v_\omega$ and $v_{\omega^2}$. Not specifying the details of the degeneracy lifting mechanism, we will not discuss such a scenario in this work and only concentrate on the scenario with $v_1$ the true vacuum that was discussed in previous paragraphs.

The collapse of DWs happens when the volume pressure force becomes greater than the tension force. Assuming that DWs have reached the scaling regime, we can estimate the decay time

$$t \approx C_{\rm ann} \mathcal{A} \frac{\sigma}{\epsilon} \simeq C_{\rm ann} \mathcal{A} \frac{\sigma}{|\Delta V_{\not{Z}_3;S^2}|}, \tag{46}$$

with $\mathcal{A} \sim 1.2$ for $N = 3$ and $C_{\rm ann}$ a coefficient of $\mathcal{O}(1)$ that takes value $C_{\rm ann} = 5.02 \pm 0.44$, based on the simulation of axion models with $N = 3$. We can estimate the decay time as $t \sim C_0 (\chi m_{3/2})^{-1}$ by the following observations: $\sigma \sim m_{\rm soft}^3$ while $V_{\rm bias} \sim C_1 (\chi m_{3/2}) m_{\rm soft}^3$ with $C_0$, $C_1$ some constant coefficients. So, the decay time can be dominantly determined by the value of $\chi m_{3/2}$.

It is well known that DWs cannot be formed from the beginning if the bias terms are sufficiently large. The prediction of percolation theory gives a necessary condition for the formation of large scale DWs with the presence of bias term [132]

$$\frac{V_{\rm bias}}{V_0} < \ln\left(\frac{1 - p_c}{p_c}\right) = 0.795. \tag{47}$$

We check that this constraint is safely satisfied for the NMSSM scalar potential with small bias terms, for example, for $\mu' S^2/2$ case with the choice $\chi m_{3/2} \sim 10^{-12}$ eV adopted in our numerical studies.

Similarly, the $\mu_0 \equiv \chi m_{3/2} H_u H_d$ term that explicitly violates the $Z_3$ discrete symmetry will also cause the collapse of DWs. (We have $\mu_{\rm eff} = \lambda s + \mu_0 \approx \lambda s$ as the term

$\lambda s$ gives the dominant contribution to $\mu_{\rm eff}$.) The scalar potential relevant to such a term is given by

$$
\begin{aligned}
\Delta\mathcal{L}_{H_uH_d} \supseteq\ & \{[|\mu_0|^2+\mu_0\lambda(S+S^*)](|H_u|^2+|H_d|^2) \\
& + (B\mu_0 H_u\cdot H_d + \mathrm{H.c.})\}, \\
& \left[1+\frac{1}{3M_P^2}2\{|S|^2+|H_u|^2+|H_d|^2\}\right], \\
& -4\left[\frac{\chi^2}{3M_P^2}+\frac{\chi^3(H_uH_d+\bar{H}_u\bar{H}_d)}{9M_P^4}\right. \\
& \left.+\frac{\chi^4(|H_u|^2+|H_d|^2)}{9M_P^4}\right]\lambda^2|S|^2|H_d|^2|H_u|^2, \\
& -8\frac{\chi^2}{9M_P^4}\lambda^2|S|^2|H_d|^2|H_u|^2. \qquad (48)
\end{aligned}
$$

We can calculate the energy difference between adjacent vacuum

$$
\begin{aligned}
\Delta_{1,\omega}V_{\not{Z}_3;H_uH_d} &= \Delta_{1,\omega^2}V_{\not{Z}_3;H_uH_d} \\
&= 2[\lambda\mu_0 v^2 s + m_{\rm soft}\mu_0 v_u v_d]\left(1-\cos\frac{2\pi}{3}\right) \\
&\simeq 3\lambda\mu_0 s v^2 + 3m_{\rm soft}\mu_0\frac{v^2}{\tan\beta} \\
&\approx 3\mu_0 v^2\left(\mu_{\rm eff}+\frac{m_{\rm soft}}{\tan\beta}\right), \\
\Delta_{\omega,\omega^2}V_{\not{Z}_3;S^2} &= 0. \qquad (49)
\end{aligned}
$$

From the electroweak symmetry breaking condition in Eq. (10), we have $\mu_{\rm eff}\gg\frac{m_{\rm soft}}{\tan\beta}$ so that the contribution to $\Delta_{1,\omega;\omega^2}V$ from soft SUSY breaking $B$ term is always negligible. Requiring the $v_1$ vacua to be the true minimum leads to the constraints $\mu_0<0$. Additional subleading contributions to biases $\Delta_H V\sim\lambda_H^4$ from hidden $\mathrm{SU}(N)_h$ are also used to spoil the degeneracy between the $\omega$ and $\omega^2$ vacua. Again, in the scaling regime, the decay time of DWs can be estimated as

$$
t\approx C_{\rm ann}\mathcal{A}\frac{\sigma}{\epsilon}\sim C_{\rm ann}\mathcal{A}\frac{\sigma}{|\Delta V_{\not{Z}_3;H_uH_d}|}. \qquad (50)
$$

Similarly, the decay time can be dominantly determined by the value of $\chi m_{3/2}$. The constraint (47) is also safely satisfied for the NMSSM scalar potential with tiny explicit $Z_3$ breaking coefficient $\chi m_{3/2}\sim10^{-12}$ eV for $\mu_0 H_u H_d$ case.

The collapse of DWs should occur before they overclose the Universe and their decay products should at the same time not destroy light elements created at the epoch of BBN. So, we have the constraints

$$
\begin{aligned}
t &\lesssim \min(0.01\ \ \mathrm{sec}, t_{\rm dom}) \\
&= \min(1.4\times10^{13}\ \ \mathrm{eV}^{-1}, t_{\rm dom}), \qquad (51)
\end{aligned}
$$

with

$$
\begin{aligned}
t_{\rm dom} &= \frac{3M_P^2}{4\mathcal{A}\sigma}\simeq 2.93\times10^3\ \ \mathrm{sec}\,\mathcal{A}^{-1}\left(\frac{\sigma}{\mathrm{TeV}^3}\right)^{-1} \\
&\simeq 0.45\times10^{19}\ \ \mathrm{eV}^{-1}\mathcal{A}^{-1}\left(\frac{\sigma}{\mathrm{TeV}^3}\right)^{-1}. \qquad (52)
\end{aligned}
$$

The BBN constraint is always more stringent than that of $t_{\rm dom}$ unless $\sigma\gtrsim10^6$ TeV$^3$.

### A. GWs from the collapse of DWs

The collisions of the DWs can deviate from the spherical symmetry, so the energy stored inside DWs can not only be released into light degrees of freedom but also into GWs when DWs collapse. The intensity of the GWs decreases as the Universe expands since the amplitude is redshifted for each mode. So, one may use a dimensionless quantity $\Omega_{\rm gw}(f)$ to characterize the intensity of GWs. The density parameter $\Omega_{\rm gw}(f)$ of the GWs can be defined as the fraction between the GWs energy density per logarithmic frequency interval and the total energy density of the Universe

$$
\Omega_{\rm gw}(f)\equiv\frac{1}{\rho_c}\frac{d\rho_{\rm gw}}{d\log f}, \qquad (53)
$$

with $\rho_{\rm gw}$ as the energy density of the GWs, $\rho_c$ the critical energy density, and $f$ the frequency of GWs. As the GWs are predominantly sourced by horizon-scale structures in the DW network, the GWs emitted are expected to peak at the frequency corresponding to the horizon scale at the decay time.

Due to the redshift by the cosmic expansion, the present density parameter $\Omega_{\rm gw}^0$ and the frequency of the GWs $f_0$ can be obtained from the quantities at the formation of the GWs (denoted with "*") as

$$
\Omega_{\rm gw}^0=\Omega_{\rm gw}^*\left(\frac{a_*}{a_0}\right)^4\left(\frac{H_*}{H_0}\right)^2, \qquad f_0=f_*\left(\frac{a_*}{a_0}\right). \qquad (54)
$$

The peak frequency today can be estimated by

$$
\begin{aligned}
f_{\rm peak} &= \frac{a(t_*)}{a(t_0)}H_*\frac{f_*}{H_*}, \\
&= 1.14\ \mathrm{nHz}\left(\frac{10.75}{g_*,s}\right)^{1/3}\left(\frac{g_*}{10.75}\right)^{1/2}\left(\frac{10^{13}\ \mathrm{eV}^{-1}}{t}\right)^{1/2}, \qquad (55)
\end{aligned}
$$

with $g_*$ and $g_{*s}$ the effective relativistic degrees of freedom for the energy density and the entropy density, respectively. Here we use the following formula to transform the temperature into the Hubble parameter

$$H = \frac{1}{2t} = \sqrt{\frac{8\pi^3}{90} g_*} \frac{T^2}{M_P} = 1.66 g_*^{\frac{1}{2}} \frac{T^2}{M_P}, \tag{56}$$

in the radiation dominant era with $M_P = 1.22 \times 10^{19}$ GeV.

For $s, m_{\text{soft}} \sim \mathcal{O}(1)$ TeV, $\lambda, \kappa \sim \mathcal{O}(1)$, and $\tan\beta \sim \mathcal{O}(10)$, the value of $\chi m_{3/2}$ should be of order $10^{-12}$ eV for $\sigma \sim \mathcal{O}(\text{TeV})^3$. Given that the gravitino mass $m_{3/2}$ cannot be much lighter than $\mathcal{O}(1)$ eV, the $\chi$ parameter should be much smaller than unity. Such tiny value can be the consequence of higher-dimensional operators in the Kahler potential with $\text{U}(1)_R$ symmetry. For example, assuming $R(\Phi) = 16/3$ and $R(\Phi') = 4$, the Kahler potential adopted in [133]

$$K \supseteq \Phi^\dagger \Phi + \Phi'^\dagger \Phi' + \left( \frac{\alpha}{M_P^2} \Phi \Phi'^\dagger H_u H_d + \frac{\alpha'}{M_P^2} \Phi \Phi'^\dagger S^2 + \text{H.c.} \right) + \cdots, \tag{57}$$

can naturally lead to $\chi \sim 10^{-24}$ for $m_{3/2} \sim \mathcal{O}(1)$ GeV and $\alpha, \alpha' \sim \mathcal{O}(1)$ without the tadpole problem, when the R-symmetry breaking scale lies of order $\langle \Phi \rangle \sim \langle \Phi' \rangle \sim 10^6$ GeV. Such Kahler potential can originate from the real function $\mathcal{N}(X, \bar{X})$

$$\mathcal{N}(X, \bar{X}) = -|X^0|^2 + \sum_{\Phi, \Phi' \cdots} |X^\alpha|^2 + \left( \alpha \frac{X^\Phi \bar{X}^{\Phi'}}{|X_0^2|} H_u H_d + \alpha' \frac{X^\Phi \bar{X}^{\Phi'}}{|X_0^2|} S^2 + \text{H.c.} \right), \tag{58}$$

after gauge fixing.

The power of the GWs radiation can be estimated as $\rho_{\text{gw}} \sim P t / t^3 \sim G \mathcal{A} \sigma^2$ by the quadrupole formula, given by $P \sim G \dddot{\mathcal{Q}}_{ij} \dddot{\mathcal{Q}}_{ij} \sim M_{\text{wall}}^2 / t^2$ with $M_{\text{wall}} = \sigma \mathcal{A} t^2$ the energy of DWs. The quadrupole formula cannot be directly applied to DWs, since it is only valid in the far-field regime [134]. To obtain the peak amplitude of GWs produced by long-lived DWs, we use the fact that numerical simulations show that the value

$$\tilde{\epsilon}_{\text{gw}} \equiv \frac{1}{G \mathcal{A}^2 \sigma^2} \left( \frac{d\rho_{\text{gw}}}{d \ln f} \right)_{\text{peak}} \tag{59}$$

almost keeps the constant value $\tilde{\epsilon}_{\text{gw}} \simeq 0.7 \pm 0.4$ [135] after DWs enter into the scaling regime. So, the peak amplitude at the annihilation time of DWs can be estimated as

$$\Omega_{\text{gw}}(H_*)_{\text{peak}} = \frac{1}{\rho_c(H_*)} \left( \frac{d\rho_{\text{gw}}(H_*)}{d \log f} \right)_{\text{peak}} = \frac{8\pi G^2 \mathcal{A}^2 \sigma^2 \tilde{\epsilon}_{\text{gw}}}{2 H_*^2}. \tag{60}$$

Here the production of GWs is assumed to suddenly terminate at $H = H_*$ and happens during the radiation dominated era. The present density parameter $\Omega_{\text{gw}}^0$ can be estimated as [134]

$$\begin{aligned} \Omega_{\text{gw}}(t_0)_{\text{peak}} h^2 &= \Omega_{\text{rad}} h^2 \left( \frac{g_*(T_{\text{ann}})}{g_{*0}} \right) \left( \frac{g_{*s0}}{g_{*s}(T_{\text{ann}})} \right)^{4/3} \Omega_{\text{gw}}(t_{\text{ann}}), \\ &\simeq 3.06 \times 10^{-14} \left( \frac{10.75}{g_*} \right)^{1/3} \left( \frac{t}{10^{13}\ \text{eV}^{-1}} \right)^2 \left( \frac{\sigma}{1\ \text{TeV}^3} \right)^2, \end{aligned} \tag{61}$$

with $\Omega_{\text{rad}} h^2 = 4.15 \times 10^{-5}$ the density parameter of radiations at the present time. The constraint $t \lesssim t_{\text{dom}}$ can set an upper bound for $\Omega_{\text{gw}}(t_0)_{\text{peak}} h^2$ with

$$\Omega_{\text{gw}}^{\text{upper}}(t_0)_{\text{peak}} h^2 \lesssim 10^{-8}, \tag{62}$$

for the case with $\sigma \gtrsim 10^6$ TeV$^3$.

The full GW spectrum today can be estimated by

$$\Omega_{\text{gw}}(f) h^2 = \Omega_{\text{gw}}(t_0)_{\text{peak}} h^2 \begin{cases} \left( \frac{f}{f_{\text{peak}}} \right)^3, & f < f_{\text{peak}} \\ \frac{f_{\text{peak}}}{f}, & f > f_{\text{peak}} \end{cases}. \tag{63}$$

In previous discussions, it is assumed that the annihilation of DWs happens during the radiation dominated era. If it happens before reheating, the above estimations can be modified accordingly. Numerical results indicate that such a possibility requires very low reheating temperature of order MeV to explain the recent nHZ GWs background signals. Low reheating temperature of order GeV can be preferred to avoid the overproduction of very light gravitino and the increase of entropy due to the production of moduli fields [136]. However, this low temperature seems to be problematic with baryogenesis, according to which it is commonly assumed that the reheating temperature is at least of the order the electroweak scale ($10^2$ GeV) [137]. Although there are some mechanisms that can explain the baryon asymmetry at very low temperatures, reheating temperature of order MeV still seems not favored.

### B. Tension between nHz PTA GW background data and muon $g-2$ anomaly

The PTA observations on the frequency of stochastic GW background can set stringent constraints for the collapse time of DWs, which can be mainly determined by the value of $\chi m_{3/2}$ [see the discussions below Eq. (46)]. With the very narrow range of the DW collapse time derived from PTA data, the upper bound for the present density parameter $\Omega^0_{\rm gw}$ can set an upper bound for the tension $\sigma$ of DWs by the formula (62), which can be further translated into the upper bound for the soft SUSY breaking parameters $m_{\rm soft}$. We can estimate the upper bound of $m_{\rm soft}$ to be $m_{\rm soft} \lesssim 200$ TeV. Such upper bound on $m_{\rm soft}$ can shed light on the UV SUSY breaking mechanisms.

Most importantly, given the BBN upper bounds for the collapse time of DWs, the lower bound for the present density parameter $\Omega^0_{\rm gw}$ can set the lower bounds for $m_{\rm soft}$, which can be estimated to be of order TeV. Such lower bounds can have tension with the SUSY explanation of the muon $g-2$ anomaly because large SUSY contributions to $\Delta a_\mu$ in general need relatively light superpartners.

We know that the SUSY contributions to muon $g-2$ are dominated by the chargino–sneutrino and the neutralino–smuon loop in MSSM. At the leading order of $\tan\beta$ and $m_W/m_{\rm SUSY}$, they are evaluated as [138]

$$\Delta a_\mu(\tilde\mu_L,\tilde\mu_R,\tilde B) = \frac{\alpha_Y}{4\pi}\frac{m_\mu^2 M_1\mu}{m^2_{\tilde\mu_L}m^2_{\tilde\mu_R}}\tan\beta\cdot f_N\left(\frac{m^2_{\tilde\mu_L}}{M_1^2},\frac{m^2_{\tilde\mu_R}}{M_1^2}\right), \quad (64)$$

$$\Delta a_\mu(\tilde B,\tilde H,\tilde\mu_R) = -\frac{\alpha_Y}{4\pi}\frac{m_\mu^2}{M_1\mu}\tan\beta\cdot f_N\left(\frac{M_1^2}{m^2_{\tilde\mu_R}},\frac{\mu^2}{m^2_{\tilde\mu_R}}\right), \quad (65)$$

$$\Delta a_\mu(\tilde B,\tilde H,\tilde\mu_L) = \frac{\alpha_Y}{8\pi}\frac{m_\mu^2}{M_1\mu}\tan\beta\cdot f_N\left(\frac{M_1^2}{m^2_{\tilde\mu_L}},\frac{\mu^2}{m^2_{\tilde\mu_L}}\right), \quad (66)$$

$$\Delta a_\mu(\tilde W,\tilde H,\tilde\mu_L) = -\frac{\alpha_2}{8\pi}\frac{m_\mu^2}{M_2\mu}\tan\beta\cdot f_N\left(\frac{M_2^2}{m^2_{\tilde\mu_L}},\frac{\mu^2}{m^2_{\tilde\mu_L}}\right), \quad (67)$$

$$\Delta a_\mu(\tilde W,\tilde H,\tilde\nu_\mu) = \frac{\alpha_2}{4\pi}\frac{m_\mu^2}{M_2\mu}\tan\beta\cdot f_C\left(\frac{M_2^2}{m^2_{\tilde\nu}},\frac{\mu^2}{m^2_{\tilde\nu}}\right). \quad (68)$$

In the previous expressions, we denote $m_\mu$ as the muon mass, $m_{\rm SUSY}$ the SUSY breaking masses, and $\mu$ the Higgsino mass, respectively. The loop functions are defined as

$$f_C(x,y) = xy\left[\frac{5-3(x+y)+xy}{(x-1)^2(y-1)^2} - \frac{2\log x}{(x-y)(x-1)^3} + \frac{2\log y}{(x-y)(y-1)^3}\right], \quad (69)$$

$$f_N(x,y) = xy\left[\frac{-3+x+y+xy}{(x-1)^2(y-1)^2} + \frac{2x\log x}{(x-y)(x-1)^3} - \frac{2y\log y}{(x-y)(y-1)^3}\right], \quad (70)$$

which are monochromatically increasing for $x>0$, $y>0$ with $0 \le f_{C,N}(x,y) \le 1$. They satisfy $f_C(1,1)=1/2$ and $f_N(1,1)=1/6$ in the limit of degenerate masses. The SUSY contributions to the muon $g-2$ will be enhanced for small soft SUSY breaking masses and large value of $\tan\beta$. It can be seen from the previous formulas that large SUSY contributions to $\Delta a_\mu$ require light sleptons, light electroweakinos, and large $\tan\beta$. However, current LHC experiments have already set stringent constraints on colored sparticles, such as the 2.2 TeV bound for gluino and the 1.4 TeV bound for squarks, making the explanation of the recent muon $g-2$ data fairly nontrivial. SUSY explanations of the muon $g-2$ anomaly can be seen in the literatures; see Refs. [139–154] and [155–159]. The inclusion of the singlet component in NMSSM will in general give negligible contributions to $\Delta a_\mu$ because of the suppressed coupling of singlino to the MSSM sector. However, the lightest neutral CP-odd Higgs scalar could give non-negligible contributions to $a_\mu$ if it is quite light [160]. The positive two-loop contribution is numerically more important for a light CP-odd Higgs at approximately 3 GeV and the sum of both one-loop and two-loop contributions is maximal around $m_{a_1} \sim 6$ GeV. Such a light $a_1$ can be constrained stringent by various low-energy experiments, such as CLEO and B-physics.

Given the estimated $\mathcal{O}$(TeV) lower bounds for soft SUSY breaking parameters by PTA GW data, it is interesting to survey numerically if the parameter regions allowed by the muon $g-2$ explanation in $Z_3$-NMSSM can be consistent with the DW explanation of such stochastic GW background signals.

### C. Numerical results

We calculated numerically the tension of DWs, generated from the spontaneously discrete symmetry breaking in the $Z_3$-invariant NMSSM, within the parameter regions allowed by low-energy experimental data, including the following constraints other than those already encoded in the NMSSMTools_5.6.2 package:

(i) The CP-even component $S_2$ dominated combination of $H_u$ and $H_d$ doublets corresponds to the SM Higgs, which should lie in the combined mass range for the Higgs boson, 122 GeV $< M_h <$ 128 GeV [161,162]. We adopt the uncertainty $\pm 3$ GeV instead of default 2 GeV because large $\lambda$ may induce additional $\mathcal{O}(1)$ GeV correction to $M_h$ at the two-loop level [163].

(ii) Bounds for low mass and high mass resonances at LEP, Tevatron, and LHC are taken into account by using the package HiggsBounds-5.5.0 [164] and HiggsSignal-2.3.0 [165].

(iii) Updated LHC exclusion bounds [166–168] for sparticle searches and the lower mass bounds of charginos and sleptons from the LEP [169]. All relevant EW SUSY searches are taken into account, via CheckMATE2 [170–172]. We discard the parameter points whose $R$ values obtained from the CheckMATE2.0 are larger than 1, i.e., excluded at 95% CL.

(iv) Constraints from B physics, such as $B \to X_s \gamma$, $B_s \to \mu^+\mu^-$, and $B^+ \to \tau^+\nu_\tau$, etc. [173–176].

(v) Vacuum stability bounds on the soft SUSY breaking parameters, including the semianalytic bounds for nonexistence of a deeper charge/color breaking minimum [177] and/or a metastable EW vacuum with a tunneling lifetime longer than the age of the Universe [178].

(vi) The relic abundance of the dark matter (DM) should be below the upper bounds by the Planck result $\Omega_{\rm DM}h^2 = 0.1199 \pm 0.0027$ [179] in combination with the WMAP data [180] (with a 10% theoretical uncertainty). Besides, we require that the Spin-Independent (SI) and Spin-Dependent (SD) DM direct detection constraints, for example, the LUX [181], XENON1T [182,183], and PandaX-4T [184,185] should be satisfied. We should note that such DM constraints can in fact be relaxed in gauge mediation scenarios, as the gravitino mass can be as light as $\mathcal{O}(1)$ eV, which can account for the present DM relic abundance for $m_{3/2} \sim$ keV. In our conservative survey, such DM constraints are included in our numerical results.

(vii) The muon $g-2$ anomaly needs to be explained within the $2\sigma$ range for the recent updated combined data. We require the new physics contributions to $\Delta a_\mu$ to lie within

$$\Delta a_\mu^{\rm NP} \in [15.3, 34.5] \times 10^{-10}. \tag{71}$$

We use NMSSMTools_5.6.2 [109,186] to numerically scan the parameter spaces for the muon $g-2$ explanation with the following low-energy NMSSM inputs:

$$\begin{aligned}
&0 < \lambda < 0.2, \quad -0.2 < \kappa < 0.2, \quad 20 < \tan\beta < 60, \quad -1000 < \mu_{\rm eff} < 1000\ \text{GeV}\\
&-1000\ \text{GeV} < A_\lambda < 1000\ \text{GeV}, \quad -500\ \text{GeV} < A_\kappa < 500\ \text{GeV},\\
&|M_1| < 1000\ \text{GeV}, \quad 70\ \text{GeV} < M_2 < 2000\ \text{GeV}, \quad M_3 = 10\ \text{TeV},\\
&1000\ \text{GeV} < M_{L_3} < 3000\ \text{GeV}, \quad 1400\ \text{GeV} < M_{E_3} < 3000\ \text{GeV}, \quad |A_{E_3}| < 2000\ \text{GeV},\\
&M_{Q_3} = 10\ \text{TeV}, \quad M_{U_3} = 10\ \text{TeV}, \quad M_{D_3} = 10\ \text{TeV}, \quad A_{U_3} = 10\ \text{TeV}, \quad A_{D_3} = 10\ \text{TeV},\\
&80\ \text{GeV} < M_{L_{1;2}} < 500\ \text{GeV}, \quad 80\ \text{GeV} < M_{E_{1;2}} < 400\ \text{GeV}, \quad |A_{E_{1;2}}| < 500\ \text{GeV},\\
&M_{Q_{1;2}} = 10\ \text{TeV}, \quad M_{U_{1;2}} = 10\ \text{TeV}, \quad M_{D_{1;2}} = 10\ \text{TeV}.
\end{aligned} \tag{72}$$

Instead of using the soft SUSY breaking parameters $m_{H_u}^2$, $m_{H_d}^2$, and $m_S^2$, one usually trades them for $m_Z$, $\tan\beta$, and $\mu_{\rm eff} = \lambda s$ by implementing the scalar potential minimization conditions. The squarks and gluino are chosen to be heavy to evade the stringent LHC constraints on the colored sparticles. The bias terms that trigger the collapse of DWs are related to the explicit $Z_3$ breaking $\chi m_{3/2}$ parameter. We check that the presence of small explicitly $Z_3$ violation bias terms with $\chi m_{3/2} \sim 10^{-12}$ eV will not alter the $Z_3$ invariant NMSSM spectrum.

The present-day relic abundance of stochastic GW backgrounds can be calculated numerically with the tensions of the DWs and the related bias terms for each parameter point that satisfy the constraints from (i) to (v). The BBN and $t_{\rm dom}$ constraints in (51) are also taken into account.

We have the following discussions for our numerical results:

(i) The $\chi S^2$ case:

From our estimation, we adopt the following range of $\chi m_{3/2}$,

$$10^{-16}\ \text{eV} < \chi m_{3/2} < 10^{-12}\ \text{eV}, \tag{73}$$

in our numerical scan.

The peak amplitude of the GW backgrounds at the present time versus the corresponding peak frequency are shown in the upper panel of Fig. 1. The corresponding GW spectra (with the NANOGrav observation data denoted by the red contour) are shown in the lower panel of Fig. 1. We can see that the GW spectrum for many of the survived parameter points can have overlap with the reconstructed posterior distributions for the NANOGrav observations of nHz GWs so as that the DW explanation from $Z_3$-NMSSM is viable.

We show in the upper panel of Fig. 2 our numerical results on the tensions of the DWs versus their collapse time. The parameter points, whose GWs spectra can overlap with the NANOGrav signal region, are marked with the color blue. From the distributions of the blue points, we can see that the PTA data can impose stringent constraints for the

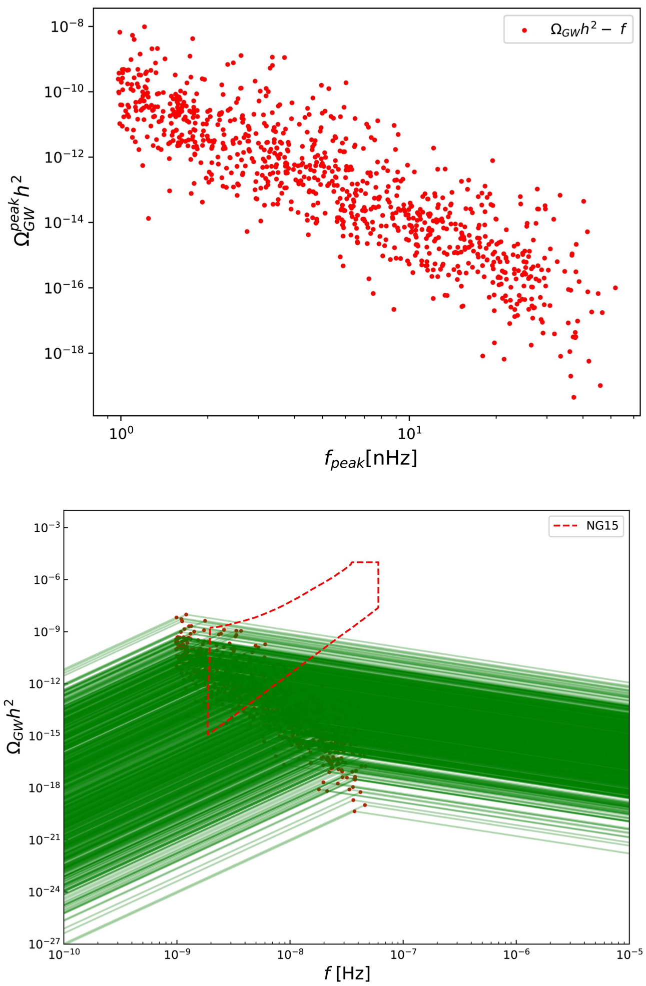


FIG. 1. The upper panel shows the present-day peak amplitude of stochastic GW background $\Omega_{\rm gw}h^2$ versus the peak frequency $f_{\rm peak}$ for the parameter points of $\chi S^2$ case. The corresponding GW spectra (with the NANOGrav observation data denoted by the red contour) are shown in the lower panel. Each parameter point satisfies the experimental constraints (i)–(v).

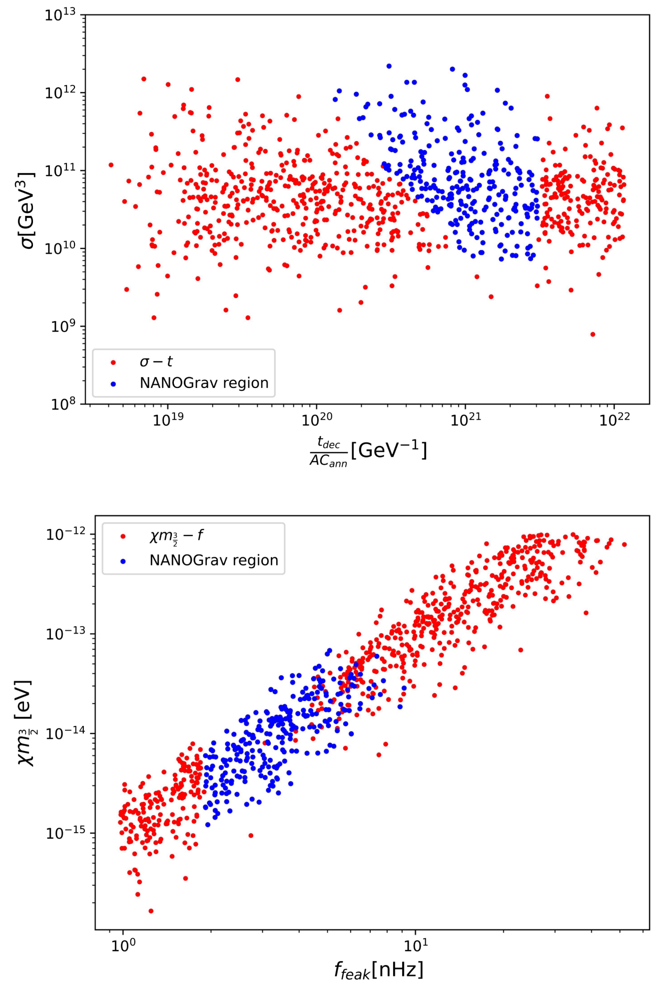


FIG. 2. The tensions of the DWs versus their collapse time for the parameter points of $\chi S^2$ case are shown in the upper panel. The parameter points that can account for the NANOGrav GW background observations are marked with blue color. Each parameter point satisfies the experimental constraints (i)–(v). The values of $\chi m_{3/2}$ versus the peak frequencies are shown in the lower panel.

muon $g-2$ favored parameter space. As discussed in the previous section, the lower bounds from $\sigma$ (hence giving lower bounds for $m_{\rm soft}$) can exclude some regions that can explain the muon $g-2$ with smaller $m_{\rm soft}$. From the lower panel of Fig. 2, we can see that the range of $\chi m_{3/2}$ that can account for the NANOGrav nHZ GW background data should lie within $10^{-15}$ eV $\sim 10^{-13}$ eV. As anticipated, there is approximate linear dependence between $\chi m_{3/2}$ and $f_{\rm peak}$.

We show in the upper panel of Fig. 3 the SUSY contributions to muon $g-2$, the value $\Delta a_\mu$, versus the present-day peak amplitude of stochastic GW background $\Omega_{\rm gw}h^2$ for those survived points whose GWs spectra can overlap with the NANOGrav signal region. We can see that there are still large parameter regions that can explain the muon $g-2$ anomaly within $1\sigma$ range. Large values of $\Omega_{\rm gw}h^2$ always favor large $\sigma$, hence large $m_{\rm soft}$. As the SUSY contributions to $\Delta a_\mu$ in general favor small $m_{\rm soft}$ of order $\mathcal{O}(10^2)$ GeV, the preferred ranges of $\Delta a_\mu$ that can account for the muon $g-2$ anomaly favor low $\Omega_{\rm gw}h^2$. The ranges of the $\lambda$ and $\kappa$ parameters of NMSSM that can account for the NANOGrav GW background observations are shown in the lower panel of Fig. 3. Both of them are upper bounded to be be less than 0.18.

(ii) The $\chi H_u H_d$ case:

From our estimation, we adopt the following range of $\chi m_{3/2}$

$$10^{-13}\ {\rm eV} < \chi m_{3/2} < 10^{-9}\ {\rm eV}, \tag{74}$$

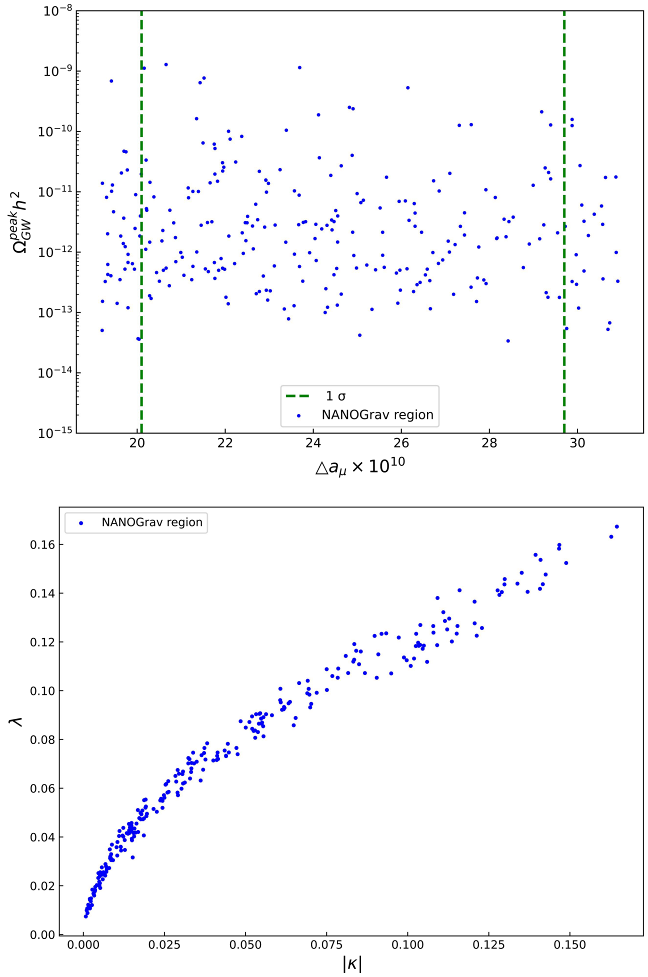


FIG. 3. The tensions of the DWs versus their collapse time for the parameter points of $\chi S^2$ case are shown in the upper panel. The parameter points that can account for the NANOGrav GW background observations are marked with the color blue. Each parameter point satisfies the experimental constraints (i)–(v). The values of $\chi m_{3/2}$ versus the peak frequencies are shown in the lower panel.

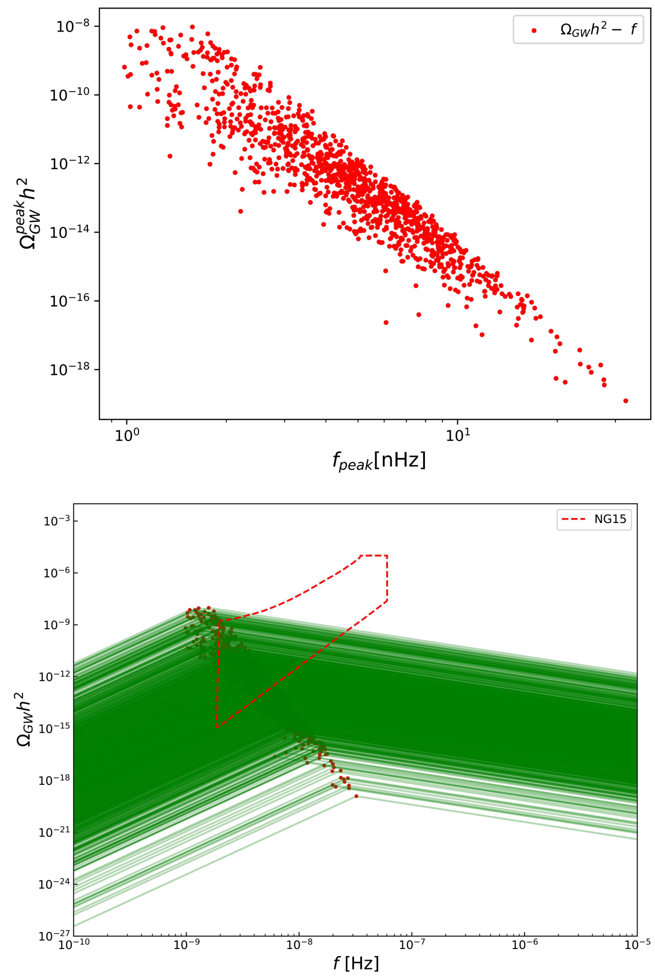


FIG. 4. The upper panel shows the present-day peak amplitude of stochastic GW background $\Omega_{\rm gw}h^2$ versus the peak frequency $f_{\rm peak}$ for the parameter points of $\chi H_u H_d$ case. The corresponding GW spectra (with the NANOGrav observation data denoted by the red contour) are shown in the lower panel. Each parameter point satisfies the experimental constraints (i)–(v).

in our numerical scan.

We can have similar discussions for this case. The peak amplitude of the GW backgrounds at the present time versus the corresponding peak frequency is shown in the upper panel of Fig. 4. The corresponding GW spectra are shown in the lower panel of Fig. 4. Again, the GW spectrum for many the surviving parameter points can have overlap with the reconstructed posterior distributions for the NANOGrav observations of nHz GWs. So, it is obvious that the collapse of DWs from approximate $Z_3$-invariant NMSSM with the small explicitly $Z_3$ breaking $\chi H_u H_d$ term (and its soft SUSY breaking B-terms) can explain the PTA observations.

In the upper panel of Fig. 5, we show our numerical results on the tensions of the DWs versus their collapse time. From the distributions of the blue points, which denote those parameter points whose GWs spectrum can be consistent with the NANOGrav signal region, we can see that the PTA data can again impose stringent constraints for the muon $g-2$ favored parameter space. Similarly, the lower bounds from $\sigma$ that lead to lower bounds for $m_{\rm soft}$ can exclude some regions that can explain the muon $g-2$ with light sparticle masses. From the lower panel of Fig. 5, we can see that the range of $\chi m_{3/2}$ that can account for the NANOGrav nHZ GW background data should lie within $10^{-11}$ eV $\sim 10^{-9}$ eV. The correlations between $\chi m_{3/2}$ and $f_{\rm peak}$ are not obviously linear, as the bias terms in this case can differ in a relatively wide range, which involves $v^2, \tan\beta$, and $m_{\rm soft}$.

We show in the upper panel of Fig. 6 the SUSY contributions to muon $g-2$, the value $\Delta a_\mu$, versus the present-day peak amplitude of stochastic GW

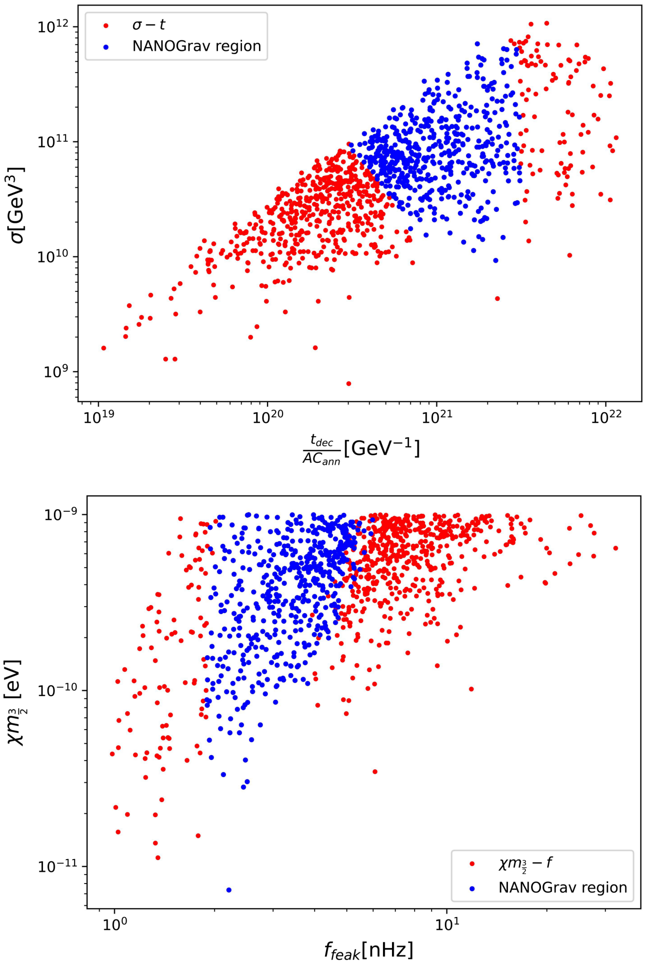


FIG. 5. The tensions of the DWs versus their collapse time for the parameter points of $\chi H_u H_d$ case are shown in the upper panel. The parameter points that can account for the NANOGrav GW background observations are marked with the color blue. Each parameter point satisfies the experimental constraints (i)–(v). The values of $\chi m_{3/2}$ versus the peak frequencies are shown in the lower panel.

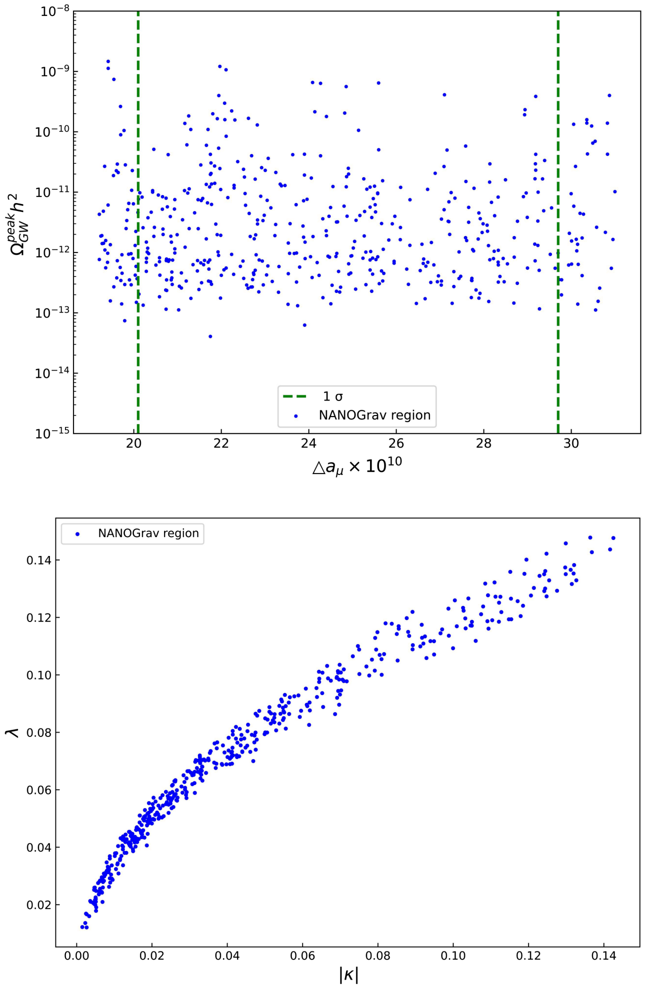


FIG. 6. The tensions of the DWs versus their collapse time for the parameter points of $\chi H_u H_d$ case are shown in the upper panel. The parameter points that can account for the NANOGrav GW background observations are marked with the color blue. Each parameter point satisfies the experimental constraints (i)–(v). The values of $\chi m_{3/2}$ versus the peak frequencies are shown in the lower panel.

background $\Omega_{\mathrm{gw}} h^2$ for those survived points whose GWs spectra can overlap with the NANOGrav observations. Again, there are still large parameter regions that can explain the muon $g-2$ anomaly within $1\sigma$ range. The NANOGrav GW background data can set upper bounds for $\lambda$ and $\kappa$ parameters of NMSSM, which should be less than 0.14 (see the lower panel of Fig. 6).

## V. CONCLUSIONS

The spontaneous breaking of the discrete $Z_3$ symmetry in approximate $Z_3$-invariant next-to minimal supersymmetric standard model by the VEVs of Higgs fields can lead to the formation of DWs in the Universe. Such potentially problematic DWs can collapse and lead to the stochastic GW background signals observed by PTA collaborations due to some explicitly $Z_3$ breaking terms in the NMSSM effective superpotential and scalar potential. In the presence of a hidden sector, such terms may originate from the geometric superconformal breaking with holomorphic quadratic correction to frame function when the global scale-invariant superpotential is naturally embedded into the canonical superconformal supergravity models. The smallness of such $Z_3$ breaking mass parameters in the NMSSM may be traced back to the original superconformal invariance.

Naive estimations indicate that SUSY explanation to the muon $g-2$ anomaly can have tension with the constraints on SUSY by PTA data, because large SUSY contributions to $\Delta a_\mu$ in general need relatively light superpartners of order $\mathcal{O}(10^2)$ GeV while the lower bounds for present $\Omega_{\mathrm{gw}}^0$ can also set the lower bounds for the tension of DWs $\sigma$, leading approximately to $\mathcal{O}(1)$ TeV lower bound for $m_{\rm soft}$ with the BBN upper bounds for the decay time of DWs. We calculate numerically the signatures of GWs produced from the collapse of DWs by the corresponding bias terms. We find that the observed nHZ stochastic GW background by NANOGrav, etc., can indeed be explained with proper tiny values of $\chi m_{3/2} \sim 10^{-14}$ eV for $\chi S^2$ case (and $\chi m_{3/2} \sim 10^{-10}$ eV for $\chi H_u H_d$ case), respectively. Such tiny values can be natural consequences of higher-dimensional operators in the Kahler potential with $\mathrm{U}(1)_R$ symmetry, which do not have the notorious tadpole problem. Besides, there are still some parameter points whose GWs spectra intersect with the NANOGrav signal region can explain the muon $g-2$ anomaly to $1\sigma$ range.

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