Test of the vector portal with dark fermions in the charge-exchange reactions in the NA64 experiment at CERN SPS

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We consider an experiment to search for dark sector particles in invisible (or semi-invisible) decays of neutral mesons $M^0 = \pi^0$, η, η', ω, $f_2(1270)$, which produced the charge-exchange reactions $\pi^- + (A,Z) \to M^0 + (A, Z-1); M^0 \to$ *invisible* of high-energy pions (or kaons) on a nuclei target in the NA64 experiment at the CERN SPS. This reaction chain would lead to a striking signature of the signal event—the complete disappearance of the beam energy in the setup [Phys. Rev. D **91**, 015004 (2015)]. Using data obtained from the measurements at IHEP (Protvino) and Fermilab (Batavia) we show that the integral cross sections σ for the production of the M^0 s slightly deviate from the phenomenological formula $\sigma \sim Z^{2/3}$, where Z is the nuclei charge. In particular, we present the formulas for the differential and integral cross sections that explicitly depend on the Mandelstam and Z variables. Derived formulas are used to predict the M^0 yield as a function of beam energy for several target nuclei, and to estimate the projection sensitivity for the proposed search in the dark vector portal model. Sensitivity to different decay modes of M^0s is explored.

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I. INTRODUCTION

Nowadays searches for dark matter (DM) and the study of its properties is a hot topic in particle physics, astrophysics, and cosmology. In the last decade, a broad experimental program has been developed to detect nongravitational DM interactions, including direct searches for DM by measuring the recoil energy from DM-nucleus scattering, indirect searches for particles from the DM annihilation, and accelerator-based measurements [1–5]. For the thermal DM in sub-GeV mass range, in order to explain the relic DM density one has to assume the existence of a new feeble interaction between the ordinary and dark matter. Several hidden sector scenarios have been widely discussed in the literature when such interaction is transmitted through the Higgs [6,7], tensor [8–10], vector [11–16], sterile neutrino [17], and axion or axionlike (ALPs) [18–20] portals. Studies at accelerators have mostly focused on testing of models in which DM couples to the SM through a new gauge vector boson, so-called, dark photon A', that could be massive and kinetically mixes with the ordinary photon [21,22], thus interacting universally with lepton and quarks. Such experiments searched for light sub-GeV DM by looking either for visible or invisible decays of the A', or another similar mediator. The searches for invisible decays typically use missing energy (momentum) techniques developed for leptonic (e or μ) beams [23–25].

Recently, significant attention has been received by alternative light DM models, when the interaction between DM and the SM is primarily leptophobic (or hadrophilic), i.e., the corresponding mediator couples predominantly to quarks, see, e.g., Refs. [26–28]. Taking into account our poor knowledge about the nature of DM, such scenarios are certainly worth studying. This also emphasizes the need for a broad experimental program using both hadron and lepton beams. One possibility to probe the leptophobic dark sector is to use neutrino or beam dump experiments [29–32]. Another possibility is to look for invisible decays of neutral mesons into dark sector particles at e^+e^- colliders [33,34]. Experimental studies of invisible decays of neutral hadrons were performed by several collaborations. In particular, the BES III collaboration [34,35] set constraints on the branching fraction of the invisible decays of η, η', ω, and ϕ mesons. The *BABAR* Collaboration [36,37] has studied the invisible decay modes of heavy quarkonia. The NA62 Collaboration [38] established the limits on invisible decays of π^0. Future experiments on dark sector physics planing to collect large statistics of mesons, such as HIKE [39] at K-meson factory at CERN, REDTOP [40,41], HIAF [42]), and the Forward Physics Facility at CERN [43] have been also proposed.

In this work we discuss the experiment proposed in Ref. [44], to search for leptophobic dark sector in the decays of neutral mesons $M^0 = \pi^0, \eta, \eta', K^0_{L,S},\ldots$ produced in the NA64 experiment at the CERN SPS. The great advantage of using $M^0 \to invisible$ decays is that their invisible decay rate into a couple of neutrinos in the SM is highly suppressed in the SM, $\Gamma(M^0 \to \nu\nu)/\Gamma(M^0 \to total) \ll 10^{-16}$ for masses $m_\nu \simeq 0.8$ eV [45] and $m_{M^0} \simeq m_{K^0}$. We discuss the NA64h sensitivity considering as an example the leptophobic vector portal model with dark fermions and show that the rate of $M^0 \to invisible(semi\text{-}invisible)$ decays could be enhanced up to a measurable level due to the contribution from the dark sector physics. Thus, the new experiment, called NA64h, could complement and significantly expand dark sector physics program of NA64 currently running with leptonic, $e^\pm$ and μ, beams.

The first theoretical analysis of parameter space of DM which can be constrained based on using invisible meson decays was performed for neutral kaons in Refs. [46–50], and for vector meson decays in Ref. [51]. In particular, in Ref. [51] it was estimated the yield of vector mesons with a lepton beam scattered at a heavy nuclei target. In Ref. [52] a novel idea to search for invisible decay of the vector ρ^0 meson produced at the accelerators with hadronic beams have been further developed in context of NA64h by using the signature of a small recoil energy in the ρ^0 production. Below, we use a model-independent estimate for the M^0 yield, which is based on experimental data of diffraction processes $\pi^- + (A,Z) \to M^0 + (A, Z-1)$ in the Regge regime. The data have been collected in a series of experiments [53–60]. We show that the missing energy technique combined with the charge exchange mechanism of the M^0 production by scattered charged pions results in sensitive constraints on invisible or semivisible M^0 decays. The paper is organized as follows. In Sec. II we briefly discuss the missing energy conception to analyze invisible M^0 decay modes in the NA64h experiment. In Sec. III we present a model-independent estimate for the charge exchange cross sections for different Z charges of target nuclei. We also discuss invisible and semi-invisible decays of neutral mesons in Sec. IV. The results from invisible and semi-invisible neutral meson decay mode to DM fermions and implementation to DM parameter space are presented in Sec. V. Finally, in Sec. VI we present our conclusions.

II. FRAMEWORK

The missing energy technique was proposed as a powerful conception to search for light DM in fixed-target experiments, which is highly complementary to DM direct detection and collider searches [23]. For the first time it was implemented in the NA64 for dark sector searches with electron [61], muon [62], and positron [63] beams at the CERN SPS. New experiments, such as LDMX [51,64–67] and M^3 [68,69] were also proposed to search for light DM with electron and muon beams by using the similar approach, see, also BDX [70–72] and ILC [73,74].

A new search for dark sector in invisible decays of neutral mesons in NA64 was proposed in Ref. [44]. The idea is to use the charge-exchange reactions

$$\pi^- + (A,Z) \to M^0 + (A, Z-1); \quad M^0 \to invisible \tag{1}$$

of high-energy pions (or kaons) at a nuclei target (A,Z) as a source of M^0s with their subsequent invisible decay into dark sector particles. In our further estimate, we consider a simple experimental version of the NA64h setup consisting of four consecutive modules of a hadronic calorimeter (HCAL1-4) used to measure the total energy deposited in the detector. Each of the HCAL modules has 48 layers, each of (2.5 mm Fe $+4$ mm scintillator), and $\sim 7.5\lambda$ (nuclear absorption length) in total. The first module HCAL1 serves as an active target irradiated by the primary 40–50 GeV π^- beam with the intensity $\lesssim 10^{-6}$ π per SPS spill of 4.8 s, which produce M^0 in charge exchange reaction on Fe (56,26) nuclei. The signature of the signal event from the $M^0 \to invisible$, would be (i) a good quality single incoming track with the primary beam momentum measured by a magnetic spectrometer; (ii) the track is identified as a minimum ionizing particle (MIP) at the entrance to the HCAL1 with an additional preshower detector, (iii) no activity in the HCAL above the "zero-energy" threshold which is $\simeq 1$ GeV, or $\sim 30\%$ of the energy deposit ~ 3 GeV by the MIP (a muon) in the HCAL1. The main background is expected from the $\mu \to e\nu\nu$, and $K \to \mu\nu$ backward (in the rest frame) decays in flight from μ's and K's present in the beam, whose admixture is, respectively, $\simeq 10^{-3}$ and $\simeq 10^{-2}$ [75]. The low energy electrons from the μ decay could be suppressed by the preshower detector, while the K contamination can be rejected by the Cherenkov counters in the upstream part of the beam line. Contributions from additional sources of background, such as accidental coincidence with low energy particles in the beam, energy leakage, etc.. are expected to be small. Thus, the striking signature of this reaction chain of Eq. (1) would be the complete disappearance of the primary beam energy of, say, 50 GeV, in the setup which could be probed with hadronic beams at NA64 with missing energy technique. As we deal with the diffraction process, we assume the energy deposit at small momentum transfer in t-channel (see, below) is small and does not significantly affect the signal efficiency.

The yield of neutral mesons M^0 produced by the π^- beam scattering at a fixed target is

$$N_{M^0} \simeq \pi\mathrm{OT} \cdot \frac{\rho_T N_A}{A} L_T \sigma_{2\to 2}(\pi^- + (A,Z) \to M^0 + (A,Z-1)), \tag{2}$$

where A is the atomic weight number, N_A is Avogadro's number, πOT is the number of negative charged pions accumulated on target, ρ_T is the target density, L_T is the effective thickness of the target which in conservative scenario is assumed to be equal to effective pion interaction length in the target [76], $\sigma_{2\to 2}(\pi^- + (A,Z) \to M^0 + (A,Z-1))$ is the cross section of charge-exchange reaction.

We estimate limits on the invisible branching ratio of produced neutral mesons, M^0, at 90% confidence level (C.L.) assuming zero observed signal events and background-free case, which implies $\mathrm{Br}(M^0 \to \mathrm{inv}) \leq 2.3/N_{M^0}$ for invisible and for semi-invisible is $\mathrm{Br}(M^0 \to \text{semi-inv}) \leq 2.3/N_{M^0}$, where N_{M^0} is a number of the produced neutral mesons from Eq. (2), and the number of accumulated pions on target $(\pi\mathrm{OT}) \simeq 3 \times 10^9$ [77].

For estimate of the projected sensitivity of the NA64h we will use $\sim 10^{12}$ πOT, which can be accumulated during 3-4 month of data taking, and 5×10^{12} πOT for optimistic scenario [44]. Finally, let us note that the knowledge of precise values for exchange cross sections of Eq. (1) for more accurate estimate of the neutral meson yield entering Eq. (2) would be very useful. Potentially, the corresponding measurements could be performed in the experiment *in situ* by using M^0 visible decay modes.

III. CHARGE-EXCHANGE REACTIONS

Charge-exchange reactions $\pi^- + (A,Z) \to M^0 + (A,Z-1)$ at nuclear target (A,Z) with $M^0 = \pi^0$, η, η', ω, $f_2(1270)$ [53–60] give a unique possibility to shed light on hadron structure, Regge phenomenology [78,79], and color transparency [80]. In particular, the amplitude $\mathcal{M}(s,t)$ of the charge-exchange reaction depending on the s and t Mandelstam variables, can be factorized as [78,79]: $\mathcal{M}(s,t) \sim A(t)(s/s_0)^{\alpha_r(t)}$. Here $A(t)$ is a phenomenological function fitted from data, $s_0 = 10\ \mathrm{GeV}^2$ is the input total energy for the evolution of the cross section, $\alpha_r(t)$ is the Regge trajectory. $\alpha_r(t)$ is normally straight line parametrized as $\alpha_r(t) = \alpha(0) + \alpha' t$. In case of small $|t| \ll s$ one can neglect by the t dependence in the Regge trajectory. Indeed, all data on the charge-exchange reactions are well described with constant Regge trajectory $\alpha_r(0) \simeq \alpha(0) \sim 0.5$. The differential cross section in the case of the proton target is parametrized as [55–57]:

$$\frac{d\sigma_H(s,t)}{dt} = \frac{d\sigma_H(s,t)}{dt}\bigg|_{t=0} [1 - g(s)c(s)t]\ \exp[c(s)t], \tag{3}$$

where

$$\frac{d\sigma_H(s,t)}{dt}\bigg|_{t=0} = A\left(\frac{s}{s_0}\right)^{2\alpha_r(0)-2}, \tag{4}$$

A is the normalization factor, $g(s) = g_0 + g_1 \log(s/s_0)$ and $c(s) = c_0 + c_1 \log(s/s_0)$ are the s-running couplings. For the specific reaction with π^0, η, η', ω, and $f_2(1270)$ production the sets of parameters are shown in Table I (some of them have been fixed in Refs. [55–57]. For proton/neutron target these processes were studied in Ref. [81]. In Fig. 1 we present the results of the fit of the parameters defining the parametrizations for the differential cross sections of the charge-exchange reactions on the proton target: $\pi^- + p \to M^0 + n$ for the cases of the $M^0 = \omega$, $f_2(1270)$, and η' meson production using IHEP data for the $d\sigma_H(s,t)/dt$ at $P = 39.1$ GeV beam [59].

For the integral cross section in case of the proton target we get

TABLE I. Parameters of the differential cross sections.

Meson	A	α_r	c_0 (GeV^{-2})	c_1 (GeV^{-2})	g_0	g_1
π^0	430 ± 20	0.48 ± 0.01	12.7 ± 0.3	1.57 ± 0.12	2.55 ± 0.09	-0.23 ± 0.06
η	36 ± 2	0.37 ± 0.02	6 ± 0.2	1.60 ± 0.10	4.6 ± 0.3	-0.5 ± 0.2
η'	1.37 ± 0.37	0.325 ± 0.01	6.84	1.7	3.7	0
ω	2 ± 0.5	0.53 ± 0.01	6.5	1.23	5.5	0
f_2	60 ± 20	0.53 ± 0.01	8	2.6	4.60	−2

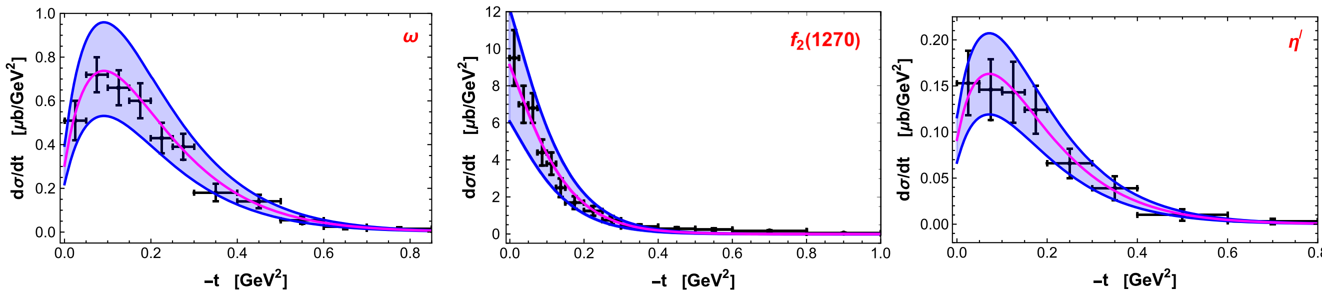

FIG. 1. Fit of the parameters for the ω, $f_2(1270)$, and η' mesons using IHEP data for the differential cross section of their production at $P = 39.1$ GeV beam [59].

$$\sigma_H(s) = A\left(\frac{s}{s_0}\right)^{2\alpha_r(0)-2}\frac{1+g(s)}{c(s)}. \qquad (5)$$

Extension to arbitrary nuclei N with charge Z is normally done by multiplying with factor $Z^{2/3}$. However, we found that this behavior should be slightly corrected as $Z^{2/3-0.15Z^{-2/3}}$. I.e., the integral cross section for the neutral meson production at nuclei with charge Z reads:

$$\sigma_N(s) = \sigma_H(s)Z^{2/3-0.15Z^{-2/3}}. \qquad (6)$$

E.g., our prediction for the ratio of cross sections of production on different nuclei does not depend on the total energy s and produced pseudoscalar meson, while depends on Z: $\sigma_N(s)/\sigma_H(s) = Z^{2/3-0.15Z^{-2/3}}$. E.g., the ratio of the total cross section of productions on carbon and hydrogen is 3.04, which is in good agreement with data: 3.2 ± 0.1 [59]. In Figs. 2–6 for each type of the meson [π^0, η, η', ω, $f_2(1270)$] we present two plots for the integral cross sections of the charge-exchange reactions $\pi^- + (A,Z) \to M^0(\to 2\gamma) + (A,Z-1)$ with $M^0 = \pi^0, \eta, \eta', \pi^- + (A,Z) \to \omega(\to \pi^0\gamma) + (A,Z-1)$, and $\pi^- + (A,Z) \to f_2(1270)(\to 2\pi^0) + (A,Z-1)$ on the different nuclear targets including H, Li, Be, C, Al, Fe, and Cu. In the left panel we present a comparison of the parametrization (5) and (6) with data for beam momentum $P = 40$ GeV or for

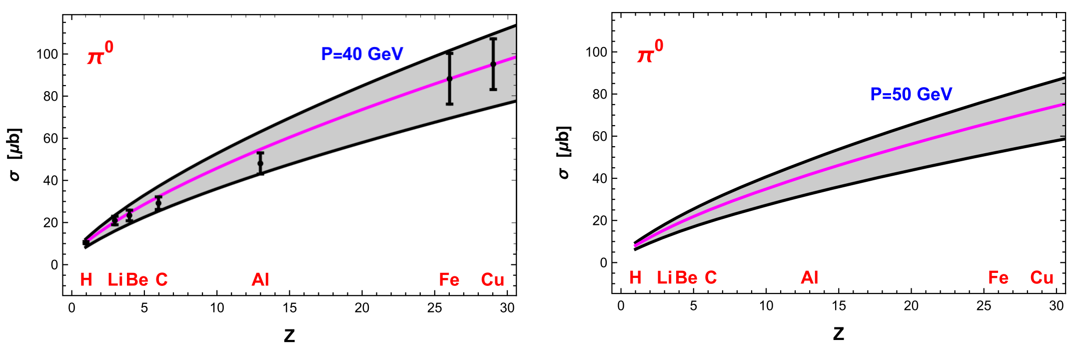

FIG. 2. Results for the integral cross section of the π^0 production $\pi^- + (A,Z) \to \pi^0(\to 2\gamma) + (A,Z-1)$: comparison of parametrization (5) and (6) with data for the beam momentum $P = 40$ GeV [58] (left), predictions for $P = 50$ GeV (right).

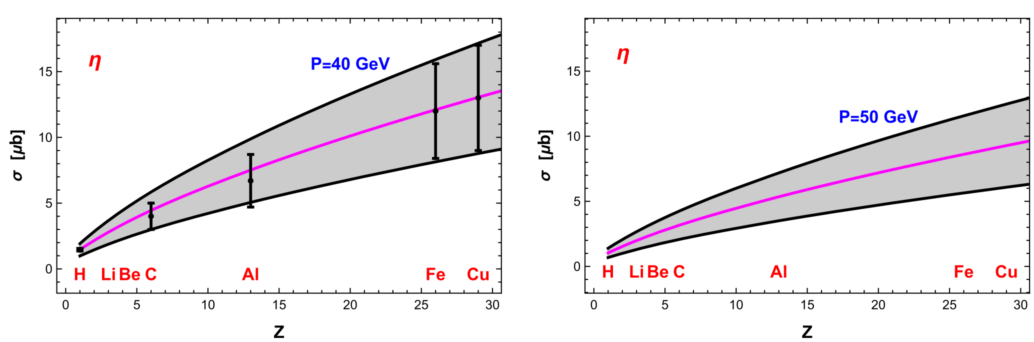

FIG. 3. Results for the integral cross section of the η production $\pi^- + (A,Z) \to \eta(\to 2\gamma) + (A,Z-1)$: comparison of parametrization (5) and (6) with data for the beam momentum $P = 40$ GeV [58] (left), predictions for $P = 50$ GeV (right).

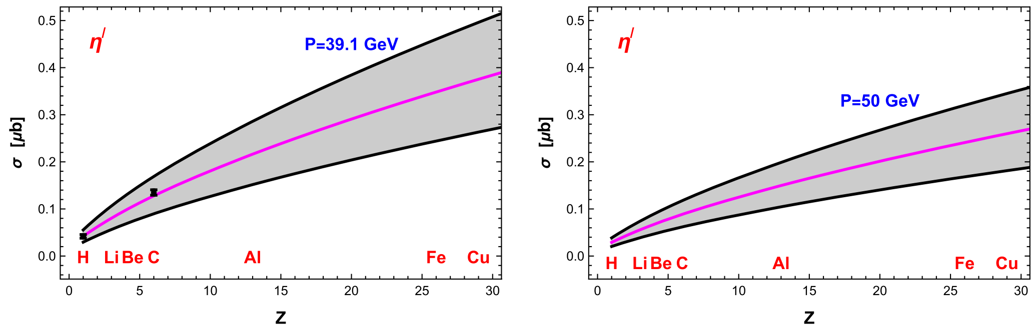

FIG. 4. Results for the integral cross section of the η' production $\pi^- + (A, Z) \to \eta'(\to 2\gamma) + (A, Z-1)$: comparison of parametrization (5) and (6) with data for the beam momentum $P = 39.1$ GeV [59] (left), predictions for $P = 50$ GeV (right).

$P = 39.1$ GeV. In the right panel we present our predictions for the $P = 50$ GeV. For the case of Fe target we made the predictions using the Z dependence of the integral cross section established in Eq. (6).

In Tables II and III we display our predictions for the integral cross sections for the reactions $\pi^- + (A, Z) \to M^0 + (A, Z-1)$, $\pi^- + (A, Z) \to P^0(\to 2\gamma) + (A, Z-1)$, $\pi^- + (A, Z) \to \omega(\to \pi^0\gamma) + (A, Z-1)$, and $\pi^- + (A, Z) \to f_2(1270)(\to 2\pi^0) + (A, Z-1)$ at $P = 50$ GeV. Here $P^0 = \pi^0, \eta, \eta'$. We take the central values of the branchings of the neutral mesons from the Particle Data Group [76]:

$$\begin{aligned} \mathrm{Br}(\pi^0 \to 2\gamma) &= 0.99, \\ \mathrm{Br}(\eta \to 2\gamma) &= 0.3936, \\ \mathrm{Br}(\eta' \to 2\gamma) &= 0.02307, \\ \mathrm{Br}(\omega \to \pi^0\gamma) &= 0.0835, \\ \mathrm{Br}(f_2 \to 2\pi^0) &= 0.281. \end{aligned} \tag{7}$$

One can see good agreement of the parametrization (5) and (6) with data. However, we would like to note that for mesons with masses $m_{M^0} > m_{\pi^0}$ the current precision for their total production cross section is at the level ≃30–35%,

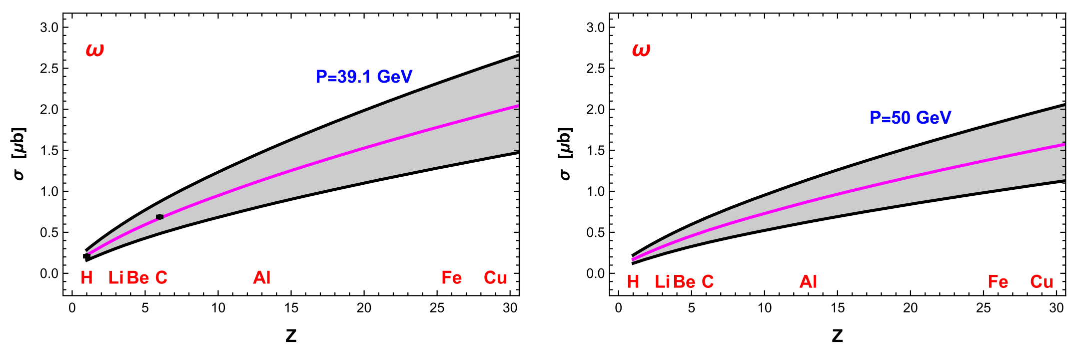

FIG. 5. Results for the integral cross section of the ω production $\pi^- + (A, Z) \to \omega(\to \pi^0\gamma) + (A, Z-1)$: comparison of parametrization (5) and (6) with data for the beam momentum $P = 39.1$ GeV [59] (left), predictions for $P = 50$ GeV (right).

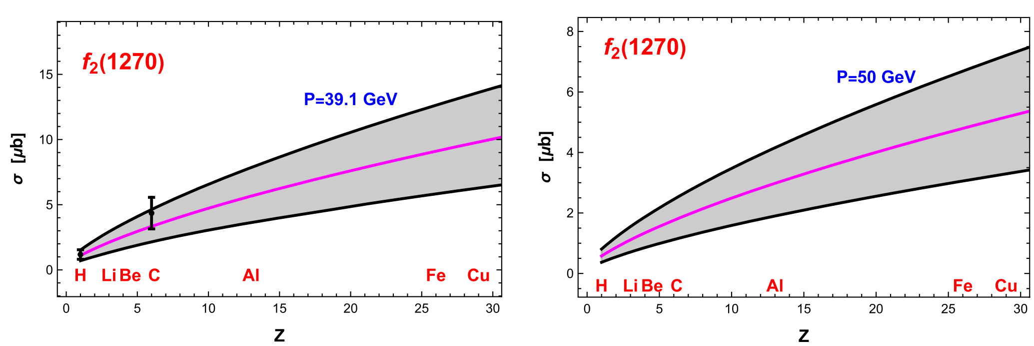

FIG. 6. Results for the integral cross section of the $f_2(1270)$ production $\pi^- + (A, Z) \to f_2(1270)(\to 2\pi^0) + (A, Z-1)$: comparison of parametrization (5) and (6) with data for the beam momentum $P = 39.1$ GeV [59] (left), predictions for $P = 50$ GeV (right).

TABLE II. Predictions for the integral cross sections for the $\pi^- + (A,Z) \to M^0 + (A,Z-1)$ reactions at beam momentum $P = 50$ GeV in μb units.

Meson	H	Li	Be	C	Al	Fe	Cu
π^0	8.1 ± 1.4	15.6 ± 2.6	18.8 ± 3.1	24.7 ± 4.1	41.9 ± 6.8	67.4 ± 11	72.6 ± 11
η	2.6 ± 0.9	5.1 ± 1.7	6.1 ± 2.1	8.0 ± 2.8	13.6 ± 4.7	21.9 ± 7.5	23.6 ± 8.1
η'	1.3 ± 0.4	2.4 ± 0.8	2.9 ± 1.0	3.8 ± 1.3	6.5 ± 2.1	10.4 ± 3.5	11.3 ± 3.7
ω	2.0 ± 0.7	3.9 ± 1.2	4.7 ± 1.5	6.2 ± 1.9	10.5 ± 3.2	16.9 ± 5.1	18.2 ± 5.6
f_2	2.1 ± 0.8	4.0 ± 1.5	4.8 ± 1.9	6.3 ± 2.4	10.6 ± 4.2	17.1 ± 6.7	18.4 ± 7.3

TABLE III. Predictions for the integral cross sections for the $\pi^- + (A,Z) \to P^0(\to 2\gamma) + (A,Z-1)$, $\pi^- + (A,Z) \to \omega(\to \pi^0\gamma) + (A,Z-1)$, and $\pi^- + (A,Z) \to f_2(\to 2\pi^0) + (A,Z-1)$, reactions at beam momentum $P = 50$ GeV in μb units. Here $P^0 = \pi^0, \eta, \eta'$.

Meson	H	Li	Be	C	Al	Fe	Cu
π^0	8.1 ± 1.4	15.6 ± 2.6	18.8 ± 3.1	24.7 ± 4.1	41.9 ± 6.8	67.4 ± 11	72.6 ± 11
η	1.0 ± 0.4	2.0 ± 0.7	2.4 ± 0.8	3.2 ± 1.0	5.4 ± 1.8	8.6 ± 3.0	9.3 ± 3.2
η'	0.03 ± 0.01	0.06 ± 0.01	0.07 ± 0.03	0.09 ± 0.03	0.15 ± 0.05	0.24 ± 0.08	0.26 ± 0.09
ω	0.17 ± 0.05	0.33 ± 0.10	0.39 ± 0.12	0.52 ± 0.15	0.87 ± 0.27	1.41 ± 0.43	1.52 ± 0.46
f_2	0.58 ± 0.23	1.11 ± 0.44	1.34 ± 0.53	1.76 ± 0.69	2.98 ± 1.18	4.80 ± 1.89	5.17 ± 2.04

see Tables II and III. This will constraints the sensitivity of the future searches for $M^0 \to invisible$ decays, and thus, this precision should be improved by at least a factor of $\gtrsim 3$.

In Fig. 1 we present the results of the fit of the parameters defining the parametrizations for the differential cross sections at proton target for the cases of the ω, $f_2(1270)$, and η' meson production using IHEP data for the $d\sigma_H(s,t)/dt$ at $P = 39.1$ GeV beam [59]. In Figs. 2–6 for each type of the meson [π^0, η, η', ω, $f_2(1270)$] we present two plots: comparison of the results for beam momentum $P = 40$ GeV or for $P = 39.1$ GeV obtained using formulas (5) and (6) with data [58,59] (left) and predictions for the $P = 50$ GeV. In case of the Fe target we made the predictions using the formula for the integral cross section derived for arbitrary Z (right). One can see good agreement of parametrization (5) and (6) with data.

IV. INVISIBLE NEUTRAL MESON DECAYS TO DARK FERMIONS THROUGH DARK PHOTON PORTAL

In this section, we will discuss invisible neutral meson decays to dark fermions through a dark photon portal. The dark photon portal can be introduced via kinetic mixing with the SM photons [22]. In particular, the gauge invariant coupling of the dark photon A' and the SM photon A has the form

$$\mathcal{L}_{\rm mix} = \frac{\epsilon}{2} F_{\mu\nu} A'^{\mu\nu} \tag{8}$$

where ϵ is the mixing parameter, $F_{\mu\nu}$ and $A'_{\mu\nu}$ are the stress tensors of the A and A' fields, respectively. We would like to mention that the derivation of the dark photon via the Stueckelberg mechanism was considered in Ref. [15,82].

The interaction of the dark photon with the charged current of SM fermions and with dark fermion current has a form:

$$\mathcal{L} \supset \epsilon e A'_\mu J^\mu + g_D A'_\mu \bar{\chi}\gamma^\mu\chi, \tag{9}$$

where g_D is the coupling of the dark photon with dark fermions, e is the electric charge, and J_μ the electromagnetic current composed of the SM fermions. Note the coupling of the A'_μ with J^μ is obtained after the shift of electromagnetic field $A_\mu \to A_\mu + \epsilon A'_\mu$ leading to the removal of the mixing term (8).

The decay width of the A' to the dark fermions is

$$\Gamma_{A'\to\bar{\chi}\chi} = \frac{\alpha_D}{3} m_{A'}(1 + 2y_\chi^2)(1 - 4y_\chi^2)^{1/2}, \tag{10}$$

where $y_\chi = m_\chi/m_{A'}$, $\alpha_D = g_D^2/4\pi$, $m_{A'}$ and m_χ are masses of dark photon and dark fermions, respectively.

One should note that the shift $A^\mu \to A^\mu + \epsilon A'^\mu$ leads to a possibility of invisible or semivisible decays of neutral mesons whose production cross section was studied before.

A. Vector meson

We derive effective Lagrangian describing transition of neutral vector meson to dark photon

$$\mathcal{L}_{V-A'} = e\epsilon g_V m_V V_\mu A'^\mu, \tag{11}$$

using Lagrangian defining the $V-\gamma$ coupling [15,83] and shift of electromagnetic field $A^\mu \to A^\mu + \epsilon A'^\mu$. Here g_V is the vector meson decay coupling, which for the ω meson is $g_\omega = 0.132$ GeV.

The width of the decay of vector meson into the dark fermion pair $V \to \bar{\chi}\chi$ is given by

$$\Gamma(V \to \bar{\chi}\chi) = \frac{\alpha_D (\epsilon e)^2}{3} g_V^2 \frac{(m_V^2 + 2m_\chi^2)\sqrt{m_V^2 - 4m_\chi^2}}{(m_{A'}^2 - m_V^2)^2 + \Gamma^2_{A' \to \bar{\chi}\chi} m_{A'}^2}, \quad (12)$$

where $m_{A'}$ and m_χ are the masses of intermediate dark photon and DM fermion, respectively, m_V is the mass of vector meson [51]. Here we use the Breit-Wigner propagator for the dark photon A' assuming that its total width is dominated by the $A' \to \bar{\chi}\chi$ mode.

B. Pseudoscalar mesons

Using the Wess-Zumino-Witten (WZW) effective action [84,85] producing the chiral anomaly transition of the π^0 into two photons and couplings of quarks with A and A' we can generate the amplitudes describing decays $\pi^0 \to \gamma\gamma$, $\pi^0 \to \gamma A'$, and $\pi^0 \to A'A'$:

$$A(\pi^0 \to \gamma\gamma) = \frac{\alpha}{\pi F_\pi} \epsilon^{\mu\nu\alpha\beta} \epsilon_\mu k_{1\nu} \epsilon_\alpha k_{2\beta}, \quad (13)$$

$$A(\pi^0 \to \gamma A') = \frac{\alpha\epsilon}{\pi F_\pi} \epsilon^{\mu\nu\alpha\beta} \epsilon_\mu k_{1\nu} \epsilon_\alpha k'_{2\beta}, \quad (14)$$

$$A(\pi^0 \to A'A') = \frac{\alpha\epsilon^2}{\pi F_\pi} \epsilon^{\mu\nu\alpha\beta} \epsilon_\mu k'_{1\nu} \epsilon_\alpha k'_{2\beta} \quad (15)$$

where ϵ_μ and ϵ_α are polarization vectors of photon and dark photon respectively, k_1 and k_2 are the momenta of the final states, $F_\pi = 92.4$ MeV is the pion decay constant, $\alpha = 1/137.036$ is fine structure constant. The decay widths of the π^0 into $\gamma\gamma$ [86], $\gamma A'$ [87], and $A'A'$ are given by

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2}{64\pi^3} \frac{m_\pi^3}{F_\pi^2}, \quad (16)$$

$$\Gamma(\pi^0 \to \gamma A') = \frac{\alpha^2\epsilon^2}{32\pi^3} \frac{m_\pi^3}{F_\pi^2} \left(1 - \frac{m_{A'}^2}{m_\pi^2}\right)^3, \quad (17)$$

$$\Gamma(\pi^0 \to A'A') = \frac{\alpha^2\epsilon^4}{64\pi^3} \frac{m_\pi^3}{F_\pi^2} \left(1 - \frac{4m_{A'}^2}{m_\pi^2}\right)^{3/2}. \quad (18)$$

Here in case of the decays into two identical particles we take into account the combinatorial factor $1/2$. One can see that these decay rates obey the relations:

$$\Gamma(\pi^0 \to \gamma\gamma) : \Gamma(\pi^0 \to \gamma A') : \Gamma(\pi^0 \to A'A') = 1 : 2\epsilon^2 \left(1 - \frac{m_{A'}^2}{m_\pi^2}\right)^3 : \epsilon^4 \left(1 - \frac{4m_{A'}^2}{m_\pi^2}\right)^{3/2}. \quad (19)$$

Due to the fact that ϵ mixing parameter is small, we want to note that semi-invisible decay channel $\pi^0 \to \gamma A'$ provides a better sensitivity for dark photon.

Besides, we need to take into account possible decay of the dark photon A' into dark fermion-antifermion pair $\gamma\chi\bar{\chi}$. It leads to a probability of the three-body decay $\pi^0 \to \gamma\chi\bar{\chi}$ of the neutral pion. The corresponding decay distribution reads

$$d\Gamma(\pi \to \gamma\chi\bar{\chi}) = \frac{\alpha^2\epsilon^2\alpha_D}{192\pi^4 F_\pi^2 m_\pi^3} (m_\pi^2 - q^2)^3 (q^2 + 2m_\chi^2) \times (q^2 - 4m_\chi^2)^{\frac{1}{2}} \frac{1}{(m_{A'}^2 - q^2)^2 + \Gamma^2_{A' \to \bar{\chi}\chi} m_{A'}^2} \frac{dq^2}{\sqrt{q^2}}, \quad (20)$$

where q^2 should be integrated from $4m_\chi^2$ to m_π^2, the decay width $\Gamma_{A' \to \bar{\chi}\chi}$ is defined in Eq. (10).

In previous section, it was shown that in the case of pion charge-exchange scattering at a nuclear target one has a sizable yield contribution of the $f_2(1270)$ mesons having spin-parity 2^+. For this meson the dominant decay mode is the one into two pions [76,88]. Therefore, we will take into account that the $f_2(1270)$ meson gives an additional yield to neutral pions.

For an estimate of the yield of the η and η' mesons we should to take into account their mixing. Here we will follow the scheme of the octet-singlet mixing proposed in Refs. [89,90]. In particular, in this scheme the two-photon decay rates of the η and η' mesons are given by [89,90]

$$\Gamma(\eta \to \gamma A') = \frac{9\alpha^2\epsilon^2}{16\pi^3} m_\eta^3 \left(1 - \frac{m_{A'}^2}{m_\eta^2}\right)^3 \left[\frac{C_8 \cos\theta_0}{f_8 \cos(\theta_8 - \theta_0)} - \frac{(1 - \Lambda_3) C_0 \sin\theta_8}{f_0 \cos(\theta_8 - \theta_0)}\right]^2, \quad (21)$$

$$\Gamma(\eta' \to \gamma A') = \frac{9\alpha^2\epsilon^2}{16\pi^3} m_{\eta'}^3 \left(1 - \frac{m_{A'}^2}{m_{\eta'}^2}\right)^3 \left[\frac{C_8 \sin\theta_0}{f_8 \cos(\theta_8 - \theta_0)} + \frac{(1 - \Lambda_3) C_0 \cos\theta_8}{f_0 \cos(\theta_8 - \theta_0)}\right]^2, \quad (22)$$

where $C_8 = (e_u^2 + e_d^2 - 2e_s^2)/\sqrt{6}$ and $C_0 = (e_u^2 + e_d^2 + e_s^2)/\sqrt{3}$ are the charge factors, $\theta_8 = -(21.2 \pm 1.6)^\circ$ and $\theta_0 = -(9.2 \pm 1.7)^\circ$ are the mixing angles, $f_8 = (1.26 \pm 0.04)\sqrt{2}F_\pi$ and $f_0 = (1.17 \pm 0.03)\sqrt{2}F_\pi$ are the octet and single leptonic decay constants, Λ_3 is the OZI-rule violating parameter $-0.28 < \Lambda_3 < 0.02$. One should stress that for the η' meson it is more interesting to the study decay $\eta' \to \rho^0\gamma$ with next conversion of virtual ρ^0 meson into dark photon. Branching of the process $\eta' \to \rho^0\gamma$ is ~30% of η'. The amplitude and decay width of this process are given respectively by [90,91]

$$A(\eta' \to \rho^0\gamma) = e g_{\eta'\rho\gamma}\epsilon^{\mu\nu\alpha\beta}\epsilon_\mu p_\nu \epsilon_\alpha k_\beta, \tag{23}$$

$$\Gamma(\eta' \to \rho^0\gamma) = \frac{\alpha}{8m_{\eta'}^3} g_{\eta'\rho\gamma}^2 (m_{\eta'}^2 - m_\rho^2)^3, \tag{24}$$

where ϵ_μ and ϵ_α are the polarization vectors of photon and ρ^0 meson respectively, p and k are the momenta of the η' and ρ^0 meson, $g_{\eta'\rho\gamma} = 1.257\ \mathrm{GeV}^{-1}$ is the effective $\eta'\rho\gamma$ coupling fixed from data [76]. This value is in good agreement with the theoretical prediction done in Refs. [89,90].

Using kinetic mixing Lagrangian, we can receive two different decays. The first one is decay of η' meson into ρ^0 meson and dark photon A'

$$\Gamma(\eta' \to \rho^0 A') = \frac{\alpha\epsilon^2 g_{\eta'\rho\gamma}^2}{8m_{\eta'}^3}\lambda^{\frac{3}{2}}(m_{\eta'}^2, m_\rho^2, m_{A'}^2), \tag{25}$$

and with next decay where dark photon transits to dark fermions

$$\Gamma(\eta' \to \rho^0\bar{\chi}\chi) = \frac{\alpha\alpha_D\epsilon^2 g_{\eta'\rho\gamma}^2}{24\pi m_{\eta'}^3}\lambda^{\frac{3}{2}}(m_{\eta'}^2, m_\rho^2, q^2)(q^2 - 4m_\chi^2)^{\frac{1}{2}}[q^2 + 2m_\chi^2]\frac{1}{(m_{A'}^2 - q^2)^2 + \Gamma_{A'\to\bar{\chi}\chi}^2 m_{A'}^2}\frac{dq^2}{\sqrt{q^2}}, \tag{26}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Källen kinematical triangle function. Needed to note that these semi-invisible decays do not have additional α suppression factor which we have in case of decays $\eta(\eta') \to \gamma A'$.

The second decay is process with intermediate conversion of the ρ^0 mesons into dark photon is given by the expression

$$\Gamma(\eta' \to (\rho^0 \to A')\gamma) = \frac{\alpha\epsilon^2}{8m_{\eta'}^3} g_{\eta'\rho\gamma}^2 (m_{\eta'}^2 - m_{A'}^2)^3 \frac{g_\rho^2}{(m_\rho^2 - m_{A'}^2)^2 + \Gamma_\rho^2 m_\rho^2}. \tag{27}$$

The differential decay width of $\eta' \to \gamma\chi\bar{\chi}$ with ρ meson resonance transition is

$$d\Gamma(\eta' \to (\rho^0 \to A' \to \chi\bar{\chi})\gamma) = \frac{\alpha^2\epsilon^2\alpha_D}{6m_{\eta'}^3}(q^2 - m_{\eta'}^2)^3(q^2 + 2m_\chi^2)(q^2 - 4m_\chi^2)^{\frac{1}{2}}\left[\frac{g_\rho^2}{(m_{A'}^2 - q^2)^2 + \Gamma_{A'\to\bar{\chi}\chi}^2 m_{A'}^2}\frac{g_{\eta'\rho\gamma}^2}{(m_\rho^2 - q^2)^2 + \Gamma_\rho^2 m_\rho^2}\right]\frac{dq^2}{\sqrt{q^2}}. \tag{28}$$

Where important to note that η meson also can decay to $\bar{\chi}\chi\gamma$ through intermediate ρ^0 meson which transit to dark fermions by mixing with dark photon [see Lagrangian in Eq. (11)]. The coupling of $g_{\eta\rho\gamma} = 1.52\ \mathrm{GeV}^{-1}$ in framework of T. Feldmann, P. Kroll, and B. Stech (FKS) scheme and $1.42\ \mathrm{GeV}^{-1}$ from experimental data [89,90]. Wherein from analysis of decay $\eta \to \gamma\pi\pi$ is known that contribution due to intermediate ρ^0 vector meson is huge, for decay of $\eta' \to \gamma\pi\pi$ is dominate [92,93]. In case of decay to dark fermions we have more large area of integration for light masses of dark fermions. From these decays, we should obtain a stricter restriction than we have from the process of decay pseudoscalar meson into two photon by the chiral anomaly [84] with one photon mixed with dark photon.

Besides, in analogy with contribution $f_2(1270)$ to neutral pion yield, we can receive additional yield of π^0 and η mesons from decays $\eta \to 3\pi^0$, $\eta' \to \pi^0\pi^0\eta$, and $\eta' \to \pi^+\pi^-\eta$ with very sizable branchings: $\mathrm{Br}(\eta \to 3\pi^0) = 32.57\%$, $\mathrm{Br}(\eta' \to \pi^0\pi^0\eta) = 22.4\%$, and $\mathrm{Br}(\eta' \to \pi^+\pi^-\eta) = 42.5\%$. For full analysis, we take into account both channel's transition to DM for η' meson where transition through ρ^0 meson is the main contribution.

V. BOUNDS

In this section we discuss possible bounds for the dark photon portal, which can be obtained by searching for missing energy/momentum signals in the proposed experiment NA64$_h$ with the negatively charged pion beam. Using cross section for the production of neutral mesons in the charge-exchange process as a result of scattering of the negative pion beam on the Fe target with the beam momentum $P = 50$ GeV we can estimate yields of neutral mesons. The estimated yields of mesons are presented in Table IV. For 90% C.L. of parameter space bounds we will use formulas $\mathrm{Br}(M^0 \to \mathrm{inv}) \le 2.3/N_{M^0}$ for invisible and for semi-invisible is $\mathrm{Br}(M^0 \to \text{semi-inv}) \le 2.3/N_{M^0}$ limits, where N_{M^0} is yields of mesons. The dark portal with dark fermions includes four parameters: ϵ kinematic mixing parameter of photon and dark photon, $m_{A'}$ is the mass of dark photon, $\alpha_D = g_D^2/4\pi$ coupling interaction with dark fermions, and m_χ is the dark fermion mass. These parameters can be combined into dimensionless parameter $y = \alpha_D\epsilon^2(m_\chi/m_{A'})^4$ [64,94], which is convenient to use for the thermal target DM parameter space. In particular, using this

TABLE IV. Yields N_{M^0} of mesons in the reaction of charge exchange at the scattering of negative pion beam onto the iron target at the beam energy of 50 GeV. The meson yields are based on the average value of charge exchange cross sections presented in Table II.

	N_{π^0}	N_η	$N_{\eta'}$	N_ω	N_{f_2}
NA64$_h$ (3×10^9 πOT)	0.35×10^6	0.089×10^6	0.054×10^6	0.088×10^6	0.089×10^6
NA64$_h$ (5×10^{12} πOT)	5.86×10^8	1.48×10^8	0.9×10^8	1.47×10^8	1.48×10^8

dimensionless parameter $y = \alpha_D \epsilon^2 (m_\chi/m_{A'})^4$, we can compare the existing and projected limits from the NA64$_h$ proposal to dark photon portal with dark fermions with the typical relic DM parameter space.

In the previous section it was mentioned that the real yield of neutral pions will be larger. It implies that η, η', and $f_2(1270)$ have dominant hadronic decays in which the neutral pion is is one of the possible final states. For an estimate of bounds on dark photon parameter space from pion decay we will use yield equal $N_{\pi^0} = 1.14\times 10^9$ for statistics of 5×10^{12} πOT. We obtain factor two for the yield of π^0 from Table IV. Full yield of η mesons is changed insignificantly in comparison with data from Table IV and will be equal to $N_\eta = 1.9\times 10^8$ of η mesons for the NA64 experiment with statistics in 5×10^{12} πOT.

The analysis of limits from semi-invisible mode of pseudoscalar meson decays for dark photon parameter space (ϵ and $m_{A'}$) is shown in Fig. 7. We analyzed all possible channels of semi-invisible decay and obtained results that decays with final or intermediate ρ^0 meson give more strict limits to parameter ϵ of kinetic mixing of dark photon and SM photon. Limits from the famous decay which is due to WZW chiral anomaly of pseudoscalar mesons into two photons and take into account that one photon mix with dark photon which also were presented in Ref. [95] are suppressed as $\propto \alpha^2\epsilon^2$. Wherein decays with finite or intermediate ρ^0 meson are $\propto \alpha g^2_{\eta\rho\gamma}\epsilon^2$ or $\propto \alpha^2 g^2_{\eta\rho\gamma}\epsilon^2$. These factors coupled with the Breit-Wigner form of propagator produce less suppression factor for branching ratio for η and η' mesons. We will use notation $\eta \to \gamma\bar{\chi}\chi$ for limit from branching ratio obtained from Eq. (28), for η' meson we will use notation $\eta' \to semi\text{-}inv$ and use limit from branching ratio obtained from Eq. (26) and Eq. (28).

The limits for dark photon parameter space (ϵ and $m_{A'}$) using yields of neutral meson presented in Table IV are shown in Fig. 8. In particular, in the left panel it is shown the bound from semi-invisible pseudoscalar mesons decay to semi-invisible mode and for invisible decay ($\omega \to \chi\bar{\chi}$) through the transition in the dark photon by mixing with the ordinary photon. In derivation of the constraint, we use the benchmark values $m_{A'} = 3m_\chi$ and $\alpha_D = 0.5$. For low statistics 3×10^9 πOT for the NA64$_h$ experiment one can see that all neutral meson decays cannot provide sizable bounds in comparison with existing limits to mixing coupling from NA64$_e$ and *BABAR* experiments. For proposal statistics 5×10^{12} πOT we can test a new area of parameter space dark photon model by study invisible and semi-invisible modes of η, η' and ω mesons. The strong limits from vector meson transition to dark photon is connected with decay width $\omega \to \chi\bar{\chi}$ that is proportional only to $\propto \alpha\alpha_D\epsilon^2$. Besides for η' mesons, the dominant contribution to the decay width $semi\text{-}inv$ is absent for the decay $\eta' \to \gamma\rho^0$ process. The decay $\eta' \to semi\text{-}inv$ with ρ^0 transition is shown by the black dashed line. For η meson limit for parameter mixing ϵ also is strong and can test new area of parameter space (ϵ and $m_{A'}$). With statistics of 5×10^{12} πOT, the NA64$_h$ can probe the parameter space for the kinetic mixing of dark photons which is currently unexplored. In the right panel in Fig. 8 it is seen that limits from projected η/η' factories REDTOP [40] (with projected yields $\sim 3.9\times 10^{14}$ for η and $\sim 7.9\times 10^{11}$ for η') and HIAF (with projected yields $\sim 10^{15}$ for η and $\sim 10^{13}$ for η') [42] can constrain the mixing parameter ϵ at level 10^{-6} and 10^{-7} for the A' mass range up to masses of η/η' mesons. Herewith we need to note that the neutrino floor limit [96] for this process is sufficiently strong and the main signal of missing energy should be associated with the transition to dark sector. In Fig. 8 the regarding bounds are shown by dot-dashed lines. For η meson neutrino floor [96] is very close to the projected sensitivity of future η/η' factories.

Using projected statistics for the neutral meson yield in the NA64, we can predict the typical bound on the semi-invisible on the invisible branching ratio

$$\begin{aligned} \mathrm{Br}(\pi^0 \to \gamma + A') &< 3.16\times 10^{-9} \quad (\text{from NA64}_h); \\ \mathrm{Br}(\eta \to semi\text{-}inv) &< 9.4\times 10^{-9} \quad (\text{from NA64}_h); \\ \mathrm{Br}(\eta' \to semi\text{-}inv) &< 4.7\times 10^{-9} \quad (\text{from NA64}_h); \\ \mathrm{Br}(\omega \to \mathrm{inv}) &< 8.1\times 10^{-9} \quad (\text{from NA64}_h); \end{aligned} \tag{29}$$

in the framework of 90% C.L. of missing energy signature implying zero signal events and background free case. This branching is calculated for case if A' dark photon decay to dark fermions with $\alpha_D = 0.5$ and $m_{A'} = 3m_\chi$. The proposed REDTOP [40] and HIAF [42] factories are expected to work at the level of neutrino floor, $\mathrm{Br}(\eta \to \gamma\nu\bar{\nu}) \simeq 2\times 10^{-15}$, for η'. (see right panel in Fig. 8). The existing limit for this branching ratio of neutral pion from the NA62 experiment is $\lesssim 1.9\times 10^{-7}$. This limit was obtained by NA62 with $N_{\pi^0} = 4.12\times 10^8$ πOT [97].

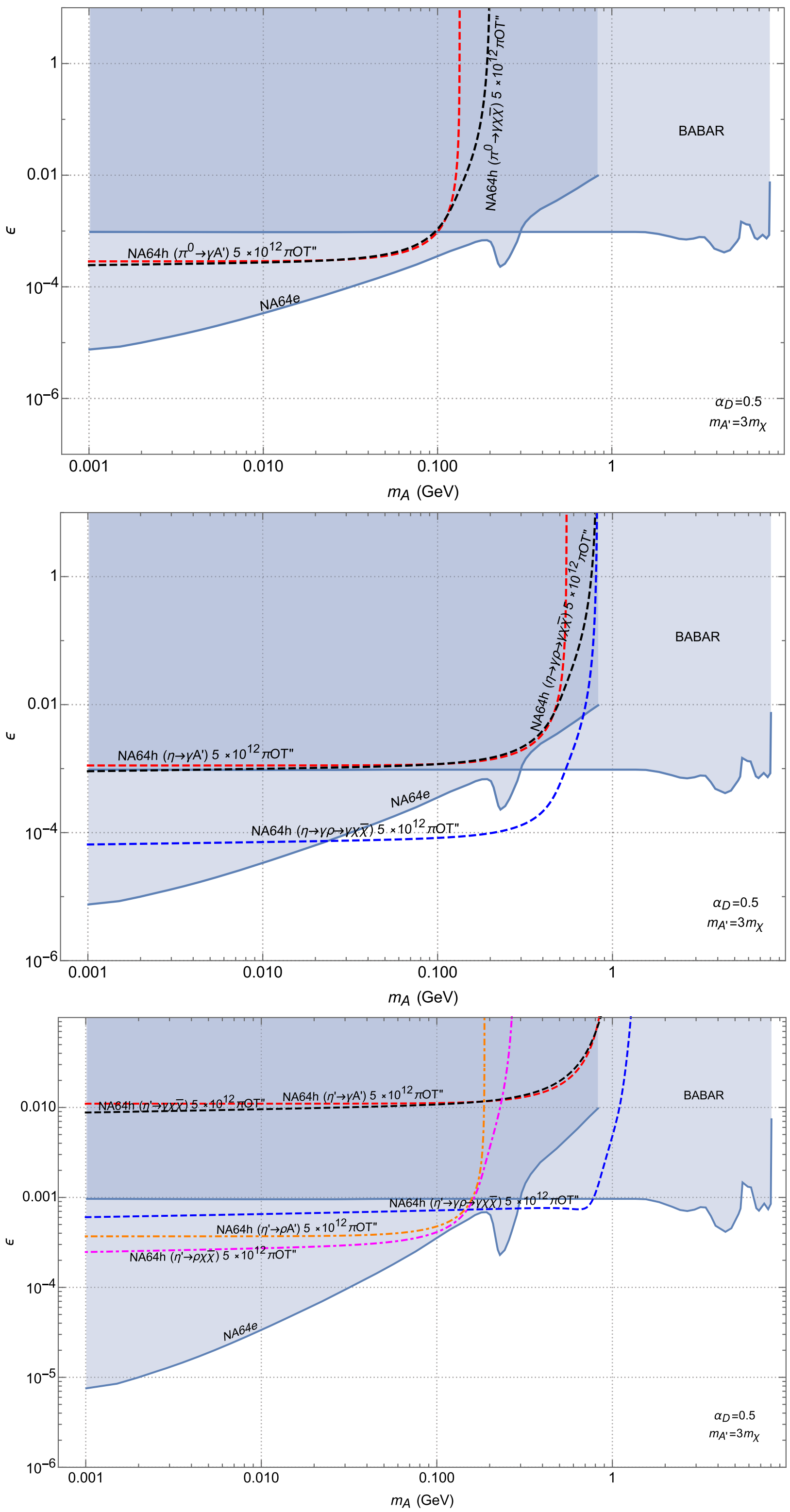

FIG. 7. Bounds for dark photon ϵ parameter mixing obtained for 90% C.L. and for case where $m_{A'} = 3m_\chi$ and $\alpha_D = 0.5$. In these panels, we show existed limit from current data of NA64$_e$ experiment [61] and constraints from production of DM in e^+e^- collision at *BABAR* [37]. In the top panel we plot the bounds from semi-invisible mode of neutral pion decays $\pi^0 \to \gamma A'$ and $\pi^0 \to \gamma\bar{\chi}\chi$. In the central panel we depict the bounds from semi-invisible mode of η meson decays $\eta \to \gamma A'$, $\eta \to \gamma\bar{\chi}\chi$ and $\eta \to \gamma\rho^* \to \gamma\bar{\chi}\chi$. In the bottom panel we show the bounds from semi-invisible mode of η' meson decays $\eta' \to \gamma A'$, $\eta' \to \gamma\bar{\chi}\chi$, $\eta' \to \gamma(\rho^0 \to A') \to \gamma\bar{\chi}\chi$ and $\eta' \to \rho^0 A'$, $\eta' \to \rho^0\bar{\chi}\chi$. All constraints are presented for proposal statistics 5×10^{12} πOT of NA64 experiment.

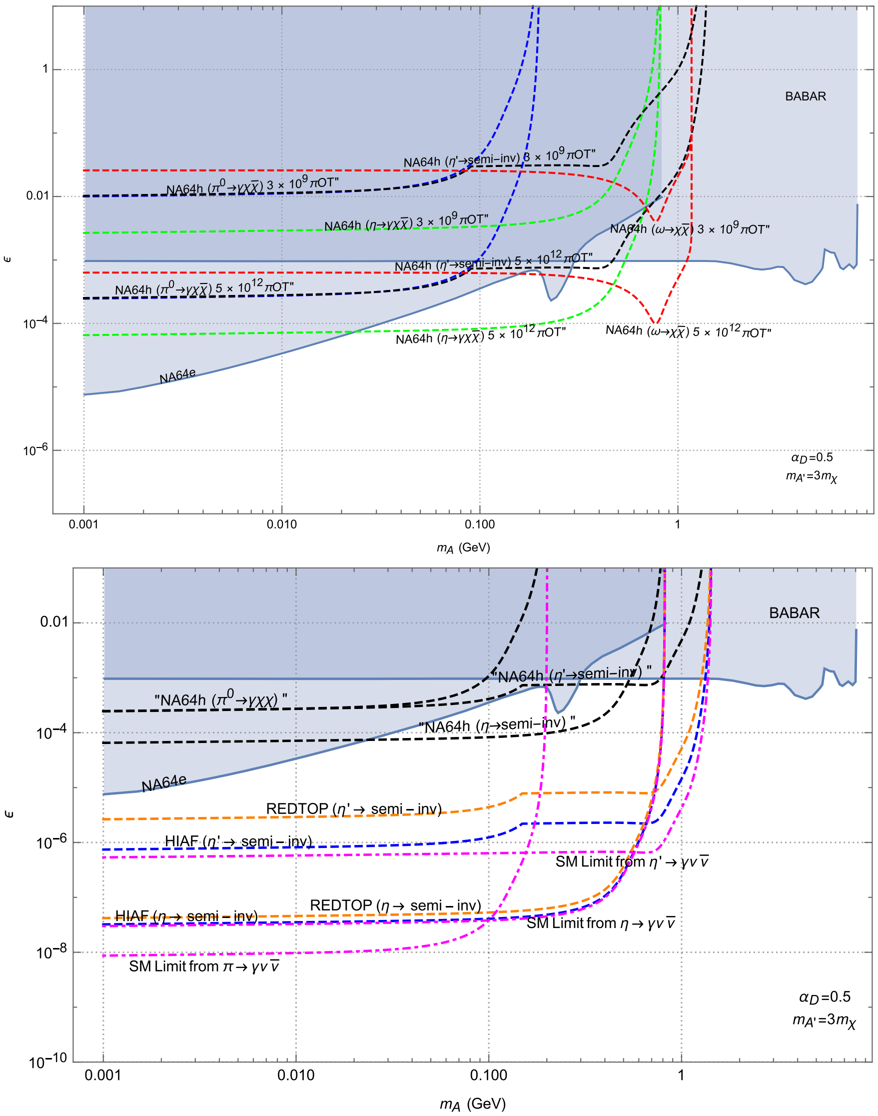

FIG. 8. Bounds for dark photon ϵ parameter mixing obtained for 90% C.L. and for case where $m_{A'} = 3m_\chi$ and $\alpha_D = 0.5$. In both panels, we show existed limit from current data of NA64$_e$ experiment [61] and constraints from production of DM in e^+e^- collision at *BABAR* [37]. Top: bounds from semi-invisible pseudoscalar decays ($\pi^0 \to \gamma\chi\bar{\chi}, \eta \to \gamma\chi\bar{\chi}, \eta' \to semi\text{-}inv$) and invisible decay ($\omega \to \chi\bar{\chi}$) for statistics 3×10^9 πOT (few days of data taking) and for 5×10^{12} πOT as proposal statistics for NA64$_h$ experiment. Bottom: bounds from semi-invisible pseudoscalar decays ($\pi^0 \to \gamma\chi\bar{\chi}, \eta \to \gamma\chi\bar{\chi}, \eta' \to \gamma\chi\bar{\chi}$) for proposal/projected statistics of NA64$_h$, REDTOP [40] and HIAF [42] experiments. The dot-dashed lines show limit of neutrino floor from decay light pseudoscalar mesons to $\gamma\nu\bar{\nu}$ predicted in the framework of SM [96].

In the framework of SM decay of pion into photon and pair of neutrinos is $\mathrm{Br}(\pi^0 \to \gamma\nu\bar{\nu}) \simeq 2 \times 10^{-18}$ [96].

In Fig. 9 we present the constraints on the dimensionless parameter $y = \alpha_D \epsilon^2 (m_\chi / m_{A'})^4$ and the mass of dark fermions m_χ [64]. For the existing and projected limits of NA64$_h$ one can compare this parameter with the typical relic DM parameter space. The yields of pseudoscalar mesons are too small to test unconstrained area for dark photon model if we consider decay from WZW term with taking into account mixing with dark photon. Including decay of η and η' mesons with intermediate or finite state ρ^0 vector meson, we can test and constrain new area for dark photon model. Besides, vector meson invisible decay can give a relatively strong bound to the dark photon model with dark fermions [51,52]. For the case of a negative pion beam we predict a sufficiently large yield of ω vector bosons (see Table. IV). The NA64$_h$ limit from ω in charge exchange reaction for statistics 5×10^{12} πOT will give a limit between the projected bounds from invisible meson decay limits computed for the electron beam experiments NA64$_e$ with 5×10^{12} electron on target (EOT) and projected bounds LDMX with 10^{16} EOT [51], respectively.

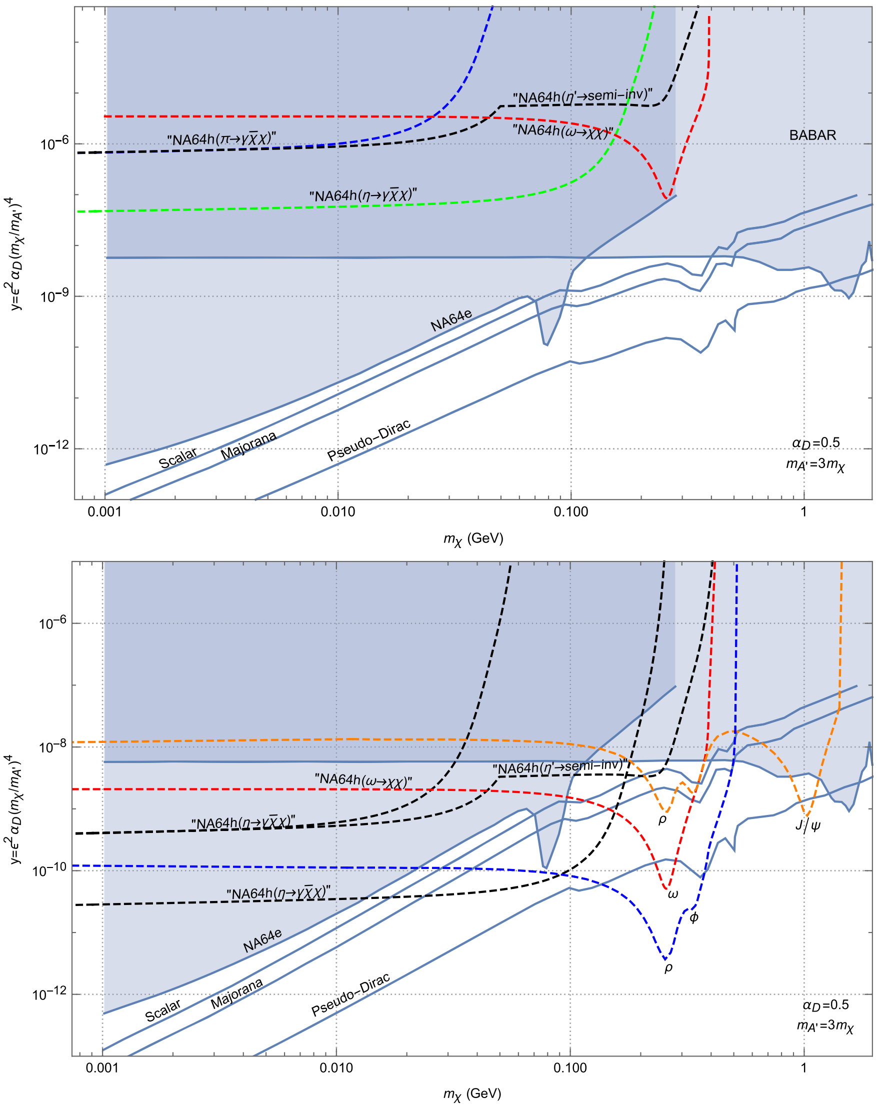

FIG. 9. The projected 90% C.L. exclusion limits for benchmark scenarios of invisible and semi-invisible neutral meson decays to dark matter in dark photon mediator portal model. The constrains are shown for the benchmark values $m_{A'} = 3m_\chi$ and $\alpha_D = 0.5$. In both panels, we show existed limit from last data of NA64$_e$ experiment [61] and constraints from production of DM in e^+e^- collision at *BABAR* [37]. Top: Limits from semi-invisible pseudoscalar decays ($\pi^0 \to \gamma\chi\bar{\chi}, \eta \to \gamma\chi\bar{\chi}, \eta' \to semi\text{-}inv$) and invisible decay ($\omega \to \chi\bar{\chi}$) using statistics 3×10^9 πOT. Bottom: Limits for the same decays for 5×10^{12} πOT of NA64$_h$ experiment in comparison with bound from invisible vector meson decays obtained for NA64$_e$ and LDMX experiments [51].

The process of the ω production in charge exchange reaction and ρ^0 meson production presented in Ref. [52] gives relatively weak bounds but in the case of invisible decays it is associated with the same signature of missing energy. Pseudoscalar η meson will test area near of peak from analysis NA64 experiment with positron [63]. Information from semi-invisible η meson decay and ω are covered possible limits which can be obtained from η' meson study.

In Fig. 10 we show typical parameter space associated with various benchmark values of dark photon parameters. The variations of existing constraints from data of the NA64$_e$ experiment [61] and from production of DM in e^+e^- collision at *BABAR* [37] were done based on analysis of $R = m_{A'}/m_\chi$ dependence presented in Ref. [94]. Changing the typical values of α_D affects the constraints of vector meson only in the area near the mass of vector mesons. Pseudoscalar meson constraints are shifted with the existing constraints from the Bremsstrahlung process at NA64$_e$ experiment. Relatively small values of α_D lead to the suppressed branching ratio for semi-invisible decay of pseudoscalar mesons. Note that the dependence on mass ration $R = m_{A'}/m_\chi$ is crucial for all channels. For $R \gtrsim 5$ the pseudoscalar semi-invisible decay can rule out a new region of the parameter space of dark photon model with dark fermions.

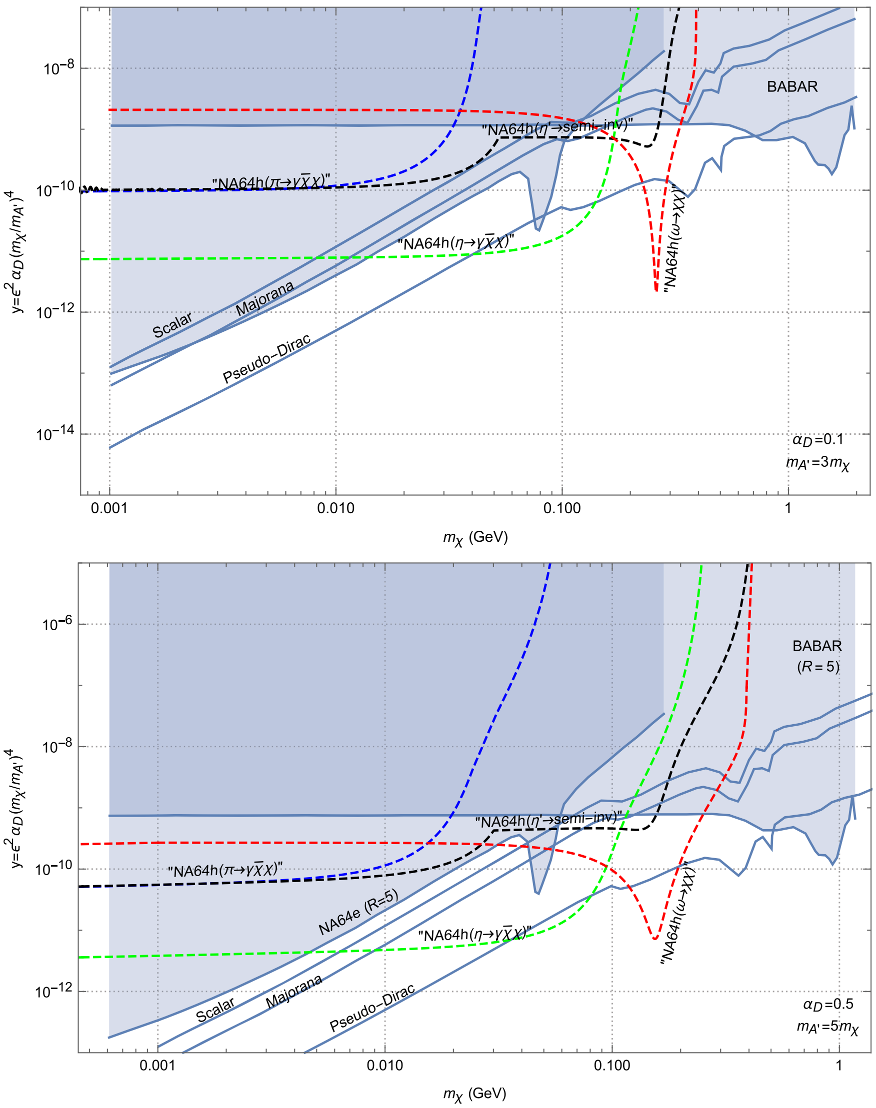

FIG. 10. The projected 90% C.L. exclusion for benchmark scenarios of invisible and semi-invisible neutral meson decays to dark matter in dark photon mediator portal model. In both panels, we show existed limit from last data of $NA64_e$ experiment [61] and constraints from production of DM in e^+e^- collision at *BABAR* [37] and limits from invisible pseudoscalar decays ($\pi^0 \to \gamma\chi\bar{\chi}$, $\eta \to \gamma\chi\bar{\chi}$, $\eta' \to semi\text{-}inv$) and invisible decay ($\omega \to \chi\bar{\chi}$) using statistics 5×10^{12} πOT. Top: for parameter $\alpha_D = 0.1$ and $m_{A'} = 3m_\chi$. Bottom: for parameter $\alpha_D = 0.5$ and $m_{A'} = 5m_\chi$.

VI. CONCLUSION

The yield of neutral mesons associated with the charge exchange process was calculated in the framework of model-independent Regge approach using previous experimental data for charge exchange reactions. For mesons with masses $m_{M^0} > m_{\pi^0}$ the current precision for their total production cross section is at the level ≃30–35%. This will constrain the sensitivity of the future searches for neutral meson decays into the invisible or the semi-invisible modes, and thus, this precision should be improved by at least a factor of $\gtrsim 3$.

Based on averaged yields of neutral mesons in charge exchange reactions, we made an estimate of semi-invisible decays of light pseudoscalar mesons and invisible decay of vector ω meson. The obtained results for the parameter space of the dark photon model with dark fermions show that vector meson production in negative pion beam scattering on a fixed target has an advantage in comparison with the leptonic beam. We also shown that η and η' mesons can be used for probing dark vector portal. Besides, the studies of meson decay with sufficiently large statistics provide an opportunity to test dark vector portal for unconstrained area of the parameter space of the scenario. In addition, the hadronic beam opens a new possibility to test dark matter physics by analyzing rare invisible or semi-invisible decays of mesons by using the missing energy/momentum technique.

Also we would like to note that this calculation is associated with the charge exchange process. However, the realistic number of neutral pions with small recoil energy to the target can be larger in the process of pion scattering on the atomic target. In particular, neutral meson production can be also associated with two-meson production in the double-Regge region [98] or by the Primakoff effect from Bremsstrahlung photons. The full picture requires simulations in GEANT4 [99]. Needed to note that neutral vector meson decay to invisible mode to DM is more convenient for analysis dark photon portal model because in this case all the energy of vector meson will be missed. For semi-invisible decays of neutral pseudoscalar meson we have another kinematic picture.

Besides, we need to note that the experimental study of DM physics by searching for invisible or semi-invisible meson decays with the missing energy technique needs to deepen our knowledge in meson production physics with the hadronic and leptonic beam at high energy. Model-independent approaches that are based on experimental data are required.

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