Study on the mixing of Ξ_c and Ξ_c' by the transition $\Xi_b \to \Xi_c^{(\prime)}$

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Recently, the LHCb Collaboration has observed the decays $\Xi_b^0 \to \Xi_c^+ D_s^-$ and $\Xi_b^- \to \Xi_c^0 D_s^-$. They measured the relative branching fractions times the ratio of beauty-baryon production cross sections $\mathcal{R}(\frac{\Xi_b^0}{\Lambda_b}) \equiv \frac{\sigma(\Xi_b^0)}{\sigma(\Lambda_b^0)} \times \frac{B(\Xi_b^0 \to \Xi_c^+ D_s^-)}{B(\Lambda_b^0 \to \Lambda_c^+ D_s^-)}$ and $\mathcal{R}(\frac{\Xi_b^-}{\Lambda_b}) \equiv \frac{\sigma(\Xi_b^-)}{\sigma(\Lambda_b^0)} \times \frac{B(\Xi_b^- \to \Xi_c^0 D_s^-)}{B(\Lambda_b^0 \to \Lambda_c^+ D_s^-)}$. Once the ratio $\frac{\sigma(\Xi_b^0)}{\sigma(\Lambda_b^0)}$ or $\frac{\sigma(\Xi_b^-)}{\sigma(\Lambda_b^0)}$ is known, one can determine the relative branching fractions which can be used to exam the mixing of Ξ_c and Ξ_c'. In previous literature, Ξ_c and Ξ_c' were assumed to belong to $\mathrm{SU}(3)_F$ antitriple and sextet, respectively. However, recent experimental measurements, such as the ratio $\Gamma(\Xi_{cc} \to \Xi_c \pi^+)/\Gamma(\Xi_{cc} \to \Xi_c' \pi^+)$, indicate the spin-flavor structures of Ξ_c and Ξ_c' are a mixture of $\Xi_c^{\bar{3}}$ and Ξ_c^6. The exact value of mixing angle θ is still under debate. In theoretical models, the mixing angle was fitted to be about $16.27° \pm 2.30°$ or $85.54° \pm 2.30°$ based on decay channels $\Xi_{cc} \to \Xi_c^{(\prime)}$. While in lattice calculation, a small angle ($1.2° \pm 0.1°$) is preferred. To address such discrepancy and test the mixing of Ξ_c and Ξ_c', here we propose the analysis of semileptonic and nonleptonic decays of $\Xi_b \to \Xi_c$ and $\Xi_b \to \Xi_c'$. We calculate the decay rate of $\Xi_b \to \Xi_c$ and $\Xi_b \to \Xi_c'$ based on the light-front quark model and study the effect of the mixing angle on the ratios of weak decays $\Xi_b \to \Xi_c$ and $\Xi_b \to \Xi_c'$. In particular, we find the transition $\Xi_b \to \Xi_c'$ can be an ideal channel to verify the mixing and extract the mixing angle because in theory, the decay rate would be extremely tiny without mixing. Our calculation suggests a measurement of $\Xi_b \to \Xi_c'$ can be feasible in the near future, which will help to test flavor mixing angle θ and elucidate the mechanism of decay of heavy baryons.

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I. INTRODUCTION

Recently, the LHCb Collaboration observed the decays $\Xi_b^0 \to \Xi_c^+ D_s^-$ and $\Xi_b^- \to \Xi_c^0 D_s^-$ [1]. They measured the relative branching fractions times the ratio of the corresponding beauty-baryon production cross sections $\mathcal{R}(\frac{\Xi_b^0}{\Lambda_b}) \equiv \frac{\sigma(\Xi_b^0)}{\sigma(\Lambda_b^0)} \times \frac{B(\Xi_b^0 \to \Xi_c^+ D_s^-)}{B(\Lambda_b^0 \to \Lambda_c^+ D_s^-)} = 15.8 \pm 1.1 \pm 0.6 \pm 7.7$ and $\mathcal{R}(\frac{\Xi_b^-}{\Lambda_b}) \equiv \frac{\sigma(\Xi_b^-)}{\sigma(\Lambda_b^0)} \times \frac{B(\Xi_b^- \to \Xi_c^0 D_s^-)}{B(\Lambda_b^0 \to \Lambda_c^+ D_s^-)} = 16.9 \pm 1.3 \pm 0.9 \pm 4.3$, which suggests experimental measurement of transitions $\Xi_b \to \Xi_c$ and $\Xi_b \to \Xi_c'$ can be promising in the near future. These channels can be used to probe the possible mixing of Ξ_c and Ξ_c' states.

Historically, Ξ_c and Ξ_c' were identified as $\mathrm{SU}(3)_F$ antitriple and sextet of flavor symmetry, respectively. For these kinds of heavy baryons, usually the two light quarks q (denotes u or d) and s are assumed to form a subsystem called diquark [2,3]. In previous literature, a presupposition is the spin-flavor structure of Ξ_c is $[qs]_0 c$ and that of Ξ_c' is $[qs]_1 c$, where the subscript 0 or 1 represents the total spin of the qs subsystem.

However, recent experimental data such as the ratio of the branching fraction $\Xi_{cc} \to \Xi_c' \pi^+$ relative to that of the decay $\Xi_{cc} \to \Xi_c \pi^+$ [4] indicate the spin-flavor structures of Ξ_c and Ξ_c' are not pure $[qs]_0 c$ and $[qs]_1 c$ (they are denoted as $\Xi_c^{\bar{3}}$ and Ξ_c^6, respectively, where the superscripts $\bar{3}$ and 6 correspond to $\mathrm{SU}(3)_F$ antitriple and sextet) but mixtures of them. In this case, we can use a mixing angle θ $(0 < \theta < \pi)$ to describe the composition of $\Xi_c^{\bar{3}}$ and Ξ_c^6 in flavor-spin wave functions of Ξ_c and Ξ_c', i.e., $|\Xi_c\rangle = \cos\theta|\Xi_c^{\bar{3}}\rangle + \sin\theta|\Xi_c^6\rangle$ and $|\Xi_c'\rangle = -\sin\theta|\Xi_c^{\bar{3}}\rangle + \cos\theta|\Xi_c^6\rangle$. When θ is set to 0, i.e., $\Xi_c = \Xi_c^{\bar{3}}$ and $\Xi_c' = \Xi_c^6$, the structures of Ξ_c and Ξ_c' restore the original setting supposed by the authors of Refs. [2,3]. Such mixing is a reflection of the breaking of SU(3) flavor symmetry, which is due to the mass difference between s and u, d quarks.

To determine the mixing angle between Ξ_c and Ξ_c', different approaches has been proposed, while a consensus has not been achieved. One approach is to analyze transition rates of these state based on phenomenological

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models. In Ref. [5], we fixed the mixing angle to be $16.27° \pm 2.30°$ or $85.54° \pm 2.30°$ by fitting the data $\Gamma(\Xi_{cc} \to \Xi_c^{\prime +}\pi^+)/\Gamma(\Xi_{cc} \to \Xi_c \pi^+)$. Geng *et al.* [6,7] obtain the value is 24.66° from the mass spectra. Another approach is lattice simulation. Recent lattice calculation [8] prefers $1.2° \pm 0.1°$, which is much smaller than those from phenomenological analysis.

To better understand the mixing mechanism and obtain a precise measurement of mixing angle of Ξ_c and Ξ_c', more experimental data involving Ξ_c and Ξ_c' are needed. Inspired by recent data from LHCb on $\Xi_b \to \Xi_c D_s$, here we apply light-front quark model [9–29] framework (LFQM) to study the semileptonic and nonleptonic decays of $\Xi_b \to \Xi_c$ and $\Xi_b \to \Xi_c'$, and estimate the feasibility to extract the mixing angle from these channels.

The light-front quark model has been widely used to study light-quark systems. In our earlier work, we extended the light-quark model to study baryon with heavy quark-diquark picture [30–32]. Later the three-quark picture of baryon was adopted [33–35]. In this study, we also apply the three-quark picture to analyze decays of heavy-baryon. In the calculation, we apply the spectator approximation to the light diquark components.

This paper is organized as follows: after the Introduction, in Sec. II, we discuss the form factors for the transition $\Xi_b \to \Xi_c^{(\prime)}$ in the light-front quark model. Our numerical results for $\Xi_b \to \Xi_c^{(\prime)}$ are presented in Sec. III. The Sec. IV is devoted to our conclusion and discussions.

II. THE FORM FACTORS OF $\Xi_b \to \Xi_c$ AND $\Xi_b \to \Xi_c'$ IN LFQM

For decays of heavy baryons, when energy transfer is relatively small, the nonperturbative gluon exchange is the dominant part of strong interaction. Nonperturbative interaction can generate composite states such as diquark, and the quark model with spectator approximation is valid. Instead, if the energy transfer is large and it involves hard gluon exchange, we cannot neglect perturbative QCD interaction between light quarks and heavy quarks, and the spectator approximation fails. In that case, the perturbative QCD approach [36] is more effective for describing decays of heavy baryons.

We can apply invariant velocity transfer ρ to quantify the energy transfer in the decays of heavy baryons [37]. If $\rho < 1.2$, the perturbative QCD approach is not valid [38], and the soft gluon exchange leads to nonperturbative interaction. In our study, for nonleptonic decay of bottom baryons such as $\Xi_b^0 \to \Xi_c^+ D_s^-$, $m(\Xi_b^0) = 5791$ MeV, $m(\Xi_c^+) = 2468$ MeV, $m(D_s^-) = 1968$ MeV, we can find $\rho \approx 1.25$, suggesting that the nonperturbative gluon exchange can be the dominant interaction. Therefore, we use the diquark picture with spectator approximation to describe decays of heavy baryons. Previous theoretical studies also suggested that such approximation works well for decays of heavy bottom baryons [33].

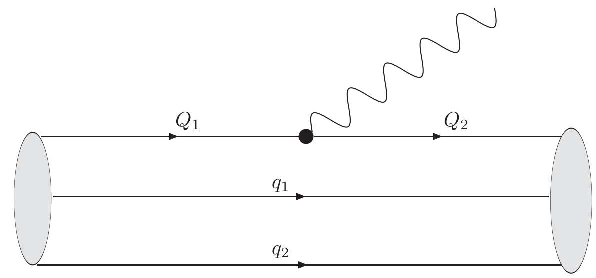

FIG. 1. The Feynman diagrams for $\Xi_b \to \Xi_c^{(\prime)}$ transitions, where Q_1 and Q_2 denote heavy quark in the initial and final states, respectively. q_1 and q_2 represent light quark states. • denotes $V - A$ current vertex.

The leading Feynman diagram responsible for the weak decay $\Xi_b \to \Xi_c^{(\prime)}$ is shown in Fig. 1. During this process, b quark decays to c quark, and diquark sq stands as the spectator. Here, spectator approximation is directly applied to diquark state sq because its spin configuration does not change during the transition at the leading order, which greatly alleviates the theoretical difficulties for calculating the hadronic transition matrix elements.

The form factors for the weak transition $\Xi_b \to \Xi_c$ are defined as

$$\begin{aligned}
&\langle \Xi_c(P', S', S_z') | \overline{Q'}\gamma_\mu(1-\gamma_5)Q | \Xi_b(P, S, S_z)\rangle \\
&= \bar{u}_{\Xi_c}(P', S_z')\left[\gamma_\mu f_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M_{\Xi_b}} f_2(q^2)\right. \\
&\quad \left. + \frac{q_\mu}{M_{\Xi_b}} f_3(q^2)\right] u_{\Xi_b}(P, S_z) \\
&\quad - \bar{u}_{\Xi_c}(P', S_z')\left[\gamma_\mu g_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{M_{\Xi_b}} g_2(q^2)\right. \\
&\quad \left. + \frac{q_\mu}{M_{\Xi_b}} g_3(q^2)\right]\gamma_5 u_{\Xi_b}(P, S_z),
\end{aligned} \tag{1}$$

where P and P' denote four-momentum of Ξ_b and Ξ_c, respectively. $q \equiv P - P'$.

In terms of the spin-flavor structures of Ξ_b and Ξ_c, we assumed $|\Xi_c\rangle$ is a mixture of state $|\Xi_c^{\bar{3}}\rangle$ and $|\Xi_c^{6}\rangle$, with a mixing angle θ [5],

$$|\Xi_c\rangle = \cos\theta|\Xi_c^{\bar{3}}\rangle + \sin\theta|\Xi_c^{6}\rangle, \tag{2}$$

and for $|\Xi_c'\rangle$,

$$|\Xi_c'\rangle = -\sin\theta|\Xi_c^{\bar{3}}\rangle + \cos\theta|\Xi_c^{6}\rangle. \tag{3}$$

With heavy-quark symmetry, we can also express $|\Xi_b\rangle$ as a superposition of flavor eigenstates,

$$|\Xi_b\rangle = \cos\theta|\Xi_b^{\bar{3}}\rangle + \sin\theta|\Xi_b^6\rangle. \tag{4}$$

Here, we assume that mixing angles θ are the same for b-quark and c-quark systems. In the heavy quark limit, the mixing angle $\tan\theta_Q \propto \frac{1}{m^2\theta_Q}$ ($Q = b$, c quark) [39]. Since the heavy quark symmetry is violated due to the charm-bottom mass difference, we might expect a large difference in the mixing angle θ_Q between the b and c systems. However, calculations based on QCD sum rules [39] and heavy quark effective theory [40] show that the suppression of the mixing angle in the heavy quark limit is not realized up to the physical b quark mass, and mixing angles in the b and c systems are relatively close to each other. This implies that in the language of heavy quark effective theory, in the leading order, $\theta_Q \propto O((\frac{1}{m\theta_Q})^0)$; hence, the dependence of mixing angle on heavy quark mass is not strong. Inspired by these theoretical studies, we use a simplified assumption that $\theta_b \approx \theta_c = \theta$.

Using eigenstates of $SU(3)_F$, $|\Xi_b^6\rangle$, $|\Xi_b^{\bar{3}}\rangle$, $|\Xi_c^{\bar{3}}\rangle$, $|\Xi_c^6\rangle$, the matrix element $\langle\Xi_c(P', S', S_z')|\bar{c}\gamma_\mu(1-\gamma_5)b|\Xi_b(P, S, S_z)\rangle$ can be written as

$$\cos^2\theta\langle\Xi_c^{\bar{3}}|\bar{c}\gamma_\mu(1-\gamma_5)b|\Xi_b^{\bar{3}}\rangle + \sin^2\theta\langle\Xi_c^6|\bar{c}\gamma_\mu(1-\gamma_5)b|\Xi_b^6\rangle. \tag{5}$$

where we neglect the transition matrix elements for $\Xi_b^{\bar{3}} \to \Xi_c^6$ and $\Xi_b^6 \to \Xi_c^{\bar{3}}$ because they are forbidden in the leading-order approximation of light-front quark model. In certain phenomenological approaches, the transitions $\Xi_b^{\bar{3}} \to \Xi_c^6$ and $\Xi_b^6 \to \Xi_c^{\bar{3}}$ maybe exist in higher order of α_s but they should be very suppressed like the decays of $\Lambda_b \to \Sigma_c$.

For the transition matrix elements $\langle\Xi_c^{\bar{3}}|\bar{s}\gamma_\mu(1-\gamma_5)c|\Xi_b^{\bar{3}}\rangle$, the corresponding form factors are denoted as f_i^s, g_i^s (See Eq. (1) for the definition of form factors). Similarly, for $\langle\Xi_c^6|\bar{s}\gamma_\mu(1-\gamma_5)c|\Xi_b^6\rangle$, the form factors are denoted as f_i^v, g_i^v. Combining Eqs. (1) and (5), we have

$$\begin{aligned} f_i &= \cos^2\theta f_i^s + \sin^2\theta f_i^v, \\ g_i &= \cos^2\theta g_i^s + \sin^2\theta g_i^v. \end{aligned} \tag{6}$$

Following the procedures given in Refs. [28–31], the transition matrix element can be computed using light-front approach with specific forms of vertex functions of $\Xi_b^{\bar{3}}$, $\Xi_c^{\bar{3}}$, Ξ_b^6, and Ξ_c^6. The calculations for $\Xi_b^{\bar{3}} \to \Xi_c^{\bar{3}}$ and $\Xi_b^6 \to \Xi_c^6$ are same as those for $\Lambda_b \to \Lambda_c$ and $\Sigma_b \to \Sigma_c$, respectively. The detailed derivation and the expressions f_i^s, g_i^s, f_i^v, g_i^v can be found in our earlier paper [33].

For the transition matrix element $\langle\Xi_c'(P', S', S_z')|\bar{c}\gamma_\mu \times (1-\gamma_5)b|\Xi_b(P, S, S_z)\rangle$, by using $|\Xi_b^6\rangle$, $|\Xi_b^{\bar{3}}\rangle$, $|\Xi_c^{\bar{3}}\rangle$, $|\Xi_c^6\rangle$, it can be written as

$$\begin{aligned} &-\cos\theta\sin\theta\langle\Xi_c^{\bar{3}}|\bar{c}\gamma_\mu(1-\gamma_5)b|\Xi_b^{\bar{3}}\rangle \\ &+\cos\theta\sin\theta\langle\Xi_c^6|\bar{c}\gamma_\mu(1-\gamma_5)b|\Xi_b^6\rangle. \end{aligned} \tag{7}$$

The form factors of transition matrix element are also defined as in Eq. (1). Here, we just add a symbol "′" on f_1, f_2, g_1, and g_2 to distinguish the quantities for $\Xi_b \to \Xi_c'$ and those for $\Xi_b \to \Xi_c$. They are given by

$$\begin{aligned} f_i' &= \cos\theta\sin\theta(f_i^v - f_i^s), \\ g_i' &= \cos\theta\sin\theta(g_i^v - g_i^s). \end{aligned} \tag{8}$$

III. NUMERICAL RESULTS

A. The results for $\Lambda_b \to \Lambda_c$

In Ref. [33], we extended three-quark picture to study the decay of $\Lambda_b \to \Lambda_c$ and $\Sigma_b \to \Sigma_c$. In that paper, we employ a pole form in Eq. (9) to fix the form factor of the transition,

$$F(q^2) = \frac{F(0)}{\left(1-\frac{q^2}{M_{\mathcal{B}_i}^2}\right)\left[1 - a\left(\frac{q^2}{M_{\mathcal{B}_i}^2}\right) + b\left(\frac{q^2}{M_{\mathcal{B}_i}^2}\right)^2\right]}, \tag{9}$$

where $M_{\mathcal{B}_i}$ is the mass of the initial baryon.

Later, we find it does not work for the decay of double charmed baryon so in Refs. [34,35], we employed a polynomial to parametrize these form factors f_i^s, g_i^s, f_i^v and g_i^v ($i = 1, 2$),

$$F(q^2) = F(0)\left[1 + a\left(\frac{q^2}{M_{\mathcal{B}_i}^2}\right) + b\left(\frac{q^2}{M_{\mathcal{B}_i}^2}\right)^2\right]. \tag{10}$$

In this paper, we first check whether the above polynomial formula also can be used for the transition $\Lambda_b \to \Lambda_c$. With the same parameters (quark masses and β parameters in wave functions) in Ref. [33], we refit the parameters in Eq. (10). The fitted values of a, b and $F(0)$ in the form factors are presented in Table I. We also depict two results in Fig. 2. The left graph (a) is the result from Ref. [33], and the right one (b) is the result this time. Comparing these plots, one can find they are nearly equal to each other. With these form factors, we recalculate the decay rates Γ, the integrated longitudinal and transverse asymmetries a_l and a_t and their ratio $R = a_l/a_t$ of semileptonic for the

TABLE I. The form factors given in polynomial form for the transition $\Lambda_b \to \Lambda_c$.

F	$F(0)$	a	b
f_1^s	0.488	1.94	1.70
f_2^s	−0.181	2.44	2.52
g_1^s	0.470	1.86	1.58
g_2^s	−0.048	2.75	3.07

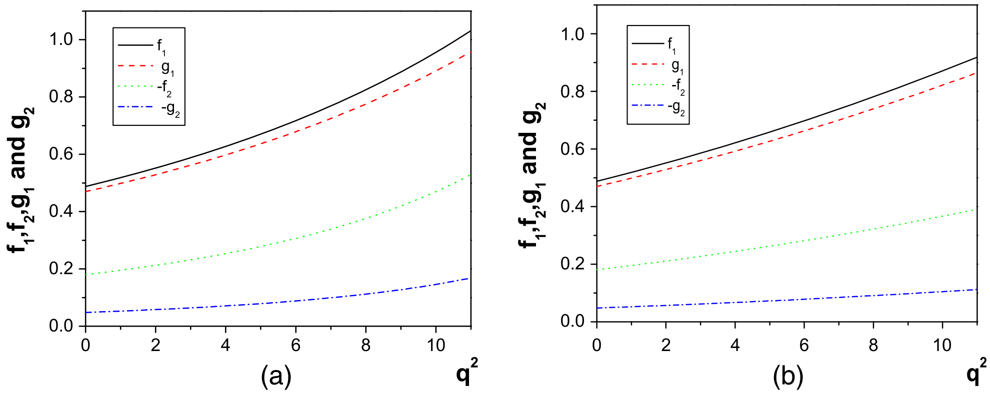

FIG. 2. Form factors for the decay $\Lambda_c \to \Lambda_c$ (a) pole form and (b) polynomial form.

transition $\Lambda_b \to \Lambda_c$, and the results are presented in Table II. We also collect the results in Refs. [30,33] for comparison. From the values in Table II, we can conclude that two extrapolative forms of the form factors do not significantly affect the results of transitions of $\Lambda_b \to \Lambda_c$. We also reevaluate the width and up-down asymmetry α of several nonleptonic decay channels. The results of the nonleptonic decays in Table III are also very consistent. For the detailed expressions used in the calculation of these physical quantities, one can refer to Refs. [41–44] and the Appendix of the Ref. [33].

In summary, we find a polynomial parametrization of form factors (Eq. (10)) can be used to describe decays of general types of heavy baryons, not only double charmed baryons, but also single bottom and charmed baryons. In Tables II and III, we compare new results based on the polynomial parameterization with previous results [30,33] based on a pole form (Eq. (9)). We find that the difference of results due to different parametrization of form factors is reasonably small, up to about 10%. Therefore, in the following analysis, we will use the polynomial parametrization.

TABLE II. The widths (in unit of 10^{10} s^{-1}) and polarization asymmetries of $\Lambda_b \to \Lambda_c l \bar{\nu}_l$.

	Γ	a_L	a_T	R	P_L
The results in [33]	4.22	−0.962	−0.766	1.54	−0.885
The results in [30]	5.15	−0.932	−0.601	1.47	−0.798
This work	4.36	−0.933	−0.668	1.49	−0.826

TABLE III. The widths (in unit of 10^{10} s^{-1}) and up-down asymmetries of nonleptonic decays $\Lambda_b \to \Lambda_c M$.

	The results in Ref. [33]		The results in [30]		This work	
	Γ	α	Γ	α	Γ	α
$\Lambda_b^0 \to \Lambda_c^+ \pi^-$	0.261	−0.999	0.307	−1	0.261	−0.999
$\Lambda_b^0 \to \Lambda_c^+ \rho^-$	0.769	−0.875	0.848	−0.883	0.774	−0.875
$\Lambda_b^0 \to \Lambda_c^+ K^-$	0.0209	−0.999	0.0247	−1	0.0210	−0.999
$\Lambda_b^0 \to \Lambda_c^+ K^{*-}$	0.0398	−0.836	0.0440	−0.846	0.0402	−0.837
$\Lambda_b^0 \to \Lambda_c^+ a_1^-$	0.758	−0.710	0.838	−0.726	0.770	−0.710
$\Lambda_b^0 \to \Lambda_c^+ D_s^-$	0.927	−0.974	0.932	−0.982	0.928	−0.987
$\Lambda_b^0 \to \Lambda_c^+ D_s^{*-}$	1.403	−0.327	1.566	−0.360	1.446	−0.330
$\Lambda_b^0 \to \Lambda_c^+ D^-$	0.0355	−0.979	0.0410	−0.986	0.0357	−0.989
$\Lambda_b^0 \to \Lambda_c^+ D^{*-}$	0.0630	−0.371	0.0702	−0.403	0.0649	−0.374

B. The results for $\Xi_b \to \Xi_c$ and $\Xi_b \to \Xi_c'$

After checking the effectiveness of the polynomial parametrization, we calculate the form factors for $\Xi_b^{\bar{3}} \to \Xi_c^{\bar{3}}$ and $\Xi_b^6 \to \Xi_c^6$ in a spacelike region and extrapolate them to a timelike region by the Eq. (10). The parameters in the expressions of the form factors are same as those for $\Lambda_b \to \Lambda_c$ in Ref. [33] except $\beta_{23} \approx 2.9\beta_{us} = 0.994$ GeV. We list the numerical values of parameters in polynomial parametrization of form factors in Table IV. We also depict them in Fig. 3. One can find the form factors for the transition $\Xi_b^{\bar{3}} \to \Xi_c^{\bar{3}}$ are close to those for $\Lambda_b \to \Lambda_c$.

Presetting different mixing angles, we evaluate the rates and some asymmetries parameters for the semilepotonic decay and nonleptonic decays of $\Xi_b \to \Xi_c$. In Tables V and VI, we explicitly list the theoretical predictions with different mixing angles. The dependence of differential decay widths $d\Gamma/d\omega$ ($\omega = \frac{P \cdot P'}{mm'}$) on ω are depicted in Fig. 4(a). In order to minimize the model dependence of results, we also calculate the ratio of these values to those for $\Lambda_b \to \Lambda_c l \bar{\nu}_l$, which are presented in the parentheses in Table V.[1] One may notice when $\theta = 0°$ and $\theta = 16.27°$, the polarization asymmetries of $\Xi_b \to \Xi_c l \bar{\nu}_l$ are very close to each other, and the decay widths have a slight difference. For a large mixing angle such as $\theta = 85.54°$, the mixing angle apparently changes the decay rate and polarization asymmetries relative to the case at $\theta = 0°$ in the decay $\Xi_b \to \Xi_c l \bar{\nu}_l$. In terms of $\mathcal{R}(\frac{\Xi_b^0}{\Lambda_b})$, we find the ratio $\frac{\sigma(\Xi_b^0)}{\sigma(\Lambda_b^0)} \approx 18.4$ when θ takes 16.27°.

[1]In Tables VI–VIII, the value in the parentheses is the ratio with respect to the corresponding value for the process $\Lambda_b \to \Lambda_c$.

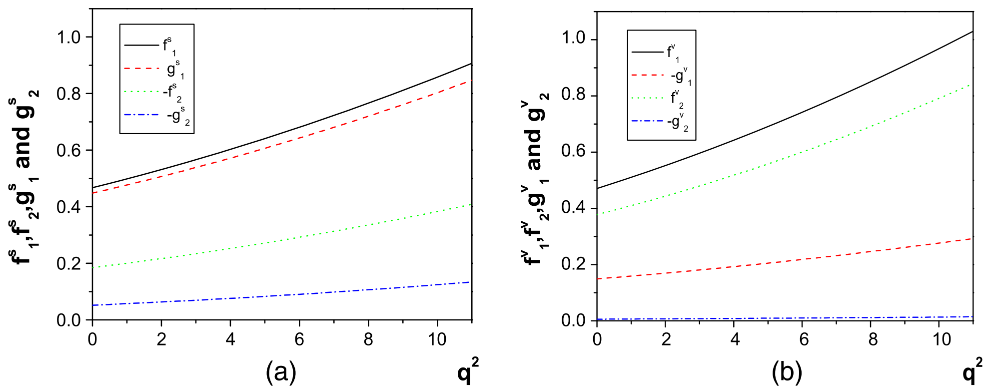

FIG. 3. Form factors for the decay $\Xi_b^{\bar{3}} \to \Xi_c^{\bar{3}}$ (a) and $\Xi_b^6 \to \Xi_c^6$ (b).

TABLE IV. The form factors given in polynomial form for the transitions $\Xi_b^{\bar{3}} \to \Xi_c^{\bar{3}}$ and $\Xi_b^6 \to \Xi_c^6$.

F	$F(0)$	a	b
f_1^s	0.467	2.19	2.09
f_2^s	−0.185	2.70	3.01
g_1^s	0.448	2.09	1.92
g_2^s	−0.052	3.32	4.54
f_1^v	0.471	2.57	2.40
f_2^v	0.378	2.56	2.81
g_1^v	−0.149	2.07	2.00
g_2^v	−0.006	2.95	2.98

For the transition $\Xi_b \to \Xi_c'$, when $\theta = 0°$, the theoretical results are zero in our calculation since the spin of spectator does not change. Certainly, there may exist a very small decay rate for the transition of spin flip in some other approaches, but the situation should be similar to the case of the transition $\Lambda_b \to \Sigma_c$, which is very small so that it is not listed in Particle Data Group databook [45].

If the mixing angle $\theta = 16.27°$, the semileptonic decay of $\Xi_b \to \Xi_c'$ will be one order smaller than that of $\Xi_b \to \Xi_c$, and the nonleptonic decay of $\Xi_b \to \Xi_c'$ will be ten to twenty times smaller than that of $\Xi_b \to \Xi_c$. The dependence of the differential decay widths $d\Gamma/d\omega$ ($\omega = \frac{P \cdot P'}{mm'}$) on ω are depicted in Fig. 4(b). Comparing Figs. 4(a) and 4(b),

TABLE V. The widths (in unit of 10^{10} s^{-1}) and polarization asymmetries of $\Xi_b \to \Xi_c l \bar{\nu}_l$. For each value, we also list in parentheses its ratio with respect to the corresponding value in the process $\Lambda_b \to \Lambda_c l \bar{\nu}_l$.

	Γ	a_L	a_T	R	P_L
$\theta = 0°$	3.68 (0.844)	−0.962 (1.03)	−0.745 (1.12)	1.63 (1.09)	−0.879 (1.06)
$\theta = 16.27°$	3.13 (0.718)	−0.973 (1.04)	−0.756 (1.13)	1.78 (1.19)	−0.895 (1.08)
$\theta = 85.54°$	1.15 (0.264)	0.685 (−0.734)	−0.237 (0.355)	6.69 (4.49)	0.565 (−0.684)

TABLE VI. The widths (in unit 10^{10} s^{-1}) and up-down asymmetries of nonleptonic decays $\Xi_b \to \Xi_c M$. For each value, we also list in parentheses its ratio with respect to the corresponding value in the process $\Lambda_b \to \Lambda_c M$.

	$\theta = 0°$		$\theta = 16.27°$		$\theta = 85.54°$	
	Γ	α	Γ	α	Γ	α
$\Xi_b^0 \to \Xi_c \pi^-$	0.247 (0.946)	−0.999 (1.000)	0.224 (0.858)	−0.988 (0.989)	0.143 (0.548)	0.557 (−0.558)
$\Xi_b^0 \to \Xi_c \rho^-$	0.725 (0.937)	−0.877 (1.002)	0.651 (0.841)	−0.880 (1.006)	0.388 (0.501)	0.567 (−0.648)
$\Xi_b^0 \to \Xi_c K^-$	0.0198 (0.943)	−0.999 (1.000)	0.0179 (0.852)	−0.987 (0.988)	0.0116 (0.552)	0.551 (−0.552)
$\Xi_b^0 \to \Xi_c K^{*-}$	0.0375 (0.933)	−0.839 (1.002)	0.0336 (0.836)	−0.846 (1.011)	0.0195 (0.485)	0.571 (−0.682)
$\Xi_b^0 \to \Xi_c a_1^-$	0.712 (0.925)	−0.715 (1.007)	0.641 (0.832)	−0.733 (1.032)	0.338 (0.439)	0.583 (−0.821)
$\Xi_b^0 \to \Xi_c D_s^-$	0.857 (0.923)	−0.972 (0.986)	0.796 (0.858)	−0.941 (0.954)	0.612 (0.659)	0.455 (−0.461)
$\Xi_b^0 \to \Xi_c D_s^{*-}$	1.271 (0.879)	−0.338 (1.024)	1.090 (0.754)	−0.377 (1.142)	0.418 (0.289)	0.643 (1.948)
$\Xi_b^0 \to \Xi_c D^-$	0.0330 (0.924)	−0.977 (0.988)	0.0305 (0.854)	−0.948 (0.959)	0.0230 (0.644)	0.466 (−0.471)
$\Xi_b^0 \to \Xi_c D^{*-}$	0.0575 (0.886)	−0.382 (1.021)	0.0495 (0.763)	−0.420 (1.122)	0.0199 (0.183)	0.636 (1.701)

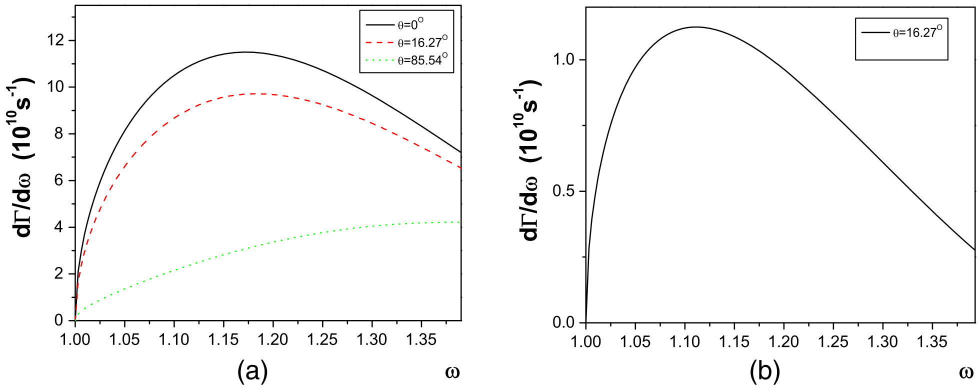

FIG. 4. Differential decay rates $d\Gamma/d\omega$ for the decay $\Xi_b \to \Xi_c l\bar{\nu}_l$ (a) and $\Xi_b \to \Xi'_c l\bar{\nu}_l$ (b).

one also find the ratio of the differential decay widths at every ω is also about 10. In Ref. [1], the signal yields for Ξ_b^0 and Ξ_b^+ decay are 462 ± 29 and 175 ± 14, respectively. It seems that we have opportunity to observe the decays, such as $\Xi_b \to \Xi'_c l\nu_l$, $\Xi_b \to \Xi'_c D_s$, $\Xi_b \to \Xi'_c D_s^*$ and (or) $\Xi_b \to \Xi'_c \rho$, in the near future. These processes are very optimal to determine the mixing angle θ since $\Xi_b \to \Xi'_c$ is very small when $\theta = 0°$. Once an observation $\Xi_b \to \Xi'_c$ is confirmed, we will have strong evidence to believe that a modest value of mixing angle θ.

TABLE VII. The widths (in unit of $10^{10}\ \mathrm{s}^{-1}$) and polarization asymmetries of $\Xi_b \to \Xi'_c l\bar{\nu}_l$. For each value, we also list in parentheses its ratio with respect to the corresponding value in the process $\Lambda_b \to \Lambda'_c l\bar{\nu}_l$.

	Γ	a_L	a_T	R	P_L
$\theta = 16.27°$	0.300 (0.0668)	−0.0729 (0.0781)	−0.622 (0.931)	0.997 (0.669)	−0.348 (0.421)

TABLE VIII. The widths (in unit $10^{10}\ \mathrm{s}^{-1}$) and up-down asymmetries of nonleptonic decays $\Xi_b \to \Xi'_c M$ with $\theta = 16.27°$. For each value, we also list in parentheses its ratio with respect to the corresponding value in the process $\Lambda_b \to \Lambda'_c M$ with $\theta = 16.27°$.

	Γ	α
$\Xi_b^0 \to \Xi'_c \pi^-$	0.0142 (0.0544)	0.0138 (−0.0138)
$\Xi_b^0 \to \Xi'_c \rho^-$	0.0450 (0.0581)	0.100 (−0.114)
$\Xi_b^0 \to \Xi'_c K^-$	0.00112 (0.0533)	0.0180 (−0.0180)
$\Xi_b^0 \to \Xi'_c K^{*-}$	0.00238 (0.0592)	0.124 (−0.148)
$\Xi_b^0 \to \Xi'_c a_1^-$	0.0487 (0.0632)	0.194 (−0.273)
$\Xi_b^0 \to \Xi'_c D_s^-$	0.0373 (0.0402)	0.0902 (−0.0914)
$\Xi_b^0 \to \Xi'_c D_s^{*-}$	0.107 (0.0740)	0.325 (−0.985)
$\Xi_b^0 \to \Xi'_c D^-$	0.00149 (0.0417)	0.0818 (−0.0827)
$\Xi_b^0 \to \Xi'_c D^{*-}$	0.00474 (0.0730)	0.317 (−0.848)

In Tables VII and VIII, we only list the results at $\theta = 16.27°$. For other mixing angles, the decay rates are different, but the polarization asymmetries are same. The results can be understood from the expressions of the form factors f'_i and g'_i in Eq. (8). To calculate the decay rate of $\Xi_b \to \Xi'_c$ for other value of θ, one can simply rescale the width in Tables VII or VIII by a factor $\sin\theta\cos\theta/(\sin 16.27° \cos 16.27°)$.

IV. CONCLUSIONS AND DISCUSSIONS

In this work, we study the transition rates of $\Xi_b \to \Xi_c^{(\prime)}$ in the light front quark model with a three-quark picture of baryon. To correctly compute the transition rates, it is important to properly assess the spin-flavor structures of initial and final states. In the three-quark picture, the two light quarks constitute a diquark, which combines with a heavy quark to form baryons Ξ_b and Ξ_c. In previous studies, the light qs pair in Ξ_c (Ξ'_c) are assumed be to the eigenstates (scalar or pseudovector) of SU(3) flavor symmetry. However, experimental measurements, such as the ratio $\Gamma(\Xi_{cc}^{++} \to \Xi_c^+\pi)/\Gamma(\Xi_{cc}^{++} \to \Xi_c^{\prime +}\pi)$ [4], suggest the spin-flavor structures of Ξ_c and Ξ'_c should be a mixture of $\Xi_c^{\bar{3}}$ and Ξ_c^6. Therefore, here we introduce a mixing angle θ to define the state $|\Xi_c\rangle$ as $\cos\theta|\Xi_c^{\bar{3}}\rangle + \sin\theta|\Xi_c^6\rangle$, and $|\Xi'_c\rangle$ as $-\sin\theta|\Xi_c^{\bar{3}}\rangle + \cos\theta|\Xi_c^6\rangle$. Because of heavy quark symmetry, we can similarly define $|\Xi_b\rangle$ as a mixture of states with the same mixing angle: $\cos\theta|\Xi_b^{\bar{3}}\rangle + \sin\theta|\Xi_b^6\rangle$.

Based on proposed spin-flavor structures of Ξ_b, Ξ_c and Ξ'_c, we calculate semileptonic and nonleptonic decay of $\Xi_b \to \Xi_c$ with specific value of mixing angle ($\theta = 0°$, 16.27° and 85.54°) used in previous analysis [5]. We find the decay width and the asymmetry parameter at $\theta = 85.54°$ are very different with those without mixing ($\theta = 0°$). However, for the mixing angle $\theta = 16.27°$, only the decay width has a significant decrease relative to the result without mixing.

Following this theoretical framework, we also evaluate the rates of semileptonic decay and nonleptonic decay of $\Xi_b \to \Xi'_c$. Since the form factors $f'_i = \cos\theta\sin\theta(f_i^v - f_i^s)$, $g'_i = \cos\theta\sin\theta(g_i^v - g_i^s)$, the theoretical decay rates are proportional to the value $\cos^2\theta\sin^2\theta$; hence, we only display results with $\theta = 16.27°$ for the purpose of illustration. Our numerical results indicate that the decay rates of these decays are about ten to twenty times smaller than the values of the transitions $\Xi_b \to \Xi_c$, which can be feasible to measure experimentally in the near future. Without a mixing of flavor eigenstates, the rates of the transitions $\Xi_b \to \Xi'_c$ should be very tiny (about the same order of the transition of $\Lambda_b \to \Sigma_c$). However, a modest mixing angle such as $\theta = 16.27°$ will make the transition rate large enough for measurements, which makes these channels ideal for studying the mixing of Ξ_c and Ξ'_c (Ξ_b and Ξ'_b). We hope measurement of those decay channels of $\Xi_b \to \Xi'_c$ will be available in the near future, which will help us to test flavor mixing angle θ and elucidate the mechanism of decay of heavy baryons.

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