Nontrivial self-consistent backreaction of quantum fields in 2D dilaton gravity

E. T. Akhmedov, P. A. Anempodistov,* and K. V. Bazarov†

Moscow Institute of Physics and Technology, Institutskii Pereulok 9, 141700 Dolgoprudny, Russia and NRC "Kurchatov Institute," 123182 Moscow, Russia

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We consider $(1+1)$-dimensional dilatonic black hole with two horizons, canonical temperatures of which do not coincide. We show that the presence of quantum fields in such a background leads to a substantial backreaction on the metric: 2D dilatonic analog of the semiclassical Einstein equations are solved self-consistently, and we demonstrate that taking into account backreaction leads to a geometry with two horizons with coinciding temperatures.

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I. INTRODUCTION

Spacetimes with multiple horizons are interesting backgrounds to consider from the perspective of consistency and/or stability when quantum fields are added. The notable example is the Schwarzschild–de Sitter spacetime. It is known that quantum fields in such a background in Euclidean signature inevitably lead to the conical singularity [1–5], whose presence poses a question of the consistency of quantum field theory in the backgrounds with multiple horizons.

Here, however, we adopt a different logic. Namely, one can consider quantum fields in any state, which has a sensible stress-energy tensor (e.g., conserved, etc.) in backgrounds with Lorentzian signature. However, if a quantum field is taken in a spacetime with a horizon, e.g., in a thermal state that does not coincide with the canonical temperature (Unruh, Hawking, or Gibbons-Hawking, correspondingly), then its regularized stress-energy tensor diverges at the horizon. We interpret such a situation as leading just to a strong backreaction on the geometry by the quantum fields (see, e.g., [6–15]).

Furthermore, if the geometry in question has multiple horizons, and the canonical temperatures of these horizons do not coincide, any thermal state of quantum fields leads to divergence of the regularized stress-energy tensor on at least one of the horizons. Then, in the situation with multiple horizons, the regularized stress-energy tensor inevitably has a divergence, which causes strong backreaction on the background geometry. In this paper, we consider a solvable example of 2D quantum field theory where such a conclusion can be checked. We restrict our attention to the case when quantum fields are considered on a classical gravitational background, i.e., gravitational field is considered to be classical.

In [16] the authors have considered the two-dimensional analog of Schwarzschild–de Sitter spacetime and argued that for the thermal states the stress-energy tensor diverges at least on one of the horizons. They also conclude that the divergence of the stress-energy tensor signals that the state of quantum fields is unphysical. Then, they attempt to construct a state that is regular on both horizons. We, however, adopt a different logic, as we have mentioned above and which is in accordance with [17]. The point is that thermal states in stationary backgrounds have a notable advantage in comparison with other states: according to the fluctuation-dissipation theorem, they are stable under quantum corrections for self-interacting fields.[1]

The problem of backreaction of matter fields on background geometries has been addressed by many authors (see, e.g., [6,7,18] for the related works). In these papers, the Einstein equations

$$G_\nu{}^\mu + \Lambda \delta_\nu{}^\mu = 8\pi G \langle :\hat{T}_\nu{}^\mu: \rangle, \tag{1.1}$$

*anempodistov.pa@phystech.edu
†bazarov.kv@phystech.edu

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[1]In [16] they consider a Gaussian theory in the background in question. In this paper, we also restrict our attention to the Gaussian theory, but keep in mind the effect of self-interactions. Also, in two dimensions, the situation with the fluctuation-dissipation theorem is different from higher dimensions due to kinematic reasons, but we consider the 2D case as the model example.

are solved perturbatively:

(i) First, solution $g^{(0)}$ of the Einstein equations in the vacuum (i.e., with the zero on the rhs) is considered (usually a black hole with $\Lambda = 0$).

(ii) Second, the stress-energy tensor for quantum fields in the background $g^{(0)}$ is calculated.

(iii) Third, the Einstein equations are solved perturbatively for a new metric g on the lhs, while on the rhs one plugs the stress-energy tensor calculated with the metric $g^{(0)}$.

In this approach, the new metric g is usually divergent at the position of the horizon.

In our paper, we take a different approach. Namely, we solve the 2D analog of Einstein equations (in dilatonic gravity theory) self-consistently, i.e., the stress-energy tensor is calculated for the same metric that is plugged on the lhs of these equations. Of course, for this method to work, one has to know the form of the expectation value of the stress-energy tensor in a generic background. That is the reason why at this stage we have to confine our analysis to the 2D situation, where the exact form of the regularized stress-energy tensor is known at any point in spacetime for an arbitrary metric.

Note that there is another method to tackle the backreaction problem using the effective action approach (see, e.g., [19–28]). The advantage of our method is that we can explicitly calculate the quantum average of stress-energy tensor for any level population [see Eq. (3.10)].

Usually to make the gravity in 2D dynamical one has to add the dilaton field,

$$S^{\rm grav} = \frac{1}{16\pi G}\int d^2x\sqrt{-g}e^{-2\phi}\left[R - 4\omega(\partial_\mu\phi)^2 + 4\lambda^2\right], \quad (1.2)$$

where ω is an arbitrary constant. For particular values of this ω parameter, this action reduces to the known theories:

(i) the $\omega = 0$ case is the Jackiw-Teitelboim theory;

(ii) if $\omega = -\frac{1}{2}$ one obtains planar general relativity;

(iii) if $\omega = -1$ one has the first-order string theory.

To make the kinetic term canonical, one can always redefine the dilaton field, but then the dilaton coupling is modified.

The equations of motion following from the variation of this action with respect to the metric and dilaton field, correspondingly, are as follows:

$$e^{2\phi}T^{\rm grav}_{\mu\nu} \equiv -2(\omega+1)D_\mu\phi D_\nu\phi + D_\mu D_\nu\phi - g_{\mu\nu}D_\mu D^\nu\phi + (\omega+2)g_{\mu\nu}D_\mu\phi D^\mu\phi - g_{\mu\nu}\lambda^2 = 0, \quad (1.3)$$

$$R - 4\omega D_\mu D^\mu\phi + 4\omega D_\mu\phi D^\mu\phi + 4\lambda^2 = 0. \quad (1.4)$$

The comprehensive analysis of solutions of these equations of motion for arbitrary ω can be found in [29] (see [30–34] for a review of this model, as well as discussion of various quantum effects). For a particular choice of the integration constant for the dilaton, the solution of these equations of motion can be represented in the following form:

$$\phi = -\log(ar)^{\frac{1}{2(\omega+1)}},$$
$$ds^2 = -\left[a^2r^2 - (ar)^{\frac{\omega}{\omega+1}}\right]dt^2 + \frac{dr^2}{a^2r^2 - (ar)^{\frac{\omega}{\omega+1}}}, \quad (1.5)$$

for some a related to λ and to be defined below.

We begin in Sec. II by discussing the properties of this solution for the case when $\omega = -\frac{4}{3}$. The most important property of such a solution is the presence of two horizons with different temperatures. Then, in Sec. III we add Gaussian quantum fields that live in the background of this geometry and demonstrate that the regularized stress-energy tensor diverges at least on one of the horizons. In Sec. IV we solve semiclassical gravitational equations and show that the resulting backreacted geometry has two horizons with coinciding temperatures.

II. STARTING GEOMETRY

Solutions (1.5) with different ω correspond to different physical situations, and we want to consider the most similar one to the black hole in de Sitter spacetime. We are looking for the multiple horizon scenario, where horizons have different temperatures (surface gravity). To model this situation, we propose to consider the case $\omega = -\frac{4}{3}$. Then the metric takes the form

$$ds^2 = -[a^2r^2 - a^4r^4]dt^2 + \frac{dr^2}{a^2r^2 - a^4r^4}, \quad (2.1)$$

while the dilaton field acquires the form

$$\phi = \frac{3}{2}\log(ar), \quad (2.2)$$

where $a = -\sqrt{\frac{2\lambda^2}{3}}$. The Ricci scalar for the metric in question is $R = -\frac{4}{3}\lambda^2[1 - 6(ar)^2]$. This metric is defined for $r > 0$ and possesses two horizons: at $r = 0$ and $r = 1/a$. The canonical temperatures at the horizons are

$$T = \begin{cases} \frac{a}{2\pi}, & r \to 1/a, \\ 0, & r \to 0. \end{cases} \quad (2.3)$$

Note that the zero of the function $[a^2r^2 - a^4r^4]$ at $r = 0$ is degenerate. As a result, the corresponding temperature is vanishing. The Penrose-Carter diagram for this spacetime is depicted in Fig. 1. For such spacetimes that we consider, it is useful to use other (tortoise) coordinates,

$$ds^2 = -[a^2r^2 - a^4r^4](dt^2 - dr_*^2), \quad (2.4)$$

where

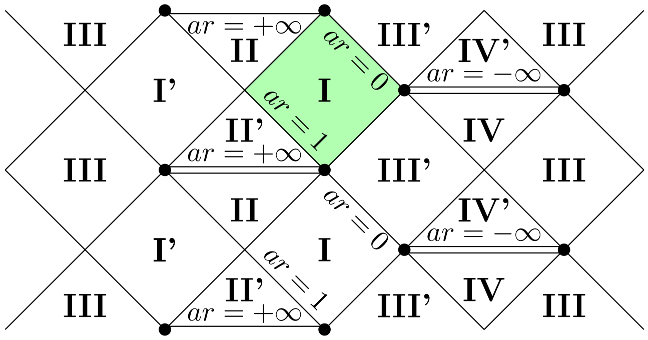

FIG. 1. Maximal analytical extension of the metric (2.1). There are infinite copies of this diagram extending both in vertical and horizontal directions. Coordinates (2.1) cover only chart I (highlighted by green on the diagram). The boundaries of the chart are lightlike surfaces corresponding to $ar = 0$ and $ar = 1$.

$$r_* = -\frac{1}{a^2 r} + \frac{1}{a}\operatorname{arctanh}(ar). \tag{2.5}$$

Or, using Eddington-Finkelstein coordinates,

$$u = t - r_*, \qquad v = t + r_*, \tag{2.6}$$

we can rewrite the metric in the form

$$\begin{aligned} ds^2 &= C(u,v)dudv, \\ C(u,v) &= -a^2 r^2(u,v) + a^4 r^4(u,v), \end{aligned} \tag{2.7}$$

where $r(u,v)$ is determined from

$$\frac{v-u}{2} = -\frac{1}{a^2 r} + \frac{1}{a}\operatorname{arctanh}(ar). \tag{2.8}$$

Using substitution

$$r = \frac{1}{a\cosh(aX)}, \tag{2.9}$$

we can write the metric in the form

$$ds^2 = -e^{2\nu(X)}dt^2 + dX^2, \tag{2.10}$$

where

$$e^{2\nu(X)} = \frac{\sinh^2(aX)}{\cosh^4(aX)}. \tag{2.11}$$

In these coordinates, the dilaton field is equal to

$$\phi(X) = -\frac{3}{2}\log\cosh(aX). \tag{2.12}$$

In this paper, we use all these coordinate systems in different situations, depending on where it is convenient to use one of them.

III. ADDING QUANTUM FIELDS

The next step in our reasoning is to add quantum fields. We consider real conformal field φ with the action

$$S = S^{\text{grav}} + S^{\text{matter}} = S^{\text{grav}} - \frac{1}{2}\int d^2x \partial_\mu\varphi\partial^\mu\varphi, \tag{3.1}$$

just as in, e.g., [19–23], but rather than integrating the field φ out, we explicitly quantize it and calculate the regularized stress-energy tensor. We will adopt the canonical quantization.

It is useful to use the Eddington-Finkelstein coordinates (2.7), in which the equations of motion for the scalar field take the form

$$\partial_U\partial_V\varphi(U,V) = 0. \tag{3.2}$$

Thus, we can write down the mode decomposition of the field operator as

$$\hat{\varphi}(U,V) = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}}\left[\hat{a}^\dagger_\omega e^{i\omega U} + \hat{b}^\dagger_\omega e^{i\omega V} + \text{H.c.}\right] \tag{3.3}$$

Commutation relations for the creation and annihilation operators for the left and right moving modes are

$$\left[\hat{a}^\dagger_\omega, \hat{a}_{\omega'}\right] = \delta(\omega-\omega'), \qquad \left[\hat{b}^\dagger_\omega, \hat{b}_{\omega'}\right] = \delta(\omega-\omega'). \tag{3.4}$$

We take the quantum field in the thermal state, for which

$$\langle\hat{a}^\dagger_\omega\hat{a}_{\omega'}\rangle = \langle\hat{b}^\dagger_\omega\hat{b}_{\omega'}\rangle = \frac{1}{e^{\beta\omega}-1}\delta(\omega-\omega'), \tag{3.5}$$

where $\beta = 1/T$ is the inverse temperature.

Then, the regularized stress-energy tensor is given by [35]

$$-\langle : T^{\text{matter}}_{\mu\nu} : \rangle = \Theta_{\mu\nu} + \frac{R}{48\pi}g_{\mu\nu}, \tag{3.6}$$

where

$$\begin{aligned} \Theta_{uu} &= \frac{1}{48\pi}\left(\frac{2\pi}{\beta}\right)^2 - \frac{1}{12\pi}C^{1/2}\partial_u^2 C^{-1/2}, \\ \Theta_{vv} &= \frac{1}{48\pi}\left(\frac{2\pi}{\beta}\right)^2 - \frac{1}{12\pi}C^{1/2}\partial_v^2 C^{-1/2}, \\ \Theta_{uv} &= \Theta_{vu} = 0, \end{aligned} \tag{3.7}$$

and $C(u,v)$ here is equal to (2.7). Then, plugging the expression for the conformal factor $C(u,v)$, we find that

$$\Theta_{uu} = \begin{cases} \frac{1}{48\pi}\left[\left(\frac{2\pi}{\beta}\right)^2 - a^2\right], & \text{as } r \to \frac{1}{a}, \\ \frac{1}{48\pi}\left(\frac{2\pi}{\beta}\right)^2, & \text{as } r \to 0, \end{cases} \tag{3.8}$$

and the same expression for Θ_{vv}. We see that one can choose the inverse temperature β such that $\Theta_{\mu\nu}$ is vanishing only on one of the horizons. If $\Theta_{\mu\nu}$ has a nonzero value on a horizon, it implies that T^{μ}_{ν} and $T^{\mu\nu}$ blow up on this horizon, which leads to the substantial backreaction on the geometry.

In the coordinates (2.10), the stress-energy tensor has the following form[2]:

$$-\langle :T^{\text{matter}}_{tt}: \rangle = \frac{\pi}{6\beta^2} + \frac{e^{2\nu(X)}}{24\pi}\left[\nu'(X)^2 + 2\nu''(X)\right],$$
$$-\langle :T^{\text{matter}}_{XX}: \rangle = \frac{e^{-2\nu(X)}\pi}{6\beta^2} - \frac{\nu'(X)^2}{24\pi}. \tag{3.9}$$

Below, we will use these expressions to find a self-consistent solution of the modified equations of motion.

We consider only thermal states in our calculations. Such a state has level population given by the Planckian distribution. However, our method allows one to extend the calculation for any level population. For example, for generic level population $n(\omega)$ energy density is given by the following expression:

$$-\langle :T^{\text{matter}}_{tt}: \rangle = \int_{-\infty}^{\infty} \frac{\omega d\omega}{2\pi}\left[n(\omega) + \Theta(-\omega)\right] + \frac{e^{2\nu(X)}}{24\pi}\left[\nu'(X)^2 + 2\nu''(X)\right], \tag{3.10}$$

where

$$\langle \hat{a}^{\dagger}_{\omega}\hat{a}_{\omega'} \rangle = \langle \hat{b}^{\dagger}_{\omega}\hat{b}_{\omega'} \rangle = n(\omega)\delta(\omega - \omega'), \tag{3.11}$$

and $\Theta(-\omega)$ is the Heaviside step function. If $n(\omega) = [e^{\beta\omega} - 1]^{-1}$, then

$$\int_{-\infty}^{\infty} \frac{\omega d\omega}{2\pi}\left[\frac{1}{e^{\beta\omega} - 1} + \Theta(-\omega)\right] = \frac{\pi}{6\beta^2} \tag{3.12}$$

and we obtain (3.9).

IV. BACKREACTION OF QUANTUM FIELDS ON THE BACKGROUND GEOMETRY

Now, let us continue with the solution of the 2D dilatonic analog of the semiclassical Einstein equations (1.1) self-consistently, i.e., by plugging in it an arbitrary metric and regularized stress-energy tensor of the matter fields calculated in the background of the same metric. In the 2D dilaton gravity that we consider here, these equations of course look different from (1.1).

Let us start with writing down nonzero components of the gravitational stress-energy tensor in the unitary gauge (2.10)[3]:

$$e^{2\phi(X)}T^{\text{grav}}_{tt} = e^{2\nu(X)}\left[\lambda^2 - \frac{2}{3}\phi'(X)^2 + \phi''(X)\right],$$
$$e^{2\phi(X)}T^{\text{grav}}_{XX} = -\lambda^2 - \phi'(X)\nu'(X) + \frac{4}{3}\phi'(X)^2. \tag{4.1}$$

Then, the 2D dilatonic analog of Einstein equations (1.1) reduces to

$$\lambda^2 - \frac{2}{3}\phi'(X)^2 + \phi''(X) = 8\pi G e^{2\phi(X)}\left(\frac{e^{-2\nu(X)}\pi}{6\beta^2} + \frac{1}{24\pi}\left[\nu'(X)^2 + 2\nu''(X)\right]\right),$$
$$-\lambda^2 - \phi'(X)\nu'(X) + \frac{4}{3}\phi'(X)^2 = 8\pi G e^{2\phi(X)}\left(\frac{e^{-2\nu(X)}\pi}{6\beta^2} - \frac{\nu'(X)^2}{24\pi}\right), \tag{4.2}$$

where now $\phi(X)$ and $\nu(X)$ are unknown functions to be determined from these equations.

When $\beta = 0$ (the case of interest for us, as is explained below), these equations have an obvious solution, with $\nu(X)$ being a linear function of X and ϕ being a constant. The constants in the solution should be suitably adjusted according to the values of λ and G. However, we are not looking for such a solution. Because our solution should have an appropriate asymptotic behavior at one of the horizons, we want to find what happens under backreaction (due to the quantum fields) with the solution with two horizons.

To find the solution of (4.2) with appropriate asymptotic behavior, we have to apply numerical methods. To do that, we need to specify boundary conditions. We will assume that the horizon at $r = 0$ $(X = \infty)$ remains unchanged. Comparing with (3.8), one can see that this condition implies that $1/\beta = 0$ (in other words, the state of the field is the Fock space ground state). Hence, Eq. (4.2) for the dimensionless variable $x = |a|X$ (with $a = -\sqrt{\frac{2\lambda^2}{3}}$) reads[4]

$$\frac{3}{2} - \frac{2}{3}\phi'(x)^2 + \phi''(x) = \frac{e^{2\phi(x)}}{3}\left[\nu'(x)^2 + 2\nu''(x)\right],$$
$$-\frac{3}{2} - \phi'(x)\nu'(x) + \frac{4}{3}\phi'(x)^2 = -\frac{e^{2\phi(x)}}{3}\nu'(x)^2. \tag{4.3}$$

[2]Note that the stress-energy tensor is conserved $D_\mu\langle :T^{\mu\nu}_{\text{matter}}: \rangle = 0$.

[3]Note, that the third equation $\frac{\delta S^{\text{grav}}}{\delta\phi(X)} = 0$ is satisfied if $T^{\text{grav}}_{\mu\nu} = 8\pi G\langle :T^{\text{matter}}_{\mu\nu}: \rangle$.

[4]Here we have set $G = 1$.

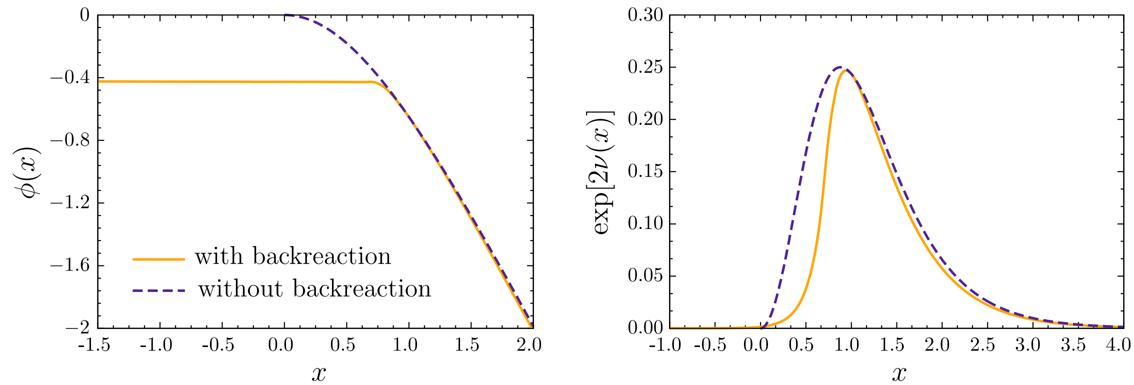

FIG. 2. Numerical plot of the dilaton field $\phi(x)$ and for the $e^{2\nu(x)}$ with and without taking into account backreaction. Note that the solution without backreaction is defined only for $X > 0$, while the solution with the backreaction is defined on the whole X axis.

The solution of these equations for the dilaton field and for the $e^{2\nu(x)}$ have the form shown in Fig. 2. These forms of the solutions could have been guessed from the following perspective: the solutions of (4.3) should interpolate from the form of (2.12) and (2.11) at $x \gg x_0$ (where x_0 is a reference gluing point) to the solution

$$\phi(x) \approx \phi_0 = \text{const}, \qquad \nu(x) \approx \frac{3}{\sqrt{2}} e^{-\phi_0} x + \text{const}, \qquad (4.4)$$

at $x \ll x_0$, as follows from (4.3). As can be seen from Fig. 2, these are precisely solutions obeying such boundary conditions.

There is one important point to note here. The solutions (2.12) and (2.11) are defined in the region $X > 0$ [from single valuedness of (2.9)], but the backreacted geometry of Fig. 2 is defined on the entire axis $X \in (-\infty; +\infty)$. The new geometry has two horizons at $X = \pm\infty$, and canonical temperatures at these two horizons are vanishing.[5] Thus, the backreacted geometry with the quantum state under consideration is, in fact, stable.

V. CONCLUSIONS

We have started with the geometry that possessed two horizons with different canonical temperatures. The fact that the two horizons have different temperatures leads to substantial backreaction of quantum fields on the background metric—the divergence of the stress-energy tensor on one of the horizons does not allow us to neglect the influence of quantum fields on the metric of the spacetime at this horizon. This happens if one considers fields to be in a thermal state, and one could, in principle, define a very specific state for which the regularized stress-energy tensor is regular on both horizons (see, e.g., [16]). However, unlike the thermal one, any another state will not necessarily survive quantum corrections in self-interacting theory (at least in dimensions higher than 2).

Then, we have written down the 2D dilatonic analog of the semiclassical Einstein equations in a self-consistent manner, i.e., the stress-energy tensor on the rhs of these equations was calculated for an arbitrary metric, which is to be determined from the resulting equations. Then, we have solved these equations numerically with the boundary condition that one of the horizons is unchanged.

We have found quite a remarkable result that the account of the backreaction changes the geometry in a way that the two horizons have the same (vanishing) temperature. Hence, after the introduction of quantum fields and accounting for the backreaction, we obtain a geometry that is stable for the vacuum state (the thermal state with vanishing temperature) of the quantum field.

[5]This can be seen from examination of the contribution $C^{1/2}\partial_u^2 C^{-1/2}$ to the stress-energy tensor at $X = \pm\infty$.

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[1] Bernard S. Kay and Robert M. Wald, Theorems on the uniqueness and thermal properties of stationary, nonsingular, quasifree states on space-times with a bifurcate killing horizon, Phys. Rep. **207**, 49 (1991).
[2] Feng-Li Lin and Chopin Soo, Black hole in de Sitter space, in *6th International Symposium on Particles, Strings and Cosmology* (1998), pp. 89–91, arXiv:hep-th/9807084.
[3] Claudio Teitelboim, Gravitational thermodynamics of Schwarzschild–de Sitter space, in *Meeting on Strings and Gravity: Tying the Forces Together* (2001), pp. 291–299, arXiv:hep-th/0203258.
[4] S. Shankaranarayanan, Temperature and entropy of Schwarzschild–de Sitter space-time, Phys. Rev. D **67**, 084026 (2003).
[5] T. Roy Choudhury and T. Padmanabhan, Concept of temperature in multi-horizon spacetimes: Analysis of Schwarzschild–de Sitter metric, Gen. Relativ. Gravit. **39**, 1789 (2007).
[6] Pei-Ming Ho and Yoshinori Matsuo, On the near-horizon geometry of an evaporating black hole, J. High Energy Phys. 07 (2018) 047.
[7] Pei-Ming Ho, Hikaru Kawai, Yoshinori Matsuo, and Yuki Yokokura, Back reaction of 4D conformal fields on static geometry, J. High Energy Phys. 11 (2018) 056.
[8] Prokopii A. Anempodistov, Remarks on the thermofield double state in 4D black hole background, Phys. Rev. D **103**, 105008 (2021).
[9] K. V. Bazarov, Notes on peculiarities of quantum fields in space-times with horizons, Classical Quantum Gravity **39**, 217001 (2022).
[10] E. T. Akhmedov, P. A. Anempodistov, K. V. Bazarov, D. V. Diakonov, and U. Moschella, Heating up an environment around black holes and inside de Sitter space, Phys. Rev. D **103**, 025023 (2021).
[11] Valter Moretti, Direct zeta function approach and renormalization of one loop stress tensors in curved space-times, Phys. Rev. D **56**, 7797 (1997).
[12] Valter Moretti and Devis Iellici, Optical approach for the thermal partition function of photons, Phys. Rev. D **55**, 3552 (1997).
[13] V. P. Frolov, A. Pinzul, and A. I. Zelnikov, Vacuum polarization at finite temperature on a cone, Phys. Rev. D **51**, 2770 (1995).
[14] O. Diatlyk, Hawking radiation of massive fields in 2D, Phys. Rev. D **104**, 065011 (2021).
[15] Dmitrii Diakonov, Is the Euclidean path integral always equal to the thermal partition function?, arXiv:2310.08522.
[16] D. Markovic and W. G. Unruh, Vacuum for a massless scalar field outside a collapsing body in de Sitter space-time, Phys. Rev. D **43**, 332 (1991).
[17] Emil T. Akhmedov and Kirill V. Bazarov, Backreaction issue for the black hole in de Sitter spacetime, Phys. Rev. D **107**, 105012 (2023).
[18] Olaf Baake and Jorge Zanelli, Quantum backreaction for overspinning BTZ geometries, Phys. Rev. D **107**, 084015 (2023).
[19] Steven B. Giddings, Toy models for black hole evaporation, in *International Workshop on Theoretical Physics: 6th Session: String Quantum Gravity and Physics at the Planck Energy Scale* (1992), arXiv:hep-th/9209113.
[20] Curtis G. Callan, Jr., Steven B. Giddings, Jeffrey A. Harvey, and Andrew Strominger, Evanescent black holes, Phys. Rev. D **45**, R1005 (1992).
[21] S. P. de Alwis, Quantum black holes in two-dimensions, Phys. Rev. D **46**, 5429 (1992).
[22] Jorge G. Russo, Leonard Susskind, and Larus Thorlacius, The endpoint of Hawking radiation, Phys. Rev. D **46**, 3444 (1992).
[23] Jorge G. Russo, Leonard Susskind, and Larus Thorlacius, Cosmic censorship in two-dimensional gravity, Phys. Rev. D **47**, 533 (1993).
[24] Maxim Fitkevich, Dmitry Levkov, and Yegor Zenkevich, Dilaton gravity with a boundary: From unitarity to black hole evaporation, J. High Energy Phys. 06 (2020) 184.
[25] Sergey N. Solodukhin, Two-dimensional quantum corrected eternal black hole, Phys. Rev. D **53**, 824 (1996).
[26] Yohan Potaux, Debajyoti Sarkar, and Sergey N. Solodukhin, Quantum states and their back-reacted geometries in 2D dilaton gravity, Phys. Rev. D **105**, 025015 (2022).
[27] Damir Sadekov, Generalization of 2D gravity with the simplest noninvariant states, Phys. Rev. D **105**, 125003 (2022).
[28] Andrew Svesko, Evita Verheijden, Erik P. Verlinde, and Manus R. Visser, Quasi-local energy and microcanonical entropy in two-dimensional nearly de Sitter gravity, J. High Energy Phys. 08 (2022) 075.
[29] Jose P. S. Lemos and Paulo M. Sa, The black holes of a general two-dimensional dilaton gravity theory, Phys. Rev. D **49**, 2897 (1994); **51**, 5967(E) (1995).
[30] M. Cadoni, M. Oi, and A. P. Sanna, Evaporation and information puzzle for 2D nonsingular asymptotically flat black holes, J. High Energy Phys. 06 (2023) 211.
[31] S. Nojiri and S. D. Odintsov, Quantum dilatonic gravity in $d = 2, 4$ and 5 dimensions, Int. J. Mod. Phys. A **16**, 1015 (2001).
[32] S. Nojiri and S. D. Odintsov, Anomaly-induced effective actions in even dimensions and reliability of s-wave approximation, Phys. Lett. B **463**, 57 (1999).
[33] P. van Nieuwenhuizen, S. Nojiri, and S. D. Odintsov, Conformal anomaly for 2D and 4D dilaton coupled spinors, Phys. Rev. D **60**, 084014 (1999).
[34] S. Nojiri and S. D. Odintsov, Quantum (in)stability of 2D charged dilaton black holes and 3D rotating black holes, Phys. Rev. D **59**, 044003 (1999).
[35] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, UK, 1984).