Scalaron dynamics from the UV to the IR regime

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We consider a scenario where the scalaron of $f(\mathcal{R})$ models is related to the volume modulus of string compactifications leaving only one scalar degree of freedom at low energy. The coefficient of the leading curvature squared contribution to the low-energy effective action of gravity determines the mass of the scalaron. We impose that this mass is small enough to allow for the scalaron to drive Starobinski's inflation. After inflation, the renormalization group evolution of the couplings of the $f(\mathcal{R})$ theory, viewed as a scalar-tensor theory, provides the link with the infrared regime. We consider a scenario where the corrections to the mass of the scalaron are large and reduce it below the electron mass in the infrared, so that the scalaron plays a central role in the low-energy dynamics of the Universe. In particular this leads to a connection between the scalaron mass and the measured vacuum energy provided its renormalization group running at energies higher than the electron mass never drops below the present day value of the dark energy.

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I. INTRODUCTION

The dark sector of the Universe, comprising both dark matter and dark energy, is as elusive as ever [1,2]. One interesting possibility would be that the dark energy sector could have a complete gravitational origin [3,4]. Usually, this is understood in such a way that gravity, i.e., Einstein's general relativity, should be modified and for instance become massive [5]. This is fraught with conceptual difficulties [6]. On the other hand, a less trodden path could be that gravity is simply modified by the presence of small corrections to the Einstein-Hilbert action whose role would be to modify the quantum nature of the vacuum. We introduce a scenario whereby the leading Ricci scalar squared $\mathcal{R}^2$ corrections dominate at low energy as long as the compactification scale is large enough and the nonperturbative effects involved in the moduli stabilisation occur at low enough energy compared to the compactification scale. This circumvents the problem of hierarchy between the $\mathcal{R}^2$ and the higher-order terms noticed in [7] and give a valid description of the dynamics of the Universe from the inflation scale down to the infrared (IR). This is the approach we will follow in this paper where we will argue that string compactifications of physical relevance should enrich the gravitational sector of the theory with such $f(\mathcal{R})$ corrections [8–10].

Our starting point is the gravitational action of string theory in ten dimensions. Irrespective of the details of the mechanism, dimensions must be reduced from ten to four and in this process the universal volume modulus representing the typical size of the compactification manifold appears [11]. After compactification, the low-energy gravitational action contains terms involving the Riemann tensor of the four-dimensional Universe and the volume modulus. The volume modulus has no potential and therefore appears as a very dangerous massless field whose coupling to matter would exceed the bounds from gravitational physics in the Solar System, e.g., the Cassini bound on the Yukawa coupling of such a light scalar to fermions [12]. In this paper, we will not try to generate the nontrivial scalar potentials which could lead to the screening of the volume modulus in the Solar System [13]. We will simply assume that the volume modulus is stabilized [14] and acquires a given mass which is large enough to evade tests of short-range interactions such as provided by the Eöt-Wash experiment [15]. Once stabilized, we find that the gravitational effective action in four dimensions becomes a $f(\mathcal{R})$ model where the scalaron field [16,17] is then identified with the volume modulus. To do so we restrict ourselves to ghost-free effective actions and assume that the ghost fields [18] induced by higher derivative operators in the gravitational field have a mass which is rejected at the cutoff scale of the model, i.e., the compactification scale. This leaves a well-ordered $f(\mathcal{R})$ action where, in the small curvature regime below the string and compactification scales, the Ricci scalar squared $\mathcal{R}^2$ term dominates.

Written as a scalar-tensor theory depending on the scalaron, and imposing the existence of a supersymmetric origin for this low-energy action, we find the most general Kähler potentials and superpotentials which are compatible with the Ricci scalar squared $\mathcal{R}^2$ structure. They involve a two-parameter family of Kähler potentials including the familiar no-scale case and a fully determined and unique superpotential. Of course, this theory is determined in the ultraviolet (UV) at the compactification scale. It has all the features to generate primordial inflation like in the original Starobinski scenario [7,16,17]. Here inflation is determined by the coefficient $c_{\rm UV}$ of the Ricci scalar squared $\mathcal{R}^2$ correction in the UV. From the compactification point of view, this is a bottom-up constraint as no preferred value of $c_{\rm UV}$ originates from the compactification. Notice that here inflation appears as a consequence of both the volume modulus stabilization and the fact that it happens in a regime where all the higher-order corrections involving higher powers of the curvature are negligible.

We then use a renormalization group approach to tie up the regime described above after the end of inflation and the low-energy regime of the theory. In this Wilsonian approach, the Lagrangian of the theory is determined in particular by the vacuum energy and the coefficient c of the $\mathcal{R}^2$ term which evolve with the scale as particles are integrated out. In a given energy range μ corresponding to a given temperature T of the Universe, the Wilsonian action and its coefficients depend on μ. We take as the initial point of the renormalization group evolution at the reheat temperature and integrate out particles as μ is decreased toward low energies. We analyze the renormalization group evolution of the Ricci scalar squared $\mathcal{R}^2$ theory down to the IR limit at energies well below the masses of all massive particles present in the Universe. We identify this long-distance limit as the vacuum energy which engenders dark energy. The scalaron mass parameter at a given scale μ is given by the inverse of the square of the coefficient of the $\mathcal{R}^2$. We assume following [9] that the scalaron mass in the IR is lower than the electron mass and therefore plays a role in the dynamics of the vacuum at low energy. Unfortunately the mass of the scalaron in the IR is not determined completely as it depends on the UV properties of the model. The dynamics can only be closed by imposing reasonable assumptions on the IR properties of the theory. In particular, following the reasoning of [19], we will require that the cosmological constant $\rho_\Lambda(\mu)$ at the energy scale μ should be large enough that any bound structure in the Universe with such an energy scale, e.g., a gas cloud with a temperature $\mu \simeq T$, does not collapse within the age of the Universe. This is guaranteed as long as $\rho_\Lambda(\mu) \gtrsim -\rho_\Lambda$, where ρ_Λ is the dark energy of the Universe. Using these constraints for $\mu \simeq m_e$ at the electron mass corresponding to the x-ray emitting gas in a galaxy cluster, we found in [9] that the mass of the scalaron is tightly bounded once the Eöt-Wash experiment bound from gravity tests is taken into account. This almost determines the value of $c_{\rm IR}$ in the IR and therefore using the renormalization group evolution the cosmological constant at the end of inflation. This set of phenomenological constraints provides a bottom-up approach to the physics from the compactification of string theory down to four dimensions. If the scalaron had very large mass in the IR, i.e., larger than the electron mass, then no such constraints would stand. We also notice that the interval of masses for the scalaron in the IR is compatible with scenarios where the scalaron plays the role of dark matter [20].

This paper is arranged as follows. In Sec. II, we consider the compactification from ten dimensions to four dimensions and the identification of the volume modulus with the scalaron. In Sec. III, we impose that the low-energy effective action describing the Ricci scalar squared $\mathcal{R}^2$ theory comes from a supergravity model and determines the Kähler potential and the superpotential. We find that the Kähler potential is a two-parameter deformation of the no-scale model describing the volume modulus at tree level. Finally, in Sec. IV we describe the renormalization evolution of the Ricci scalar squared $\mathcal{R}^2$ model down to the IR. We conclude with Sec. V. We give an explicit example of potential for the volume modulus in an Appendix. We also discuss the renormalization group and the thermodynamic decoupling of scalarons in two appendixes.

II. THE VOLUME MODULUS AS SCALARON

In this section we present a scenario where the Ricci scalar squared $\mathcal{R}^2$ model leading to Starobinski's inflation is induced from the compactification of extra dimensions such as the reduction from ten dimensions to four dimensions of string theory. For this we first detail in Sec. II A how, in the so-called supergravity frame, the volume modulus does not have a kinetic term. We then use this in Sec. II D for the scalaron with the universal volume modulus. We discuss volume stabilization in Sec. II C and show in Sec. II E that higher derivative corrections induced by string theory are negligible. We discuss in Sec. IV C the validity of the quadratic approximation used to derive the Starobinski model during inflation where the excursion of the volume modulus is large [21].

A. Dimensional reduction

We consider a $4+d$ space-time and compactify on a d manifold of vanishing Ricci scalar [22]. We will consider first the case of the Einstein-Hilbert action

$$S_{4+d} = M_{2+d}^{d+2} \int d^{4+d}x \sqrt{-g^{4+d}} R_{4+d}, \tag{2.1}$$

where R_{4+d} is the Ricci scalar of the metric g^{4+d}. We consider a metric of the form

$$g_{ab}^{4+d} = g_{\mu\nu}^{4} dx^\mu dx^\nu + \sigma^2(x) g_{ij}^{d} dx^i dx^j. \tag{2.2}$$

Defining the volume of the compactification as

$$V_0 = \int d^d x \sqrt{g^d}, \tag{2.3}$$

we find after dimensional reduction the effective action

$$S_4 = M_{2+d}^{d+2} V_0 \int d^4x \sqrt{-g^4} \sigma^d \left(R_4 + d(d-1)(\partial \ln \sigma)^2\right). \tag{2.4}$$

We can go to the Einstein frame by defining

$$g^4_{\mu\nu} = \sigma^{-d} g^E_{\mu\nu}, \tag{2.5}$$

leading to

$$S_4 = M_{2+d}^{d+2} V_0 \int d^4x \sqrt{-g^E} \left(R^E - \frac{d(d+2)}{2} (\partial \ln \sigma)^2\right). \tag{2.6}$$

This action can be transformed into

$$S_4 = M_{2+d}^{d+2} V_0 \int d^4x \sqrt{-g} \sigma^{-2\alpha} R, \tag{2.7}$$

where

$$g^E_{\mu\nu} = \sigma^{-2\alpha} g_{\mu\nu} \tag{2.8}$$

with

$$\alpha = -\frac{1}{2}\sqrt{\frac{d(d+2)}{3}}. \tag{2.9}$$

In the rest of the paper, we identify the volume modulus as

$$\mathcal{V} = \sigma^{-2\alpha}, \tag{2.10}$$

which has no kinetic terms. When $d = 6$ we have $\alpha = -2$ and

$$\mathcal{V} = \left(\frac{V}{V_0}\right)^{\frac{2}{3}}, \tag{2.11}$$

where V is the volume of the extra dimensions.

Defining $m_{\rm Pl}^2 = 2M_{2+d}^{d+2} V_0$, we can identify the action (2.7) with the supergravity action in the supergravity frame, following Chap. 31 of [23]:

$$S_4 = \frac{m_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left(f(T,\bar{T}) R + 6 f_{T\bar{T}} \partial_\mu T \partial^\mu \bar{T}\right), \tag{2.12}$$

where $f(T,\bar{T}) = e^{-K(T,\bar{T})/3m_{\rm Pl}^2}$ is the Kähler potential

$$K = -3m_{\rm Pl}^2 \ln \frac{T+\bar{T}}{m_{\rm Pl}}. \tag{2.13}$$

Notice that $f_{T\bar{T}} = 0$ as $f(T,\bar{T}) = \frac{T+\bar{T}}{m_{\rm Pl}}$ is linear in the modulus T such that

$$V = V_0 \left(\frac{T+\bar{T}}{m_{\rm Pl}}\right)^{\frac{3}{2}}. \tag{2.14}$$

We couple matter to the Jordan frame metric $g_{\mu\nu}$. When matter is supersymmetric defined by the superfield C, the scalar part reads

$$S_4 = \frac{m_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \big(f(T,\bar{T}) R + 6 f_{T\bar{T}} \partial_\mu T \partial^\mu \bar{T} + 6 f_{C\bar{C}} \partial_\mu C \partial^\mu \bar{C}\big), \tag{2.15}$$

where $f_{C\bar{C}} = -\frac{1}{3m_{\rm Pl}^2}$ leading to

$$f(T,\bar{T}) = \frac{T+\bar{T}}{m_{\rm Pl}} - \frac{|C|^2}{3m_{\rm Pl}^2} \tag{2.16}$$

corresponding to

$$K(T,\bar{T}) = -3m_{\rm Pl}^2 \ln\left(\frac{T+\bar{T}}{m_{\rm Pl}} - \frac{|C|^2}{m_{\rm Pl}^2}\right). \tag{2.17}$$

Notice that in the supergravity frame defined by the metric $g_{\mu\nu}$, the matter fields C are canonically normalized corresponding to the Jordan frame for matter. In the Einstein frame, the action reads

$$S_4 = \int d^4x \sqrt{-g^E} \left(\frac{m_{\rm Pl}^2}{2} R^E - K_{i\bar{j}} \partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{j}}\right), \tag{2.18}$$

where $\phi^i = (T, C)$. The Kähler potential (2.17) is the one we use in this work.

The crucial ingredient that we will use in the following is that there exists a frame, here identified as the supergravity frame, where the volume modulus has no kinetic terms. This is not special to the volume indeed. Indeed, using a Weyl transformation one can always remove the kinetic terms of one field at the price of having a nontrivial rescaling of the metric. What is special about the volume modulus is that this Weyl transformation coincides with the change of metric which transforms the Einstein frame where the kinetic terms are nonvanishing to the Jordan frame where the kinetic terms vanish and the volume modulus decouples from matter; see (2.15) where the C field does not couple to the volume modulus. Had it been otherwise, the construction presented below would be altered and integrating out the volume modulus leading to the identification with a $f(\mathcal{R})$ theory would have been impossible.

B. The volume modulus and $f(\mathcal{R})$ theories

We are interested in the low-energy dynamics of the volume modulus obtained, for instance, after the compactification of the ten-dimensional effective field theory of type-IIA string theory down to four dimensions on a Calabi-Yau manifold of Hodge numbers $h_{1,1}$ and $h_{2,1}$. This leads to $h_{1,1}$ hypermultiplets and $h_{2,1}+1$ vector multiplets. The hypermultiplets always contain the volume modulus; see for instance [24]. We will assume that the multimodulus dynamics reduces, after integrating out all the extra scalars, to a system involving the volume modulus only. The reason for this hypothesis, as will be clear below, is that the volume modulus has the correct dimensionless coupling to matter fields $\beta = 1/\sqrt{6}$; see [7,8] for instance. The effective dynamics of the volume modulus are then assumed to be determined by a scalar potential $V_{\rm vac}(\mathcal{V})$ determined by nonperturbative effects. As shown in the previous section there is a frame where the volume modulus has no kinetic term. This frame is not the same as the string or Einstein frame but a Jordan frame where matter couples to the metric $g_{\mu\nu}$; see the previous section for details about the different frames. After dimensional reduction, the action reads

$$S = \int d^4x\sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left(V_{\rm vac}(\mathcal{V}) + \mathcal{V}\mathcal{R}\right) + S_{\rm matter}(g_{\mu\nu},\psi), \tag{2.19}$$

where $\mathcal{V}$ is the volume modulus and $S_{\rm matter}(g_{\mu\nu},\psi)$ is the matter action. As mentioned above the field $\mathcal{V}$ has no intrinsic dynamics as no kinetic terms are present in the Jordan frame of Sec. II A.

Let assume that there exists a function $f(\chi)$ of an auxiliary field χ such that

$$\chi = -\frac{dV_{\rm vac}(\mathcal{V})}{d\mathcal{V}}, \qquad \mathcal{V}_0 f(\chi) = V_{\rm vac}(\mathcal{V}) - \mathcal{V}\frac{dV_{\rm vac}(\mathcal{V})}{d\mathcal{V}} \tag{2.20}$$

with $\mathcal{V}_0$ a given constant. Hence, the function $\mathcal{V}_0 f(\chi)$ is the Legendre transform of the potential $V_{\rm vac}(\mathcal{V})$. Under the assumption that $f''(\chi)\neq 0$ one can identify the volume modulus as

$$\frac{\mathcal{V}}{\mathcal{V}_0} = f'(\chi). \tag{2.21}$$

The gravitational part of the action (2.19) becomes

$$\mathcal{S}_g = \frac{m_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\big(f(\chi) - \chi f'(\chi) + f'(\chi)\mathcal{R}\big), \tag{2.22}$$

where the four-dimensional Planck mass depends now on $\mathcal{V}_0$ as

$$m_{\rm Pl}^2 = \mathcal{V}_0 M_{\rm Pl}^2. \tag{2.23}$$

One can then integrate out the χ field as it has no dynamics and is a simple auxiliary field. An extremum of the resulting action exists provided $f''(\chi)\neq 0$ which gives

$$\chi = \mathcal{R}, \tag{2.24}$$

and as a result one obtains that the action (2.19) is a $f(\mathcal{R})$ theory in the Jordan frame

$$S = \frac{m_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}f(\mathcal{R}) + S_{\rm matter}(g_{\mu\nu},\psi). \tag{2.25}$$

We then have, under the assumption (2.20), that the volume modulus dependence in (2.19) is equivalent to a $f(\mathcal{R})$ theory.

C. Volume stabilization

The effective potential $V_{\rm vac}(\mathcal{V})$ follows from nonperturbative effects which involve an intricate interplay with the dynamics of other moduli [7,21]. In the large volume limit corresponding to $\mathcal{V}\to\infty$ the potential in the Einstein frame vanishes corresponding to the absence of cosmological constant in ten dimensions. Here we will assume that the volume modulus is stabilized at a finite value $\mathcal{V}_0$ in the supergravity frame such that locally in the vicinity of the minimum[1]

$$V_{\rm vac}(\mathcal{V}) = -V_{\rm vac} - \frac{m^2}{2}(\mathcal{V}-\mathcal{V}_0)^2. \tag{2.26}$$

This is only valid close to the minimum. For large deviations from the minimum, correction terms must be considered first and eventually, in the large volume limit, this expression becomes not valid any more and should be replaced by a function such that the potential in the Einstein frame $V_E(\mathcal{V}) = V_{\rm vac}(\mathcal{V})/\mathcal{V}^2$ converges to zero for large $\mathcal{V}$. This regime is not the one we are interested in as we focus on the domain around the stabilized value $\mathcal{V}_0$ and will give sufficient conditions on the expansion of $V_{\rm vac}(\mathcal{V})$ in its vicinity to guarantee that Starobinski's inflation is under control.

Using (2.21), we obtain $\chi = m^2(\mathcal{V}-\mathcal{V}_0)$ and $\mathcal{V}_0 f(\chi) = -V_{\rm vac}\frac{m^2}{2}(\mathcal{V}^2-\mathcal{V}_0^2)$ leading to a $f(\mathcal{R})$ action in the Jordan frame

$$S = \int d^4x\sqrt{-g}\left(-\frac{V_{\rm vac}}{\mathcal{V}_0} + \frac{m_{\rm Pl}^2}{2}\mathcal{R} + \delta c_0\mathcal{R}^2\right) + S_{\rm matter}(g,\psi), \tag{2.27}$$

[1]The potential in the Einstein frame $V_E(\mathcal{V}) = \mathcal{V}_0^2 V_{\rm vac}(\mathcal{V})/\mathcal{V}^2$ has a local minimum for $\mathcal{V} = \mathcal{V}_0 - 2V_{\rm vac}/(m^2\mathcal{V}_0)$ which is close to $\mathcal{V}_0$ in the realistic cases discussed in Sec. IV. During inflation the Einstein potential is nearly constant and the end of inflation occurs around the local minimum. For an explicit example inspired from string theory, see Appendix A.

and the coefficient of the induced $\mathcal{R}^2$ is given by

$$\delta c_0 = \frac{m_{\rm Pl}^2}{4m^2\mathcal{V}_0}, \quad (2.28)$$

depending on the mass term for the volume modulus. Hence, we find that a theory of the $\mathcal{R}^2$ type follows from stabilizing the volume modulus and restricting its scalar potential to a simple quadratic term. Moreover, the curvature of the scalar potential around the stabilized value $\mathcal{V}_0$ determines the coefficient of the $\mathcal{R}^2$ term.

D. The volume modulus as a scalaron

After having stabilized the volume modulus, the action in (2.27) is an $f(\mathcal{R})$ action with

$$f(\mathcal{R}) = -2\Lambda + \mathcal{R} + \frac{\mathcal{R}^2}{2m^2\mathcal{V}_0}, \quad (2.29)$$

where $\Lambda = V_{\rm vac}/(\mathcal{V}_0 m_{\rm Pl}^2)$. Performing the usual change of frame (see [4] for a review)

$$g_{\mu\nu}^E = \frac{\mathcal{V}}{\mathcal{V}_0} g_{\mu\nu} = f'(\mathcal{R}) g_{\mu\nu}; \qquad \frac{\mathcal{V}}{\mathcal{V}_0} \coloneqq \exp(-2\beta\varphi); \qquad \beta = \frac{1}{\sqrt{6}}, \quad (2.30)$$

one gets in the Einstein frame

$$S = \int d^4x \sqrt{-g^E} \frac{m_{\rm Pl}^2}{2} \left(\mathcal{R}_E - \frac{1}{2} \partial_\mu\varphi \partial^\mu\varphi - V(\varphi) \right) + S_{\rm matter}\left(e^{2\beta\varphi} g_{\mu\nu}^E, \psi\right) \quad (2.31)$$

with

$$V(\phi) = \frac{\mathcal{R} f'(\mathcal{R}) - f(\mathcal{R})}{(f'(\mathcal{R}))^2}. \quad (2.32)$$

We see in this frame that the matter fields ψ have the universal coupling to the volume through the scaling factor $e^{2\beta\phi/m_{\rm Pl}}$ with the strength controlled by $\beta = 1/\sqrt{6}$.

The relation in (2.30) relates the volume $\mathcal{V}$ to the scalaron φ. Under scale transformations $x^\mu \to x^\mu/\lambda$, the scalaron field transforms as $\varphi \to \varphi - 3/\beta \ln \lambda$; i.e., φ is the Goldstone boson associated to the breaking of scale invariance. As a result the evolution of φ from $-\infty$ to ∞ gives a "reading" in time of the renormalization group evolution from the ultraviolet to the infrared. In [25] a cosmon field was considered from the volume using (2.30). The difference here is that the scalar field is the scalaron from a $f(\mathcal{R})$ theory.

E. The curvature expansion

The action in (2.19) is part of the low-energy effective action after compactification and can receive higher derivative corrections. We now review the derivative expansion of the compactification to four dimensions of the gravitational sector of ten-dimensional string theory. The discussion is given in the Jordan frame which is obtained by the Weyl scaling from the string or Einstein frame as discussed in Sec. II A.

The curvature squared models are part of a four-dimensional string theory low-energy effective action which has a derivative expansion of the type in the Jordan frame

$$S_{4d}^{\rm eff} = \int d^4x \sqrt{-g} \left(\frac{M_{\rm Pl}^2}{2} \mathcal{V}_0 \mathcal{R} + c_0 \mathcal{V}_0 \mathcal{R}^2 \right) + \delta S_{4d}^{\rm higher} + S_{\rm matter}(g_{\mu\nu}, \psi), \quad (2.33)$$

where the higher derivative contribution is given by

$$\delta S_{4d}^{\rm high} = \int d^4x \sqrt{-g} M_{\rm Pl}^2 \alpha_3 \frac{R^2}{M^2} \times \left(\tilde{l}_6^2 R + \mathcal{V}_0 \frac{\ell_s^4}{\tilde{l}_6^4} \sum_{p\geq 3} \frac{\alpha_{p+1}}{\alpha_3} (\ell_s^2 R)^{p-1} \right). \quad (2.34)$$

Here we have assumed that the volume modulus is fixed at its value $\mathcal{V}_0$. The R^2 couplings are modulus dependent and in the large volume limit are dominated by the volume compactification (see [26,27] for instance for a discussion of R^2 terms from string theory). These expressions are symbolic, and R represents either the Riemann tensor, the Ricci tensor or the Ricci scalar. We now briefly review the analysis of these higher derivative corrections as given in Appendix A of [10] and Sec. 5.1 of [8] (which we follow for the notations), where it is argued that the higher derivative corrections to the Einstein-Hilbert action are suppressed in the large volume expansion.

The dimensionless coefficient c_0 of the scalar $\mathcal{R}^2$ term is given by

$$c_0 = \alpha_3 \frac{M_{\rm Pl}^2}{M^2}, \quad (2.35)$$

where the mass scales $M_{\rm Pl}^2$ and M^2 depend on the string coupling constant g_s, the string scale ℓ_s and the compactification scale ℓ_6:

$$M_{\rm Pl}^2 \simeq \frac{l_6^6}{g_s^2 \ell_s^8} \left(1 + \sum_{n\geq 3} \alpha_n \frac{\ell_s^{2n}}{\tilde{l}_6^{2n}} \right); \qquad M = \frac{\tilde{l}_6^2}{\ell_s^3}, \quad (2.36)$$

and the volume-dependent curvature scale

$$\tilde{l}_6 = \mathcal{V}_0^{1/6} l_6, \quad (2.37)$$

which controls the typical averaged value of the ten-dimensional curvature R over the compactification manifold M_6:

$$\int_{M_6} \frac{d^6x}{V_6}\sqrt{-g}R^n \sim \tilde{l}_6^{-2n}. \tag{2.38}$$

A condition for the curvature expansion to make sense is that $\tilde{l}_6 \gg \ell_s$, where ℓ_s is the string scale. The higher-order terms in the curvature expansion depend on the couplings

$$d_p \simeq \sum_{n\geq \max(p+1,3)} \alpha_n \left(\frac{\ell_s}{\tilde{l}_6}\right)^{2(n-p-1)}, \tag{2.39}$$

and the series are dominated by the first term in the regime $\tilde{l}_6 \gg \ell_s$. This gives

$$d_0 \simeq \alpha_3 \left(\frac{\ell_s}{\tilde{l}_6}\right)^4, \qquad d_1 \simeq \alpha_3 \left(\frac{\ell_s}{\tilde{l}_6}\right)^2, \qquad p\geq 2\colon\ d_p \simeq \alpha_{p+1}. \tag{2.40}$$

This immediately gives us indications that the $\mathcal{R}^2$ models are a valid description of the physics at low energy up to the string scale $M_s = 1/\ell_s$ as long as curvature is small in string units, i.e., $\ell_s^2 R \ll 1$. This suppresses all the higher curvature terms and leaves only the dimension-four R^2 terms and the dimension-six R^3 corrections as relevant at low energy. At the four derivative order, the Riemann tensor square terms can be traded for a combination of the Ricci scalar square and the Ricci tensor square $(R_{\mu\nu})^2$ using the standard Gauss-Bonnet identity in four dimensions. The $(R_{\mu\nu})^2$ term induces a massive ghost spin-2 state, with a mass determined by the coefficient of the coupling in the effective action [18]. The coefficients of these terms are affected by the metric field redefinition $g_{\mu\nu} \to g_{\mu\nu} + \alpha R_{\mu\nu} + \beta \mathcal{R} g_{\mu\nu}$ so that the coefficient of the $\mathcal{R}^2$ is shifted by $(\alpha + 3\beta)/3$ and the $R^2_{\mu\nu}$ term by α [28,29]. By considering the dynamics of supersymmetry breaking involving the volume modulus as in Sec. 5.2.2 of [8], we can consider a situation where $\beta \gg \alpha$ and that the coefficient of the $R^2_{\mu\nu}$ term is such that the mass of the massive spin 2 is at the order of the UV cutoff of the theory. In fact, we require that the only physical compactifications are the ones which lead to the absence of ghosts or such that the ghosts are rejected at the UV cutoff scale of the theory. As a result, the remaining higher derivative corrections in (2.33) are the powers of the Ricci scalar $\mathcal{R}$.

The $\mathcal{R}^2$ model only applies up to the curvature scale $\mathcal{R} \leq \tilde{l}_6^{-2}$, which is typically much greater than practical curvature scales. This suppresses the dimension-six cubic $\mathcal{R}^3$ corrections compared to the quadratic one, and we can focus on the $\mathcal{R}^2$ action in (2.33):

$$S^{\rm eff}_{4d} = \int d^4x\sqrt{-g}\left(\frac{M^2_{\rm Pl}}{2}[V_{\rm vac}(\mathcal{V}) + \mathcal{V}\mathcal{R}] + c_0\mathcal{V}_0\mathcal{R}^2\right) + S_{\rm matter}(g_{\mu\nu},\psi). \tag{2.41}$$

We can now apply the analysis of Sec. II B this time starting with the action (2.41). The resulting $f(\mathcal{R})$ action in (2.27) after stabilization of the volume becomes

$$S^{\rm eff}_{4d} = \int d^4x\sqrt{-g}\left[\frac{m^2_{\rm Pl}}{2}\mathcal{R} + \left(c_0\mathcal{V}_0 + \delta c_0\right)\mathcal{R}^2\right] + S_{\rm matter}(g,\psi). \tag{2.42}$$

The coefficient of the $\mathcal{R}^2$ is given by the c_0 coefficient in (2.35) from the compactification of the string-induced corrections and the corrections induced from the volume stabilization in (2.28):

$$c_0\mathcal{V}_0 + \delta c_0 = \frac{\alpha_3 M^2_{\rm Pl}}{M^2}\mathcal{V}_0 + \frac{m^2_{\rm Pl}}{4m^2\mathcal{V}_0} = \frac{m^2_{\rm Pl}}{2}\left(\frac{\alpha_3}{M^2} + \frac{1}{2m^2\mathcal{V}_0}\right). \tag{2.43}$$

Notice that both terms are large as $m \ll m_{\rm Pl}$ and the compactification scale is such that $\tilde{l}_6 \gg \ell_s$. We can then just apply the formalism of Sec. II D in the Jordan frame with this time the function

$$f(\mathcal{R}) = \mathcal{R} + \left(\frac{1}{2m^2\mathcal{V}_0} + \frac{\alpha_3}{M^2}\right)\mathcal{R}^2. \tag{2.44}$$

This allows us to identify the volume modulus as

$$\mathcal{V} = \mathcal{V}_0 + \left(\frac{1}{m^2} + \frac{2\alpha_3\mathcal{V}_0}{M^2}\right)\mathcal{R}. \tag{2.45}$$

This relation is not surprising as the volume modulus has no dynamics in the original Jordan frame and can therefore be integrated out exactly. As the volume modulus mixes with the Ricci scalar of the string compactification, this implies that the volume modulus is stabilized up to fluctuations which are parametrized by the Ricci scalar. As the Ricci scalar is also related to the scalaron, we find that the volume modulus and the scalaron become one and only one field. We can also identify the scalaron used in the Weyl transformation to the Einstein frame as

$$\frac{\mathcal{V}}{\mathcal{V}_0} = e^{-2\beta\varphi}. \tag{2.46}$$

The volume modulus deviates from its stabilized value by an amount which is parametrized by the scalaron. In the regime where the curvature is small compared to the large scale M, we find that such a deviation is small and the scalaron has a small excursion in Planck units.

In the following, we will reverse engineer the construction and obtain the low-energy $N = 1$ supergravity description for a scalar field whose coupling to matter is given by $\beta = 1/\sqrt{6}$. This will determine the Kähler potential of the associated chiral superfield. By imposing that the scalar potential is the one of the Starobinski model, we will determine the superpotential. The Kähler potential that we obtain is the one of the volume modulus with deformations parametrized by two parameters only. In a sense, this confirms that the effective $N = 1$ supergravity description of Starobinski's model is related to the volume modulus and that its effective dynamics are determined by a superpotential whose shape is uniquely determined. Finding explicit string theory models whose behavior mimics these results is a challenge left for future work.

III. SUPERSYMMETRY

A. The volume modulus and its Kähler potential

So far we have not taken into account one crucial ingredient: supersymmetry. The compactifications that we consider should result in a supergravity theory in four dimensions. Written in terms of a modulus superfield T, its Kähler potential and its superpotential $W(T, \bar{T})$, the resulting $N = 1$ supergravity theory should be such that the matter fermions ψ associated to another superfield C have a coupling given by $\beta = 1/\sqrt{6}$. The archetypical example of this type of behavior is given by the Kähler potential

$$K(T, \bar{T}) = -3m_{\rm Pl}^2 \ln\left(\frac{T+\bar{T}}{m_{\rm Pl}} - \frac{C\bar{C}}{3m_{\rm Pl}^2}\right) \simeq -3m_{\rm Pl}^2 \ln\frac{T+\bar{T}}{m_{\rm Pl}} + m_{\rm Pl}\frac{C\bar{C}}{T+\bar{T}}. \quad (3.1)$$

As we will recall below, this gives rise to the expected $\beta = \frac{1}{\sqrt{6}}$ coupling. Moreover, a constant superpotential

$$W = W_0 \quad (3.2)$$

gives rise to a vanishing scalar potential for the volume modulus.

In the following, we will generalize the Kähler potential and superpotential by imposing that the coupling to matter is determined by β and that the scalar potential follows from the $\mathcal{R}^2$ theory as expressed in the Einstein frame via the scalaron.

B. From supergravity to the scalaron

We define the scalaron field as the canonically normalized real part of T, the universal Kähler modulus determining the volume of compactification; i.e., we require that

$$d\varphi = -\sqrt{2K_{T\bar{T}}(T, T)}\frac{dT}{m_{\rm Pl}}. \quad (3.3)$$

Notice the minus sign. Here the Kähler potential is left unspecified and will be determined by requiring that matter couples to the scalarons like in $f(\mathcal{R})$ theories.

Let us consider matter fields C representing the superfield associated to the Weyl fermions ψ_C. We assume that the Kähler potential of these matter fields is in

$$K \supset \frac{C\bar{C}}{(T+\bar{T})}. \quad (3.4)$$

In supergravity this implies that the kinetic terms for the fermions are

$$i\frac{\partial\bar{\psi}_C\sigma^\mu D_\mu\psi_C}{(T+\bar{T})}, \quad (3.5)$$

which can be normalized according to

$$\psi = \frac{\psi_C}{(T+\bar{T})^{1/2}}. \quad (3.6)$$

Now a mass term for the fermions depends on the superpotential and the Kähler potential

$$\mathcal{L} \supset e^{K(T,\bar{T})/2m_{\rm Pl}^2}\frac{\partial W}{\partial C^2}\psi_C^2 \quad (3.7)$$

corresponding to a Majorana mass term. Dirac mass terms can be constructed by taking pairs of superfield coupled to a Higgs field. In terms of the normalized fermions this becomes

$$\mathcal{L} \supset e^{K(T,\bar{T})/2m_{\rm Pl}^2}(T+\bar{T})\frac{\partial W}{\partial C^2}\psi^2. \quad (3.8)$$

As the superpotential cannot depend on the modulus field T at the perturbative level, this mass term can be written as

$$\mathcal{L} \supset A(\varphi)m\psi^2, \quad (3.9)$$

where $m = \frac{\partial W}{\partial C^2}$ is the modulus-independent mass and

$$A(\varphi) = e^{K(T,\bar{T})/2m_{\rm Pl}^2}(T+\bar{T})|_{T(\varphi)} \quad (3.10)$$

is a factor dependent on the normalized field ϕ.

C. The Kähler potential from the $f(\mathcal{R})$ matter coupling

We can try to find the Kähler potentials such that the normalized real part of T, i.e., $\mathrm{Re}(T) = m_{\rm Pl}t$, has a coupling given by

$$A(\varphi) = e^{\beta\varphi} = e^{\frac{K(T,\bar{T})}{2m_{\rm Pl}^2}}(T+\bar{T})|_{T(\varphi)}. \quad (3.11)$$

We set $K(T, \bar{T})|_{T=\bar{T}=t} = k(t)m_{\rm Pl}^2$. We find from (3.3) that

$$\left(\frac{d\varphi(t)}{dt}\right)^2 = \frac{1}{2}\frac{d^2k(t)}{dt^2}, \qquad (3.12)$$

where we used that

$$\frac{d^2k(t)}{dt^2} = 4K_{T\bar{T}}(T,\bar{T})|_{T=\bar{T}=t}. \qquad (3.13)$$

Imposing (3.11) gives the relation

$$k(t) = 2\beta\varphi(t) - 2\ \ln(2t), \qquad (3.14)$$

which leads to the differential equation

$$\left(\frac{d\varphi(t)}{dt}\right)^2 = \beta\frac{d^2\varphi(t)}{dt^2} + \frac{1}{t^2}. \qquad (3.15)$$

We leave β indeterminate, and we will show how β is uniquely determined to the value $\beta = 1/\sqrt{6}$. This equation has for solutions for all values of a and b

$$\varphi(t) = \frac{a+\log(4)}{2\beta} + \frac{1}{2}\left(-\beta + \sqrt{\beta^2+4}\right)\ln(t) - \beta\ln\left(t^{\frac{\sqrt{\beta^2+4}}{\beta}} - b\right). \qquad (3.16)$$

The Kähler potential reads (after redefining the arbitrary constant a)

$$k(t) = a - \left(2+\beta^2 - \beta\sqrt{\beta^2+4}\right)\ \log(t) - 2\beta^2\log\left(t^{\frac{\sqrt{4+\beta^2}}{\beta}} - b\right). \qquad (3.17)$$

Requesting that $k(t) = -3\ \ln(2t) + O(1)$ for large t gives

$$2+\beta^2+\beta\sqrt{4+\beta^2} = 3, \qquad (3.18)$$

whose unique solution is $\beta = 1/\sqrt{6}$. Hence, we have found that the only coupling constant β in the identification between the coupling of fermion fields in supergravity and a scalaron coupling is the one compatible with the $f(\mathcal{R})$ structure.

The resulting Kähler potential is given by

$$k(t) = a - \frac{4}{3}\ln(t) - \frac{1}{3}\ln(t^5 - b) = a - 3\ \ln t + \frac{1}{3}\sum_{n\geq 1}\frac{b^n}{t^{5n}}. \qquad (3.19)$$

The associated scalaron solution reads

$$2\beta\varphi(t) = a + \log(4) + \frac{2}{3}\log(t) - \frac{1}{3}\log(t^5 - b) = a + \log(4) - \log t + \frac{1}{3}\sum_{n\geq 1}\frac{b^n}{t^{5n}}. \qquad (3.20)$$

When $b < 0$ we have that $\varphi(t) \to -\infty$ for $t \to 0$ and $t \to +\infty$ with $\varphi(t_*) = 0$ for

$$64e^{3a}t_*^2 = t_*^5 - b. \qquad (3.21)$$

When $b > 0$ we must have $t > b^{-1/5}$, and $\varphi(t) \to +\infty$ for $t \to 1/b^{1/5}$ and $t \to +\infty$.

In conclusion, the most general Kähler potential which is compatible with the coupling to matter of $f(\mathcal{R})$ models is a modification of the no-scale models by a series of corrections in $1/t^5$.

D. The scalaron potential and the reconstruction of the superpotential

The $\mathcal{R}^2$ model is associated to a specific scalar potential for the scalaron $V(\varphi)$. This is then equivalent to having a scalar potential $V(\varphi(t))$ using the mapping $\varphi(t)$. In the following we will focus on the theory

$$S = \frac{m_{\mathrm{Pl}}^2}{2}\int d^4x f(\mathcal{R}), \qquad (3.22)$$

where we introduce an explicit cosmological constant

$$f(\mathcal{R}) = -2\Lambda + \mathcal{R} + c\mathcal{R}^2. \qquad (3.23)$$

There the coefficient c is related to the coefficient c_0 and the correction δc_0 discussed previously by

$$c_0 + \delta c_0 = \frac{m_{\mathrm{Pl}}^2 c}{2}. \qquad (3.24)$$

We have then the identification

$$e^{-2\beta\varphi} = 1 + 2c\mathcal{R} \qquad (3.25)$$

and finally the scalar potential

$$V(\varphi) = \frac{(1-e^{2\beta\varphi})^2}{4c} + 2\Lambda e^{4\beta\varphi}, \qquad (3.26)$$

which becomes a function of t using (3.16):

$$V(t) = \frac{(32\Lambda c + 8)t^{\frac{4}{3}}e^{2a}}{(t^5-b)^{\frac{2}{3}}c} - \frac{4t^{\frac{2}{3}}e^a}{(t^5-b)^{\frac{1}{3}}c} + \frac{1}{2c}. \qquad (3.27)$$

Interestingly, this potential has a minimum for the value

$$e^{2\varphi_{\min}} = \frac{1}{1+8c\Lambda} \qquad (3.28)$$

for which the potential energy becomes

$$V(\varphi_{\min}) = \frac{2\Lambda}{1+8c\Lambda}. \tag{3.29}$$

When $\Lambda > 0$, this implies that supersymmetry is broken at the minimum of the potential. We will concentrate on this case below and therefore supersymmetry will always be spontaneously broken in the supergravity models that we will consider.

Let us recall that in $N = 1$ supergravity and using Planck units in the following equations (the Planck scale can be easily reinstated by simple dimensional analysis), the scalar potential reads

$$V(t) = e^{k(t)}\left(k^{T\bar{T}}|\partial_T W + \partial_T k W|^2 - 3|W|^2\right), \tag{3.30}$$

where $k^{T\bar{T}} = 1/\partial_T\partial_{\bar{T}}k$. We have also $k_{T\bar{T}} = \frac{k''}{4}$, where $k'' = \frac{d^2k}{dt^2}$. In the following we will focus on a real superpotential when $T = \bar{T} = t$ is real, since we are focusing on the volume dependence which is the real part of the T modulus. We can always extend this by analytic continuation (in an open subset of the complex plane $\mathbb{C}$ containing the real line $\mathbb{R}$) to an holomorphic superpotential $\mathfrak{w}(T)$ matching $w(t)$ on the real line. Equation (3.30) becomes

$$\left(\frac{dW(t)}{dt} + \frac{1}{2}\frac{dk(t)}{dt}W(t)\right)^2 = \frac{k''(t)}{4}\left(3W(t)^2 + e^{-k(t)}V(t)\right). \tag{3.31}$$

It is convenient to define the function

$$w(t) = W(t)e^{\frac{k(t)}{2}}, \tag{3.32}$$

which is nothing but $w(t) = e^{G(t)/2}$, where $G(t) = k(t) + \ln|W(t)|^2$ characterizes the full $N = 1$ Lagrangian and shows the underlying Kähler invariance $k(T) \to k(T) + f(T) + \bar{f}(\bar{T})$, $W(T) \to e^{-f(T)}W(T)$, where f is a holomorphic function of T. Using this function we have

$$\left(\frac{dw(t)}{dt}\right)^2 - \frac{3}{4}k''(t)w(t)^2 = \frac{k''(t)}{4}V(t). \tag{3.33}$$

Given the function $V(t)$ obtained from the $f(\mathcal{R})$ theory, we can solve this differential equation and find the most general superpotential compatible with the $\mathcal{R}^2$ structure.

E. The superpotential associated to $\mathcal{R}^2$

As a sanity check, let us consider a no-scale model with $V(t) = 0$ and $k(t) = -3\ln t$, i.e., $b = 0$ in (3.19). From the differential equation (3.31) we retrieve the familiar no-scale model where $W(t) = w_0$ is a constant and another solution $W(t) = w_0 t^3$ for which the potential also vanishes.

For the superpotential associated to $\mathcal{R}^2$ with $k(t)$ given in (3.19), and the potential (3.27), the differential equation has the unique solution (this is unique up to an overall sign)

$$\begin{aligned} w_\pm(t) = \frac{\pm 1}{\sqrt{-6c}}\Bigg(& 1 - \frac{4\mathrm{e}^a}{t} + \frac{32(9\Lambda c+1)\mathrm{e}^{2a}}{9t^2} + \frac{128(9\Lambda c+1)\mathrm{e}^{3a}}{81t^3} \\ &+ \frac{(290304(\Lambda c)^2 + 50688\Lambda c + 2048)\mathrm{e}^{4a}}{729t^4} + \frac{(-3151872(\Lambda c)^2 - 313344\Lambda c + 4096)\mathrm{e}^{5a}}{6561t^5} \\ &+ \frac{1}{t^6}\left(\frac{(213663744(\Lambda c)^3 + 78446592(\Lambda c)^2 + 7626752\Lambda c + 172032)\mathrm{e}^{6a}}{6561} - \frac{4b\mathrm{e}^a}{3}\right) + O\left(\frac{1}{t^7}\right)\Bigg). \end{aligned} \tag{3.34}$$

The superpotential is obtained by $W_\pm(t) = w_\pm(t)\exp(-k(t)/2)$. This is uniquely determined from the Kähler potential and the $f(\mathcal{R})$ functional.

Since $k(t) \simeq -3\log(t)$ for large t we get a diverging expression at large volume for the superpotential. This large t behavior can be removed using a Kähler transformation (see Chap. XXIII in [30]) $K(T,\bar{T}) \to K(T,\bar{T}) + f(T) + \bar{f}(\bar{T})$ and $W(T,\bar{T}) \to e^{-f(T)}W(T,\bar{T})$ with $f(T) = \frac{3}{4}\log(T/m_{\rm Pl}) + \log(\pm i)$. This shifts the Kähler potential to

$$K(T,\bar{T}) = m_{\rm Pl}^2\left(a - \frac{4}{3}\log\left(\frac{T+\bar{T}}{m_{\rm Pl}}\right) - \frac{1}{3}\log\left(\left(\frac{T+\bar{T}}{m_{\rm Pl}}\right)^5 - b\right) + \frac{3}{4}\log\left(\frac{T\bar{T}}{m_{\rm Pl}^2}\right)\right) \tag{3.35}$$

with

$$\begin{aligned} W(t) = m_{\rm Pl}^3\frac{e^{-\frac{a}{2}}}{\sqrt{6c}}\Bigg(& 1 - \frac{4e^a}{t} + \frac{32(9\Lambda c+1)e^{2a}}{9t^2} + \frac{128(9\Lambda c+1)e^{3a}}{81t^3} + \frac{512e^{4a}(567(\Lambda c)^2 + 99\Lambda c + 4)}{729t^4} \\ &+ \frac{1}{t^5}\left(\frac{(-6303744(\Lambda c)^2 - 626688\Lambda c + 8192)e^{5a}}{13122} - \frac{b}{6}\right) + \cdots\Bigg). \end{aligned} \tag{3.36}$$

The Kähler potential is lacunar and the superpotential is an infinite series in $1/t$ where $\mathrm{Re}(T) = m_{\mathrm{Pl}}t$. This is valid when $b \neq 0$ where the first correction in b appears at the t^{-5} order.

When $b = 0$, the Kähler potential is $k(t) = a - 3\log(t)$, the $f(\mathcal{R})$ potential is

$$V(t)|_{b=0} = \frac{(32\Lambda c + 8)\mathrm{e}^{2a}}{t^2 c} - \frac{4\mathrm{e}^a}{tc} + \frac{1}{2c}, \tag{3.37}$$

and the superpotential is given by

$$\begin{aligned} W(t)|_{b=0} = m_{\mathrm{Pl}}^3 \frac{e^{-\frac{a}{2}}}{\sqrt{6c}} \Bigg(& 1 - \frac{4e^a}{t} + \frac{32(9\Lambda c + 1)e^{2a}}{9t^2} + \frac{128(9\Lambda c + 1)e^{3a}}{81t^3} \\ & + \frac{(290304(\Lambda c)^2 + 50688\Lambda c + 2048)e^{4a}}{729t^4} - \frac{2048(1539(\Lambda c)^2 + 153\Lambda c - 2)e^{5a}}{6561t^5} \\ & + \frac{(213663744(\Lambda c)^3 + 78446592(\Lambda c)^2 + 7626752\Lambda c + 172032)e^{6a}}{6561t^6} + \cdots \Bigg). \end{aligned} \tag{3.38}$$

We find an expansion in te^{-a} where a is the constant of integration in the Kähler potential. Actually shifting the value of a amounts to redefining the Planck mass m_{Pl}.

We notice that the cosmological constant Λ and the coefficient c of the $\mathcal{R}^2$ term appear in the combination Λc. We now turn to the quantum evolution of c, from the ultraviolet (the inflationary period) to the infrared regime (the late time behavior). We will relate the evolution of c along the renormalization group flow to the values of the evolution of the cosmological constant.

IV. THE QUANTUM EVOLUTION OF *c*

The model that we have considered so far is valid in the ultraviolet at high energy, i.e., at the compactification scale. We will recall how inflation appears in this setting and then link this behavior to the properties of the scalaron in the infrared regime.

A. Starobinski inflation

The $\mathcal{R}^2$ model is a very good candidate for inflation. During the inflationary regime the potential in dimensionful units, defined as $V_{\mathrm{inflation}}(\varphi) = m_{\mathrm{Pl}}^2 V(\varphi)/2$, is

$$V_{\mathrm{inflation}}(\varphi) = \frac{m_{\mathrm{Pl}}^2}{8c}(e^{2\beta\varphi} - 1)^2 + \Lambda m_{\mathrm{Pl}}^2 e^{4\beta\varphi}. \tag{4.1}$$

Inflation takes place in a quasi–de Sitter region of field space with $\varphi \to -\infty$ such that $e^{2\beta\varphi} \ll 1$. This is the large volume regime where $t \to \infty$. Notice that the cosmological constant Λ plays no role in this regime. We have therefore

$$V_{\mathrm{inflation}}(\varphi) \simeq \frac{m_{\mathrm{Pl}}^2}{8c}(1 - 2e^{2\beta\varphi}). \tag{4.2}$$

The slow roll parameters are then

$$\epsilon = \frac{1}{2}\left(\frac{V'_{\mathrm{inflation}}}{V_{\mathrm{inflation}}}\right)^2 \simeq 8\beta^2 e^{4\beta\varphi} \tag{4.3}$$

and

$$\eta = \frac{V''_{\mathrm{inflation}}}{V_{\mathrm{inflation}}} \simeq -8\beta^2 e^{2\beta\varphi}. \tag{4.4}$$

Observable scales by the cosmic microwave background (CMB) are N e-folding before the end of inflation where $\epsilon_{\mathrm{end}} = 1$. For these scales, the η parameter dominates over ϵ:

$$\epsilon_\star \ll |\eta_\star|, \tag{4.5}$$

and the spectral index is then given by

$$n_s - 1 = 2\eta_\star \simeq -16\beta^2 e^{2\beta\varphi_\star}. \tag{4.6}$$

This determines $\varphi_\star$ as $n_s - 1 \simeq -0.0351$ according to Planck [31]. The number of e-foldings is $a_{\mathrm{end}}/a_\star = e^N$, where a is the scale factor of the Universe with

$$N = \int_{\varphi_\star}^{\varphi_{\mathrm{end}}} \frac{V_{\mathrm{inflation}}}{V'_{\mathrm{inflation}}} d\varphi. \tag{4.7}$$

We find

$$N \simeq \frac{1}{8\beta^2} e^{-2\beta\varphi_\star}, \tag{4.8}$$

and therefore

$$n_s - 1 = -\frac{2}{N}, \tag{4.9}$$

which determines $N \simeq 56$ as $n_s - 1 \simeq -0.0351$. At the end of inflation we have

$$e^{-2\beta\varphi_{\rm end}} = 2\sqrt{2}\beta, \tag{4.10}$$

where $\epsilon(\varphi_{\rm end}) = 1$. Notice that the approximation $\varphi \to -\infty$ is not really valid toward the end of inflation. In principle this means that numerics is required to evaluate when $\epsilon = 1$ and inflation stops. After the end of inflation the field oscillates around zero and eventually reaches $\varphi \ll 1$.

B. Validity of the curvature expansion

At the end of inflation, the field φ settles at the minimum of the potential for $\varphi \ll 1$ where it becomes massive with

$$m_\varphi^2 = \frac{\beta^2}{c} + 16\beta^2\Lambda, \tag{4.11}$$

where the second term is negligible at the end of inflation. The value of c at the end of inflation can be deduced from the normalization of the CMB spectrum as

$$\frac{V_{\rm inflation}(\varphi_\star)^3}{m_{\rm Pl}^6(V'_{\rm inflation}(\varphi_\star))^2} \simeq 2\times 10^{-11} \tag{4.12}$$

evaluated at the value of $\varphi_\star$ determined in (4.6), which implies that

$$c^{\rm end} m_{\rm Pl}^2 \simeq \frac{5}{8}\times 10^{10}\beta^2 N^2. \tag{4.13}$$

This is the value of c at the end of inflation. This gives a mass for m_φ around $2\times 10^{-7} m_{\rm Pl}$.

Now we can come back to the curvature expansion and check that the expansion in powers of the Ricci scalar is valid. The expansion is valid as long as $\tilde{l}_6^2\mathcal{R} \ll 1$ and $l_s^2\mathcal{R} \ll 1$. During inflation we have $2c\mathcal{R} \approx \exp(-2\beta\varphi)$ and during inflation

$$2\sqrt{2}\beta \le e^{-2\beta\varphi} \le 8\beta^2 N, \tag{4.14}$$

implying that

$$\tilde{l}_6^2 \ll \frac{c}{4\beta^2 N}. \tag{4.15}$$

As c is given by (4.13) we find that

$$\tilde{l}_6 \le m_{\rm Pl}^{-1}\sqrt{\frac{5N}{32}}10^5, \tag{4.16}$$

implying that the compactification scale must be

$$\tilde{l}_6^{-1} \ge 3.4\times 10^{-6} m_{\rm Pl}, \tag{4.17}$$

which is close to the grand unified theory scale. This confirms that the curvature expansion in (2.34) is valid.

C. Validity of the quadratic expansion for the scalar potential

We have shown that the quadratic terms around the minimum value $\mathcal{V}_0$ of the nonperturbative scalar potential $V_{\rm vac}(\mathcal{V})$ as a function of the volume modulus $\mathcal{V}$ in (2.26) are enough to generate the inflationary Starobinski potential. One issue with this description is that the excursion of the volume modulus (4.14) during inflation must be large and could jeopardize the quadratic approximation. Using (2.11) we have

$$\frac{V}{V_6} = \left(\frac{\mathcal{V}}{\mathcal{V}_0}\right)^{3/2}, \tag{4.18}$$

where $V_6 = \ell_6^6$ is the stabilized volume of the compactification manifold. Using dimensional analysis, we can write the nonperturbative potential as

$$\begin{aligned} V_{\rm vac}(\mathcal{V}) &= M^2 F(M^6 V) = M^2 F\left(M^6 V_6\left(\frac{\mathcal{V}}{\mathcal{V}_0}\right)^{3/2}\right) \\ &= M^2 G\left(\alpha\frac{\mathcal{V}}{\mathcal{V}_0}\right), \end{aligned} \tag{4.19}$$

where $G(x) \equiv F(x^{3/2})$ is a nonperturbative function which is assumed to vanish when $\mathcal{V} \to \infty$. Here we have used, in the spirit of effective field theories, that the low-energy dynamics of the volume modulus are determined by a single nonperturbative scale M and the function $G(x)$. We have introduced the dimensionless parameter $\alpha = (M^6 V_6)^{2/3}$. Now the function $-G$ is assumed to have minimum for $x = \alpha$, so we can expand in Taylor series

$$\begin{aligned} V_{\rm vac}(\mathcal{V}) &= M^2 G(\alpha) + \frac{g_2}{2}\alpha^2 M^2\left(\frac{\mathcal{V}}{\mathcal{V}_0} - 1\right)^2 \\ &\quad + M^2\sum_{n\ge 3}\frac{g_n}{n!}\alpha^n\left(\frac{\mathcal{V}}{\mathcal{V}_0} - 1\right)^n, \end{aligned} \tag{4.20}$$

where we assume that $g_n = \mathcal{O}(1)$. The first terms lead to the quadratic Lagrangian (2.26) with

$$V_{\rm vac} = -M^2 G(\alpha), \qquad m^2 = -g_2\alpha^2 M^2, \tag{4.21}$$

while the higher-order terms are negligible as long as

$$M\ell_6 \ll (8\beta^2 N)^{1/4} \tag{4.22}$$

corresponding to a small suppression of the nonperturbative scale M compared to the compactification scale ℓ_6^{-1}.

D. Postinflationary era

After inflation which occurs at high energy, we follow a Wilsonian approach, reviewed in [32–35] for instance, and consider the effective action obtained by integrating out all the momentum scales larger than a given scale μ, $\mu \ll |p|$ and much smaller than the compactification scale, to obtain the Wilson effective action at leading order:

$$S_W = \frac{m_{\rm Pl}^2}{2}\int d^4x f(\mathcal{R};\mu); \tag{4.23}$$

the Wilsonian Lagrangian reads

$$f(\mathcal{R}) = -2\Lambda(\mu) + \mathcal{R} + c(\mu)\mathcal{R}^2. \tag{4.24}$$

Having fixed the Planck mass $m_{\rm Pl}^2$, which is then independent of μ and fixes the scales, the parameters in the Wilson effective action S_W are μ dependent. The dependence of the Lagrangian parameters in (4.23) and (4.24) is given by the renormalization group equation that we will present below and review briefly in Appendix B. The initial values of the renormalization group are taken at the end of inflation corresponding to the reheat temperature $T_{\rm reh}$ taken to be larger than any physical masses of the particles in the spectrum of the theory. During the cosmological evolution, the change of the scalaron mass and couplings follow the renormalization group flow with respect to the scale μ which is cosmological time dependent, i.e., should be adapted to describe each cosmological era.

Although the Wilson effective action depends on the scale μ, the physical observables are independent of that scale. In our case the scale-independent observables are the energy density $\rho_{\rm vac}$ in (4.36) and the physical scalar mass m_φ obtained both in the IR corresponding to the limit $\mu \to 0$. In particular, the physical scalaron mass sets the range of the new scalar interaction to matter which can be tested using experiments such as Eöt-Wash [15].

The couplings evolve each time a particle species of mass m is integrated out [32], i.e., when $\mu \le m$. In the history of the Universe, particles in the thermal bath are integrated out when the temperature falls below the mass m. This allows us to estimate $\mu \sim T$. Let us review below how the renormalization evolution of the vacuum energy and the coupling c can be inferred. For explicit details on the regularization procedure, see Appendix B. Let us consider the quantum corrections to m_φ. As φ couples to fermions like $\frac{\beta m_\psi}{m_{\rm Pl}}\varphi\bar\psi\psi$, the one-loop contribution to the effective mass of φ is

$$\delta\bar m_\varphi^2(\mu) = -\frac{\beta^2 m_\psi^2}{m_{\rm Pl}^2}\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2+m_\psi^2}. \tag{4.25}$$

The divergent integral has to be renormalized. Using dimensional regularization and the decoupling scheme where the effects of the corrections are only nonvanishing at the renormalization scale $\mu \ge m_\psi$, we have

$$\delta\bar m_\varphi^2(\mu) = -\frac{\beta^2 m_\psi^4}{16\pi^2 m_{\rm Pl}^2}\ln\frac{\mu^2}{m_\psi^2}\theta(\mu - m_\psi), \tag{4.26}$$

where we have introduced the Heaviside function $\theta(x) = 1$ for $x > 0$ and 0 otherwise. This implies that the quantum corrections reduce the mass of the scalar when μ decreases and, therefore, increase the value of $c(\mu)$ from (4.11). In fact the scalaron couples to both massive scalars and massive vector bosons, implying that the correction to the mass at one-loop order is given by the supertrace

$$\delta\bar m_\varphi^2(\mu) = -\frac{\beta^2}{16\pi^2 m_{\rm Pl}^2}\mathcal{S}tr\left(M^4\ln\frac{\mu^2}{M^2}\theta(\mu - M)\right), \tag{4.27}$$

where M is the mass matrix of all the massive particles with a mass less than μ. This corresponds to a renormalization group equation (see Appendix B for a physical discussion about the regularization method):

$$\frac{d\bar m_\varphi^2(\mu)}{d\ln\mu} = -\frac{\beta^2}{8\pi^2 m_{\rm Pl}^2}\mathcal{S}tr(M^4)\theta(\mu - M). \tag{4.28}$$

Using the renormalization group equation for the cosmological constant

$$\frac{d\Lambda(\mu)}{d\ln\mu} = -\frac{1}{32\pi^2 m_{\rm Pl}^2}\mathcal{S}tr(M^4)\theta(\mu - M), \tag{4.29}$$

where again only particles with a mass less than μ contribute. Combining these expressions using (4.11) we get the renormalization group equation for $c(\mu)^{-1}$ with $\beta = 1/\sqrt{6}$:

$$\frac{dc(\mu)^{-1}}{d\ln\mu} = \frac{7}{16\pi^2 m_{\rm Pl}^2}\mathcal{S}tr(M^4) \equiv -14\frac{d\Lambda(\mu)}{d\ln\mu}. \tag{4.30}$$

This implies that

$$c_{\rm IR}^{-1} = c_{\rm end}^{-1} + 7(\Lambda_{\rm end} - \Lambda_{\rm IR}) - 14\sum_i \Lambda_i, \tag{4.31}$$

where we have introduced the sum over the jumps of the cosmological jumps when a phase transition happens corresponding to a jump of the value of the cosmological constant by Λ_i. This can be complemented with the evolution of the cosmological constant to the deep IR where μ is much lower than all the particle masses and give

$$\Lambda_{\rm IR} = \Lambda_{\rm end} + \sum_i \Lambda_i, \tag{4.32}$$

where $\Lambda_{\rm IR}$ is the cosmological constant obtained after integrating out all the quantum fluctuations and sending

$\mu \to 0$ in the Wilson effective action. This corresponds to the cosmological constant in the full 1PI effective action of the theory. We then obtain our final relation

$$c_{\rm IR}^{-1} = c_{\rm end}^{-1} - 7(\Lambda_{\rm end} - \Lambda_{\rm IR}). \tag{4.33}$$

We have used $1 + \frac{1}{\beta^2} = 7$. Notice that a positive cosmological constant at the end of inflation would naturally lead to a smaller value of c in the IR, i.e., an increase in the Lagrangian effective mass of the scalaron. We remark, as well, that the influence of the higher derivative terms that arise from the nonrenormalizability of the theory are negligible in the IR. As the cosmological constant $\Lambda_{\rm end}$ at the end of inflation is not directly determined by the experimental data, i.e., the dynamics of the Starobinski model is not influenced by the cosmological constant which only plays a role toward the end of inflation, we cannot calculate $c_{\rm IR}$ by following the renormalization group evolution from the UV to the IR. We have to resort in Sec. IV E to low-energy stability arguments to bound the value of $c_{\rm IR}$.

Finally let us comment on supersymmetry breaking in these models. Supersymmetry is broken dynamically during inflation, and then from the end of inflation onward once the scalaron settles at its minimum, the vacuum energy corresponding to the minimum of the scalar potential becomes

$$V_{\rm min} = \frac{\Lambda m_{\rm Pl}^2}{1 + 8\Lambda c} \tag{4.34}$$

which evolves with the renormalization flow. In the IR, this coincides with

$$V_{\rm min} \simeq \Lambda_{\rm IR} m_{\rm Pl}^2 \tag{4.35}$$

as we will see below that $c_{\rm IR}\Lambda_{\rm IR} \ll 1$. Hence, at low energy supersymmetry is spontaneously broken, although extremely softly, by the small vacuum energy of the Universe.

E. Low-energy stability

The value of c in the IR cannot be directly deduced from its value at the end of inflation without a detailed knowledge of high-energy physics and all the phase transitions between inflation and the present Universe.

On the other hand, the value of c in the infrared regime can be bounded by phenomenological stability arguments [9]. We will assume that the scalaron becomes light in the IR with the physical mass m_φ, given by the value of effective mass $\bar{m}_\varphi(\mu)$ for $\mu = 0$, which is directly related to $c_{\rm IR}$ by the relation in (4.11). In practice, we will take the mass of the scalaron much smaller than the electron and neutrino masses. In this model the scalaron at low energy is assumed to be the lightest massive particle in the Universe. Defining $\rho_\Lambda = \Lambda m_{\rm Pl}^2/2$, the renormalization group evolution in the deep IR below all particle masses gives

$$\rho_{\rm vac} = \rho_\Lambda(m_e) + \frac{m_\varphi^4}{64\pi^2}\ln\frac{m_e^2}{m_\varphi^2} - 2\sum_{f=1}^{3}\frac{m_f^4}{64\pi^2}\ln\frac{m_e^2}{m_f^2}, \tag{4.36}$$

where the observational value of the vacuum energy is simply $\rho_{\rm vac} \simeq 2.7 \times 10^{-11}$ eV^4. The vacuum energy at the energy of the electron mass has been denoted by $\rho_\Lambda(m_e)$. This encapsulates our lack of knowledge of the physics at scales larger than m_e. The neutrinos also contribute at low energy, and we have for the two possible hierarchies of neutrino masses. For both ordering the neutrino contribution is bounded (see Sec. V of [9]):

$$2 \times 10^4 \rho_{\rm vac} \le \sum_{f=1}^{3}\frac{m_f^4}{64\pi^2}\log\left(\frac{m_e^2}{m_f^2}\right) \le 2 \times 10^5 \rho_{\rm vac}. \tag{4.37}$$

This is obviously a large contribution which exemplifies the nature of the cosmological constant problem even at low energy.

Let us now invoke the stability argument of [9]. The vacuum energy $\rho_\Lambda(\mu)$ cannot be too negative; otherwise, any bound structure in the Universe whose constituents have a typical energy μ would collapse faster than the age of the Universe. We will therefore impose that $\rho_\Lambda(\mu) \ge -\rho_\Lambda$ to guarantee the stability of the Universe.

The x-ray emitting gas of a galaxy cluster has a typical temperature of $T_{\rm X} \sim 1$ keV. These systems typically appeared at a redshift $z \gtrsim 0.1$ and already have a lifetime of the order of the age of the Universe. At these energies, the vacuum energy corresponds $\rho_\Lambda(m_e)$ and the absence of collapse of the clusters over the age of the Universe implies that $\rho_\Lambda(m_e) \gtrsim -\rho_{\rm vac}$. This implies that [31,36–40]

$$m_\varphi \lesssim \bar{m}_\nu = \left(m_1^4 + m_2^4 + m_3^4\right)^{\frac{1}{4}} \simeq 0.1 \text{ eV}. \tag{4.38}$$

Similarly, the scalaron could appear as contributing to a fifth force in gravitational experiments [18]:

$$V(r) = -\frac{G_N M}{r}\left(1 + \frac{1}{3}e^{-m_\varphi r}\right). \tag{4.39}$$

The absence of evidence for short-range forces in the Eöt-Wash experiment [15,41,42] provides an upper bound on the range of scalar forces $d \le 52$ μm corresponding to the strong lower bound

$$m_\varphi \gtrsim 3.8 \times 10^{-3} \text{ eV}. \tag{4.40}$$

We have thus an interval of masses for the nearly massless scalaron. This is a fairly narrow interval provided the scalaron is less massive than the electron mass. This implies in particular

$$\Lambda_{\rm IR} c_{\rm IR}^2 \simeq 8\pi G_N, \tag{4.41}$$

a relation that can be tested by low-energy laboratory experiments [8,9]. Notice that this would also give directly the value of the cosmological constant at the end of Starobinski's inflation:

$$c_{\rm end}\Lambda_{\rm end} = \frac{1}{7} \tag{4.42}$$

as the other contributions to (4.33) are negligible. This determines the product Λc which appears in the superpotential (3.38) leading to Starobinski's inflation. Numerically using (4.13) we have $\Lambda_{\rm end} \simeq \frac{8}{35} \times 10^{-10}\beta^{-2}N^{-2}m_{\rm Pl}^2$. This determines the energy scale of the cosmological constant during inflation:

$$E_{\rm inf} \equiv (\Lambda_{\rm end} m_{\rm Pl}^2)^{\frac{1}{4}} \simeq \left(\frac{8\times 10^{-10}}{35\beta^2 N^2}\right)^{\frac{1}{4}} m_{\rm Pl} \simeq 9\times 10^{14}\ \text{GeV} \tag{4.43}$$

corresponding to a sub-Planckian regime of the effective field theory after compactification with a cosmological constant $E_{\rm inf}$ close to the grand unified scale.

From a UV point of view, this value only reinforces the fact that the physics at high energy seems to be largely constrained by the physics at low energy. This is the case of the mass of the scalaron during inflation which is constrained by the CMB data. Here we found that the physics of the vacuum in the IR, i.e., the vacuum stability combined with gravitational tests, determines indirectly the value of the cosmological constant in the UV. Of course, our analysis does not provide any explanation for this value from a top-bottom point of view.

Finally, let us mention that the narrow interval of mass 3.8×10^{-3} eV $\lesssim m_\varphi \lesssim 0.1$ eV is compatible with the value $m_\varphi \simeq 4.4\times 10^{-3}$ eV for which the scalaron could be at the origin of the observed dark matter abundance [20,43–45]. In this scenario, the coupling of the scalaron to the Higgs field, coming from the coupling to matter that we have discussed at length, implies that, at low energy compared to the inflation scale, the vacuum expectation value of the scalaron φ is displaced from the origin by an amount depending on the electroweak scale $v \sim 250$ GeV.[2] As the electroweak transition begins, the scalaron starts oscillating with a decreasing amplitude, eventually converging to the origin. This misalignment mechanism is similar to what happens for axions and leads an abundance of dark matter which fits the observed value for $m_\varphi \simeq 4.4\times 10^{-3}$ eV [45]. Combining both scenarios, this would lead to a possible signal in gravitational experiments below a distance $d \lesssim 45$ μm. The possibility of testing the existence of a new interaction mediated by the scalaron whose existence could play a role in both dark energy and dark matter is certainly worth pursuing.

[2]For more generic initial conditions after inflation taking into account the quantum fluctuations of the scalaron during inflation, the whole interval up to $m_\varphi \simeq 0.1$ eV could accommodate dark matter.

V. CONCLUSION

The dark sector of the Universe and in particular dark energy could be the result of the gravitational dynamics of the Universe. This could follow from massive gravity for instance or scalar theories which would mimic the behavior of the cosmological constant in the late time limit. Another possibility which has been mostly overlooked is that the dark energy could result from the IR limit of the vacuum energy of a theory whose spectrum would include at least one light degree of freedom coming from the gravitational sector of the model. This would influence the renormalization group evolution of the vacuum energy and could combine its effect to the contributions of the neutrinos to generate the right amount of dark energy. In this article, we consider such a scenario where the light field is the volume modulus of string compactifications whose effective field theory at low energy is a $f(\mathcal{R})$ model of the Ricci scalar squared $\mathcal{R}^2$ type. This is true once the volume modulus is stabilized and as long as the curvature of the Universe is lower than the string and compactification scales. Using the reasonable assumption that the vacuum energy in the IR is never too negative to imply the collapse of structures in the Universe, we find that the mass of the scalaron in the IR is tied to the measured cosmological constant in a way which could be testable with future tests of gravity in the submillimeter range. En route, we describe how the scalaron's, i.e., the volume modulus', effective field theory after compactification can be described as a $N = 1$ supergravity with a two-parameter family of Kähler potentials, including the familiar no-scale models, and a unique superpotential that we determine its series expansion in the large volume limit. The link between the UV where the scalaron can lead to inflation like in the original Starobinski model and the vacuum properties in the IR is provided by the renormalization group evolution of the scalaron mass and the vacuum energy (a similar approach has been considered in [46]). In particular, if the scalaron both is responsible from inflation in the UV and participates in the dynamics of dark energy in the form of vacuum energy in the IR, then the scalaron effective field theory after compactification is almost uniquely determined; i.e., inflation determines the mass of the scalaron and dark energy the cosmological constant in the UV. Of course this bottom-up approach only provides a set of likely constraints on the set of possibilities for these couplings after compactifications. No dynamical principle determines their values, which are simply fixed by observations.

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APPENDIX A: AN EXPLICIT EXAMPLE

Our analysis can be exemplified using the effective potential for the volume modulus described in [7]:

$$V_E(\mathcal{V}) = \mathcal{V}^{-2}\left(U - \alpha(\ln \mathcal{V})^{3/2}\right) + \gamma\mathcal{V}^{-x} \tag{A1}$$

in the Einstein frame. Here we have $0 < x < 2$. In the supergravity frame, this corresponds to the effective potential

$$V_{\rm vac}(\mathcal{V}) = \mathcal{V}_0^{-2}\left(U - \alpha(\ln\mathcal{V})^{\frac{3}{2}}\right) + \gamma\mathcal{V}_0^{-2}\mathcal{V}^{2-x}, \tag{A2}$$

where $\mathcal{V}_0$ is the minimum of $V_{\rm vac}(\mathcal{V})$ determined by

$$\frac{3}{2}\alpha(\ln\mathcal{V}_0)^{\frac{1}{2}} = (2-x)\gamma\mathcal{V}_0^{2-x}. \tag{A3}$$

The effective potential can then be written as

$$V_{\rm vac}(\mathcal{V}) = \gamma\mathcal{V}_0^{-x}\left(\frac{2(2-x)}{3(\ln\mathcal{V}_0)^{\frac{1}{2}}}\left(\frac{U}{\alpha} - (\ln\mathcal{V})^{\frac{3}{2}}\right) + \left(\frac{\mathcal{V}}{\mathcal{V}_0}\right)^{2-x}\right). \tag{A4}$$

In the Einstein frame the potential is given by

$$V_E(\mathcal{V}) = \left(\frac{\mathcal{V}_0}{\mathcal{V}}\right)^2 V_{\rm vac}(\mathcal{V}). \tag{A5}$$

Let us assume that during inflation when the observable scales in the cosmic microwave background leave the horizon the volume $\mathcal{V} \approx \mathcal{V}_\star$ is such that $x\ln\frac{\mathcal{V}_\star}{\mathcal{V}_0} \ll 1$. During inflation, the Einstein frame potential reduces to

$$V_E(\varphi) \approx \gamma\left(1 - 2\left(\frac{\ln\mathcal{V}_\star}{\ln\mathcal{V}_0}\right)^{1/2}\frac{\mathcal{V}_0}{\mathcal{V}_\star}e^{2\beta\varphi}\right), \tag{A6}$$

where $\mathcal{V}/\mathcal{V}_0 = e^{-2\beta\phi}$. Notice that in that regime the potential in the supergravity frame is simply

$$V_{\rm vac}(\mathcal{V}) \approx V_{\rm vac}(\mathcal{V}_*) + \gamma\left(2\left(\frac{\ln\mathcal{V}_\star}{\ln\mathcal{V}_0}\right)^{\frac{1}{2}} - \frac{\ln\mathcal{V}_\star}{\ln\mathcal{V}_0}\frac{\mathcal{V}_0^2}{\mathcal{V}_\star{}^2} + \left(\frac{\mathcal{V}}{\mathcal{V}_0} - \left(\frac{\ln\mathcal{V}_\star}{\ln\mathcal{V}_0}\right)^{\frac{1}{2}}\frac{\mathcal{V}_0}{\mathcal{V}_\star}\right)^2\right), \tag{A7}$$

which is a quadratic potential with an effective minimum that is not situated at $\mathcal{V}_0$.

We deduce that

$$\eta = -8\beta^2\left(\frac{\ln\mathcal{V}_\star}{\ln\mathcal{V}_0}\right)^{1/2}\frac{\mathcal{V}_0}{\mathcal{V}_\star}e^{2\beta\varphi_\star} \tag{A8}$$

and approximately the number of e-foldings:

$$N = \frac{1}{8\beta^2}\left(\frac{\ln\mathcal{V}_0}{\ln\mathcal{V}_\star}\right)^{1/2}\frac{\mathcal{V}_\star}{\mathcal{V}_0}e^{-2\beta\varphi_\star}, \tag{A9}$$

which determines $n_s - 1 = -\frac{2}{N}$. Notice that the integral determining N is dominated by the behavior of the integrand close to $\phi_\star$ where the approximation to the potential is accurate. Consistency implies that

$$\frac{\mathcal{V}_\star}{\mathcal{V}_0} = e^{-2\beta\varphi_\star} = 8\beta^2\left(\frac{\ln\mathcal{V}_\star}{\ln\mathcal{V}_0}\right)^{1/2}\frac{\mathcal{V}_0}{\mathcal{V}_\star}N, \tag{A10}$$

which determines $\mathcal{V}_\star/\mathcal{V}_0$.

Toward the end of inflation $\mathcal{V}$ differs from $\mathcal{V}_\star$. On the other hand, the potential in the supergravity frame is again quadratic around the true minimum $\mathcal{V}_0$. The distortion to the quadratic shape affects only the evolution of the volume modulus between these two epochs of inflation. This will hardly change the relation between the number of e-foldings N and the spectral index n_s as N is essentially determined by the shape of the potential around $\mathcal{V}_\star$. In conclusion, the potential in the supergravity frame is well approximated by a quadratic form around $\mathcal{V}_\star$ during the creation of the observable structures and the inflationary potential by the Starobinski potential.

APPENDIX B: THE RENORMALIZATION GROUP

In the main text, we discuss the evolution of the mass of the scalaron under the renormalization group between high and low energies. In particle physics, the renormalization scale is usually identified with the typical energy of a given collision. In the cosmological context that we have considered, the interpretation of the scale μ needs to be discussed more precisely. As we are considering the renormalization group in the decoupling subtraction scheme (see for instance [32] or the recent textbook [35] for the methods reviewed here for the decoupling substraction scheme), the scale μ corresponds to the largest momentum scale for virtual particles running in loops. Particles contribute to the running of the coupling constant as long as they have not been integrated out, i.e., as long as μ is larger than their mass. When the scale μ goes through the threshold at the mass m, the particle is removed from the particle content of the model while a threshold correction is added to the coupling constant. In cosmology, we use this Wilsonian setting in the context of particles such as the ones in the standard model with a typical momentum given by the temperature of the plasma T. At each epoch in the history of the Universe, particles are integrated when the temperature falls below their masses. This allows us to identify $\mu \sim T$.

Let us illustrate the decoupling substraction scheme in the simple case of a massive scalar of mass m. The vacuum energy is given by

$$\rho = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\omega_p, \tag{B1}$$

where $\omega_p = \sqrt{\vec{p}^2 + m^2}$. This can be written as

$$\rho = \int \frac{dEd^3p}{(2\pi)^4}\frac{\vec{p}^2 + m^2}{E^2 + \vec{p}^2 + m^2}. \tag{B2}$$

It is convenient to introduce the Euclidean vector $p_E = (E, \vec{p})$ and from rotation invariance we have

$$\int \frac{d^4p_E}{(2\pi)^4}\frac{\vec{p}^2}{\vec{p}_E^2 + m^2} = \frac{3}{4}\int \frac{d^4p_E}{(2\pi)^4}\frac{\vec{p}_E^2}{\vec{p}_E^2 + m^2}. \tag{B3}$$

Using the 't Hooft–Weltmann regularization procedure [47] $\int d^4p_E p_E^n = 0$, $n \neq -4$, we find that the vacuum energy is related to the Feynman propagator at coinciding points:

$$\rho = \frac{m^2}{4}G_F(0), \qquad G_F(0) = \int \frac{d^4p_E}{(2\pi)^4}\frac{1}{\vec{p}_E^2 + m^2}. \tag{B4}$$

We can now calculate

$$G_F(0) = \frac{S_3}{(2\pi)^4}\int_0^\infty dp_E \frac{p_E^3}{p_E^2 + m^2}, \tag{B5}$$

where $S_3 = 2\pi^2$ and finally

$$G_F(0) = \frac{m^2}{8\pi^2}\int dx \frac{x^3}{x^2 + 1}. \tag{B6}$$

Making $x^3 = x(x^2 + 1) - x$ and $\int dx\, x = 0$ in dimensional regularization, we get

$$G_F(0) = -\frac{m^2}{8\pi^2}\int dx \frac{x}{x^2 + 1}. \tag{B7}$$

We now regularize the divergence by applying a cutoff at a scale $x_{\max} = \frac{\mu^2}{m^2}$ corresponding to a Lorentz invariant cutoff in $p_E \le \mu$; i.e., we only integrate over the quantum fluctuations with momenta up to μ. We therefore find

$$\rho(\mu) = -\frac{m^4}{64\pi^2}\ln\left(1 + \frac{\mu^2}{m^2}\right). \tag{B8}$$

Two regimes are particularly important:

$$\mu \gg m, \qquad \rho(\mu) \simeq \frac{m^4}{64\pi^2}\ln\frac{m^2}{\mu^2},$$
$$\mu \ll m, \qquad \rho \simeq 0. \tag{B9}$$

Hence the particle only participates in the vacuum energy when $\mu \gtrsim m$ as we advocated. This corresponds to the renormalization group equation

$$\frac{d\rho(\mu)}{d\ln\mu} = -\frac{m^4}{32\pi^2}\theta(\mu - m) \tag{B10}$$

that we have used in the main text. With this we can write the Wilson effective action. When $\mu < m$, the scalar is integrated out and the effective action contains only the vacuum energy

$$S_W = -\int d^4x\sqrt{-g}\rho_{\rm vac}, \tag{B11}$$

where $\rho_{\rm vac}$ is the vacuum energy when all the fluctuations have been integrated out, i.e., $\mu \to 0$, and S_w can be identified with the 1PI effective action for vanishing external sources. When $\mu > m$ the Wilsonian action is

$$S_W = \int d^4x\sqrt{-g}\left(-\rho_{\rm vac} - \frac{m^4}{64\pi^2}\ln\frac{m^2}{\mu^2} - \frac{(\partial\phi)^2}{2} - \frac{m^2}{2}\phi^2\right). \tag{B12}$$

This result generalizes to the cases in the main text where the renormalization group allows one to evolve both $\rho(\mu)$ and $c(\mu)$ when massive particles are integrated out. This allows us to evaluate the vacuum energy at the end of inflation from the IR vacuum energy and all the contributions from massive particles which are integrated out when the temperature crosses $T = m$. This is the main point used in the paper.

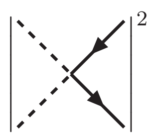
Although the running of the coupling constant that we used is only between the end of inflation down to low energy, the effective action can also be used during inflation as the physical modes have physical momenta outside the horizon corresponding to $k \le H_{\rm inf}$ associated to a scale μ at the Hubble scale. Integrating out the modes inside the horizon with $k > H$ is at the heart of the stochastic description of inflation pioneered by Starobinski [48]. Finally for the nonrelativistic protons of the ionized hydrogen gas in a galaxy cluster with momenta $k_p \sim \sqrt{m_p T}$ the scale is then $\mu \sim k_p \sim m_e$ when $T \simeq 1$ keV corresponding to the vacuum energy $\rho(m_e)$ as used in the main text.

APPENDIX C: THERMODYNAMICAL DECOUPLING OF THE SCALARON

The scalaron could in principle be in thermal equilibrium with the particles in the thermal bath and acquire a large momentum of order T. This could happen via the interaction of the scalaron with the thermal bath from the coupling

$$\mathcal{L}_2 = -\frac{\beta^2}{2}\frac{m_\Psi}{m_{\rm Pl}^2}\varphi^2\bar{\Psi}\Psi, \tag{C1}$$

where Ψ is a massive particle when the Universe has the temperature T. This would lead to chemical equilibrium where scalarons would be created by annihilation of pairs of fermions. Considering the radiation era where Ψ is relativistic, the cross section for $\varphi + \varphi \to \Psi + \bar{\Psi}$,

$\sigma \sim$  , (C2)

is of order $\sigma \simeq \beta^4 \frac{m_\Psi^2}{m_{\rm Pl}^4}$ and the interaction rate which could maintain thermal equilibrium is, for the relativistic particles Ψ, given by $\Gamma \sim gT^3\sigma$, where g is the number of relativistic species. The chemical equilibrium is maintained as long as the reaction rate is larger than the Hubble rate and no cosmological dilution takes place, i.e., $\Gamma > H \sim \frac{\sqrt{g}T^2}{m_{\rm Pl}}$ corresponding to the bound

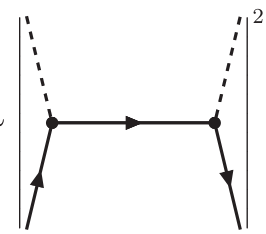
$$T \gtrsim T_{\rm dec} = \frac{1}{\sqrt{g}\beta^4}\frac{m_{\rm Pl}^3}{m_\Psi^2} \tag{C3}$$

with a decoupling temperature typically larger than the Planck scale. As a result, the scalaron is never in thermal equilibrium with the thermal bath.

Elastic processes could also raise the typical momentum of scalarons to a value of order T. This follows from the Yukawa coupling

$$\mathcal{L}_{\rm yuk} = -\frac{\beta m_\Psi}{m_{\rm Pl}}\varphi\bar{\Psi}\Psi \tag{C4}$$

allowing for the kinetic reaction $\varphi + \Psi \to \varphi + \Psi$ mediated by Ψ:

$\sigma \sim$  . (C5)

This could take scalarons with initially low momenta such as $k_{\rm ex}(T)$ and by momentum transfer of order T leads to momenta for the scalarons of order $k \sim T$. The interaction rate $\Gamma = g\beta^4 \frac{m_\Psi^4}{m_{\rm Pl}^4} T$ is very small, implying that there is far less than one interaction per Hubble time. As a result, the scalarons never receive a momentum transfer of order T.

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