Holographic three-point functions from higher curvature gravities in arbitrary dimensions

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The three-point function of the energy-momentum tensor in a conformal field theory is completely determined by three parameters, called $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$. We carry out a holographic calculation of these three parameters in general $d \geq 4$ dimensions from higher curvature gravities up to and including the quartic order. The result is valid both for massless and perturbative higher curvature gravities. It is known that in four dimensional conformal field theory (CFT) the a-charge is a linear combination of $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, our result reproduces this but also shows that a similar relation does not exist for general $d > 4$. We then compute the Weyl anomaly in $d = 6$ and found all the three c-charges are linear combinations of $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, which is consistent with that the a-charge is not. We also find the previously conjectured relation between t_2, t_4, h'' does not hold in general massless gravities.

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I. INTRODUCTION

In conformal field theory (CFT), conformal invariance typically requires the correlation functions to have fairly rigid forms. For example, in flat spacetime the two-point function of the energy-momentum tensor T_{ab} is completely determined by the parameter C_T [1,2]

$$\langle 0|T_{ab}(x)T_{cd}(y)|0\rangle = C_T \frac{\mathcal{I}^{(0)}_{abcd}(x-y)}{|x-y|^{2d}}, \tag{1}$$

where d is the spacetime dimension. Similarly, after imposing further constraints that arise from the conservation of energy, the three-point function of T_{ab} is controlled by three parameters $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ [1,2], i.e.,

$$\langle 0|T_{ab}(x)T_{cd}(y)T_{ef}(z)|0\rangle = \frac{\mathcal{A}\mathcal{I}^{(1)}_{abcdef} + \mathcal{B}\mathcal{I}^{(2)}_{abcdef} + \mathcal{C}\mathcal{I}^{(3)}_{abcdef}}{|x-y|^d|y-z|^d|z-x|^d}. \tag{2}$$

Note that the $\mathcal{I}^{(0)}$, $\mathcal{I}^{(1)}$, $\mathcal{I}^{(2)}$ and $\mathcal{I}^{(3)}$ are tensorial structures whose explicit forms are inessential in our discussion here. These parameters are generally independent in $d \geq 4$, but for $d = 3$ and $d = 2$, the $\mathcal{I}^{(i)}$ tensors become degenerate and the number of independent coefficients is two and one respectively.

In a curved spacetime background, CFT in even dimensions becomes anomalous in that the trace of T_{ab} acquires a nonzero expectation value known as the trace anomaly [3]

$$(4\pi)^{d/2}\langle T^a_a\rangle = -aE^{(d)} + \sum_i c_i I^{(d)}_i, \tag{3}$$

where $E^{(d)}$ and $I^{(d)}_i$ are the Euler density and Weyl invariants in (even) d dimensions respectively. The coefficients a and c_i are known as central charges. In general even dimensions, C_T is a linear combination of the c_i's [1]; it is thus simply proportional to the only c in $d = 4$. Although the concept of conformal anomaly no longer applies in odd dimensions, the quantity C_T, a linear combination of c_i's in even dimensions, survives. The a-charge can also be generalized to odd dimensions as the universal coefficient of the entanglement entropy across a spherical entangling surface, which coincides with a-charge in even dimensions [4]. Thus the a-charge, C_T, and three-point function parameters $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ are important characteristics of CFT in general dimensions as they control the energy-momentum tensor correlators up to and including three points. These parameters are not independent. It was shown by Ward identity that C_T is a linear combination of $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ [1,2], namely

$$C_T = \frac{\pi^d}{\Gamma(d/2)} \frac{(d-1)(d+2)\mathcal{A} - 2\mathcal{B} - 4(d+1)\mathcal{C}}{d(d+2)}. \tag{4}$$

In $d = 4$, a similar relation exists for a [1]

$$a = \frac{\pi^6}{2880}(13\mathcal{A} - 2\mathcal{B} - 40\mathcal{C}). \tag{5}$$

It thus is tempting to expect that analogous relation of (4) would exist also for the a-charge.

The AdS/CFT correspondence provides a new way to study the large N limit of CFTs from the weakly coupled bulk gravity theory in the anti–de sitter (AdS) background in $D = d + 1$ dimensions [5]. Einstein gravity extended with higher-order curvature invariants is insightful to study as they capture new features of the dual CFT. The large number of higher-derivative terms provide unlimited data that can not only reveal the CFT in the large N limit, but also some universal properties of CFT.

There has been extensive studies of the holographic correlators in the pure gravity sector with higher curvature extensions, e.g., Refs. [6–10]. For general higher curvature gravity, the corresponding linearized equation of motion around AdS background contains at most four derivatives of the metric, the graviton spectrum thus contains two extra modes, the massive scalar mode and ghost-like spin-2 mode [11]. Unitarity of the dual CFT requires the decoupling of the ghost mode, while the decoupling of the scalar mode is required by the holographic a-theorem [12]. Thus we usually require the decoupling of both modes, either exactly or perturbatively, resulting in massless gravity or perturbative gravity, respectively. Due to computational difficulties, three-point function parameters of higher curvature gravity are usually calculated indirectly *via* the energy flux parameters t_2 and t_4 [7,13], together with C_T, they contain all the information about the parameters $(\mathcal{A}, \mathcal{B}, \mathcal{C})$. Three-point function parameters for massless cubic gravity was calculated in Ref. [9] for $d = 3$ and $d = 4$, while the results in arbitrary dimensions or with higher-order curvature invariants are still absent in the literature.

Several features of (a, C_T) and $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ have been known. It was shown that to distinguish a and c holographically in $d = 4$, one needs to introduce at least the Gauss-Bonnet density, in which case only two of the three three-point function parameters are independent; one needs to further consider cubic curvature invariants to obtain all the three independent parameters [10]. In additional to these algebraic relations, holography also provides a hidden differential relation between the c and a charges for all massless higher curvature gravities [14,15]

$$C_T = \frac{\Gamma(d+2)}{\pi^d (d-1)^2} L \frac{\partial a}{\partial L}, \tag{6}$$

where L is the effective radius of AdS. Furthermore, it was conjectured that a relation exists between t_2, t_4, h'' for massless gravity [16]

$$a_{(d)} t_2 + b_{(d)} t_4 = -\frac{h''(-L^{-2})}{L^2 h'(-L^{-2})}, \tag{7}$$

where $h(\lambda)$ is the AdS vacuum equation of motion, to be defined later. This relation was later shown to be true for quasitopological gravity in Ref. [17], the corresponding $a_{(d)}$ and $b_{(d)}$ was determined. It is thus interesting to check for more general curvature invariants in diverse dimensions.

The main purpose of this work is to calculate holographically the parameters ($\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$) of the three-point function from the general higher curvature invariants up to and including the quartic order in arbitrary $D = d + 1$ dimensions with $d \geq 4$. We employ an indirect method by considering the one-point function of an energy flux operator parametrized by t_2 and t_4, which are directly related to the three-point function parameters. We consider both the finite higher-derivative coupling case and perturbative case. For the former, we need to impose the massless conditions. For the latter the number of nontrivial terms can be reduced by some appropriate order-by-order field redefinitions of the metric.

From our results, we can make some interesting statements:

(1) After setting $d = 3$ for the general d results, the value of t_4 coincides with that obtained in Ref. [9], even though our method should only be valid for $d \geq 4$.
(2) The a-charge and ($\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$) turn out to be in general linearly independent for $d \geq 5$. In other words, there is no generalization of (5) beyond $d = 4$.
(3) Instead, all the three c-charges in $d = 6$ are linear combinations of ($\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$).
(4) The conjectured relation (7) does not hold for general massless gravity. We also verify this conjecture for quasitopological gravity up to and including the quartic order.

The three-point function, and hence $(\mathcal{A}, \mathcal{B}, \mathcal{C})$, contains three different possible structures, which can be enumerated holographically by Einstein, quadratic, and cubic curvature terms [10,18,19]. It was claimed by Ref. [9] without proof that higher order curvature polynomials will not provide further information. In fact, this can be easily proved, and we present it in Appendix D. Nevertheless, the quartic-order calculation provides a useful consistent check.

The paper is organized as follows. In Sec. II we review higher curvature gravity and list all the Reimann curvature polynomial invariants up to and including the quartic order. In Sec. III we first briefly review the method we use to calculate the energy flux parameters, following Ref. [7], then we present our results and cross check them with the known special cases. Further discussions on our results are given in Sec. IV, where we also calculate the a-charge in general dimensions and the three c-charges in $d = 6$. We conclude this paper and make further comments in Sec. V. Some lengthy expressions and digressions are given in five Appendix sections.

II. A BRIEF REVIEW ON HIGHER CURVATURE GRAVITIES

We consider Einstein gravity extended with higher-order curvature invariant polynomials in $D = d + 1$ dimensions, up to and including the quartic order. The general form of action is

$$S = \int \mathrm{d}^{d+1}x\sqrt{|g|}\mathcal{L}(R_{abcd}, g_{ab}) = \int \mathrm{d}^{d+1}x\sqrt{|g|}\left[R + \frac{d(d-1)}{L_0^2} + \sum_{i,j} e_{i,j}\mathcal{R}_j^{(i)}\right], \tag{8}$$

where $\mathcal{R}_j^{(i)}$ is the jth Riemann scalar polynomial of order i, coefficients $e_{i,j}$ are the coupling constants, and L_0 is the bare AdS radius. All the Riemann curvature polynomial invariants studied in this paper are given explicitly below

$$\mathcal{R}^{(2)} = \{R^2, R_{ab}R^{ab}, R_{abcd}R^{abcd}\}, \tag{9}$$

$$\begin{aligned}\mathcal{R}^{(3)} = \{&R^3, R_{ab}R^{ab}R, R_a{}^bR_b{}^cR_c{}^a, R_{abcd}R^{ac}R^{bd}, RR_{abcd}R^{abcd}, R^{ab}R_a{}^{cde}R_{bcde},\\ &R_{ab}{}^{cd}R_{cd}{}^{ef}R_{ef}{}^{ab}, R_a{}^e{}_c{}^fR^{abcd}R_{bedf}\},\end{aligned} \tag{10}$$

$$\begin{aligned}\mathcal{R}^{(4)} = \{&R^4, R_{ab}R^{ab}R^2, R_a{}^cR^{ab}R_{bc}R, (R_{ab}R^{ab})^2, R_a{}^cR^{ab}R_b{}^dR_{cd}, RR^{ab}R^{cd}R_{acbd},\\ &R^{ab}R_c{}^eR^{cd}R_{adbe}, R^2R_{abcd}R^{abcd}, RR^{ab}R_{cdeb}R^{cde}{}_a, R_{ab}R^{ab}R_{cdef}R^{cdef},\\ &R_a{}^cR^{ab}R_{defc}R^{def}{}_b, R^{ab}R^{cd}R_{efbd}R^{ef}{}_{ac}, R^{ab}R^{cd}R_{ecfd}R^e{}_a{}^f{}_b, R^{ab}R^{cd}R_{ebfd}R^e{}_a{}^f{}_c,\\ &RR_{ab}{}^{ef}R^{abcd}R_{cdef}, RR_a{}^e{}_c{}^fR^{abcd}R_{bedf}, R^{ab}R_a{}^c{}_b{}^dR_{efgd}R^{efg}{}_c,\\ &R^{ab}R_{cd}{}^g{}_aR^{cdef}R_{efgb}, R^{ab}R_c{}^g{}_{ea}R^{cdef}R_{dgfb}, (R_{abcd}R^{abcd})^2,\\ &R_{abc}{}^eR^{abcd}R_{fghe}R^{fgh}{}_d, R_{ab}{}^{ef}R^{abcd}R_{cdgh}R_{ef}{}^{gh}, R_{ab}{}^{ef}R^{abcd}R_{ce}{}^{gh}R_{dfgh},\\ &R_{ab}{}^{ef}R^{abcd}R_c{}^g{}_e{}^hR_{dgfh}, R_a{}^e{}_c{}^fR^{abcd}R_{bgdh}R_e{}^g{}_f{}^h, R_a{}^e{}_c{}^fR^{abcd}R_e{}^g{}_b{}^hR_{fgdh}\}.\end{aligned} \tag{11}$$

In other words, there are 3, 8, 26 terms for the quadratic, cubic and quartic orders respectively. The indices (i, j) of the coefficients $e_{i,j}$ in (8) are labeled based on the above order.

Before proceeding, we shall briefly review some general properties of higher curvature gravity. The equation of motion is given by $\mathcal{E}^{ab} = 0$, with

$$\mathcal{E}^{ab} = -\frac{1}{\sqrt{|g|}}\frac{\delta S}{\delta g_{ab}} = P^a{}_{cde}R^{bcde} - \frac{1}{2}g^{ab}\mathcal{L} - 2\nabla_c\nabla_dP^{acdb},$$
$$P^{abcd} = \frac{\partial\mathcal{L}}{\partial R_{abcd}}. \tag{12}$$

The equation admits the maximally symmetric solution $\bar{g}_{ab}$ whose Riemann tensor is given by $R_{abcd} = 2\lambda\bar{g}_{a[c}\bar{g}_{d]b}$, where λ is to be determined. When evaluated on such a background, the tensor P^{abcd} takes the following simple form [20]

$$P^{abcd}|_{\bar{g}_{ab}} = 2k_1\bar{g}^{a[c}\bar{g}^{d]b}. \tag{13}$$

Defining $\mathcal{L}(\lambda) \equiv \mathcal{L}|_{\bar{g}_{ab}}$, we have

$$\mathcal{L}'(\lambda) = \left(P^{abcd}\frac{\mathrm{d}R_{abcd}}{\mathrm{d}\lambda}\right)_{\bar{g}_{ab}} = 2d(d+1)k_1. \tag{14}$$

Substituting into (12), some useful identities emerge [16,20]

$$\mathcal{L}(\lambda) = 4d\lambda k_1, \qquad h(\lambda) \equiv \mathcal{L}(\lambda) - \frac{2\lambda}{d+1}\mathcal{L}'(\lambda) = 0. \tag{15}$$

Note that the above only holds when λ is taken to the on-shell value in AdS vaccum, while (14) is defined for generic off-shell λ. Hence (15) give the vacuum equation of motion of λ. When taking a derivative with respect to λ, all the parameters in the theory including L_0 should be treated as being independent of λ.

The linearized theory is governed by the tensor

$$C^{abcdefgh} = \frac{\partial^2\mathcal{L}}{\partial R_{abcd}\partial R_{efgh}}. \tag{16}$$

Evaluating on the maximally symmetric background, it takes the form [20]

$$C^{abcdefgh}|_{\bar{g}_{ab}} = k_2(\bar{g}^{[a|e|}\bar{g}^{b]f}\bar{g}^{[c|g|}\bar{g}^{d]h} + \bar{g}^{[a|g|}\bar{g}^{b]h}\bar{g}^{[c|e|}\bar{g}^{d]f}) + 4k_3\bar{g}^{a[c}\bar{g}^{|b|d]}\bar{g}^{e[g}\bar{g}^{|f|h]} + 4k_4\delta^{[a}_{(p}\bar{g}^{b][c}\delta^{d]}_{q)}\bar{g}^{p[e}\bar{g}^{f][g}\bar{g}^{|q|h]}. \quad (17)$$

To be precise, the above tensor structure does not satisfy the cyclic Bianchi identity inherited from the Riemann tensor. We should impose this identity and redefine the tensor as follows

$$C^{abcdefgh} \to C^{abcdefgh} = C^{abcdefgh} - C^{a[bcd]e[fgh]}. \quad (18)$$

The linearized theory of any higher curvature theory is completely described by the coefficients k_i, $i = 1, 2, 3, 4$. For a Lagrangian that is a polynomial of curvature invariants, these four coefficients are linear functions of coupling constants of the polynomial invariants. For a specific Lagrangian, the coefficients k_i can be obtained efficiently with the method proposed in Ref. [20]. The effective Newton constant $\kappa_{\rm eff}$ and the masses of the scalar mode m_s and ghost-like spin-2 mode m_g are given below [20]

$$\kappa_{\rm eff}^{-1} = 4k_1 - 8l(d-2)k_2, \quad (19)$$

$$m_g^2 = \frac{-k_1 + 2(d-2)lk_2}{2k_2 + k_4},$$
$$m_s^2 = \frac{(d-1)k_1 - 4l[k_2 + d(d+1)k_3 + dk_4]}{2k_2 + 4dk_3 + (d+1)k_4}. \quad (20)$$

The quantity $1/\kappa_{\rm eff}$ appears as a coefficient of the linearized equation of motion of the massless graviton after the massive modes are decoupled, namely

$$\mathcal{E}_L^{ab} \sim \frac{1}{\kappa_{\rm eff}}(\Box - 2\lambda)h^{ab}. \quad (21)$$

The central charge C_T of the dual CFT is related to $\kappa_{\rm eff}$ as

$$C_T = \frac{\Gamma(d+2)}{\pi^{d/2}(d-1)\Gamma(d/2)}\frac{L^{d-1}}{\kappa_{\rm eff}}. \quad (22)$$

To decouple both the massive modes, we require

$$2k_2 + k_4 = 2k_2 + 4dk_3 + (d+1)k_4 = 0. \quad (23)$$

At the quadratic order, we have $k_1 = \lambda d(d+1)e_{2,1} + \lambda de_{2,2} + 2\lambda e_{2,3}$, $k_2 = e_{2,3}$, $k_3 = e_{2,1}/2$ and $k_4 = e_{2,2}/2$, the above conditions yield precisely the Gauss-Bonnet density. At the cubic order, we have the following constraints on the coupling constants [12]

$$d(d+1)e_{3,2} + 3de_{3,3} + (2d-1)e_{3,4} + 4d(d+1)e_{3,5} + 4(d+1)e_{3,6} + 24e_{3,7} - 3e_{3,8} = 0,$$
$$12d(d+1)e_{3,1} + d(d+9)e_{3,2} + 3de_{3,3} + (2d+3)e_{3,4} + 16e_{3,5} + 4e_{3,6} + 3e_{3,8} = 0. \quad (24)$$

For the quartic order the constraints become too lengthy and we record them in Appendix A.

An alternative approach to higher-order gravity is to treat it as an effective theory of quantum corrections to Einstein gravity. In this approach, the massive modes are decoupled perturbatively since at the zeroth order the coefficient k_1 is nonzero while $k_{2,3,4}$ are first order, making the kinetic terms of the massive modes first order. This applies to effective field theory where the first massive state appear at some high energy cutoff scale M [21]. In this approach, one can perform field redefinitions of the metric order-by-order by appropriate higher curvature terms, so as to eliminate some Ricci tensor and scalar terms in the Lagrangian. This gives the following residual sets of Riemann polynomials:

$$\mathcal{R}'^{(2)} = \{R_{abcd}R^{abcd}\}, \quad (25)$$

$$\mathcal{R}'^{(3)} = \{R_{ab}{}^{cd}R_{cd}{}^{ef}R_{ef}{}^{ab}, R_a{}^e{}_c{}^f R^{abcd}R_{bedf}\}, \quad (26)$$

$$\mathcal{R}'^{(4)} = \{(R_{abcd}R^{abcd})^2, R_{abc}{}^e R^{abcd}R_{fghe}R^{fgh}{}_d, R_{ab}{}^{ef}R^{abcd}R_{cdgh}R_{ef}{}^{gh}, R_{ab}{}^{ef}R^{abcd}R_{ce}{}^{gh}R_{dfgh}, R_{ab}{}^{ef}R^{abcd}R_c{}^g{}_e{}^h R_{dgfh}, R_a{}^e{}_c{}^f R^{abcd}R_{bgdh}R_e{}^g{}_f{}^h, R_a{}^e{}_c{}^f R^{abcd}R_e{}^g{}_b{}^h R_{fgdh}\}. \quad (27)$$

The coupling constants of these terms are invariant under the field redefinition.

In many of our calculations in this paper, we find that it is not necessary to impose the massless conditions explicitly. Therefore many of our results are valid for both approaches to higher-derivative gravities.

III. HOLOGRAPHIC CALCULATION OF ENERGY FLUX PARAMETERS

We now turn to the main subject of this work. We shall determine holographically the three-point function parameters $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ of the energy-momentum tensor in the dual CFT. To calculate the three-point function directly from higher curvature gravities, one needs to perturb the metric to the third order in the Lagrangian, which is quite challenging even for Einstein gravity [22]. We therefore employ an alternative way to determine these parameters.

We follow Refs. [6,7] by considering a specific frame and polarization in which the three-point function describes a hypothetical conformal collider experiment proposed in Ref. [13]. In this experiment one first creates a localized excitation with the operator $\mathcal{O} \sim \varepsilon_{ij}T^{ij}$ where ε_{ij} is the polarization tensor, then measures the energy flux at the null infinity of the direction indicated by the unit vector $\vec{n}$. The energy-flux operator $\mathcal{E}(\vec{n})$ is

$$\mathcal{E}(\vec{n}) = \lim_{r\to+\infty} \int_{-\infty}^{+\infty} \mathrm{d}t T^t{}_i(t, r\vec{n}) n_i. \tag{28}$$

Its expectation value takes the form

$$\langle \mathcal{E}(\vec{n}) \rangle = \frac{\langle 0|\mathcal{O}^\dagger \mathcal{E}(\vec{n}) \mathcal{O}|0\rangle}{\langle 0|\mathcal{O}^\dagger \mathcal{O}|0\rangle}. \tag{29}$$

For $d \geq 3$, by $\mathrm{O}(d-1)$ invariance the most general form of the energy flux can be determined by two parameters t_2 and t_4 [13]

$$\langle \mathcal{E}(\vec{n}) \rangle = \frac{E}{\Omega_{d-1}} \left[1 + t_2 \left(\frac{\varepsilon^*_{ij} \varepsilon_{il} n^j n^l}{\varepsilon^*_{ij} \varepsilon_{ij}} - \frac{1}{d-1} \right) + t_4 \left(\frac{|\varepsilon_{ij} n^i n^j|^2}{\varepsilon^*_{ij} \varepsilon_{ij}} - \frac{2}{d^2-1} \right) \right]. \tag{30}$$

Note that for $d = 3$, the coefficient of t_2 vanishes identically and we are left with only the t_4 term. To isolate the contribution from null infinity, it is convenient to define a new set of coordinate

$$y^+ = -\frac{L^2}{x^+}, \qquad y^- = x^- - \frac{x^{\bar{i}} x_{\bar{i}}}{x^+}, \qquad y^{\bar{i}} = L \frac{x^{\bar{i}}}{x^+}, \tag{31}$$

where $x^\pm = t \pm x^{d-1}$, L is some energy scale, to be chosen as the bulk AdS radius, and $\bar{i}$ denotes the index of the $(d-2)$ dimensional subspace, *i.e.*, $1 \leq \bar{i} \leq d-2$. This is in fact a conformal transformation with conformal factor $(y^+/L)^2$, after which the energy-flux operator becomes

$$\mathcal{E}(\vec{n}) = L^2 \Omega^{d-1} \int_{-\infty}^{+\infty} \mathrm{d}y^- T_{--}(y^+ = 0, y^-, y^{\bar{i}} = y'^{\bar{i}}), \tag{32}$$

where

$$\Omega = \frac{2}{1+n^{d-1}}, \qquad y'^{\bar{i}} = \frac{L n^{\bar{i}}}{1+n^{d-1}}. \tag{33}$$

On the other hand, the excitation operator $\mathcal{O}$ takes the form

$$\mathcal{O} = \int \mathrm{d}^d x\, e^{-iEt} \varepsilon_{ij} T^{ij} \psi(x/\sigma), \tag{34}$$

where $\psi(x)$ is some distribution that's localized at $x = 0$, and $E\sigma \gg 1$ is assumed so that the operator is localized. Thus one can see that the numerator of (29) is indeed the three-point function with indices contracted with specific polarizations, thus one can relate (t_2, t_4) to $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ [6]

$$t_2 = \frac{(2(d+1))(\mathcal{A}(d-2)(d+1)(d+2) + 3\mathcal{B}d^2 - 4\mathcal{C}(2d+1)d)}{d(\mathcal{A}(d-1)(d+2) - 2\mathcal{B} - 4\mathcal{C}(d+1))}, \tag{35}$$

$$t_4 = -\frac{(d+1)(\mathcal{A}(2d^2-3d-3)(d+2) + 2\mathcal{B}(d+2)d^2 - 4\mathcal{C}(d+1)(d+2)d)}{d(\mathcal{A}(d-1)(d+2) - 2\mathcal{B} - 4\mathcal{C}(d+1))}. \tag{36}$$

Using the above and (4) one can solve the parameters $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ in terms of (t_2, t_4, C_T), thus the problem is converted to the calculation of (t_2, t_4, C_T) quantities.

To calculate the energy flux parameters holographically, we consider the following AdS metric in Poincaré patch

$$\mathrm{d}s^2_{\mathrm{AdS}} = \frac{L^2}{z^2}(\mathrm{d}z^2 + \mathrm{d}x^i \mathrm{d}x^i) = \frac{L^2}{z^2}(\mathrm{d}z^2 - \mathrm{d}x^+ \mathrm{d}x^- + \mathrm{d}x^{\bar{i}} \mathrm{d}x^{\bar{i}}). \tag{37}$$

Inspired by (31), we define the new bulk coordinates as follows

$$u = L\frac{z}{x^+}, \qquad y^+ = -\frac{L^2}{x^+}, \qquad y^- = x^- - \frac{z^2}{x^+} - \frac{x^{\bar{i}} x^{\bar{i}}}{x^+}, \qquad y^{\bar{i}} = L\frac{x^{\bar{i}}}{x^+}, \tag{38}$$

which is an isometric transformation in the bulk

$$\mathrm{d}s^2_{\mathrm{AdS}} = \frac{L^2}{z^2}(\mathrm{d}z^2 - \mathrm{d}x^+ \mathrm{d}x^- + \mathrm{d}x^{\bar{i}} \mathrm{d}x^{\bar{i}}) = \frac{L^2}{u^2}(\mathrm{d}u^2 - \mathrm{d}y^+ \mathrm{d}y^- + \mathrm{d}y^{\bar{i}} \mathrm{d}y^{\bar{i}}), \tag{39}$$

and reproduces (31) at the boundary $u = 0$. According to holographic dictionary, the energy-momentum tensor is dual to the metric perturbation h_{ab}. Specifically, the energy flux operator in (32) is sourced by $\hat{h}_{++} = L^2 \Omega^3 \delta(y^+) \delta^{d-2}(y^{\bar{i}} - y'^{\bar{i}})$, so that the bulk solution is

$$h_{++} \propto \frac{L^2}{u^2} \int \mathrm{d}y'^- \frac{u^d}{[u^2 - y^+(y^- - y'^-) + (y^{\bar{i}} - y'^{\bar{i}})(y^{\bar{i}} - y'^{\bar{i}}) + i\epsilon]^d} \propto \frac{L^2}{u^2} \frac{\delta(y^+) u^d}{[u^2 + (y^{\bar{i}} - y'^{\bar{i}})(y^{\bar{i}} - y'^{\bar{i}})]^{d-1}}, \tag{40}$$

where the overall factor is unimportant so we ignore it here. Remarkably, the above insertion for $\mathcal{E}(\vec{n})$ can be done using an exact solution instead of perturbation by considering the shockwave solution

$$\mathrm{d}s^2_{\text{shockwave}} = \mathrm{d}s^2_{\text{AdS}} + \frac{L^2}{u^2}\delta(y^+)W(u, y^{\bar{i}})(\mathrm{d}y^+)^2, \tag{41}$$

where W satisfies the equation of motion

$$\partial_u^2 W - \frac{d-1}{u}\partial_u W + \partial_i\partial_i W = 0. \tag{42}$$

It is important to note that this equation will not be altered by higher curvature terms [23]. We now only need to consider the second-order perturbation around the shockwave background, instead of the general third order around the AdS background. This simplifies the calculation greatly. Comparing (41) with (40), the desired bulk solution of W is given by

$$W \propto \frac{u^d}{[u^2 + (y^{\bar{i}} - y'^{\bar{i}})(y^{\bar{i}} - y'^{\bar{i}})]^{d-1}}. \tag{43}$$

One can verify that it indeed satisfies (42).

For the excitation operator $\mathcal{O}$, we choose the polarization to be $\varepsilon_{x^1x^2} = \varepsilon_{x^2x^1} = 1$ with all other components vanishing, so that the only nonvanishing component of the metric perturbation is $h_{x^1x^2}$. This implies that this particular holographic procedure requires $d \geq 4$, even though the CFT energy flux (30) can be defined in $d = 3$. Since h_{++} is localized at $y^+ = 0$, we are only interested in the behavior of $h_{x^1x^2}$ on this surface. It can be shown [7,13] that after transforming to (u, y^a) coordinate, $h_{y^1y^2}$ is also localized, namely

$$h_{y^1y^2}(u, y^+ = 0, y^-, y^{\bar{i}}) \sim e^{-iEy^-/2}\delta(u - L)\delta^{d-2}(y^{\bar{i}}). \tag{44}$$

The transformation also introduces other components $h_{y^+y^1}$, $h_{y^+y^2}$, $h_{y^+y^+}$, but as we shall see later, they can be eliminated by imposing the transverse and traceless condition

$$h^a_a = 0, \qquad \nabla_a h^{ab} = 0. \tag{45}$$

Defining ϕ by $h_{y^1y^2} = (L^2/u^2)\phi$ and imposing the transverse and traceless condition, the equation of motion of ϕ is

$$\partial_u^2\phi - \frac{d-1}{u}\partial_u\phi - 4\partial_{y^+}\partial_{y^-}\phi + \partial_{\bar{i}}\partial_{\bar{i}}\phi = 0, \tag{46}$$

up to interaction terms with the shockwave.

With these preliminaries, we are ready to evaluate the energy flux. This can be done by turning on the perturbations on the shockwave metric and evaluate the on-shell action, and then extract the terms of the form $W\phi^2$. Note that by (44) the bulk coordinate u is localized at $u = L$, so we do not need to consider the boundary action. After imposing the transverse and traceless condition, using the equations of motion (42) and (46), and integration by parts, the on-shell effective action becomes

$$S^{(3)} = -\frac{1}{L^{d+1}}\int \mathrm{d}^{d+1}x\phi\partial^2_{y^-}\phi W(\hat{C}_T + \hat{t}_2T_2 + \hat{t}_4T_4)\Big|_{u=L, y^{\bar{i}}=0}. \tag{47}$$

The basis functions T_2 and T_4 depend only on the shockwave metric function W, namely

$$T_2 = \frac{1}{2d(d-1)W}[u^2(\partial^2_{y^1}W + \partial^2_{y^2}W) - 2u\partial_u W], \tag{48}$$

$$\begin{aligned} T_4 = \frac{2}{d(d-1)(d-2)(d+1)W}&[(d-1)u^2(\partial^2_{y^1}W + \partial^2_{y^2}W) \\ &+ u^4\partial^2_{y^1}\partial^2_{y^2}W - (d+2)u\partial_u W \\ &- u^3(\partial_u\partial^2_{y^1}W + \partial_u\partial^2_{y^2}W) - u^2\partial_{\hat{i}}\partial_{\hat{i}}W], \end{aligned} \tag{49}$$

where the index $\hat{i}$ covers the remaining $(d-4)$ directions, i.e., $3 \leq \hat{i} \leq d-1$. Substitute the solution (43) of W into the above leads to

$$T_2 = \frac{n_1^2 + n_2^2}{2} - \frac{1}{d-1}, \qquad T_4 = 2n_1^2n_2^2 - \frac{2}{d^2-1}. \tag{50}$$

While T_2 and T_4 are independent of the detail of the action, the coefficients $\hat{C}_T$, $\hat{t}_2$, $\hat{t}_4$ are determined by the coupling constants of higher curvature gravities. Specifically, we find

$$\begin{aligned} \hat{C}_T = L^{d-1}\Big\{&1 + 2L^{-2}[-d(d+1)e_{2,1} - de_{2,2} + 2(d-3)e_{2,3}] + L^{-4}[3(d+1)^2d^2e_{3,1} + 3(d+1)d^2e_{3,2} + 3d^2e_{3,3} + 3d^2e_{3,4} \\ &- 2(d+1)(2d-7)de_{3,5} - 2(2d-7)de_{3,6} + (60-24d)e_{3,7} + 3(3d-5)e_{3,8}] + L^{-6}\Big[-2d^3(d+1)^3e_{4,1} \\ &- 2d^3(d+1)^2e_{4,2} - 2d^3(d+1)e_{4,3} - 2d^3(d+1)e_{4,4} - 2d^3e_{4,5} - 2d^3(d+1)e_{4,6} - 2d^3e_{4,7} + 2(d-4)d^2(d+1)^2e_{4,8} \\ &+ 2(d-4)d^2(d+1)e_{4,9} + 2(d-4)d^2(d+1)e_{4,10} + 2(d-4)d^2e_{4,11} + 2(d-4)d^2e_{4,12} - 2d^3e_{4,13} + 2(d-4)d^2e_{4,14} \\ &+ 4d(d+1)(3d-8)e_{4,15} - d(d+1)(5d-8)e_{4,16} + 2(d-4)d^2e_{4,17} + 4d(3d-8)e_{4,18} - d(5d-8)e_{4,19} \\ &+ 8(d-3)d(d+1)e_{4,20} + 8(d-3)de_{4,21} + 16(3d-7)e_{4,22} + 8(3d-7)e_{4,23} + 2(d^2-10d+14)e_{4,24} \\ &- 2(d^2-7d+14)e_{4,25} + \frac{1}{2}(8d^2-52d+56)e_{4,26}\Big]\Big\}, \end{aligned} \tag{51}$$

$$
\begin{aligned}
\hat{t}_2 = d(d-1)L^{d-1}\{&4L^{-2}e_{2,3} + L^{-4}[-4d(d+1)e_{3,5} - 4de_{3,6} - 36(d+2)e_{3,7} - 3(7d+2)e_{3,8}] \\
&\times L^{-6}[4(d+1)^2d^2e_{4,8} + 4(d+1)d^2e_{4,9} + 4(d+1)d^2e_{4,10} + 4d^2e_{4,11} + 4d^2e_{4,12} + 4d^2e_{4,14} \\
&+ 4d^2e_{4,17} + 36(d+1)(d+2)de_{4,15} + 3(d+1)(7d+2)de_{4,16} + 36(d+2)de_{4,18} \\
&+ 3(7d+2)de_{4,19} + 16(d+1)de_{4,20} + 16de_{4,21} + 96(3d+5)e_{4,22} + 48(3d+5)e_{4,23} \\
&+ 4(13d-6)e_{4,24} + (24-48d)e_{4,25} + 8(7d-3)e_{4,26}]\},
\end{aligned} \tag{52}
$$

$$
\begin{aligned}
\hat{t}_4 = 6d(d+1)(d-1)(d+2)L^{d-1}\{&L^{-4}[2e_{3,7} + e_{3,8}] + L^{-6}[-2d(d+1)e_{4,15} - d(d+1)e_{4,16} \\
&- 2de_{4,18} - de_{4,19} - 16e_{4,22} - 8e_{4,23} - 2e_{4,24} + 2e_{4,25} - 2e_{4,26}]\}.
\end{aligned} \tag{53}
$$

To obtain the final result, we need to divide the cubic action by the two-point function $\langle T_{12}T_{12}\rangle$, which is proportional to C_T. The latter can be calculated using (22) and for the case we study, we find

$$
C_T = \frac{2\Gamma(d+2)}{\pi^{d/2}(d-1)\Gamma(d/2)}\hat{C}_T. \tag{54}
$$

Thus we have

$$
\frac{S^{(3)}}{\langle T_{12}T_{12}\rangle} \sim 1 + \frac{\hat{t}_2}{\hat{C}_T}\left(\frac{n_1^2+n_2^2}{2} - \frac{1}{d-1}\right) + \frac{\hat{t}_4}{\hat{C}_T}\left(2n_1^2n_2^2 - \frac{2}{d^2-1}\right). \tag{55}
$$

On the other hand, specializing to the polarization $\varepsilon_{x^1x^2} = \varepsilon_{x^2x^1} = 1$, the two terms involving $\vec{n}$ in (30) become

$$
\frac{\varepsilon_{ij}^*\varepsilon_{il}n^jn^l}{\varepsilon_{ij}^*\varepsilon_{ij}} = \frac{n_1^2+n_2^2}{2}, \qquad \frac{|\varepsilon_{ij}n^in^j|^2}{\varepsilon_{ij}^*\varepsilon_{ij}} = 2n_1^2n_2^2. \tag{56}
$$

By comparing (55) to (30), we arrive at the final result

$$
t_2 = \frac{\hat{t}_2}{\hat{C}_T}, \qquad t_4 = \frac{\hat{t}_4}{\hat{C}_T}, \tag{57}
$$

where the hatted variables are given by (51)–(53).

Now we examine some known special cases. First, for Lovelock gravities up to and including the quartic order, we set the coupling constants to

$$
\{e_{2,i}\} = \mu_2\left[\prod_{i=2}^{3}(d-i)\right]^{-1}\{1,-4,1\}, \tag{58}
$$

$$
\{e_{3,i}\} = \mu_3\left[\prod_{i=2}^{5}(d-i)\right]^{-1}\{1,-12,16,24,3,-24,4,-8\}, \tag{59}
$$

$$
\begin{aligned}
\{e_{4,i}\} = \mu_4\left[\prod_{i=2}^{7}(d-i)\right]^{-1}\{&1,-24,64,48,-96,96, \\
&-384,6,-96,-24,192,96,-192,192,16, \\
&-32,192,-192,384,3,-48,6,48,-96,48, \\
&-96\}.
\end{aligned} \tag{60}
$$

Substituting them into our results (51)–(53) and (57), we obtain

$$
\begin{aligned}
t_2 &= \frac{4(d-1)d}{(d-3)(d-2)}\frac{\mu_2L^{-2} - 3\mu_3L^{-4} + 6\mu_4L^{-6}}{1 - 2\mu_2L^{-2} + 3\mu_3L^{-4} - 4\mu_4L^{-6}}, \\
t_4 &= 0.
\end{aligned} \tag{61}
$$

Energy flux parameters for general Lovelock gravity was derived in Ref. [10]. Explicitly we have (after adapting to our conventions)

$$
t_2 = \frac{2d(d-1)}{(d-2)(d-3)}\frac{h''(-L^{-2})}{L^2h'(-L^{-2})}, \qquad t_4 = 0. \tag{62}
$$

In our case the function $h(\lambda)$ is given by

$$
h(\lambda) = d(d-1)\left(\frac{1}{L_0^2} + \lambda + \mu_2\lambda^2 + \mu_3\lambda^3 + \mu_4\lambda^4\right). \tag{63}
$$

Substituting this into (62) we get exactly identical result with (61).

Secondly, after specializing our result to general massless cubic curvature gravity in $d=4$ and eliminating $e_{3,7}$, $e_{3,8}$ by the massless condition (24), we arrive at

$$
t_2 = \frac{48(2340e_{3,1} + 552e_{3,2} + 144e_{3,3} + 123e_{3,4} + 316e_{3,5} + 80e_{3,6})}{L^4 - 2(60e_{3,1} + 8e_{3,2} + e_{3,4} + 4e_{3,5})}, \tag{64}
$$

$$t_4 = -\frac{360(600e_{3,1} + 140e_{3,2} + 36e_{3,3} + 31e_{3,4} + 80e_{3,5} + 20e_{3,6})}{L^4 - 2(60e_{3,1} + 8e_{3,2} + e_{3,4} + 4e_{3,5})}, \tag{65}$$

which reproduces the result of Ref. [9].

We can now obtain all the three-point function parameters ($\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$) by inverting (4), (35), and (36). The final expressions of ($\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$) in general dimension d are recorded in Appendix B.

IV. DISCUSSIONS

Having obtained all the three-point function parameters $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ holographically from higher curvature gravities up to and including the quartic order, we can compare our results to those in literature and study their implications.

First, it is interesting to mention that even though our result, based on its derivation, should be valid only for $d \geq 4$, there exists a smooth $d = 3$ limit. In particular, if we restrict to massless cubic gravity and set $d = 3$, our result gives rise to the following value for t_4

$$t_4 = -\frac{120(360e_{3,1} + 96e_{3,2} + 27e_{3,3} + 25e_{3,4} + 64e_{3,5} + 18e_{3,6})}{L^4 - 2(36e_{3,1} + 6e_{3,2} + e_{3,4} + 4e_{3,5})} \tag{66}$$

which is precisely the one obtained in Ref. [9] that was derived using a different method. We thus expect that our new result of t_4, arising from the quartic massless gravity, is also valid at $d = 3$. From our general results, setting $d = 3$ also gives a nonzero value for t_2; however, in $d = 3$ the symmetry group reduces to O(2), and hence there can only be one energy flux parameter t_4. Furthermore, specializing $\hat{t}_2$ in $d = 3$, we find it is actually linearly independent of $\hat{C}_T$ and $\hat{t}_4$. Therefore, it is interesting to explore the physical meaning of the value of t_2 of general d in the $d = 3$ limit.

Second, we examine the conjecture (7). Since for massless gravity $h'(-L^{-2})$ is proportional to C_T [16], it is equivalent to that $\hat{t}_2$, $\hat{t}_4$ and $h''(-L^{-2})$ are linearly dependent, i.e.,

$$a'_{(d)}\hat{t}_2 + b'_{(d)}\hat{t}_4 = h''(-L^{-2}). \tag{67}$$

As mentioned earlier, the conjecture (7) was already proved for quasitopological gravity, it's easy to check for cubic and quartic quasitopological gravity in general dimension using our result of $\hat{t}_2$ and $\hat{t}_4$. We find the coefficients are given by

$$a_{(d)} = -\frac{(d-3)(d-2)}{2(d-1)d},$$
$$b_{(d)} = -\frac{(d-2)(7d^3 - 19d^2 - 8d + 8)}{2(d-1)d(d+1)(d+2)(2d-1)}, \tag{68}$$

they are in consistent with Ref. [17]. A brief review on quasitopological (QT) gravities can be found in Appendix E, where we find that there are 15 such theories in quartic gravities. However, for the case of general massless higher curvature gravity we find that $\hat{t}_2$, $\hat{t}_4$ and $h''(-L^{-2})$ are actually linearly independent, thus we have disproved the conjecture (7) for general massless gravity.

Third, we focus on identities involving the a-charge (5) and (6). As mentioned earlier, the a-charge can be generalized to arbitrary dimensions as the entanglement entropy across a spherical region S^{d-2}. It was shown with a conformal map that the entanglement entropy over the spherical region in Minkowski background equals to the thermal entropy of $R \times H^{d-1}$ background. The latter can be calculated holographically by the black hole entropy of a locally AdS hyperbolic topological black hole [4]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Sigma^2_{d-1,k=-1},$$
$$f(r) = \frac{r^2}{L^2} - 1. \tag{69}$$

The black hole entropy can be calculated from the Wald entropy in a standard way [24,25]

$$S_{\text{Wald}} = -2\pi \int d^{d-1}x\sqrt{\sigma}P^{abcd}\epsilon_{ab}\epsilon_{cd}\Big|_{r=L}, \tag{70}$$

where σ_{ab} is the induced metric of the horizon, and $\epsilon = dt \wedge dr$ is the binormal of the horizon, satisfying $\epsilon_{ab}\epsilon^{ab} = -2$. For the topological black hole (69), the integrand is a constant proportional to the area of the horizon so the value diverges, thus one may assign the entropy density on the horizon to the a-charge as follows [15]

$$a = \frac{\pi^{d/2}}{2\pi\Gamma(d/2)}\frac{S_{\text{Wald}}}{\Omega_{d-1,-1}}$$
$$= -\frac{\pi^{d/2}}{\Gamma(d/2)}L^{d-1}(P^{abcd}\epsilon_{ab}\epsilon_{cd})_{r=L}. \tag{71}$$

It can be shown that with this definition the value coincides with a-charge in even dimensions. It is straightforward to calculate the a-charge from (71), we obtain

$$
\begin{aligned}
a = \frac{2\pi^{d/2}}{\Gamma(d/2)} L^{d-1}\{ & 1 - 2L^{-2}[d(d+1)e_{2,1} + de_{2,2} + 2e_{2,3}] + 3L^{-4}[d^2(d+1)^2 e_{3,1} \\
& + d^2(d+1)e_{3,2} + d^2 e_{3,3} + d^2 e_{3,4} + 2d(d+1)e_{3,5} + 2de_{3,6} + 4e_{3,7} + (d-1)e_{3,8}] \\
& + 4L^{-6}[-d^3(d+1)^3 e_{4,1} - d^3(d+1)^2 e_{4,2} - d^3(d+1)e_{4,3} - d^3(d+1)e_{4,4} - d^3 e_{4,5} \\
& - d^3(d+1)e_{4,6} - d^3 e_{4,7} - 2d^2(d+1)^2 e_{4,8} - 2d^2(d+1)e_{4,9} - 2d^2(d+1)e_{4,10} - 2d^2 e_{4,11} \\
& - 2d^2 e_{4,12} - d^3 e_{4,13} - 2d^2 e_{4,14} - 4d(d+1)e_{4,15} - d(d^2-1)e_{4,16} - 2d^2 e_{4,17} - 4de_{4,18} \\
& - (d-1)de_{4,19} - 4d(d+1)e_{4,20} - 4de_{4,21} - 8e_{4,22} - 4e_{4,23} - 2(d-1)e_{4,24} \\
& - (d^2 - d + 2)e_{4,25} + (2-3d)e_{4,26}]\}.
\end{aligned} \tag{72}
$$

With both C_T and a evaluated, it follows immediately that after applying the massless condition we have (6).

Specializing our results to $d = 4$, we reproduce (5) and therefore we verify the relation holographically in higher curvature gravity up to and including the quartic order. However, for $d \geq 5$, we find that a and ($\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$) are in general linearly independent. In other words, the $d = 4$ relation (5) does not have a higher-dimensional generalization. This somewhat unexpected result instructs us to further consider central charges in $d = 6$, where there are three c-charges. Details and explicit values of the c-charges in $d = 6$ can be found in Appendix C. We find that all three c-charges turn out be linear combinations of ($\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$), namely

$$c_1 = \frac{\pi^9}{233280}(-98\mathcal{A} + 9\mathcal{B} + 174\mathcal{C}), \tag{73}$$

$$c_2 = \frac{\pi^9}{6531840}(1226\mathcal{A} - 153\mathcal{B} + 882\mathcal{C}), \tag{74}$$

$$c_3 = \frac{\pi^6}{3024} C_T|_{d=6} = \frac{\pi^9}{145152}(20\mathcal{A} - \mathcal{B} - 14\mathcal{C}). \tag{75}$$

In terms of t_2, t_4, we have

$$
\begin{aligned}
t_2 &= \frac{15(23c_1 - 44c_2 + 144c_3)}{16c_3}, \\
t_4 &= -\frac{105(c_1 - 2c_2 + 6c_3)}{2c_3},
\end{aligned} \tag{76}
$$

which coincides with the CFT results derived from free Dirac fermion, real scalar, and antisymmetric two-form fields [26]. Our results suggest that this is indeed a universal property of $d = 6$ CFT.

V. CONCLUSION

In this work we considered general higher curvature gravity up to and including the quartic order and calculated holographically the three-point function parameters ($\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$) of the dual field theory in general d dimensions. We adopted an indirect method by calculating the central charge C_T and the energy flux parameters t_2, t_4, which are known to be directly related to the three-point function parameters. We therefore obtained the complete list of the holographic results of (a, C_T) and $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ for general d dimensions.

Despite of the fact that our method should be valid only for $d \geq 4$, we not only reproduce the previously known result in $d = 4$, but also the correct value of t_4 of Ref. [9] after setting $d = 3$. Therefore it may be of interest to explore the physical meaning of t_2 in this case since at $d = 3$ the parameter t_2 does not contribute to the energy flux.

We also examined the relation between t_2, t_4, h'' proposed by Ref. [16] and checked its validity for cubic and quartic quasitopological gravity, but found that it does not hold for general massless gravities. We also found that the $d = 4$ identity (5) cannot be generalized to higher dimensions. The generalized a-charge is linearly independent of $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ when $d \geq 5$. We calculated the c-charges in $d = 6$ and found they are all linear combinations of $(\mathcal{A}, \mathcal{B}, \mathcal{C})$. Considering the fact that the number of c-charges proliferates as d increases, their relations to $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ are hard to conjecture and require a new investigation.

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APPENDIX A: MASSLESS CONDITION FOR QUARTIC CURVATURE GRAVITY

In this appendix, we give the conditions on the coupling constants for higher curvature massless gravity. The condition for the decoupling of the massive scalar mode is

$$
\begin{aligned}
&(d+1)^2 d^2 e_{4,2} + 3(d+1)d^2 e_{4,3} + 2(d+1)d^2 e_{4,4} + 6d^2 e_{4,5} + 4(d+1)^2 d^2 e_{4,8} \\
&\quad + (3d^2 - 2d + 1)e_{4,13} + (4d^2 + 9d - 1)e_{4,14} + 2(2d^2 + 3d - 2)e_{4,17} \\
&\quad + (d+1)(2d-1)de_{4,6} + 2(2d-1)de_{4,7} + 4(d+1)^2 de_{4,9} + 2(d+1)(2d+1)de_{4,10} \\
&\quad + 2(2d+5)de_{4,11} + 24(d+1)de_{4,15} - 3(d+1)de_{4,16} + 16(d+1)de_{4,20} + (4(d+1)^2 - 6)e_{4,12} \\
&\quad + 12(2d+1)e_{4,18} - (d+4)e_{4,19} + 16(d+1)e_{4,21} + 2(2d-7)e_{4,24} + 4(d+4)e_{4,25} \\
&\quad + 2(4d-5)e_{4,26} + 96e_{4,22} + 48e_{4,23} = 0. \qquad \text{(A1)}
\end{aligned}
$$

The condition for the decoupling of the massive ghostlike spin-2 mode is

$$
\begin{aligned}
&24(d+1)^2 d^2 e_{4,1} + (d+1)(d+21)d^2 e_{4,2} + 3(d+5)d^2 e_{4,3} + 2(d+9)d^2 e_{4,4} + 6d^2 e_{4,5} + (2d^2 + 17d + 3)de_{4,6} \\
&\quad + (3d^2 + 10d - 3)e_{4,13} + 3(d^2 + 5d - 4)e_{4,16} + 2(2d+3)de_{4,7} + 40(d+1)de_{4,8} + 4(d+7)de_{4,9} \\
&\quad + 2(d+17)de_{4,10} + 10de_{4,11} + 5de_{4,19} + 4de_{4,25} + 2(4d+3)e_{4,12} + 3(3d+1)e_{4,14} + 6(d+2)e_{4,17} + 48e_{4,15} \\
&\quad + 12e_{4,18} + 64e_{4,20} + 16e_{4,21} + 10e_{4,24} + 14e_{4,26} = 0. \qquad \text{(A2)}
\end{aligned}
$$

APPENDIX B: EXPLICIT EXPRESSIONS FOR ($\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$)

In Sec. III, we obtained the energy flux parameters t_2 and t_4 and also C_T. From these, we can read off the three-point function parameters. They are given by

$$
\begin{aligned}
\mathcal{A} = {}& \frac{2d^3 \pi^{-d} L^{d-1} \Gamma(d+2)}{(d-1)^3 (d+1)} \{-1 + 2L^{-2}[d(d+1)e_{2,1} + de_{2,2} + 8e_{2,3}] - 3L^{-4}[d^2(d+1)^2 e_{3,1} \\
&+ d^2(d+1)e_{3,2} + d^2 e_{3,3} + d^2 e_{3,4} + 6d(d+1)e_{3,5} + 6de_{3,6} + 36e_{3,7} + (d^2 - 2d - 7)e_{3,8}] \\
&+ L^{-6}[4d^3(d+1)^3 e_{4,1} + 4d^3(d+1)^2 e_{4,2} + 4d^3(d+1)e_{4,3} + 4d^3(d+1)e_{4,4} + 4d^3 e_{4,5} \\
&+ 4d^3(d+1)e_{4,6} + 4d^3 e_{4,7} + 20d^2(d+1)^2 e_{4,8} + 20d^2(d+1)e_{4,9} + 20d^2(d+1)e_{4,10} \\
&+ 20d^2 e_{4,11} + 20d^2 e_{4,12} + 4d^3 e_{4,13} + 20d^2 e_{4,14} + 112d(d+1)e_{4,15} \\
&+ d(d+1)(3d^2 - 5d - 22)e_{4,16} + 20d^2 e_{4,17} + 112de_{4,18} + d(3d^2 - 5d - 22)e_{4,19} \\
&+ 64d(d+1)e_{4,20} + 64de_{4,21} + 512e_{4,22} + 256e_{4,23} + 4(3d^2 - 4d - 26)e_{4,24} \\
&- 8(d^2 - 4d - 13)e_{4,25} + 4(3d^2 - 26)e_{4,26}]\}. \qquad \text{(B1)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{B} = {}& \frac{d\pi^{-d} L^{d-1} \Gamma(d+2)}{(d-1)^3 (d+1)} \{-2(d^3 - d^2 + 1) + 4L^{-2}[d(d+1)(d^3 - d^2 + 1)e_{2,1} + d(d^3 - d^2 + 1)e_{2,2} \\
&- (d^4 - 7d^3 + 5d^2 - d - 6)e_{2,3}] + L^{-4}[-6d^2(d+1)^2(d^3 - d^2 + 1)e_{3,1} - 6d^2(d+1)(d^3 - d^2 + 1)e_{3,2} \\
&- 6d^2(d^3 - d^2 + 1)e_{3,3} - 6d^2(d^3 - d^2 + 1)e_{3,4} + 4d(d+1)(d^4 - 8d^3 + 6d^2 - d - 7)e_{3,5} \\
&+ 4d(d^4 - 8d^3 + 6d^2 - d - 7)e_{3,6} - 12(d^5 - 3d^4 + 11d^3 - 7d^2 + 6d + 10)e_{3,7} \\
&- 3(3d^5 - 3d^4 - 11d^3 + 9d^2 - 4d - 10)e_{3,8}] + L^{-6}[8d^3(d+1)^3(d^3 - d^2 + 1)e_{4,1} + 8d^3(d+1)^2(d^3 - d^2 + 1)e_{4,2} \\
&+ 8d^3(d+1)(d^3 - d^2 + 1)e_{4,3} + 8d^3(d+1)(d^3 - d^2 + 1)e_{4,4} + 8d^3(d^3 - d^2 + 1)e_{4,5} \\
&+ 8d^3(d+1)(d^3 - d^2 + 1)e_{4,6} + 8d^3(d^3 - d^2 + 1)e_{4,7} - 4d^2(d+1)^2(d^4 - 9d^3 + 7d^2 - d - 8)e_{4,8} \\
&- 4d^2(d+1)(d^4 - 9d^3 + 7d^2 - d - 8)e_{4,9} - 4d^2(d+1)(d^4 - 9d^3 + 7d^2 - d - 8)e_{4,10} \\
&- 4d^2(d^4 - 9d^3 + 7d^2 - d - 8)e_{4,11} - 4d^2(d^4 - 9d^3 + 7d^2 - d - 8)e_{4,12} + 8d^3(d^3 - d^2 + 1)e_{4,13} \\
&- 4d^2(d^4 - 9d^3 + 7d^2 - d - 8)e_{4,14} + 4d(d+1)(3d^5 - 9d^4 + 35d^3 - 23d^2 + 18d + 32)e_{4,15} \\
&+ d(d+1)(d+2)(9d^4 - 25d^3 + 13d^2 + 3d - 16)e_{4,16} - 4d^2(d^4 - 9d^3 + 7d^2 - d - 8)e_{4,17} \\
&+ 4d(3d^5 - 9d^4 + 35d^3 - 23d^2 + 18d + 32)e_{4,18} + d(d+2)(9d^4 - 25d^3 + 13d^2 + 3d - 16)e_{4,19}
\end{aligned}
$$

$$
\begin{aligned}
&- 16d(d+1)(d^4-7d^3+5d^2-d-6)e_{4,20} - 16d(d^4-7d^3+5d^2-d-6)e_{4,21}\\
&+ 32(3d^5-6d^4+14d^3-8d^2+15d+14)e_{4,22} + 16(3d^5-6d^4+14d^3-8d^2+15d+14)e_{4,23}\\
&+ 4(5d^5-3d^4-29d^3+23d^2-22d-28)e_{4,24} - 16(d^5-2d^4-6d^3+5d^2-7d-7)e_{4,25}\\
&+ 8(2d^5+2d^4-17d^3+12d^2-8d-14)e_{4,26}]\}. \qquad \text{(B2)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{C} = \frac{d^2\pi^{-d}L^{d-1}\Gamma(d+2)}{2(d-1)^3(d+1)}\{&-2d^2+2d+1+2L^{-2}[d(d+1)(2d^2-2d-1)e_{2,1}\\
&+ d(2d^2-2d-1)e_{2,2} - 2(d^3-8d^2+5d+6)e_{2,3}]\\
&+ L^{-4}[-3d^2(d+1)^2(2d^2-2d-1)e_{3,1} - 3d^2(d+1)(2d^2-2d-1)e_{3,2}\\
&- 3d^2(2d^2-2d-1)e_{3,3} - 3d^2(2d^2-2d-1)e_{3,4}\\
&+ 2d(d+1)(2d^3-18d^2+12d+13)e_{3,5} + 2d(2d^3-18d^2+12d+13)e_{3,6}\\
&+ 36(d^3-6d^2+3d+5)e_{3,7} - 3(d^4-2d^3-8d^2+2d+15)e_{3,8}]\\
&+ L^{-6}[4d^3(d+1)^3(2d^2-2d-1)e_{4,1} + 4d^3(d+1)^2(2d^2-2d-1)e_{4,2}\\
&+ 4d^3(d+1)(2d^2-2d-1)e_{4,3} + 4d^3(d+1)(2d^2-2d-1)e_{4,4} + 4d^3(2d^2-2d-1)e_{4,5}\\
&+ 4d^3(d+1)(2d^2-2d-1)e_{4,6} + 4d^3(2d^2-2d-1)e_{4,7}\\
&- 4d^2(d+1)^2(d^3-10d^2+7d+7)e_{4,8} - 4d^2(d+1)(d^3-10d^2+7d+7)e_{4,9}\\
&- 4d^2(d+1)(d^3-10d^2+7d+7)e_{4,10} - 4d^2(d^3-10d^2+7d+7)e_{4,11}\\
&- 4d^2(d^3-10d^2+7d+7)e_{4,12} + 4d^3(2d^2-2d-1)e_{4,13} - 4d^2(d^3-10d^2+7d+7)e_{4,14}\\
&- 4d(d+1)(9d^3-56d^2+29d+46)e_{4,15} + d(d+1)(d+2)(3d^3-10d^2-8d+23)e_{4,16}\\
&- 4d^2(d^3-10d^2+7d+7)e_{4,17} - 4d(9d^3-56d^2+29d+46)e_{4,18}\\
&+ d(d+2)(3d^3-10d^2-8d+23)e_{4,19} - 16d(d+1)(d^3-8d^2+5d+6)e_{4,20}\\
&- 16d(d^3-8d^2+5d+6)e_{4,21} - 64(3d^3-16d^2+7d+14)e_{4,22}\\
&- 32(3d^3-16d^2+7d+14)e_{4,23} + 4(2d^4+d^3-35d^2+3d+56)e_{4,24}\\
&- 4(d^4-5d^3-29d^2+9d+56)e_{4,25} + 4(d^4+9d^3-40d^2-3d+56)e_{4,26}]\}. \qquad \text{(B3)}
\end{aligned}
$$

APPENDIX C: CENTRAL c-CHARGES IN $d=6$

In $d=6$ there are three Weyl invariants I_i [27]

$$
\begin{aligned}
I_1 &= C_{abcd}C^{aefd}C_e{}^{bc}{}_f,\\
I_2 &= C_{abcd}C^{cdef}C_{ef}{}^{ab},\\
I_3 &= C_{abcd}\nabla^2 C^{abcd} + 4C_{abcd}R^a_e C^{ebcd}\\
&\quad - \frac{6}{5}C_{abcd}C^{abcd}R + \nabla_a J^a \qquad \text{(C1)}
\end{aligned}
$$

where C_{abcd} is the Weyl tensor, and the explicit form of $\nabla_a J^a$ is irrelevant since it is a total divergence and can be canceled by a local counterterm. This gives three c-charges in $d=6$.

The central charges of cubic curvature gravity in $d=6$ was computed in Ref. [28], we therefore extend the result to quartic order. We employ the reduced Fefferman-Graham expansion trick to calculate the central charges [12,29]. We find that the a-charge is given by (72) specialized to $d=6$, and the three c-charges are

$$
\begin{aligned}
c_1 = \frac{4}{3}\pi^3L^5[&-3+4L^{-2}(63e_{2,1}+9e_{2,2}-e_{2,3}) + 3L^{-4}(-5292e_{3,1}-756e_{3,2}-108e_{3,3}-108e_{3,4}\\
&- 28e_{3,5}-4e_{3,6}+20e_{3,7}-39e_{3,8}) + 24L^{-6}(37044e_{4,1}+5292e_{4,2}+756e_{4,3}+756e_{4,4}\\
&+ 108e_{4,5}+756e_{4,6}+108e_{4,7}+588e_{4,8}+84e_{4,9}+84e_{4,10}+12e_{4,11}+12e_{4,12}+108e_{4,13}\\
&+ 12e_{4,14}-84e_{4,15}+231e_{4,16}+12e_{4,17}-12e_{4,18}+33e_{4,19}-28e_{4,20}-4e_{4,21}-12e_{4,22}\\
&- 6e_{4,23}+13e_{4,24}+4e_{4,25}+12e_{4,26})], \qquad \text{(C2)}
\end{aligned}
$$

$$c_2 = \frac{1}{3}\pi^3 L^5[-3 + 4L^{-2}(63e_{2,1} + 9e_{2,2} + 7e_{2,3}) + 3L^{-4}(-5292e_{3,1} - 756e_{3,2} - 108e_{3,3} - 108e_{3,4} - 476e_{3,5} - 68e_{3,6} + 20e_{3,7} - 7e_{3,8}) + 24L^{-6}(37044e_{4,1} + 5292e_{4,2} + 756e_{4,3} + 756e_{4,4} + 108e_{4,5} + 756e_{4,6} + 108e_{4,7} + 2940e_{4,8} + 420e_{4,9} + 420e_{4,10} + 60e_{4,11} + 60e_{4,12} + 108e_{4,13} + 60e_{4,14} - 84e_{4,15} + 63e_{4,16} + 60e_{4,17} - 12e_{4,18} + 9e_{4,19} + 196e_{4,20} + 28e_{4,21} - 44e_{4,22} - 22e_{4,23} + 13e_{4,24} + 12e_{4,25} + 20e_{4,26})], \tag{C3}$$

$$c_3 = \frac{\pi^6}{3024} C_T\Big|_{d=6}. \tag{C4}$$

APPENDIX D: GENERAL STRUCTURE OF THREE-POINT FUNCTION

Holographic three-point functions can be extracted from the cubic effective action of graviton on AdS background. For higher curvature gravities whose the Lagrangian depends on the metric g_{ab} and R_{abcd}, the metric dependence becomes implicit if one chooses $R_{ab}{}^{cd}$ as the independent variable [30], *i.e.*, $\mathcal{L} = \mathcal{L}(R_{ab}{}^{cd})$. The general form of the cubic effective action can then be obtained by varying the action to the third order

$$\begin{aligned} S^{(3)} &= \frac{1}{3!}\left[\delta^3 \int \mathrm{d}^{d+1}x\sqrt{|g|}\mathcal{L}(R_{ab}{}^{cd})\right]_{\bar{g}_{ab}} \\ &= \frac{1}{6}\int \mathrm{d}^{d+1}x\sqrt{|g|}\Big[\bar{P}^{ab}{}_{cd}\bar{\delta}^3(R_{ab}{}^{cd}) \\ &\quad + 3\bar{C}^{ab}{}_{cd}{}^{ef}{}_{gh}\bar{\delta}^2(R_{ab}{}^{cd})\bar{\delta}(R_{ef}{}^{gh}) \\ &\quad + \bar{G}^{ab}{}_{cd}{}^{ef}{}_{gh}{}^{ij}{}_{kl}\bar{\delta}(R_{ab}{}^{cd})\bar{\delta}(R_{ef}{}^{gh})\bar{\delta}(R_{ij}{}^{kl}) \\ &\quad - \frac{3}{2}\bar{P}^{ab}{}_{cd}\bar{\delta}(R_{ab}{}^{cd})h_{ef}h^{ef} - \frac{4}{L^2}k_1 h^b_a h^c_b h^a_c\Big], \end{aligned} \tag{D1}$$

where $h_{ab} = \delta g_{ab}$, $\bar{g}_{ab}$ is the AdS metric, tensors P^{abcd} and $C^{abcdefgh}$ are given by (12) and (16) respectively. The tensor $G^{abcdefghijkl}$ is defined by

$$G^{abcdefghijkl} = \frac{\partial^3\mathcal{L}}{\partial R_{abcd}\partial R_{efgh}\partial R_{ijkl}}. \tag{D2}$$

We have also imposed the transverse and traceless condition. It now becomes clear that there are three different structures of the three-point function, controlled by the tensors P^{abcd}, $C^{abcdefgh}$, $G^{abcdefghijkl}$ evaluated on the AdS background respectively. The AdS background has $\bar{R}_{abcd} = -2/L^2\bar{g}_{a[c}\bar{g}_{d]b}$, it follows that when evaluated on this background, these three tensors can only be built from the metric $\bar{g}_{ab}$, thus their forms are all rigidly fixed, with theory-dependent coefficients, e.g., (13) and (17). For Einstein gravity only the contribution from P^{abcd} is nonzero, while for quadratic gravity the $G^{abcdefghijkl}$ contribution vanishes. All three tensors are nonzero for cubic gravity, which can therefore enumerate all the possible structures. For the quartic or higher orders, no new structures arise; they just modify the coefficients in these three tensors.

APPENDIX E: QUASITOPOLOGICAL GRAVITY

We shall briefly review QT gravity and present dimension-generic cubic and quartic QT combinations. There has been an extensive study on QT gravity (e.g., Refs. [7,31–35]). A quasitopological (QT) gravity is a type of gravity theories whose equation of motion on the special spherically symmetric metric ansatze

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Sigma^2_{d-1,k} \tag{E1}$$

is algebraic in $f(r)$, *i.e.*, does not involve derivatives of $f(r)$. This condition is equivalent to [36]

$$\nabla_a P^{abcd}|_f = 0 \tag{E2}$$

where $\ldots|_f$ denotes evaluating on the metric ansatze (E1). This makes the black hole solutions of QT gravities easy to obtain, thus QT gravity serves as a simplified model of general higher curvature gravity.

At a given curvature polynomial order, it is straightforward to derive QT curvature combinations using constraints arise from (E2). For our purpose we only consider dimension-generic case here, but it is important to note that at specific dimensions there may be more possible combinations than dimension-generic case. We have two linearly independent cubic combinations with coupling constants given by

$$\{e^{Q,1}_{3,i}\} = \{d^2 - 2d + 3, -12(d-1)^2, 16(d-2)d, 24(d^2 - 3d + 2), 3(d^2 - 2d - 1), -24(d^2 - 3d + 1), 4d^2 - 14d + 6, 0\}, \tag{E3}$$

$$\{e^{Q,2}_{3,i}\} = \{3(d+1), 12 - 36d, 48(d-1), 24(d+1), 9d - 15, -24(d-1), 0, 8(2d^2 - 7d + 3)\}. \tag{E4}$$

Note that Lovelock combination is a special case of QT gravity, we thus have only one nontrivial cubic QT combination. For the quartic order we have 15 independent combinations, the full set of them is too lengthy to be presented here so we only show the first two of them.

$$\{e_{4,i}^{Q,1}\} = \{1, -2(d+1), 8(d-1), d^2 - 4d + 7, 2 - 2d^2, 0, 0, \ldots\}, \tag{E5}$$

$$\{e_{4,i}^{Q,2}\} = \{0, 2(d+1), 0, -2(d^2 - 2d + 5), 0, -16(d-1), 8(d^2 - 1), -d^2 + 3d - 4, 4(d-1)^2, d^3 - 4d^2 + 5d + 2, -2(d-1)^2(d+1), 0, 0, \ldots\}. \tag{E6}$$

The full set of quartic QT combinations can be found in the Supplemental Material `r4QTG.wl` [37], which is a Wolfram Language file with further instructions included in the usage messages.

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