Post-Newtonian gravitational waves with cosmological constant Λ from the Einstein-Hilbert theory

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We study the compact binary dynamics in the post-Newtonian approach implemented to the Einstein-Hilbert action adding the cosmological constant Λ at first post-Newtonian (1PN) order. We consider very small values of Λ finding that it plays the role of a PN factor to derive the Lagrangian of a compact two-body system at the center of mass frame at 1PN. Furthermore, the phase function $\phi(t)$ is obtained from the balance equation, and the two polarizations h_+ and $h_\times$ are also calculated. We observe changes due to Λ only at very low frequencies, and we notice that it plays the role of "stretch" the spacetime such that both amplitudes become smaller. However, given its nearly negligible value, Λ has no relevance at higher frequencies whatsoever.

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I. INTRODUCTION

The direct detection of gravitational waves (GWs) has opened new perspectives to understand the nature and behavior of the Universe from an astrophysical point of view [1,2]. These observations strengthen the general relativity (GR) predictions given by Einstein in 1916 [3]. On the other hand, from a plethora of gravitational phenomena, the very small value of the astrophysical cosmological constant $\Lambda \simeq 10^{-52}\ \mathrm{m}^{-2}$ [4] is probably the reason for not considering its contribution into the Einstein field equations (EFE); however, there are in fact astronomical observations that suggest that Λ might cause the current accelerated expansion of the Universe [5–8]. For instance, the analysis of the cosmological microwave background radiation [9,10] must include the effect of Λ; nonetheless, studies of probable observational effects inside the solar system, due to the Λ, are nearly negligible to be detected [11,12]. Certainly, the standard cosmological Λ cold dark matter (ΛCDM) model has been successfully tested throughout several sources of observations, and it remains the most simple yet accurate scenario. However, there are still areas of unresolved phenomenology and ignorance.

Moreover, the post-Newtonian (PN) expansion is implemented in GR to obtain approximate solutions of the EFE. This method consists in expanding the metric at various orders around small values of the velocity ratio v/c, where v is the typical internal velocity of the system (or the relative velocity in the binary case) [13]. Here, the Newtonian theory is recovered when taking the limit of the speed of light to infinity, or the velocity ratio to zero. Einstein first made use of the PN approximation (at first order) to compute the perihelion precession of the Mercury's orbit [14,15]. Nowadays, this method is mostly utilized to study the propagation of GWs of the relativistic two-body problem (see, for instance, [16]). Note that this approach is only valid at the very near zone of the source $\mathcal{R}$, namely, in the region when the evaluation point r $(0 < r \ll \mathcal{R})$ is much smaller than the emitted wavelength λ; in other words, the condition $r \ll \lambda$ must be satisfied, where there are no effects of time retardation in this region [17–19]. On the other hand, in the external domain $d < r < \infty$, we introduce the post-Minkowskian (PM) approximation, where d is the radius of the smallest sphere comprehending the whole system, and here, the gothic metric $\mathfrak{g}^{\mu\nu} \coloneqq \sqrt{-g}g^{\mu\nu}$ is written as an expansion of powers of the Newtonian constant of gravitation G. Note that there is an overlapping region $d < r < \mathcal{R}$; therefore, the coefficients of the external domain can be expressed in terms of powers of the PN approximation by matching relations [18]. The DIRE (Direct Integration of Relaxed Einstein Equation) can be used to compute a wave equation of the EFE in an exact form as long as the harmonic gauge holds to obtain waveforms as powers of PN orders [20,21].

The starting point is the Einstein-Hilbert (EH) action for the gravitational field with cosmological constant Λ given by

$$S[g] = \int_M d^4x\sqrt{-g}\left[\frac{16\pi G}{c^4}(R - 2\Lambda) + \mathcal{L}_m\right], \qquad (1)$$

where G is the Newton's gravitational constant, c is the speed of light in vacuum, the metric determinant is $(-g) \coloneqq \det(g_{\mu\nu})$, $\mathcal{L}_m$ represents an arbitrary Lagrangian density that describes matter, $R \coloneqq g^{\mu\nu}R_{\mu\nu}$ and $R_{\mu\nu} \coloneqq R^{\alpha}{}_{\mu\alpha\nu}$

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are the scalar curvature and Ricci curvature tensor, respectively. They are derived from the curvature tensor $R^{\alpha}{}_{\beta\gamma\delta} = \partial_\gamma \Gamma^{\alpha}{}_{\beta\delta} - \partial_\delta \Gamma^{\alpha}{}_{\beta\gamma} + \Gamma^{\rho}{}_{\beta\delta}\Gamma^{\alpha}{}_{\rho\gamma} - \Gamma^{\rho}{}_{\beta\gamma}\Gamma^{\alpha}{}_{\rho\delta}$, where the Christoffel symbols are given in terms of the metric tensor and its partial derivatives $\Gamma^{\rho}{}_{\alpha\beta} = \frac{1}{2} g^{\rho\gamma}(\partial_\alpha g_{\gamma\beta} + \partial_\beta g_{\gamma\alpha} - \partial_\gamma g_{\alpha\beta})$. Thus, the EFE are given by

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{2}$$

where the source term is obtained by $T^{\mu\nu} \coloneqq -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g_{\mu\nu}}$.

The main aim of this paper consists in exploring the effects on the propagation of GWs due to the presence of Λ, having a two-body problem examined to the 1PN order in the post-Newtonian method. We then compare our results with those with $\Lambda = 0$ at 2PN [22,23], having the same inspiralling compact binary systems. Furthermore, previous studies have explored the effects of Λ in the linearized GR; the authors expanded the metric around a flat Minkowski spacetime [24,25]. However, the present work is the first one to include such a constant in the two-body problem.

We begin our analysis in Sec. II where we solve the EFE through the DIRE approach utilizing the gothic metric and imposing the harmonic gauge; we then compute the tensor waveforms h^{ij} at the near zone contribution of the faraway components through the Epstein Wagoner (EW) tensors at 1PN order at the center of mass frame of a binary compact system. Many explicit calculations are presented in Appendices A–E. Then, in Sec. IV, the circular orbit properties are explained, and the PN parameters γ and x are introduced. Also, we calculate the energy loss rate. After that, by taking into account the balance equation (79), we obtain the orbital phase $\phi(t)$ of the two-body system at 1PN approximation. Note that these results can also be derived using the symmetric trace-free tensor (STF) (see Appendix D). Moreover, in Sec. V, we compute the polarizations waveforms $h_+(t)$ and $h_\times(t)$. We finish the paper by making some remarks in Sec. VI.

A. Conventions

We consider a 4-dimensional spacetime manifold M. Spacetime indices are designated by Greek letters $\mu, \nu, \ldots = \{0, i\}$ where i labels spatial components of tensors, and 0 indicates the temporal component. These indices are raised and lowered with the spacetime metric $g_{\mu\nu}$ which signature is given by $(-1, 1, 1, 1)$. The repeated indices mean sum throughout the paper unless otherwise stated. The symmetric and trace-free part of a tensor $T^{i_1 i_2 i_3 \cdots i_n}$ is denoted as $T^{\langle i_1 i_2 i_3 \cdots i_n \rangle}$. The time derivative of an object is represented by a dot over the corresponding variable.

II. RELAXED EFE AND WAVEFORM

To solve the EFE (2) in the weak-field limit, we use the DIRE approach [20] (see also [21,26]). First, we introduce the gothic metric $\mathfrak{g}^{\mu\nu} \coloneqq \sqrt{-g} g^{\mu\nu}$. Then, we define the tensor,

$$H^{\alpha\mu\beta\nu} \coloneqq \mathfrak{g}^{\mu\nu}\mathfrak{g}^{\alpha\beta} - \mathfrak{g}^{\alpha\nu}\mathfrak{g}^{\beta\mu}, \tag{3}$$

where the following identity holds:

$$\partial_\mu \partial_\nu H^{\mu\alpha\nu\beta} = (-g)\left(2G^{\alpha\beta} + \frac{16\pi G}{c^4} t^{\alpha\beta}_{\mathrm{LL}}\right). \tag{4}$$

Here, $t^{\alpha\beta}_{\mathrm{LL}}$ is the Landau-Lifshitz energy-momentum tensor [27],

$$\begin{aligned} \frac{16\pi G}{c^4}(-g) t^{\alpha\beta}_{LL} = {} & g_{\lambda\mu} g^{\nu\rho} \partial_\nu \mathfrak{g}^{\alpha\lambda} \partial_\rho \mathfrak{g}^{\beta\mu} + \frac{1}{2} g_{\lambda\mu} g^{\alpha\beta} \partial_\rho \mathfrak{g}^{\lambda\nu} \partial_\nu \mathfrak{g}^{\rho\mu} \\ & - g_{\mu\nu}(g^{\lambda\alpha}\partial_\rho \mathfrak{g}^{\beta\nu} + g^{\lambda\beta}\partial_\rho \mathfrak{g}^{\alpha\nu})\partial_\lambda \mathfrak{g}^{\rho\mu} \\ & + \frac{1}{8}(2g^{\alpha\lambda}g^{\beta\mu} - g^{\alpha\beta}g^{\lambda\mu})(2g_{\nu\rho}g_{\sigma\tau} \\ & - g_{\rho\sigma}g_{\nu\tau})\partial_\lambda \mathfrak{g}^{\nu\tau}\partial_\mu \mathfrak{g}^{\rho\sigma}, \end{aligned} \tag{5}$$

and the Einstein tensor $G_{\mu\nu}$ is defined as

$$G_{\mu\nu} \coloneqq R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda \mathfrak{g}^{-1/2} \mathfrak{g}_{\mu\nu}, \tag{6}$$

with $\mathfrak{g} \coloneqq \det(\mathfrak{g}^{\mu\nu}) = (-g)$. In order to study the field outside the source, we expand the gothic metric around the Minkowski metric as follows:

$$\mathfrak{g}^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}, \tag{7}$$

where $h^{\mu\nu}$ stands as a potential. We select the harmonic gauge [17–19]: $\partial_\mu \mathfrak{g}^{\mu\nu} = 0$; in this case, the relation (4) becomes the wave equation,

$$\Box h^{\alpha\beta} = \frac{16\pi G}{c^4} \mu^{\alpha\beta}, \tag{8}$$

where $\mu^{\alpha\beta}$ is the source of the system, which is given by

$$\mu^{\alpha\beta} = (-g) T^{\alpha\beta} + \frac{c^4}{16\pi G} \Lambda^{\alpha\beta}_{\mathrm{GR}}, \tag{9}$$

and in this approach, the cosmological constant is taken as

$$\begin{aligned} \Lambda^{\alpha\beta}_{\mathrm{GR}} \coloneqq {} & \frac{16\pi G}{c^4}(-g) t^{\alpha\beta}_{LL} - 2\Lambda \mathfrak{g}^{-1/2} \mathfrak{g}^{\alpha\beta} + \partial_\mu h^{\alpha\mu} \partial_\nu h^{\beta\nu} \\ & - h^{\mu\nu} \partial_\mu \partial_\nu h^{\alpha\beta}. \end{aligned} \tag{10}$$

The previous expression (8) is known as the relaxed EFE.

Notice that the harmonic gauge $\partial_\mu \mathfrak{g}^{\mu\nu} = 0$ and (8), together, are equivalent to the Einstein field equations [17–19]. Additionally, at this point of the analysis, we stress that (7) represents a change of variable since the transformation between $g^{\mu\nu}$ and $h^{\mu\nu}$ is one-to-one and invertible. More explicitly, we have

$$h^{\mu\nu} = \sqrt{-g}g^{\mu\nu} - \eta^{\mu\nu}, \tag{11}$$

$$g^{\mu\nu} = [\det(\eta^{\alpha\beta} + h^{\alpha\beta})]^{-1/2}(\eta^{\mu\nu} + h^{\mu\nu}). \tag{12}$$

A. The tensor waveforms at 1PN

In this subsection, we compute the near-zone contribution waveform of h^{ij}. The near zone contribution of the faraway zone term is identified as h^{ij}_{N}, which is given by [20,21]

$$h^{ij}_{\mathrm{N}}(x) = \frac{2G}{Rc^4}\frac{d^2}{dt^2}\sum_{l=0}^{\infty}\hat{N}_{k_1}\cdots\hat{N}_{k_l}I^{ijk_1\cdots k_l}_{\mathrm{EW}}, \tag{13}$$

where $\hat{N}_{k_l}$ is the unit normal vector pointing from the source to the detector, and R is the distance between the source and the detector. All terms of $I^{ijk_1\cdots k_l}_{\mathrm{EW}}$ are known as the EW moments, and they are given explicitly as

$$I^{ij}_{\mathrm{EW}} \coloneqq \frac{1}{c^2}\int_M \mu^{00}x^i x^j d^3x, \tag{14}$$

$$I^{ijk}_{\mathrm{EW}} \coloneqq \frac{1}{c^3}\int_M (2\mu^{0(i}x^{j)}x^k - \mu^{0k}x^i x^j)d^3x, \tag{15}$$

$$I^{ijk_1\cdots k_l}_{\mathrm{EW}} \coloneqq \frac{2}{l!c^2}\frac{d^{l-2}}{d(ct)^{l-2}}\int_M \mu^{ij}x^{k_1}x^{k_2}\cdots x^{k_l}d^3x. \tag{16}$$

To determine the GWs at 1PN order, we have to compute up to the fourth index of the EW moments, keeping in mind the transverse-traceless (TT) gauge [21] of the spatial tensor, that is,

$$h^{ij}_{\mathrm{N}}(x) = \frac{2G}{Rc^4}\frac{d^2}{dt^2}\{I^{ij} + \hat{N}_k I^{ijk} + \hat{N}_k\hat{N}_l I^{ijkl}\}_{\mathrm{TT}}. \tag{17}$$

Moreover, the TT operator acting on a tensor object A^{ij} is such that [18,19]

$$A^{ij}_{\mathrm{TT}} = \left(P^{ik}P^{jl} - \frac{1}{2}P^{ij}P^{kl}\right)A_{kl}, \tag{18}$$

$$P^{ij} \coloneqq \delta^{ij} - \hat{N}^i\hat{N}^j, \tag{19}$$

where P^{ij} is an operator projection and satisfying the properties $P^{ii} = 2$, $P^{ij}P_{ij} = 2$, and $P^{ij}P^{ik} = P^{jk}$.

In order to compute the EW moments, we need to know the explicit form of the source components $\mu^{\alpha\beta}$ to the 1PN order. To do so, we have to obtain the terms h^{00}, h^{0i}, and h^{ij} at the lowest order from the relaxed EFE, where the energy-momentum tensor has the following form [27]:

$$T^{\mu\nu} = \frac{1}{\sqrt{-g}}\sum_a m_a\frac{d\tau_a}{dt}\frac{dx^\mu_a}{d\tau_a}\frac{dx^\nu_a}{d\tau_a}\delta^3(\vec{x} - \vec{x}_a(t)), \tag{20}$$

where $\delta^3(\vec{x} - \vec{x}_a(t))$ is the 3-dimensional Dirac delta, m_a is the mass of the particle a, τ_a is the proper time of the particle a, and t is the time coordinate. Therefore, from the relaxed EFE (8), the equations of motion become

$$\nabla^2 h^{00} = \frac{16\pi G}{c^2}\sum_a m_a\delta^3(\vec{x} - \vec{x}_a(t)) + 2\Lambda + O\left(\Lambda h, \frac{1}{c^4}\right), \tag{21}$$

$$\nabla^2 h^{0i} = O\left(\Lambda h, \frac{1}{c^3}\right), \tag{22}$$

$$\nabla^2 h^{ij} = -2\Lambda\delta^{ij} + O\left(\Lambda h, \frac{1}{c^4}\right). \tag{23}$$

Notice that Eq. (21) represents the equation of motion for h^{00} at the lowest order where in the first term of the right-hand side involves a PN term $1/c^2$. Therefore, we can say that the cosmological constant Λ plays the role of a PN factor.

It is worth mentioning that in this approach the field $h_{\mu\nu}$ and Λ are considered as perturbations; namely, we are considering a very small value of the cosmological constant [24]. Hence, the equations of motion of all $h^{\alpha\beta}$ are taken at the lowest order and neglecting the terms $O(\Lambda h^{\mu\nu})$, and the harmonic gauge must be met, i.e., $\partial_\mu h^{\mu\nu} = 0$. We point out that the lowest order of h^{0i} and h^{ij} components are $O(c^{-3}, \Lambda h)$ and $O(1, \Lambda)$, respectively. Therefore, the solutions of the $h^{\mu\nu}$ components read

$$h^{00} = -\frac{4G}{c^2}\sum_a\frac{m_a}{r_a} + \frac{\Lambda}{3}|\vec{x}|^2 + O(\Lambda h, c^{-4}), \tag{24}$$

$$h^{0i} = O(\Lambda h, c^{-3}), \tag{25}$$

$$h^{ij} = \delta^{ij}\left[-\frac{1}{2}\Lambda(|\vec{x}|^2 - x_i^2)\right] + O(\Lambda h, c^{-4}), \tag{26}$$

with no sum in the term x_i from h^{ij} (26). Notice that the trace of the spatial component reads $h^{ii} = -\Lambda|\vec{x}|^2$. In explicit matrix form, the solution $h^{\mu\nu}$ is written as

$$h^{\mu\nu} = \begin{pmatrix} -\frac{4G}{c^2}\sum_a \frac{m_a}{r_a} + \frac{\Lambda}{3}|\vec{x}|^2 & 0 & 0 & 0 \\ 0 & -\frac{1}{2}\Lambda(y^2+z^2) & 0 & 0 \\ 0 & 0 & -\frac{1}{2}\Lambda(x^2+z^2) & 0 \\ 0 & 0 & 0 & -\frac{1}{2}\Lambda(x^2+y^2) \end{pmatrix}, \tag{27}$$

where x, y and z are the Cartesian coordinates. This solution satisfies the harmonic gauge condition $\partial_\mu h^{\mu\nu} = 0$ at lowest order; i.e., the condition $\partial_\mu h^{\mu 0} = 0$ holds as long as we add h^{0i} at order $O(\Lambda h, c^{-3})$ and h^{00} [given in Eq. (24)], while for the case $\partial_\mu h^{\mu i} = 0$ we only need to consider h^{ij}, which is given in Eq. (26). This constraint is a fundamental feature of the methodology of Blanchet and Damour [17], and this implies that the solution expressed in (27) has cylindrical symmetry around the corresponding principal axis. Note that this property is a remnant of the rotational symmetry; hence, the breaking of this symmetry is an artifact of the harmonic gauge condition [24]. It is worth stressing that the solution given in (27) leads to the linearized Schwarchild-de Sitter metric, written in a set of appropriate coordinates that correspond to the harmonic gauge [24]. Therefore, the spacetime that we are considering at the lowest order is de Sitter or anti-de Sitter.

In [28] (see also [29–35]), the linearization of gravity is performed introducing a de Sitter background in the form of the spacetime metric (the cosmological constant Λ does not comes from the action). In such linearized Einstein equations, the perturbation is not sharp; there is a tail term. Similarly, in our work, we observe in (27) that due to Λ the solution of the field h^{00} has a sharp and a tail term, where Λ is found in the latter. Moreover, note that h^{ij} at the lowest order only has a tail term; however, at higher orders, in h^{0i} and as well as h^{ij}, both contributions will appear, i.e., sharp and tail terms.

From the definition of the gothic metric and its expansion (7), we can obtain the components of the metric at the same order of the perturbation $h^{\mu\nu}$. Accordingly, we have

$$(-g) = \det(\mathfrak{g}^{\mu\nu}) = \det(\eta^{\mu\nu} + h^{\mu\nu}) = 1 + h + O(h^2), \tag{28}$$

with $h \coloneqq h^{\mu\nu}\eta_{\mu\nu}$ and $\mathfrak{g}_{\mu\nu} = \eta_{\mu\nu} - h_{\mu\nu} + O(h^2)$. Thus, the components of the metric at lowest order read

$$\begin{aligned} g_{00} &= \sqrt{-g}\mathfrak{g}_{00} \\ &= -\left(1 + \frac{1}{2}h^{00} + \frac{1}{2}h^{ii}\right) + O(h^2, \Lambda h), \end{aligned} \tag{29}$$

$$\begin{aligned} g_{0i} &= \sqrt{-g}\mathfrak{g}_{0i} \\ &= O(\Lambda h, c^{-3}), \end{aligned} \tag{30}$$

$$\begin{aligned} g_{ij} &= \sqrt{-g}\mathfrak{g}_{ij} \\ &= \delta_{ij} - h_{ij} - \frac{1}{2}\delta_{ij}h^{00} + \frac{1}{2}h^{kk}\delta_{ij} + O(h^2, \Lambda h). \end{aligned} \tag{31}$$

Here, we have used $h = -h^{00} + h^{ii}$. When substituting the components of the metric into those of the source (9), it yields

$$\begin{aligned} \mu^{00} &= c^2\sum_a m_a\left(1 - \frac{3}{4}h^{00} + \frac{1}{4}h^{ii} + \frac{v_a^2}{2c^2}\right)\delta^3(\vec{x} - \vec{x}_a(t)) \\ &\quad + \frac{c^4}{16\pi G}\left[-\frac{7}{8}\partial_i h^{00}\partial_i h^{00} + 2\Lambda\right] + O(\Lambda h, c^{-2}), \end{aligned} \tag{32}$$

$$\mu^{0i} = c\sum_a m_a v_a^i \delta^3(\vec{x} - \vec{x}_a(t)) + O(\Lambda h, c^{-2}), \tag{33}$$

$$\begin{aligned} \mu^{ij} &= \sum_a m_a v_a^i v_a^j \delta^3(\vec{x} - \vec{x}_a(t)) + \frac{c^4}{16\pi G}\left[\frac{1}{4}\partial^i h^{00}\partial^j h^{00}\right. \\ &\quad \left. - \frac{1}{8}\delta^{ij}\partial_k h^{00}\partial_k h^{00}\right] - 2\Lambda\delta^{ij} + O(\Lambda h, c^{-2}), \end{aligned} \tag{34}$$

with v_a as the velocity of the particle a. We stress that the cosmological constant only appears in the last terms of μ^{00} and μ^{ij}. In the case of $\Lambda = 0$, one recovers the usual gravitational case at 1PN order [26].

III. EVALUATION OF THE EPSTEIN-WAGONER MOMENTS

From the expressions (14)–(16), one can observe that the EW moments are written as volume integrals, evaluated at the retarded time $\tau = t - R/c$, and the boundary region ∂M of the near zone is given by $\mathcal{R}$. In this section, we present some steps, in detail, in order to obtain the EW moments; particularly, we focus in the terms that contain Λ, since the integrals that do not have this parameter have already been evaluated in [21]. Also, in various occasions, we integrate by parts, and we use the identity [19,21]

$$\int_M \partial_k F^{ij\cdots k} d^3x = \int_{\partial M} F^{ij\cdots k}|_{\mathcal{R}} \hat{N}_k \mathcal{R}^2 d\Omega, \tag{35}$$

with ∂M as the boundary of the 3-dimensional manifold M at the near region, and $\mathcal{R}^2\hat{n}^k d\Omega^2 = dS^k$ is the surface element at this border. Moreover, we are only interested in finding the tensorial terms that survive after applying the TT projector. Thus, the subsequent identities follow from the projection operator ${P^i}_j$, given in Eqs. (18) and (19),

$$(\delta^{ij})_{\mathrm{TT}} = 0, \tag{36}$$

$$(\hat{N}^i F^j)_{\mathrm{TT}} = 0, \tag{37}$$

where the indices i and j apply to the final components of the waveform (not the integrands), and F denotes a general term. All these results and procedures are explained in detail in [19,21].

A. Two-index moment I^{ij}_{EW}

The first step to compute the moment I^{ij}_{EW} begins using the following useful identity:

$$\partial_k h^{00}\partial_k h^{00} x^i x^j = \partial_k(h^{00}\partial_k h^{00} x^i x^j) - h^{00}(\nabla^2 h^{00} x^i x^j + \partial^i h^{00} x^j + \partial^j h^{00} x^i), \tag{38}$$

and after some algebraic manipulation, we obtain

$$-h^{00}\partial^i h^{00} x^j - h^{00}\partial^j h^{00} x^i = -\frac{1}{2}\partial^i[(h^{00})^2 x^j] - \frac{1}{2}\partial^j[(h^{00})^2 x^i] + (h^{00})\delta^{ij}. \tag{39}$$

Substituting Eq. (32) into (14), then using Eqs. (38) and (39), neglecting the boundary terms and considering the TT components of the far zone tensor perturbation, it yields

$$I^{ij}_{\mathrm{EW}} \overset{\mathrm{TT}}{=} \frac{1}{c^2}\int_M \left\{ c^2 \sum_a m_a \left(1 - \frac{3}{4}h^{00} + \frac{1}{4}h^{ii} + \frac{v_a^2}{2c^2}\right)\delta^3(\vec{x} - \vec{x}_a(t))\right\} x_a^{\langle i} x_a^{j\rangle} d^3x + \frac{c^4}{c^2 16\pi G}\frac{7}{8}\int_M h^{00}\nabla^2 h^{00} x^{\langle i}x^{j\rangle} d^3x. \tag{40}$$

Next, we use the results (21) and (24), so the last term becomes

$$\begin{aligned}
\frac{c^2}{16\pi G}\frac{7}{8}\int_M h^{00}\nabla^2 h^{00} x^{\langle i}x^{j\rangle} d^3x &\overset{\mathrm{TT}}{=} \frac{c^2}{16\pi G}\frac{7}{8}\int_M \left(-\frac{4G}{c^2}\sum_a \frac{m_a}{r_a} + \frac{\Lambda}{3}|\vec{x}|^2\right)\nabla^2 h^{00} x^{\langle i}x^{j\rangle} d^3x \\
&\overset{\mathrm{TT}}{=} \frac{c^2}{16\pi G}\frac{7}{8}\int_M \left(-\frac{4G}{c^2}\sum_a \frac{m_a}{r_a}\nabla^2 h^{00} x^{\langle i}x^{j\rangle}\right) d^3x \\
&\overset{\mathrm{TT}}{=} -\frac{7G}{2c^2}\sum_a\sum_{b\neq a}\frac{m_b}{r_{ab}} x_a^{\langle i}x_a^{j\rangle},
\end{aligned} \tag{41}$$

where we have neglected all the terms at order $O(\Lambda h)$. Finally, plugging in (24) and (26) into the first term of (40) and putting together the previous result, the two-index EW moment becomes

$$I^{ij}_{\mathrm{EW}} \overset{\mathrm{TT}}{=} \sum_a m_a\left[1 - \frac{G}{2c^2}\sum_{b\neq a}\frac{m_b}{r_{ab}} - \frac{\Lambda}{2}|\vec{x}_a|^2 + \frac{v_a^2}{2c^2}\right] x_a^i x_a^j + O(\Lambda h, c^{-4}). \tag{42}$$

B. Three-index moment I^{ijk}_{EW}

In this case since there is no cosmological constant contribution, the computation of the integral is direct, yielding

$$I^{ijk}_{\mathrm{EW}} \overset{\mathrm{TT}}{=} \frac{1}{c}\sum_a m_a[2v_a^{(i}x_a^{j)}x_a^k - v_a^k x_a^i x_a^j] + O(\Lambda h, c^{-3}). \tag{43}$$

C. Four-index moment I^{ijkl}_{EW}

Plugging in the spatial component of the source (34) into the corresponding four indices integral (16) and considering the TT gauge, we obtain

$$I^{ijkl}_{\mathrm{EW}} \overset{\mathrm{TT}}{=} \frac{1}{c^2}\sum_a m_a v_a^i v_a^j x_a^k x_a^l + \frac{c^2}{64\pi G}\int_M \partial^i h^{00}\partial^j h^{00} x^k x^l d^3x. \tag{44}$$

Next, we integrate by parts the last term, neglecting all the boundary terms, applying the TT gauge and using (24), yielding

$$\begin{aligned}
\int_M \partial^i h^{00}\partial^j h^{00} x^k x^l d^3x &\overset{\mathrm{TT}}{=} -\int_M h^{00}\partial^i\partial^j h^{00} x^k x^l d^3x \\
&\overset{\mathrm{TT}}{=} \frac{4G}{c^2}\int_M \sum_a \frac{m_a}{r_a}\partial^i\partial^j h^{00} x^k x^l d^3x.
\end{aligned} \tag{45}$$

Here, in the last term, we have neglected expressions at order $O(\Lambda h)$, and also, the TT gauge projection was not used. On the other hand, using again the result (24), one obtains

$$\partial_i\partial_j h^{00} = \frac{4G}{c^2}\sum_a m_a\left[\frac{1}{|\vec{x}-\vec{x}_a|^3}\delta_{ij} - \frac{3}{|\vec{x}-\vec{x}_a|^5}(\vec{x}-\vec{x}_a)_i(\vec{x}-\vec{x}_a)_j\right] + \frac{2}{3}\Lambda\delta_{ij}, \tag{46}$$

with $\vec{x}\neq\vec{x}_a$. After applying the TT gauge, the cosmological constant term vanishes. Therefore, it turns out that the four indices EW moment has no Λ; thus, from [21], the integral reads

$$I^{ijkl}_{\rm EW} \overset{\rm TT}{=} \frac{1}{c^2}\sum_a m_a v_a^i v_a^j x_a^k x_a^l + \sum_a\sum_{b\neq a}\left[\frac{Gm_a m_b}{12rc^2}x^i x^j\left(\frac{x^k x^l}{|\vec{x}|^2} - \delta^{kl} - 6\frac{x_a^k x_a^l}{|\vec{x}|^2}\right)\right] + O(\Lambda, hc^{-4}). \tag{47}$$

We remark that in the three- and four-index moments there are no contributions of the cosmological constant under the TT projection; however, Λ will have an impact on higher order approximations.

D. Center of mass at 1PN order with Λ

To express the waveform in terms of the relative variables, we move to the center of mass frame $X^i_{\rm CM} = 0$, that is,

$$\begin{aligned} X^i_{\rm CM} &\coloneqq \frac{1}{m}\int_M \mu^{00} x^i d^3\vec{x} \\ &= \frac{1}{m}\sum_a m_a x_a^i - \frac{G}{2c^2 m}\sum_a m_a \sum_{b\neq a}\frac{m_b}{r_{ab}}x_a^i + \frac{1}{2c^2 m}\sum_a m_a v_a^2 x_a^i - \frac{\Lambda}{2m}\sum_a m_a|\vec{x}_a|^2 x_a^i \\ &\quad + O(\Lambda h, c^{-3}), \end{aligned} \tag{48}$$

where we have used Eqs. (24) and (32), and the spatial trace of Eq. (26). On the other hand, considering only two particles in interaction, we find at 1PN order that the coordinates of each body in the center of mass frame are given by

$$\vec{r}_1 = \frac{\mu}{m_1}\vec{r} + \frac{\mu\Delta m}{2m^2c^2}\left(v^2 - \frac{Gm}{r} - \frac{\Lambda c^2 r^2}{2}\right)\vec{r} + O(c^{-2}\Lambda, \Lambda^2, c^{-3}), \tag{49}$$

$$\vec{r}_2 = -\frac{\mu}{m_2}\vec{r} + \frac{\mu\Delta m}{2m^2c^2}\left(v^2 - \frac{Gm}{r} - \frac{\Lambda c^2 r^2}{2}\right)\vec{r} + O(\Lambda c^{-2}, \Lambda^2, c^{-3}), \tag{50}$$

with $\vec{r} = \vec{r}_1 - \vec{r}_2$ as the relative position, $r = |\vec{r}|$, $\vec{v} = \vec{v}_1 - \vec{v}_2$ as the relative velocity, $v = |\vec{v}|$, $\mu = m_1 m_2/m$ as the reduced mass of the binary system, $m = m_1 + m_2$, and $\Delta m \coloneqq m_1 - m_2$.

Computing the time derivative of the positions given by Eqs. (49) and (50) leads to the velocities of each particle,

$$\vec{v}_1 = \frac{\mu}{m_1}\vec{v} + \frac{\mu\Delta m}{2m^2c^2}\left[\left(v^2 - \frac{Gm}{r} - \frac{\Lambda c^2 r^2}{2}\right)\vec{v} - \left(\frac{Gm}{r^2} + \frac{\Lambda c^2 r}{2}\right)\dot{r}\,\vec{r}\right] + O(\Lambda c^{-2}, \Lambda^2, c^{-4}), \tag{51}$$

$$\vec{v}_2 = -\frac{\mu}{m_2}\vec{v} + \frac{\mu\Delta m}{2m^2c^2}\left[\left(v^2 - \frac{Gm}{r} - \frac{\Lambda c^2 r^2}{2}\right)\vec{v} - \left(\frac{Gm}{r^2} + \frac{\Lambda c^2 r}{2}\right)\dot{r}\,\vec{r}\right] + O(\Lambda c^{-2}, \Lambda^2, c^{-4}). \tag{52}$$

Now, we substitute the positions and velocities (49)–(52) into the EW moments (42), (43), and (47); finally, we plug in the later results in Eq. (13), obtaining the waveform of a compact two-body system in a general motion,

$$\begin{aligned} h^{ij}_{\rm N,TT} = \frac{2G\mu}{Rc^4}\frac{d^2}{dt^2}\Bigg\{&\left[1 + \frac{1}{2c^2}(1-3\nu)(v^2 - \Lambda c^2 r^2) - \frac{Gm}{3rc^2}(2-9\nu)\right]r^i r^j - \frac{\Delta m}{mc^2}(2v^{(i}r^{j)}(\hat{N}\cdot\vec{r}) \\ &- (\hat{N}\cdot\vec{v})r^i r^j) + \frac{1}{c^2}(1-3\nu)(\hat{N}\cdot\vec{r})^2\left(v^i v^j - \frac{Gm}{3r^3}r^i r^j\right)\Bigg\}_{\rm TT}, \end{aligned} \tag{53}$$

with $\nu \coloneqq \mu/m = m_1 m_2/m^2$ as the symmetric mass ratio. Then, performing the time derivatives present in the right-hand side (53) and using the relative 1PN acceleration (B38) (which is computed in Appendix B) where required, we arrive at the final form of the near zone waveform,

$$h^{ij}_{\rm N,TT}(t,x) = \frac{2G\mu}{c^4 R}\left\{\tilde{Q}^{ij} + \frac{1}{c}P^{1/2}\tilde{Q}^{ij} + \frac{1}{c^2}P\tilde{Q}^{ij} + O(c^{-3}, c^{-1}\Lambda, \Lambda^2)\right\}, \tag{54}$$

with

$$\tilde{Q}^{ij} = 2\left(v^i v^j - \frac{Gm}{r^3}r^i r^j\right) + \frac{\Lambda}{3}c^2 r^i r^j, \tag{55}$$

$$P^{1/2}\tilde{Q}^{ij} = \Delta m\left[3\frac{Gm}{r^3}(\hat{N}\cdot\vec{r})\left(2v^{(i}r^{j)} - \frac{\dot{r}}{r}r^i r^j\right) + (\vec{v}\cdot\hat{N})\left(-2v^i v^j + \frac{Gm}{r^3}r^i r^j\right) - 2\Lambda c^2(\hat{N}\cdot\vec{r})v^{(i}r^{j)} - \frac{\Lambda}{3}c^2(\hat{N}\cdot\vec{v})r^i r^j\right], \quad (56)$$

$$\begin{aligned}
P\tilde{Q}^{ij} = {} & \frac{1}{3}\left[3(1-3\nu)v^2 - 2(2-3\nu)\frac{Gm}{r}\right]v^i v^j + \frac{4}{3}(5+3\nu)\frac{Gm}{r^2}\dot{r}v^{(i}r^{j)} \\
& + \frac{1}{3}\frac{Gm}{r^3}\left[-(10+3\nu)v^2 + 3(1-3\nu)\dot{r}^2 + 29\frac{Gm}{r}\right]r^i r^j + \frac{2}{3}(1-3\nu)(\vec{v}\cdot\hat{N})^2\left(3v^i v^j - \frac{Gm}{r^3}r^i r^j\right) \\
& + \frac{4}{3}(1-3\nu)(\vec{v}\cdot\hat{N})(\vec{r}\cdot\hat{N})\frac{Gm}{r^3}\left[-8v^{(i}r^{j)} + 3\frac{\dot{r}}{r}r^i r^j\right] \\
& + \frac{1}{3}(1-3\nu)(\vec{r}\cdot\hat{N})^2\frac{Gm}{r^3}\left[-14v^i v^j + 30\frac{\dot{r}}{r}v^{(i}r^{j)} + \left(3\frac{v^2}{r^2} - 15\frac{\dot{r}^2}{r^2} + 7\frac{Gm}{r^3}\right)r^i r^j\right] \\
& - \frac{17\Lambda c^2}{9}(1+3\nu)\frac{Gm}{r}r^i r^j - \Lambda c^2\left[2\left(\frac{2}{3}-\nu\right)v^2 + (1-3\nu)\left(\frac{Gm}{r^3}(r_i)^2 + (v_i)^2\right)\right]r^i r^j \\
& + \Lambda c^2[2(1-3\nu)r_i\dot{r}_i - (6-14\nu)r\dot{r}]v^{(i}r^{j)} - \Lambda c^2(1-3\nu)r^2 v^i v^j + \frac{4}{3}\Lambda c^2(1-3\nu)(\hat{N}\cdot\vec{r})^2 v^i v^j \\
& - \frac{13}{9}\Lambda c^2(1-3\nu)\frac{Gm}{r^3}(\hat{N}\cdot\vec{r})^2 r^i r^j + \frac{8}{3}\Lambda c^2(1-3\nu)(\hat{N}\cdot\vec{v})(\hat{N}\cdot\vec{r})r^{(i}v^{j)},
\end{aligned} \quad (57)$$

where the repeated indices do not indicate sum. For instance, for $i = j = 1$, we have

$$\begin{aligned}
\left(\frac{Gm}{r^3}(r_i)^2 + (v_i)^2\right)r^i r^j &= \left(\frac{Gm}{r^3}(r_1)^2 + (v_1)^2\right)r^1 r^1 \\
&= \left(\frac{Gm}{r^3}x^2 + (v_x)^2\right)x^2,
\end{aligned} \quad (58)$$

with x as the Cartesian coordinate of the relative position $\vec{r}$ and v_x as their respective velocity component (see Appendix B). Notice that omitting the cosmological constant the wave expression becomes the case of the gravitational radiation at 1PN order [19].

IV. CIRCULAR ORBIT

In this section, we study the interaction of the compact two-body system given the particular case of a circular orbit, which is the most simple case to analyze. Here, we have to consider that $\dot{r} = \ddot{r} = 0$, as well as we denote $\dot{\phi} := \omega$ as the orbital frequency. From the results (B44) and (B45) obtained in Appendix B, we arrive at the following expression:

$$\begin{aligned}
\omega^2 = {} & \frac{Gm}{r^3} - \frac{\Lambda}{3}c^2 - \frac{Gm}{c^2 r}\left[\frac{Gm}{r^3}(3-\nu) - \frac{c^2\Lambda}{6}(10-3\nu)\right] \\
& + O(c^{-4}, \Lambda c^{-2}, \Lambda^2).
\end{aligned} \quad (59)$$

Additionally, we know that the velocity is given by

$$\begin{aligned}
v^2 &= (r\omega)^2 \\
&= \frac{Gm}{r} - \frac{\Lambda}{3}c^2 r^2 - \frac{Gm}{c^2 r}\left[\frac{Gm}{r}(3-\nu) - \frac{c^2\Lambda}{6}r^2(10-3\nu)\right] \\
&\quad + O(c^{-4}, \Lambda c^{-2}, \Lambda^2).
\end{aligned} \quad (60)$$

Bearing in mind this particular case of circular orbit, we point out the substitution of the velocity (60) into the coordinates of each body in interaction given by (49) and (50); we find that the terms corresponding to the 1PN order do not vanish. In contrast with the case of absence of Λ, there is no contribution in the coordinates of each body for a circular motion at 1PN order [22]. The energy of the system is [see (B39)]

$$\begin{aligned}
E = {} & mc^2 - \frac{G\mu m}{2r}\left[1 - \frac{1}{4}(7-\nu)\frac{Gm}{c^2 r}\right] \\
& - \frac{1}{3}\mu\Lambda c^2 r^2 + \frac{\Lambda}{6}\left(\frac{13}{2} + 5\nu\right) \\
& - \frac{11}{2}\Lambda\mu(1-3\nu)(x^2 v_x^2 + y^2 v_y^2 + z^2 v_z^2).
\end{aligned} \quad (61)$$

Recalling the orbital plane coordinates,

$$x = r\cos\phi, \quad (62)$$

$$y = r\sin\phi, \quad (63)$$

$$z = 1, \quad (64)$$

then if $\dot{r} = 0$, this leads to

$$v_x = \dot{x} = -r\omega \sin\phi, \tag{65}$$

$$v_y = \dot{y} = r\omega \cos\phi, \tag{66}$$

$$v_z = \dot{z} = 0. \tag{67}$$

Now, we can obtain the following identity:

$$\begin{aligned} x^2 v_x^2 + y^2 v_y^2 + z^2 v_z^2 &= 2r^4\omega^2 \cos^2\phi \sin\phi \\ &= \frac{1}{2} r^2 v^2 \sin^2(2\phi) \\ &= \frac{1}{2} Gmr \sin^2(2\phi) + O(c^{-2}, \Lambda c^2), \end{aligned} \tag{68}$$

and here we have used (60). Consequently, we substitute (68) into the energy (61), resulting in

$$\begin{aligned} E = mc^2 &- \frac{G\mu m}{2r}\left[1 - \frac{1}{4}(7-\nu)\frac{Gm}{c^2 r}\right] - \frac{\Lambda}{3}\mu c^2 r^2 \\ &+ \frac{\Lambda}{6}\mu G m r\left(\frac{13}{2} + 5\nu\right) - \frac{11}{4}\Lambda \mu G m r (1-3\nu)\sin^2(2\phi) \\ &+ O(c^{-4}, \Lambda c^{-2}, \Lambda^2). \end{aligned} \tag{69}$$

A. Energy loss rate

The flux of energy (see Appendix C) that comes from the tensor wave is

$$P = \frac{c^3 R^2}{32\pi G}\int \dot{h}^{ij}_{\mathrm{TT}} \dot{h}_{ij}^{\mathrm{TT}} d^3x, \tag{70}$$

with R as the distance from the source to the detector. In order to compute P, we can proceed in two different ways, in which we consider the particular case of a circular orbit of a compact two-body system, i.e., $\dot{r} = 0$. A first approach is to differentiate h^{ij} [from (54)] with respect time, where the 1PN equation of motion (B38) can be utilized, and we substitute this outcome into (70). The other method is taking the appropriate time derivatives of the STF moments (D4)–(D6) (obtained in Appendix D), and we plug them into (D7). By doing so, the rate of loss of energy of such system is

$$\begin{aligned} P = -\frac{G}{c^5}\frac{32}{5}(\nu m)^2 \Bigg[& \frac{G^3 m^3}{r^5} - \frac{G^2 m^2}{r^2}\Lambda c^2 \\ & - \frac{G^4 m^4}{c^2 r^6}\left(\frac{2927}{336} + \frac{5}{4}\nu\right) + \frac{G^3 m^3}{r^3}\Lambda\left(\frac{2423}{252} + \frac{31}{6}\nu\right) \\ & + O(c^{-4}, c^{-2}\Lambda, \Lambda^2)\Bigg]. \end{aligned} \tag{71}$$

Notice that both ways agree with each other.

B. Post-Newtonian parameters

For future purposes, we introduce some PN parameters. The first one is defined as $\gamma \coloneqq \frac{Gm}{c^2 r}$. Hence, the orbital frequency of a circular orbit (59) takes the following form:

$$\omega^2 = \frac{Gm}{r^3}[1 - (3-\nu)\gamma] - \frac{\Lambda c^2}{3}\left[1 - \frac{\gamma}{2}(10 - 3\nu)\right]. \tag{72}$$

Then, the relative distance can be expressed as

$$\begin{aligned} r = \left(\frac{Gm}{\omega^2}\right)^{1/3} & [1 - (3-\nu)\gamma]^{1/3} \\ & \times \left\{1 + \frac{\Lambda c^2}{3\omega^2}\left[1 - \frac{\gamma}{2}(10 - 3\nu)\right]\right\}^{-1/3}. \end{aligned} \tag{73}$$

Therefore, the first PN parameter becomes

$$\begin{aligned} \gamma &= \frac{Gm}{c^2 r} \\ &= \left(\frac{\omega G m}{c^3}\right)^{2/3}[1 - (3-\nu)\gamma]^{-1/3}\left\{1 + \frac{\Lambda c^2}{3\omega^2}\left[1 - \frac{\gamma}{2}(10-3\nu)\right]\right\}^{1/3} \\ &= x^2\left[1 + \frac{1}{3}(3-\nu)x^2 + \frac{\Lambda}{9}\frac{G^2 m^2}{c^4 x^6} - \frac{1}{54}\frac{\Lambda G^2 m^2}{c^4 x^4}(6-\nu) + O(x^4, \Lambda c^{-4} x^{-8}, \Lambda^2)\right], \end{aligned} \tag{74}$$

where with the second PN parameter $x \coloneqq (\frac{\omega G m}{c^3})^{1/3}$, the inverse squared frequency given by

$$\omega^{-2} = \frac{r^3}{Gm}[1 + (3-\nu)\gamma + O(\gamma^2, \Lambda c^2, \Lambda)], \tag{75}$$

which is obtained from (72) using Taylor series expansion, and the relations of both PN parameters $\gamma \simeq x^2$ were introduced. On the other hand, we introduce the PN parameter γ from (74) into the radiated power (71) and the energy (69), yielding

$$P = -\frac{32}{5}\frac{c^5}{G}\nu^2\gamma^5\left[1-\gamma\left(\frac{2927}{336}+\frac{5}{4}\nu\right)-\frac{\Lambda G^2m^2}{c^4\gamma^3}+\frac{\Lambda G^2m^2}{c^4\gamma^2}\left(\frac{2423}{252}+\frac{31}{6}\nu\right)+O(\gamma^2,\Lambda c^{-4}\gamma^{-1},\Lambda^2)\right]$$
$$= -\frac{32}{5}\frac{c^5}{G}\nu^2x^{10}\left[1-\left(\frac{1247}{336}+\frac{35}{12}\nu\right)x^2-\frac{\Lambda G^2m^2}{c^4x^6}-\frac{\Lambda G^2m^2}{432c^4x^4}(97-2692\nu)+O(x^4,\Lambda c^{-4}x^2,\Lambda^2)\right], \tag{76}$$

$$E = mc^2-\frac{\mu}{2}c^2\gamma\left[1-\frac{1}{4}(7-\nu)\gamma\right]-\frac{1}{3}\mu\Lambda\frac{G^2m^2}{c^2\gamma^2}+\frac{1}{6}\Lambda\mu\frac{G^2m^2}{c^2\gamma}\left(\frac{13}{2}+5\nu\right)-\frac{11}{4}\Lambda\mu\frac{G^2m^2}{c^2\gamma}(1-3\nu)\sin^2(2\phi_{0\mathrm{PN}})$$
$$= mc^2-\frac{\mu}{2}c^2x^2\left[1-\left(\frac{3}{4}+\frac{1}{12}\nu\right)x^2+\frac{7}{9}\Lambda\frac{G^2m^2}{c^4x^6}-\frac{\Lambda G^2m^2}{54c^4x^4}(103+95\nu)\right.$$
$$\left.+\frac{11}{2}\frac{\Lambda G^2m^2}{c^4x^4}(1-3\nu)\sin^2(2\phi_{0\mathrm{PN}})+O(x^4,\Lambda c^{-4}x^2,\Lambda^2)\right]. \tag{77}$$

Here, we have to remark that the energy depends explicitly of the orbital phase ϕ (inside the sine function). However, at this point of the analysis, this information is not available (at least at 1PN order). As a consequence of this fact, we introduce the Newtonian orbital phase $\phi_{0\mathrm{PN}}$ since at this point of the analysis, this is the quantity that we have in hand. Furthermore, it is worth mentioning that the explicit appearance of the orbital phase ϕ in (77) is due that the spatial components of the solution of the relaxed EFE do not have rotational symmetry due to the gauge artifact [24]. Thus, the Newtonian orbital phase $\phi_{0\mathrm{PN}}$ in terms of the PN parameter x that we use in (77) is given by

$$\phi_{0\mathrm{PN}} = -\frac{1}{32\nu}x^{-5}\left[1-\frac{25}{99}\frac{\Lambda G^2m^2}{c^4}x^{-6}\right]. \tag{78}$$

C. Energy loss rate of a circular motion of a two-body system

It is well known that the loss of energy is in the form of the GWs; therefore, this configuration becomes a binary quasicircular scenario. Then, to obtain the orbital phase of the GW ϕ, we must use the balance equation, namely,

$$\frac{dE}{dt} = -P. \tag{79}$$

Next, the time derivative of the energy (77) becomes

$$\frac{dE}{dt} = -\mu c^2x\dot{x}\left[1-\frac{1}{2}\left(3+\frac{1}{3}\nu\right)x^2-\frac{14}{9}\frac{\Lambda G^2m^2}{c^4x^6}+\frac{\Lambda G^2m^2}{18c^4x^4}(103+95\nu)\right.$$
$$\left.-\frac{33}{2}\frac{\Lambda G^2m^2}{c^4x^4}(1-3\nu)\sin^2(2\phi_{0\mathrm{PN}})+O(x^4,\Lambda c^{-4}x^{-2},\Lambda^2)\right]$$
$$-\frac{11}{2}\mu\Lambda Gmcx(1-3\nu)\sin(4\phi_{0\mathrm{PN}})+O(\Lambda x^2,\Lambda^2), \tag{80}$$

where the PN parameter x was used to express the frequency as $\dot{\phi}=\omega=\frac{c^3x^3}{Gm}$. We equate the formulas (76) and (80) leading to the following expression to solve for the unknown PN parameter x, that is,

$$\int \mu c^2x\left[1-\frac{1}{2}\left(3+\frac{1}{3}\nu\right)x^2-\frac{14}{9}\frac{\Lambda G^2m^2}{c^4x^6}+\frac{\Lambda G^2m^2}{18c^4x^4}(103+95\nu)-\frac{33\Lambda G^2m^2}{2c^2x^4}(1-3\nu)\sin^2(2\phi_{0\mathrm{PN}})\right]$$
$$\times\left\{\frac{32}{5}\frac{c^5}{G}\nu^2x^{10}\left[1-\left(\frac{1247}{336}+\frac{35}{12}\nu\right)x^2-\frac{\Lambda G^2m^2}{c^4x^6}-\frac{\Lambda G^2m^2}{432c^4x^4}(97-2692\nu)\right]\right.$$
$$\left.+\frac{11\mu\Lambda Gmcx}{2}(1-3\nu)\sin(4\phi_{0\mathrm{PN}})\right\}^{-1}dx = -(t_c-t). \tag{81}$$

Expanding in Taylor series, we arrive at the following expression:

$$\Theta(t) = \frac{1}{256}x^{-8}\left[1+\frac{256}{192}\left(\frac{743}{336}+\frac{11}{4}\nu\right)x^2 - \frac{20}{63}\frac{\Lambda G^2m^2}{c^4}x^{-6} - \frac{1}{54}\frac{\Lambda G^2m^2}{c^4}(573+2444\nu)x^{-4} + \frac{132\Lambda G^2m^2}{c^4}(1-3\nu)x^8\int\frac{1}{x^{13}}\sin^2(2\phi_{0\mathrm{PN}})dx + O(x^4, \Lambda c^{-4}x^{-2}, \Lambda^2)\right], \tag{82}$$

where $\Theta(t) := \frac{c^3\nu}{5Gm}(t_c - t)$, and t_c is the time of coalescence. The inversion of the later equation reads

$$x = \frac{1}{2}\Theta^{-1/8} + \left(\frac{743}{16128}+\frac{11}{192}\nu\right)\Theta^{-3/8} - \frac{80}{63}\frac{\Lambda G^2m^2}{c^4}\Theta^{5/8} - \frac{1}{54}\frac{\Lambda G^2m^2}{c^4}(573+2444\nu)\Theta^{3/8} + \frac{33}{1024}\frac{\Lambda G^2m^2}{c^4}(1-3\nu)\Theta^{-9/8}I(\Theta), \tag{83}$$

with $I(\Theta) := \int\frac{1}{x^{13}}\sin^2(2\phi_{0\mathrm{PN}})dx$. Additionally, notice that if we introduce $\phi_{0\mathrm{PN}}$ into $I(\Theta)$ and we expand it at PN order, the lowest order of such integral is $O(\Lambda c^{-4}x^{-14})$, then the next order is $O(\Lambda c^{-4}x^{-24})$, and so on. This implies that the lowest order of the term $\frac{132\Lambda G^2m^2}{c^4}(1-3\nu)x^8\int\frac{1}{x^{13}}\sin^2(2\phi_{0\mathrm{PN}})dx$ must be $O(\Lambda c^{-4}x^{-6})$; consequently, the next order is $O(\Lambda c^{-4}x^{-16})$. Hence, under this approach, the integral diverges. As a consequence of this fact, it is convenient to evaluate the complete integral considering no expansion (see Appendix E for the explicit calculation). Note that the precise solution is a complex function; however, given our next numerical examples, only the real part is considered since its imaginary upshot is very small compared to its real counterpart.

D. Computation of the phase of oscillation of the GW of a compact two-body system

First, to compute the phase of oscillation ϕ, we know that

$$\frac{d\phi}{dt} = \frac{d\phi}{d\Theta}\frac{d\Theta}{dt} = -\frac{c^3\nu}{5Gm}\frac{d\phi}{d\Theta}. \tag{84}$$

Therefore, we have

$$\frac{d\phi}{d\Theta} = -\frac{5Gm}{c^3\nu}\frac{d\phi}{dt} = -\frac{5Gm}{c^3\nu}\omega = -\frac{5Gm}{c^3\nu}\frac{c^3x^3}{Gm} = -\frac{5}{\nu}x^3, \tag{85}$$

where we have used the relation $\omega = \frac{c^3x^3}{Gm}$. Moreover, from (83), we obtain

$$x^3 = \frac{1}{8}\Theta^{-3/8} + \left(\frac{743}{21504}+\frac{11}{256}\nu\right)\Theta^{-5/8} - \frac{20}{21}\frac{\Lambda G^2m^2}{c^4}\Theta^{3/8} - \frac{1}{72}\frac{\Lambda G^2m^2}{c^4}(572+2444\nu)\Theta^{1/8} + \frac{99}{4096}(1-3\nu)\frac{\Lambda G^2m^2}{c^4}\Theta^{-11/8}I(\Theta). \tag{86}$$

Finally, we integrate Eq. (85), resulting in the following expression:

$$\phi(t) = \phi_0 - \frac{1}{\nu}\left[\Theta^{5/8} + \left(\frac{3715}{8064}+\frac{55}{96}\nu\right)\Theta^{3/8} - \frac{800}{231}\frac{\Lambda G^2m^2}{c^4}\Theta^{11/8} - \frac{5}{81}\frac{\Lambda G^2m^2}{c^4}(572+2444\nu)\Theta^{9/8} + \frac{495}{4096}(1-3\nu)\frac{\Lambda G^2m^2}{c^4}\int\Theta^{-11/8}I(\Theta)d\Theta\right], \tag{87}$$

where ϕ_0 is the value of the phase at the instant of coalescence. First, note that in the limit $\Lambda \to 0$, Eq. (87) matches the known phase of the waveform propagation of a GW [22,23]. Second, we present Fig. 1, which is the graphic representation of the Newtonian phase $\phi_{0\mathrm{PN}}(t)$,

$$\phi_{0\mathrm{PN}} = -\frac{5}{\nu}\left[\frac{1}{5}\Theta^{5/8} - \frac{160}{231}\frac{\Lambda G^2m^2}{c^4}\Theta^{11/8}\right], \tag{88}$$

where we consider the binary compact system with both identical masses, such $\mathrm{m} = 10^{31}$ kg; and $\phi_0 = 0$, $\Lambda = 10^{-52}\ \mathrm{m}^{-2}$ [4], and $t_c = 1s$. The orange line includes Λ, while the blue one does not ($\Lambda = 0$). Note that both lines are superimposed on each other; thus, the effect of Λ in $\phi_{0\mathrm{PN}}$ is negligible. To bear out this result, it is convenient to carry out a numerical comparison of the phase that contains Λ given by (78) with respect to the standard Newtonian phase without a cosmological constant given by

$$\phi_{0\mathrm{PN},\Lambda=0} = -\frac{1}{\nu}\Theta^{5/8}. \tag{89}$$

Thus, the relative correction that the cosmological constant Λ causes at $t = 0$ (time where it reaches its maximum value) reads

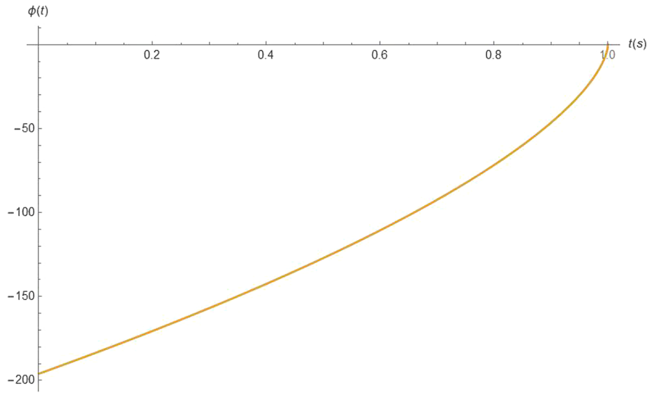

FIG. 1. Plot of the GW Newtonian phase $\phi_{0\mathrm{PN}}(t)$ (88) for a binary compact system of identical masses with $m = 10^{31}$ kg, $\phi_0 = 0$, $\Lambda = 10^{-52}\ \mathrm{m}^{-2}$, and $t_c = 1s$. The blue line includes Λ, while the orange one does not ($\Lambda = 0$). Note that both lines are superimposed on each other; thus, the effect of Λ in $\phi_{0\mathrm{PN}}$ is negligible.

$$\left|\frac{\phi_{0\mathrm{PN}} - \phi_{0\mathrm{PN},\Lambda=0}}{\phi_{0\mathrm{PN},\Lambda=0}}\right| = 5.74 \times 10^{-42}. \tag{90}$$

Lastly, we observe that at 1PN order, that is, Eq. (87), the behavior of $\phi(t)$ due to Λ in $\phi_{1\mathrm{PN}}$ is not modified whatsoever.

V. GRAVITATIONAL WAVEFORMS IN THE TIME DOMAIN

To compute the time domain, we must introduce the orthonormal triad $\hat{N}$, $\hat{p}$, and $\hat{q}$; with $\hat{N}$ as the unit vector, which is a radial vector pointing from the source to the observer; $\hat{p}$ lies on the intersection of the orbital plane with the plane of the sky, i.e., the plane which is orthonormal to the direction $\hat{N}$, and $\hat{q} = \hat{N} \times \vec{p}$. We also have to consider the parameters ι and ϕ that are the inclination angle relative to $\hat{N}$ and the orbital phase of the motion of the body 1, measured counterclockwise from the line of nodes, which is given by the line of intersection between the two planes (plane of sky and orbital plane), and $\hat{n}$ is the unitary vector of $\vec{r}$. Thus, we have that

$$\hat{p} = (1, 0, 0), \tag{91}$$

$$\hat{q} = (0, \cos\iota, -\sin\iota), \tag{92}$$

$$\hat{n} = \hat{p}\cos\phi + (\hat{q}\cos\iota + \hat{N}\sin\iota)\cos\phi, \tag{93}$$

$$\hat{\lambda} = -\hat{p}\sin\phi + (\hat{q}\cos\iota + \hat{n}\sin\iota)\cos\phi, \tag{94}$$

where $\vec{v} = r\omega\hat{\lambda}$ for circular orbits. The gravitational waveforms in the time domain are linear combinations of the polarizations waveforms $h_+(t)$ and $h_\times(t)$ defined by the projections

$$h_+ = \frac{1}{2}(\hat{p}_i\hat{p}_j - \hat{q}_i\hat{q}_j)h^{ij}, \tag{95}$$

$$h_\times = \frac{1}{2}(\hat{p}_i\hat{q}_j + \hat{q}_i\hat{p}_j)h^{ij}. \tag{96}$$

We have already computed the waveforms extracted after applying their projections [see (54)], and recall that we have taken the particular case for a circular motion $\dot{r} = 0$. Therefore, both polarizations become

$$h_+ = \frac{2G\mu}{c^2R}\left(\frac{Gm\omega}{c^3}\right)^{2/3}\{H_+^0 + xH_+^{1/2} + x^2H_+^1 + O(x^3, \Lambda c^{-1}, \Lambda^2)\}, \tag{97}$$

$$h_\times = \frac{2G\mu}{c^2R}\left(\frac{Gm\omega}{c^3}\right)^{2/3}\{H_\times^0 + xH_\times^{1/2} + x^2H_\times^1 + O(x^3, \Lambda c^{-1}, \Lambda^2)\}, \tag{98}$$

with

$$H_+^0 = -(1+\cos^2\iota)\cos 2\phi + \frac{\Lambda c^2}{\omega^2}\left(-\frac{1}{12}\sin^2\iota + \frac{5}{36}(1+\cos^2\iota)\cos 2\phi\right), \tag{99}$$

$$H_+^{1/2} = -\frac{\Delta m}{m}\frac{1}{8}\sin\iota[(5+\cos^2\iota)\cos\phi - 9(1+\cos^2\iota)\cos 3\phi]\left(1 - \frac{\Lambda c^2}{3\omega^2}\right), \tag{100}$$

$$H_+^1 = \frac{1}{6}\{[19 + 9\cos^2\iota - 2\cos^4\iota] - \nu[19 - 11\cos^2\iota - 6\cos^4\iota]\}\cos 2\phi - \frac{4}{3}\sin^2\iota(1+\cos^2\iota)(1-3\nu)\cos 4\phi + \frac{\Lambda c^2}{\omega^2}\left\{\frac{13}{24} - \frac{9}{16}\cos^2\iota + \frac{1}{48}\cos^4\iota + \frac{275}{72}\nu\sin^2\iota + \cos 2\phi\left[-\frac{371}{432} - \frac{35}{144}\cos^2\iota - \frac{35}{108}\cos^4\iota + \nu\left(\frac{331}{144} + \frac{65}{144}\cos^2\iota + \frac{35}{36}\cos^4\iota\right)\right] + \cos 4\phi\left[\frac{5}{18} + \frac{11}{54}\cos^2\iota - \frac{13}{27}\cos^4\iota + \nu\left(-\frac{5}{6} - \frac{69}{72}\cos^2\iota + \frac{13}{9}\cos^4\iota\right)\right]\right\}, \tag{101}$$

$$H_\times^0 = -2\cos\iota\sin 2\phi + \frac{\Lambda c^2}{9\omega^2}\cos\iota\sin 2\phi, \tag{102}$$

$$H_\times^{1/2} = -\frac{\Delta m}{m}\frac{3}{8}\sin 2\iota\left[\left(1+\frac{2}{9}\frac{\Lambda c^2}{\omega^2}\right)\sin\phi - \left(3-\frac{20}{9}\frac{\Lambda c^2}{\omega^2}\right)\sin 3\phi\right], \tag{103}$$

$$H_\times^1 = \cos\iota\left[\left\{\left(\frac{17}{3}-\frac{4}{3}\cos^2\iota\right)+\nu\left(-\frac{13}{3}+4\cos^2\iota\right)\right\}\sin 2\phi - \frac{8}{3}(1-3\nu)\sin^2\iota\sin 4\phi\right.$$
$$\left. + \frac{\Lambda c^2}{\omega^2}\left\{\left(-\frac{92}{27}+\frac{1}{3}\cos^2\iota\right)+\nu\left(\frac{79}{18}-\frac{13}{6}\cos^2\iota\right)\right\}\sin 2\phi + \frac{\Lambda c^2}{\omega^2}\left(\frac{359}{216}-\frac{359}{72}\nu\right)\sin^2\iota\sin 4\phi\right]. \tag{104}$$

The following identities, which come from the combinations of the definitions (95) and (96) with (91)–(94), were utilized to compute the above polarizations h_+, $h_\times$:

$$(\hat{n}^i\hat{n}^j)_+ = \frac{1}{4}\sin^2\iota + \frac{1}{4}[1+\cos^2\iota]\cos 2\phi, \tag{105}$$

$$(\hat{\lambda}^i\hat{\lambda}^j)_\times = -\frac{1}{2}\cos\iota\sin 2\phi, \tag{106}$$

$$(\hat{n}^{(i}\hat{\lambda}^{j)})_+ = -\frac{1}{4}[1+\cos^2\iota]\sin 2\phi, \tag{107}$$

$$(\hat{n}^i\hat{n}^j)_\times = \frac{1}{2}\cos\iota\sin 2\phi, \tag{108}$$

$$(\hat{\lambda}^i\hat{\lambda}^j)_+ = \frac{1}{4}\sin^2\iota - \frac{1}{4}[1+\cos^2\iota]\cos 2\phi, \tag{109}$$

$$(\hat{n}^{(i}\hat{\lambda}^{j)})_\times = \frac{1}{2}\cos\iota\cos 2\phi, \tag{110}$$

$$((r_i)^2 r^{(i}r^{j)})_\times = \frac{1}{4}r^4\cos\iota\sin 2\phi, \tag{111}$$

$$((r_i)^2 r^{(i}r^{j)})_+ = \frac{1}{16}r^4[3+5\cos 2\phi - 3\cos^2\iota(1-\cos 2\phi)], \tag{112}$$

$$[(v_i)^2 r^{(i}r^{j)}]_\times = [r_i v_i v^{(i}r^{j)}]_\times = \frac{1}{4}r^4\omega^2\cos\iota\sin 2\phi, \tag{113}$$

$$[(v_i)^2 r^{(i}r^{j)}]_+ = [r_i v_i v^{(i}r^{j)}]_+ = \frac{1}{16}r^4\omega^2\sin^2\iota(1-\cos 4\phi), \tag{114}$$

$$\hat{N}\cdot\hat{n} = \sin\iota\sin\phi, \tag{115}$$

$$\hat{N}\cdot\hat{\lambda} = \sin\iota\cos\phi. \tag{116}$$

Here, all repeated indices do not indicate sum. From Eqs. (99)–(104), one can observe that all H_i's present nearly the same structure; that is, they have a constant term multiplied by a trigonometry function, which contains the phase ϕ, except the first term of H_+^0 and the first four terms of H_+^1 with Λ. Thus, considering constant frequencies $\phi = \omega(t - R/c)$, we can say that the presence of Λ makes the amplitude of the waveforms change in magnitude, and their roots (points where the function vanishes) are modified in the h_+ polarization (see Figs. 3 and 4). We close this section with four remarks:

(i) The first case is presented in Fig. 2 with $\omega = 10^{-17}\ \mathrm{s}^{-1}$; one can see that the two lines (blue with $\Lambda \simeq 10^{-52}\ \mathrm{m}^{-2}$ [4]; orange $\Lambda = 0$) in both polarizations h_+ and $h_\times$ are superimposed on each other. Thus, the effect of Λ is almost negligible. To observe this, we compute the highest value of the approximation error of the two polarizations that contains the presence of the cosmological constant Λ with respect to the standard polarizations (without Λ), obtaining as a result 0.8% and 0.5% for h_+ and $h_\times$, respectively.

(ii) In the second case with $\omega = 10^{-18}\ \mathrm{s}^{-1}$ (see Fig. 3), we can notice that Λ begins to have importance. We observe that Λ modifies the amplitudes of h_+ and $h_\times$, and it reduces their sizes compared to the case with null Λ. This is a direct consequence that the cosmological constant "stretches" the spacetime making that the objects within it move away from each other. In this example, it turns out that the highest value of the relative change of the polarizations containing the Λ terms with to respect the standard polarizations (without Λ) are 80% for h_+ and 50% for $h_\times$.

(iii) For the particular frequencies $\omega_0 = c\sqrt{5\Lambda}/6 = 1.12\times 10^{-18} s^{-1}$ and $\omega_0 = c\sqrt{2\Lambda}/6 = 7.07\times 10^{-19} s^{-1}$, the amplitudes of h_+ and $h_\times$ are canceled respectively, at 0PN order. Thus, if the system oscillates at one of this particular frequencies, Λ would annihilate such amplitudes at Newtonian order. Nevertheless, we

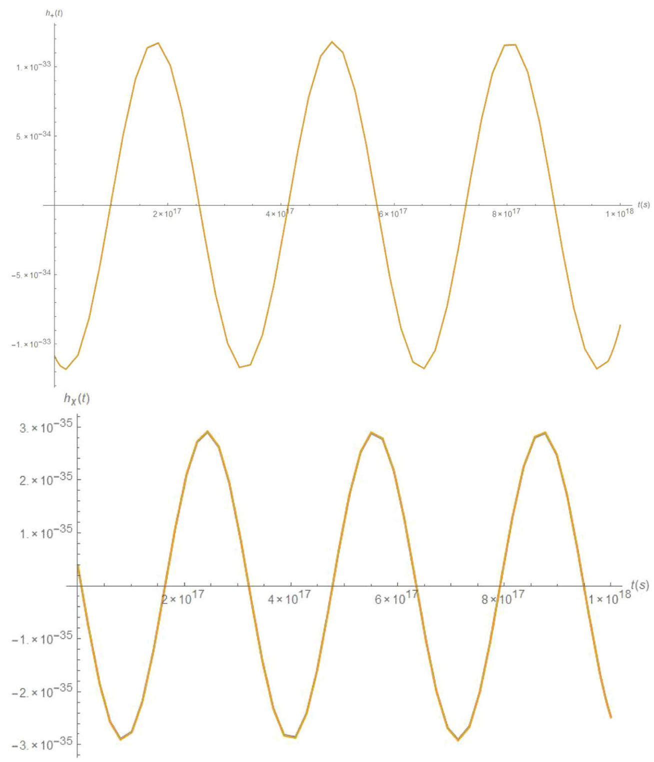

FIG. 2. Plots of the gravitational waveform h_+ (top) and $h_\times$ (bottom) for a binary compact system of identical masses at 1PN order with parameter values $m = 10^{31}$ kg, $R = 200 \times 10^{22}$ m, $\omega = 10^{-17}$ s^{-1}, $\Lambda = 10^{-52}$ m^{-2} and the inclination angle $\iota = \pi/2$ (top), $\iota = 0$ (bottom). The blue line includes Λ, while the orange one does not ($\Lambda = 0$). Note that two lines, in both polarizations, are superimposed on each other; thus, the effect of Λ is negligible.

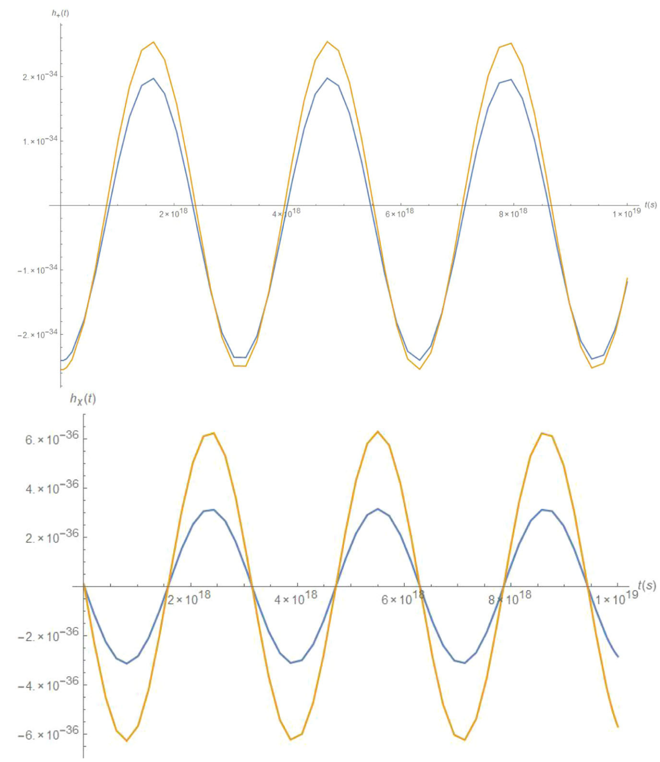

FIG. 3. Plots of the gravitational waveforms h_+ (top) and $h_\times$ (bottom) for a binary compact system of identical masses at 1PN order. The parameters are given by $m = 10^{31}$ Kg, $R = 200 \times 10^{22}$ m, $\omega = 10^{-18}$ s^{-1} and the inclination angle $\iota = \pi/2$ (top), $\iota = 0$ (bottom). The blue line includes Λ, while the orange one does not ($\Lambda = 0$). Note that with this particular frequency, the effect of Λ starts to be observable.

observe that the spacetime is still altered by Λ in an oscillatory way since the correction of the waveforms at 0.5PN order or higher, in fact, prevail [see (97) and (98)].

(iv) Then, for $\omega < \omega_0$, as shown in Fig. 4, the effect of Λ becomes very evident. The ripples of the spacetime are now "stretched" by the cosmological constant. In fact, one can drop all expressions without Λ from (97) and (98), and we will get nearly the same output; therefore, the waveforms of the GW depend mostly on those terms with $\Lambda \neq 0$. Furthermore, observe that in the case of the plot of h_+ the crest and trough are displaced downward as a consequence of the shift constants, such as the coefficient $-c^2 \sin^2 \iota/(12\omega^2)$ that multiplies Λ in (99). Finally, in this instance, the highest values of the relative corrections between the polarizations that contain Λ terms and the standard polarizations (without Λ) are considerably much larger, having as a result 8000% for h_+ and 5000% for $h_\times$.

On the other hand, there is an exception among all aforementioned examples, that is case iii. There is no difference between the plots of 0PN and 1PN orders due to the very small value of the frequency ω. The corresponding terms at 1PN are practically negligible in comparison to the 0PN ones. Nonetheless, for the case of higher frequencies, i.e., $\omega \geq 10^{-18}$ s^{-1}, there might be a difference between results at 0PN and 1PN, but the presence of Λ is negligible for numerical purposes. Hence, we can confirm that at higher orders of the post-Newtonian method, the presence of Λ will not affect the polarizations h_+ and $h_\times$.

Finally, in [28], it is mentioned that the perturbation is not sharp, and there is also a tail term. The analysis given in [36] shows that the sharp term is comparable with the tail term, no matter how small Λ is. In our work, as it is mentioned in the cases given in remarks ii, iii, and iv, the effect of Λ is notorious.

VI. CONCLUDING REMARKS

In this paper, we have studied from scratch the propagation of GWs including the cosmological constant Λ in a binary compact system. Using the direct integration of the relaxed EFE at 1PN, and taking into account that the terms $O(\Lambda h)$ were dropped given that, from the beginning, we assume that $\Lambda \simeq 10^{-52}$ m^{-2} [4] is very small and

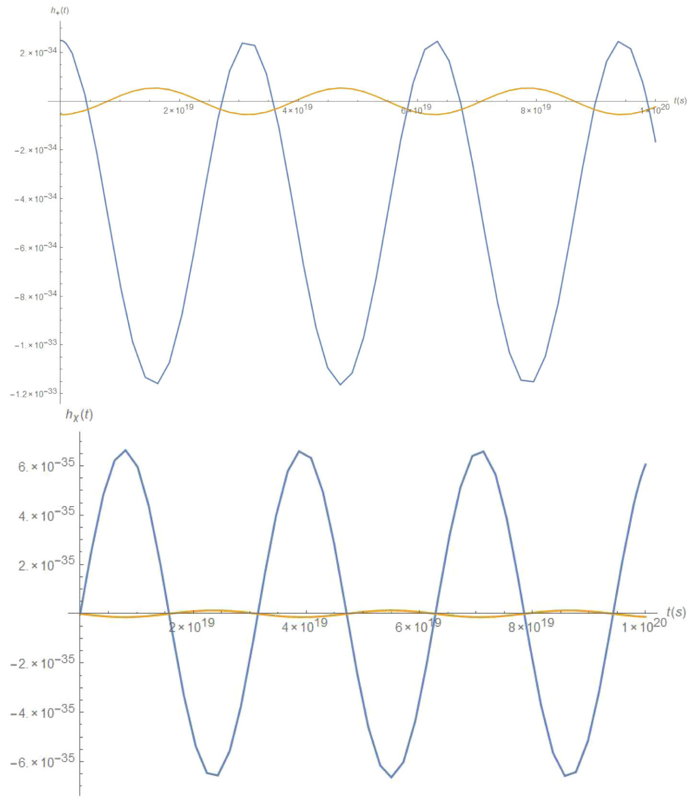

FIG. 4. Plots of the waveforms h_+ (top) and $h_\times$ (bottom) for a binary compact system of identical masses at 1PN order. The parameter values are given by $m = 10^{31}$ kg, $R = 200 \times 10^{22}$ m, $\omega = 10^{-19}$ s^{-1} with inclination angle $\iota = \pi/2$ (top), $\iota = 0$ (bottom). The blue line includes Λ, while the orange one does not ($\Lambda = 0$). Note that with this particular frequency, the effect of Λ becomes very evident.

positive [24]. We also compute the waveforms (54), where the equations of motion at 1PN (B38) were derived from the Lagrangian taken at the center of mass frame, expressed in (B36). Furthermore, observing the solutions for h^{00} and $\overset{(2)}{g}_{00}$ given by (24) and (A12), correspondingly, we find that Λ can be interpreted as a PN factor since globally we can factorize $1/c^2$, and this power of c is the 1PN approximation.

Focusing on the particular case of a binary quasicircular motion, we derive the energy and the radiated power given by (69) and (71), respectively. Then, we substitute these results into the balance equation (79), where the PN parameters γ and x were introduced, in order to obtain the phase (87) in the time domain at 1PN order. We notice that this expression depends on explicitly on their own quasicircular orbital phase $\phi(t)$ of lower order; nevertheless, we can use the Newtonian phase (78) to compute the integral (E3) (given in Appendix E). On the other hand, from Fig. 1, we can observe that ϕ behaves the same with or without Λ, therefore adding the cosmological constant does not affect the phase. However, the impact of Λ starts becoming noticeable on the amplitudes of the polarizations h_+ and $h_\times$ (see Figs. 3 and 4) when taking a constant frequency $\omega < 10^{-18}$ s^{-1}. Moreover, we find that given the particular frequencies $\omega_0 = c\sqrt{5\Lambda}/6$ and $\omega_0 = c\sqrt{2\Lambda}/6$ the amplitudes of h_+ (97) and $h_\times$ (98) vanish at Newtonian order; nonetheless, at higher orders, the propagation of the GWs holds.

In the near future, we can extend our study now considering $O(\Lambda h)$ terms [25], given that in the early stages of the Universe (inflationary period [37–39]) the value of Λ could have been much larger. Also, we may investigate heavier objects, such as a system of black holes at the center of two galaxies weighing billions of solar masses, since they emit GWs with lower frequencies. This, indeed, opens the possibility to explore detectable signals from the most recent NANOGrav survey [40]. Furthermore, complementing our work applied to the scalar-Gauss-Bonnet-gravity [41] could shed light to understand the behavior of Λ together with the scalar field. On the other hand, we can explore the coordinate transformation from the Cartesian coordinates given in the spatial components of (27) which leads to the Schwarzschild-de-Sitter metric [24] (see also [25,42]). We can also expand this PN approach from the very beginning in the Brans-Dicke theory [43] (see also [44]).

ACKNOWLEDGMENTS

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APPENDIX A: METRIC AT NEWTONIAN ORDER

This appendix is devoted to compute the components of the metric at Newtonian order. We follow the method developed in [45]. To begin, we make the expansion of the metric in the PN approximation as follows:

$$g_{00} = -1 + \overset{(2)}{g}_{00} + \cdots \tag{A1}$$

$$g_{0i} = \overset{(3)}{g}_{0i} + \cdots \tag{A2}$$

$$g_{ij} = \delta_{ij} + \overset{(2)}{g}_{ij} + \cdots, \tag{A3}$$

where the number over the objects means the power of the factor of the velocity ratio v/c. The temporal and spatial components of the Ricci tensor take the following form:

$$\overset{(2)}{R}_{00} = -\frac{1}{2}\nabla^2 \overset{(2)}{g}_{00}, \tag{A4}$$

$$\begin{aligned}\overset{(2)}{R}_{ij} &= \frac{1}{2}\left[\partial_i\left(\frac{1}{2}\partial_j \overset{(2)}{g}_{00} - \frac{1}{2}\partial_j \overset{(2)}{g}_{kk} + \partial_k \overset{(2)}{g}_{jk}\right)\right. \\ &\quad \left. + \partial_j\left(\frac{1}{2}\partial_i \overset{(2)}{g}_{00} - \frac{1}{2}\partial_i \overset{(2)}{g}_{kk} + \partial_k \overset{(2)}{g}_{ik}\right) - \nabla^2 \overset{(2)}{g}_{ij}\right] \\ &= \frac{1}{2}[\partial_i \Gamma_j + \partial_j \Gamma_i - \nabla^2 \overset{(2)}{g}_{ij}],\end{aligned} \tag{A5}$$

where we define $\Gamma_i := \frac{1}{2}\partial_i \overset{(2)}{g}_{00} - \frac{1}{2}\partial_i \overset{(2)}{g}_{kk} + \partial_k \overset{(2)}{g}_{ik}$. Since we are considering a system of n compact bodies, the expression energy-momentum tensor that describes it is given by

$$T^{\mu\nu} = \frac{1}{\sqrt{-g}}\sum_a m_a \frac{d\tau_a}{dt}\frac{dx_a^\mu}{d\tau_a}\frac{dx_a^\nu}{d\tau_a}\delta^3(\vec{x} - \vec{x}_a(t)), \tag{A6}$$

with τ_a as the proper time of the particle a and t as the time coordinate. Furthermore, the EFE (2) can be rewritten as follows:

$$\begin{aligned}R_{\mu\nu} &= \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}T g_{\mu\nu}\right) + \Lambda g_{\mu\nu} \\ &:= S_{\mu\nu},\end{aligned} \tag{A7}$$

with $S_{\mu\nu}$ playing the role as the source of the EFE. The sources components S_{00} and S_{ij} at lowest order correspondingly read

$$S_{00} = \frac{4\pi G}{c^4}\overset{(0)}{T}_{00} - \Lambda, \tag{A8}$$

$$S_{ij} = \frac{4\pi G}{c^4}\delta_{ij}\overset{(0)}{T}_{00} + \Lambda\delta_{ij}, \tag{A9}$$

where we use the result $\overset{(0)}{T}_{00} \simeq -\overset{(0)}{T}$. From the EFE, we find the following set of equations:

$$-\frac{1}{2}\nabla^2 \overset{(2)}{g}_{00} = \frac{4\pi G}{c^4}\overset{(0)}{T}_{00} - \Lambda, \tag{A10}$$

$$\frac{1}{2}[\partial_i\Gamma_j + \partial_j\Gamma_i - \nabla^2 \overset{(2)}{g}_{ij}] = \frac{4\pi G}{c^4}\delta_{ij}\overset{(0)}{T}_{00} + \Lambda\delta_{ij}. \tag{A11}$$

Recalling that $\overset{(0)}{T}_{00} = \sum_a m_a \delta^3(\vec{x} - \vec{x}_a(t))$, the solution of (A10) is shown as

$$\overset{(2)}{g}_{00} = \frac{2G}{c^2}\sum_a \frac{m_a}{r_a} + \frac{\Lambda}{3}|\vec{x}|^2, \tag{A12}$$

with $\vec{x}, \vec{x}_a$ as the vector field point and the vector position of the particle a, respectively, and $r_a := |\vec{x} - \vec{x}_a|$. On the other hand, to solve (A11), it is convenient to choose the gauge $\Gamma_i = 0$. Therefore, the solution is given by

$$\overset{(2)}{g}_{ij} = \left[\frac{2G}{c^2}\sum_a \frac{m_a}{r_a} - \frac{\Lambda}{2}\left(\frac{1}{3}|\vec{x}|^2 + x_i^2\right)\right]\delta_{ij}, \tag{A13}$$

with no sum over the index i. Notice that $\overset{(2)}{g}_{ij}$ is a diagonal matrix, but it is not proportional to the identity. The solution (A13) satisfies (A11) as long as the gauge[1] $\Gamma_i = 0$ holds. Furthermore, observe that the solutions (A12) and (A13) meet the gauge $\Gamma_i = 0$ as well.

APPENDIX B: TWO-BODY LAGRANGIAN OF A COMPACT SYSTEM

In this section, the interaction of a compact two-body system at the first post-Newtonian correction order with cosmological constant is considered. To begin the analysis, we propose the following ansatz of the components of the metric as follows:

$$g_{00} = -e^{2U} + O(c^{-4}, \Lambda c^{-2}), \tag{B1}$$

$$g_{0i} = 4g_i + O(c^{-5}, \Lambda c^{-3}), \tag{B2}$$

[1]See [46] to notice that the gauge $\Gamma_i = 0$ is equivalent to fix the spatial components of the De Donder gauge condition $\partial_\mu(\sqrt{-g}g^{\mu i}) = 0$, which is used in the linearized form of the waveform corresponding to $\partial_\mu h^{\mu i} = 0$.

$$g_{ij} = \delta_{ij} e^{-[2U + \frac{\Lambda}{2}(|\vec{x}|^2 + (x_i)^2)]} + O(c^{-4}, \Lambda c^{-2}), \qquad \text{(B3)}$$

with no sum over the index i in the last term of g_{ij}. Introducing the objects (B1)–(B3) into the temporal and spatial-temporal components of the Ricci tensor we find

$$R_{00} = \nabla^2 U + \frac{1}{c}\partial_t\left(\frac{1}{c}3\partial_t U + 4\partial_i g_i + \frac{2}{c}\Lambda\vec{x}\cdot\dot{\vec{x}}\right), \qquad \text{(B4)}$$

$$R_{0i} = -2\nabla^2 g_i + 2\partial_i\left[\frac{1}{c}\dot{U} + \partial_j g_j + \frac{3}{4c}\Lambda(\vec{x}\cdot\dot{\vec{x}}) - \frac{\Lambda}{4c}(x_i\dot{x}_i)\right], \qquad \text{(B5)}$$

again with no sum over the index i in the last term of R_{0i}. Recalling that the source of the EFE can be defined from (A7), we find that at Newtonian order, namely, when the factor $v/c \to 0$,

$$S_{00} = \frac{4\pi G}{c^4}(T_{00} + T_{ii}) - \Lambda, \qquad \text{(B6)}$$

$$S_{0i} = \frac{8\pi G}{c^4}T_{0i}. \qquad \text{(B7)}$$

Consequently, plugging back the components of the Ricci tensor (B4) and (B5), and the components of the source (B6) and (B7) into the EFE (A7) yields

$$\nabla^2 U + \frac{1}{c}\partial_t\left(\frac{1}{c}3\partial_t U + 4\partial_i g_i + \frac{2}{c}\Lambda\vec{x}\cdot\dot{\vec{x}}\right) = \frac{4\pi G}{c^4}(T_{00} + T_{ii}) + g_{00}\Lambda, \qquad \text{(B8)}$$

$$-2\nabla^2 g_i + 2\partial_i\left[\frac{1}{c}\dot{U} + \partial_j g_j + \frac{3}{4c}\Lambda(\vec{x}\cdot\dot{\vec{x}}) - \frac{\Lambda}{4c}(x_i\dot{x}_i)\right] = \frac{8\pi G}{c^4}T_{0i}, \qquad \text{(B9)}$$

with no sum in the index i in the last term of the second equation. Using the energy-momentum tensor given in (A6), the components of the matter sources shown in (B8) and (B9) become

$$T^{00} = c^2\sum_a m_a\left[1 + U + \frac{v^2}{2c^2}(1 + \Lambda|\vec{x}|^2) + \Lambda|\vec{x}|^2\right]\delta^3(\vec{x} - \vec{x}_a(t)), \qquad \text{(B10)}$$

$$T^{0i} = -c\sum_a m_a v_{ai}\delta^3(\vec{x} - \vec{x}_a(t)), \qquad \text{(B11)}$$

$$T^{ii} = \sum_a m_a v_a^2[1 + \Lambda|\vec{x}|^2]\delta^3(\vec{x} - \vec{x}_a(t)). \qquad \text{(B12)}$$

On the other hand, to solve (B8), it is convenient to use the "Coulomb-like" gauge [45] considering the presence of the cosmological constant as follows:

$$\frac{1}{c}3\dot{U} + 4\partial_i g_i + \frac{2}{c}\Lambda\vec{x}\cdot\dot{\vec{x}} = 0. \qquad \text{(B13)}$$

Therefore, regarding this particular gauge, (B8) takes the following form:

$$\nabla^2 U = \frac{4\pi G}{c^4}(T_{00} + T_{ii}) + \Lambda g_{00}. \qquad \text{(B14)}$$

Next, we define the object ξ_i that satisfies the subsequent relation,

$$\nabla^2\xi_i = \nabla^2 g_i - \partial_i\left[\frac{1}{c}\dot{U} + \partial_j g_j + \frac{3}{4c}\Lambda(\vec{x}\cdot\dot{\vec{x}}) - \frac{\Lambda}{4c}(x_i\dot{x}_i)\right], \qquad \text{(B15)}$$

with no sum in the i index, and we define

$$g_i \coloneqq \xi_i + \frac{1}{4c}\partial_i\dot{\chi} + \frac{\Lambda}{18c}\partial_i(x^3\dot{x} + y^3\dot{y} + z^3\dot{z}). \qquad \text{(B16)}$$

Remarkably, the addition of the last term of (B16) ensures that the Coulomb-like gauge (B13) holds. Moreover, the third term with Λ has no rotational symmetry; hence, the Coulomb-like gauge breaks the rotational symmetry of the component of g_{0i} at order $O(\Lambda c^{-1})$. We then apply the Laplacian operator to both sides of previous relation, having

$$\begin{aligned} -\frac{1}{4c}\partial_i\nabla^2\dot{\chi} &= \nabla^2\xi_i - \nabla^2 g_i + \frac{\Lambda}{3c}\partial_i(\vec{x}\cdot\dot{\vec{x}}) \\ &= -\partial_i\left[\frac{1}{c}\dot{U} + \partial_j g_j\right] \\ &\quad - \partial_i\left[\frac{3}{4c}\Lambda(\vec{x}\cdot\dot{\vec{x}}) - \frac{1}{4c}\Lambda(x_i\dot{x}_i)\right] \\ &\quad + \frac{\Lambda}{3c}\partial_i(\vec{x}\cdot\dot{\vec{x}}), \end{aligned} \qquad \text{(B17)}$$

with no sum in the index i and where we have used (B15). Integrating the above relation, we obtain

$$-\frac{1}{4c}\nabla^2\dot{\chi} = -\left[\frac{1}{c}\dot{U} + \partial_j g_j + \frac{3\Lambda}{4c}(\vec{x}\cdot\dot{\vec{x}}) - \frac{\Lambda}{4c}(x_i\dot{x}_i) - \frac{\Lambda}{3c}(\vec{x}\cdot\dot{\vec{x}})\right], \qquad \text{(B18)}$$

with no sum over the index i. Note that the last expression represents three equations given that $i = 1, 2, 3$; and this fact is a direct consequence that the rotational symmetry is broken due to the presence of Λ. However, if we add them all together, we obtain

$$-\frac{1}{4c}\nabla^2\dot{\chi} = -\left[\frac{1}{c}\dot{U} + \partial_j g_j\right] - \frac{1}{3c}\Lambda(\vec{x}\cdot\dot{\vec{x}}). \tag{B19}$$

Then, we apply the Coulomb-like gauge (B13), having

$$\frac{1}{4c}\nabla^2\dot{\chi} = \partial_t\left[\frac{1}{4c}U - \frac{\Lambda}{12c}|\vec{x}|^2\right]. \tag{B20}$$

Next, we integrate with respect to time both sides,

$$\nabla^2\chi = U - \frac{\Lambda}{3}|\vec{x}|^2. \tag{B21}$$

On the other hand, replacing (B15) in (B9) yields

$$\nabla^2\xi_i = -\frac{4\pi G}{c^4}T_{0i}. \tag{B22}$$

Now, to solve (B8), we begin solving it at lowest order. So, that relation becomes

$$\begin{aligned}\nabla^2 U &= \frac{4\pi G}{c^4}(T_{00}+T_{ii}) - \Lambda \\ &\simeq \frac{4\pi G}{c^2}\sum_a m_a\delta^3(\vec{x}-\vec{x}_a(t)) - \Lambda \\ &= \nabla^2\left[-\frac{G}{c^2}\sum_a\frac{m_a}{r_a} - \frac{\Lambda}{6}|\vec{x}|^2\right],\end{aligned} \tag{B23}$$

with $r_a \coloneqq |\vec{x}-\vec{x}_a(t)|$, and note that we have used the relations $\delta^3(\vec{x}-\vec{x}_a(t)) = -\frac{1}{4\pi}\nabla^2(\frac{1}{r_a})$ and $\nabla^2|\vec{x}|^2 = 6$. Thus, the solution at lowest order is

$$U = -\frac{G}{c^2}\sum_a\frac{m_a}{r_a} - \frac{\Lambda}{6}|\vec{x}|^2. \tag{B24}$$

We remark that the substitution of (B24) into the components of the metric (B1) and (B3) leads to the same results previously given in (29) and (31) using the DIRE approach. This, in fact, reflects that both approaches PN and PM are related to each other in the near zone of the faraway waveform. Furthermore, from (A12) and (A13), one can realize that the ansatz given by (B1) and (B3) is satisfied providing that $U = -\frac{1}{2}\overset{(2)}{g}_{00} = -\frac{G}{c^2}\sum_a\frac{m_a}{r_a} - \frac{\Lambda}{6}|\vec{x}|^2$; hence, such ansatz matches the solution of $\overset{(2)}{g}_{00}$ and $\overset{(2)}{g}_{ij}$.

Moreover, from (B21) and (B22), and using the result (B24), we find that

$$\chi = -\frac{G}{2c^2}\sum_a m_a r_a + \frac{\Lambda}{24}|\vec{x}|^4, \tag{B25}$$

$$\xi_i = -\frac{G}{c^3}\sum_a m_a\frac{v_{ai}}{r_a}. \tag{B26}$$

This leads to

$$\begin{aligned}g_i &= -\frac{G}{8c^3}\sum_a\frac{m_a}{r_a}[7v_{ai} + \hat{n}_{ai}(\hat{n}_a\cdot\hat{v}_a)] \\ &\quad + \frac{\Lambda}{24c}|\vec{x}|^2\left[\frac{dx_i}{dt} + 2\frac{x_i}{|\vec{x}|^2}(\vec{x}\cdot\vec{v})\right] - \frac{\Lambda}{6c}(x_i)^2\dot{x}_i,\end{aligned} \tag{B27}$$

with no sum in the index i in the last term, where $\hat{n}_a = \vec{r}_a/r_a$ and $\hat{v}_a = d\vec{r}_a/dt$. These results (B24) and (B27) do comply the Coulomb-like gauge (B13) at order $O(c^{-3}, \Lambda c^{-1})$. Subsequently, using the previous result at Newtonian order, one can get the following result:

$$\begin{aligned}U &= -\frac{G}{c^2}\sum_a\frac{m_a}{|\vec{x}-\vec{x}_a(t)|} \\ &\quad - \frac{G}{c^4}\sum_a\frac{m_a}{|\vec{x}-\vec{x}_a(t)|}\left(\frac{3}{2}v_a^2 - G\sum_{b\neq a}\frac{m_b}{r_{ab}}\right) \\ &\quad - \frac{G\Lambda}{3c^2}\sum_a\frac{m_a|\vec{x}_a(t)|^2}{|\vec{x}-\vec{x}_a(t)|} \\ &\quad - \frac{\Lambda}{6}|\vec{x}|^2 + \frac{\Lambda}{c^2}G\sum_a m_a r_a + O(c^{-4}, \Lambda c^{-2}).\end{aligned} \tag{B28}$$

Here, $\vec{r}_{ab} \coloneqq \vec{r}_a - \vec{r}_b$. On the other hand, considering a compact two-body system, the two-body Lagrangian can be obtained à la Droste-Fichtenholz, a technique which, at this order, is equivalent to the Fokker Lagrangian [23]. To obtain the Lagrangian that describes the interaction of compact bodies, i.e., which are regarded as point particles, we begin computing the equations of motion of a particle of mass m_1 moving in the near zone which follows the geodesic equation. This action is given by

$$\begin{aligned}S &\coloneqq \int dt L_{m_1} = -m_1 c\int dt\left(-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}\right)^{1/2} \\ &= -m_1c^2\int dt\left(-g_{00} - 2g_{0i}\frac{v_1^i}{c} - g_{ij}\frac{v_1^i v_1^j}{c^2}\right)^{1/2},\end{aligned} \tag{B29}$$

where L_{m_1} is the Lagrangian of the geodesic of the mass m_1, and v_1^i means the velocity of the particle 1. We expand the integrand given by such Lagrangian to 1PN order as follows:

$$\begin{aligned} L_{m_1} &= -m_1c^2e^U\left[1-8g_i\frac{v_1^i}{c}e^{-2U}-e^{-4U}e^{-\frac{\Lambda}{2}|\vec{x}|^2}\left(e^{-\frac{\Lambda}{2}x_1^2}\frac{v_{1x}^2}{c^2}+e^{-\frac{\Lambda}{2}y_1^2}\frac{v_{1y}^2}{c^2}+e^{-\frac{\Lambda}{2}z_1^2}\frac{v_{1z}^2}{c^2}\right)\right]^{1/2} \\ &= -m_1c^2\left[1-\frac{1}{2c^2}v_1^2+U-\frac{1}{8c^4}v_1^4+\frac{3}{2c^2}v^2U+\frac{1}{2}U^2-\frac{4}{c}g_iv_1^i+\frac{\Lambda}{4c^2}v_1^2|\vec{x}_1|^2+\frac{\Lambda}{4c^2}(x_1^2v_{1x}^2+y_1^2v_{1y}^2+z_1^2v_{1z}^2)\right] \\ &\quad +O(c^{-4},\Lambda c^{-2},\Lambda^2), \end{aligned} \tag{B30}$$

where we utilized the ansatz of the metric given in (B1)–(B3) and evaluate the Lagrangian at the position of the mass m_1, implying that we must assess the potentials U and g_i on the trajectory, where their self part $\propto m_1$ or m_1^2 diverges, ignoring all the contributions to the field from the body m_1. These ill-defined (formally infinite) potentials that diverge are regularized (see, for instance, [22]) yielding

$$U=-\frac{Gm_2}{c^2r_{12}}\left(1+\frac{3}{2c^2}v_2^2\right)-\frac{G\Lambda m_2}{3c^2r_{12}}|\vec{x}_2|^2-\frac{\Lambda}{6}|\vec{x}_1|^2+\frac{\Lambda G}{c^2}m_2r_{12}, \tag{B31}$$

$$g_i=-\frac{Gm_2}{8c^3r_{12}}[7v_{2i}+\hat{n}_{2i}(\hat{n}_2\cdot\vec{v}_2)]+\frac{\Lambda}{24c}|\vec{x}_1|^2[v_{1i}+2(\hat{n}_1\cdot\vec{v}_1)\hat{n}_{1i}]-\frac{\Lambda}{6c}(x_{i1})^2\dot{x}_{1i}. \tag{B32}$$

Substituting the regularized potentials (B31) and (B32) into the Lagrangian (B30) leads to

$$\begin{aligned} L_{m_1} &= -m_1c^2+\frac{1}{2}m_1v_1^2+\frac{Gm_1m_2}{r_{12}}+\frac{1}{8c^2}m_1v_1^4+\frac{Gm_1m_2}{2c^2r_{12}}[3(v_1^2+v_2^2)-7\vec{v}_1\cdot\vec{v}_2-(\hat{n}_{12}\cdot\vec{v}_1)(\hat{n}_{12}\cdot\vec{v}_2)]-\frac{G^2m_1m_2^2}{2c^2r_{12}^2} \\ &\quad +\frac{G\Lambda m_1m_2}{3r_{12}}\left(r_2^2-\frac{1}{2}r_1^2\right)+\frac{\Lambda c^2m_1r_1^2}{6}-\Lambda Gm_1m_2r_{12}+\frac{\Lambda}{6}m_1r_1^2[v_1^2+2(\hat{n}_1\cdot\vec{v}_1)^2] \\ &\quad -\frac{11}{12}\Lambda m_1(x_1^2v_{1x}^2+y_1^2v_{1y}^2+z_1^2v_{1z}^2)+O(c^{-4},\Lambda c^{-2},\Lambda^2). \end{aligned} \tag{B33}$$

The Fichtenholz Lagrangian that describes the motion of two compact bodies is constructed in such a way to give the same equations of motion as (B33) when $m_1\to 0$. Hence, the Lagrangian that governs the motion of two compact bodies in interaction is given by

$$\begin{aligned} L &= -m_1c^2-m_2c^2+\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2+\frac{Gm_1m_2}{r_{12}}+\frac{1}{8c^2}m_1v_1^4+\frac{1}{8c^2}m_2v_2^4 \\ &\quad +\frac{Gm_1m_2}{2c^2r_{12}}[3(v_1^2+v_2^2)-7\vec{v}_1\cdot\vec{v}_2-(\hat{n}_{12}\cdot\vec{v}_1)(\hat{n}_{12}\cdot\vec{v}_2)]-\frac{G^2m_1m_2}{2c^2r_{12}^2}(m_1+m_2) \\ &\quad +\frac{\Lambda}{6}c^2(m_1r_1^2+m_2r_2^2)+G\Lambda m_1m_2\left(\frac{r_1^2+r_2^2}{6r_{12}}-r_{12}\right)+\frac{\Lambda}{6}m_1r_1^2v_1^2+\frac{\Lambda}{6}m_2r_2^2v_2^2 \\ &\quad +\frac{\Lambda}{3}m_1(\hat{n}_1\cdot\vec{v}_1)^2r_1^2+\frac{\Lambda}{3}m_2(\hat{n}_2\cdot\vec{v}_2)^2r_2^2-\frac{11}{12}\Lambda[m_1(x_1^2v_{1x}^2+y_1^2v_{1y}^2+z_1^2v_{1z}^2)+m_2(x_2^2v_{2x}^2+y_2^2v_{2y}^2+z_2^2v_{2z}^2)] \\ &\quad +O(c^{-4},\Lambda c^{-2},\Lambda^2), \end{aligned} \tag{B34}$$

with $\hat{n}_1=\frac{\vec{x}-\vec{x}_1(t)}{|\vec{x}-\vec{x}_1(t)|}$, $\hat{n}_2=\frac{\vec{x}-\vec{x}_2(t)}{|\vec{x}-\vec{x}_2(t)|}$. The first two lines of the Lagrangian correspond to the case of a null Λ. Besides, this formula depends explicitly of the components of the position and the velocity of the particles. On the other hand, this computation can be repeated taking into account the interaction of n particles, giving as a result

$$
\begin{aligned}
L = &\sum_a m_a v_a^2 + \sum_{a\neq b}\frac{G m_a m_b}{2 r_{ab}} + \sum_a \frac{1}{8} m_a v_a^4 - \sum_{a\neq b}\frac{G m_a m_b}{4 r_{ab}}[7\vec{v}_a\cdot\vec{v}_b + (\hat{n}_{ab}\cdot\vec{v}_a)(\hat{n}_{ab}\cdot\vec{v}_b)] \\
&+\frac{3G}{2}\sum_a\sum_{b\neq a}\frac{m_a m_b v_a^2}{r_{ab}} - \frac{G^2}{2}\sum_a\sum_{b\neq a}\sum_{c\neq a}\frac{m_a m_b m_c}{r_{ab} r_{ac}} + \frac{\Lambda}{6}c^2\sum_a m_a r_a^2 + \frac{G\Lambda}{6}\sum_a \frac{m_a m_b}{r_{ab}} r_a^2 - \frac{G\Lambda}{2}\sum_{a\neq b} m_a m_b r_{ab} \\
&+\frac{\Lambda}{6}\sum_a m_a r_a^2 v_a^2 + \frac{\Lambda}{3}\sum_a m_a (\hat{n}_a\cdot\vec{v}_a)^2 r_a^2 - \frac{11}{12}\Lambda\sum_a m_a (x_a^2 v_{ax}^2 + y_a^2 v_{ay}^2 + z_a^2 v_{az}^2) + O(c^{-4}, c^{-2}\Lambda, \Lambda^2),
\end{aligned} \tag{B35}
$$

where $a = 1, \ldots, N$ labels the particle, r_{ab} is the distance between the particle a and b, and $\hat{n}_{ab}$ is the unit vector from a to b. Considering the center of mass frame given by (49) and (50), the Lagrangian (B34) becomes

$$
\begin{aligned}
L = &-mc^2 + \frac{1}{2}\mu v^2 + \frac{G\mu m}{r} + \frac{1}{8c^2}\mu v^4 (1-3\nu) + \frac{G\mu m}{2c^2 r}\left[(3+\nu)v^2 + \nu(\hat{n}\cdot\vec{v})^2 - \frac{Gm}{r}\right] + \frac{\Lambda}{6}c^2\mu r^2 \\
&-\frac{1}{6}G\Lambda\mu r(5+2\nu) + \frac{1}{6}\Lambda\mu(1-3\nu)r^2 v^2 + \frac{\Lambda}{3}\mu(\hat{n}\cdot\vec{v})^2(1-3\nu)r^2 \\
&-\frac{11}{12}\Lambda\mu(1-3\nu)(x^2 v_x^2 + y^2 v_y^2 + z^2 v_z^2) + O(c^{-4}, \Lambda c^{-2}, \Lambda^2),
\end{aligned} \tag{B36}
$$

with $\vec{r} = \vec{x}_1 - \vec{x}_2$, $\hat{n} = \frac{\vec{x}_1 - \vec{x}_2}{|\vec{x}_1 - \vec{x}_2|}$, and $\vec{v} \coloneqq \vec{v}_1 - \vec{v}_2$ are the relative vector distance, the relative unit vector, and the relative velocity vector between the particles 1 and 2, correspondingly, and x, y, z are the Cartesian components of the relative vector position $\vec{r}$ with v_x, v_y, and v_z as their respective velocities components. The objects without the vector symbol stand only for the magnitude of the vector. The two-body Lagrangian (B36) is one of the main results of this work. It describes the interaction of a compact two-body system with relativistic correction considering the presence of the cosmological constant. Using the Euler-Lagrange equations,

$$
\frac{d}{dt}\left(\frac{\partial L}{\partial v^i}\right) - \frac{\partial L}{\partial r^i} = 0, \tag{B37}
$$

the equation of motion of a compact two-body system is displayed as follows:

$$
\begin{aligned}
a^i = &-\frac{Gm}{r^2}\hat{n}^i + \frac{Gm}{c^2 r^2}\left\{\left[\frac{Gm}{r}(4+2\nu) - v^2(1+3\nu) + \frac{3}{2}\nu(\hat{n}\cdot\vec{v})^2\right]\hat{n}^i + (4-2\nu)(\hat{n}\cdot\vec{v})v^i\right\} + \frac{\Lambda}{3}c^2 r\hat{n}^i \\
&+\Lambda(1-3\nu)\left[-\frac{5}{3}r(\hat{n}\cdot\vec{v}) + \frac{11}{3}(r_i\dot{r}_i)\right]v^i - Gm\Lambda\left[2\left(\frac{3}{4}+\nu\right) + \frac{11}{6}(1-3\nu)\frac{(r_i)^2}{r^2}\right]\hat{n}^i \\
&-\Lambda r(1-3\nu)\left[\frac{1}{2}v^2 + \frac{11}{6}(v_i)^2\right]\hat{n}^i + O(c^{-4}, \Lambda c^{-2}, \Lambda^2),
\end{aligned} \tag{B38}
$$

with no sum over the repeated index i; so this implies that is not possible to express the equations of motion using vector notation, unlike the case where $\Lambda = 0$. The energy of the system is given by $\frac{\partial L}{\partial v^i}v^i - L$, that is,

$$
\begin{aligned}
E = &\, mc^2 + \frac{1}{2}\mu v^2 - \frac{G\mu m}{r} - \frac{\Lambda}{6}c^2\mu r^2 + \frac{1}{c^2}\left\{\frac{3}{8}\mu(1-3\nu)v^4 + \frac{G\mu m}{2r}\left[(3+\nu)v^2 + \nu(\hat{n}\cdot\vec{v})^2 + \frac{Gm}{r}\right]\right\} \\
&+\frac{G\Lambda}{6}\mu m r(5+2\nu) + \frac{1}{3}(1-3\nu)\Lambda\mu r^2\left[\frac{1}{2}v^2 + (\hat{n}\cdot\vec{v})^2\right] - \frac{11}{12}\Lambda\mu(1-3\nu)(x^2 v_x^2 + y^2 v_y^2 + z^2 v_z^2).
\end{aligned} \tag{B39}
$$

Both in the acceleration (B38) and the energy (B39) hinge on explicitly of the Cartesian components of the relative position and velocity as a consequence that the cosmological constant breaks the rotational symmetry of the system. Next, taking the orbital plane coordinates,

$$
\hat{n} = (\cos\phi, \sin\phi, 0), \tag{B40}
$$

$$\hat{\phi} = (-\sin\phi, \cos\phi, 0), \tag{B41}$$

$$\hat{z} = (0, 0, 1), \tag{B42}$$

we have that the components of the acceleration can be written as

$$a^i = (\ddot{r} - r\dot{\phi}^2)\hat{n}^i + \frac{1}{r}\frac{d}{dt}(r^2\dot{\phi})\phi^i. \tag{B43}$$

Therefore, comparing (B38) and (B43), and adding the three different equations for each component, given that $i = 1,\ 2,\ 3$, we obtain

$$\begin{aligned}\ddot{r} = {}& r\dot{\phi}^2 - \frac{Gm}{r^2} + \frac{\Lambda}{3}c^2 r + \frac{Gm}{c^2 r^2}\left[\frac{1}{2}(6-7\nu)\dot{r}^2 \right.\\ & \left. - (r\dot{\phi})^2(1+3\nu) + \frac{2Gm}{r}(2+\nu)\right]\\ & - \frac{1}{3}\Lambda r(1-3\nu)\dot{r}^2 + \frac{1}{9}\Lambda r^3(1-3\nu)\dot{\phi}^2\\ & - \frac{\Lambda}{18}Gm(38+3\nu) + O(c^{-4}, \Lambda c^2, \Lambda^2),\end{aligned} \tag{B44}$$

$$\frac{d}{dt}(r^2\dot{\phi}^2) = 2(2-\nu)\frac{Gm}{c^2}\dot{r}\,\dot{\phi} + 2\Lambda(1-3\nu)r^3\dot{r}\,\dot{\phi}. \tag{B45}$$

APPENDIX C: RADIATED POWER FORMULA

In this section, we show that the radiated power formula does not contain Λ provided that the condition $\Lambda h \to 0$ is satisfied. Thus, from (10), we can observe that the radiated power, taking into account Λ, reads

$$\begin{aligned}P &= c\int\left[(-g)t^{0k}_{\mathrm{LL}} - 2\frac{c^4}{16\pi G}\Lambda\mathfrak{g}^{-1/2}\mathfrak{g}^{0k}\right]dS_k\\ &= c\int\left[(-g)t^{0k}_{\mathrm{LL}} - \frac{c^4}{8\pi G}\Lambda\mathfrak{g}^{-1/2}(\eta^{0k} + h^{0k})\right]dS_k\\ &= c\int\left[(-g)t^{0k}_{\mathrm{LL}} - \frac{c^4}{8\pi G}\mathfrak{g}^{-1/2}(\Lambda h^{0k})\right]dS_k\\ &\simeq c\int(-g)t^{0k}_{\mathrm{LL}}dS_k,\end{aligned} \tag{C1}$$

where dS_k is an outward-directed surface element on the 2-dimensional surface S. Considering the shortwave approximation (see, for example, [19]), which is based on expansion of the gravitational potentials in powers of $\lambda/R \ll 1$, with λ as the wavelength of the source and R as the distance between the source and the observation point, we write

$$h^{\alpha\beta} = (\lambda/R)f_1^{\alpha\beta} + (\lambda/R)^2 f_2^{\alpha\beta} + \cdots, \tag{C2}$$

where $f_n^{\alpha\beta}$ with $n = 1, 2, 3, \ldots$, is a function of the retarded time $\tau \coloneqq t - \frac{R}{c}$. Substituting (C2) in t^{0k}_{LL}, given by (5), from there, we replace it into (C1), yielding

$$P = \frac{c^3 R^2}{32\pi G}\int \dot{h}^{ij}_{\mathrm{TT}}\dot{h}^{\mathrm{TT}}_{ij}d^3x. \tag{C3}$$

Observe that the assumption $\Lambda h \to 0$, due to the very small value of Λ, implies that the flux of the radiation power P does not contain Λ, giving as a result the expression (70). Nonetheless, in the waveform (54), Λ does appear explicitly.

APPENDIX D: ENERGY LOSS RATE OBTAINED FROM THE SYMMETRIC TRACE-FREE MULTIPOLE DECOMPOSITION

It is well known that the EW multipoles are related with the symmetric trace-free multipoles at 1PN order as follows [21,47]:

$$I^{ij}_{\mathrm{STF}} = I^{\langle ij\rangle}_{\mathrm{EW}} + \frac{1}{21}\left(11 I^{\langle ij\rangle kk}_{\mathrm{EW}} - 12 I^{k\langle ij\rangle k}_{\mathrm{EW}} + 4 I^{kk\langle ij\rangle}_{\mathrm{EW}}\right), \tag{D1}$$

$$I^{ijk}_{\mathrm{STF}} = 3I^{ijk}_{\mathrm{EW}}, \tag{D2}$$

$$J^{ij}_{\mathrm{STF}} = \frac{1}{2}\epsilon^{\langle i}{}_{kl}I^{j\rangle kl}_{\mathrm{EW}}. \tag{D3}$$

Then, recalling the results of the EW multipoles (42), (43), and (47), and considering the interaction of only two particles at the center of mass frame coordinates (49) and (50), the STF moments (D1)–(D3) become

$$\begin{aligned}I^{ij}_{\mathrm{STF}} = {}& \mu r^{\langle i}r^{j\rangle} + \frac{\mu}{7c^2}(-5+8\nu)\frac{Gm}{r}r^{\langle i}r^{j\rangle}\\ & + \frac{\mu}{c^2}\frac{29}{42}(1-3\nu)v^2 r^{\langle i}r^{j\rangle} + \frac{11}{21c^2}\mu(1-3\nu)r^2 v^{\langle i}v^{j\rangle}\\ & - \frac{\Lambda\mu}{2}(1-3\nu)r^2 r^{\langle i}r^{j\rangle},\end{aligned} \tag{D4}$$

$$I^{ijk}_{\mathrm{STF}} = -3\frac{\mu\Delta m}{mc^2}v^{\langle i}r^j r^{k\rangle}, \tag{D5}$$

$$J^{ij}_{\mathrm{STF}} = \frac{\mu\Delta m}{c^2 m}\epsilon^{kl\langle i}r^{j\rangle}r_k v_l. \tag{D6}$$

At 1PN approximation, the radiated power in terms of the STF multipoles is given by [21,22]

$$\begin{aligned}P = -\frac{G}{c^5}\Bigg\{&\frac{1}{5}\dddot{I}^{ij}_{\mathrm{STF}}\dddot{I}^{\mathrm{STF}}_{ij} + \frac{1}{c^2}\left[\frac{1}{189}\overset{(4)}{I}{}^{\mathrm{STF}}_{ijk}\overset{(4)}{I}{}^{ijk}_{\mathrm{STF}}\right.\\ &\left. + \frac{16}{45}\dddot{J}^{\mathrm{STF}}_{ij}\dddot{J}^{ij}_{\mathrm{STF}}\right] + O(c^{-4}, c^{-2}\Lambda, \Lambda^2)\Bigg\}.\end{aligned} \tag{D7}$$

Therefore, plugging back the results (D4)–(D6) into (D7) yields the energy loss rate of a circular motion of a binary compact system given by (71).

APPENDIX E: COMPUTATION OF THE INTEGRAL $I(\Theta)$

In this appendix, we compute the integral

$$I(\Theta) = \int \frac{\sin^2(2\phi_{0\mathrm{PN}})}{x^{13}} dx, \tag{E1}$$

considering the Newtonian phase (78) neglecting the additional Λ term since $(\Lambda G^2 m^2)/c^4 \ll 1$; i.e., we only take $\phi_{0\mathrm{PN}} \simeq -x^{-5}/(32\nu)$, and the post-Newtonian parameter $x = \Theta^{-1/8}/2$. Therefore,

$$\phi_{0\mathrm{PN}} = -\frac{\Theta^{5/8}}{\nu}. \tag{E2}$$

Then, we have

$$\begin{aligned} I(\Theta) &= \int \frac{\sin^2(2\phi_{0\mathrm{PN}})}{x^{13}} dx \\ &= -512 \int d\Theta \Theta^{1/2} \sin^2\left(\frac{2\Theta^{5/8}}{\nu}\right) \\ &= \frac{\nu}{1500}\Theta^{1/4} \times \left[-\frac{500}{\nu}\Theta^{5/4} + 105\nu\cos\left(\frac{4\Theta^{5/8}}{\nu}\right) + 300\nu\Theta^{5/8}\sin\left(\frac{2\Theta^{5/8}}{\nu}\right)\right. \\ &\quad \left. + 21\nu\left(E_{\frac{3}{5}}\left(\frac{4i\Theta^{5/8}}{\nu}\right) + E_{\frac{3}{5}}\left(-\frac{4i\Theta^{5/8}}{\nu}\right)\right)\right], \end{aligned} \tag{E3}$$

with $E_n(x) \coloneqq x^{n-1}\Gamma(1-n, x)$ as the exponential integral function with $\Gamma(1-n, x) = \int_x^\infty t^{-n} e^{-t} dt$ as the incomplete Gamma function. On the other hand, we also compute the following expression:

$$\begin{aligned} \int \Theta^{-11/8} I(\Theta) d\Theta = -\frac{2\nu}{3375}\Theta^{-1/8}\Bigg[& 500\Theta^{5/4} + 135\nu^2\cos\left(\frac{4\Theta^{5/8}}{\nu}\right) \\ & + 18\nu^2\left(E_{\frac{6}{5}}\left(\frac{4i\Theta^{5/8}}{\nu}\right) + E_{\frac{6}{5}}\left(-\frac{4i\Theta^{5/8}}{\nu}\right)\right) + 63\nu^2\left(E_{\frac{3}{5}}\left(\frac{4i\Theta^{5/8}}{\nu}\right) + E_{\frac{3}{5}}\left(-\frac{4i\Theta^{5/8}}{\nu}\right)\right)\Bigg]. \end{aligned} \tag{E4}$$

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