Probing high frequency gravitational waves with pulsars

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We study graviton-photon conversion in the magnetosphere of a pulsar and explore the possibility of detecting high frequency gravitational waves with pulsar observations. It is shown that conversion of one polarization mode of photons to that of gravitons can be enhanced significantly due to strong magnetic fields around a pulsar. We also derive the conservative upper bound on stochastic gravitational waves in a frequency range of 10^8–10^9 Hz and 10^{13}–10^{27} Hz by using data of observations of the Crab pulsar and the Geminga pulsar, respectively. Our method widely fills the gap among existing high frequency gravitational wave experiments and boosts the frequency frontier in gravitational wave observations.

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I. INTRODUCTION

Detection of gravitational waves from a binary black hole merger with LIGO [1] opened the era of gravitational astronomy/cosmology. It is important to push forward multifrequency gravitational wave observations to investigate our universe [2]. In the lower frequency range, we have promising methods for observing gravitational waves like the cosmic microwave background observations (10^{-18}–10^{-16} Hz) and pulsar timing arrays (10^{-9}–10^{-7} Hz) [3,4]. Indeed, recently, pulsar timing arrays of NANOGrav, PPTA, EPTA, and IPTA detected the common temporally correlated signal across pulsars [5–8], which may be signals of stochastic gravitational waves. On the other hand, detection of high frequency gravitational waves above kHz is still under development and even new ideas are required [9]. From a theoretical viewpoint, there are many sources of high frequency gravitational waves: scattering of particles in thermal bath [10–14], inflaton annihilation into gravitons [15–17], bremsstrahlung during the reheating [18–20], preheating after inflation [21–23], decay of heavy particles into the graviton pair [24], mergers of subsolarmass primordial black holes [25]. Thus the detection of high frequency gravitational waves provides us with rich information on the fundamental physics models.

One natural direction to seek methods of detecting high frequency gravitational waves is to utilize tabletop experiments, since gravitational wave detectors often become sensitive when its size is comparable to wavelength of gravitational waves. For example, the magnon gravitational wave detector utilizing resonant excitation of magnons by gravitational waves was proposed for detecting gravitational waves around GHz [26–28]. Gravitational waves can also be converted into photons under the background magnetic field [29,30]. Along this direction, new high frequency gravitational wave detection methods with the use of axion detection experiments have been proposed intensively [9,12,31–34]. Another possible way is to utilize astrophysical observations of photons with various frequencies. In Refs. [35–37] it has been proposed that the observation of microwave/x-ray/gamma-ray photons gives constraint on the stochastic gravitational waves at the corresponding frequency, depending on the strength of the primordial/Galactic magnetic field.[1] In this paper, we propose a new detection method of high frequency gravitational waves with observations of pulsars.

Pulsars are extremely fast rotating neutron stars that originated from supernova explosions and possess strong

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[1]The inverse process, i.e., the cosmic background photon conversion into gravitons has also been investigated [38–42].

magnetic fields with $\mathcal{O}(10^{12})$ G [43].[2] This strong magnetic field accelerates electrons through electric field to energies of approximately $\mathcal{O}(1)$ TeV, and they have a power-law energy spectrum starting at $\mathcal{O}(1)$ MeV and a cutoff structure at the energy of $\mathcal{O}(1)$ TeV [43]. It is known that photons with pulsed and stationary components are produced by curvature radiation, inverse Compton scattering and synchrotron radiation, and by the high-energy electrons. Here, we point out that photons converted from gravitational waves by the magnetic field is contained in the observed photons from pulsars. We calculate the graviton-photon conversion probability around a pulsar with a specific model of magnetosphere. Then, requiring that photons converted from gravitons in the magnetic field of a pulsar do not exceed the observed photons emitted from the pulsar, we evaluate the potential use of data of pulsar observations to search for high frequency gravitational waves at frequencies from 10^8 Hz to 10^{27} Hz.

II. PHOTON PROPAGATION IN MAGNETIZED PLASMA

Let us derive the modified dispersion relation of photons in magnetized plasma. Around a pulsar, there exists magnetic fields and charged particles such as electrons and protons. We consider cold plasma, namely its thermal motion is negligible. When electromagnetic fields propagate in plasma medium, charged particles are fluctuated. A charged particle with a mass $m_{(i)}$ and a charge $e_{(i)}$ (i specifies species) obeys the equation:

$$m_{(i)}\frac{\partial \boldsymbol{v}_{(i)}}{\partial t} = e_{(i)}(\boldsymbol{E} + \boldsymbol{v}_{(i)} \times \bar{\boldsymbol{B}}), \tag{1}$$

where $\bar{\boldsymbol{B}}$ represents the component of the background magnetic field, which is perpendicular to the propagation direction. The velocity of the charged particle $\boldsymbol{v}_{(i)}$ and the electric field $\boldsymbol{E}$ are treated as perturbations. Accordingly, we have electric current

$$\boldsymbol{j} = en_{(i)}\boldsymbol{v}_{(i)}, \tag{2}$$

where $n_{(i)}$ is the number density of the charged particle. Then the Maxwell equations for perturbations are given by

$$\mathrm{rot}\,\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{3}$$

$$\mathrm{rot}\,\boldsymbol{B} = \boldsymbol{j} + \frac{\partial \boldsymbol{E}}{\partial t}. \tag{4}$$

We will only consider a background magnetic field perpendicular to the propagation direction of photons, because only such a configuration contributes to graviton-photon conversion. Then, without loss of generality, we take the direction of the magnetic field and of the propagation of photons along y-axis and z-axis, respectively. Assuming a function form of $(\boldsymbol{E}, \boldsymbol{B}, \boldsymbol{v}_{(i)}) \propto e^{-i(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}$, from Eqs. (1)–(4), one can deduce the following dispersion relation,

$$\begin{pmatrix} 1 - \frac{\omega_{p,(i)}^2}{\omega^2-\omega_{c,(i)}^2} - \frac{k^2}{\omega^2} & 0 & +i\frac{\omega_{c,(i)}}{\omega}\frac{\omega_{p,(i)}^2}{\omega^2-\omega_{c,(i)}^2} \\ 0 & 1 - \frac{\omega_{p,(i)}^2}{\omega^2} - \frac{k^2}{\omega^2} & 0 \\ -i\frac{\omega_{c,(i)}}{\omega}\frac{\omega_{p,(i)}^2}{\omega^2-\omega_{c,(i)}^2} & 0 & 1 - \frac{\omega_{p,(i)}^2}{\omega^2-\omega_{c,(i)}^2} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = 0. \tag{5}$$

In the expression, rewrote the electric field by the vector potential $\boldsymbol{A}$, i.e., $\boldsymbol{E} = -\partial_t \boldsymbol{A} = i\omega \boldsymbol{A}$.[3] The plasma frequency and the cyclotron frequency are defined by $\omega_{p,(i)} = \sqrt{\frac{4\pi\alpha n_{(i)}}{m_{(i)}}}$ and $\omega_{c,(i)} = \frac{e\bar{B}}{m_{(i)}}$, respectively.

III. GRAVITON-PHOTON CONVERSION

We now consider graviton-photon conversion around a pulsar. We first consider the mixing between gravitons and photons in vacuum and promote the result into magnetized plasma background later. The action of photons in QED is

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\mathrm{pl}}^2}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha^2}{90m_e^4}\left((F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right)\right], \tag{6}$$

where M_{pl} represents the reduced Planck mass, R is the Ricci scalar, g is the determinant of a metric $g_{\mu\nu}$, α is the fine structure constant, and m_e is the electron mass. The field strength of electromagnetic fields is defined by $F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ where $\mathcal{A}_\mu$ is the vector potential.

[2]Here, for simplicity we do not consider magnetars which have stronger magnetic field than the critical value $\sim 4 \times 10^{13}$ G because of the nonlinear QED effects (e.g., see Refs. [44,45]).

[3]We neglect the scalar potential, if any, because we are interested in only propagation degree of freedom of electromagnetic fields.

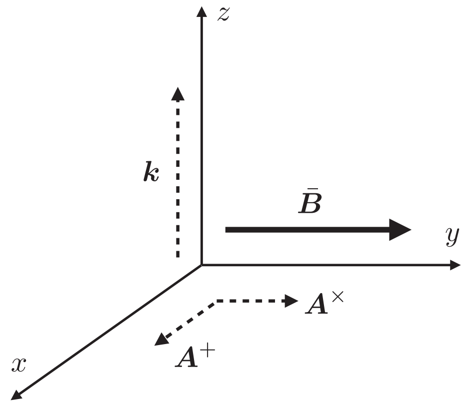

FIG. 1. The configuration of linear polarization modes to the propagation direction $\boldsymbol{k}$ and the background magnetic field $\bar{\boldsymbol{B}}$ is shown.

$\tilde{F}^{\mu\nu} = -1/2\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ ($\epsilon^{0123} = -1$) is the dual of the field strength. The third term is the Euler-Heisenberg term from the vacuum polarization [46]. We now expand the vector potential and the metric as

$$\mathcal{A}_\mu(x) = \bar{A}_\mu + A_\mu(x), \tag{7}$$

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \frac{2}{M_{\rm pl}}h_{\mu\nu}(x). \tag{8}$$

Here, $\bar{A}_\mu$ consists of background magnetic fields $\bar{B}^i = \epsilon^{ijk}\partial_j\bar{A}_k$ around a pulsar, $\eta_{\mu\nu}$ stands for the Minkowski metric, and $h_{\mu\nu}(x)$ is a traceless-transverse tensor representing gravitational waves. Below we take the gauge $A_0 = 0$. We will only consider the transverse mode, A_x and A_y as shown in Fig. 1, and neglect their mixing with A_z in Eq. (5) since the mixing is small in the case of our interest.

In the presence of background magnetic fields, one can expand the action (6) at the second order of perturbations,

$$\begin{aligned}\delta S^{(2)} = \int d^4x\bigg[&-\frac{1}{2}(\partial_\mu h_{ij})^2 - \frac{1}{2}(\partial_\mu A_i)^2 + \frac{2}{M_{\rm pl}}\epsilon_{ijk}\bar{B}^k h^{jl}\partial_i A^l \\ &+ \frac{\alpha^2}{90m_e^4}(16\bar{B}^i\bar{B}^j(\delta_{ij}(\partial_k A_l)^2 - (\partial_k A_i)(\partial_k A_j) \\ &- (\partial_i A_k)(\partial_j A_k)) + 28((\partial_0 A_i)\bar{B}_i)^2)\bigg],\end{aligned} \tag{9}$$

where higher order terms of α have been dropped. The third term represents the mixing between gravitons and photons due to the background magnetic field. Note that only magnetic fields perpendicular to the propagation direction of gravitons and/or photons contribute to the mixing. Our configurations are shown in Fig. 1. We consider gravitons and/or photons propagating along z-direction and the background magnetic field orthogonal to the propagation direction, which is taken to be y-direction, $\bar{\boldsymbol{B}} = (0, \bar{B}, 0)$. One can also choose the polarization bases for the vector and the tensor as

$$e_i^+ = \begin{pmatrix}1\\0\\0\end{pmatrix}, \qquad e_i^\times = \begin{pmatrix}0\\1\\0\end{pmatrix},$$
$$\epsilon_{ij}^+ = \frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}, \qquad \epsilon_{ij}^\times = \frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}. \tag{10}$$

Using the bases (10), the electromagnetic field and the gravitational wave can be expanded as follows:

$$A_i = e^{-i(\omega t - kz)}A^\sigma(z)e_i^\sigma, \tag{11}$$

$$h_{ij} = e^{-i(\omega t - kz)}h^\sigma(z)\epsilon_{ij}^\sigma, \tag{12}$$

We can now derive coupled equations of motion for the photon and the graviton of each polarization modes. Since there exists ionized particles around a pulsar, we also take into account the modification to the dispersion relation of photons (5). From Eqs. (5) and (9)–(12), we obtain

$$\left[i\partial_z + \begin{pmatrix} -\frac{1}{2\omega}\frac{\omega^2\omega_{p,(i)}^2}{\omega^2-\omega_{c,(i)}^2} + \frac{1}{2\omega}\frac{16\alpha^2\bar{B}^2\omega^2}{45m_e^4} & i\frac{B}{\sqrt{2}M_{\rm pl}} \\ -i\frac{B}{\sqrt{2}M_{\rm pl}} & 0\end{pmatrix}\right]\begin{pmatrix}A^+(z)\\h^+(z)\end{pmatrix} \simeq 0, \tag{13}$$

$$\left[i\partial_z + \begin{pmatrix} -\frac{\omega_{p,(i)}^2}{2\omega} + \frac{1}{2\omega}\frac{28\alpha^2\bar{B}^2\omega^2}{45m_e^4} & i\frac{B}{\sqrt{2}M_{\rm pl}} \\ -i\frac{B}{\sqrt{2}M_{\rm pl}} & 0\end{pmatrix}\right]\begin{pmatrix}A^\times(z)\\h^\times(z)\end{pmatrix} \simeq 0. \tag{14}$$

As we derive the equations, we have assumed that the scale of conversion between photons and gravitons is much longer than k^{-1} and photons are ultrarelativistic, i.e., $\omega \simeq k$. Also, we neglected spatial dependence of $\bar{B}$ by considering enough small region Δr where $\bar{B}$ can be regarded as a constant. since it is not of our interest. By diagonalizing the matrix in Eqs. (13) and (14), one can solve the equations and obtain the conversion rate between electromagnetic fields and gravitational waves of the plus modes:

$$P^{(+)}_{(i)}(A^+ \leftrightarrow h^+) = \frac{\frac{8\bar{B}^2\omega^2}{M_{\mathrm{pl}}^2}}{\left(\frac{\omega^2\omega_{p,(i)}^2}{\omega^2-\omega_{c,(i)}^2} - \frac{16\alpha^2\bar{B}^2\omega^2}{45m_e^4}\right)^2 + \frac{8\bar{B}^2\omega^2}{M_{\mathrm{pl}}^2}} \times \sin^2\left(\frac{\sqrt{\left(\frac{\omega^2\omega_{p,(i)}^2}{\omega^2-\omega_{c,(i)}^2} - \frac{16\alpha^2\bar{B}^2\omega^2}{45m_e^4}\right)^2 + \frac{8\bar{B}^2\omega^2}{M_{\mathrm{pl}}^2}}}{4\omega}\Delta r\right), \tag{15}$$

and of the cross modes:

$$P^{(\times)}_{(i)}(A^\times \leftrightarrow h^\times) = \frac{\frac{8\bar{B}^2\omega^2}{M_{\mathrm{pl}}^2}}{\left(\omega_{p,(i)}^2 - \frac{28\alpha^2\bar{B}^2\omega^2}{45m_e^4}\right)^2 + \frac{8\bar{B}^2\omega^2}{M_{\mathrm{pl}}^2}} \times \sin^2\left(\frac{\sqrt{\left(\omega_{p,(i)}^2 - \frac{28\alpha^2\bar{B}^2\omega^2}{45m_e^4}\right)^2 + \frac{8\bar{B}^2\omega^2}{M_{\mathrm{pl}}^2}}}{4\omega}\Delta r\right). \tag{16}$$

It is worth noting that there is the contribution of $\omega_{c,(i)}$ in the conversion rate of the plus mode, while there is not for the cross mode. In the next section, we will see that the contribution gives rise to big difference of the conversion rate between the polarization. We will also evaluate photon flux from conversion of gravitational waves around a pulsar and derive the conservative upper limit on high frequency gravitational waves by using observed photon spectrum.

IV. PHOTON FLUX FROM GRAVITATIONAL WAVES AROUND A PULSAR

We now estimate photon flux converted from gravitational waves around a pulsar. To this end, based on the Goldreich-Julian model [47], we simply model the magnetosphere of a pulsar; magnetic fields has a dipolelike distribution:

$$\bar{B}(r) = \sqrt{\frac{2}{3}}\bar{B}_0\left(\frac{r}{r_0}\right)^{-3}, \tag{17}$$

where $\bar{B}_0$ is an amplitude of the magnetic field at the surface of a neutron star with a radius r_0. We took an average of the direction of magnetic fields by assuming equipartition, so that the component of magnetic fields perpendicular to propagation direction of photons and gravitons obtained the factor of $\sqrt{2/3}$. Inspired by the Goldreich-Julian model [47], the number density of electrons or protons is parametrized as follows,

$$n_{(i)}(r) = 7\times 10^{-2} \times \left(\frac{1\mathrm{s}}{T}\right)\left(\frac{\bar{B}(r)}{1\mathrm{G}}\right)\ \mathrm{cm}^{-3}. \tag{18}$$

Here T is the rotation period of a neutron star.

We first show that the amplitude of the conversion rate of the plus mode (15) is always higher than that of the cross mode (16). Around a pulsar, one can estimate the each relevant parameter as follows:

$$\omega_{p,(i)} = 1.5\times 10^{11} \times \left(\frac{511\ \mathrm{keV}}{m_i}\right)^{1/2}\left(\frac{\bar{B}}{10^{12}\ \mathrm{G}}\right)^{1/2} \times \left(\frac{10\ \mathrm{ms}}{T}\right)^{1/2}\ \mathrm{Hz}, \tag{19}$$

$$\omega_{c,(i)} = 1.8\times 10^{19} \times \left(\frac{511\ \mathrm{keV}}{m_i}\right)\left(\frac{\bar{B}}{10^{12}\ \mathrm{G}}\right)\ \mathrm{Hz}, \tag{20}$$

$$\omega_{\mathrm{QED}} \equiv \frac{\alpha\bar{B}\omega}{m_e^2} = 3.4\times 10^{16} \times \left(\frac{\bar{B}}{10^{12}\ \mathrm{G}}\right)\left(\frac{\omega/2\pi}{10^{19}\ \mathrm{Hz}}\right)\ \mathrm{Hz}, \tag{21}$$

$$\Omega \equiv \sqrt{\frac{8\bar{B}\omega}{M_{\mathrm{pl}}}} = 2.5\times 10^9 \times \left(\frac{\bar{B}}{10^{12}\ \mathrm{G}}\right)^{1/2}\left(\frac{\omega/2\pi}{10^{19}\ \mathrm{Hz}}\right)^{1/2}\ \mathrm{Hz}, \tag{22}$$

where ω_{QED} represents the magnitude of the nonlinear QED effect and Ω is the coupling strength of the graviton-photon mixing. First of all, from Eqs. (19)–(21), one can find that the term $\sqrt{\frac{\omega^2\omega_{p,(i)}^2}{\omega^2-\omega_{c,(i)}^2}}$, which appeared in Eq. (15), is always much smaller than ω_{QED} for typical values of the parameters characterizing magnetosphere around a pulsar. Note that this is true not only for electrons but also for protons. Then, we can neglect the irrelevant term and approximate the conversion rate of the plus mode as

$$P^{(+)}(r,\omega) \simeq \frac{\frac{8\bar{B}^2\omega^2}{M_{\rm pl}^2}}{\left(\frac{16\alpha^2\bar{B}^2\omega^2}{45m_e^4}\right)^2 + \frac{8\bar{B}^2\omega^2}{M_{\rm pl}^2}} \times \sin^2\left(\frac{\sqrt{\left(\frac{16\alpha^2\bar{B}^2\omega^2}{45m_e^4}\right)^2 + \frac{8\bar{B}^2\omega^2}{M_{\rm pl}^2}}}{4\omega}\Delta r\right). \tag{23}$$

Notably, when $\omega_{p,(i)} > \omega_{\rm QED}$, the amplitude of the above conversion rate is significantly higher than that of the cross mode (16). Even when $\omega_{p,(i)} < \omega_{\rm QED}$, the amplitude of Eq. (23) is higher than that of Eq. (16) by a numerical factor. Thus hereafter, we will only consider the conversion of the plus mode.[4] It should be noticed that, when we consider graviton-photon conversion around a pulsar, we need to take into account r dependence of the background magnetic field. Thus we cannot simply use the formula (23). To illustrate this, let us suppose that, starting from the pure graviton state at $r \to -\infty$, the magnetic field is adiabatically turned on and peaked around $r \sim 0$ and then adiabatically turned off at $r \to \infty$. Then the probability to find the photon at $r \to \infty$ vanishes. In reality, however, the oscillation length is longer than R (R is the end of the magnetosphere, which is usually characterized by the light cylinder $R = T/2\pi$) for $f \lesssim 10^{22}$ Hz hence the adiabaticity is violated within $|r| < R$. Thus the graviton-photon conversion takes place in the pulsar magnetosphere. For higher frequency, it is not very clear which fraction of gravitons are converted to photons after passing through the magnetosphere. One possibility is that the conversion happens at the boundary of the light cylinder, at which the change of the magnetic field is rather sudden. Still, for very high frequency, the oscillation length becomes so small that any spatial variation of the magnetic field is regarded as adiabatic. While the existence of the light cylinder is a general feature in any model of magnetosphere, determining its detailed structure is challenging due to various factors, such as the dynamics and kinematics of charged particles. Assessing the effect of the boundary condition of the magnetosphere on the conversion probability is beyond the scope of our current study and is left for future work. In the following we numerically integrate Eq. (13) to calculate the conversion rate $P^{(+)}_{r_0-R}$ when gravitons propagate from r_0 to R. For $f \lesssim 10^{22}$ Hz, this gives a reasonable estimate of the probability to find photons at the detector far from the pulsar. For higher frequency, one should be cautious about the derived result.

Now let us estimate flux of the photons converted from stochastic gravitational waves and show the ability of pulsars as gravitational wave detectors. Around a pulsar, there would exist stochastic gravitational waves, which can be characterized with the characteristic amplitude defined by [48]

$$\langle h_{ij}h^{ij}\rangle = \frac{M_{\rm pl}^2}{2}\int_{f=0}^{f=\infty} d(\log f) h_c^2(f). \tag{24}$$

Due to graviton-photon conversion in magnetosphere of a pulsar, photons are generated. Its flux F is given by

$$F = \frac{2\pi^2 M_{\rm pl}^2 f h_c^2 R^2}{d^2} P^{(+)}_{r_0-R}, \tag{25}$$

where d is the distance between a pulsar and the Earth.

We use observed spectra of the Crab pulsar and the Geminga pulsar [49–52] to derive the (rough) upper limit with our method on stochastic gravitational waves. The upper bounds are derived by using the upper error bar values (2σ) in the data of the observations, which would give the maximum amount of the photon flux emitted from pulsars [43], irrespective of gravitational waves. In other words, we evaluated the upper bound on gravitational waves by requiring that the photon flux converted from gravitational waves does not exceed the actually observed photon flux. We emphasize that this approach offers a conservative estimate of the upper bound for fixed parameters of pulsars. The parameters characterizing the pulsars are $r_0 = 10$ km, $\bar{B}_0 = 7.6 \times 10^{12}$ G, $T = 33$ ms, $d = 2.0$ kpc for the Crab pulsar [53,54] and $r_0 = 10$ km, $\bar{B}_0 = 1.6 \times 10^{12}$ G, $T = 237$ ms, $d = 0.25$ kpc for the Geminga pulsar [55,56]. Note that there are relatively large uncertainties in some of these parameters, and they may cause an order of magnitude change in the final result. The result is shown in Fig. 2. There is no observation of spectra between 10^9–10^{13} Hz due to absorption by atmosphere. Nevertheless, one sees that our new method widely fill the gap among the existing experiments and boost the highest observable frequency. The upper limits from EDGES and ARCADE2 have large uncertainty depending on the amplitude of cosmological magnetic fields. The upper bound from our method around 10^8 Hz are in the middle of the uncertainty. There are constraints from ALPS and OSQAR around 10^{15} Hz, and CAST around 10^{18} Hz [31]. Their limits around the frequency regions are still stronger than ours. One can explore the higher frequency region above 10^{18} Hz up to 10^{27} Hz with our method, where there has not been any constraints.

[4] In small parameter region which satisfies $\omega_{p,(i)} \simeq \sqrt{28/45}\omega_{\rm QED}$, the amplitude of the conversion rate of the cross mode can be of order unity. However, since whether such resonance occurs or not highly depends on the model of magnetosphere, we do not consider such a possibility to give a conservative result.

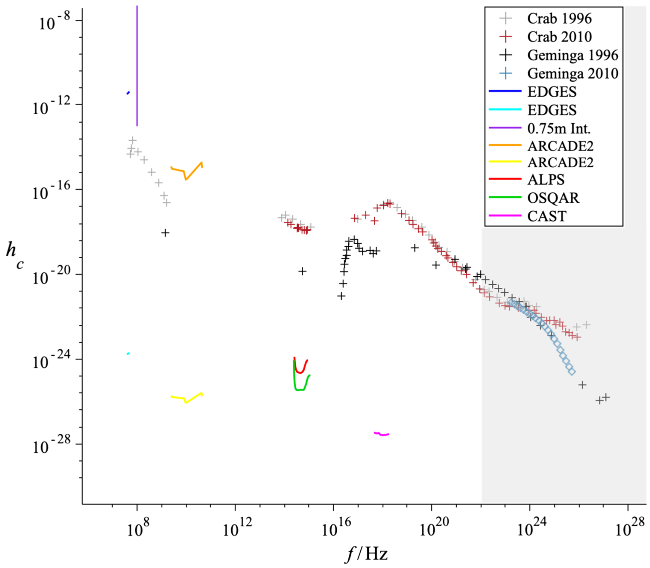

FIG. 2. The (rough) upper bound [Here the upper bound is conservative in some sense. It is actually the strain equivalent of the observed photon flux, namely, the bound is derived under the assumption that the observed photon flux is dominated by other sources rather than gravitational waves. On the other hand, the upper bound is rough in a sense that the it may fluctuate by an order of magnitude due to uncertainties of parameters characterizing pulsars. See the Sec. IV for more detailed discussion regarding these points.] on h_c of stochastic gravitational waves for frequency f[Hz] from the Crab pulsar observations in 1996s [49], 2010s [50], and from the Geminga pulsar observations in 1996 [49], 2010 [51] are depicted with the gray, brawn, black, and navy points, respectively. The blue (cyan) and the orange (yellow) lines respectively represents the constraints with EDGES and ARCADE2 for maximal (minimum) amplitude of cosmological magnetic fields [36]. The violet line is the upper limit from 0.75 m interferometer [57]. The red, green, and pink lines represents constraints with ALPS, OSQAR, and CAST, respectively [31]. The gray shaded region represents the frequency range where the results could be affected by the boundary condition of the magnetosphere, as explained in the main body.

V. CONCLUSION

We studied graviton-photon conversion in magnetosphere of a pulsar. It turned out that graviton-photon conversion can be significantly effective for photons of a polarization mode perpendicular to magnetic fields, which is called plus mode in this paper, compared to a polarization mode parallel to magnetic fields (cross mode) due to the large magnetic field around a pulsar. This enhancement of graviton-photon conversion rate does not happens for typical magnetic fields in our universe such as cosmological magnetic fields background [30]. It is also noted that the enhancement is absent for axion-photon conversion where only a polarization mode parallel to magnetic fields (cross mode) of photons are mixed with axions. Therefore, it is characteristic only for graviton-photon conversion in existence of strong magnetic fields and plasma.

We also demonstrated ability of pulsar observations as high frequency gravitational wave detectors by deriving the conservative upper bound on stochastic gravitational waves in frequency range from 10^8 Hz to 10^9 Hz and from 10^{13} Hz to 10^{27} Hz with data of observations of the Crab pulsar and the Geminga pulsar. As one can see from Fig. 2, our method enables us to fill the gap among existing high frequency gravitational wave observations. Moreover, the frequency frontier in gravitational wave observations is significantly extended from 10^{18} Hz to 10^{27} Hz.[5] To achieve the actual detection of gravitational waves using our method, the information of photon spectra derived from concrete gravitational wave sources would be needed. While this paper focused on stochastic gravitational waves as a demonstration, it is also possible to observe eventlike gravitational waves in principle. In such case, the information of the arrival direction would be useful for extracting the gravitational wave signal from data.

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[5]Recently, [58] appeared on arXiv, which also considered graviton-photon conversion around magnetosphere of planets. They gave constraints around 10^{11} Hz – 10^{15} Hz and 10^{17} Hz – 10^{23} Hz.

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