Linear response theory for spin alignment of vector mesons in thermal media

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We present a calculation of the spin alignment for unflavored vector mesons in thermalized quark-gluon plasma based on the Kubo formula in linear response theory. This is achieved by expanding the system to the first order of the coupling constant and the spatial gradient. The effect strongly relies on the vector meson's spectral functions which are determined by the interaction and medium properties. The spectral functions are calculated for the one-quark-loop self-energy with meson-quark interaction. The numerical results show that the correction to the spin alignment from the thermal shear tensor is of the order $10^{-4} \sim 10^{-5}$ for the chosen values of quark-meson coupling constant, if the magnitude of thermal shear tensor is 10^{-2}.

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I. INTRODUCTION

Rotation and spin polarization are inherently connected and can be converted to each other as demonstrated in the Barnett effect [1] and Einstein-de Haas effect [2] in materials. The same phenomenon known as global polarization can also exist in peripheral heavy-ion collisions at high energies in which the huge orbital angular momentum is partially distributed into the strong interaction matter in the form of particles' spin polarization [3–7]. The global polarization of hyperons has been observed in experiments [8,9] and been extensively studied in recent years [10–15].

Unlike the spin polarization of hyperons that can be measured through their weak decay, vector mesons can only decay by strong interaction which respects parity symmetry, which makes their spin polarization inaccessible in experiments. For spin-1 vector mesons, the only spin observables that can be measured are some elements of the spin density matrix $\rho_{\lambda_1\lambda_2}$ with λ_1 and λ_2 denoting spin states along the spin quantization direction. One of them is ρ_{00} that can be measured through the decay daughter's polar angle distribution in the rest frame of the vector meson. If ρ_{00} is not $1/3$, it means that the spin-0 state is not equally occupied among three spin states, which is called the spin alignment. The global spin alignment in heavy-ion collisions was first suggested by Liang and Wang [16]. The global spin alignments of ϕ and K^{*0} mesons were first measured by STAR collaboration in Au + Au collisions $\sqrt{s_{NN}} = 200$ GeV in 2008 [17], but no signals were found. With the accumulation of experimental data, STAR Collaboration finally found a large spin alignment for ϕ mesons in Au + Au collision at lower energies but not for K^{*0} [18].

Such a large spin alignment for ϕ mesons cannot be fully accounted by conventional mechanism [19–23]. Some of us proposed that local fluctuations of vector fields in strong interaction may give a large deviation of ρ_{00} from $1/3$ for ϕ mesons [24]. Such a prediction was made in a nonrelativistic quark coalescence model [19,25] that works for static or nearly static mesons in principle. Such a nonrelativistic quark coalescence model has been promoted to a relativistic version [26,27] based on quantum transport theory [28–31] with the help of covariant Wigner functions for massive particles [32–39] and matrix valued spin-dependent distributions [40,41]. With fluctuation parameters of strong interaction fields extracted from transverse momentum-integrated data for ρ_{00} as a function of the collision energy, the calculated transverse momentum dependence of ρ_{00} agrees with STAR's data for ϕ mesons [18]. The rapidity dependence of ρ_{00} has also been predicted with same parameters before preliminary data of STAR was released: the main feature of the data can be described by the theoretical result [42]. For recent reviews on the spin alignment of vector mesons, see, e.g., Refs. [43–45].

Recently, the contribution from the thermal shear tensor to the spin alignment of the vector meson has been calculated using the linear response theory [46] and kinetic theory [47].

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The authors of Ref. [46] argued that this contribution is quite large based on an estimate of the energy shift and width of the vector meson in medium without really calculating them. This work was inspired by Refs. [48–52] pointing out that there is a coupling between the spin polarization and the thermal shear tensor which can partially resolve the local polarization puzzle of Λ hyperons.

In this paper, we will calculate the spin alignment of vector mesons from the Kubo formula in linear response theory [53–56] in thermalized quark-gluon plasma (QGP). Vector mesons are assumed to be thermalized, and quarks and antiquarks are assumed to be unpolarized. The interaction is described by the vertex between vector meson and quark-antiquark [57–60]. In Sec. II, we present two-point Green's functions of different kinds for vector mesons in the closed-time-path (CTP) formalism [28–31,61,62]. In Sec. III, we give an introduction on spin density matrices for vector mesons from Wigner functions. In Sec. IV, we present the Dyson-Schwinger equation for retarded Green's functions. We give the expression for retarded self-energies of vector mesons from one-quark-loop. In Sec. V, we give the general form of spectral functions in medium for vector mesons from retarded Green's functions. In Sec. VI, we use the Kubo formula in the linear response theory [53–56] to calculate the correction to the two-point Green's function proportional to the thermal shear tensor. From it we are able to calculate the correction to ρ_{00} in Sec. VII. We adopt the hard-thermal-loop (HTL) [29,63–67] and quasiparticle approximations [68] to calculate spectral functions. The HTL approximation provides a toy model to illustrate the physics inside this problem since we have analytical formula for spectra functions. Then we consider a more realistic quasiparticle approximation for spectral functions. Under a few approximations or assumptions, we obtain an analytical expression for the correction to ρ_{00}, which depends on the width and energy shift from the self-energy. The numerical results for the tensor coefficients in the correction to ρ_{00} are presented. The conclusion and discussion are given in Sec. VIII.

In this paper, we adopt following notational conventions: $g^{\mu\nu} = \text{diag}(1,-1,-1,-1)$ where μ, $\nu = 0,\ 1,\ 2,\ 3$, $x^\mu = (x^0,\mathbf{x}) = (x_0,\mathbf{x})$, $x\cdot y = x^\mu y_\mu$, $x_{(\mu}y_{\nu)} = (1/2)(x_\mu y_\nu + x_\nu y_\mu)$, $\hbar = k_B = 1$. Greek letters denote components of four-vectors while lowercase Latin letters as subscripts denote components three-vectors. The four-momentum p^μ is not necessarily on-shell unless we add an index "on." The summation of repeated indices is implied if not stated explicitly. The definition of two-point Green's functions G and Σ in this paper differs by a factor $i = \sqrt{-1}$ from the usual one in quantum field theory, which are related by $G = i\tilde{G}$ and $\Sigma = i\tilde{\Sigma}$.

II. TWO-POINT GREEN'S FUNCTIONS

In this section we will give an introduction to two-point Green's functions for vector mesons on the CTP as shown in Fig. 1. The CTP formalism is a field-theory based method for many-body systems in off-equilibrium as well in equilibrium [28–31,61,62]. When it is used for systems in equilibrium, it is actually the real time formalism of the thermal (finite temperature and density) field theory [56,69]. Wigner functions can be obtained from two-point Green's functions and are related to spin density matrices, which will be addressed in the next section. We refer the readers to Section II.2 of Ref. [70] for a very brief introduction to two-point Green's functions on the CTP.

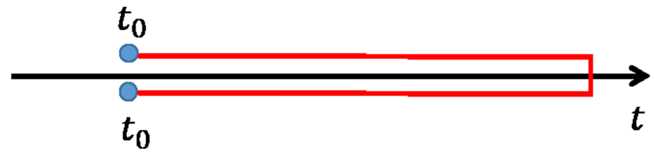

FIG. 1. Illustration of the closed-time-path upon which the nonequilibrium quantum field theory is built.

The Lagrangian density for unflavored vector mesons with spin-1 and mass m_V reads

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m_V^2}{2}A_\mu A^\mu - A_\mu j^\mu. \tag{1}$$

where $A^\mu(x)$ is the real vector field for the meson, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor, and j^μ is the source coupled to $A^\mu(x)$.

The two-point Green's function on the CTP is defined as

$$G^{\mu\nu}_{\text{CTP}}(x_1,x_2) = \langle T_C A^\mu(x_1)A^{\nu\dagger}(x_2)\rangle, \tag{2}$$

where $\langle\cdots\rangle$ denotes the ensemble average and T_C denotes time order operator on the CTP contour. Depending on whether the field A^μ lives on the positive or negative time branch, we have four components $G^{\mu\nu}_{\text{CTP}}$,

$$\begin{aligned}
G^{\mu\nu}_F(x_1,x_2) &\equiv G^{\mu\nu}_{++}(x_1,x_2)\\
&= \theta(t_1-t_2)\langle A^\mu(x_1)A^\nu(x_2)\rangle\\
&\quad + \theta(t_2-t_1)\langle A^\nu(x_2)A^\mu(x_1)\rangle,\\
G^{\mu\nu}_<(x_1,x_2) &= G^{\mu\nu}_{+-}(x_1,x_2) = \langle A^\nu(x_2)A^\mu(x_1)\rangle,\\
G^{\mu\nu}_>(x_1,x_2) &= G^{\mu\nu}_{-+}(x_1,x_2) = \langle A^\mu(x_1)A^\nu(x_2)\rangle,\\
G^{\mu\nu}_{\bar{F}}(x_1,x_2) &\equiv G^{\mu\nu}_{--}(x_1,x_2)\\
&= \theta(t_2-t_1)\langle A^\mu(x_1)A^\nu(x_2)\rangle\\
&\quad + \theta(t_1-t_2)\langle A^\nu(x_2)A^\mu(x_1)\rangle.
\end{aligned} \tag{3}$$

From the constraint $G^{\mu\nu}_F + G^{\mu\nu}_{\bar{F}} = G^{\mu\nu}_< + G^{\mu\nu}_>$, only three of them are independent. In the so-called physical representation [28,71,72], three independent two-point Green's functions are

$$
\begin{aligned}
G_R^{\mu\nu}(x_1,x_2) &= (G_F^{\mu\nu} - G_<^{\mu\nu})(x_1,x_2) \\
&\approx \theta(t_1-t_2)(G_>^{\mu\nu} - G_<^{\mu\nu})(x_1,x_2), \\
G_A^{\mu\nu}(x_1,x_2) &= (G_F^{\mu\nu} - G_>^{\mu\nu})(x_1,x_2) \\
&\approx \theta(t_2-t_1)(G_<^{\mu\nu} - G_>^{\mu\nu})(x_1,x_2), \\
G_C^{\mu\nu}(x_1,x_2) &= G_>^{\mu\nu}(x_1,x_2) + G_<^{\mu\nu}(x_1,x_2),
\end{aligned} \tag{4}
$$

where the subscripts "A" and "R" denote the advanced and retarded Green's function respectively. The two-point Green's functions in Eqs. (3)–(4) can be used to express any two-point functions defined on the CTP contour such as the self energy $\Sigma^{\mu\nu}(x_1,x_2)$. When dealing with the vacuum contributions to $G_{R,A}^{\mu\nu}$, the last equalities in the first and second line of Eq. (4) do not exactly hold since a singular term $\sim\delta(t_1-t_2)$ is missing.

III. WIGNER FUNCTIONS AND SPIN DENSITY MATRICES

In this section, we will introduce how one can obtain spin density matrices for vector mesons from Wigner functions. We refer the readers to some recent reviews [45,73] for details of the topic.

The second quantization of the vector field is in the form

$$
A^\mu(x) = \sum_{\lambda=0,\pm1}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p^V} \times [\epsilon^\mu(\lambda,\mathbf{p})a(\lambda,\mathbf{p})e^{-ip_{\rm on}\cdot x} + \epsilon^{\mu*}(\lambda,\mathbf{p})a^\dagger(\lambda,\mathbf{p})e^{ip_{\rm on}\cdot x}], \tag{5}
$$

where $p_{\rm on}^\mu = (E_p^V,\mathbf{p})$ is the on-shell momentum of the vector meson, $E_p^V = \sqrt{|\mathbf{p}|^2+m_V^2}$ is the vector meson's energy, λ denotes the spin state, $a(\lambda,\mathbf{p})$ and $a^\dagger(\lambda,\mathbf{p})$ are annihilation and creation operators respectively, and $\epsilon^\mu(\lambda,\mathbf{p}) \equiv \epsilon_\mu(\lambda,p_{\rm on})$ represents the polarization vector obeying the following relations

$$
\begin{aligned}
p_{\rm on}^\mu\epsilon_\mu(\lambda,p_{\rm on}) &= 0, \\
\epsilon(\lambda,p_{\rm on})\cdot\epsilon^*(\lambda',p_{\rm on}) &= -\delta_{\lambda\lambda'}, \\
\sum_\lambda \epsilon^\mu(\lambda,p_{\rm on})\epsilon^{\nu,*}(\lambda,p_{\rm on}) &= -\Delta^{\mu\nu}(p_{\rm on}),
\end{aligned} \tag{6}
$$

where $\Delta^{\mu\nu}(p) = g^{\mu\nu} - p^\mu p^\nu/p^2$ is the projector perpendicular to p^μ. One can check that the quantum field A^μ defined in Eq. (5) is Hermitian, $A^\mu = A^{\mu\dagger}$.

The Wigner function can be defined from $G_{\mu\nu}^<(x_1,x_2)$ [or equivalently $G_{\mu\nu}^>(x_1,x_2)$] by taking a Fourier transform with respect to the relative position $y = x_1 - x_2$,

$$
\begin{aligned}
G_{\mu\nu}^<(x,p) &\equiv \int d^4y e^{ip\cdot y} G_{\mu\nu}^<(x_1,x_2) \\
&= \int d^4y e^{ip\cdot y}\langle A_\nu^\dagger(x_2)A_\mu(x_1)\rangle.
\end{aligned} \tag{7}
$$

Inserting the quantized field (5) into the definition of the Wigner function (7), we obtain

$$
\begin{aligned}
G_{\mu\nu}^{(0)<}(x,p) = 2\pi\sum_{\lambda_1,\lambda_2}\delta(p^2-m_V^2) &\times \{\theta(p^0)\epsilon_\mu(\lambda_1,\mathbf{p})\epsilon_\nu^*(\lambda_2,\mathbf{p})f_{\lambda_1\lambda_2}^{(0)}(x,\mathbf{p}) \\
&+ \theta(-p^0)\epsilon_\mu^*(\lambda_1,-\mathbf{p})\epsilon_\nu(\lambda_2,-\mathbf{p}) \\
&\times [\delta_{\lambda_2\lambda_1} + f_{\lambda_2\lambda_1}^{(0)}(x,-\mathbf{p})]\},
\end{aligned} \tag{8}
$$

where the superscript "(0)" denotes the leading order contribution in $\hbar$ or gradient expansion, and the MVSD [40,41] at the leading order for the vector meson is defined as

$$
f_{\lambda_1\lambda_2}^{(0)}(x,\mathbf{p}) \equiv \int \frac{d^4u}{2(2\pi)^3}\delta(p\cdot u) \times e^{-iu\cdot x}\left\langle a_V^\dagger\left(\lambda_2,\mathbf{p}-\frac{\mathbf{u}}{2}\right)a_V\left(\lambda_1,\mathbf{p}+\frac{\mathbf{u}}{2}\right)\right\rangle. \tag{9}
$$

Note that $f_{\lambda_1\lambda_2}^{(0)}(x,\mathbf{p})$ is actually the (unnormalized) spin density matrix $\rho_{\lambda_1\lambda_2}$, which can be decomposed into the scalar, polarization (P_i) and tensor polarization (T_{ij}) parts as [45,73]

$$
f_{\lambda_1\lambda_2}^{(0)} = {\rm Tr}(f^{(0)})\left(\frac{1}{3}+\frac{1}{2}P_i\Sigma_i + T_{ij}\Sigma_{ij}\right)_{\lambda_1\lambda_2}, \tag{10}
$$

where $i,j=1,2,3$, ${\rm Tr}(f^{(0)}) = \sum_\lambda f_{\lambda\lambda}^{(0)}$, and Σ_i and Σ_{ij} are 3×3 traceless matrices defined as

$$
\Sigma_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}, \quad \Sigma_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}0&-i&0\\i&0&-i\\0&i&0\end{pmatrix},
$$
$$
\Sigma_3 = \begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}, \quad \Sigma_{ij} = \frac{1}{2}(\Sigma_i\Sigma_j+\Sigma_j\Sigma_i) - \frac{2}{3}\delta_{ij}. \tag{11}
$$

Let us define an integrated or on-shell Wigner function

$$
\begin{aligned}
W^{\mu\nu}(x,p_{\rm on}) &= \frac{E_p}{\pi}\int_0^\infty dp_0 G_<^{\mu\nu}(x,p) \\
&= \sum_{\lambda_1,\lambda_2}\epsilon^\mu(\lambda_1,\mathbf{p})\epsilon^{\nu*}(\lambda_2,\mathbf{p})f_{\lambda_1\lambda_2}(x,\mathbf{p}).
\end{aligned} \tag{12}
$$

It is easy to check that the second equality holds for the leading order Wigner function $G^{(0)<}_{\mu\nu}(x,p)$ given by Eq. (8). But we assume that it hold at any order. One can check that $W^{\mu\nu}(x,p_{\rm on})$ is always transverse to the on-shell momentum, $p^{\rm on}_{\mu}W^{\mu\nu}(x,p_{\rm on})=0$. The on-shell Wigner function can be decomposed into the scalar ($\mathcal{S}$), polarization ($W^{[\mu\nu]}$) and tensor polarization ($\mathcal{T}^{\mu\nu}$) parts as [45,73]

$$\begin{aligned} W^{\mu\nu}(x,p_{\rm on}) &= W^{[\mu\nu]}+W^{(\mu\nu)} \\ &= -\frac{1}{3}\Delta^{\mu\nu}(p_{\rm on})\mathcal{S}+W^{[\mu\nu]}+\mathcal{T}^{\mu\nu}, \end{aligned} \tag{13}$$

where each part is defined as

$$\begin{aligned} W^{[\mu\nu]} &\equiv \frac{1}{2}(W^{\mu\nu}-W^{\nu\mu}), \\ W^{(\mu\nu)} &\equiv \frac{1}{2}(W^{\mu\nu}+W^{\nu\mu}), \\ \mathcal{T}^{\mu\nu} &\equiv W^{(\mu\nu)}+\frac{1}{3}\Delta^{\mu\nu}(p_{\rm on})\mathcal{S}. \end{aligned} \tag{14}$$

With Eq. (13) one can show that both $W^{[\mu\nu]}$ and $\mathcal{T}^{\mu\nu}$ are traceless, $g_{\mu\nu}W^{[\mu\nu]}=g_{\mu\nu}\mathcal{T}^{\mu\nu}=0$. Inserting Eq. (10) into Eq. (12), we have

$$\begin{aligned} \mathcal{S} &= \mathrm{Tr}(f) = -\Delta^{\mu\nu}(p_{\rm on})W_{\mu\nu}, \\ W^{[\mu\nu]} &= \frac{1}{2}\mathrm{Tr}(f)\sum_{\lambda_1,\lambda_2}\epsilon^{\mu}(\lambda_1,\mathbf{p})\epsilon^{\nu*}(\lambda_2,\mathbf{p})P_i\Sigma^{i}_{\lambda_1\lambda_2}, \\ \mathcal{T}^{\mu\nu} &= \mathrm{Tr}(f)\sum_{\lambda_1,\lambda_2}\epsilon^{\mu}(\lambda_1,\mathbf{p})\epsilon^{\nu*}(\lambda_2,\mathbf{p})T_{ij}\Sigma^{ij}_{\lambda_1\lambda_2}. \end{aligned} \tag{15}$$

We see that $W^{[\mu\nu]}$ is related to P_i while $\mathcal{T}^{\mu\nu}$ is related to T_{ij}.

We can extract $f_{00}\propto\rho_{00}$ by projecting

$$L^{\mu\nu}(p_{\rm on}) = \epsilon^{\mu,*}(0,\mathbf{p})\epsilon^{\nu}(0,\mathbf{p})+\frac{1}{3}\Delta^{\mu\nu}(p_{\rm on}), \tag{16}$$

onto $W^{\mu\nu}$ in Eq. (12) as

$$\begin{aligned} L_{\mu\nu}(p_{\rm on})W^{\mu\nu} &= \sum_{\lambda_1,\lambda_2}L_{\mu\nu}(p_{\rm on})\epsilon^{\mu}(\lambda_1,\mathbf{p})\epsilon^{\nu*}(\lambda_2,\mathbf{p})f_{\lambda_1\lambda_2}(x,\mathbf{p}) \\ &= f_{00}(x,\mathbf{p})+\frac{1}{3}\sum_{\lambda_1,\lambda_2}\epsilon^{\mu}(\lambda_1,\mathbf{p})\epsilon^{*}_{\mu}(\lambda_2,\mathbf{p})f_{\lambda_1\lambda_2}(x,\mathbf{p}) \\ &= f_{00}(x,\mathbf{p})-\frac{1}{3}\mathrm{Tr}(f). \end{aligned} \tag{17}$$

In (16), $\epsilon^{\mu}(0,\mathbf{p})$ is the polarization vector along the spin quantization direction. With the first line of Eq. (15) and Eq. (17), we obtain

$$\frac{L_{\mu\nu}(p_{\rm on})W^{\mu\nu}}{-\Delta^{\mu\nu}(p_{\rm on})W_{\mu\nu}} = \frac{f_{00}(x,\mathbf{p})}{\mathrm{Tr}[f(x,\mathbf{p})]}-\frac{1}{3} = \rho_{00}-\frac{1}{3}. \tag{18}$$

The above formula relates the Wigner function to ρ_{00}, which we will use to calculate the correction to ρ_{00} in Sec. VII.

IV. DYSON-SCHWINGER EQUATION ON CTP

In this section we will give an introduction to the Dyson-Schwinger equation (DSE) on the CTP which incorporates retarded and advanced self-energies to be used for spectral functions in the next section.

We start from the integral form of the Dyson-Schwinger equation (DSE) on the CTP for the vector meson [27,74]

$$\begin{aligned} G^{\mu\nu}(x_1,x_2) &= G^{\mu\nu}_{(0)}(x_1,x_2) \\ &\quad + \int_C dx'_1 dx'_2 G^{\mu}_{(0),\rho}(x_1,x'_1) \\ &\quad \times \Sigma^{\rho}{}_{\sigma}(x'_1,x'_2)G^{\sigma\nu}(x'_2,x_2), \end{aligned} \tag{19}$$

where $dx'_{1,2}\equiv d^4x'_{1,2}$, $\int_C$ denotes the integral on the CTP contour, $G^{\mu\nu}_{(0)}$ and $G^{\mu\nu}$ are the bare and full propagator respectively, and $\Sigma^{\rho\sigma}$ is the self-energy. In Eq. (19) we have suppressed the index "CTP" in two-point functions $G^{\mu\nu}_{(0)}$, $G^{\mu\nu}$ and $\Sigma^{\rho\sigma}$. Contracting $(G^{\mu\nu}_{(0)})^{-1}$ on both sides of Eq. (19) and writing the DSE in the matrix form, we obtain

$$\begin{aligned} &-i[g^{\mu}{}_{\rho}(\partial^2_{x_1}+m_V^2)-\partial^{\mu}_{x_1}\partial^{x_1}_{\rho}]\begin{pmatrix} G^{\rho\nu}_F & G^{\rho\nu}_< \\ G^{\rho\nu}_> & G^{\rho\nu}_{\bar F}\end{pmatrix}(x_1,x_2) \\ &= \begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix} g^{\mu\nu}\delta^{(4)}(x_1-x_2) \\ &\quad + \int dx' \begin{pmatrix} \Sigma^{\mu}_{F,\rho} & -\Sigma^{\mu}_{<,\rho} \\ \Sigma^{\mu}_{>,\rho} & -\Sigma^{\mu}_{\bar F,\rho}\end{pmatrix}(x_1,x')\begin{pmatrix} G^{\rho\nu}_F & G^{\rho\nu}_< \\ G^{\rho\nu}_> & G^{\rho\nu}_{\bar F}\end{pmatrix}(x',x_2), \end{aligned} \tag{20}$$

where the integral over x'_2 is a normal one (not on the CTP). Under a unitary transformation, Eq. (20) can be put into the physical representation

$$\begin{aligned} &-i[g^{\mu}{}_{\rho}(\partial^2_{x_1}+m_V^2)-\partial^{\mu}_{x_1}\partial^{x_1}_{\rho}]\begin{pmatrix} 0 & G^{\rho\nu}_A \\ G^{\rho\nu}_R & G^{\rho\nu}_C\end{pmatrix}(x_1,x_2) \\ &= \begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix} g^{\mu\nu}\delta^{(4)}(x_1-x_2) \\ &\quad + \int dx' \begin{pmatrix} 0 & \Sigma^{\mu}_{A,\rho}\star G^{\rho\nu}_A \\ \Sigma^{\mu}_{R,\rho}\star G^{\rho\nu}_R & \Sigma^{\mu}_{C,\rho}\star G^{\rho\nu}_A+\Sigma^{\mu}_{R,\rho}\star G^{\rho\nu}_C\end{pmatrix}(x_1,x_2), \end{aligned} \tag{21}$$

where we used the shorthand notation $O_1\star O_2(x_1,x_2)\equiv O_1(x_1,x')O_2(x',x_2)$. We can assume the system is isotropic, i.e. $G^{\mu\nu}(x_1,x_2)=G^{\mu\nu}(x_1-x_2)$, and the spatial inhomogeneity of the system, as required by the Kubo formula, is induced by a perturbation. One can obtain the Dyson-Schwinger equation for retarded and advanced Green's functions in momentum space (propagators)

$$i[g^{\mu}{}_{\rho}(p^2-m_V^2)-p^{\mu}p_{\rho}]G^{\rho\nu}_{A/R}(p) = g^{\mu\nu}+\Sigma^{\mu}_{A/R,\rho}(p)G^{\rho\nu}_{A/R}(p). \quad (22)$$

The free retarded and advanced propagators are given by

$$G^{\rho\nu}_{(0)A/R}(p) = -i\frac{1}{p^2-m_V^2\mp ip_0\epsilon}\left(g^{\mu\nu}-\frac{p^{\mu}p^{\nu}}{m_V^2}\right). \quad (23)$$

One can check that $G^{\rho\nu}_{(0)A/R}(p)$ satisfies Eq. (22) neglecting the last term in the right-hand-side.

The coupling between the vector meson and quark-antiquark in QGP or the $\bar{q}qV$ vertex is assumed to be $g_V B\bar{\psi}_q\gamma^{\mu}\psi_q A_{\mu}$ [57–60]. Here B denotes the Bethe-Salpeter wave function and can be parametrized as [75,76]

$$B(p-p',p') = \frac{1-\exp[-(p-2p')^2/\sigma^2]}{(p-2p')^2/\sigma^2}, \quad (24)$$

where $p-p'$ and p' are momenta of the quark and antiquark respectively. We see that the wave function only depends on the relative momentum.

We can assume that only when the distance between the quark and antiquark is zero can they form a meson, thus we have $1/\sigma\to 0$ and $B=1$. Then the vector meson's self-energy to the lowest order of the coupling constant g_V from the quark one-loop is shown in Fig. (2). Applying Eq. (4), we can construct retarded and advanced self-energies as

$$\begin{aligned}\Sigma^{\mu\nu}_R(x_1,x_2) &= \Sigma^{\mu\nu}_F(x_1,x_2)-\Sigma^{\mu\nu}_<(x_1,x_2)\\ &= g_V^2\mathrm{Tr}[\gamma^{\mu}S_F(x_1,x_2)\gamma^{\nu}S_F(x_2,x_1)]\\ &\quad - g_V^2\mathrm{Tr}[\gamma^{\mu}S_<(x_1,x_2)\gamma^{\nu}S_>(x_2,x_1)],\\ \Sigma^{\mu\nu}_A(x_1,x_2) &= \Sigma^{\mu\nu}_F(x_1,x_2)-\Sigma^{\mu\nu}_>(x_1,x_2)\\ &= g_V^2\mathrm{Tr}[\gamma^{\mu}S_F(x_1,x_2)\gamma^{\nu}S_F(x_2,x_1)]\\ &\quad - g_V^2\mathrm{Tr}[\gamma^{\mu}S_>(x_1,x_2)\gamma^{\nu}S_<(x_2,x_1)], \quad (25)\end{aligned}$$

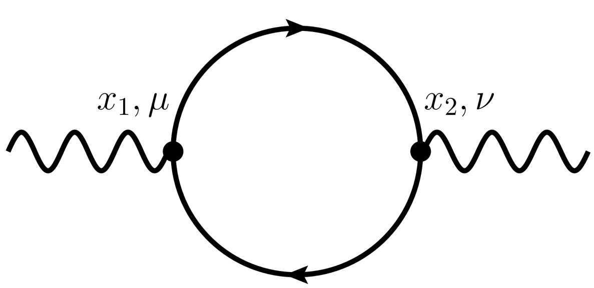

FIG. 2. The vector meson's self-energy $\Sigma_{\mu\nu}(x_1,x_2)$ from the quark loop (one-loop) contribution.

where $S(x_1,x_2)=\langle T_C\psi(x_1)\bar{\psi}(x_2)\rangle$ is the two-point Green's function of quarks on the CTP. We have included a negative sign for the quark loop in Eq. (25). Under the assumption that the system is homogeneous in position space, we obtain self-energies in momentum space

$$\begin{aligned}\Sigma^{\mu\nu}_R(p) &= g_V^2\int\frac{d^4k}{(2\pi)^4}\{\mathrm{Tr}[\gamma^{\mu}S_F(k)\gamma^{\nu}S_F(k-p)]\\ &\quad -\mathrm{Tr}[\gamma^{\mu}S_<(k)\gamma^{\nu}S_>(k-p)]\}.\\ \Sigma^{\mu\nu}_A(p) &= g_V^2\int\frac{d^4k}{(2\pi)^4}\{\mathrm{Tr}[\gamma^{\mu}S_F(k)\gamma^{\nu}S_F(k-p)]\\ &\quad +\mathrm{Tr}[\gamma^{\mu}S_>(k)\gamma^{\nu}S_<(k-p)]\}, \quad (26)\end{aligned}$$

The retarded self-energy is our starting point for derivation of spectral functions for vector mesons.

V. SPECTRAL FUNCTIONS FOR VECTOR MESONS

In this section, we will derive spectral functions for vector mesons from the retarded self-energy. We use the CTP formalism in grand-canonical equilibrium which is also called the real time formalism of the thermal field theory. The vacuum and thermal equilbrium contributions are incorporated in the same framework. We assume that quarks and antiquarks are unpolarized and their distributions are the Fermi-Dirac distribution (A2).

Evaluating the retarded self-energy in Eq. (26) using the quark propagators in Appendix A, we obtain

$$\Sigma^{\mu\nu}_R(p) = -ig_V^2\frac{1}{4\pi^3}(2I_1^{\mu\nu}+I_2^{\mu\nu})-ig_V^2 I^{\mu\nu}_{\mathrm{vac}}, \quad (27)$$

where $I_1^{\mu\nu}$ and $I_2^{\mu\nu}$ are medium parts while $I^{\mu\nu}_{\mathrm{vac}}$ is the vacuum part. The derivation of $I_1^{\mu\nu}$, $I_2^{\mu\nu}$, and $I^{\mu\nu}_{\mathrm{vac}}$ are presented in Appendix B. From Eq. (B10) we have

$$\begin{aligned}\Sigma^{0i}_R(p) &= \Sigma^{i0}_R(p) = \hat{\mathbf{p}}_i\frac{p_0}{|\mathbf{p}|}\Sigma^{00}_R(p),\\ \Sigma^{ij}_R(p) &= \hat{\mathbf{p}}_i\hat{\mathbf{p}}_j\frac{p_0^2}{|\mathbf{p}|^2}\Sigma^{00}_R(p)+(\delta^{ij}-\hat{\mathbf{p}}^i\hat{\mathbf{p}}^j)\Sigma_{\perp}(p), \quad (28)\end{aligned}$$

where $\Sigma_{\perp}(p)$ denotes the transverse part of $\Sigma^{ij}_R(p)$. Using above relations, we can greatly simplify the result of full propagators.

From Eq. (A1), one can see the only difference between the retarded and advanced propagators is the sign of the small positive number ϵ, so the retarded and advanced propagators or self-energies are complex conjugate to each other, $\Sigma^{\mu\nu}_A=-\Sigma^{\mu\nu*}_R$ (note that there is an i factor in the definition of the self-energy). It can be checked that $\Sigma^{\mu\nu}_R$ is transverse to p^{μ} as required by the current conservation. We note that the vacuum contribution and its real part is divergent and can be renormalized [68]. The imaginary part of the vacuum contribution corresponds to the pair production or annihilation processes.

Inserting Eq. (27) into Eq. (22) and introducing

$$\tilde{\Sigma}_R^{\mu\nu}(p) = -i\Sigma_R^{\mu\nu}(p) = -g_V^2 \frac{1}{4\pi^3}(2I_1^{\mu\nu} + I_2^{\mu\nu}) - g_V^2 I_{\text{vac}}^{\mu\nu}, \quad (29)$$

we obtain

$$[G_R^{-1}(p)]^{\mu\nu} = i[g^{\mu\nu}(p^2 - m_V^2) - p^\mu p^\nu - \tilde{\Sigma}_R^{\mu\nu}(p)]. \quad (30)$$

From the definition of $I_{1,2}^{\mu\nu}$, we find that they are written in terms of projectors related to three momentum $\mathbf{p}$. Therefore, we assume $G_R^{\mu\nu}$ has the same structure with $\Sigma_R^{\mu\nu}$ and can be written as

$$\begin{aligned} G_R^{00} &= iA, \\ G_R^{0i} &= G_R^{i0} = i\hat{\mathbf{p}}_i B, \\ G_R^{ij} &= i[(\delta_{ij} - \hat{\mathbf{p}}_i\hat{\mathbf{p}}_j)C + \hat{\mathbf{p}}_i\hat{\mathbf{p}}_j D], \end{aligned} \quad (31)$$

where A, B, C, D are functions of p and are not independent since $G_{\lessgtr}^{\mu\nu}$ are transverse to p^μ. By solving $(G_R^{-1})^\mu{}_\rho G_R^{\rho\nu} = g^{\mu\nu}$, we find

$$\begin{aligned} A &= \frac{1}{m_V^2}\frac{p_0^2 - m_V^2 + (p_0^2/|\mathbf{p}|^2)\tilde{\Sigma}_R^{00}}{p^2 - m_V^2 + (p^2/|\mathbf{p}|^2)\tilde{\Sigma}_R^{00}}, \\ C &= \frac{1}{p^2 - m_V^2 + \tilde{\Sigma}_\perp}, \end{aligned} \quad (32)$$

where $\tilde{\Sigma}_\perp = -i\Sigma_\perp$. Other two functions B and D can be expressed in terms of A and will be discussed later. We can also define $\tilde{G}_{R,A}^{\mu\nu}(p) = -iG_{R,A}^{\mu\nu}(p)$ to remove the factor i in the definition of $G_{R,A}^{\mu\nu}(p)$. The advanced full propagator $\tilde{G}_A^{\mu\nu}$ can be obtained by $\tilde{G}_A^{\mu\nu} = \tilde{G}_R^{\mu\nu *}$. It should be emphasized that this relation holds only for an unpolarized case.

We can construct $G_<^{\mu\nu}$ from $G_A^{\mu\nu}$ and $G_R^{\mu\nu}$ as [54,72]

$$\begin{aligned} G_<^{\mu\nu}(p) &= in_B(p_0)[\tilde{G}_R^{\mu\nu}(p) - \tilde{G}_A^{\mu\nu}(p)] \\ &= -2n_B(p_0)\text{Im}\tilde{G}_R^{\mu\nu}(p), \end{aligned} \quad (33)$$

where $n_B(p_0) = 1/(e^{\beta p_0 - \beta\mu_V} - 1)$ is the Bose-Einstein distribution with the inverse temperature $\beta \equiv 1/T$ and the vector meson's chemical potential μ_V ($\mu_V = 0$ for the unflavored meson). Note that there is an i factor in the definition of the propagator without tilde. From Eq. (33), we find the real part of A, B, C, D have no contributions to the spectral function, and the imaginary part of A, B, D have following constraints from $p_\mu G_<^{\mu\nu} = 0$,

$$\begin{aligned} p_0\text{Im}A - |\mathbf{p}|\text{Im}B &= 0, \\ p_0\text{Im}B - |\mathbf{p}|\text{Im}D &= 0. \end{aligned} \quad (34)$$

Inserting Eq. (31) into Eq. (33), one can obtain

$$G_<^{\mu\nu}(p) = -2n_B(p_0)[\Delta_T^{\mu\nu}\rho_T(p) + \Delta_L^{\mu\nu}\rho_L(p)], \quad (35)$$

or equivalently

$$\text{Im}\tilde{G}_R^{\mu\nu}(p) = \Delta_T^{\mu\nu}\rho_T(p) + \Delta_L^{\mu\nu}\rho_L(p). \quad (36)$$

In Eqs. (35), (36), we defined

$$\begin{aligned} \Delta_T^{\mu\nu} &= -g^{\mu 0}g^{\nu 0} + g^{\mu\nu} + \frac{\mathbf{p}^\mu\mathbf{p}^\nu}{|\mathbf{p}|^2}, \\ \Delta_L^{\mu\nu} &= \Delta^{\mu\nu} - \Delta_T^{\mu\nu} \equiv g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} - \Delta_T^{\mu\nu}, \end{aligned} \quad (37)$$

as the transverse and longitudinal projector respectively with $\mathbf{p}^\mu = (0, \mathbf{p})$, and $\rho_{T,L}$ are spectral functions in the transverse and longitudinal directions given by

$$\begin{aligned} \rho_T(p) &= -\text{Im}C = -\text{Im}\frac{1}{p^2 - m_V^2 + \tilde{\Sigma}_\perp(p) + i\text{sgn}(p_0)\epsilon}, \\ \rho_L(p) &= -\frac{p^2}{|\mathbf{p}|^2}\text{Im}A \\ &= -\text{Im}\frac{1}{p^2 - m_V^2 + \frac{p^2}{|\mathbf{p}|^2}\tilde{\Sigma}_{00}(p) + i\text{sgn}(p_0)\epsilon}, \end{aligned} \quad (38)$$

where $\tilde{\Sigma}_\perp$ and $\tilde{\Sigma}_{00}$ are from $\tilde{\Sigma}_R^{\mu\nu}$: $\tilde{\Sigma}_\perp \equiv -(1/2)\Delta_{\mu\nu}^T\tilde{\Sigma}_R^{\mu\nu}$ and $\tilde{\Sigma}_{00} = \tilde{\Sigma}_R^{00}$, $\text{sgn}(p_0)$ is the sign of p_0, and ε is an infinitesimal positive number. One can check in Eq. (37) that $p_\mu\Delta_T^{\mu\nu} = p_\mu\Delta_L^{\mu\nu} = 0$. In Eq. (38), one can verify that the real parts of $\tilde{\Sigma}_\perp$ and $\tilde{\Sigma}_{00}$ contribute to the mass correction while the imaginary parts of $\tilde{\Sigma}_\perp$ and $\tilde{\Sigma}_{00}$ determines the width or life-time of the quasiparticle mode. For free vector mesons, the spectral functions are $\rho_T^{(0)} = \rho_L^{(0)} = \pi\text{sgn}(p_0)\delta(p^2 - m_V^2)$, which give $G_<^{\mu\nu}(p)$ for the free vector meson following Eq. (35) and $\text{Im}\tilde{G}_R^{\mu\nu}(p)$ for the free vector meson following Eq. (36).

VI. KUBO FORMULA IN LINEAR RESPONSE THEORY

In this section we use the Kubo formula in linear response theory to calculate the nonequilibrium correction to $G_<^{\mu\nu}(p)$. The Kubo formula has been derived in Zubarev's approach to nonequilibrium density operator [55,77,78].

According to the Kubo formula, the linear response of the expectation value of an operator $\hat{O}$ to the perturbation $\partial_\mu\beta_\nu$ has the form

$$\langle\hat{O}(x)\rangle=\langle\hat{O}\rangle_{\rm LE}+\partial_\mu\beta_\nu(x)\lim_{K^\mu\to 0}\frac{\partial}{\partial K_0}\times{\rm Im}\left[iT(x)\int_{-\infty}^{t}d^4x'\langle[\hat{O}(x),\hat{T}^{\mu\nu}(x')]\rangle_{\rm LE}e^{-iK\cdot(x'-x)}\right], \tag{39}$$

where $\langle\hat{O}(x)\rangle\equiv{\rm Tr}[\hat{\rho}\,\hat{O}(x)]$ and $\langle\hat{O}(x)\rangle_{\rm LE}\equiv{\rm Tr}[\hat{\rho}_{\rm LE}\hat{O}(x)]$ with $\hat{\rho}$ and $\hat{\rho}_{\rm LE}$ being the nonequilibrium and local equilibrium density operator respectively [78], $\beta^\mu(x)\equiv u^\mu(x)/T(x)$ with $u^\mu(x)$ and $T(x)$ being the local velocity and temperature respectively, K^μ is the momentum roughly equals to π/L with L being the length of the system, and

$$\hat{T}^{\mu\nu}=\hat{F}^{\mu}{}_{\alpha}\hat{F}^{\alpha\nu}+m_V^2\hat{A}^\mu\hat{A}^\nu-g^{\mu\nu}\left(-\frac{1}{4}\hat{F}_{\rho\eta}\hat{F}^{\rho\eta}+\frac{1}{2}m_V^2\hat{A}_\rho\hat{A}^\rho\right), \tag{40}$$

is the energy-momentum tensor for the vector field. Detailed derivation of Eq. (39) is given in Ref. [78].

Now we set $\hat{O}(x)$ to be the operator corresponding to $G_<^{\mu\nu}$

$$\hat{G}_<^{\mu\nu}(x,p)=\int d^4y e^{ip\cdot y}\hat{A}^\nu\left(x-\frac{y}{2}\right)\hat{A}^\mu\left(x+\frac{y}{2}\right), \tag{41}$$

which gives $G_<^{\mu\nu}=\langle\hat{G}_<^{\mu\nu}(x,p)\rangle$. In Eqs. (40) and (41) we explicitly show the "hat" on the field operator $\hat{A}^\mu$ which we have suppressed in Sec. II and III just to emphasize their operator's nature in the Kubo formula (39). When inserting Eqs. (40) and (41) into Eq. (39), the vector field $\hat{A}^\mu$ can be approximated as the free field at the leading order in space-time gradient, since $\partial_\mu\beta_\nu(x)$ is already of the next-to-leading order.

Substituting $\hat{G}_<^{\mu\nu}(x,p)$ in (41) into Eq. (39), one obtains the next-to-leading order term of $G_<^{\mu\nu}$ as

$$\begin{aligned}\delta G_<^{\mu\nu}(x,p)&\equiv\langle\hat{G}_<^{\mu\nu}(x,p)\rangle-\langle\hat{G}_<^{\mu\nu}(x,p)\rangle_{\rm LE}\\&=4T\lim_{K^\mu\to 0}\frac{\partial}{\partial K_0}{\rm Im}\int\frac{dp_1^0dp_2^0}{2\pi}\frac{n_B(p_1^0)-n_B(p_2^0)}{p_1^0-p_2^0+K^0+i\epsilon}\\&\quad\times\delta\left(p^0-\frac{p_1^0+p_2^0}{2}\right)\partial_\gamma\beta_\lambda(x)\\&\quad\times\sum_{a,b=L,T}\rho_a(p_1)\rho_b(p_2)I_{ab}^{\mu\nu\gamma\lambda}(p_1,p_2)\end{aligned} \tag{42}$$

where $p_1=(p_1^0,\mathbf{p}-\mathbf{K}/2)$, $p_2=(p_2^0,\mathbf{p}+\mathbf{K}/2)$, $n_B(p_0)$ is the Bose-Einstein distribution defined after Eq. (33), and $\rho_{L,T}$ are given in Eq. (38). Note that integral ranges for $p_{1,2}^0$ are different from Ref. [46]. The tensor $I_{ab}^{\mu\nu\gamma\lambda}(p_1,p_2)$ can be expressed in terms of projectors $\Delta_{L,T}^{\mu\nu}$ as

$$\begin{aligned}I_{ab}^{\mu\nu\gamma\lambda}(p_1,p_2)&=(p_1^\lambda p_2^\gamma+p_1^\gamma p_2^\lambda)\Delta_{a,\alpha}^{\nu}(p_1)\Delta_b^{\mu\alpha}(p_2)+(p_{1,\alpha}p_2^\alpha-m_V^2)[\Delta_a^{\gamma\nu}(p_1)\Delta_b^{\mu\lambda}(p_2)+\Delta_a^{\lambda\nu}(p_1)\Delta_b^{\mu\gamma}(p_2)]\\&\quad-[p_1^\gamma p_2^\alpha\Delta_{a,\alpha}^{\nu}(p_1)\Delta_b^{\mu\lambda}(p_2)+p_2^\gamma p_1^\alpha\Delta_a^{\lambda\nu}(p_1)\Delta_{b,\alpha}^{\mu}(p_2)]-[p_{1,\alpha}p_2^\lambda\Delta_a^{\gamma\nu}(p_1)\Delta_b^{\mu\alpha}(p_2)+p_1^\lambda p_{2,\alpha}\Delta_a^{\alpha\nu}(p_1)\Delta_b^{\mu\gamma}(p_2)]\\&\quad-g^{\gamma\lambda}[g_{\beta\alpha}(p_{2,\rho}p_1^\rho-m_V^2)-p_{1,\beta}p_{2,\alpha}]\Delta_a^{\alpha\nu}(p_1)\Delta_b^{\mu\beta}(p_2).\end{aligned} \tag{43}$$

Then we integrate Eq. (42) over p_0 from 0 to $+\infty$ to exclude the contribution from antiparticles. As we have mentioned above, the limit $K^\mu\to 0$ should be taken in the last step, thus the integral of Eq. (42) can be simplified as

$$\int_0^{+\infty}dp_0\delta G_<^{\mu\nu}(x,p)\approx 2T\xi_{\gamma\lambda}\int_0^\infty dp_1^0\frac{\partial n_B(p_1^0)}{\partial p_1^0}\sum_{a,b=L,T}\rho_a(p_1^0,\mathbf{p})\rho_b(p_1^0,\mathbf{p})I_{ab}^{\mu\nu\gamma\lambda}(p_1^0,\mathbf{p},p_1^0,\mathbf{p}), \tag{44}$$

where $\xi_{\gamma\lambda}=\partial_{(\gamma}\beta_{\lambda)}$ denotes the thermal shear tensor.

The spin alignment coupled with the thermal shear tensor is given by

$$\delta\rho_{00}=\frac{L^{\mu\nu}(p_{\rm on})\int_0^{+\infty}dp_0[G_{\mu\nu}^<(x,p)+\delta G_{\mu\nu}^<(x,p)]}{-\Delta^{\mu\nu}(p_{\rm on})\int_0^{+\infty}dp_0[G_{\mu\nu}^<(x,p)+\delta G_{\mu\nu}^<(x,p)]}, \tag{45}$$

where $G_{\mu\nu}^<(x,p)$ is given in Eq. (35) while $\delta G_{\mu\nu}^<(x,p)$ is given in (44), and $L^{\mu\nu}(p_{\rm on})$ is defined in Eq. (16). The above formula is the starting point for us to evaluate the correction to ρ_{00} from the shear stress tensor in the next section.

We should note about the difference between the average taken in $G_{\mu\nu}^<(x,p)$ given by Eq. (35) [as well as other avarages in Sec. II] and the one taken in $\langle\hat{G}_<^{\mu\nu}(x,p)\rangle$ in Eq. (42). The local equilibrium average is implied for the former, while the nonequilibrium average is implied for the latter. For notational simplicity, we do not put "LE" index to local equilibrium averages in this paper except in the Kubo formula Eqs. (39) and (42).

VII. SPIN ALIGNMENT CORRECTION FROM SHEAR TENSOR

In this section, we will calculate the spin alignment correction from the shear tensor. To this end, we adopt two approximations to evaluate self-energies and spectral functions of unflavored vector mesons: the HTL and quasiparticle approximation.

A. HTL approximation

Under the HTL approximation, the external momentum of the vector meson's self-energy is of order $g_V T$ which is called "soft" while the quark loop momentum is of order T which is called "hard" [29,63–67]. This condition is not satisfied for the real vector meson in the thermal environment at RHIC and LHC with $p_0 > m_V \gg T$. The reason that we still consider the HTL approximation is that the self-energies in this approximation is analytical and the calculation of the spin density matrix is transparent. In other words, we treat the HTL approximation as a toy model to show the underlying physics.

We can consider massless quarks for simplicity. Note that the vacuum term is not included since the imaginary part of the vacuum term corresponds to the process that one particle decomposes into two on-shell quarks, i.e. $p^0 > k^0$, which is beyond the HTL approximation. The vacuum contribution is proportional to $p^2\Delta^{\mu\nu}$ as required by the Ward identity, which is of order $g_V^4 T^2$ since $p \sim g_V T$. The self-energy in the HTL approximation reads

$$\tilde{\Sigma}_{00}(p) = 3m_T^2\left(1-\frac{p_0}{2|\mathbf{p}|}\ln\frac{p_0+|\mathbf{p}|+i\epsilon}{p_0-|\mathbf{p}|+i\epsilon}\right),$$
$$\tilde{\Sigma}_{\perp}(p) = -\frac{3}{2}m_T^2\frac{p_0^2}{|\mathbf{p}|^2}\left(1-\frac{p_0^2-|\mathbf{p}|^2}{2p_0|\mathbf{p}|}\ln\frac{p_0+|\mathbf{p}|+i\epsilon}{p_0-|\mathbf{p}|+i\epsilon}\right), \quad (46)$$

where $m_T^2 = g_V^2 T^2/9$ denotes the thermal mass. The real and imaginary parts of $\tilde{\Sigma}^{00}$ and $\tilde{\Sigma}_\perp$ can be obtained as

$$\mathrm{Re}\tilde{\Sigma}_{00}(p) = 3m_T^2\left(1-\frac{p_0}{2|\mathbf{p}|}\ln\left|\frac{p_0+|\mathbf{p}|}{p_0-|\mathbf{p}|}\right|\right),$$
$$\mathrm{Re}\tilde{\Sigma}_{\perp}(p) = -\frac{3}{2}m_T^2\frac{p_0^2}{|\mathbf{p}|^2}\left(1-\frac{p_0^2-|\mathbf{p}|^2}{2p_0|\mathbf{p}|}\ln\left|\frac{p_0+|\mathbf{p}|}{p_0-|\mathbf{p}|}\right|\right),$$
$$\mathrm{Im}\tilde{\Sigma}_{00}(p) = \pi\frac{3}{2}m_T^2\frac{p_0}{|\mathbf{p}|}\theta(|\mathbf{p}|^2-p_0^2),$$
$$\mathrm{Im}\tilde{\Sigma}_{\perp}(p) = -\pi\frac{3}{4}m_T^2\frac{p_0(p_0^2-|\mathbf{p}|^2)}{|\mathbf{p}|^3}\theta(|\mathbf{p}|^2-p_0^2), \quad (47)$$

where $\theta(x)$ is the Heaviside step function. We see that the imaginary parts are nonvanishing only in spacelike region of p.

Under the HTL approximation, one can get the inequality $p_0 \sim m_T \sim g_V T \ll m_V$, which provides a natural power counting in $\alpha \equiv m_T/m_V$. We also assume $p_0^2 < |\mathbf{p}|^2$, so there is no pole contribution. Then the spectral function $\rho_{L/T}$ in (38) can be approximated as

$$\rho_T(p) = \frac{\mathrm{Im}\tilde{\Sigma}_\perp(p)}{m_V^4} + \mathcal{O}(\alpha^5)$$
$$= -\pi\frac{3}{4}\frac{m_T^2}{m_V^4}\frac{p_0 p^2}{|\mathbf{p}|^3}\theta(|\mathbf{p}|^2-p_0^2) + \frac{1}{m_V^2}\mathcal{O}(\alpha^3),$$
$$\rho_L(p) = \frac{p^2}{|\mathbf{p}|^2}\frac{\mathrm{Im}\tilde{\Sigma}^{00}(p)}{m_V^4} + \mathcal{O}(\alpha^5)$$
$$= \pi\frac{3}{2}\frac{m_T^2}{m_V^4}\frac{p_0 p^2}{|\mathbf{p}|^3}\theta(|\mathbf{p}|^2-p_0^2) + \frac{1}{m_V^2}\mathcal{O}(\alpha^3). \quad (48)$$

for $p^0 \ll m_V$. Using Eq. (48) in Eq. (45), we can get the leading order term of $\delta\rho_{00}$

$$\delta\rho_{00}^{(0)} \approx -T\frac{\xi_{\gamma\lambda}\int_0^{|\mathbf{p}|}dp_0\frac{\partial n(p_0)}{\partial p_0}\rho_a(p)\rho_b(p)[I_{ab}^{22\gamma\lambda}(p)-\frac{1}{3}I_{ab}^{ii\gamma\lambda}(p)]}{(g_{\mu 0}g_{\nu 0}-g_{\mu\nu})\int_0^{|\mathbf{p}|}dp_0 n(p_0)[\Delta_L^{\mu\nu}\rho_L(p)+\Delta_T^{\mu\nu}\rho_T(p)]} + \frac{\int_0^\infty dp^0 n_B(p_0)[(\Delta_T^{22}-\frac{1}{3}\Delta_T^{ii})\rho_T(p)+(\Delta_L^{22}-\frac{1}{3}\Delta_L^{ii})\rho_L(p)]}{\{\int_0^{|\mathbf{p}|}dp^0 n_B(p_0)[\Delta_T^{kk}\rho_T(p)+\Delta_L^{kk}\rho_L(p)]\}^2}$$
$$\times T\xi_{\gamma\lambda}\int_0^{|\mathbf{p}|}dp^0\frac{\partial n_B(p_0)}{\partial p_0}\sum_{a,b=T,L}\rho_a(p)\rho_b(p)I_{ab}^{jj\gamma\lambda}(p;p), \quad (49)$$

where $\Delta_{L,T}^{\mu\nu}$ are projectors defined in Eq. (37). In Eq. (49), the polarization vector can be approximated as $\epsilon^\mu(0,p_{\mathrm{on}}) = (0,0,1,0)+\mathcal{O}(\alpha)$ with $|\mathbf{p}| \ll E_p \approx m_V$, where we choose y direction as the spin quantization direction. So $\Delta^{\mu\nu}(p_{\mathrm{on}})$ can be approximated as $g^{\mu\nu}-g^{\mu 0}g^{\nu 0}$. The leading order $I_{ab,(0)}^{\mu\nu\gamma\lambda}$ is given by

$$I_{ab}^{\mu\nu\gamma\lambda}(p) \approx -m_V^2[\Delta_a^{\gamma\nu}(p)\Delta_b^{\mu\lambda}(p)+\Delta_a^{\lambda\nu}(p)\Delta_b^{\mu\gamma}(p)]$$
$$+ m_V^2 g^{\gamma\lambda}g_{\beta\alpha}\Delta_a^{\alpha\nu}(p)\Delta_b^{\mu\beta}(p), \quad (50)$$

which is $\mathcal{O}(m_V^2)$. Finally we can estimate

$$\delta\rho_{00}^{(0)} \sim \frac{m_V^{-2}\alpha^2\times m_V^{-2}\alpha^2\times m_V^2}{m_V^{-2}\alpha^2}\times\xi \sim \alpha^2\xi, \quad (51)$$

where $\xi \equiv |\xi_{\gamma\lambda}|$ is the magnitude of the thermal shear tensor. If we set the parameters' values as $g_V = 1$, $T = 150$ MeV, $m_V = 1020$ MeV, the coupling between the spin alignment and the shear tensor is about $\alpha^2 \sim \mathcal{O}(10^{-2})$. If we further use $\xi \sim 0.01$, then we obtain $\delta\rho_{00}^{(0)} \sim \mathcal{O}(10^{-4})$, which is much smaller than the contribution from the coalescence model via strong force fields [26].

B. Vector mesons as resonances

Now we consider the realistic case that vector mesons are resonances so that the coalescence and dissociation processes can happen. In this case, we have $p^0 > E_k$ and $(p-k)^2 > 0$. The small imaginary numbers in the quark loop integral become $\pm i(E_k \pm p_0)\epsilon \propto i\epsilon$ in $J_\pm(p; n_1, n_2)$ in Eq. (B4). Therefore, the vector meson's self-energies read

$$\begin{aligned}
\tilde{\Sigma}_{00}(p) = & -g_V^2 \frac{1}{4\pi^2}\left\{-4J_0(-1,2) + 2\frac{p_0}{|\mathbf{p}|}[J_+(p;0,1) - J_-(p;0,1)] + \frac{p^2}{2|\mathbf{p}|}[J_+(p;-1,1) + J_-(p;-1,1)]\right. \\
& \left. + \frac{2}{|\mathbf{p}|}[J_+(p,1,1) + J_-(p,1,1)]\right\} - g_V^2 I_{\mathrm{vac}}^{00}, \\
\tilde{\Sigma}_\perp(p) = & -g_V^2 \frac{1}{16\pi^2}\frac{1}{|\mathbf{p}|^3}\{(8p^2|\mathbf{p}| + 16|\mathbf{p}|^3)J_0(-1,2) - (p^4 + 2p^2|\mathbf{p}|^2)[J_+(p;-1,1) + J_-(p;-1,1)] \\
& - 4p_0^2[J_+(p;1,1) + J_-(p;1,1)] - 4p_0 p^2[J_+(p;0,1) - J_-(p;0,1)] \\
& + 4|\mathbf{p}|^2[J_+(p;-1,3) + J_-(p;-1,3)]\} - g_V^2 I_{\mathrm{vac}}^\perp,
\end{aligned} \tag{52}$$

where $I_{\mathrm{vac}}^\perp \equiv (1/2)(\delta_{ij} - \hat{\mathbf{p}}_i\hat{\mathbf{p}}_j)I_{\mathrm{vac}}^{ij}$. Note that the vacuum contributions to real parts of self-energies are canceled by renormalization. When evaluating imaginary parts, we note that $J_0(n_1, n_2)$ is real and $\mathrm{Im}J_+$ is nonzero in the region $p^2 + 2p_0E_k < 2|\mathbf{k}||\mathbf{p}|$, which cannot be satisfied under the quasiparticle approximation with $p_0 > |\mathbf{p}|$ and $E_k > |\mathbf{k}|$. So the imaginary parts come from $\mathrm{Im}J_-(p; n_1, n_2)$ within the range $-2|\mathbf{k}||\mathbf{p}| \le p^2 - 2p_0E_k \le 2|\mathbf{k}||\mathbf{p}|$ as

$$\mathrm{Im}J_-(p;n_1,n_2) = -\pi\int_{E_{\min}}^{E_{\max}} dE_k (E_k^2 - m_q^2)^{(n_2-1)/2} E_k^{n_1+1}\left[\frac{1}{e^{(E_k-\mu_q)/T}+1} + \frac{1}{e^{(E_k+\mu_q)/T}+1}\right], \tag{53}$$

where μ_q is the chemical potential of quarks, m_q is the quark mass, and $E_{\max/\min}$ is

$$E_{\max/\min} = \sqrt{\left(\pm\frac{\mathbf{p}}{2} + \frac{p_0}{2}\sqrt{1 - \frac{4m_q^2}{p^2}}\right)^2 + m_q^2}. \tag{54}$$

We see that imaginary parts exist only when $p^2 > 4m_q^2$. Then the imaginary parts of self-energies read

$$\begin{aligned}
\mathrm{Im}\tilde{\Sigma}_{00}(p) = & -g_V^2\frac{1}{2\pi^2|\mathbf{p}|}\mathrm{Im}\left[J_-(p;1,1) - p_0J_-(p;0,1) + \frac{p^2}{4}J_-(p;-1,1)\right] - g_V^2\mathrm{Im}I_{\mathrm{vac}}^{00}, \\
\mathrm{Im}\tilde{\Sigma}_\perp(p) = & -g_V^2\frac{1}{16\pi^2|\mathbf{p}|^3}\mathrm{Im}[-(p^4 + 2|\mathbf{p}|^2p^2)J_-(p;-1,1) - 4p_0^2J_-(p;1,1) + 4p_0p^2J_-(p;0,1) + 4|\mathbf{p}|^2J_-(p;-1,3)] \\
& - g_V^2\mathrm{Im}I_{\mathrm{vac}}^\perp.
\end{aligned} \tag{55}$$

Note that vacuum contributions are included in imaginary parts of self-energies, which correspond to pair production and annihilation (dissociation and combination) processes involving on-shell particles in the initial and final states (the meson, quark and antiquark are all on-shell).

1. Quasiparticle approximation

We take the quasiparticle approximation (QPA) for the vector meson that g_V is not very large and the self-energies are assumed to be small compared with m_V^2. In this case, the spectral functions in Eq. (38) have narrow peaks around E_p^V. In the region near $p_0 = E_p^V$, we can approximate the self-energies as their on-shell values, i.e., $\tilde{\Sigma}_{00}(p) \approx \tilde{\Sigma}_{00}(p_{\mathrm{on}})$ and $\tilde{\Sigma}_\perp(p) \approx \tilde{\Sigma}_\perp(p_{\mathrm{on}})$. Then spectral functions for transverse/longitudinal modes can be approximated as

$$\begin{aligned}
\rho_{T/L}(p) = & \rho_{T/L}^{\mathrm{pole}}(p) + \rho_{T/L}^{\mathrm{cut}}(p) \\
\approx & \pi\,\mathrm{sgn}(p_0)\delta[p_0^2 - (E_p^V + \Delta E_{T/L})^2]\theta(4m_q^2 - p^2) \\
& + \frac{m_V\Gamma_{T/L}}{[p_0^2 - (E_p^V + \Delta E_{T/L})^2]^2 + m_V^2\Gamma_{T/L}^2} \\
& \times\theta(p^2 - 4m_q^2),
\end{aligned} \tag{56}$$

where $\Gamma_{T/L}$ are widths and $\Delta E_{T/L}$ are energy shifts for transverse/longitudinal modes approximated as

$$\Gamma_T = \frac{1}{m_V}\mathrm{Im}\tilde{\Sigma}_{\perp}(p_{\mathrm{on}}),$$
$$\Gamma_L = \frac{m_V}{|\mathbf{p}|^2}\mathrm{Im}\tilde{\Sigma}^{00}(p_{\mathrm{on}}),$$
$$\Delta E_T = \sqrt{E_{V,p}^2 - \mathrm{Re}\tilde{\Sigma}_{\perp}(p_{\mathrm{on}})} - E_p^V,$$
$$\Delta E_L = \sqrt{E_{V,p}^2 - \frac{m_V^2}{|\mathbf{p}|^2}\mathrm{Re}\tilde{\Sigma}_{00}(p_{\mathrm{on}})} - E_p^V. \quad (57)$$

We see in Eq. (56) that $\rho_{T/L}^{\mathrm{pole}}(p)$ denote pole contributions while $\rho_{T/L}^{\mathrm{cut}}(p)$ denote cut contributions.

We plot widths and energy shifts in Fig. 3 as functions of $|\mathbf{p}|$ at $g_V = 1$, 2. We choose two sets of values for the strange quark chemical potential and temperature corresponding to the freeze-out conditions at $\sqrt{s_{NN}} \approx 20$ and 200 GeV in heavy-ion collisions [79,80]: $\mu_s \approx \mu_B/3 \approx 64.5$ MeV and $T \approx 155.7$ MeV (black) and $\mu_s \approx \mu_B/3 \approx 7.4$ MeV and $T \approx 158.4$ MeV (red). Other parameters are set to $g_V = 1$, $m_V = 1.02$ GeV, and $m_s = 419$ MeV. We can check $\rho_{T/L}^{\mathrm{pole}}(p) = 0$ for these values of parameters since $4m_q^2 - p^2 < 0$ at the corrected mass-shell $p^2 = m_V^2 - \mathrm{Re}\tilde{\Sigma}_{\perp}(p_{\mathrm{on}})$ and $p^2 = m_V^2 - (m_V^2/|\mathbf{p}|^2)\mathrm{Re}\tilde{\Sigma}_{00}(p_{\mathrm{on}})$ for transverse and longitudinal modes respectively. One can see in Fig. 3 that the width and energy shift are almost independent of freezeout conditions at the collision energy 20 and 200 GeV.

We find that the $\Gamma_{T/L}$ and $\Delta E_{T/L}$ are much smaller than m_V, which allows us to introduce the following power counting scheme

$$\frac{\Delta E_{T/L}}{E_p^V} \sim \frac{\Gamma_{T/L}}{E_p^V} \sim \epsilon \ll 1, \quad (58)$$

where we have introduced ϵ as a small power counting parameter. Since Γ_T and Γ_L are positive definite, we expect that their difference is a second-order contribution $\Delta\Gamma/E_p^V \equiv (\Gamma_T - \Gamma_L)/E_p^V \sim \mathcal{O}(\epsilon^2)$, while $E_p^V(1/\Gamma_T - 1/\Gamma_L) = E_p^V(\Gamma_L - \Gamma_T)/(\Gamma_T\Gamma_L) \sim \mathcal{O}(1)$. On the other hand, such a cancellation may not happen for ΔE_T and ΔE_L, because they may have different signs. Therefore $(\Delta E_T - \Delta E_L)/E_p^V \lesssim \mathcal{O}(\epsilon)$ could be a first-order contribution. According to hydrodynamic simulation of the strong interaction matter in heavy-ion collisions, the thermal shear tensor $\xi \equiv |\xi_{\gamma\lambda}|$ is a small quantity of $\mathcal{O}(10^{-2})$, which can be treated as another power counting parameter. With Eq. (56) for spectral functions, one can prove that the term with the p_0 integral of $\delta G_<^{\mu\nu}(x,p)$ in the denominator of the right-hand-side of Eq. (45) is of the order $\xi E_p^V/\Gamma_{T/L} \sim \xi/\epsilon$, while the term with the p_0 integral of $G_<^{\mu\nu}(x,p)$ is $\mathcal{O}(1)$. In order

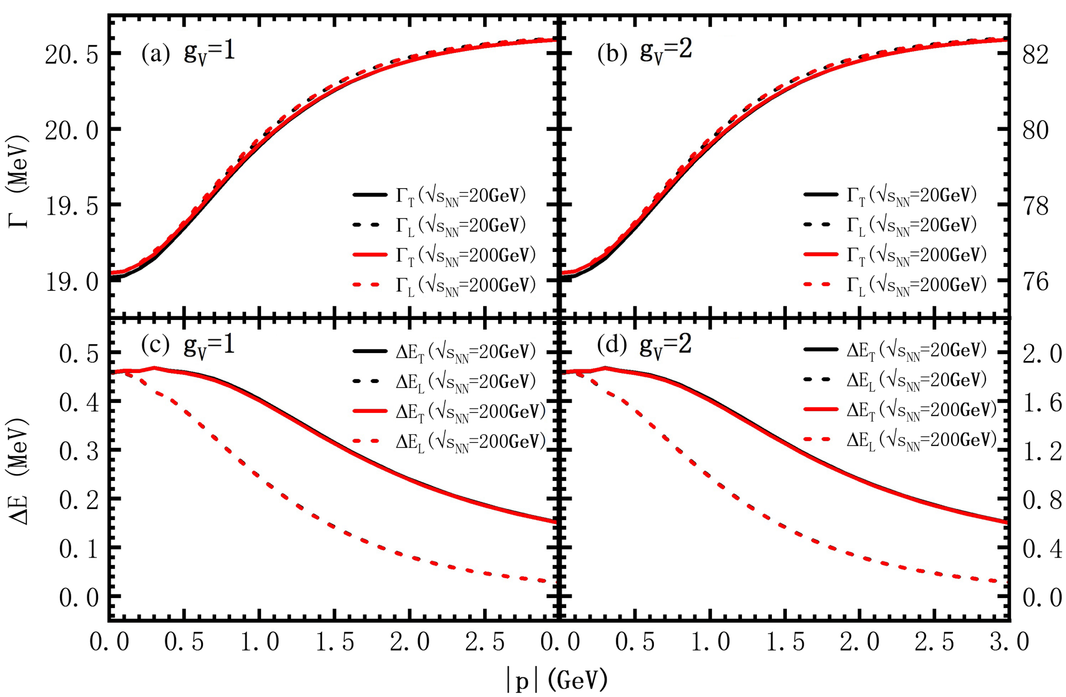

FIG. 3. The width Γ (a,b) and energy shift ΔE (c,d) for transverse (solid lines) and longitudinal (dashed lines) modes as functions of $|\mathbf{p}|$ at $g_V = 1$ (a,c) and $g_V = 2$ (b,d). Two sets of values are chosen for the s-quark chemical potential and temperature corresponding to the freezeout conditions at $\sqrt{s_{NN}} \approx 20$ GeV and 200 GeV: $\mu_s = 64.5$ MeV, $T = 155.7$ MeV (black) and $\mu_s = 7.4$ MeV and $T = 158.4$ MeV (red).

for the linear response theory to work, one has to require $\xi/\epsilon \ll 1$.

It is clear that the integrands in Eq. (45) are suppressed by spectral functions in the region of p^μ far from the mass-shell. Therefore we can make an approximation by expanding p_0 in the integrands around the on-shell energy E_p^V in powers of $\delta p_0 = p_0 - E_p^V$ except spectral functions. To the first order in δp_0, the p_0 integral of $G_<^{\mu\nu}(p)$ gives

$$\int_0^{+\infty} dp_0 G_<^{\mu\nu}(p) = -2\sum_{a=T,L}\int_0^{+\infty} dp_0 \rho_a^{\rm cut}(p)\left(1+\delta p_0\frac{\partial}{\partial E_p^V}\right) \times [n_B(p_{\rm on})\Delta_a^{\mu\nu}(p_{\rm on})], \tag{59}$$

while the p_0 integral of $\delta G_<^{\mu\nu}(p)$ from the linear response to the shear tensor gives

$$\int_0^{+\infty} dp_0 \delta G_<^{\mu\nu}(x,p) = 2T\xi_{\gamma\lambda}\sum_{a,b=L,T}\int_0^{+\infty} dp_0 \rho_a^{\rm cut}(p)\rho_b^{\rm cut}(p) \times \left(1+\delta p_0\frac{\partial}{\partial E_p^V}\right)\left[\frac{\partial n_B(p_{\rm on})}{\partial E_p^V} I_{ab}^{\mu\nu\gamma\lambda}(p_{\rm on},p_{\rm on})\right]. \tag{60}$$

Detailed calculations for the integrand in Eq. (60) are given in Appendix C. The integrals over p_0 in Eqs. (59) and (60) can be completed and the results are listed in Appendix D. Then $\delta\rho_{00}(\mathbf{p})$ is calculated by substituting Eqs. (59) and (60) into Eq. (45). Up to linear order in ϵ or ξ, the result reads

$$\begin{aligned}\delta\rho_{00}(\mathbf{p}) \approx{}& -\frac{1}{3}[1+n_B(E_p^V)]\Big\{-L_{\mu\nu}(p_{\rm on})\Delta_T^{\mu\nu}(p_{\rm on})C_0(\mathbf{p}) + \xi_{\gamma\lambda}L_{\mu\nu}(p_{\rm on})\Delta_T^{\mu\nu}(p_{\rm on}) \\ &\times\left[\frac{p_{\rm on}^\gamma p_{\rm on}^\lambda}{(E_p^V)^2}C_1(\mathbf{p}) + \frac{g^{\lambda0}p_{\rm on}^\gamma + g^{\gamma0}p_{\rm on}^\lambda - E_p^V g^{\gamma\lambda}}{2E_p^V}(C_T(\mathbf{p})-C_L(\mathbf{p}))\right] \\ &+\xi_{\gamma\lambda}L_{\mu\nu}(p_{\rm on})[\Delta_T^{\gamma\nu}(p_{\rm on})\Delta_L^{\lambda\mu}(p_{\rm on}) + \Delta_L^{\gamma\nu}(p_{\rm on})\Delta_T^{\lambda\mu}(p_{\rm on})]C_2(\mathbf{p}) \\ &+\xi_{\gamma\lambda}L_{\mu\nu}(p_{\rm on})[\Delta_L^{\gamma\nu}(p_{\rm on})\Delta_L^{\lambda\mu}(p_{\rm on})C_L(\mathbf{p}) + \Delta_T^{\gamma\nu}(p_{\rm on})\Delta_T^{\lambda\mu}(p_{\rm on})C_T(\mathbf{p})]\Big\} + \mathcal{O}(\epsilon^2)\end{aligned} \tag{61}$$

where the dimensionless coefficients are defined as

$$\begin{aligned}C_0 &= \frac{1+n_B(E_p^V)+T/E_p^V}{1+n_B(E_p^V)}\frac{\Delta E_T-\Delta E_L}{T}, \\ C_1 &= \frac{(E_p^V)^2}{m_V}\left(\frac{1}{\Gamma_T}-\frac{1}{\Gamma_L}\right) + n_B(E_p^V)\frac{(E_p^V)^2}{m_V T}\left(\frac{\Delta E_L}{\Gamma_L}-\frac{\Delta E_T}{\Gamma_T}\right), \\ C_2 &= \frac{4m_V E_p^V(\Gamma_L\Delta E_T+\Gamma_T\Delta E_L)}{4(E_p^V)^2(\Delta E_T-\Delta E_L)^2+m_V^2(\Gamma_L+\Gamma_T)^2}, \\ C_{T/L} &= \frac{2E_p^V\Delta E_{T/L}}{m_V\Gamma_{T/L}}.\end{aligned} \tag{62}$$

Noting that ρ_{00} could deviate from $1/3$ due to a nonzero C_0 independent of the shear tensor. Such a deviation arises from the possible difference between spectral functions for transverse and longitudinal modes [81]. In the power counting scheme, we can check that $C_0 \sim \mathcal{O}(\epsilon)$ and other coefficients C_i with $i = 1,2,T,L$ are all $\mathcal{O}(1)$. The numerical results show that $C_0 \sim \mathcal{O}(10^{-3})$ and other coefficients C_i with $i = 1,2,T,L$ are $\mathcal{O}(10^{-1}\sim 10^{-2})$ for $g_V = 1,\ 2$. The dominant term that is proportional to the shear tensor is the C_1 term, which is controlled by $1/\Gamma_T - 1/\Gamma_L$ for the current values of g_V.

2. Numerical results

In this subsection we will numerically calculate spectral functions and $\delta\rho_{00}$ using Eqs. (38) and (55). We will compare numerical results with the QPA results using Eq. (61). The parameters are set to the same values as in Sec. VII B 1. We can express $\delta\rho_{00}(\mathbf{p})$ as

$$\delta\rho_{00}(\mathbf{p}) = \delta\rho_{00}^{(\xi=0)}(\mathbf{p}) + \xi_{\mu\nu}C^{\mu\nu}(\mathbf{p}), \tag{63}$$

where $C^{\mu\nu}$ are dimensionless constants.

The numerical results for $\delta\rho_{00}^{(\xi=0)}(\mathbf{p})$ are shown in Fig. 4. The QPA results using Eq. (61) are shown for comparison. We choose two configurations for the mometum direction with respect to the spin quantization one: transverse or parallel configuration. The analytic results using Eq. (61) are also shown for comparison. The results of the configuration with an arbitrary angle are between these two limits. We see that the magnitude of $\delta\rho_{00}^{(\xi=0)}(\mathbf{p})$ is about 10^{-3} for the values of parameters we choose.

The numerical results for the tensor coefficient $C^{\mu\nu}(\mathbf{p})$ are shown in Figs. 5 and 6 for transverse and parallel configurations respectively. The QPA results using Eq. (61) are shown for comparison. We see that the magnitude of $C^{\mu\nu}(\mathbf{p})$ is about $10^{-2}\sim 10^{-3}$ for the values of parameters

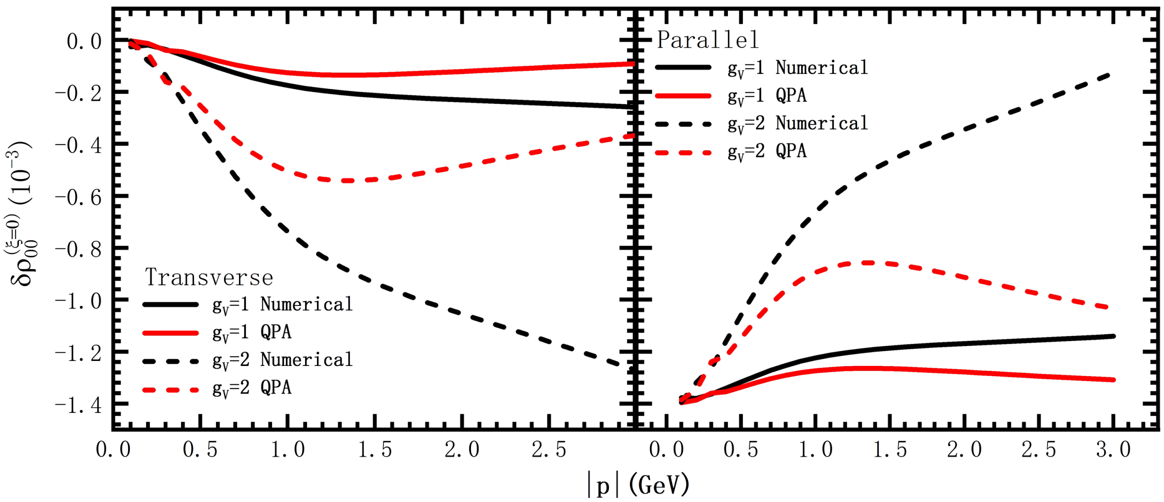

FIG. 4. The numerical results for $\delta\rho_{00}^{(\xi=0)}$ in Eq. (63) for the transverse (left) and parallel (right) configurations in which the momentum is transverse and parallel to the spin quantization direction z respectively. The results under the quasiparticle approximation (QPA) using Eq. (61) are shown for comparison.

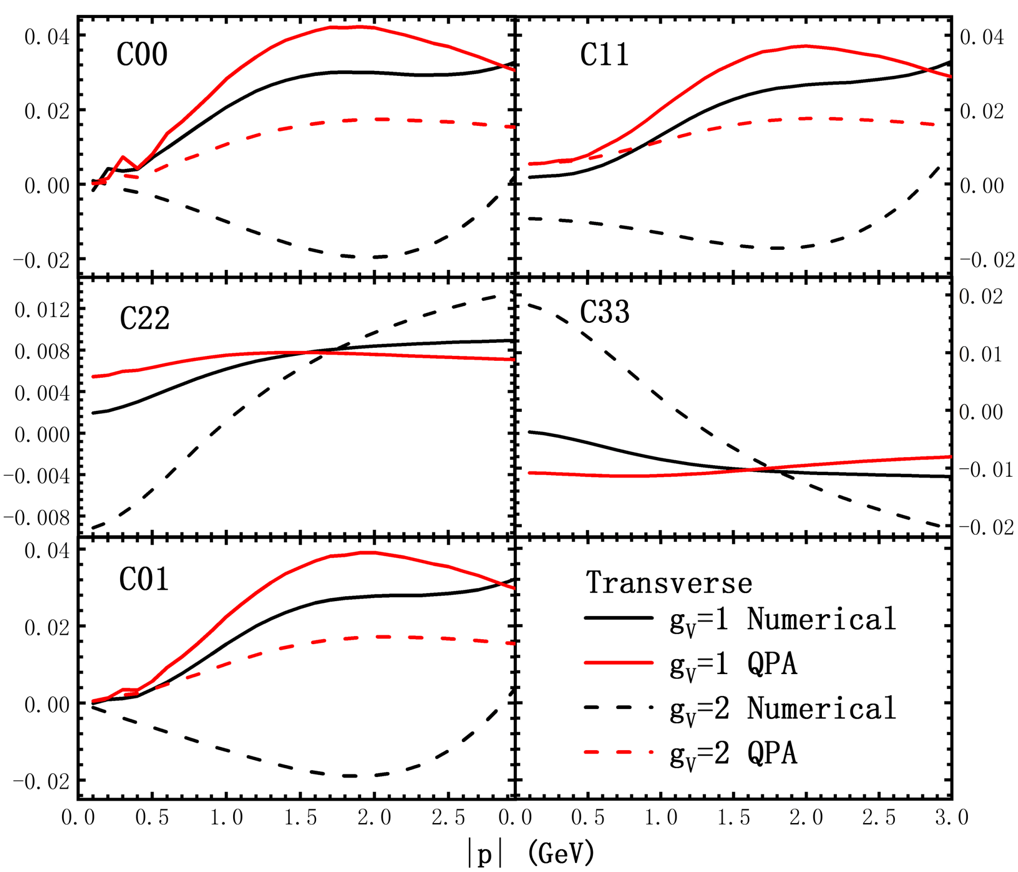

FIG. 5. The numerical results for $C^{\mu\nu}$ in Eq. (63) for the transverse configuration in which the momentum is perpendicular to the spin quantization direction z. The results under the quasiparticle approximation (QPA) using Eq. (61) are shown for comparison.

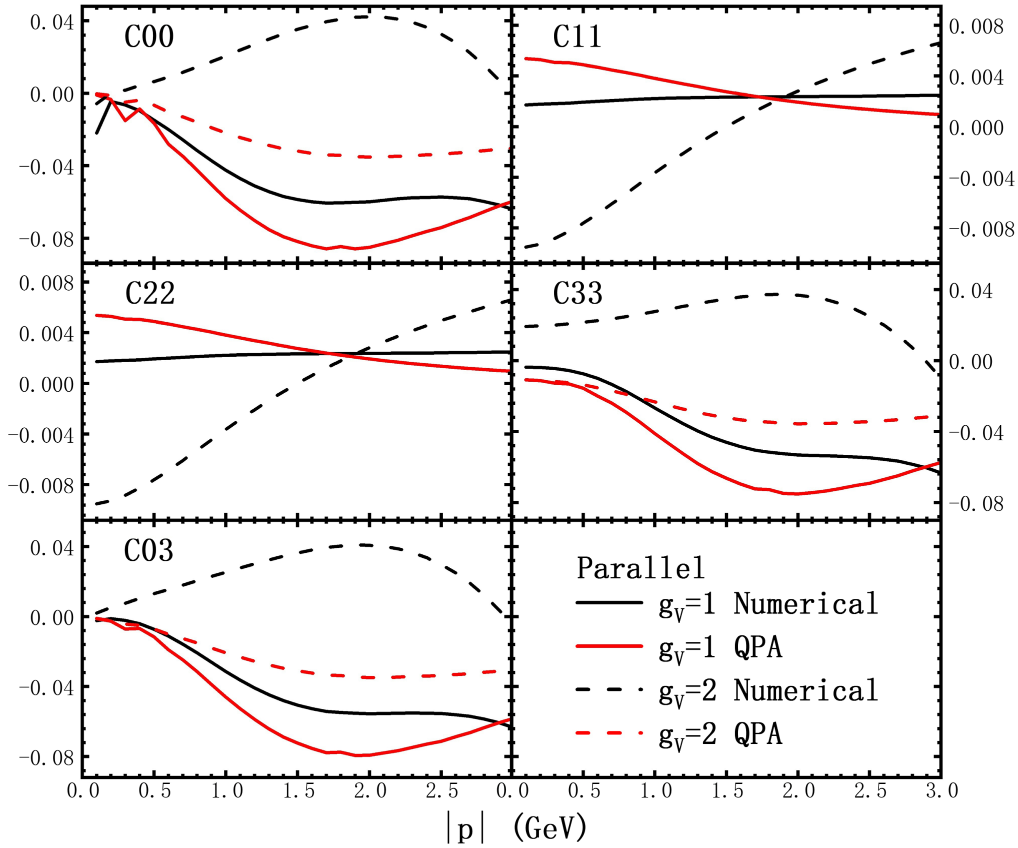

FIG. 6. The numerical results for $C^{\mu\nu}$ in Eq. (63) for the parallel configuration for the spin quantization and momentum directions. The spin quantization is chosen to be in the z direction. The results under the quasiparticle approximation (QPA) using Eq. (61) are shown for comparison.

we choose, which is consistent with the result of Ref. [47] in the order of magnitude.

VIII. DISCUSSION AND CONCLUSION

We study thermal medium effects for the spin alignment of vector mesons from the meson-quark interaction in the thermalized QGP, in which quarks, antiquarks and vector mesons are assumed to be thermalized. Quarks and antiquarks are also assumed to be unpolarized. We calculate the retarded self-energy of the vector meson from the quark loop. The spectral function can be obtained from the retarded two-point Green's function including the contribution of the retarded self-energy. Other types of two-point Green's functions with interaction can all be expressed in spectral functions. Then we calculate the linear response of the two-point Green's function to the thermal shear tensor using the Kubo formula, which provides a correction to the Green's function. Such an effect is caused by interaction.

Finally the correction to ρ_{00} can be expressed in terms of spectral functions through one-loop self-energies. In order to obtain an analytical formula for the correction to ρ_{00}, we take the quasiparticle approximation: (a) the energy shifts and widths from real and imaginary parts of self-energies are much smaller than energies of vector mesons; (b) the difference between widths for transverse and longitudinal modes is much smaller than widths themselves. This approximation is supported by numerical results with the parameters we have chosen. Under this approximation we derive an analytical formula for the correction to ρ_{00} to the linear order in the expansion parameter in terms of energy shifts and widths. The numerical results show that dimensionless coefficients of the thermal shear tensor are of $\mathcal{O}(10^{-2} \sim 10^{-3})$ for the chosen values of quark-meson coupling constant. The magnitude of the contribution from the thermal shear tensor to ρ_{00} is then $\mathcal{O}(10^{-4} \sim 10^{-5})$ if the thermal shear tensor is $\mathcal{O}(10^{-2})$.

Our results are based the one-loop self-energy with meson-quark interaction in the QGP. One can also consider other interactions, such as $\rho\pi\pi$ or ϕKK couplings, in the nuclear matter [81–83].

ACKNOWLEDGMENTS

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APPENDIX A: QUARK PROPAGATORS

The propagators for unpolarized quarks at the leading order are given by

$$\begin{aligned}
S^{(0)<}_{\alpha\beta}(x,p) &= (p\cdot\gamma+m_q)_{\alpha\beta}\frac{\pi}{E_p}\{-\delta(p_0-E_p)f^{(+)}_{FD}(E_p)+\delta(p_0+E_p)[1-f^{(-)}_{FD}(E_p)]\},\\
S^{(0)>}_{\alpha\beta}(x,p) &= (p\cdot\gamma+m_q)_{\alpha\beta}\frac{\pi}{E_p}\{\delta(p_0-E_p)[1-f^{(+)}_{FD}(E_p)]-\delta(p_0+E_p)f^{(-)}_{FD}(E_p)\},\\
S^{(0)F}_{\alpha\beta}(x,p) &= \frac{i(p\cdot\gamma+m_q)_{\alpha\beta}}{p^2-m_q^2+i\epsilon}-\frac{\pi}{E_p}(p\cdot\gamma+m_q)_{\alpha\beta}[\delta(p_0-E_p)f^{(+)}_{FD}(E_p)+\delta(p_0+E_p)f^{(-)}_{FD}(E_p)],\\
S^{(0)A}_{\alpha\beta}(x,p) &= \frac{i(p\cdot\gamma+m_q)_{\alpha\beta}}{p^2-m_q^2-ip_0\epsilon},\\
S^{(0)R}_{\alpha\beta}(x,p) &= \frac{i(p\cdot\gamma+m_q)_{\alpha\beta}}{p^2-m_q^2+ip_0\epsilon},
\end{aligned} \tag{A1}$$

where ϵ is a small positive number, $m_q = m_{\bar{q}}$ is the quark mass, and

$$f^{\pm}_{FD}(E_k) = \frac{1}{\exp(\beta E_k - \beta\mu_{q/\bar{q}})+1}, \tag{A2}$$

are Fermi-Dirac distributions for quarks/antiquarks as functions of the energy $E_k = \sqrt{|\mathbf{k}|^2+m_q^2}$ and chemical potentials $\mu_{q/\bar{q}}$.

APPENDIX B: RETARDED SELF-ENERGY

We evaluate the retarded self-energy in Eq. (26) using quark propagators in (A1). The result reads

$$\begin{aligned}
\Sigma^{\mu\nu}_R(p) &= g_V^2\int\frac{d^4k}{(2\pi)^4}\{\mathrm{Tr}[\gamma^\mu S_F(k)\gamma^\nu S_F(k-p)]-\mathrm{Tr}[\gamma^\mu S_<(k)\gamma^\nu S_>(k-p)]\}\\
&= -g_V^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\{\gamma^\mu(k\cdot\gamma+m_q)\gamma^\nu[(k-p)\cdot\gamma+m_q]\}\frac{i}{(k-p)^2-m_q^2-i(k_0-p_0)\epsilon}\\
&\quad\times\frac{\pi}{E_k}\{\delta(k_0-E_k)f^{(+)}_{FD}(E_k)+\delta(k_0+E_k)f^{(-)}_{FD}(E_k)\}+(\mu\leftrightarrow\nu, p\to -p, \epsilon\to-\epsilon)\\
&\quad+g_V^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\{\gamma^\mu(k\cdot\gamma+m_q)\gamma^\nu[(k-p)\cdot\gamma+m_q]\}\\
&\quad\times\left[\frac{i}{k^2-m_q^2+i\epsilon}\cdot\frac{i}{(k-p)^2-m_q^2+i\epsilon}-\frac{\pi^2}{E_kE_{k-p}}\delta(k^0+E_k)\delta(k^0-p^0-E_{k-p})\right]\\
&= -ig_V^2\frac{1}{4\pi^3}(2I_1^{\mu\nu}+I_2^{\mu\nu})-ig_V^2I^{\mu\nu}_{\mathrm{vac}},
\end{aligned} \tag{B1}$$

where $I_1^{\mu\nu}$, $I_2^{\mu\nu}$, and $I^{\mu\nu}_{\mathrm{vac}}$ are defined as

$$\begin{aligned}
I_1^{\mu\nu} &= \int d^3k k^\mu_{\mathrm{on}}k^\nu_{\mathrm{on}}\frac{1}{E_k}[f^{(+)}_{FD}(E_k)+f^{(-)}_{FD}(E_k)]\left[\frac{1}{(k_{\mathrm{on}}+p)^2-m_q^2+i(E_k+p_0)\epsilon}+\frac{1}{(k_{\mathrm{on}}-p)^2-m_q^2-i(E_k-p_0)\epsilon}\right],\\
I_2^{\mu\nu} &= \int d^3k\frac{1}{E_k}[p^\mu k^\nu_{\mathrm{on}}+p^\nu k^\mu_{\mathrm{on}}-g^{\mu\nu}(p\cdot k_{\mathrm{on}})][f^{(+)}_{FD}(E_k)+f^{(-)}_{FD}(E_k)]\\
&\quad\times\left[\frac{1}{(k_{\mathrm{on}}+p)^2-m_q^2+i(E_k+p_0)\epsilon}-\frac{1}{(k_{\mathrm{on}}-p)^2-m_q^2-i(E_k-p_0)\epsilon}\right],
\end{aligned} \tag{B2}$$

$$I^{\mu\nu}_{\text{vac}} = i\int \frac{d^4k}{(2\pi)^4}\text{Tr}[\gamma^\mu(k\cdot\gamma+m_q)\gamma^\nu((k-p)\cdot\gamma+m_q)] \times\left[\frac{i}{k^2-m_q^2+i\epsilon}\cdot\frac{i}{(k-p)^2-m_q^2+i\epsilon}-\frac{\pi^2}{E_kE_{k-p}}\delta(k^0+E_k)\delta(k^0-p^0-E_{k-p})\right], \tag{B3}$$

where $k^\mu_{\text{on}} = (E_k, \mathbf{k})$ is an on-shell momentum for the quark or antiquark. The tensors $I^{\mu\nu}_1$ and $I^{\mu\nu}_2$ can be expressed in special functions $J_\pm(p; n_1, n_2)$ and $J_0(n_1, n_2)$ defined as

$$\begin{aligned} J_\pm(p;n_1,n_2) &\equiv \int_0^\infty d|\mathbf{k}|E_k^{n_1}|\mathbf{k}|^{n_2}[f^{(+)}_{FD}(E_k)+f^{(-)}_{FD}(E_k)]\ln\frac{p^2\pm 2p_0E_k+2|\mathbf{k}||\mathbf{p}|\pm i(E_k\pm p_0)\epsilon}{p^2\pm 2p_0E_k-2|\mathbf{k}||\mathbf{p}|\pm i(E_k\pm p_0)\epsilon}, \\ J_0(n_1,n_2) &\equiv \int_0^\infty d|\mathbf{k}|E_k^{n_1}|\mathbf{k}|^{n_2}[f^{(+)}_{FD}(E_k)+f^{(-)}_{FD}(E_k)]. \end{aligned} \tag{B4}$$

We now evaluate each element of $I^{\mu\nu}_1$ separately. The result for I^{00}_1 is

$$\begin{aligned} I^{00}_1 &= 2\pi\int d|\mathbf{k}||\mathbf{k}|^2E_k[f^{(+)}_{FD}(E_k)+f^{(-)}_{FD}(E_k)]\int_{-1}^1 d\cos\theta \\ &\quad\times\left[\frac{1}{p^2+2p_0E_k-2|\mathbf{k}||\mathbf{p}|\cos\theta+i(E_k+p_0)\epsilon}+\frac{1}{p^2-2p_0E_k+2|\mathbf{k}||\mathbf{p}|\cos\theta-i(E_k-p_0)\epsilon}\right] \\ &= \pi\frac{1}{|\mathbf{p}|}[J_+(p;1,1)+J_-(p;1,1)]. \end{aligned} \tag{B5}$$

In evaluating I^{0i}, we decompose the vector $\mathbf{k}$ into the component parallel and perpendicular to $\mathbf{p}$ as $\mathbf{k} = \hat{\mathbf{p}}(\mathbf{k}\cdot\mathbf{p}) + \mathbf{k}_T$ with $\mathbf{k}_T\cdot\mathbf{p} = 0$. The integral over the component perpendicular to $\mathbf{p}$ vanishes. The result for I^{0i}_1 is

$$\begin{aligned} I^{0i}_1 &= I^{i0}_1 \\ &= 2\pi\hat{\mathbf{p}}_i\int d|\mathbf{k}|d\theta\sin\theta\cos\theta|\mathbf{k}|^3[f^{(+)}_{FD}(E_k)+f^{(-)}_{FD}(E_k)] \\ &\quad\times\left[\frac{1}{p^2+2p_0E_k-2|\mathbf{k}||\mathbf{p}|\cos\theta+i(E_k+p_0)\epsilon}+\frac{1}{p^2-2p_0E_k+2|\mathbf{k}||\mathbf{p}|\cos\theta-i(E_k-p_0)\epsilon}\right] \\ &= \frac{\pi p^2}{2|\mathbf{p}|^2}\hat{\mathbf{p}}_i[J_+(p;0,1)-J_-(p;0,1)]+\frac{\pi p_0}{|\mathbf{p}|^2}\hat{\mathbf{p}}_i[J_+(p;1,1)+J_-(p;1,1)]. \end{aligned} \tag{B6}$$

To evaluate I^{ij}, we notice that it is symmetric in i and j, so we can decomposes it into components proportional to $\hat{\mathbf{p}}_i\hat{\mathbf{p}}_j$ and $\delta_{ij}-\hat{\mathbf{p}}_i\hat{\mathbf{p}}_j$ using

$$\mathbf{k}_i\mathbf{k}_j \to \hat{\mathbf{p}}_i\hat{\mathbf{p}}_j(\mathbf{k}\cdot\mathbf{p})+\frac{1}{2}\mathbf{k}_T^2(\delta_{ij}-\hat{\mathbf{p}}_i\hat{\mathbf{p}}_j). \tag{B7}$$

Then we obtain the result for I^{ij} as

$$
\begin{aligned}
I_1^{ij} &= 2\pi\hat{\mathbf{p}}_i\hat{\mathbf{p}}_j\int d|\mathbf{k}|\frac{1}{E_k}|\mathbf{k}|^4\int d\theta\sin\theta\cos^2\theta[f_{FD}^{(+)}(E_k)+f_{FD}^{(-)}(E_k)]\\
&\quad\times\left[\frac{1}{p^2+2p_0E_k-2\mathbf{k}\cdot\mathbf{p}+i(E_k+p_0)\epsilon}+\frac{1}{p^2-2p_0E_k+2\mathbf{k}\cdot\mathbf{p}-i(E_k-p_0)\epsilon}\right]+\pi(\delta_{ij}-\hat{\mathbf{p}}_i\hat{\mathbf{p}}_j)\int d|\mathbf{k}|\frac{1}{E_k}|\mathbf{k}|^4\\
&\quad\times\int d\theta\,\sin\theta\sin^2\theta[f_{FD}^{(+)}(E_k)+f_{FD}^{(-)}(E_k)]\left[\frac{1}{p^2+2p_0E_k-2\mathbf{k}\cdot\mathbf{p}+i(E_k+p_0)\epsilon}+\frac{1}{p^2-2p_0E_k+2\mathbf{k}\cdot\mathbf{p}-i(E_k-p_0)\epsilon}\right]\\
&= \hat{\mathbf{p}}_i\hat{\mathbf{p}}_j\frac{\pi}{4|\mathbf{p}|^3}\{-8p^2|\mathbf{p}|J_0(-1,2)+p^4[J_+(p;-1,1)+J_-(p;-1,1)]\\
&\quad+4p_0^2[J_+(p;1,1)+J_-(p;1,1)]+4p_0p^2[J_+(p;0,1)-J_-(p;0,1)]\}\\
&\quad+(\delta_{ij}-\hat{\mathbf{p}}_i\hat{\mathbf{p}}_j)\frac{\pi}{8|\mathbf{p}|^3}\{8p^2|\mathbf{p}|J_0(-1,2)-p^4[J_+(p;-1,1)+J_-(p;-1,1)]\\
&\quad-4p_0^2[J_+(p;1,1)+J_-(p;1,1)]-4p_0p^2[J_+(p;0,1)-J_-(p;0,1)]+4|\mathbf{p}|^2[J_+(p;-1,3)+J_-(p;-1,3)]\}. \qquad \text{(B8)}
\end{aligned}
$$

In Eqs. (B5), (B6) and (B8) we have express the result of $I_1^{\mu\nu}$ in terms of special functions $J_\pm(p;n_1,n_2)$ and $J_0(n_1,n_2)$ in Eq. (B4).

The derivation of $I_2^{\mu\nu}$ is similar and straightforward. Here we just list the result as follows

$$
\begin{aligned}
I_2^{00} &= -4\pi J_0(-1,2)+2\pi\frac{p_0}{|\mathbf{p}|}[J_+(p;0,1)-J_-(p;0,1)]+\frac{\pi p^2}{2|\mathbf{p}|}[J_+(p;-1,1)+J_-(p;-1,1)],\\
I_2^{0i} &= I_2^{i0} = -4\pi\hat{\mathbf{p}}_i\frac{p_0}{|\mathbf{p}|}J_0(-1,2)+\frac{\pi}{2}\hat{\mathbf{p}}_i\frac{p_0p^2}{|\mathbf{p}|^2}[J_+(p;-1,1)+J_-(p;-1,1)]+\pi\left(1+\frac{p_0^2}{|\mathbf{p}|^2}\right)\hat{\mathbf{p}}_i[J_+(p;0,1)-J_-(p;0,1)],\\
I_2^{ij} &= \hat{\mathbf{p}}_i\hat{\mathbf{p}}_j\left\{-4\pi J_0(-1,2)+\frac{\pi}{2}\frac{p^2}{|\mathbf{p}|}[J_+(p;-1,1)+J_-(p;-1,1)]+2\pi\frac{p_0}{|\mathbf{p}|}[J_+(p;0,1)-J_-(p;0,1)]\right\}\\
&\quad+(\delta_{ij}-\hat{\mathbf{p}}_i\hat{\mathbf{p}}_j)\left\{4\pi J_0(-1,2)-\frac{\pi}{2}\frac{p^2}{|\mathbf{p}|}[J_+(p;-1,1)+J_-(p;-1,1)]\right\}. \qquad \text{(B9)}
\end{aligned}
$$

Using the results for elements of $I_1^{\mu\nu}$ and $I_2^{\mu\nu}$, we obtain the elements of $2I_1^{\mu\nu}+I_2^{\mu\nu}$ in Eq. (B1),

$$
\begin{aligned}
2I_1^{00}+I_2^{00} &= -4\pi J_0(-1,2)+2\pi\frac{1}{|\mathbf{p}|}[J_+(p;1,1)+J_-(p;1,1)]+2\pi\frac{p_0}{|\mathbf{p}|}[J_+(p;0,1)-J_-(p;0,1)]\\
&\quad+\frac{\pi p^2}{2|\mathbf{p}|}[J_+(p;-1,1)+J_-(p;-1,1)],\\
2I_1^{0i}+I_2^{0i} &= \hat{\mathbf{p}}_i\frac{p_0}{|\mathbf{p}|}(2I_1^{00}+I_2^{00})\\
2I_1^{ij}+I_2^{ij} &= \hat{\mathbf{p}}_i\hat{\mathbf{p}}_j\frac{p_0^2}{|\mathbf{p}|^2}(2I_1^{00}+I_2^{00})+(\delta_{ij}-\hat{\mathbf{p}}_i\hat{\mathbf{p}}_j)\frac{\pi}{4|\mathbf{p}|^3}\{8(p_0^2+|\mathbf{p}|^2)|\mathbf{p}|J_0(-1,2)\\
&\quad-p^2(p_0^2+|\mathbf{p}|^2)[J_+(p;-1,1)+J_-(p;-1,1)]-4p_0^2[J_+(p;1,1)+J_-(p;1,1)]\\
&\quad-4p_0p^2[J_+(p;0,1)-J_-(p;0,1)]+4|\mathbf{p}|^2[J_+(p;-1,3)+J_-(p;-1,3)]\}. \qquad \text{(B10)}
\end{aligned}
$$

The vacuum contribution $I_{\mathrm{vac}}^{\mu\nu}$ for $p^0>0$ can be evaluated by dimensional regularization as

$$
\begin{aligned}
I_{\mathrm{vac}}^{\mu\nu} &= i\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\{\gamma^\mu(k\cdot\gamma+m_q)\gamma^\nu[(k-p)\cdot\gamma+m_q]\}\frac{i}{k^2-m_q^2+i\epsilon}\cdot\frac{i}{(k-p)^2-m_q^2+i\epsilon}\\
&= (g^{\mu\nu}p^2-p^\mu p^\nu)\frac{1}{2\pi^2}\int_0^1 dxx(1-x)\left[\frac{2}{\epsilon}-\log(-x(1-x)p^2+m_q^2-i0^+)-\gamma+\log(4\pi)+\mathcal{O}(\epsilon)\right]\\
&= p^2\Delta^{\mu\nu}\frac{1}{2\pi^2}\int_0^1 dxx(1-x)\left[\frac{2}{\epsilon}-\log|-x(1-x)p^2+m_q^2|-\gamma+\log(4\pi)+\mathcal{O}(\epsilon)\right]\\
&\quad+ip^2\Delta^{\mu\nu}\frac{1}{2\pi^2}\int_0^1 dxx(1-x)\pi\theta[x(1-x)p^2-m_q^2], \qquad \text{(B11)}
\end{aligned}
$$

where $\epsilon = 4 - d$ (d is an arbitrary space-time dimension in regularization) and $\gamma \approx 0.5772$. Here the term proportional to delta functions in $I^{\mu\nu}_{\rm vac}$ is vanishing. The real part of $I^{\mu\nu}_{\rm vac}$ can be canceled by introducing a renormalization term with the condition $\mathrm{Re} I^{\mu\nu}_{\rm vac}(p^2 = m_V^2) = 0$. The imaginary part $I^{\mu\nu}_{\rm vac}$ is nonzero when $\sqrt{p^2} > 2m_q$ and contributes to the spectral density. This corresponds to pair production or annihilation processes.

APPENDIX C: EXPANSION OF PART OF INTEGRAND IN EQ. (44) AT MASS-SHELL

In this appendix, we will expand p_0 around E_p^V in powers of δp_0 for the integrand in the second term of Eq. (44) except spectral functions. The integrand can be written as

$$I^{\mu\nu\gamma\lambda} = \frac{\partial n_B(p_0)}{\partial p_0}\rho_a(p)\rho_b(p)I^{\mu\nu\gamma\lambda}_{ab}(p,p) = \rho_a(p)\rho_b(p)\left(1+\delta p_0\frac{\partial}{\partial E_p^V}\right)\left[\frac{\partial n_B(p_{\rm on})}{\partial p_{\rm on}}I^{\mu\nu\gamma\lambda}_{ab}(p_{\rm on},p_{\rm on})\right] \tag{C1}$$

where a summation over $a, b = L, T$ is implied.

First we expand $I^{\mu\nu\gamma\lambda}_{ab}(p,p)$ in (43) at $p_0 = E_p^V$. We can express $p^\mu = p^\mu_{\rm on} + \delta p_0 g^{\mu 0}$, where $\delta p_0 = p_0 - E_p^V$. We note that $\Delta^{\mu\nu}_T(p)$ does not depend on p_0, but $\Delta^{\mu\nu}_L(p) = \Delta^{\mu\nu}(p) - \Delta^{\mu\nu}_T(p)$ depends on p_0 through $\Delta^{\mu\nu}$. Then to the first order in δp_0 they can be expanded as

$$\begin{aligned}
\Delta^{\mu\nu}_T(p) &= \Delta^{\mu\nu}_T(p_{\rm on}),\\
\Delta^{\mu\nu}(p) &\approx \Delta^{\mu\nu}(p_{\rm on}) + \frac{2E_p^V}{m_V^4}p^\mu_{\rm on}p^\nu_{\rm on}\delta p_0 - \frac{1}{m_V^2}(p^\mu_{\rm on}g^{\nu 0} + p^\nu_{\rm on}g^{\mu 0})\delta p_0,\\
\Delta^{\mu\nu}_L(p) &\approx \Delta^{\mu\nu}_L(p_{\rm on}) + \frac{2E_p^V}{m_V^4}p^\mu_{\rm on}p^\nu_{\rm on}\delta p_0 - \frac{1}{m_V^2}(p^\mu_{\rm on}g^{\nu 0} + p^\nu_{\rm on}g^{\mu 0})\delta p_0.
\end{aligned} \tag{C2}$$

Then $I^{\mu\nu\gamma\lambda}_{ab}(p,p)$ can be expanded to the leading order in δp_0 as

$$\begin{aligned}
I^{\mu\nu\gamma\lambda}_{ab}(p,p) &= 2p^\lambda p^\gamma \Delta^\nu_{a,\alpha}(p)\Delta^{\mu\alpha}_b(p) + (p^2 - m_V^2)[\Delta^{\gamma\nu}_a(p)\Delta^{\mu\lambda}_b(p) + \Delta^{\lambda\nu}_a(p)\Delta^{\mu\gamma}_b(p)] - g^{\gamma\lambda}g_{\beta\alpha}(p^2 - m_V^2)\Delta^{\alpha\nu}_a(p)\Delta^{\mu\beta}_b(p),\\
I^{\mu\nu\gamma\lambda}_{ab}(p_{\rm on},p_{\rm on}) &= 2p^\lambda_{\rm on}p^\gamma_{\rm on}\Delta^\nu_{a,\alpha}(p_{\rm on})\Delta^{\mu\alpha}_b(p_{\rm on}),\\
I^{\mu\nu\gamma\lambda}_{TT}(p,p) &\approx 2p^\lambda_{\rm on}p^\gamma_{\rm on}\Delta^{\mu\nu}_T(p) + 2(p^\lambda_{\rm on}g^{\gamma 0} + p^\gamma_{\rm on}g^{\lambda 0} - E_p^V g^{\gamma\lambda})\delta p_0\Delta^{\mu\nu}_T(p) + 2E_p^V\delta p_0[\Delta^{\gamma\nu}_T(p)\Delta^{\mu\lambda}_T(p) + \Delta^{\lambda\nu}_T(p)\Delta^{\mu\gamma}_T(p)],\\
I^{\mu\nu\gamma\lambda}_{TL}(p,p) &\approx 2E_p^V\delta p_0[\Delta^{\gamma\nu}_T(p)\Delta^{\mu\lambda}_L(p_{\rm on}) + \Delta^{\lambda\nu}_T(p)\Delta^{\mu\gamma}_L(p_{\rm on})],\\
I^{\mu\nu\gamma\lambda}_{LT}(p,p) &\approx 2E_p^V\delta p_0[\Delta^{\gamma\nu}_L(p_{\rm on})\Delta^{\mu\lambda}_T(p) + \Delta^{\lambda\nu}_L(p_{\rm on})\Delta^{\mu\gamma}_T(p)],\\
I^{\mu\nu\gamma\lambda}_{LL}(p,p) &= 2p^\lambda_{\rm on}p^\gamma_{\rm on}\Delta^{\mu\nu}_L(p_{\rm on}) + 4\frac{E_p^V}{m_V^4}p^\lambda_{\rm on}p^\gamma_{\rm on}p^\mu_{\rm on}p^\nu_{\rm on}\delta p_0 - 2\frac{1}{m_V^2}p^\lambda_{\rm on}p^\gamma_{\rm on}(p^\mu_{\rm on}g^{\nu 0} + p^\nu_{\rm on}g^{\mu 0})\delta p_0\\
&\quad + 2(p^\lambda_{\rm on}g^{\gamma 0} + p^\gamma_{\rm on}g^{\lambda 0} - E_p^V\eta^{\gamma\lambda})\delta p_0\Delta^{\mu\nu}_L(p_{\rm on}) + 2E_p^V\delta p_0[\Delta^{\gamma\nu}_L(p_{\rm on})\Delta^{\mu\lambda}_L(p_{\rm on}) + \Delta^{\lambda\nu}_L(p_{\rm on})\Delta^{\mu\gamma}_L(p_{\rm on})].
\end{aligned} \tag{C3}$$

The function $\partial n_B(p_0)/\partial p_0$ is expanded to the first order in δp_0 as

$$\begin{aligned}
\frac{\partial n_B(p_0)}{\partial p_0} &\approx \frac{\partial n_B(E_p^V)}{\partial E_p^V} + \frac{\partial^2 n_B(E_p^V)}{\partial^2 E_p^V}\delta p_0 + \mathcal{O}[(\delta p_0)^2]\\
&= -\beta n_B(E_p^V)[1 + n_B(E_p^V)] + \beta^2 n_B(E_p^V)[1 + n_B(E_p^V)][1 + 2n_B(E_p^V)]\delta p_0,
\end{aligned} \tag{C4}$$

To the first order in δp_0, the integrand is expanded as

$$\begin{aligned}
I^{\mu\nu\gamma\lambda}_{\rm LO} &= \rho_a(p)\rho_b(p)\frac{\partial n_B(E_p^V)}{\partial E_p^V}I^{\mu\nu\gamma\lambda}_{ab}(p_{\rm on},p_{\rm on})\\
&= 2p^\lambda_{\rm on}p^\gamma_{\rm on}\frac{\partial n_B(E_p^V)}{\partial E_p^V}\{\Delta^{\mu\nu}(p_{\rm on})\rho_L^2(p) + \Delta^{\mu\nu}_T(p_{\rm on})[\rho_T^2(p) - \rho_L^2(p)]\},
\end{aligned} \tag{C5}$$

$$
\begin{aligned}
I^{\mu\nu\gamma\lambda}_{\rm NLO} =& 2p^\lambda_{\rm on}p^\gamma_{\rm on}\frac{\partial^2 n_B(E^V_p)}{\partial^2 E^V_p}\delta p_0\{\Delta^{\mu\nu}(p_{\rm on})\rho^2_L(p)+\Delta^{\mu\nu}_T(p_{\rm on})[\rho^2_T(p)-\rho^2_L(p)]\}\\
&+2\delta p_0\rho^2_T(p)\frac{\partial n_B(E^V_p)}{\partial E^V_p}\{(p^\lambda_{\rm on}g^{\gamma 0}+p^\gamma_{\rm on}g^{\lambda 0}-E^V_p g^{\gamma\lambda})\Delta^{\mu\nu}_T(p)+E^V_p[\Delta^{\gamma\nu}_T(p)\Delta^{\mu\lambda}_T(p)+\Delta^{\lambda\nu}_T(p)\Delta^{\mu\gamma}_T(p)]\}\\
&+2\delta p_0\rho^2_L(p)\frac{\partial n_B(E^V_p)}{\partial E^V_p}\left\{2\frac{E^V_p}{m^4_V}p^\lambda_{\rm on}p^\gamma_{\rm on}p^\mu_{\rm on}p^\nu_{\rm on}-\frac{1}{m^2_V}p^\lambda_{\rm on}p^\gamma_{\rm on}(p^\mu_{\rm on}g^{\nu 0}+p^\nu_{\rm on}g^{\mu 0})+\Delta^{\mu\nu}_L(p_{\rm on})(p^\lambda_{\rm on}g^{\gamma 0}+p^\gamma_{\rm on}g^{\lambda 0}-E^V_p g^{\gamma\lambda})\right.\\
&\left.+E^V_p[\Delta^{\gamma\nu}_L(p_{\rm on})\Delta^{\mu\lambda}_L(p_{\rm on})+\Delta^{\lambda\nu}_L(p_{\rm on})\Delta^{\mu\gamma}_L(p_{\rm on})]\right\}\\
&+2E^V_p\delta p_0\rho_T(p)\rho_L(p)\frac{\partial n_B(E^V_p)}{\partial E^V_p}[\Delta^{\gamma\nu}_T(p)\Delta^{\mu\lambda}_L(p_{\rm on})+\Delta^{\lambda\nu}_T(p)\Delta^{\mu\gamma}_L(p_{\rm on})+\Delta^{\mu\lambda}_T(p)\Delta^{\gamma\nu}_L(p_{\rm on})+\Delta^{\mu\gamma}_T(p)\Delta^{\lambda\nu}_L(p_{\rm on})]. \quad \text{(C6)}
\end{aligned}
$$

APPENDIX D: INTEGRALS FOR SPECTRAL FUNCTIONS

The integrals in Eqs. (59) and (60) are given as follows

$$
\begin{aligned}
\int_0^\infty dp_0\rho_{L/T}(p) &\approx \frac{\pi}{2E^V_p}\left(1-\frac{\Delta E_{L/T}}{E^V_p}\right)+\mathcal{O}(\epsilon^2)\\
\int_0^\infty dp_0\delta p_0\rho_{L/T}(p) &\approx \frac{\pi}{2}\frac{\Delta E_{L/T}}{E^V_p}+\mathcal{O}(\epsilon^2)\\
\int_0^\infty dp_0\rho^2_{L/T}(p) &\approx \frac{\pi}{4E_p m_V\Gamma_{L/T}}\left(1-\frac{\Delta E_{L/T}}{E_p}\right)+\mathcal{O}(\epsilon)\\
\int_0^\infty dp_0\delta p_0\rho^2_{L/T}(p) &\approx \frac{\pi\Delta E_{L/T}}{4E_p m_V\Gamma_{L/T}}+\mathcal{O}(\epsilon)\\
\int_0^\infty dp_0\delta p_0\rho_L(p)\rho_T(p) &\approx \frac{\pi}{2E_p}\frac{m_V(\Gamma_L\Delta E_T+\Gamma_T\Delta E_L)}{[4E^2_p(\Delta E_L-\Delta E_T)^2+m^2_V(\Gamma_T+\Gamma_L)^2]}+\mathcal{O}(\epsilon) \quad \text{(D1)}
\end{aligned}
$$

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