95 GeV excess in a *CP*-violating μ-from-ν SSM

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The CMS and ATLAS Collaborations have recently reported their results searching for a light Higgs boson with mass around 95 GeV, based on the full run 2 dataset. In the framework of the *CP*-violating (CPV) μ-from-ν supersymmetric standard model (SSM), we discuss an $\sim 2.9\sigma$ (local) excess at 95 GeV in the light Higgs boson search in the diphoton decay mode as reported by ATLAS and CMS, together with an $\sim 2\sigma$ excess (local) in the $b\bar{b}$ final state at LEP in the same mass range. By introducing CPV phases as well as by mixing *CP*-even Higgs and *CP*-odd Higgs, a lighter Higgs boson in the μ-from-ν SSM can be produced, which can account for the "diphoton excess."

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I. INTRODUCTION

In 2012, the 125 GeV Higgs boson was discovered by the Large Hadron Collider (LHC) [1,2]; the measured mass of the Higgs boson now is [3] $m_h = 125.25 \pm 0.17$ GeV. The discovery of the Higgs boson marked a huge success for the standard model (SM), but it did not stop the search for new physics (NP) at the LHC, and one of them was the search for lighter scalar particles.

Searches for a lighter Higgs have been performed at the LEP [4–6], the Tevatron [7], and the LHC [8–16]. Interestingly, the excesses observed by CMS and LEP occurred at a similar mass range [17]. CMS has performed searches for scalar diphoton resonances at 8 and 13 TeV [16,17]; based on the 8 TeV data and the 13 TeV data, integrated luminosity is 19.7 and 35.9 fb^{-1}, respectively, which showed a 2.8σ local excess at 95.3 GeV [9,13,17]. Since the excess was performed [9,13], it has received considerable attention [18–28]. The excesses have been discussed in several models, in a natural next-to-minimal supersymmetric standard model (SSM) [29] and a general next-to-minimal supersymmetric standard model [18]. In Refs. [30,31], the authors have discussed the excesses in a *CP*-conserving μ-from-ν SSM. In the two-Higgs doublet model with an additional real singlet, the excesses have also been discussed in Refs. [32–34]. In this work, we find a suitable parameter spaces which can explain the 95 GeV excess.

As one of the extensions of the SM, the μ-from-ν supersymmetric standard model [35–42] can solve the μ problem [43] of the minimal supersymmetric standard model (MSSM) [44–48], through introducing three singlet right-handed neutrino superfields $\hat{\nu}_i^c$ ($i = 1, 2, 3$). The neutrino superfields lead the mixing of the neutral components of the Higgs doublets with the right-handed sneutrinos, that is different from the Higgs sector of the MSSM. The mixing can change the Higgs couplings and influence the decay processes of the Higgs bosons. In addition, we also introduce *CP* violation, and we also get a lighter Higgs at ~ 95 GeV with a suitable parameter space.

The paper is organized as follows. In Sec. II, we introduce the *CP*-violating (CPV) μ-from-ν SSM briefly, about the superpotential and the CPV phases. In Sec. III, we study the excess at 95 GeV in the CPV μ-from-ν SSM. In Secs. IV and V, we show the numerical analysis and the conclusion, respectively.

II. THE MODEL

The superpotential of the μ-from-ν SSM contains Yukawa couplings for neutrinos, two additional types of

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terms involving the Higgs doublet superfields $\hat{H}_u$ and $\hat{H}_d$, and the right-handed neutrino superfields $\hat{\nu}_i^c$ [35]:

$$W = \epsilon_{ab}(Y_{u_{ij}}\hat{H}_u^b\hat{Q}_i^a\hat{u}_j^c + Y_{d_{ij}}\hat{H}_d^a\hat{Q}_i^b\hat{d}_j^c + Y_{e_{ij}}\hat{H}_d^a\hat{L}_i^b\hat{e}_j^c) + \epsilon_{ab}Y_{\nu_{ij}}\hat{H}_u^b\hat{L}_i^a\hat{\nu}_j^c - \epsilon_{ab}\lambda_i e^{i\phi_{\lambda_i}}\hat{\nu}_i^c\hat{H}_d^a\hat{H}_u^b + \frac{1}{3}\kappa_{ijk}\hat{\nu}_i^c\hat{\nu}_j^c\hat{\nu}_k^c, \quad (1)$$

where $\hat{H}_u^T = (\hat{H}_u^+, \hat{H}_u^0)$, $\hat{H}_d^T = (\hat{H}_d^0, \hat{H}_d^-)$, $\hat{Q}_i^T = (\hat{u}_i, \hat{d}_i)$, and $\hat{L}_i^T = (\hat{\nu}_i, \hat{e}_i)$ (the index T denotes the transposition) represent the MSSM-like doublet Higgs superfields and $\hat{u}_i^c$, $\hat{d}_i^c$, and $\hat{e}_i^c$ are the singlet up-type quark, down-type quark, and charged lepton superfields, respectively. In addition, $Y_{u,d,e,\nu}$, λ, and κ are dimensionless matrices, a vector, and a totally symmetric tensor, respectively. $a, b = 1, 2$ are SU(2) indices with antisymmetric tensor $\epsilon_{12} = 1$, and $i, j, k = 1, 2, 3$ are generation indices. The CP is violated by the parameter λ_i, and the CP-violating phase is ϕ_{λ_i}.

In the superpotential, if the scalar potential is such that nonzero vacuum expectative values (VEVs) of the scalar components ($\tilde{\nu}_i^c$) of the singlet neutrino superfields $\hat{\nu}_i^c$ are induced, the effective bilinear terms $\epsilon_{ab}\varepsilon_i\hat{H}_u^b\hat{L}_i^a$ and $\epsilon_{ab}\mu\hat{H}_d^a\hat{H}_u^b$ are generated, with $\varepsilon_i = Y_{\nu_{ij}}\langle\tilde{\nu}_j^c\rangle$ and $\mu = \lambda_i\langle\tilde{\nu}_i^c\rangle$, once the electroweak symmetry is broken. The last term in Eq. (1) generates the effective Majorana masses for neutrinos at the electroweak scale. Therefore, the μ-from-ν SSM can generate three tiny neutrino masses at the tree level through TeV-scale seesaw mechanism [35,49–55].

It is worth explaining why the TeV-scale seesaw was chosen. Through a seesaw on the scale of the grand unified theory, one can get Yukawa couplings of the order of one for neutrinos. But we know that the Yukawa coupling of the electron is on the order of 10^{-6}, and the Yukawa couplings of neutrinos can also be around on the order of 10^{-6} instead of one. In the TeV-scale seesaw, this is sufficient to reproduce the neutrino mass, if the Yukawa coupling of the neutrino is of the same order as the Yukawa coupling of the electron [35]. Here, it is important to note that the VEVs of the left-handed sneutrinos υ_{ν_i} are generally small. We know that the Dirac masses for the neutrinos $m_{D_i} = Y_{\upsilon_{\nu_i}}\upsilon_u \lesssim 10^{-4}$ GeV in the TeV-scale seesaw. So we can get an estimate of the VEVs of the left-handed sneutrinos, $\upsilon_{\nu_i} \lesssim m_{D_i} \lesssim 10^{-4}$ GeV, which means that $\upsilon_{\nu_i} \ll \upsilon_d, \upsilon_u$ [35,36].

The general soft supersymmetry-breaking terms of the μ-from-ν SSM are given by

$$\begin{aligned}-\mathcal{L}_{\text{soft}} = {}& m^2_{\tilde{Q}_{ij}}\tilde{Q}_i^{a*}\tilde{Q}_j^a + m^2_{\tilde{u}^c_{ij}}\tilde{u}_i^{c*}\tilde{u}_j^c + m^2_{\tilde{d}^c_{ij}}\tilde{d}_i^{c*}\tilde{d}_j^c + m^2_{\tilde{L}_{ij}}\tilde{L}_i^{a*}\tilde{L}_j^a + m^2_{\tilde{e}^c_{ij}}\tilde{e}_i^{c*}\tilde{e}_j^c + m^2_{H_d}H_d^{a*}H_d^a + m^2_{H_u}H_u^{a*}H_u^a + m^2_{\tilde{\nu}^c_{ij}}\tilde{\nu}_i^{c*}\tilde{\nu}_j^c \\ &+ \epsilon_{ab}[(A_uY_u)_{ij}H_u^b\tilde{Q}_i^a\tilde{u}_j^c + (A_dY_d)_{ij}H_d^a\tilde{Q}_i^b\tilde{d}_j^c + (A_eY_e)_{ij}H_d^a\tilde{L}_i^b\tilde{e}_j^c + \text{H.c.}] \\ &+ \left[\epsilon_{ab}(A_\nu Y_\nu)_{ij}H_u^b\tilde{L}_i^a\tilde{\nu}_j^c - \epsilon_{ab}(A_\lambda e^{i\phi_\lambda}\lambda)_i\tilde{\nu}_i^c H_d^a H_u^b + \frac{1}{3}(A_\kappa\kappa)_{ijk}\tilde{\nu}_i^c\tilde{\nu}_j^c\tilde{\nu}_k^c + \text{H.c.}\right] \\ &- \frac{1}{2}(M_3\tilde{\lambda}_3\tilde{\lambda}_3 + M_2\tilde{\lambda}_2\tilde{\lambda}_2 + M_1\tilde{\lambda}_1\tilde{\lambda}_1 + \text{H.c.}).\end{aligned} \quad (2)$$

Here, the first two lines contain mass squared terms of squarks, sleptons, and Higgses. The next two lines consist of the trilinear scalar couplings. In the last line, M_3, M_2, and M_1 denote Majorana masses corresponding to SU(3), SU(2), and U(1) gauginos $\tilde{\lambda}_3$, $\tilde{\lambda}_2$, and $\tilde{\lambda}_1$, respectively. In addition to the terms from $\mathcal{L}_{\text{soft}}$, the tree-level scalar potential receives the usual D- and F-term contributions [36,37].

Once the electroweak symmetry is spontaneously broken, the neutral scalars develop in general the VEVs. The CP can violated by the VEVs of the scalar fields:

$$\langle H_d^0\rangle = e^{i\phi_{\upsilon_d}}\upsilon_d, \qquad \langle H_u^0\rangle = e^{i\phi_{\upsilon_u}}\upsilon_u, \qquad \langle\tilde{\nu}_i\rangle = \upsilon_{\nu_i}, \qquad \langle\tilde{\nu}_i^c\rangle = e^{i\phi_{\upsilon_{\nu_i^c}}}\upsilon_{\nu_i^c}. \quad (3)$$

One can define the neutral scalars as

$$H_d^0 = e^{i\phi_{\upsilon_d}}\left(\frac{h_d + iP_d}{\sqrt{2}} + \upsilon_d\right), \qquad \tilde{\nu}_i = \frac{(\tilde{\nu}_i)^{\text{Re}} + i(\tilde{\nu}_i)^{\text{Im}}}{\sqrt{2}} + \upsilon_{\nu_i},$$
$$H_u^0 = e^{i\phi_{\upsilon_u}}\left(\frac{h_u + iP_u}{\sqrt{2}} + \upsilon_u\right),$$
$$\tilde{\nu}_i^c = e^{i\phi_{\upsilon_{\nu_i^c}}}\left(\frac{(\tilde{\nu}_i^c)^{\text{Re}} + i(\tilde{\nu}_i^c)^{\text{Im}}}{\sqrt{2}} + \upsilon_{\nu_i^c}\right), \quad (4)$$

and

$$\tan\beta = \frac{\upsilon_u}{\sqrt{\upsilon_d^2 + \upsilon_{\nu_i}\upsilon_{\nu_i}}} \approx \frac{\upsilon_u}{\upsilon_d},$$
$$\upsilon = \sqrt{\upsilon_u^2 + \upsilon_d^2 + \upsilon_{\nu_i}\upsilon_{\nu_i}} \approx \sqrt{\upsilon_u^2 + \upsilon_d^2}, \quad (5)$$

In supersymmetric extensions of the SM, the R parity of a particle is defined as $R = (-1)^{L+3B+2S}$ [44–48]. R parity is violated if either the baryon number (B) or lepton number (L) is not conserved, where S denotes the spin of concerned component field. The last two terms in Eq. (1) explicitly violate lepton number and R parity. For example, if one assigns $L = 1$ to the right-handed neutrino superfields, then the last term $\frac{1}{3}\kappa_{ijk}\hat{\nu}_i^c\hat{\nu}_j^c\hat{\nu}_k^c$ in Eq. (2) violates the lepton number by three units contrary to the $\hat{L}_i\hat{L}_j\hat{e}_k^c$ term of the R parity violating MSSM which shows the $\Delta L = 1$ effect. R-parity breaking implies that the lightest supersymmetric particle is no longer stable.

A. The μ-from-ν SSM Higgs potential

The neutral scalar potential of the tree level can be written as

$$V^0 = V_{\text{soft}} + V_D + V_F \tag{6}$$

with

$$V_{\text{soft}} = m_{H_d}^2 H_d^0 H_d^{0*} + m_{H_u}^2 H_u^0 H_u^{0*} + m_{\tilde{L}_{ij}}^2 \tilde{\nu}_i \tilde{\nu}_j^* + m_{\tilde{\nu}_{ij}^c}^2 - \left((A_\lambda e^{i\phi_\lambda}\lambda)_i \tilde{\nu}_i^c H_d^0 H_u^{0*} - \frac{1}{3}(A_\kappa \kappa)_{ijk} \tilde{\nu}_i^c \tilde{\nu}_j^c \tilde{\nu}_k^c + \text{H.c.} \right), \tag{7}$$

$$V_D = \frac{G^2}{8}(\tilde{\nu}_i\tilde{\nu}_i^* + H_d^0 H_d^{0*} - H_u^0 H_u^{0*})^2, \tag{8}$$

$$\begin{aligned} V_F = {} & \lambda_i \lambda_i^* H_d^0 H_d^{0*} H_u^0 H_u^{0*} \\ & + e^{i\phi_{\lambda_i}} e^{-i\phi_{\lambda_j}} \lambda_i \lambda_j^* e^{-i\phi_{\tilde{\nu}_i^c}} e^{-i\phi_{\tilde{\nu}_j^c}} \tilde{\nu}_i^{c*} \tilde{\nu}_j^{c*} (H_d^0 H_d^{0*} + H_u^0 H_u^{0*}) \\ & + \kappa_{ijk}\kappa_{ljm}^* e^{i\phi_{\tilde{\nu}_i^c}} e^{i\phi_{\tilde{\nu}_k^c}} e^{-i\phi_{\tilde{\nu}_l^c}} e^{-i\phi_{\tilde{\nu}_m^c}} \tilde{\nu}_i^c \tilde{\nu}_k^c \tilde{\nu}_l^{c*} \tilde{\nu}_m^{c*} \\ & - (e^{-i\phi_{\lambda_j}} \lambda_j^* \kappa_{ijk} \nu_i^c \nu_k^c H_d^0 H_u^{0*} + \text{H.c.}), \end{aligned} \tag{9}$$

where $G^2 = g_1^2 + g_2^2$. The tree-level neutral scalar potential include the usual soft terms and D- and F-term contributions. In this work, we take all parameters in the potential area real.

By using the effective potential methods [56–72], one can get the one-loop effective potential:

$$V^1 = \frac{1}{32\pi^2}\left\{ \sum_{\tilde{f}} N_f m_{\tilde{f}}^4 \left(\log\frac{m_{\tilde{f}}^2}{Q^2} - \frac{3}{2} \right) - 2\sum_{f=t,b,\tau} N_f m_f^4 \left(\log\frac{m_f^2}{Q^2} - \frac{3}{2} \right) \right\}. \tag{10}$$

Here, Q is the renormalization scale, $N_t = N_b = 3$, and $N_\tau = 1$. $f = t,\ b,\ \tau$ denote the third fermions, and $\tilde{f} = \tilde{t}_{1,2} = \tilde{b}_{1,2} = \tilde{\tau}_{1,2}$ are the corresponding supersymmetric partners.

Considering the one-loop effective potential, the Higgs potential can be written as

$$V = V^0 + V^1; \tag{11}$$

we can calculate the minimization conditions of the potential and the Higgs masses in this work.

B. Higgs masses and CPV phases

In the μ-from-ν SSM, the left- and right-handed sneutrino VEVs lead to the mixing of the neutral components of the Higgs doublets with the left- and right-handed sneutrinos producing an 8×8 CP-even neutral scalar mass matrix, which can be seen in Refs. [36,37,41,73]. The mixing gives a rich phenomenology in the Higgs sector of the μ-from-ν SSM [36–40,73–76].

Here, we note that the Higgs doublets and right-handed sneutrinos are basically decoupled from the left-handed sneutrinos [73], so we did not consider the left-handed sneutrinos in the CP-even and CP-odd scalar parts.

The CP-even sector mix with the CP-odd sector, the 10×10 mixing matrix, is defined by

$$M_h^2 = \begin{pmatrix} M_S^2 & M_{SP}^2 \\ (M_{SP}^2)^2 & M_P^2 \end{pmatrix} \tag{12}$$

with M_S^2 denoting the CP-even neutral scalars, M_P^2 is the CP-odd neutral scalars, and M_{SP}^2 represents the mass submatrix for the mixing of CP-even neutral scalars and CP-odd neutral scalars.

The mass squared matrix M_h^2 can be diagonalized as

$$Z_H M_h^2 Z_H^T = m_h^2; \tag{13}$$

with CPV in the CP-even and CP-odd scalar sector the matrix Z_H can be complex.

We consider the radiative corrections in mass submatrix M_H^2; the radiative corrections from the third fermions ($f = t,\ b,\ \tau$) and their superpartners include the two-loop leading-log effects [77–79]. The CP-even neutral scalar is given as

$$M_S^2 = \begin{pmatrix} M_H^2 & M_{\text{Mix}}^2 \\ (M_{\text{Mix}}^2)^2 & M_{\tilde{\nu}_R^c}^2 \end{pmatrix}. \tag{14}$$

In detail, the mass submatrix M_H^2 is defined by

$$M_H^2 = \begin{pmatrix} M_{h_d h_d}^2 + \Delta_{11} & M_{h_d h_u}^2 + \Delta_{12} \\ M_{h_d h_u}^2 + \Delta_{12} & M_{h_u h_u}^2 + \Delta_{22} \end{pmatrix}. \tag{15}$$

The dominating contributions of radiative corrections Δ_{11}, Δ_{12}, and Δ_{22} comes from the fermions (t, b) and their superpartners

$$\begin{aligned} \Delta_{11} &= \Delta_{11}^t + \Delta_{11}^b, \\ \Delta_{12} &= \Delta_{12}^t + \Delta_{12}^b, \\ \Delta_{22} &= \Delta_{22}^t + \Delta_{22}^b. \end{aligned} \tag{16}$$

We did not consider the terms containing coupling Y_{ν_i} and υ_{ν_i}, because these terms are very small. The radiative corrections from the top quark is given by [74,80–85]

$$\Delta_{11}^{t} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \frac{(\mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{\upsilon_u}} e^{-i\phi_{\upsilon_d}} e^{i\phi_{\upsilon_{\nu_i^c}}}) A_t\mu - |\mu|^2 \cot\beta)^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2), \tag{17}$$

$$\Delta_{12}^{t} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \frac{|\mu|^2 \cot\beta - \mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{\upsilon_u}} e^{-i\phi_{\upsilon_d}} e^{i\phi_{\upsilon_{\nu_i^c}}}) A_t\mu}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \times \left(\ln \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} + \frac{\mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{\upsilon_u}} e^{-i\phi_{\upsilon_d}} e^{i\phi_{\upsilon_{\nu_i^c}}}) A_t\mu \cot\beta}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right), \tag{18}$$

$$\begin{aligned} \Delta_{22}^{t} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \Bigg\{ &\ln \frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_{t^4}} + \frac{2|A_t|^2 - \mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{\upsilon_u}} e^{-i\phi_{\upsilon_d}} e^{i\phi_{\upsilon_{\nu_i^c}}}) A_t\mu \cos\beta}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \ln \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} \\ &+ \frac{(2|A_t|^2 - \mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{\upsilon_u}} e^{-i\phi_{\upsilon_d}} e^{i\phi_{\upsilon_{\nu_i^c}}}) A_t\mu \cot\beta)^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \frac{1}{16\pi^2} \ln \frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4} \\ &\times \left(\frac{3e^2 m_t^2}{4s_w^2 m_W^2} - 32\pi\alpha_s \right) \left[\frac{1}{2} \ln \frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4} + \frac{2|A_t - \mu\cot\beta|^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}} \left(1 - \frac{(A_t - \mu\cot\beta)^2}{12 m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2} \right) \right] \Bigg\} \end{aligned} \tag{19}$$

with

$$g(m_1^2, m_2^2) = 2 - \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2}; \tag{20}$$

to save space, the mass matrix and the radiative corrections are given in the Appendix.

In the radiative corrections, the trilinear coupling $A_t = |A_t| e^{i\phi_{A_t}}$ can be complex. These seven independent phase have been defined as

$$\phi_\lambda, \quad \phi_{A_t}, \quad \phi_{\upsilon_u}, \quad \phi_{\upsilon_d}, \quad \phi_{\upsilon_{\nu_i^c}}. \tag{21}$$

III. EXCESS AT 95 GeV

The process measured at LEP reported a 2.3σ local excess in the $b\bar{b}$ final state searches, with the scalar mass at ~96 GeV. The production of a Higgs boson via Higgstrahlung is associated with the Higgs decaying to bottom quarks. Normalized to the SM expectation, the signal strength is defined as

$$\mu_{\mathrm{LEP}}^{bb} = \frac{\sigma(e^+e^- \to Zh_1 \to Zb\bar{b})}{\sigma^{\mathrm{SM}}(e^+e^- \to Zh_1 \to Zb\bar{b})} = 0.117 \pm 0.057. \tag{22}$$

The value for μ_{LEP}^{bb} can be found in [29,86] with the method introduced in [87]. h_1 is the Higgs which has mass range around ~96 GeV, and h_2 is the 125 GeV Higgs boson in the following. In the framework of the μ-from-ν SSM, we use μ_{NP}^{bb} to describe the signal strength; the expression for μ_{NP}^{bb} can be approximated as [30]

$$\begin{aligned} \mu_{NP}^{bb} &= \frac{\sigma^{\mathrm{NP}}(Z^* \to Zh_1)}{\sigma^{\mathrm{SM}}(Z^* \to Zh_1)} \times \frac{Br^{\mathrm{NP}}(h_1 \to b\bar{b})}{Br^{\mathrm{SM}}(h_1 \to b\bar{b})} \\ &\approx |C_{h_1VV}|^2 \times \frac{\Gamma_{h_1\to b\bar{b}}^{\mathrm{NP}}}{\Gamma_{h_1\to b\bar{b}}^{\mathrm{SM}}} \times \frac{\Gamma_{\mathrm{tot}}^{\mathrm{SM}}}{\Gamma_{\mathrm{tot}}^{\mathrm{NP}}} \\ &\approx \frac{|C_{h_1VV}|^2 \times |C_{h_1d\bar{d}}|^2}{|C_{h_1d\bar{d}}|^2 (Br_{h_1\to b\bar{b}}^{\mathrm{SM}} + Br_{h_1\to\tau\bar{\tau}}^{\mathrm{SM}}) + |C_{h_1u\bar{u}}|^2 (Br_{h_1\to g\bar{g}}^{\mathrm{SM}} + Br_{h_1\to c\bar{c}}^{\mathrm{SM}})}. \end{aligned} \tag{23}$$

One can find the SM branching ratios $Br^{\rm SM}$ in Ref. [3], and Γ is the decay widths. The couplings are normalized to the SM prediction of a Higgs boson of the same mass. C_{h_1} is the coupling of h_1 and gauge boson, and $C_{h_1 u\bar{u}}$ and $C_{h_1 d\bar{d}}$ are the couplings of h_1 and up- and down-type quarks. The normalized couplings are given as

$$C_{h_i d\bar{d}} = \frac{Z_H^{i1}}{\cos\beta}, \qquad C_{h_i u\bar{u}} = \frac{Z_H^{i2}}{\sin\beta},$$
$$C_{h_i VV} = Z_H^{i1}\cos\beta + Z_H^{i2}\sin\beta. \tag{24}$$

In 2019, the CMS searches for the Higgs boson decaying in the diphoton channel showed a local excess of $\sim 3\sigma$ around ~ 96 GeV [11]; the previous results is that [11,88]

$$\mu_{\gamma\gamma}^{\rm CMS} = \frac{\sigma(gg\to h_1\to\gamma\gamma)}{\sigma^{\rm SM}(gg\to h_1\to\gamma\gamma)} = 0.6\pm 0.2. \tag{25}$$

Recently, ATLAS reported their new results at 95.4 GeV based on the full run 2 dataset [15], the "diphoton excess" with a signal strength of

$$\mu_{\gamma\gamma}^{\rm ATLAS} = 0.18\pm 0.1. \tag{26}$$

Meanwhile, the corresponding CMS result for the diphoton excess is given by [12]

$$\mu_{\gamma\gamma}^{\rm CMS} = \frac{\sigma^{\rm exp}(pp\to\phi\to\gamma\gamma)}{\sigma^{\rm SM}(pp\to H_{\rm SM}\to\gamma\gamma)} = 0.33^{+0.19}_{-0.12}. \tag{27}$$

Neglecting possible correlations, one can get a combined signal strength of [13]

$$\mu_{\gamma\gamma}^{\rm exp} = \mu_{\gamma\gamma}^{\rm ATLAS+CMS} = 0.24^{+0.09}_{-0.08}. \tag{28}$$

In this work, the approximation of the diphoton rate of the h_1 can written as [29,30]

$$\begin{aligned}\mu_{NP}^{\gamma\gamma} &= \frac{\sigma^{\rm NP}(gg\to h_1)}{\sigma^{\rm SM}(gg\to h_1)}\times\frac{Br^{\rm NP}(h_1\to\gamma\gamma)}{Br^{\rm SM}(h_1\to\gamma\gamma)}\\ &\approx |C_{h_1u\bar{u}}|^2\times\frac{\Gamma^{\rm NP}_{h_1\to\gamma\gamma}}{\Gamma^{\rm SM}_{h_1\to\gamma\gamma}}\times\frac{\Gamma^{\rm SM}_{\rm tot}}{\Gamma^{\rm NP}_{\rm tot}}\\ &\approx \frac{|C_{h_1u\bar{u}}|^2\times|C_{h_1\gamma\gamma}|^2}{|C_{h_1d\bar{d}}|^2(Br^{\rm SM}_{h_1\to b\bar{b}}+Br^{\rm SM}_{h_1\to\tau\bar{\tau}})+|C_{h_1u\bar{u}}|^2(Br^{\rm SM}_{h_1\to gg}+Br^{\rm SM}_{h_1\to c\bar{c}})}.\end{aligned} \tag{29}$$

The effective coupling $C_{h_1\gamma\gamma}$ can be written as [30]

$$|C_{h_1\gamma\gamma}|^2 = \frac{|\frac{4}{3}C_{h_1t\bar{t}}A_{1/2}(\tau_t)+C_{h_1VV}A_1(\tau_W)|^2}{|\frac{4}{3}A_{1/2}(\tau_t)+A_1(\tau_W)|^2} \tag{30}$$

with $\tau_t = \frac{m_{h_1}^2}{4m_t^2} < 1$ and $\tau_t = \frac{m_{h_1}^2}{4m_W^2} < 1$. The form factors $A_{1/2}$ and A_1 are given by [89]

$$A_{1/2}(x) = 2(x+(x-1)\arcsin^2\sqrt{x})x^{-2}, \quad x\le 1, \tag{31}$$

$$A_1(x) = -(2x^2+3x+3(2x-1)\arcsin^2\sqrt{x})x^{-2}, \qquad x\le 1. \tag{32}$$

By using Eqs. (23) and (29), we calculate the two signal strengths.

IV. NUMERICAL RESULTS

In this section, we will discuss the couplings and signal strength of 96 GeV Higgs in CPV μ-from-ν SSM. The free parameters in our analysis will be

$$\lambda, \quad \tan\beta, \quad \kappa, \quad A_t, \quad A_\lambda, \quad A_\kappa, \quad A_b, \quad \upsilon_{\nu^c}. \tag{33}$$

We take $\upsilon_{\nu_1^c} = \upsilon_{\nu_2^c} = \upsilon_{\nu_3^c}$, and we have defined

$$\begin{aligned}&\kappa_{ijk} = \kappa\delta_{ij}\delta_{jk}, \quad (A_\kappa\kappa)_{ijk} = A_\kappa\kappa\delta_{ij}\delta_{jk}, \quad \lambda_i = \lambda,\\ &(A_\lambda\lambda)_i = A_\lambda\lambda, \quad Y_{e_{ij}} = Y_{e_i}\delta_{ij}, \quad (A_eY_e)_{ij} = A_eY_{e_i}\delta_{ij},\\ &Y_{\nu_{ij}} = Y_{\nu_i}\delta_{ij}, \quad (A_\nu Y_\nu)_{ij} = a_{\nu_i}\delta_{ij}, \quad m^2_{\tilde{\nu}^c_{ij}} = m^2_{\tilde{\nu}^c_i}\delta_{ij},\\ &m^2_{\tilde{Q}_{ij}} = m^2_{\tilde{Q}_i}\delta_{ij}, \quad m^2_{\tilde{u}^c_{ij}} = m^2_{\tilde{u}_i{}^c}\delta_{ij}, \quad m^2_{\tilde{d}^c_{ij}} = m^2_{\tilde{d}_i{}^c}\delta_{ij},\\ &m^2_{\tilde{L}_{ij}} = m^2_{\tilde{L}}\delta_{ij}, \quad m^2_{\tilde{e}^c_{ij}} = m^2_{\tilde{e}^c}\delta_{ij}, \quad \upsilon_{\nu_i^c} = \upsilon_{\nu^c},\end{aligned} \tag{34}$$

where i, $j = 1, 2, 3$.

A. Mass and coupling

In Fig. 1(a), we can see that, when h_1 is near 96 GeV, h_2 is closer to 125 GeV with the increase of $\tan\beta$. Although both h_1 and h_2 can conform to the experimental mass range in our parameter space, if we assume a theory uncertainty of up to 3 GeV [30], the parameter range will be larger. One can clear see that $|C_{h_2VV}|$ is much larger than $|C_{h_1VV}|$ in Fig. 1(b); the LHC measurements of the SM-like Higgs bosons couplings to fermions and massive gauge bosons are still not very precise [30,90]. If in the future some

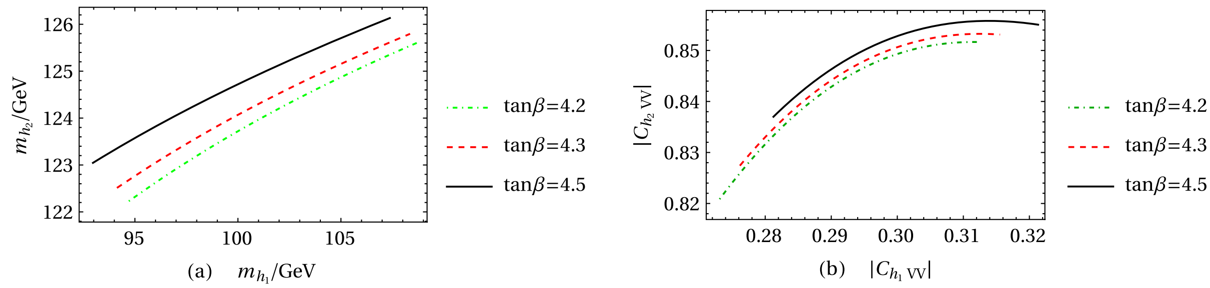

FIG. 1. Values for m_{h_2} versus m_{h_2} in (a) and the normalized couplings $|C_{h_2 VV}|$ versus $|C_{h_1 VV}|$ in (b). The input parameters are in Table I.

collider can measure these couplings to the percent level, then we can choose a more reasonable parameter space.

In Fig. 2, left column, we can see that m_{h_1} is very sensitive to υ_{ν^c}. As υ_{ν^c} becomes larger, m_{h_1} will rapidly become smaller; on the contrary, m_{h_2} will slowly become larger. In order for m_{h_1} to be around 95.4 GeV, the value of υ_{ν^c} will not have a wide range. Similar to υ_{ν^c}, m_{h_1} is also very sensitive to κ, and with the increase of λ, a larger κ value can be taken to keep m_{h_1} near 95.4 GeV. On the contrary, κ has little effect on m_{h_2}; especially when $\lambda = 0.085$ or 0.087, m_{h_2} will be very stable.

In Fig. 3, we have showed the component of h_1 and h_2; for h_1, CP-odd H_u component and CP-even right-handed sneutrinos component are the main components, and with the increase of λ, CP-odd H_d component will become smaller, while the CP-even right-handed sneutrino component will become larger. While for h_2, CP-even H_u is the main component, as λ increases, the CP-even H_u component will increase, the CP-even H_d component will gradually decrease, and the CP-odd H_d component will gradually increase.

Then, in Fig. 4, we analyze the correlation between the Higgs masses and the CPV phases. For the first row, we take $\lambda = 0.09$, $\tan\beta = 6$, and $\kappa = 0.315$, for the second row, $\lambda = 0.095$, $\tan\beta = 4.5$, and $\kappa = 0.315$, for the third row, $\lambda = 0.098$, $\tan\beta = 4.3$, and $\kappa = 0.32$, and for the last row, $\lambda = 0.085$, $\tan\beta = 3$, and $\kappa = 0.04$. Other parameters remain the same as in Table I. We should remark that there are many CPV phase values that can be constrained by the Higgs masses in Fig. 7(a), and here we select only one of

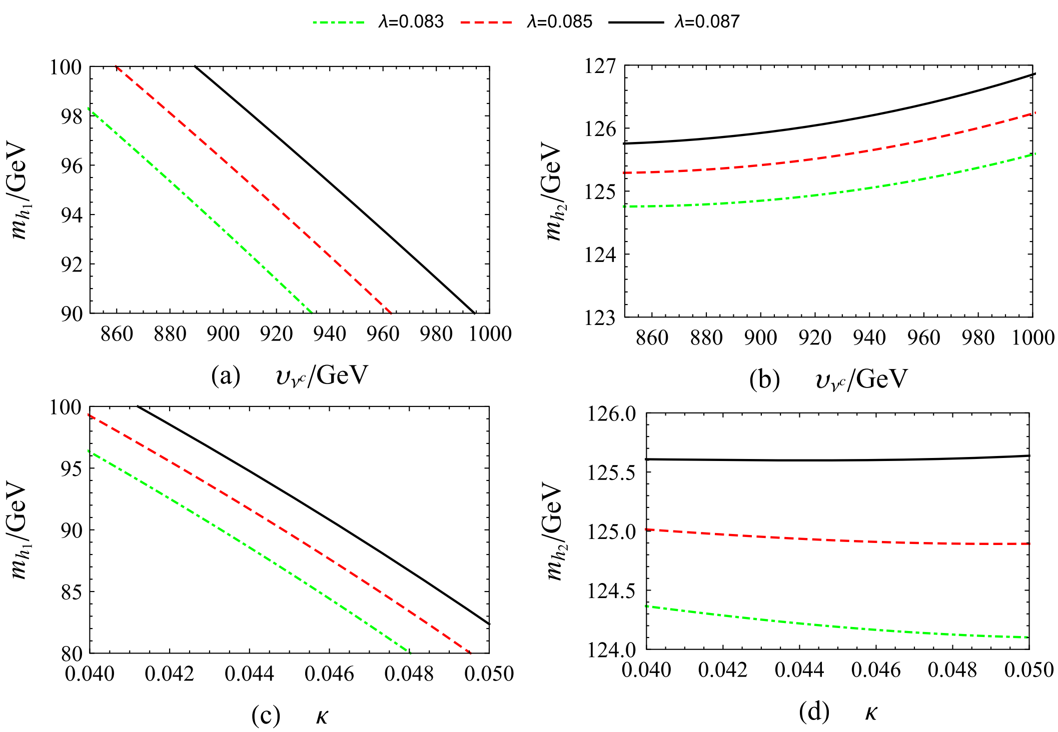

FIG. 2. Values for m_{h_1} versus the parameters υ_{ν^c} in (a) and κ in (c) and values for m_{h_2} versus the parameters υ_{ν^c} in (b) and κ in (d). We take $\tan\beta = 5$ and other parameters in Table I.

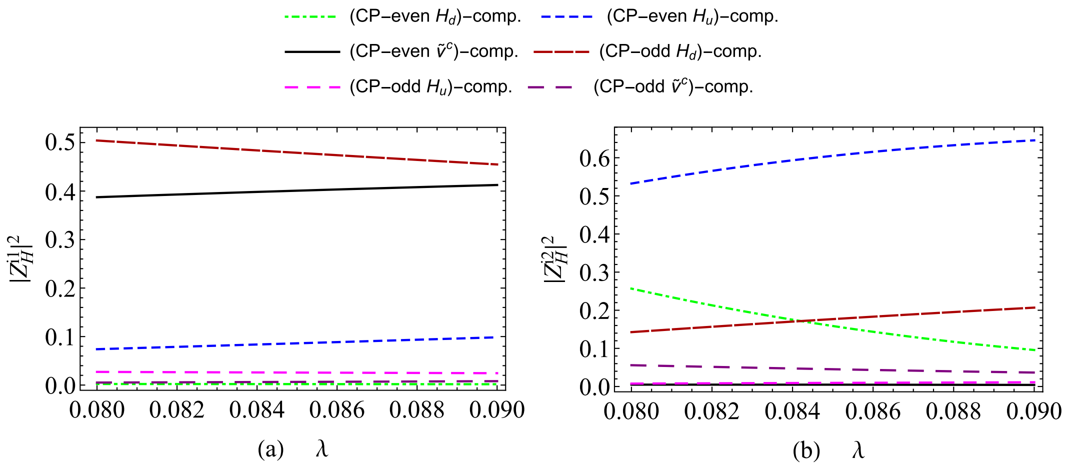

FIG. 3. The component of h_1 (a) and h_2 (b). The input parameters are in Table I, and we take $\tan\beta = 4.31$.

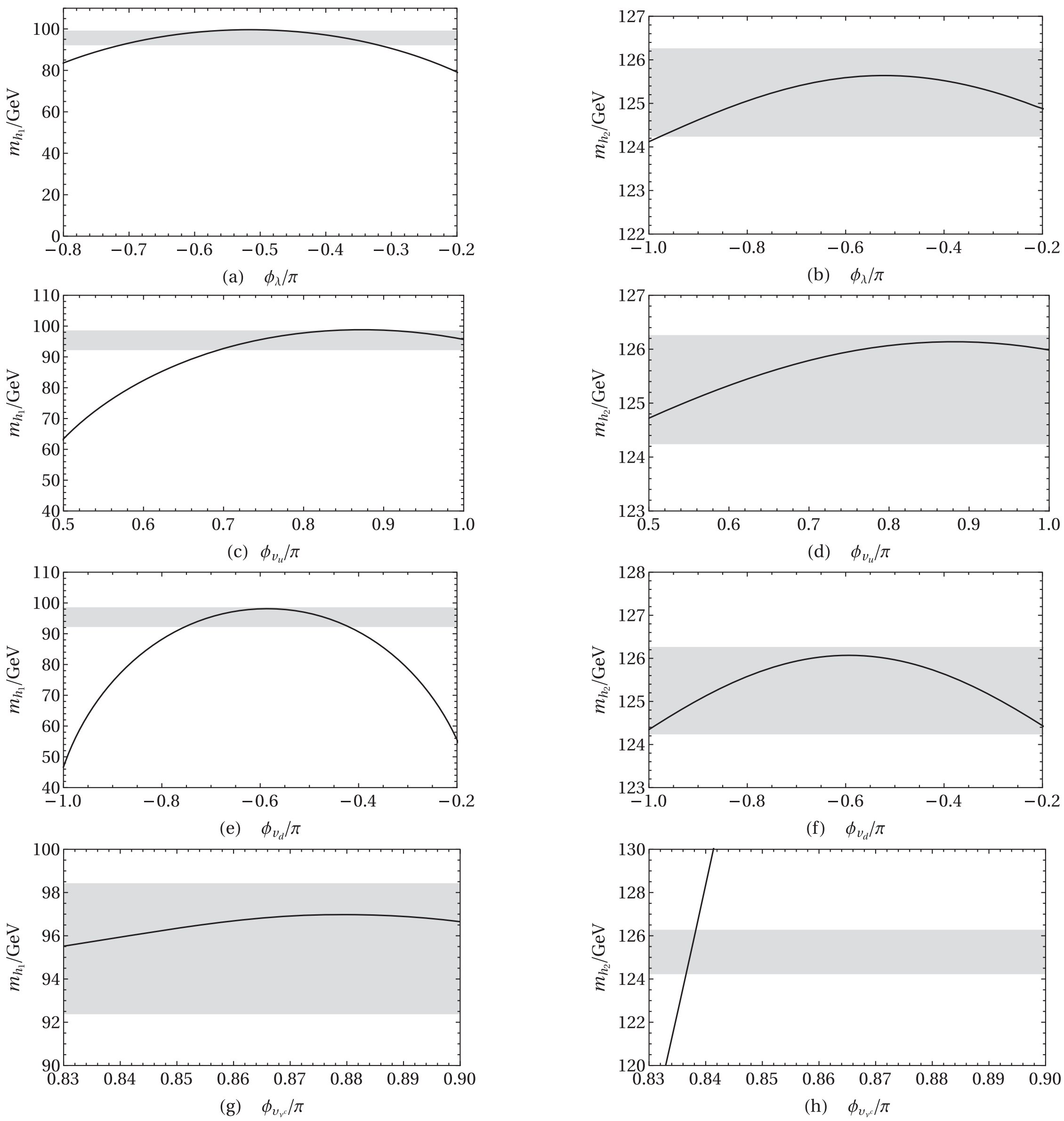

FIG. 4. The correlation of h_1 and the CPV phases in (a) (c) (e) (g); the gray area represents the experimental error of 3 GeV. . The correlation of h_2 and the CPV phases in (b) (d) (f) (h); the gray area is the theory uncertainty of 1 GeV.

TABLE I. Input parameters to fit the LEP and the CMS excesses. All dimensionful parameters are given in GeV.

A_λ	λ	A_t	υ_{ν^c}	κ	A_κ
920	[0.08;0.09]	1660	1000	0.042	−370
A_b	ϕ_λ/π	ϕ_{A_t}/π	ϕ_{υ_u}/π	ϕ_{υ_d}/π	$\phi_{\upsilon_{\nu^c}}/\pi$
700	−0.020	−0.200	0.380	−0.095	−1

them. The signal strengths and the Higgs masses do not increase or decrease all the time but oscillate like a sine function. In Fig. 4, the first row, when ϕ_λ grows from 4 to 5, the mass range of the diphoton excess can fit the experimental constraints, and the SM-like Higgs mass can be kept around 125 GeV.

Similar to the first row in Fig. 4, the second and third rows show the variation of h_1 and h_2 with the phases ϕ_{υ_u} and ϕ_{υ_d}. m_{h_1} is very sensitive to ϕ_{υ_u} and ϕ_{υ_d}. When ϕ_{υ_u} is $0.7\pi - \pi$, h_1 can be kept within the experimental error of 3 GeV. When ϕ_{υ_u} continues to increase or decrease, m_{h_1} will drop very quickly. Similarly, when ϕ_{υ_d} is $[-0.75\pi, -0.45\pi]$, m_{h_1} can conform to the experimental error. For SM-like Higgs, ϕ_{υ_u} and ϕ_{υ_d} can provide a wide range to keep m_{h_2} near 125 GeV. Different from other CPV phases, it is difficult to determine the value of $\phi_{\upsilon_\nu^c}$, and only a narrow range allows h_1 and h_2 to be simultaneously constrained by their respective experiments.

B. Signal strengths

First, in Figs. 5(a) and 5(b), we can see the correlation of the signal strengths μ_{NP}^{bb}, $\mu_{NP}^{\gamma\gamma}$, and υ_{ν^c}; with the increase of the value of υ_{ν^c}, the values of μ_{NP}^{bb} and $\mu_{NP}^{\gamma\gamma}$ will increase. And if we set $\upsilon_{\nu^c} = 1000$, the value of μ_{NP}^{bb} of $\tan\beta = 3.5$ will be larger than that of $\tan\beta = 3$, because we conclude from Fig. 1 that the Higgs masses and couplings will increase with the increase of $\tan\beta$. In the second row in Fig. 5, as the growth of κ, the value of μ_{NP}^{bb} will slowly increase, while the value of $\mu_{NP}^{\gamma\gamma}$ will slowly decrease. However, either $\tan\beta = 3$ or $\tan\beta = 3.5$ ensures that μ_{NP}^{bb} and $\mu_{NP}^{\gamma\gamma}$ are both within their respective 1σ experimental error.

The effect of CPV phases is also very obvious. Here, we take ϕ_{A_t} as an example, we take ϕ_{A_t} from $-\pi$ to π. In the first row in Fig. 6, we can see that the minimum value of h_1 appears to be around 96.2 GeV and varies periodically with ϕ_{A_t}. The maximum value of h_1 does not exceed 1 GeV larger than the minimum value. Meanwhile, the highest point of h_2 is at 125.3 GeV, with a maximum value 1.5 GeV larger than the minimum value.

The impact of ϕ_{A_t} on the normalized couplings is relatively small. In the second row in Fig. 6, we can see that the peak of $|C_{h_1VV}|$ is close to 0.313, and the highest point of $|C_{h_2VV}|$ is slightly less than 0.856. This implies that, if the "95.4 excess" is a new particle, then its coupling with gauge bosons should be much smaller than the coupling of the SM-like Higgs with gauge bosons.

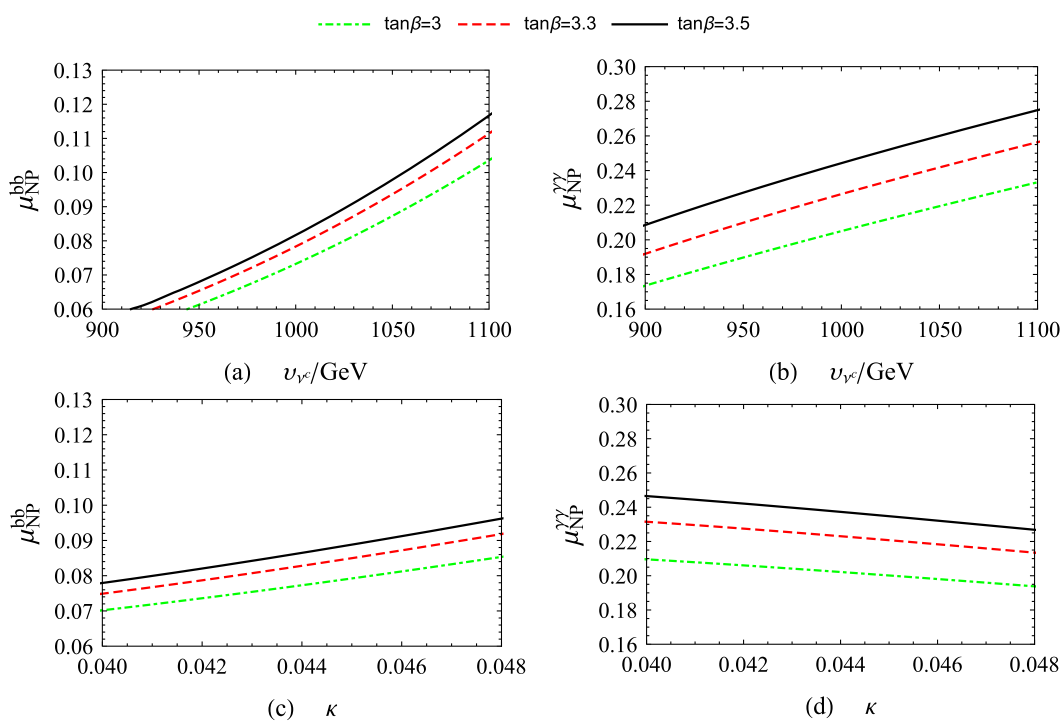

FIG. 5. The left column shows that the signal strength μ_{NP}^{bb} varies with υ_{ν^c} (a) and κ (c); the right column shows that the signal strength $\mu_{NP}^{\gamma\gamma}$ varies with υ_{ν^c} (b) and κ (d). We take $\lambda = 0.09$ and other parameters as in Table I.

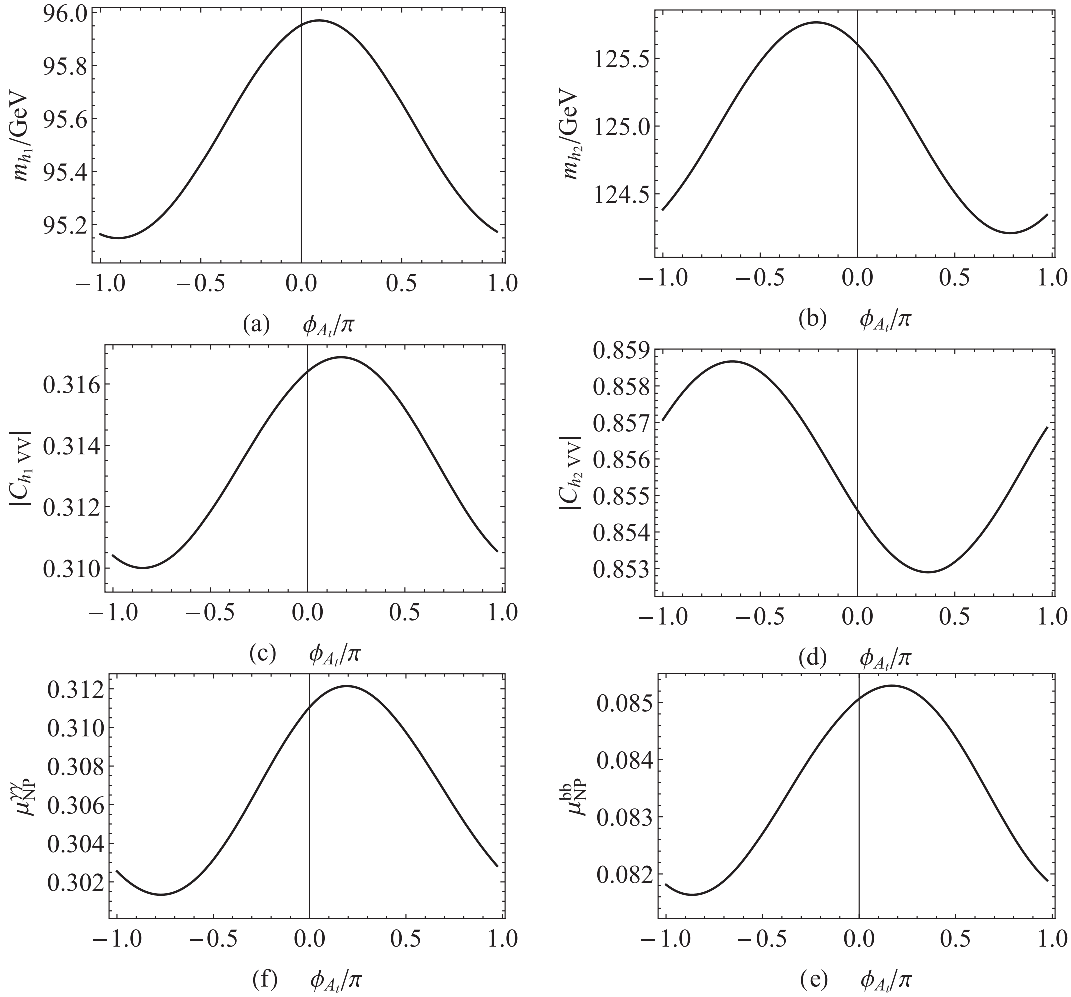

FIG. 6. The Higgs masses vary with the phase ϕ_{A_t} in (a)-(b); the correlation of couplings and the phase ϕ_{A_t} in (c)-(d); and (e)-(f) are the correlation of signal strengths and the phase ϕ_{A_t}. Here, we take $\tan\beta = 4.5$, $\lambda = 0.086$, and $\phi_{A_t} = [-\pi, \pi]$, and other parameters are the same as in Table I.

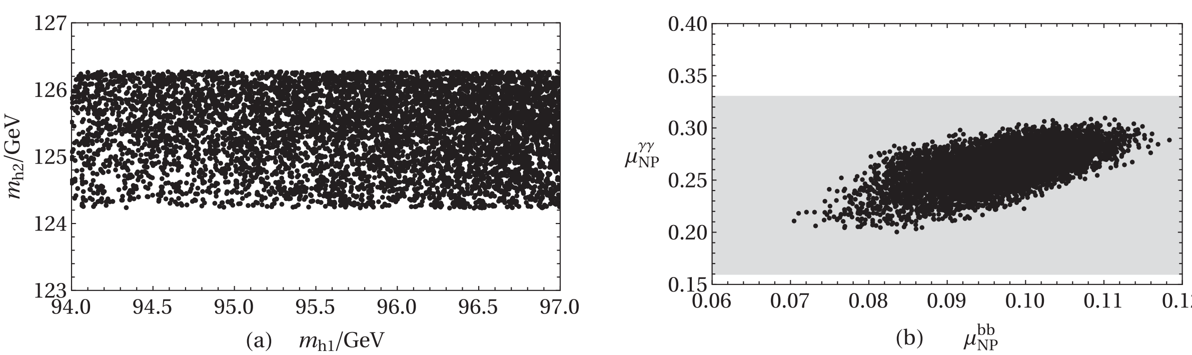

FIG. 7. The left plot (a) shows the mass range of h_1 and h_2; for h_2, we take 1 GeV theory uncertainty, for h_1, CMS gives a mass of around 95.4 GeV, and we take 3 GeV experimental error. Correlation of these two signal strengths in (b); the gray area is the 1σ experimental error. The values of parameters are in Table II.

In the last row in Fig. 6, we found that ϕ_{A_t} also has a slight effect on the two signal strengths; $\mu_{NP}^{\gamma\gamma}$ is always slightly bigger than the central value of $\mu_{\rm exp}^{\gamma\gamma}$ but does not exceed 1σ experimental error. At the same time, μ_{NP}^{bb} is lower than the central value of $\mu_{\rm exp}^{bb}$, but it also does not exceed 1σ experimental error.

Let us remark that, in the random scan plots, we must first ensure that the mass of h_2 can conform to the

TABLE II. The parameter space of the random scan plot. All dimensionful parameters are given in GeV.

λ	$\tan\beta$	κ	υ_{ν^c}	A_λ	A_t	A_κ
[0.087; 0.090]	[2.4;4.5]	[0.040;0.045]	[950;1050]	[915;920]	[1600;1700]	[−375; −370]
A_b	ϕ_λ/π	ϕ_{A_t}/π	ϕ_{υ_u}/π	ϕ_{υ_d}/π	$\phi_{\upsilon_{\nu^c}}/\pi$	M_2
[695; 710]	[−0.024; −0.022]	[−0.4; −0.1]	[0.382;0.385]	[−0.095; −0.090]	[−1; −0.9]	800

experimental constraints, because the mass determination for SM-like Higgs is already very accurate. In Fig. 7(a), we chose 1 GeV theory uncertainty for the SM-like Higgs. For h_1, we can choose the theory uncertainty up to 3 GeV, because CMS shows only that there is a 2.8σ excess at 95.3 GeV [9,13,17], which is not exactly observed. In Fig. 7(b), the $(\mu_{NP}^{\gamma\gamma}, \mu_{NP}^{bb})$ plane, we can clearly see that, with the parameter space of Table II, most points can explain the diphoton excess and the $b\bar{b}$ excess.

V. CONCLUSION

In this paper, we introduced CPV in the μ-from-ν SSM, which leads to CP-even Higgs sector mixed with CP-odd Higgs sector. We also analyzed an excess in the diphoton decay mode at ~95 GeV as reported by ATLAS and CMS, together with a ~2σ excess at LEP in the same mass range. The mixing and CPV are used to produce the lightest Higgs boson mass around 95 GeV, and the next-to-lightest Higgs boson mass around 125 GeV is the so-called SM-like Higgs. The lightest Higgs boson can explain an excess of $\gamma\gamma$ events at ~96 GeV as reported by CMS.

In the numerical part, we find a suitable parameter space, based on which we show the properties of the lightest and next-to-lightest Higgs boson. We found it very easy get both μ_{NP}^{bb} and $\mu_{NP}^{\gamma\gamma}$ to reach the central values of their respective experiments at the same time. We also analyze the influence of relevant parameters and CPV phases on the signal strengths or Higgs masses and improve the signal strengths as much as possible while ensuring that the SM-like Higgs meets the experimental constraints and the diphoton excess is around 95 GeV.

ACKNOWLEDGMENTS

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APPENDIX: MASS MATRICES

The CP-even neutral scalars have the composition $S^T = (h_d, h_u, (\tilde{\nu}_i^c)^{\text{Re}})$, and one can write the mass matrix M_S^2 as

$$M_S^2 = \begin{pmatrix} M^2_{h_d h_d} & M^2_{h_d h_u} & M^2_{h_d (\tilde{\nu}_1^c)^{\text{Re}}} & M^2_{h_d (\tilde{\nu}_2^c)^{\text{Re}}} & M^2_{h_d (\tilde{\nu}_3^c)^{\text{Re}}} \\ M^2_{h_u h_d} & M^2_{h_u h_u} & M^2_{h_u (\tilde{\nu}_1^c)^{\text{Re}}} & M^2_{h_u (\tilde{\nu}_2^c)^{\text{Re}}} & M^2_{h_u (\tilde{\nu}_3^c)^{\text{Re}}} \\ M^2_{(\tilde{\nu}_1^c)^{\text{Re}} h_d} & M^2_{(\tilde{\nu}_1^c)^{\text{Re}} h_u} & M^2_{(\tilde{\nu}_1^c)^{\text{Re}} (\tilde{\nu}_1^c)^{\text{Re}}} & M^2_{(\tilde{\nu}_1^c)^{\text{Re}} (\tilde{\nu}_2^c)^{\text{Re}}} & M^2_{(\tilde{\nu}_1^c)^{\text{Re}} (\tilde{\nu}_3^c)^{\text{Re}}} \\ M^2_{(\tilde{\nu}_2^c)^{\text{Re}} h_d} & M^2_{(\tilde{\nu}_2^c)^{\text{Re}} h_u} & M^2_{(\tilde{\nu}_2^c)^{\text{Re}} (\tilde{\nu}_1^c)^{\text{Re}}} & M^2_{(\tilde{\nu}_2^c)^{\text{Re}} (\tilde{\nu}_2^c)^{\text{Re}}} & M^2_{(\tilde{\nu}_2^c)^{\text{Re}} (\tilde{\nu}_3^c)^{\text{Re}}} \\ M^2_{(\tilde{\nu}_3^c)^{\text{Re}} h_d} & M^2_{(\tilde{\nu}_3^c)^{\text{Re}} h_u} & M^2_{(\tilde{\nu}_3^c)^{\text{Re}} (\tilde{\nu}_1^c)^{\text{Re}}} & M^2_{(\tilde{\nu}_3^c)^{\text{Re}} (\tilde{\nu}_2^c)^{\text{Re}}} & M^2_{(\tilde{\nu}_3^c)^{\text{Re}} (\tilde{\nu}_3^c)^{\text{Re}}} \end{pmatrix}, \tag{A1}$$

$$M^2_{h_d h_d} = \frac{G^2}{2}\upsilon_d^2 + \text{Re}(e^{i\phi_\lambda} e^{-i\phi_{\upsilon_u}} e^{i\phi_{\upsilon_d}} e^{i\phi_{\upsilon_{\nu_i^c}}})(A_\lambda\lambda)_i \upsilon_{\nu_i^c} \tan\beta + \text{Re}(e^{-i\phi_\lambda} e^{-i\phi_{\upsilon_u}} e^{i\phi_{\upsilon_d}} e^{i\phi_{\upsilon_{\nu_i^c}}} e^{i\phi_{\upsilon_{\nu_j^c}}} \lambda_k^* \kappa_{ijk})\upsilon_{\nu_i^c}\upsilon_{\nu_j^c} \tan\beta + \Delta_{11}, \tag{A2}$$

$$M^2_{h_d h_u} = -\frac{G^2}{2}\upsilon_d\upsilon_u - \text{Re}(e^{i\phi_\lambda} e^{-i\phi_{\upsilon_u}} e^{i\phi_{\upsilon_d}} e^{i\phi_{\upsilon_{\nu_i^c}}})(A_\lambda\lambda)_i \upsilon_{\nu_i^c} + 2|\lambda_i|^2 \upsilon_d\upsilon_u - \text{Re}(e^{-i\phi_\lambda} e^{-i\phi_{\upsilon_u}} e^{i\phi_{\upsilon_d}} e^{i\phi_{\upsilon_{\nu_i^c}}} e^{i\phi_{\upsilon_{\nu_j^c}}} \lambda_k^* \kappa_{ijk})\upsilon_{\nu_i^c}\upsilon_{\nu_j^c} + \Delta_{12}, \tag{A3}$$

$$M^2_{h_u h_u} = \frac{G^2}{2}\upsilon_u^2 + \text{Re}(e^{i\phi_\lambda} e^{-i\phi_{\upsilon_u}} e^{i\phi_{\upsilon_d}} e^{i\phi_{\upsilon_{\nu_i^c}}})(A_\lambda\lambda)_i \upsilon_{\nu_i^c} \cot\beta + \text{Re}(e^{-i\phi_\lambda} e^{-i\phi_{\upsilon_u}} e^{i\phi_{\upsilon_d}} e^{i\phi_{\upsilon_{\nu_i^c}}} e^{i\phi_{\upsilon_{\nu_j^c}}} \lambda_k^* \kappa_{ijk})\upsilon_{\nu_i^c}\upsilon_{\nu_j^c} \cot\beta + \Delta_{22}, \tag{A4}$$

$$M^2_{h_d(\tilde{\nu}^c_i)^{\rm Re}} = -{\rm Re}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})(A_\lambda\lambda)_i v_u + 2\lambda_i\lambda^*_j v_d v_{\nu^c_j} - 2{\rm Re}(e^{-i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}}e^{i\phi_{v_{\nu^c_j}}}\lambda^*_k\kappa_{ijk})v_u v_{\nu^c_j} + \Delta_{1(2+i)}, \tag{A5}$$

$$M^2_{h_u(\tilde{\nu}^c_i)^{\rm Re}} = -{\rm Re}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})(A_\lambda\lambda)_i v_d + 2\lambda_i\lambda^*_j v_u v_{\nu^c_j} - 2{\rm Re}(e^{-i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}}e^{i\phi_{v_{\nu^c_j}}}\lambda^*_k\kappa_{ijk})v_d v_{\nu^c_j} + \Delta_{2(2+i)}, \tag{A6}$$

$$M^2_{(\tilde{\nu}^c_i)^{\rm Re}(\tilde{\nu}^c_i)^{\rm Re}} = (A_\kappa\kappa)_{ijk}{\rm Re}(e^{i\phi_{v_{\nu^c_k}}}e^{i\phi_{v_{\nu^c_k}}}e^{i\phi_{v_{\nu^c_k}}})v_{\nu^c_k} + \Delta_{(2+i)(2+j)} + {\rm Re}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})(A_\lambda\lambda)_i\frac{v_d v_u}{v_{\nu^c_j}}, \tag{A7}$$

$$M^2_{(\tilde{\nu}^c_i)^{\rm Re}(\tilde{\nu}^c_j)^{\rm Re}} = |\lambda_i|^2(v_d^2+v_u^2) + \Delta_{(2+i)(2+j)}. \tag{A8}$$

The radiative corrections from the top quark and bottom quark and their corresponding supersymmetric partners, the corrections in the mass matrix, can be expressed as

$$\Delta_{ab} = \Delta^t_{ab} + \Delta^b_{ab}, \tag{A9}$$

$$\Delta^t_{1(2+i)} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2\sin^2\beta}\left\{\frac{\frac{1}{2}v_{\nu^c_j}v_d(\lambda_i\lambda^*_j+\lambda^*_i\lambda_j)\cot\beta - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})A_t\lambda_i v_d}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}\right.$$
$$\left.\times\frac{|\mu|^2\cot\beta - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})A_t\mu}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}g(m^2_{\tilde{t}_1},m^2_{\tilde{t}_2})\right\}, \tag{A10}$$

$$\Delta^t_{2(2+i)} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2\sin^2\beta}\left\{\frac{\frac{1}{2}v_{\nu^c_j}v_d(\lambda_i\lambda^*_j+\lambda^*_i\lambda_j)\cot\beta - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})A_t\lambda_i v_d}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}\right\}$$
$$\times\left\{\ln\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}} + \frac{|A_t|^2 - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})A_t\mu\cot\beta}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}g(m^2_{\tilde{t}_1},m^2_{\tilde{t}_2})\right\}, \tag{A11}$$

$$\Delta^t_{(2+i)(2+j)} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2\sin^2\beta}\left\{\frac{(\frac{1}{2}v_{\nu^c_j}v_d(\lambda_i\lambda^*_j+\lambda^*_i\lambda_j)\cot\beta - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})A_t\lambda_i v_d)^2}{(m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2})^2}g(m^2_{\tilde{t}_1},m^2_{\tilde{t}_2})\right\}, \tag{A12}$$

$$\Delta^b_{11} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2\cos^2\beta}\left\{\ln\frac{m^2_{\tilde{b}_1}m^2_{\tilde{b}_2}}{m_b^4} + \frac{2|A_b|^2 - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})A_b\mu\tan\beta}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}\ln\frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{b}_2}}\right.$$
$$\left.+\frac{(|A_b|^2 - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})A_b\mu\tan\beta)^2}{(m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2})^2}g(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2})\right\}, \tag{A13}$$

$$\Delta^b_{12} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2\cos^2\beta}\left\{\frac{|\mu|^2\tan\beta - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})A_b\mu}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}\right\}$$
$$\times\left\{\ln\frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{b}_2}} + \frac{|A_b|^2 - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})A_b\mu\tan\beta}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}g(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2})\right\}, \tag{A14}$$

$$\Delta^b_{22} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2\cos^2\beta}\left\{\frac{(|\mu|^2\tan\beta - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu^c_i}}})A_b\mu)^2}{(m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2})^2}g(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2})\right\}, \tag{A15}$$

$$\Delta^b_{1(2+i)} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2\cos^2\beta}\left\{\frac{\frac{1}{2}\upsilon_{\nu_j^c}\upsilon_u(\lambda_i\lambda_j^* + \lambda_i^*\lambda_j)\tan\beta - \mathrm{Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\lambda_i\upsilon_u}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}}\right\}$$
$$\times\left\{\ln\frac{m^2_{\tilde{b}_1}}{m^2_{\tilde{b}_2}} + \frac{|A_b|^2 - \mathrm{Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\mu\tan\beta}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}}g(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2})\right\}, \quad \text{(A16)}$$

$$\Delta^b_{2(2+i)} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2\cos^2\beta}\left\{\frac{|\mu|^2\tan\beta - \mathrm{Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\mu}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}}g(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2})\right.$$
$$\left.\times\frac{\frac{1}{2}\upsilon_{\nu_j^c}\upsilon_u(\lambda_i\lambda_j^* + \lambda_i^*\lambda_j)\tan\beta - \mathrm{Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\lambda_i\upsilon_u}{m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}}\right\}, \quad \text{(A17)}$$

$$\Delta^b_{(2+i)(2+i)} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2\cos^2\beta}\frac{\left(\frac{1}{2}\upsilon_{\nu_j^c}\upsilon_u(\lambda_i\lambda_j^* + \lambda_i^*\lambda_j)\tan\beta - \mathrm{Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\lambda_i\upsilon_u\right)^2}{(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2})^2}g(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}). \quad \text{(A18)}$$

In the same way, the CP-odd neutral scalars mass matrix is that

$$M_P^2 = \begin{pmatrix} M^2_{\sigma_d\sigma_d} & M^2_{\sigma_d\sigma_u} & M^2_{\sigma_d(\tilde{\nu}_1^c)^{\mathrm{Im}}} & M^2_{\sigma_d(\tilde{\nu}_2^c)^{\mathrm{Im}}} & M^2_{\sigma_d(\tilde{\nu}_3^c)^{\mathrm{Im}}} \\ M^2_{\sigma_u\sigma_d} & M^2_{\sigma_u\sigma_u} & M^2_{\sigma_u(\tilde{\nu}_1^c)^{\mathrm{Im}}} & M^2_{\sigma_u(\tilde{\nu}_2^c)^{\mathrm{Im}}} & M^2_{\sigma_u(\tilde{\nu}_3^c)^{\mathrm{Im}}} \\ M^2_{(\tilde{\nu}_1^c)^{\mathrm{Im}}\sigma_d} & M^2_{(\tilde{\nu}_1^c)^{\mathrm{Im}}\sigma_u} & M^2_{(\tilde{\nu}_1^c)^{\mathrm{Im}}(\tilde{\nu}_1^c)^{\mathrm{Im}}} & M^2_{(\tilde{\nu}_1^c)^{\mathrm{Im}}(\tilde{\nu}_2^c)^{\mathrm{Im}}} & M^2_{(\tilde{\nu}_1^c)^{\mathrm{Im}}(\tilde{\nu}_3^c)^{\mathrm{Im}}} \\ M^2_{(\tilde{\nu}_2^c)^{\mathrm{Im}}\sigma_d} & M^2_{(\tilde{\nu}_2^c)^{\mathrm{Im}}\sigma_u} & M^2_{(\tilde{\nu}_2^c)^{\mathrm{Im}}(\tilde{\nu}_1^c)^{\mathrm{Im}}} & M^2_{(\tilde{\nu}_2^c)^{\mathrm{Im}}(\tilde{\nu}_2^c)^{\mathrm{Im}}} & M^2_{(\tilde{\nu}_2^c)^{\mathrm{Im}}(\tilde{\nu}_3^c)^{\mathrm{Im}}} \\ M^2_{(\tilde{\nu}_3^c)^{\mathrm{Im}}\sigma_d} & M^2_{(\tilde{\nu}_3^c)^{\mathrm{Im}}\sigma_u} & M^2_{(\tilde{\nu}_3^c)^{\mathrm{Im}}(\tilde{\nu}_1^c)^{\mathrm{Im}}} & M^2_{(\tilde{\nu}_3^c)^{\mathrm{Im}}(\tilde{\nu}_2^c)^{\mathrm{Im}}} & M^2_{(\tilde{\nu}_3^c)^{\mathrm{Im}}(\tilde{\nu}_3^c)^{\mathrm{Im}}} \end{pmatrix}, \quad \text{(A19)}$$

$$M^2_{\sigma_d\sigma_d} = \mathrm{Re}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})(A_\lambda\lambda)_i\upsilon_{\nu_i^c}\tan\beta + \mathrm{Re}(e^{-i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}}e^{i\phi_{v_{\nu_j^c}}}\lambda_k^*\kappa_{ijk})\upsilon_{\nu_i^c}\upsilon_{\nu_j^c}\tan\beta + \Delta_{66}, \quad \text{(A20)}$$

$$M^2_{\sigma_d\sigma_u} = -\mathrm{Re}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})(A_\lambda\lambda)_i\upsilon_{\nu_i^c} + \mathrm{Re}(e^{-i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}}e^{i\phi_{v_{\nu_j^c}}}\lambda_k^*\kappa_{ijk})\upsilon_{\nu_i^c}\upsilon_{\nu_j^c} + \Delta_{67}, \quad \text{(A21)}$$

$$M^2_{\sigma_u\sigma_u} = \mathrm{Re}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})(A_\lambda\lambda)_i\upsilon_{\nu_i^c}\cot\beta + \mathrm{Re}(e^{-i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}}e^{i\phi_{v_{\nu_j^c}}}\lambda_k^*\kappa_{ijk})\upsilon_{\nu_i^c}\upsilon_{\nu_j^c}\cot\beta + \Delta_{77}, \quad \text{(A22)}$$

$$M^2_{\sigma_d(\tilde{\nu}_i^c)^{\mathrm{Im}}} = \mathrm{Re}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})(A_\lambda\lambda)_i\upsilon_u + 2\mathrm{Re}(e^{-i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}}e^{i\phi_{v_{\nu_j^c}}}\lambda_k^*\kappa_{ijk})\upsilon_u\upsilon_{\nu_i^c} + \Delta_{6(7+i)}, \quad \text{(A23)}$$

$$M^2_{\sigma_u(\tilde{\nu}_i^c)^{\mathrm{Im}}} = -\mathrm{Re}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})(A_\lambda\lambda)_i\upsilon_d - 2\mathrm{Re}(e^{-i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}}e^{i\phi_{v_{\nu_j^c}}}\lambda_k^*\kappa_{ijk})\upsilon_d\upsilon_{\nu_i^c} + \Delta_{7(7+i)}, \quad \text{(A24)}$$

$$M^2_{(\tilde{\nu}_i^c)^{\mathrm{Im}}(\tilde{\nu}_i^c)^{\mathrm{Im}}} = \mathrm{Re}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})(A_\lambda\lambda)_i\frac{\upsilon_d\upsilon_u}{\upsilon_{\nu_i^c}} - (A_\kappa\kappa)_{ijk}\mathrm{Re}(e^{i\phi_{v_{\nu_i^c}}}e^{i\phi_{v_{\nu_i^c}}}e^{i\phi_{v_{\nu_i^c}}})\upsilon_{\nu_k^c}$$
$$+ 4\mathrm{Re}(e^{-i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}}e^{i\phi_{v_{\nu_j^c}}}\lambda_k^*\kappa_{ijk})\upsilon_d\upsilon_u + \Delta_{(7+i)(7+i)}, \quad \text{(A25)}$$

$$M^2_{(\tilde{\nu}_i^c)^{\mathrm{Im}}(\tilde{\nu}_j^c)^{\mathrm{Im}}} = |\lambda_i|^2(\upsilon_u^2 + \upsilon_d^2) + \Delta_{(7+i)(7+i)}, \quad \text{(A26)}$$

$$\Delta^t_{66} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2\sin^2\beta}\frac{(-\mathrm{Im}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_t\mu)^2}{(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})^2}g(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}), \quad \text{(A27)}$$

$$\Delta^t_{67} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \frac{\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \mu \cot\beta}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \frac{-\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \mu}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} g(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}), \tag{A28}$$

$$\Delta^t_{77} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \frac{(\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \mu \cot\beta)^2}{(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})^2} g(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}), \tag{A29}$$

$$\Delta^t_{6(7+i)} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \frac{-\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \mu}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \frac{\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \lambda_i v_d}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} g(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}), \tag{A30}$$

$$\Delta^t_{7(7+i)} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \frac{\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \mu \cot\beta}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \frac{-\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \lambda_i v_d}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} g(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}), \tag{A31}$$

$$\Delta^t_{(7+i)(7+j)} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \frac{(-\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \lambda_i v_d)^2}{(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})^2} g(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}), \tag{A32}$$

$$\Delta^b_{66} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \cos^2\beta} \frac{(-\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \mu \tan\beta)^2}{(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2})^2} g(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}), \tag{A33}$$

$$\Delta^b_{67} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \cos^2\beta} \frac{-(\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \mu)^2 \tan\beta}{(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2})^2} g(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}), \tag{A34}$$

$$\Delta^b_{77} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \cos^2\beta} \frac{(\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \mu)^2}{(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2})^2} g(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}), \tag{A35}$$

$$\Delta^b_{6(7+i)} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \cos^2\beta} \frac{-(\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b)^2 \mu \tan\beta \lambda_i v_u}{(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2})^2} g(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}), \tag{A36}$$

$$\Delta^b_{7(7+i)} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \cos^2\beta} \frac{(\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b)^2 \mu \lambda_i v_u}{(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2})^2} g(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}), \tag{A37}$$

$$\Delta^b_{(7+i)(7+j)} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \cos^2\beta} \frac{(\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \lambda_i v_u)^2}{(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2})^2} g(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}). \tag{A38}$$

The mix mass submatrix M^2_{SP}

$$M^2_{SP} = \begin{pmatrix} M^2_{h_d\sigma_d} & M^2_{h_d\sigma_u} & M^2_{h_d(\tilde{\nu}_1^c)^{\mathrm{Im}}} & M^2_{h_d(\tilde{\nu}_2^c)^{\mathrm{Im}}} & M^2_{h_d(\tilde{\nu}_3^c)^{\mathrm{Im}}} \\ M^2_{h_u\sigma_d} & M^2_{h_u\sigma_u} & M^2_{h_u(\tilde{\nu}_1^c)^{\mathrm{Im}}} & M^2_{h_u(\tilde{\nu}_2^c)^{\mathrm{Im}}} & M^2_{h_u(\tilde{\nu}_3^c)^{\mathrm{Im}}} \\ M^2_{(\tilde{\nu}_1^c)^{\mathrm{Re}}\sigma_d} & M^2_{(\tilde{\nu}_1^c)^{\mathrm{Re}}\sigma_u} & M^2_{(\tilde{\nu}_1^c)^{\mathrm{Re}}(\tilde{\nu}_1^c)^{\mathrm{Im}}} & M^2_{(\tilde{\nu}_1^c)^{\mathrm{Re}}(\tilde{\nu}_2^c)^{\mathrm{Im}}} & M^2_{(\tilde{\nu}_1^c)^{\mathrm{Re}}(\tilde{\nu}_3^c)^{\mathrm{Im}}} \\ M^2_{(\tilde{\nu}_2^c)^{\mathrm{Re}}\sigma_d} & M^2_{(\tilde{\nu}_2^c)^{\mathrm{Re}}\sigma_u} & M^2_{(\tilde{\nu}_2^c)^{\mathrm{Re}}(\tilde{\nu}_1^c)^{\mathrm{Im}}} & M^2_{(\tilde{\nu}_2^c)^{\mathrm{Re}}(\tilde{\nu}_2^c)^{\mathrm{Im}}} & M^2_{(\tilde{\nu}_2^c)^{\mathrm{Re}}(\tilde{\nu}_3^c)^{\mathrm{Im}}} \\ M^2_{(\tilde{\nu}_3^c)^{\mathrm{Re}}\sigma_d} & M^2_{(\tilde{\nu}_3^c)^{\mathrm{Re}}\sigma_u} & M^2_{(\tilde{\nu}_3^c)^{\mathrm{Re}}(\tilde{\nu}_1^c)^{\mathrm{Im}}} & M^2_{(\tilde{\nu}_3^c)^{\mathrm{Re}}(\tilde{\nu}_2^c)^{\mathrm{Im}}} & M^2_{(\tilde{\nu}_3^c)^{\mathrm{Re}}(\tilde{\nu}_3^c)^{\mathrm{Im}}} \end{pmatrix}, \tag{A39}$$

$$M^2_{h_d\sigma_d} = \Delta_{16}, \tag{A40}$$

$$M^2_{h_d\sigma_u} = \Delta_{17}, \tag{A41}$$

$$M^2_{h_u\sigma_d} = \Delta_{26}, \tag{A42}$$

$$M^2_{h_u\sigma_u} = \Delta_{27}, \tag{A43}$$

$$M^2_{h_d(\tilde{\nu}_i^c)^{\rm Im}} = -{\rm Im}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})(A_\lambda\lambda)_i v_u + \Delta_{1(7+i)}, \tag{A44}$$

$$M^2_{h_u(\tilde{\nu}_i^c)^{\rm Im}} = -{\rm Im}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})(A_\lambda\lambda)_i v_d + \Delta_{2(7+i)}, \tag{A45}$$

$$M^2_{(\tilde{\nu}_i^c)^{\rm Re}\sigma_d} = -{\rm Im}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})(A_\lambda\lambda)_i v_u + \Delta_{(2+i)6}, \tag{A46}$$

$$M^2_{(\tilde{\nu}_i^c)^{\rm Re}\sigma_u} = {\rm Im}(e^{i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})(A_\lambda\lambda)_i v_d + \Delta_{(2+i)7}, \tag{A47}$$

$$M^2_{(\tilde{\nu}_i^c)^{\rm Re}(\tilde{\nu}_i^c)^{\rm Im}} = 2{\rm Im}(e^{-i\phi_\lambda}e^{-i\phi_{v_u}}e^{i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}}e^{i\phi_{v_{\nu_j^c}}}\lambda_k^*\kappa_{ijk})v_d v_u - (A_\kappa\kappa)_i{\rm Re}(e^{i\phi_{v_{\nu_i^c}}}e^{i\phi_{v_{\nu_i^c}}}e^{i\phi_{v_{\nu_i^c}}})v_{\nu_i^c} + \Delta_{(2+i)(7+i)}, \tag{A48}$$

$$M^2_{(\tilde{\nu}_i^c)^{\rm Re}(\tilde{\nu}_j^c)^{\rm Im}} = \Delta_{(2+i)(7+i)}, \tag{A49}$$

$$\Delta^t_{16} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2\sin^2\beta}\frac{|\mu|^2\cot\beta - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_t\mu}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}\frac{-{\rm Im}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_t\mu}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}g(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}), \tag{A50}$$

$$\Delta^t_{17} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2\sin^2\beta}\frac{|\mu|^2\cot\beta - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_t\mu}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}\frac{{\rm Im}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_t\mu\tan\beta}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}g(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}), \tag{A51}$$

$$\Delta^t_{26} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2\sin^2\beta}\frac{-{\rm Im}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_t\mu}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}} \times\left\{\ln\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}} + \frac{|A_t|^2 - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_t\mu\cot\beta}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}g(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2})\right\}, \tag{A52}$$

$$\Delta^t_{27} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2\sin^2\beta}\frac{{\rm Im}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_t\mu\cot\beta}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}} \times\left\{\ln\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}} + \frac{|A_t|^2 - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_t\mu\cot\beta}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}g(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2})\right\}, \tag{A53}$$

$$\Delta^t_{1(7+i)} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2\sin^2\beta}\frac{|\mu|^2\cot\beta - {\rm Re}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_t\mu}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}} \times\frac{-{\rm Im}(e^{i\phi_\lambda}e^{i\phi_{A_t}}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_t\lambda_i v_d}{m^2_{\tilde{t}_1}-m^2_{\tilde{t}_2}}g(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}), \tag{A54}$$

$$\Delta^t_{2(7+i)} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \frac{-\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \lambda_i v_d}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \times \left\{ \ln \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} + \frac{|A_t|^2 - \mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \mu \cot\beta}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right\}, \tag{A55}$$

$$\Delta^t_{(2+i)6} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \frac{-\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \mu}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \times \frac{\frac{1}{2} v_{\nu_i^c} v_d (\lambda_i \lambda_j^* + \lambda_i^* \lambda_j) - \mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \lambda_i v_d}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2), \tag{A56}$$

$$\Delta^t_{(2+i)7} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \frac{\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \mu \cot\beta}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \times \frac{\frac{1}{2} v_{\nu_i^c} v_d (\lambda_i \lambda_j^* + \lambda_i^* \lambda_j) - \mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \lambda_i v_d}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}, \tag{A57}$$

$$\Delta^t_{(2+i)(7+i)} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2\beta} \frac{-\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \lambda_i v_d}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \times \frac{\frac{1}{2} v_{\nu_i^c} v_d (\lambda_i \lambda_j^* + \lambda_i^* \lambda_j) - \mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{A_t}} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_t \lambda_i v_d}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}, \tag{A58}$$

$$\Delta^b_{16} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \cos^2\beta} \frac{-\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \mu \tan\beta}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} \times \left\{ \ln \frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_2}^2} + \frac{|A_b|^2 - \mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \mu \tan\beta}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \right\}, \tag{A59}$$

$$\Delta^b_{17} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \cos^2\beta} \frac{\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \mu}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \left\{ \ln \frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_2}^2} + \frac{|A_b|^2 - \mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \mu \tan\beta}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} \right\}, \tag{A60}$$

$$\Delta^b_{26} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \cos^2\beta} \frac{-\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \mu \tan\beta}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} \frac{|\mu|^2 \tan\beta - \mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \mu}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2), \tag{A61}$$

$$\Delta^b_{27} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \cos^2\beta} \frac{\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \mu}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \frac{|\mu|^2 \tan\beta - \mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \mu}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2}, \tag{A62}$$

$$\Delta^b_{1(7+i)} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \cos^2\beta} \frac{\mathrm{Im}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \lambda_i v_u}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \times \left\{ \ln \frac{m_{\tilde{b}_1}^2}{m_{\tilde{b}_2}^2} + \frac{|A_b|^2 - \mathrm{Re}(e^{i\phi_\lambda} e^{i\phi_{v_u}} e^{-i\phi_{v_d}} e^{i\phi_{v_{\nu_i^c}}}) A_b \mu \tan\beta}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} \right\}, \tag{A63}$$

$$\Delta^b_{2(7+i)} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2\cos^2\beta}\frac{\mathrm{Im}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\lambda_i v_u}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}g(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2})\frac{|\mu|^2\tan\beta - \mathrm{Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\mu}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}, \tag{A64}$$

$$\Delta^b_{(2+i)6} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2\cos^2\beta}\frac{-\mathrm{Im}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\mu\tan\beta}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}g(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2})$$
$$\times\frac{\frac{1}{2}v_{\nu_i^c}v_u(\lambda_i\lambda_j^* + \lambda_i^*\lambda_j)\tan\beta - \mathrm{Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\lambda_i v_u}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}, \tag{A65}$$

$$\Delta^b_{(2+i)7} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2\cos^2\beta}\frac{\mathrm{Im}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\mu}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}g(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2})$$
$$\times\frac{\frac{1}{2}v_{\nu_i^c}v_u(\lambda_i\lambda_j^* + \lambda_i^*\lambda_j)\tan\beta - \mathrm{Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\lambda_i v_u}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}, \tag{A66}$$

$$\Delta^b_{(2+i)(7+j)} = \frac{3G_F m_b^4}{2\sqrt{2}\pi^2\cos^2\beta}\frac{\mathrm{Im}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\lambda_i v_u}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}g(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2})$$
$$\times\frac{\frac{1}{2}v_{\nu_i^c}v_u(\lambda_i\lambda_j^* + \lambda_i^*\lambda_j)\tan\beta - \mathrm{Re}(e^{i\phi_\lambda}e^{i\phi_{v_u}}e^{-i\phi_{v_d}}e^{i\phi_{v_{\nu_i^c}}})A_b\lambda_i v_u}{m^2_{\tilde{b}_1}-m^2_{\tilde{b}_2}}. \tag{A67}$$

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