UV finite GUT with SUSY breaking

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We construct an example of a safe ultraviolet finite supersymmetric renormalizable SO(10) grand unified theory featuring a supersymmetric dynamical breaking mechanism. Our results constitute a step forward towards an ultraviolet safe extension of the Standard Model enjoying the benefits of grand unified theories.

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I. INTRODUCTION

Grand unified theories (GUT)s [1,2] constitute one of the main guiding principles to construct extensions of the Standard Model (SM) with supersymmetric versions [3] being among the most celebrated examples because they more naturally fit into the unification paradigm. However, successful models of supersymmetric (SUSY) grand unification typically feature a large number of matter fields that modify the UV character of the GUT with a loss of asymptotic freedom and the appearance of Landau poles below the Planck scale as reviewed in [4]. One can envision two ways to amend this issue. The first is to hope that (super) gravity itself could eventually intervene and rescue the high energy behavior of the theory. However, in this case the loss of asymptotic freedom is so severe that the resulting Landau pole occurs at energies that are orders of magnitudes lower than the scale where quantum gravity could be able to influence the theory.[1] A second way to address the original problem is to search for UV finite GUTs where asymptotic freedom is replaced by interacting UV fixed points [4]. Following S. Weinberg [7] achieving UV finiteness via interacting fixed points can also be referred to as asymptotically safe (or more simply safe) theories.[2] The existence of four-dimensional gauge theories featuring fermions and scalars was established in [8] along with the stability of their ground state [9]. We will concentrate here on the super safe GUT possibility and associated SUSY breaking in order to realize the Standard Model at low energies. Nonsupersymmetric GUTs with interacting UV fixed points were investigated in [10,11] and for extra dimensions in [12].

Supersymmetric nonperturbative fixed points of safe nature were investigated in [4,13,14] via a number of mathematical tools ranging from the a-maximization [15] to unitary constraints [16,17] and positivity bounds [18,19]. Using these methods we discovered in [4] one popular supersymmetric GUT that can develop safety in the UV, i.e., the renormalizable SUSY SO(10) [20–22] theory, which we upgrade here to the version [23] in order to satisfy the correct neutrino mass spectrum [24–27]. This will be the starting point of our analysis. The first step towards constructing a viable extension of the Standard Model is to include a supersymmetric breaking mechanism. This step however typically modifies the UV behavior of the theory potentially offsetting the safe nature of the model. We have therefore carefully considered different options and discovered that one can lead to SUSY breaking compatible with the safe nature of the overall theory. More specifically we will see that the viable mechanism requires first the spontaneous breaking of SO(10) to SUSY SU(5) and then SUSY can be dynamically broken. The model, however, still predicts one massless fermion generation [4] which deserves a separate study.

Summarizing, the novel part of our work consists in providing the first example of a GUT featuring a consistent SUSY breaking mechanism compatible with the safe nature of the theory.

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[1]According to [5,6] though, the real gravitational cutoff is M_{Planck} divided by the square root of the number of degrees of freedom.

[2]This means *saving* them from the occurrence of Landau poles.

The plan of the work is the following. In Sec. I we summarize the results of [4] and present the safe SUSY GUT model we will be using for the rest of the work. We then move to the introduction of the SUSY breaking sector in Sec. III following the radiative paradigm [28,29]. We first show that it is first needed to break SO(10) to SU(5) and then break supersymmetry radiatively following [30]. We further show in Sec. IV that adding the SUSY breaking sector is compatible with the overall safety of the model. We finally provide our conclusions in Sec. V.

II. A SAFE SO(10)

The safe model we considered in [4] is renormalizable and features large representations [20–22] of the gauge group. It consists of three matter fields 16-dimensional spinorial representations, plus the Higgs sector represented by the 10, 126, $\overline{126}$, and 210. The renormalizable Higgs superpotential is

$$\begin{aligned} W_H = {} & m_{10}10^2 + m_{210}210^2 + m_{126}126\,\overline{126} \\ & + \lambda_1 210^3 + \lambda_2 210\,126\,\overline{126} + \lambda_3 210\,10\,126 \\ & + \lambda_4 210\,10\,\overline{126} \end{aligned} \tag{2.1}$$

while the Yukawa part of the superpotential is

$$W_{\text{Yukawa}} = 16_i(Y_{10}10 + Y_{126}\overline{126})_{ij}16_j \tag{2.2}$$

where $i, j = 1, \ldots 3$ are generation indices. This model has been called minimal for some time, until it was shown not to be in accord with experiments due to the presence of too small neutrino masses [24–27]. This issue is amended by adding an extra 54 representation to the spectrum of the theory [23], with the added superpotential

$$\begin{aligned} W_{H54} = {} & m_{54}54^2 + \lambda_5 54^3 + \lambda_6 54\,210^2 + \lambda_7 54\,126^2 \\ & + \lambda_8 54\,\overline{126}^2 + \lambda_9 54\,10^2. \end{aligned} \tag{2.3}$$

The modified theory is still compatible with the presence of a UV safe fixed point. The reason being that the new superfield has R charge $2/3$ which modifies only slightly the large R charge of the 16_1 superfield fixed by the Novikov, Shifman, Vainshtein, Zakharov (NSVZ) constraint [31]:

$$R(16_1) = \frac{125}{6} \tag{2.4}$$

instead of $113/6$ from [4]. With this R charge and the R charges of all the other superfields equal to $2/3$ the a-theorem is satisfied:

$$\Delta a = a_{\text{UV}} - a_{\text{IR}} = 3.74 \times 10^5 > 0 \tag{2.5}$$

instead of 2.72×10^5 from [4]. This solution requires summing in (2.2) over only the second and third generations, predicting one massless generation. In the future one may consider a more involved model where also a mass for the first generation is generated upon supersymmetry breaking via new Kähler induced potential terms which do not interfere with the main results of this work.

In the reminder of this work we focus on adding the supersymmetry breaking dynamics that is a crucial step for any supersymmetric safe extension of the Standard Model, paying attention to how it impacts the UV nature of the theory.

III. BREAKING SUPERSYMMETRY SAFELY

We employ in this work the class of supersymmetry breaking scenarios known as gauge mediation [32] following closely reference [30]. According to this paradigm supersymmetry breaking occurs radiatively in a distinct sector of the theory which develops a nonsupersymmetric metastable vacuum. Whether this metastable local minimum of the full potential exists depends on the details of the model. We have the further restriction that the model must still be overall safe in the UV.

We will therefore first investigate whether SO(10) dynamics can lead to radiative supersymmetry breaking and show that the large number of matter fields hampers the result. We are led to study the case in which SO(10) spontaneously breaks first to SU(5) and then the latter radiatively breaks supersymmetry and show that the model is viable.

Before diving into the details of the specific models relevant for this work we sketch the radiative SUSY breaking scenario. In its minimal version the mechanism employs two nonsinglet superfields Φ_1 and Φ_2 with the following gauge-invariant superpotential:

$$W_{SB} = \mu\Phi_1\Phi_2 + \lambda\Phi_1^2\Phi_2. \tag{3.1}$$

The SUSY preserving global minimum occurs for $\Phi_2 = 0$ and Φ_1 as solution of $\partial W_{SB}/\partial\Phi_2 = 0$. Another possible local minimum can appear, depending on the details of the theory, for Φ_1 being a solution of $\partial W_{SB}/\partial\Phi_1 = 0$ and Φ_2 a classical flat direction. The latter is the metastable vacuum we are interested in, provided it is not a maximum in the Φ_2 direction at the quantum level. Specifically, following reference [28–30], the potential evaluated at the SUSY breaking extremum can be approximated to be:

$$V \approx \frac{|F_2|^2}{Z_2}, \tag{3.2}$$

with F_2 the auxiliary component of the Φ_2 superfield evaluated at the extremum and Z_2 the square of its wave function renormalization. Clearly, different models predict distinct quantum corrections which we shall compute in the examples below.

A. SO(10) with two 54s

To apply the SUSY breaking mechanism we sketched above we introduce in SO(10) two nongauge singlet fields that we adopt to construct the needed quadratic and cubic gauge invariant terms. The minimal dimension of the fields are then 54 to be able to construct the following superpotential

$$W_{SB} = \mu(54_1)_{AB}(54_2)_{BA} + \lambda(54_1)_{AB}(54_1)_{BC}(54_2)_{CA}, \quad (3.3)$$

where the indices A, B, and C run from one to ten.[3] To explicitly test the viability of this construction we need to compute the running of Z_2 which depends on the beta function for λ and the SO(10) gauge coupling g_{10}. The relevant system of beta functions is:

$$\frac{dg_{10}^2}{d\tau} = 133 g_{10}^4 \quad (3.4)$$

$$\frac{d(\log\lambda^2)}{d\tau} = -60 g_{10}^2 + 28\lambda^2 \quad (3.5)$$

$$\frac{d(\log Z_2)}{d\tau} = 20 g_{10}^2 - \frac{28}{5}\lambda^2 \quad (3.6)$$

with $\tau = \log\mu/(8\pi^2)$. To determine whether the potential at the nonsupersymmetric extremum in (3.2) is a minimum we need the first derivative of Z_2 to vanish and the second to be negative. The first condition relates λ and g_{10} as follows:

$$\lambda^2 = \frac{20}{\frac{28}{5}} g_{10}^2, \quad (3.7)$$

while the second condition is not satisfied since

$$\frac{1}{Z_2}\frac{d^2 Z_2}{d\tau^2} = 20\left(133 + 60 - \frac{28\times 20}{\frac{28}{5}}\right) g_{10}^4 = 1860 g_{10}^4 > 0. \quad (3.8)$$

This shows that the nonsupersymmetric extremum is, in this case, a maximum. This occurs because the one-loop coefficient for g_{10} has large contributions coming from the matter field content of the theory while the negative contribution stemming from the λ coupling is insufficient to offset the gauge-coupling contribution. One could imagine considering different representations for the SUSY breaking sector fields $\Phi_{1,2}$ however, for SO(10), it would involve even higher gauge-group representations. The latter would typically increase the contribution to the coefficient of the gauge beta function in such a way that the maximum cannot be turned into a minimum. An explicit computation along the lines shown above confirms this expectation.

We know, however, that it is possible to break supersymmetry with the mechanism discussed above in SU(5) models of grand unification [30]. All we need to do is to allow for spontaneous breaking of SO(10) to SU(5) and enact SUSY breaking at this latter stage. Fortunately, such a breaking is allowed and has already been investigated in the literature [33,34]. To further reduce the number of degrees of freedom for the target SU(5) we add the following operator

$$W_\eta = \eta \mathrm{Tr}(54_1 54_2 45), \quad (3.9)$$

featuring a new 45 superfield that has the task, after spontaneous symmetry breaking, to give mass to the two $(15 - \overline{15})$ s inside the 54s leaving behind two light 24s of SU(5) needed to break supersymmetry.[4] We add to the original superpotential (2.1), (2.3) also terms involving this new 45:

$$\delta W_{45} = m_{45}^2 45^2 + \lambda' 45^2 210 + \lambda'' 45\, 210^2 + \lambda''' 45\, 126\, \overline{126} + \lambda'''' 45^2 54. \quad (3.10)$$

It is shown in [34] that the vacuum still aligns in the SU(5) intact direction.

B. Intermediate SU(5) with two 24

We are now left, at an intermediate energy, with an SU(5) model featuring two extra light 24 superfields that, following reference [30], can be used to break supersymmetry dynamically with the part of the superpotential relevant for computing the contributions to the relevant beta functions that reads:

$$W_{SB} = \lambda Tr(24_2 24_1^2), \quad (3.11)$$

yielding[5]

$$\frac{dg_5^2}{d\tau} = 7 g_5^4 \quad (3.12)$$

$$\frac{d(\log\lambda^2)}{d\tau} = -30 g_5^2 + 21\lambda^2 \quad (3.13)$$

$$\frac{d(\log Z_2)}{d\tau} = 10 g_5^2 - \frac{21}{5}\lambda^2. \quad (3.14)$$

[3]For the model to be phenomenologically viable quartic operators should be added as shown in [30], which however don't affect what we are going to discuss below. We will come back to this point in the next sections.

[4]We stress that the presence of this term, being linear in the 45, does not affect the vacuum structure of the SO(10) model.

[5]The difference in the one-loop coefficients with respect to reference [30] derives from the fact that we have extra 24s in the SU(5) spectrum.

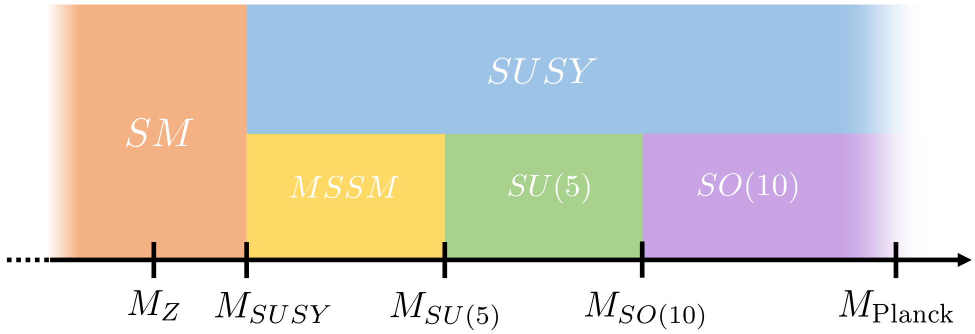

FIG. 1. Visualization of the relevant energy scales and their ordering: The onset of the SO(10) UV finite theory occurs at $M_{\mathrm{SO}(10)}$ and below this scale the theory breaks to SUSY SU(5). The minimal supersymmetric standard model (MSSM) occurs below $M_{\mathrm{SU}(5)}$ with soft masses appearing at M_{SUSY} below which one recovers the Standard Model particle states. The SUSY breaking mechanism operates at $M_{\mathrm{SU}(5)}$ generating the soft scale M_{SUSY}.

Vanishing of the first derivative of Z_2 requires:

$$\lambda^2 = \frac{10}{\frac{21}{5}} g_5^2, \tag{3.15}$$

while the value of the second derivative is now:

$$\frac{1}{Z_2}\frac{d^2 Z_2}{d\tau^2} = 10\left(7 + 30 - \frac{21\times 10}{\frac{21}{5}}\right) g_5^4 = -130 g_5^4 < 0. \tag{3.16}$$

The result guarantees that we have now a local minimum of the effective potential and that therefore we can successfully beak supersymmetry.

IV. BROKEN BUT SAFE

Here we check that the whole model, including the SUSY breaking sector, is still ultraviolet finite. To do so we first extend W_{SB} (3.3) to its more complete version [30]:

$$\begin{aligned} W_{SB-\mathrm{complete}} = \mathrm{Tr}(&-M\Omega_1\Omega_2 + \Omega_1(\mu_1 54_2 + \eta_1 54_1 54_2) \\ &+ \Omega_2(\mu_2 54_1 + \eta_2 54_1^2)), \end{aligned} \tag{4.1}$$

where $\Omega_{1,2}$ are new 54 heavy dimensional fields yielding the old W_{SB} of (3.3) upon being integrated out. We note that, while the original SUSY breaking potential is sufficient to break supersymmetry it predicts light states upsetting the coupling unification. This generalization above lifts the light states to the grand unified scale thereby preserving the coupling unification [30].

Before continuing let us summarize the scales involved. Starting from the UV, the highest scale in the superpotential is M in Eq. (4.1), which is taken to be bigger than the SO(10) scale, generated by the mass scale m_{210} in Eq. (2.1). The masses $m_{10,126}$ in (2.1), m_{54} in Eq. (2.3), m_{45} in Eq. (3.10), and $\mu_{1,2}$ in Eq. (4.1) are also of the order of m_{210}. The ratios $\mu_{1,2}^2/M$ define the SU(5) scale, while the value for F which breaks supersymmetry comes from a combination of $\mu_{1,2}$, M, and $\eta_{1,2}$ parameters appearing in Eq. (4.1), as shown in [30]. See Fig. 1 for the visualization of the relevant energy scales and their ordering.

The model that needs to be safe compared to the original one [4] that motivated this work contains now five more 54 superfields, one more 45, and the addition of the three superpotentials W_{H54} of (2.3), $W_{SB-\mathrm{complete}}$ of (4.1), and W_η in (3.9). Naturally, we also have to allow for the operators mixing the new 45 with the original fields (210, 126, $\overline{126}$, 10, and 54), i.e., δW_{45} in Eq. (3.10).

Since mass operators are irrelevant in the UV it is sufficient to consider in the UV only the trilinear terms. Given the promiscuous nature of the 45 superfield and the already established R charge for the original Higgs superfields mentioned above (210, 126, $\overline{126}$, 10 and 54), one can show that

$$R(45) = R(\Omega_1) = \frac{2}{3}, \tag{4.2}$$

where the second identity comes from comparing (3.9) with (4.1) yielding the following relations for the remaining superfields

$$R(54_2) = \frac{4}{3} - R(54_1), \qquad R(\Omega_2) = 2 - 2R(54_1). \tag{4.3}$$

Employing a-maximization for

$$a_1(R(54_1)) + a_1(R(54_2)) + a_1(R(\Omega_2)) \tag{4.4}$$

yields, in the UV, $R(54_2) = 2/3$. Therefore the overall modification to Δa with respect to the theory without SUSY breaking of [4] comes from the running of the couplings encoded in the NSVZ [31] SUSY beta function and yields:

$$R(16_1) = \frac{181}{6}, \qquad \Delta a = 1.19\times 10^6 \tag{4.5}$$

compared to $R(16_1) = 125/6$ and $\Delta a = 3.74 \times 10^5$ of the SUSY unbroken case in Eqs. (2.4) and (2.5). Overall, our result shows that the SUSY breaking sector can naturally be included in a UV safe theory without profoundly upsetting its nature.

V. CONCLUSIONS

We constructed an example of an ultraviolet super conformal SO(10) grand unified theory that dynamically breaks supersymmetry down to the Standard Model [4]. We first observed that SO(10) cannot break supersymmetry dynamically because its quantum corrected potential does not feature a nonsupersymmetric local minimum and then we showed that spontaneously breaking SO(10) to supersymmetric SU(5) allows us to use the latter dynamics to break supersymmetry. We summarize in Fig. 1 the relevant energy scales and their ordering.

We have therefore shown that one can now employ wider classes of supersymmetric grand unified theories allowing for ultraviolet interacting nonperturbative fixed points dynamically breaking supersymmetry at intermediate energies with the Standard Model in the infrared. In the future it would be interesting to investigate the E6 super grand unified theory and more generally the overall approach embedding into superstrings.

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