Investigating the M1 radiative decay behaviors and the magnetic moments of the predicted triple-charm molecular-type pentaquarks

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In this work, we systematically study the electromagnetic properties including the M1 radiative decay widths and the magnetic moments of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates, where we adopt the constituent quark model and consider both the $S-D$ wave mixing effect and the coupled channel effect. Our numerical results suggest that the M1 radiative decay widths and the magnetic moments of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates can reflect their inner structures, and the study of the electromagnetic properties is an important step to construct the family of the triple-charm molecular-type pentaquarks. With the accumulation of the experimental data during the high-luminosity phase of LHC, we expect that the present work combined with the corresponding mass spectrum information can encourage the experimental colleagues at LHCb to focus on the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates.

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I. INTRODUCTION

With the observation of more and more new hadronic states [1–11], the study of hadron spectroscopy has entered a new era, representing the high-precision frontier of particle physics. A key objective of these investigations is to provide crucial information to deepen our understanding of the nonperturbative behavior of the strong interaction. It is still full of challenges and opportunities to the whole community.

In the past decade, big progress has been made in exploring the hidden-charm molecular-type pentaquarks, thanks to the joint efforts of both theoretical [12–18] and experimental [19–22] sides. Staying in the new stage, more predictions around molecular-type pentaquarks have been given [1–11]. Among them, a typical example is the predicted triple-charm molecular-type pentaquark [23,24], which is due to the interaction of a double-charm baryon and a charmed meson. Indeed, this study is also motivated by the observation of the doubly charmed baryon $\Xi_{cc}(3620)^{++}$ reported by the LHCb Collaboration in 2017 [25]. Although the mass spectrum of triple-charm molecular-type pentaquarks has been given [23,24], we still need to make more efforts to provide theoretical suggestions for searching for them.

We can note a recent development in the study of the spectroscopy of molecular-type pentaquarks. For example, in Refs. [26–42], the authors suggested that the electromagnetic properties of molecular-type pentaquarks should be paid more attention. One main reason is that the electromagnetic properties, such as the magnetic moments of the hadron, may reflect its inner structure. More importantly, their M1 radiative decays are accessible in experiments.

Along this line, in this work we carry out the investigation of the electromagnetic properties of the predicted triple-charm molecular-type pentaquarks [23,24]. We focus mainly on quantitative calculation of their M1 radiative decay behavior, since these physical quantities can be

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measured in experiments such as LHCb during the high-luminosity phase of the LHC [43]. Although it is a challenging task to directly measure their magnetic moments, this does not prevent us from carrying out the phenomenological study of this question. We hope that the present work, combined with earlier work [23,24] on the corresponding mass spectrum, will provide more complete spectroscopic information on triple-charm molecular-type pentaquarks.

The rest of this paper is organized as follows. In Sec. II, we focus mainly on quantitative calculation of the M1 radiative decay widths of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates. In Sec. III, we discuss the magnetic moments of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates. Finally, we give the discussion and conclusion in Sec. IV.

II. THE M1 RADIATIVE DECAY WIDTHS OF THE TRIPLE-CHARM MOLECULAR-TYPE PENTAQUARKS

In Refs. [23,24], the mass spectrum of the $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquarks was predicted based on the one-boson-exchange model, taking into account the $S-D$ wave mixing effect and the coupled channel effect. Usually, the loosely bound state with the cutoff value around 1 GeV can be considered as the most promising molecular candidate according to the experience of deuteron studies [3,23,44–49]; the authors suggested that the $\Xi_{cc}D$ state with $I(J^P)=0(1/2^-)$, the $\Xi_{cc}D^*$ state with $I(J^P)=0(3/2^-)$, the $\Xi_{cc}D_1$ states with $I(J^P)=0(1/2^+,3/2^+)$, and the $\Xi_{cc}D_2^*$ states with $I(J^P)=0(3/2^+,5/2^+)$ can be recommended as the most promising candidates of the triple-charm molecular-type pentaquarks [23,24]. At present, our understanding of the properties of the triple-charm molecular-type pentaquark candidates is not sufficient. Thus, it is necessary to provide more comprehensive suggestions to encourage the experimental colleagues to explore the triple-charm molecular-type pentaquarks.

The study of the electromagnetic properties, such as the radiative decay width and the magnetic moment, can provide essential insights for understanding the inner structures of the hadronic states, which is crucial for distinguishing the spin-parity quantum numbers and the configurations of the hadronic states. Therefore, the study of the electromagnetic properties is an important step for the experimental construction of the hadron family. For example, the ratio $R_{\gamma\psi}=\frac{\mathcal{B}[X(3872)\to\gamma\psi(2S)]}{\mathcal{B}[X(3872)\to\gamma J/\psi]}$ is essential for understanding the inner structure of the charmoniumlike state $X(3872)$ [50–53]. In this study, we investigate the electromagnetic properties including the M1 radiative decay widths and the magnetic moments of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates based on their mass spectra and spatial wave functions [23,24]. In our concrete calculations, we take the constituent quark model, which is a reliable tool for discussing the electromagnetic properties of the hadrons in the past decades [26,27,30–32,35,36,54–93], particularly successfully reproducing the experimental data of the magnetic moments of the decuplet and octet baryons [54,63].

In this section, we will study the M1 radiative decay widths of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates. In the calculations, the M1 radiative decay width between the hadronic states $\Gamma_{H\to H'\gamma}$ can be related to the corresponding transition magnetic moment $\mu_{H\to H'}$ [27,30,32,35,36,65,71,73,77,79–84,88–91,93,94], which can be expressed as [32,36,37,93]

$$\Gamma_{H\to H'\gamma}=\frac{\alpha_{\rm EM}}{2J_H+1}\frac{4k^3}{e^2}\frac{\sum_{J_{H'z},J_{Hz}}\begin{pmatrix} J_{H'} & 1 & J_H \\ -J_{H'z} & 0 & J_{Hz}\end{pmatrix}^2}{\begin{pmatrix} J_{H'} & 1 & J_H \\ -J_z & 0 & J_z\end{pmatrix}^2}|\mu_{H\to H'}|^2. \tag{2.1}$$

In the above expression, $\alpha_{\rm EM}=1/137$ is the electromagnetic fine structure constant, $k=(m_H^2-m_{H'}^2)/(2m_H)$ is the momentum of the emitted photon, and the symbol $\left(\begin{smallmatrix} a & b & c \\ d & e & f\end{smallmatrix}\right)$ is the $3-j$ coefficient.

Based on the above discussions, it is necessary to calculate the corresponding transition magnetic moment when discussing the M1 radiative decay width between the hadronic states. At present, there is a lack of experimental data for the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates [95], but the transition magnetic moment and the M1 radiative decay width between the hadronic molecular states depend on the binding energies of the initial and final molecules [32,36,37,93]. For the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates, their transition magnetic moments can be determined by the following equation [32,36,37,93]:

$$\mu_{H\to H'}=\left\langle J_{H'},J_z\left|\sum_j\hat{\mu}_{zj}^{\rm spin}e^{-i\mathbf{k}\cdot\mathbf{r}_j}+\hat{\mu}_z^{\rm orbital}\right|J_H,J_z\right\rangle^{J_z={\rm min}\{J_H,J_{H'}\}} \tag{2.2}$$

when taking the same binding energy for the initial and final triple-charm molecular-type pentaquark candidates. Here, the magnetic moment operators $\hat{\mu}_{zj}^{\rm spin}$ and $\hat{\mu}_z^{\rm orbital}$ can be expressed as [26,27,30–32,35–37,54–93]

$$\hat{\mu}_{zj}^{\rm spin}=\frac{e_j}{2m_j}\hat{\sigma}_{zj}, \tag{2.3}$$

$$\hat{\mu}_z^{\rm orbital} = \left(\frac{m_m}{m_b+m_m}\frac{e_b}{2m_b} + \frac{m_b}{m_b+m_m}\frac{e_m}{2m_m}\right)\hat{L}_z, \quad (2.4)$$

respectively, where $\hat{\sigma}_{zj}$, e_j, and m_j represent the z component of the Pauli operator, the charge, and the mass of the jth constituent, respectively. The subscripts b and m represent the doubly charmed baryon and the charmed meson, while L_z stands for the z component of the orbital angular momentum operator between the doubly charmed baryons and the charmed mesons. The expression $e^{-i\mathbf{k}\cdot\mathbf{r}_j}$ represents the spatial wave function of the emitted photon.

In this study, we also discuss the magnetic moments of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates based on their mass spectra and spatial wave functions [23,24]. Similarly to the method used to estimate the transition magnetic moment between the hadrons $\mu_{H\to H'}$, the magnetic moment of the hadron μ_H within the constituent quark model can be deduced by the following relation [26,27,30–32,35–37,54–93]:

$$\mu_H = \langle J_H, J_H|\sum_j \hat{\mu}_{zj}^{\rm spin} + \hat{\mu}_z^{\rm orbital}|J_H, J_H\rangle. \quad (2.5)$$

As shown in Eqs. (2.2) and (2.5), we need to construct the color, flavor, spin, and spatial wave functions of the hadronic states when calculating their transition magnetic moments and magnetic moments [32,36,37,93]. In this work, we utilize the simple harmonic oscillator wave function to approximate describe the spatial wave functions of the doubly charmed baryons and the charmed mesons [32,36,37,93]. Explicitly, the simple harmonic oscillator wave function can be expressed as

$$\phi_{n,l,m}(\beta,\mathbf{r}) = \sqrt{\frac{2n!}{\Gamma(n+l+\frac{3}{2})}}L_n^{l+\frac{1}{2}}(\beta^2r^2)\beta^{l+\frac{3}{2}}{\rm e}^{-\frac{\beta^2r^2}{2}}r^lY_{lm}(\Omega_{\mathbf{r}}). \quad (2.6)$$

In the above formula, the symbols n, l, and m are the radial, the orbital, and the magnetic quantum numbers of the discussed hadron, respectively. $L_n^{l+\frac{1}{2}}(x)$ and $Y_{lm}(\Omega_{\mathbf{r}})$ are the associated Laguerre polynomial and the spherical harmonic function, respectively. β is a parameter in the simple harmonic oscillator wave function, and we take $\beta_{\Xi_{cc}\rho} = 0.454$ GeV, $\beta_{\Xi_{cc}\lambda} = 0.427$ GeV [96],[1] $\beta_D = 0.601$ GeV, $\beta_{D^*} = 0.516$ GeV, $\beta_{c\bar{q}|1^1P_1\rangle} = 0.475$ GeV, $\beta_{c\bar{q}|1^3P_1\rangle} = 0.482$ GeV, and $\beta_{c\bar{q}|1^3P_2\rangle} = 0.437$ GeV [97] in the following numerical analysis. For the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates, we take the precise spatial wave functions between two hadrons by solving the coupled channel Schrödinger equation in our concrete calculations. Specifically, the spatial wave functions of the isoscalar $\Xi_{cc}D$ and $\Xi_{cc}D^*$ molecular states can be obtained from the quantitative study of their mass spectrum in Ref. [23], and the spatial wave functions of the $\Xi_{cc}D_1$ and $\Xi_{cc}D_2^*$ molecular states can be obtained based on the quantitative study of their mass spectrum in Ref. [24]. For example, we present the obtained spatial wave functions of the $\Xi_{cc}D$ state with $I(J^P) = 0(1/2^-)$ and the $\Xi_{cc}D^*$ state with $I(J^P) = 0(3/2^-)$ by solving the coupled channel Schrödinger equation in Fig. 1.

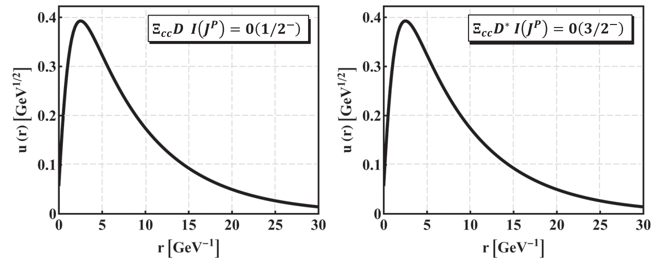

FIG. 1. The spatial wave functions of the $\Xi_{cc}D$ state with $I(J^P) = 0(1/2^-)$ and the $\Xi_{cc}D^*$ state with $I(J^P) = 0(3/2^-)$ by solving the coupled channel Schrödinger equation [23]. Here, we take the binding energies of the $\Xi_{cc}D$ state with $I(J^P) = 0(1/2^-)$ and the $\Xi_{cc}D^*$ state with $I(J^P) = 0(3/2^-)$ as -6.0 MeV.

In the specific calculations, we will expand the spatial wave function of the emitted photon $e^{-i\mathbf{k}\cdot\mathbf{r}_j}$ as the following form [98]:

$$e^{-i\mathbf{k}\cdot\mathbf{r}_j} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} 4\pi(-i)^l j_l(kr_j)Y_{lm}^*(\Omega_{\mathbf{k}})Y_{lm}(\Omega_{\mathbf{r}_j}), \quad (2.7)$$

when taking into account the contribution of the spatial wave functions of the emitted photon, the initial hadron, and the final hadron. Here, $j_l(x)$ and $Y_{lm}(\Omega_{\mathbf{x}})$ are the spherical Bessel function and the spherical harmonic function, respectively.

Within the constituent quark model, the effective masses of the quarks are the crucial input parameters that affect the reliability of the results obtained for the electromagnetic properties of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates. In this paper, we adopt $m_u = 0.336$ GeV, $m_d = 0.336$ GeV, and $m_c = 1.680$ GeV [27,30,32,35–37,61] in the realistic calculations. To validate the reliability of the above input parameters, we will first use the constituent quark model and the above input parameters to discuss the transition magnetic moments and the magnetic moments of our focused doubly charmed baryons and charmed mesons. It is important to note that the $D_1(2420)$ meson is a mixture of the 1^1P_1 and 1^3P_1 states, which satisfies the following relation [99–102]:

[1] For the Ξ_{cc} baryon, the two charmed quarks are treated as the diquark, the ρ mode corresponds to the excitation of the diquark, and the λ mode corresponds to the diquark-quark excitation [96].

TABLE I. Our obtained transition magnetic moments and magnetic moments of our focused doubly charmed baryons and charmed mesons and comparison with them obtained from other theoretical studies. Both the transition magnetic moments and the magnetic moments of the hadrons are expressed in units of $\mu_N = e/2m_p$.

Transition magnetic moments		
Processes	Our work	Other studies
$D^{*0} \to D^0\gamma$	2.173	2.134 [93], 2.233 [30]
$D^{*+} \to D^+\gamma$	−0.538	−0.559 [30], −0.540 [79]
$D_2^{*0} \to D_1^0\gamma$	1.277	...
$D_2^{*+} \to D_1^+\gamma$	−0.452	...
Magnetic moments		
Hadrons	Our work	Other studies
Ξ_{cc}^+	0.807	0.815 [83], 0.806 [103]
Ξ_{cc}^{++}	−0.124	−0.124 [103], −0.110 [79]
D^{*0}	−1.489	−1.485 [93], −1.489 [30]
D^{*+}	1.303	1.308 [93], 1.303 [30]
D_1^0	0.001	...
D_1^+	0.543	...
D_2^{*0}	−2.979	...
D_2^{*+}	2.141	...

$$\begin{pmatrix} |D_1(2430)\rangle \\ |D_1(2420)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{1P} & \sin\theta_{1P} \\ -\sin\theta_{1P} & \cos\theta_{1P} \end{pmatrix} \begin{pmatrix} |1^1P_1\rangle \\ |1^3P_1\rangle \end{pmatrix}, \tag{2.8}$$

where the mixing angle θ_{1P} in the heavy quark limit is $-54.7°$ [99–102].

In Table I, we present the transition magnetic moments and the magnetic moments of our focused doubly charmed baryons and charmed mesons. In addition, we compare our obtained numerical results with those of other theoretical studies. From Table I, we find that the transition magnetic moments and the magnetic moments of our focused doubly charmed baryons and charmed mesons are in agreement with those predicted in other theoretical studies. We, therefore, consider the effective masses of the quarks [27,30,32,35–37,61] to be reliable. This allows us to provide the credible theoretical insights for the experimental investigation of the electromagnetic properties of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charmed molecular-type pentaquark candidates.

In the following, we discuss the M1 radiative decay widths of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates, where both the $S-D$ wave mixing effect and the coupled channel effect are taken into account. The flavor wave functions of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ pentaquark systems are [23,24]

$$|\Xi_{cc}D^{(*)}\rangle = -\frac{1}{\sqrt{2}}(|\Xi_{cc}^{++}D^{(*)0}\rangle + |\Xi_{cc}^{+}D^{(*)+}\rangle), \tag{2.9}$$

$$|\Xi_{cc}D_1\rangle = -\frac{1}{\sqrt{2}}(|\Xi_{cc}^{++}D_1^{0}\rangle + |\Xi_{cc}^{+}D_1^{+}\rangle), \tag{2.10}$$

$$|\Xi_{cc}D_2^*\rangle = -\frac{1}{\sqrt{2}}(|\Xi_{cc}^{++}D_2^{*0}\rangle + |\Xi_{cc}^{+}D_2^{*+}\rangle), \tag{2.11}$$

respectively, and their corresponding spin wave functions can be constructed by the coupling of the spins of the constituent hadrons. Taking into account the $S-D$ wave mixing effect and the coupled channel effect, the relevant channels are [23,24]

States	J^P	Channels
$\Xi_{cc}D$	$1/2^-$	$\lvert ^2\mathbb{S}_{\frac{1}{2}}\rangle$
$\Xi_{cc}D^*$	$1/2^-$	$\lvert ^2\mathbb{S}_{\frac{1}{2}}/^4\mathbb{D}_{\frac{1}{2}}\rangle$
$\Xi_{cc}D^*$	$3/2^-$	$\lvert ^4\mathbb{S}_{\frac{3}{2}}/^2\mathbb{D}_{\frac{3}{2}}/^4\mathbb{D}_{\frac{3}{2}}\rangle$
$\Xi_{cc}D_1$	$1/2^+$	$\lvert ^2\mathbb{S}_{\frac{1}{2}}/^4\mathbb{D}_{\frac{1}{2}}\rangle$
$\Xi_{cc}D_1$	$3/2^+$	$\lvert ^4\mathbb{S}_{\frac{3}{2}}/^2\mathbb{D}_{\frac{3}{2}}/^4\mathbb{D}_{\frac{3}{2}}\rangle$
$\Xi_{cc}D_2^*$	$3/2^+$	$\lvert ^4\mathbb{S}_{\frac{3}{2}}/^4\mathbb{D}_{\frac{3}{2}}/^6\mathbb{D}_{\frac{3}{2}}\rangle$
$\Xi_{cc}D_2^*$	$5/2^+$	$\lvert ^5\mathbb{S}_{\frac{5}{2}}/^4\mathbb{D}_{\frac{5}{2}}/^6\mathbb{D}_{\frac{5}{2}}\rangle$

Here, the spin S, the orbital angular momentum L, and the total angular momentum J of the corresponding channel are denoted as the symbol $|^{2S+1}L_J\rangle$. In our concrete calculations, the spin-orbital wave function $|^{2S+1}L_J\rangle$ must be expanded by incorporating the spin wave function $|S, m_S\rangle$ and the orbital wave function Y_{Lm_L} when calculating the transition magnetic moment and the magnetic moment of the D-wave channel. This can be expressed as $|^{2S+1}L_J\rangle = \sum_{m_S,m_L} C^{J,M}_{Sm_S,Lm_L}|S, m_S\rangle Y_{Lm_L}$ [32,36,37,93]. In Table II, we show the hadron masses [95] used in our calculations.

Combined with the mass spectrum of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates [23,24], in the present work we analyze the M1 radiative decay widths for the $\Xi_{cc}D^*(3/2^-)\to\Xi_{cc}D(1/2^-)\gamma$, $\Xi_{cc}D_2^*(3/2^+) \to \Xi_{cc}D_1(1/2^+)\gamma$, $\Xi_{cc}D_2^*(3/2^+) \to \Xi_{cc}D_1(3/2^+)\gamma$, $\Xi_{cc}D_2^*(5/2^+) \to \Xi_{cc}D_1(3/2^+)\gamma$, $\Xi_{cc}D_1(3/2^+) \to \Xi_{cc}D_1(1/2^+)\gamma$, and $\Xi_{cc}D_2^*(5/2^+) \to \Xi_{cc}D_2^*(3/2^+)\gamma$ processes. In Table III, we present the numerical results of the transition magnetic moments and the M1 radiative decay widths between the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates discussed in three different cases: (I) neglecting the

TABLE II. A summary of the hadron masses used in our calculations. The mass of the Ξ_{cc}^+ is taken from Ref. [104], while the masses of other hadrons are sourced from the Particle Data Group [95], and the units of the hadron masses are GeV.

$m_N = 0.938$	$m_{\Xi_{cc}^+} = 3.620$	$m_{\Xi_{cc}^{++}} = 3.622$	$m_{D^0} = 1.865$
$m_{D^+} = 1.870$	$m_{D^{*0}} = 2.007$	$m_{D^{*+}} = 2.010$	$m_{D_1^0} = 2.422$
$m_{D_1^+} = 2.426$	$m_{D_2^{*0}} = 2.461$	$m_{D_2^{*+}} = 2.464$	

TABLE III. The obtained transition magnetic moments and M1 radiative decay widths between the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates by performing the single-channel scenario and the coupled channel scenario. Here, we discuss the transition magnetic moments and the M1 radiative decay widths between our discussed isoscalar triple-charm molecular-type pentaquarks with three different cases: (I) neglecting the contribution of the spatial wave functions of the emitted photon, the doubly charmed baryons, the charmed mesons, and the triple-charm molecular-type pentaquarks, (II) taking into account the contribution of the spatial wave functions of the emitted photon and the triple-charm molecular-type pentaquarks, and (III) taking into account the contribution of the spatial wave functions of the emitted photon, the doubly charmed baryons, the charmed mesons, and the triple-charm molecular-type pentaquarks. Meanwhile, $\Gamma^{\rm Max}_{H\to H'\gamma}$ is the maximum value of the M1 radiative decay width obtained by varying the binding energies of the initial and final isoscalar triple-charm molecular-type pentaquark candidates.

Single channel			
Processes	Cases	$\mu_{H\to H'}(\mu_N)$	$\Gamma_{H\to H'\gamma}$(keV)
$\Xi_{cc}D^*(3/2^-)\to\Xi_{cc}D(1/2^-)\gamma$	I	0.684	5.269, 5.269, 5.269
	II	0.461, 0.643, 0.660	2.396, 4.652, 4.898
	III	0.450, 0.627, 0.644	2.282, 4.430, 4.666
$\Xi_{cc}D_2^*(3/2^+)\to\Xi_{cc}D_1(1/2^+)\gamma$	I	0.439	0.0441, 0.0441, 0.0441
	II	0.422, 0.437, 0.437	0.0408, 0.0436, 0.0438
	III	0.419, 0.433, 0.434	0.0401, 0.0429, 0.0430
$\Xi_{cc}D_2^*(3/2^+)\to\Xi_{cc}D_1(3/2^+)\gamma$	I	0.186	0.00881, 0.00881, 0.00881
	II	0.178, 0.182, 0.181	0.00802, 0.00842, 0.00833
	III	0.176, 0.180, 0.179	0.00789, 0.00828, 0.00819
$\Xi_{cc}D_2^*(5/2^+)\to\Xi_{cc}D_1(3/2^+)\gamma$	I	0.372	0.0529, 0.0529, 0.0529
	II	0.356, 0.370, 0.371	0.0485, 0.0522, 0.0524
	III	0.353, 0.367, 0.368	0.0477, 0.0513, 0.0515
Processes	Cases	$\mu_{H\to H'}(\mu_N)$	$\Gamma^{\rm max}_{H\to H'\gamma}$(keV)
$\Xi_{cc}D_1(3/2^+)\to\Xi_{cc}D_1(1/2^+)\gamma$	I	0.197	0.0003
$\Xi_{cc}D_2^*(5/2^+)\to\Xi_{cc}D_2^*(3/2^+)\gamma$	I	0.360	0.002
Coupled channel			
Process	Cases	$\mu_{H\to H'}(\mu_N)$	$\Gamma_{H\to H'\gamma}$(keV)
$\Xi_{cc}D^*(3/2^-)\to\Xi_{cc}D/\Xi_{cc}D^*(1/2^-)$	II	0.680, −0.674, −0.670	5.127, 4.940, 4.888
	III	0.447, −0.615, −0.629	2.196, 4.094, 4.291
Process	Cases	$\mu_{H\to H'}(\mu_N)$	$\Gamma^{\rm max}_{H\to H'\gamma}$(keV)
$\Xi_{cc}D_1/\Xi_{cc}D_2^*(3/2^+)\to\Xi_{cc}D_1/\Xi_{cc}D_2^*(1/2^+)$	II	−0.267, 0.412, 0.444	0.002
	III	−0.266, 0.408, 0.439	0.002

contribution of the spatial wave functions of the emitted photon, the doubly charmed baryons, the charmed mesons, and the triple-charm molecular-type pentaquarks, (II) taking into account the contribution of the spatial wave functions of the emitted photon and the triple-charm molecular-type pentaquarks, and (III) taking into account the contribution of the spatial wave functions of the emitted photon, the doubly charmed baryons, the charmed mesons, and the triple-charm molecular-type pentaquarks. In this study, we utilize the identical binding energies for the initial and final isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates and take three representative binding energies −0.5, −6.0, and −12.0 MeV to investigate their transition magnetic moments and M1 radiative decay widths. In addition, our analysis takes into account the contribution of the $S-D$ wave mixing effect and the coupled channel effect, corresponding to the coupled channel scenario.

From the transition magnetic moments and the M1 radiative decay widths between the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates listed in Table III, we find the following.

(i) The M1 radiative decay widths between the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates can be considered as the effective physical observables reflecting their inner structures. For example, the M1 radiative decay widths of the $\Xi_{cc}D_2^*(3/2^+)\to\Xi_{cc}D_1(1/2^+)\gamma$ and $\Xi_{cc}D_2^*(3/2^+)\to\Xi_{cc}D_1(3/2^+)\gamma$ processes are obviously different, which implies that the spin-parity quantum numbers of the isoscalar $\Xi_{cc}D_1$ molecular states can be distinguished by studying the associated M1 radiative decay widths.

(ii) The contribution of the $S-D$ wave mixing effect and the coupled channel effect plays a minor role for the M1 radiative decay width of the $\Xi_{cc}D^*(3/2^-) \to \Xi_{cc}D(1/2^-)\gamma$ process. However, for the M1 radiative decay width of the $\Xi_{cc}D_1(3/2^+) \to \Xi_{cc}D_1(1/2^+)\gamma$ process, the contribution of the $S-D$ wave mixing effect and the coupled channel effect plays a major role, which depends on the spatial wave functions of the mixing channels.

(iii) The spatial wave functions of the emitted photon, the doubly charmed baryons, the charmed mesons, and the triple-charm molecular-type pentaquarks do not seem to have a significant effect on the transition magnetic moments and the M1 radiative decay widths between the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates, especially for the contribution of the spatial wave functions of the doubly charmed baryons and the charmed mesons.

(iv) When assuming the existence of the same binding energies for the initial and final isoscalar triple-charm molecular-type pentaquarks, the phase spaces are zero for the $\Xi_{cc}D_1(3/2^+) \to \Xi_{cc}D_1(1/2^+)\gamma$ and $\Xi_{cc}D_2^*(5/2^+) \to \Xi_{cc}D_2^*(3/2^+)\gamma$ processes. In this study, we further examine their M1 radiative decay widths as the binding energies of the initial and final isoscalar triple-charm molecular-type pentaquarks vary. However, their M1 radiative decay widths are greatly suppressed, which is attributed to the small phase spaces for these processes.

III. THE MAGNETIC MOMENTS OF THE TRIPLE-CHARM MOLECULAR-TYPE PENTAQUARK

In the previous section, we studied the M1 radiative decay widths between the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates, which can be considered as the effective physical observables reflecting their inner structures. In this section, we discuss the magnetic moments of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates.

In Table IV, we present the numerical results of the magnetic moments of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates. Here, we investigate the contribution of the $S-D$ wave mixing effect and the coupled channel effect corresponding to the coupled channel scenario. For the coupled channel scenario, we consider three typical binding energies −0.5, −6.0, and −12.0 MeV for the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates to discuss their magnetic moments.

TABLE IV. The obtained magnetic moments of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates by performing the single-channel scenario and the coupled channel scenario.

Molecules	$\mu_H(\mu_N)$
Single channel	
$\Xi_{cc}D(1/2^-)$	0.347
$\Xi_{cc}D^*(3/2^-)$	0.259
$\Xi_{cc}D_1(1/2^+)$	0.0691
$\Xi_{cc}D_1(3/2^+)$	0.625
$\Xi_{cc}D_2^*(3/2^+)$	−0.579
$\Xi_{cc}D_2^*(5/2^+)$	−0.0649
Coupled channel	
$\Xi_{cc}D/\Xi_{cc}D^*(1/2^-)$	0.334, 0.323, 0.320
$\Xi_{cc}D^*(3/2^-)$	0.249, 0.249, 0.248
$\Xi_{cc}D_1/\Xi_{cc}D_2^*(1/2^+)$	0.0680, 0.0681, 0.0680
$\Xi_{cc}D_1/\Xi_{cc}D_2^*(3/2^+)$	0.507, −0.0137, −0.183

As shown in Table IV, the magnetic moments of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates can be considered as the important aspect reflecting their inner structures, which can be used to distinguish their spin-parity quantum numbers. For example, the magnetic moments of the $\Xi_{cc}D_1$ state with $I(J^P)=0(1/2^+)$ and the $\Xi_{cc}D_1$ state with $I(J^P)=0(3/2^+)$ are obviously different, and the $\Xi_{cc}D_2^*$ state with $I(J^P)=0(3/2^+)$ and the $\Xi_{cc}D_2^*$ state with $I(J^P)=0(5/2^+)$ have different magnetic moments. Similarly to the case of the transition magnetic moments and the M1 radiative decay widths between the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates, the $S-D$ wave mixing effect and the coupled channel effect play a minor role for the magnetic moments of several discussed isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates, such as the $\Xi_{cc}D$ state with $I(J^P)=0(1/2^-)$, the $\Xi_{cc}D^*$ state with $I(J^P)=0(3/2^-)$, and the $\Xi_{cc}D_1$ state with $I(J^P)=0(1/2^+)$. However, the $S-D$ wave mixing effect and the coupled channel effect play an important role to mediate the magnetic moment of the $\Xi_{cc}D_1$ state with $I(J^P)=0(3/2^+)$, and the corresponding magnetic moment is sensitive to the binding energy.

Except for the constituent quark model, other models and approaches are often used to discuss the electromagnetic properties of hadrons in the past decades. Especially, chiral perturbation theory is a popular way to study the electromagnetic properties of heavy hadrons [40,105–123]. Thus, we hope that the electromagnetic properties of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates can be discussed by other models and approaches in the future, which can make our knowledge of the electromagnetic properties of the isoscalar

$\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates become more abundant.

IV. DISCUSSION AND CONCLUSION

In recent decades, the study of molecular-type multiquark states has become an influential and attractive area of research in hadron physics. Currently, more predictions around molecular-type pentaquarks have been made. Inspired by the discovery of the doubly charmed baryon $\Xi_{cc}(3620)^{++}$ by LHCb [25], the mass spectrum of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charmed molecular-type pentaquark candidates has been predicted in Refs. [23,24]. At present, our knowledge of their properties is still not enough, and further suggestions for the experimental search for the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates should be discussed.

In this work, we systematically study the electromagnetic properties including the M1 radiative decay widths and the magnetic moments of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates within the framework of the constituent quark model, where both the $S-D$ wave mixing effect and the coupled channel effect are taken into account. From our obtained numerical results, we conclude that the M1 radiative decay widths and the magnetic moments of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates can reflect their inner structures, which can be used to distinguish their spin-parity quantum numbers. Thus, the study of the electromagnetic properties is an important step to construct the family of triple-charm molecular-type pentaquarks.

As emphasized in Fig. 2, single-charm, hidden-charm, and double-charm multiquark candidates have been reported in experiments [1–11]. So far, there have been no experimental signals of the triple-charm multiquark candidates [1–11]. In future experiment, searching for the triple-charm multiquark candidates will be an interesting and important research topic of hadron physics, and the triple-charm pentaquark state is the simplest quark configuration among the triple-charm multiquark states. As a potential experimental research task, LHCb has the opportunity to investigate the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and

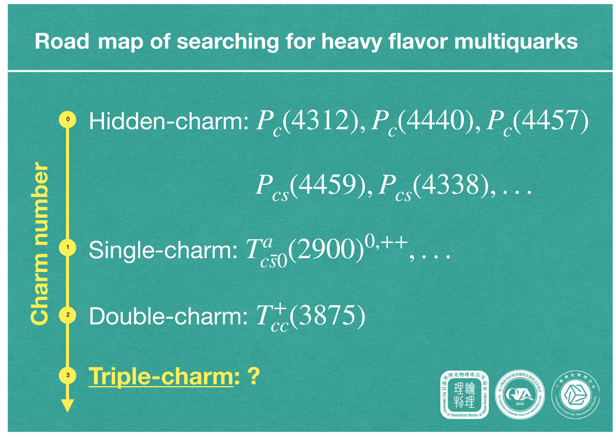

FIG. 2. Road map of searching for heavy flavor multiquarks [1–11].

$\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates with the accumulation of experimental data during the high-luminosity phase of the LHC [43]. Obviously, the present work combined with the corresponding mass spectrum information [23,24] can provide more comprehensive spectroscopic information when searching for the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates. In addition, the theoretical study of other properties of the isoscalar $\Xi_{cc}D^{(*)}$, $\Xi_{cc}D_1$, and $\Xi_{cc}D_2^*$ triple-charm molecular-type pentaquark candidates is also encouraged.

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