R7-branes as charge conjugation operators

Markus Dierigl,[1,*] Jonathan J. Heckman,[2,3,†] Miguel Montero,[4,‡] and Ethan Torres[2,§]

[1]*Arnold Sommerfeld Center for Theoretical Physics, LMU, Munich 80333, Germany*
[2]*Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA*
[3]*Department of Mathematics, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA*
[4]*Instituto de Física Teórica UAM-CSIC, c/Blas Cabrera 13-15, 28049 Madrid, Spain*

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R7-branes are a class of recently discovered nonsupersymmetric real codimension-two duality defects in type IIB string theory predicted by the swampland cobordism conjecture. For type IIB realizations of 6D SCFTs with $\mathcal{N} = (2, 0)$ supersymmetry, wrapping an R7-brane "at infinity" leads to a topological operator associated with a zero-form charge conjugation symmetry that squares to the identity. Similar considerations hold for those theories obtained from further toroidal compactification, but this can be obstructed by bundle curvature effects. Using some minimal data on the topological sector of the R7-branes, we extract the associated fusion rules for these charge conjugation operators. More broadly, we sketch a top down realization of various topological operators/interfaces associated with C, R, and T transformations. We also use holography to provide strong evidence for the existence of the R7-brane which is complementary to the cobordism conjecture. Similar considerations apply to other string-realized QFTs with symmetry operators constructed via nonsupersymmetric branes which carry a conserved charge.

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I. INTRODUCTION

Symmetries provide a powerful organizing tool in the study of quantum fields and gravity. Recently, it was shown that the structures of symmetries in physical systems are intimately tied with topological structures. In the context of quantum field theory (QFT), such generalized symmetries provide a framework for understanding many of these features [1], and this has by now led to a number of new developments both in the study of higher-form, higher-group, as well as noninvertible/categorical generalizations.[1]

Most of these developments have centered on global symmetries, but in quantum gravity, one expects that these symmetries are either explicitly gauged or broken. In the swampland program this was recently formalized in terms of the swampland cobordism conjecture, which asserts that the bordism group of quantum gravity is trivial [155].[2] In practice, one considers a long distance limit captured by the gravitational path integral and then imposes specific symmetry (spacetime and internal) constraints. Obtaining a nontrivial bordism group then amounts to the prediction of new objects, since in the full quantum gravity there must be boundaries for the bordism classes that seemed nontrivial in the low-energy effective field theory. By now, the cobordism conjecture has undergone a number of nontrivial checks in the context of supersymmetric backgrounds, and has even been used to predict the existence of new nonsupersymmetric objects [149,155,164].

String theory makes direct contact with both of these developments. In the context of QFTs, string backgrounds with localized singularities in the metric/fields/solitonic branes provide a general template for constructing and studying a wide class of strongly coupled systems decoupled from gravity. In this regard, it is worth noting that string theory remains the method for explicitly constructing interacting $D > 4$ fixed points. Indeed, the spectrum of (often supersymmetric) extended defects in such systems is encapsulated in terms of the "defect group" [6,15,19,20], where branes wrapped on noncompact cycles are screened by dynamical states obtained from branes wrapped on compact, collapsing cycles. The associated symmetry operators which act on these defects directly encode the generalized symmetry operators, and can be viewed as branes "wrapped at infinity." Since they are infinitely far away, essentially the only contribution they can make to the

*m.dierigl@lmu.de
†jheckman@sas.upenn.edu
‡miguel.montero@csic.es
§emtorres@sas.upenn.edu

[1]See e.g., [1–154] for a recent review.
[2]For recent developments, see e.g., [74,149,156–164].

field theory is via their topological sector [namely, Wess-Zumino (WZ) terms]. This was recently used to exhibit explicit examples of nontrivial fusion rules in a number of different systems (see e.g., [102,103,105,165]).

Given this, it is natural to ask whether the new branes predicted by the swampland cobordism conjecture also generate topological symmetry operators. Our aim in this note will be to show that this is indeed the case for a specific new 7-brane predicted in the context of type IIB dualities; the reflection 7-brane. As found in [149,164],[3] these "R7-branes" can be viewed as a codimension-two defect of the 10D type IIB supergravity. Winding once around this brane amounts to a reflection on either the a- or b-cycle of the F-theory torus. In terms of the type IIB world sheet theory, these reflections are associated with world sheet orientation reversal Ω and left-moving fermion parity $(-1)^{F_L}$. This object carries a $\mathbb{Z}_2$ charge of the corresponding type IIB duality group, and as such, cannot completely "disappear." Even so, there are good indications from [164] that it is strongly coupled and potentially unstable to thickening/expansion.

That being said, wrapping such a brane "at infinity" means that it cannot contribute to the stress energy tensor of a localized QFT sector. As such, we can insert these R7-branes and deduce the corresponding symmetry operator generated by these objects. In the context of 6D $\mathcal{N} = (2,0)$ superconformal field theories (SCFTs) realized via type IIB on an ADE orbifold, we show that insertion of an R7-brane realizes a zero-form symmetry which acts as a charge conjugation operation on the heavy stringlike defects of the theory. Further compactification to four dimensions leads to a corresponding charge conjugation operation which can be combined with other "branes at infinity" to implement more general symmetries such as spacetime reflections.

Beyond the case of pure geometric engineering, one can also consider D-branes probing singularities. In some cases, the contribution from the R7-brane leads to a large backreaction due to the putative symmetry being explicitly broken by the background, thus making it unsuitable as a topological symmetry operator, but in other cases this can be used to engineer related charge conjugation/reflections of the localized QFT sector. The basic considerations we consider here apply to other choices of nonsupersymmetric branes which carry a conserved charge. In these cases, we sketch how string-realized QFTs and little string theories (LSTs) admit symmetry operators obtained from wrapping these nonsupersymmetric branes "at infinity."

Turning the discussion around, one can argue that the existence of a suitable symmetry in the string-realized QFT implies the existence of a corresponding topological symmetry operator. This in turn requires the existence of a suitable object which could implement this symmetry, amounting to the requirement that a suitable brane must exist. From this perspective, the main thing to verify is that such a symmetry exists in the first place. We show that for those theories with a suitable holographic dual such as the large-N limits of the A- and D-type 6D SCFTs with $\mathcal{N} = (2,0)$ supersymmetry, charge conjugation amounts to a reflection on X, the "internal direction" of the background $\mathrm{AdS}_7 \times X$. One can also extend this reasoning to many other cases where one has a stringy realization of a D-dimensional CFT with an AdS_{D+1} dual, and more broadly, it can even be applied to more general systems such as LSTs.

[3]They were also hinted at in [166].

II. R7-BRANES AND 6D SCFTs

We now argue that some 6D SCFTs have a charge conjugation symmetry which, in the context of F-theory on an elliptically-fibered Calabi-Yau threefold, is realized via R7-branes wrapped "at infinity." That being said, we will find (by explicit analysis) that only theories with $\mathcal{N} = (2,0)$ supersymmetry have a charge conjugation symmetry which squares to $+1$, and is implemented by the R7-brane. This corresponds to the case of a trivial elliptic fibration.[4]

To begin, let us recall that in F-theory on a noncompact Calabi-Yau threefold $X \to B$, we get a 6D SCFT by contracting curves of the base B to zero size. D3-branes wrapped on finite volume curves provide effective strings with tension which tends to zero as the curves' volumes vanish. In this limit, one obtains a 6D SCFT. The full list of noncompact bases B as well as possible elliptic fibrations was determined in [167–169] (for reviews see [170,171]). The general structure of all such bases is, in the contracting limit, given by an orbifold of the form $\mathbb{C}^2/\Gamma_{U(2)}$ for $\Gamma_{U(2)}$ a finite subgroup of $U(2)$. Working in radial coordinates, this specifies a conical geometry with an $S^3/\Gamma_{U(2)}$ at each radial slice. One obtains heavy stringlike defects from D3-branes wrapped on noncompact 2-cycles which extend along the radial direction and wrap a torsional 1-cycle at the boundary $S^3/\Gamma_{U(2)}$ "at infinity." Since they wrap a torsion cycle n times these defects must be trivial, which means they are charged under a $\mathbb{Z}_n$ 2-form symmetry (only discrete 2-form symmetries are possible in 6D SCFTs [27]). More precisely, the spectrum of heavy stringlike defects which cannot be screened by dynamical strings are classified by the "defect group" (see Ref. [6]) which is given by the Abelianization of $\Gamma_{U(2)}$, namely $H_1(S^3/\Gamma_{U(2)}, \mathbb{Z}) = \mathrm{Ab}(\pi_1(S^3/\Gamma_{U(2)}, \mathbb{Z})) = \mathrm{Ab}(\Gamma_{U(2)})$.

Thus, it should be possible to construct codimension-three topological symmetry operators that link with the above heavy stringlike defects. Indeed, these can be obtained from D3-branes wrapping these same torsional

[4]Theories with $\mathcal{N} = (1,0)$ supersymmetry admit a charge conjugation symmetry which squares to $(-1)^F$, as we explain later.

1-cycles [105] in the $S^3/\Gamma_{U(2)}$ at infinity. Unlike the heavy stringlike defects implemented by D3-branes that extend along the radial direction from infinity to the singularity where the SCFT lives, the D3-branes implementing topological operators are localized at infinity. Intuitively, this means that a small deformation cannot affect the local physics, as any backreaction must traverse an infinite distance, and their correlators can only be possibly affected by the linking with the D3-branes implementing heavy stringlike defects; precisely the definition of a topological operator.

The dualities of type IIB string theory act on these heavy stringlike defects via a general conjugation operation. As described in [172] (see also [173]), the actual duality group of type IIB string theory is the Pin^+ double cover of $GL(2,\mathbb{Z})$. The reflections with negative determinant given (in terms of their action on the F-theory torus)[5] by $M_{\mathsf{F}_L} = \mathrm{diag}(-1,1)$ and $M_\Omega = \mathrm{diag}(1,-1)$, correspond respectively to left-moving fermion parity $(-1)^{F_L}$ and world sheet orientation reversal Ω. Each of these generators sends a D3-brane to an anti-D3-brane: $|\mathrm{D3}\rangle \to |\overline{\mathrm{D3}}\rangle$. This specifies a generalized charge conjugation operation on D3-branes. In the corresponding 6D SCFT, this sends each of our heavy stringlike defects (obtained from wrapped D3-branes) to its antistring counterpart. The reflections M_{F_L} and M_Ω also act nontrivially on D7-branes since we also have $|\mathrm{D7}\rangle \to |\overline{\mathrm{D7}}\rangle$.

Generically, most 6D SCFTs do not have a charge conjugation symmetry. Indeed, on the tensor branch it is common to encounter various 6D gauge theories which are coupled to tensor multiplets. To cancel 1-loop gauge anomalies generated by the chiral matter of the vector multiplet one must include suitable Green-Schwarz-Sagnotti-West terms (see [174,175]) which are schematically of the form $B^a \wedge I_a^{GS}$, where B^a is an antichiral 2-form field and I_a^{GS} is a 4-form constructed via the characteristic classes of the gauge bundles. The specific form of such couplings can be extracted from the algorithm developed in [176–178], and can also be extended to include possible couplings to background curvatures/R-symmetries/flavor symmetries. The presence of couplings such as $B^a \wedge I_a$ manifestly breaks the charge conjugation symmetry since I_a is realized via even powers of curvatures/field strengths (and therefore, must be charge conjugation invariant), whereas B^a is manifestly odd under the conjugation operation, since these fields couple directly to the D3-branes wrapping the noncompact 2-cycles of the ambient geometry. In the associated F-theory background this is also expected because the gauge theory degrees of freedom are realized via 7-branes wrapped on compact curves, and reflections generically send 7-branes to anti-7-branes.

[5]The monodromy matrices M can also be deduced from the action on the 2-form fields of type IIB that transform as a vector given by $(C_2, B_2)^T$.

The exception to this general situation are those 6D SCFTs which have no 7-branes at all. This occurs for the celebrated $\mathcal{N}=(2,0)$ theories, as obtained from a collection of -2 curves in the base with intersection form given by the corresponding ADE Dynkin diagram,

$$A_N\colon\ \underbrace{2,2,\ldots,2}_{N}, \tag{2.1}$$

$$D_N\colon\ \underbrace{2,\overset{2}{2},\ldots,2}_{N-1}, \tag{2.2}$$

$$E_6\colon\ 2,2,\overset{2}{2},2,2, \tag{2.3}$$

$$E_7\colon\ 2,2,\overset{2}{2},2,2,2, \tag{2.4}$$

$$E_8\colon\ 2,2,\overset{2}{2},2,2,2,2. \tag{2.5}$$

In fact, one can argue directly from the classification of superconformal algebras that only $\mathcal{N}=(2,0)$ theories could possibly have a charge conjugation symmetry represented by R7-branes. R7-branes have a world volume charge which is $\mathbb{Z}_2$ valued, so the charge conjugation symmetry they implement squares to $+1$. In an $\mathcal{N}=(1,0)$ theory, this is impossible, since any charge conjugation symmetry must map the supercharge Q to itself, but in six Lorentzian dimensions (or Euclidean reflection-positive), the only possible charge conjugation operator that preserves chirality squares to -1 [179,180]. So while there may be a charge conjugation symmetry for $\mathcal{N}=(1,0)$ theories, it is qualitatively different from the $\mathcal{N}=(2,0)$ case. In fact, this charge conjugation symmetry may be realized as simply any $\mathbb{Z}_4$ subgroup of the $SU(2)$ R-symmetry.

We now directly construct the corresponding topological symmetry operator for the $\mathcal{N}=(2,0)$ theories. This is realized at once in terms of an R7-brane "wrapped at infinity." In terms of the local coordinates the relevant objects are obtained as follows:

		0	1	2	3	4	5	6	7	8	9	
Defect	D3					×	×	×	×			,
Symm Op.	R7	×	×	×	×	×			×	×	×	

(2.6)

where the "0, …, 5" directions denote the 6D spacetime, the "6" direction denotes the radial direction of the base, and the "7,8,9" directions denote the S^3/Γ "at infinity."

Since both the F_L- and Ω-brane act the same way on D3-branes, we might be tempted to conclude that there is no difference in which one we use to implement this operator. However, one can wrap F1-strings or D1-branes on the noncompact 2-cycles of the ambient geometry, and this engineers pointlike defects in the $\mathcal{N} = (2,0)$ theory. The F_L- and Ω-branes act differently on these, mapping only F1-strings or D1-branes to their conjugates, respectively.[6] In any case, we see that much as in [165], either R7-brane defines a real codimension-one topological operator, and as such should be viewed as a zero-form symmetry operator. It is in fact typical of charge conjugation that it acts nontrivially on both pointlike and extended operators.

A. Fusion rules

While much is still unknown about the R7-brane, general topological/anomaly inflow arguments provide a natural candidate action for at least a subsector of the world volume degrees of freedom of this system [164]. Using this, we can then consider the fusion rules for two such symmetry operators wrapped on a 5D subspace of the 6D spacetime. For ease of exposition we focus on the Ω-brane. Similar considerations apply for the F_L-brane.

In differential cohomology terms,[7] (for physicist friendly reviews see e.g., [70,182,183] as well as the book [184]), we can rewrite our action for the Ω-brane as [164]

$$\int_{R7} \breve{H}_3 \star \breve{f}_6 + \breve{F}_5 \star \breve{f}_4 + \breve{H}_7 \star \breve{f}_2. \tag{2.7}$$

Here, $\breve{H}_3$ and $\breve{H}_7$ denote differential characters that describe the NS 2- and 6-form fields, respectively, while $\breve{F}_5$ describes the chiral RR 4-form. The remaining differential characters $\breve{f}_k$ describe $(k-1)$-form fields that are localized on the brane world volume and can absorb the charges of bulk objects, such as D3-branes, ending on the R7 (see [164] for details). The product $\star$ is defined as a map

$$\star\colon\ \breve{H}^p \times \breve{H}^q \to \breve{H}^{p+q}, \tag{2.8}$$

producing a differential cohomology class which can naturally be integrated over $(p+q-1)$-manifolds, such as the eight-dimensional world volume of the R7-brane above.

Consider the 6D $\mathcal{N} = (2,0)$ SCFTs engineered from taking type IIB on $B = \mathbb{C}^2/\Gamma_{SU(2)}$. We can now expand these fields along differential cohomology classes of S^3/Γ to obtain topological terms on the codimension-one wall, M_5, in the 6D spacetime. The cohomology groups of the boundary geometry are[8]

$$H^*(S^3/\Gamma, \mathbb{Z}) = \{\mathbb{Z}, 0, \mathrm{Ab}(\Gamma), \mathbb{Z}\}. \tag{2.9}$$

Denote the generator (or generators when Γ is of D-type) of $H^2 = \mathrm{Ab}(\Gamma)$ by t_2 (or $t_2^{i=1,2}$ for D_{4k}-type) which can be lifted to a differential cohomology class $\breve{t}_2$ in the sense that it defines its characteristic class. In the notation of Sec. 2 of [70] there is a projection $I(\breve{t}_2) = t_2$. We will pay particular attention to the middle term of (2.7), returning to the other two later, and consider the following expansions (suppressing the indices in the D-type case)

$$\breve{F}_5 = \breve{G}_3 \star \breve{t}_2, \tag{2.10}$$

$$\breve{f}_4 = \breve{g}_2 \star \breve{t}_2. \tag{2.11}$$

Reducing to M_5 then simply requires knowledge of the linking pairing $L_\Gamma = \int_{S^3/\Gamma} \breve{t}_2 \star \breve{t}_2$ which is a 2×2 matrix in the D-type case. The resulting action on M_5 can now be written as

$$L_\Gamma \int_{M_5} G_3 \cup g_2, \tag{2.12}$$

and if we assume that M_5 is torsion-free, the Künneth theorem implies that G_3 is an $\mathrm{Ab}(\Gamma)$-valued 3-form which is hardly surprising since this is precisely the background field for the 2-form symmetry of the 6D $\mathcal{N} = (2,0)$ theory. The path integral of this 5D topological field theory (TFT) can be written as

$$\mathcal{P}_2(M_5) \equiv \int Dg_2 e^{2\pi i L_\Gamma \int_{M_5} G_3 \cup g_2} = \sum_{\Sigma_3 \in H_3(M_5, \mathrm{Ab}(\Gamma))} e^{2\pi i L_\Gamma \int_{\Sigma_3} G_3}, \tag{2.13}$$

where again we point out that we have suppressed the extra indices in L_Γ^{ij} for the D_{4k} case. Since $e^{2\pi i L_\Gamma \int_{\Sigma_3} G_3}$ can be interpreted as a symmetry operator for $\mathrm{Ab}(\Gamma)^{(2)}$, we see that we are gauging this symmetry along M_5. In the language of [84], this is a 1-gauging of a 2-form symmetry. Returning to the other two terms in (2.7), we see that those produce 1-gaugings of $\mathrm{Ab}(\Gamma)^{(4)}$ and $\mathrm{Ab}(\Gamma)^{(0)}$ symmetries, denoted as $\mathcal{P}_4$ and $\mathcal{P}_0$, respectively, whose charged operators arise from wrapping NS5-branes and F1-strings on relative 2-cycles which, topologically, are cones over the boundary 1-cycles. We then can write our charge conjugation operator as

$$\mathcal{U}_\Omega(M_5) = \mathsf{C} \cdot \mathcal{P}_0 \cdot \mathcal{P}_2 \cdot \mathcal{P}_4, \tag{2.14}$$

[6]One can also directly see the full duality group action on objects of the theory by introducing a stack of probe D3-branes into the system. From the perspective of the 6D SCFT this is a specific real codimension-two defect which supports a supersymmetric gauge theory. In that gauge theory, the axiodilaton descends to a marginal coupling.

[7]For simplicity, we take the approximation of classifying type IIB charges by cohomology, but in principle one should replace this by KR-theory (see e.g., [166,181]) at the perturbative level and, ultimately, some unknown generalization of twisted K-theory which is covariant under S-duality. This subtlety will not affect our main conclusions.

[8]For ease of exposition we give the ordinary cohomology group since the lift of these generators to differential cohomology are what is relevant in the actual fusion rule calculation.

where C is the more elementary charge conjugation which simply acts on the operators of the 6D $\mathcal{N}=(2,0)$ theory in the form we mentioned above. We have that $\mathsf{C}^2=1$ because the R7 monodromy matrix, as an element in $GL(2,\mathbb{Z})$ lifts to an order-two element in $GL^+(2,\mathbb{Z})$ [74]. As discussed in [105], the operators $\mathcal{P}_k$ which enact a p-gauging of a k-form symmetry satisfy $\mathcal{P}_k^2=\mathcal{P}_k$, i.e., they are projection operators onto sectors where the flux being gauged vanishes. This does not have a well-defined inverse which is the sense in which our charge conjugation operator engineered from the R7-brane, $\mathcal{U}_\Omega$, is noninvertible. So in summary, the fusion rules of $\mathcal{U}_\Omega$ with itself are summarized as

$$\mathcal{U}_\Omega^2=\mathcal{U}_\Omega^\dagger\cdot\mathcal{U}_\Omega=\mathcal{P}_0\cdot\mathcal{P}_2\cdot\mathcal{P}_4. \quad (2.15)$$

We now consider the effect of passing a string defect operator $W_\gamma(M_2)$ with charge[9] $\gamma\in\mathrm{Ab}(\Gamma)^{(2)}$ through $\mathcal{U}_\Omega(M_5)$. This can be determined by passing a D3-brane through an R7 as in Fig. 1. We see that two D3-branes (with the orientations illustrated) emanate from the R7 as required for consistency with charge conservation. This Hanany-Witten effect is similar to the usual case of passing $[p,q]$ strings/5-branes through supersymmetric 7-branes.[10] We see also from Fig. 1 that if we regard the vertical direction as the radial direction of $\mathbb{C}^2/\Gamma$ with $r=0$ indicating the bottom of the figure, then the ending D3-brane created from the Hanany-Witten-like move is located at the asymptotic boundary. This D3-brane is nothing other than the symmetry operator associated to $\mathrm{Ab}(\Gamma)^{(2)}$, which we denote by $\mathcal{U}_{2\gamma}^{(2)}$. The world volume of this D3 is $H_3\times\{2\gamma\}$ where H_3 is a 3-manifold in the 6D spacetime such that $\partial H_3=M_2\coprod\overline{M_2}$ see Fig. 2, and we use 2γ to denote a 1-cycle in S^3/Γ with such a charge in $H_1(S^3/\Gamma)$. We thus have the fusion rule,

$$\mathcal{U}_\Omega\cdot W_\gamma(M_2)=W_{-\gamma}(M_2)\cdot\mathcal{U}_{2\gamma}^{(2)}(H_3)\cdot\mathcal{U}_\Omega. \quad (2.16)$$

This effect of a creation of another topological symmetry operator when passing a heavy operator through a 0-form symmetry operator is a common feature of noninvertible symmetries. This notably happens when passing (dis)order operators through the Kramers-Wannier duality defect in the Ising model [185–188] (see also [189–191]). Note that for D_{4k}-type theories, the action on $W_\gamma(M_2)$ is trivial since the charge of γ is labeled by $\mathbb{Z}_2\times\mathbb{Z}_2$.

[9]Technically speaking, we should write $\tilde{\gamma}\in(\mathrm{Ab}(\Gamma)^{(2)})^\vee$ where $^\vee$ denotes Pontryagin dual and $\tilde{\gamma}$ pairs perfectly with γ, but we choose not to overload the notation.

[10]The $[p,q]$ strings/5-branes also experience a Hanany-Witten effect for R7-branes, which, for example is nontrivial for $p\neq 0$ for the Ω-brane. The relevance of Hanany-Witten moves in the study of symmetry operators was noted in [102] and was further explored in [105].

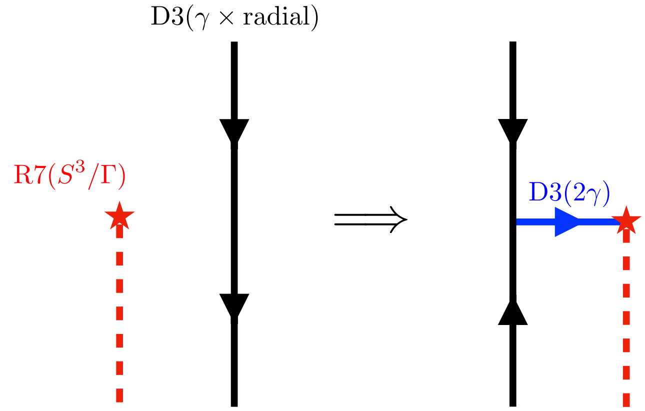

FIG. 1. Here we illustrate the effect of dragging a D3-brane (oriented black line) through an R7-brane (red star) whose cut associated to the monodromy action $C_4\to -C_4$ is denoted by the dashed red line. We also denote the submanifolds of $\mathbb{C}^2/\Gamma$ wrapped by these branes, here $\gamma\in H_1(S^3/\Gamma)$ is the torsion 1-cycle, relevant to constructing the 0-form charge conjugation operator for 6D SCFTs.

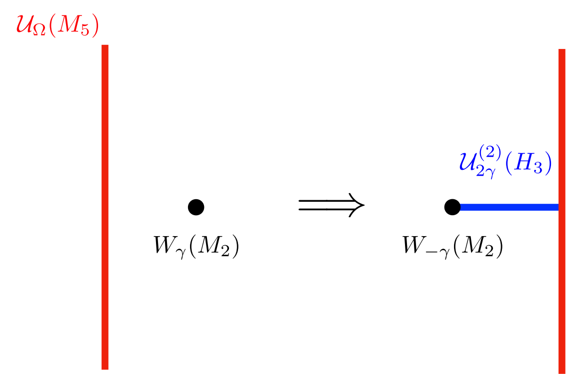

FIG. 2. Spacetime view of the Hanany-Witten process illustrated in Fig. 1 where we now indicate the spacetime submanifolds where these operators are supported. As the charged string defect operator $W_\gamma(M_2)$ passes through the charge conjugation operator $\mathcal{U}_\Omega(M_5)$, the 2-form symmetry operator $\mathcal{U}_{2\gamma}^{(2)}$ is created and stretches between $W_{-\gamma}(M_2)$ and $\mathcal{U}_\Omega(M_5)$.

The fusion rule (2.16) can simplify after one chooses a polarization for the 6D SCFT defect group, or equivalently, gauge a maximal nonanomalous subgroup of $\mathrm{Ab}(\Gamma)^{(2)}$ such that a given $W_\gamma(M_2)$ is a genuine defect operator while some $\mathcal{U}_{2\gamma'}^{(2)}$ is summed over the entire spacetime. In this case, $\mathcal{U}_{2\gamma'}^{(2)}(H_3)$ would no longer appear in the fusion rule since it is projected out of the theory.[11] For an illustrative example, take the type A_{p^2-1} 6D (2,0) theory where $\mathrm{Ab}(\Gamma)^{(2)}=\mathbb{Z}_{p^2}^{(2)}$. *A priori* this is a relative theory and we can form an

[11]Note that this is not always be possible as for instance when $|\mathrm{Ab}(\Gamma)^{(2)}|$ is a square-free integer.

absolute 6D SCFT by gauging $\mathbb{Z}_p^{(2)} \subset \mathbb{Z}_{p^2}^{(2)}$. If we denote γ as a generator of $\mathbb{Z}_{p^2}^{(2)}$, the gauging implies that we sum over networks of topological operators $\mathcal{U}_{p\gamma}^{(2)}$ such that $p\gamma \in \mathbb{Z}_p^{(2)} \subset \mathbb{Z}_{p^2}^{(2)}$. The gauged theory has the topological operators $\mathcal{U}_{\gamma \mathrm{mod} p}^{(2)}$ that generate the remaining $\mathbb{Z}_p^{(2)}$ symmetry (we leave the mod p implicit in what follows). From the string defect perspective, we start in the relative 6D theory with nongenuine defects $W_{\tilde{\gamma}}(M_2) \cdot \mathcal{U}_\gamma(M_3)$ where $\tilde{\gamma}$ generates the Pontryagin dual group $(\mathbb{Z}_{p^2}^{(2)})^\vee$, $\tilde{\gamma}(\gamma) = 1/p^2 \bmod 1$, and $\partial M_3 = M_2$. After gauging $\mathbb{Z}_p^{(2)}$, we have genuine defects $W_{p\tilde{\gamma}}(M_2) \in (\mathbb{Z}_p^{(2)})^\vee \subset (\mathbb{Z}_{p^2}^{(2)})^\vee$, while all other defects [i.e., ones nontrivial in $(\mathbb{Z}_{p^2}^{(2)})^\vee/(\mathbb{Z}_p^{(2)})^\vee$] are nongenuine. We now observe what happens when we drag a genuine and nongenuine defect across a charge conjugation operator $\mathcal{U}_\Omega$. We see that the genuine defect no longer has a topological operator attached because Eq. (2.16) now reads

$$\mathcal{U}_\Omega \cdot W_{p\tilde{\gamma}}(M_2) = W_{-p\tilde{\gamma}}(M_2) \cdot \mathcal{U}_{2p\gamma}(H_3) \cdot \mathcal{U}_\Omega \tag{2.17}$$

but $\mathcal{U}_{2p\gamma} = 1$ in the gauged theory so there is no extra topological operator attached. Meanwhile, the nongenuine defect has its attached topological operator altered by $\mathcal{U}_\gamma \mapsto \mathcal{U}_{-\gamma}$. In other words, the right-hand side of the fusion rule would automatically be accompanied by an extra $\mathcal{U}_{2\gamma}(H_3)$.

From this example, we then see that appearance of condensation operators in the definition of $U_\Omega(M_5)$ in (2.14) also follows from bottom-up considerations. This is because we are allowed to spontaneously create open topological defects of the form $U_{2\gamma}(N_3)$ on its world volume where $\partial N_3 \subset M_5$. This follows from moving $W_{\tilde{\gamma}}$ across $U_\Omega(M_5)$ and back again which means that a network of $U_{2\gamma}(N_3)$ is implicitly summed on the charge conjugation world volume M_5. For a similar point, see Fig. 5 of [65] which shows this creation property for duality defects.

For completeness, we also mention the analogous fusion rules relevant for the action of the R7 charge conjugation operator on the local operators and 4-manifold defects charged under $\mathrm{Ab}(\Gamma)^{(4)}$ and $\mathrm{Ab}(\Gamma)^{(0)}$ in the obvious notational adaptations

$$\mathcal{U}_\Omega \cdot W_\gamma(x) = W_{-\gamma}(x) \cdot \mathcal{U}_{2\gamma}^{(0)}(H_1) \cdot \mathcal{U}_\Omega, \tag{2.18}$$

$$\mathcal{U}_\Omega \cdot W_\gamma(M_4) = W_{-\gamma}(M_4) \cdot \mathcal{U}_{2\gamma}^{(4)}(H_5) \cdot \mathcal{U}_\Omega. \tag{2.19}$$

Similar remarks related to the simplification after choosing the polarization apply to these symmetries as well.

B. Using holographic CFTs to predict cobordism defects

Up to this point, we have assumed the existence of the R7-brane and have shown that it admits a natural interpretation as a charge conjugation symmetry operator in certain 6D SCFTs. We now turn the discussion around and use holography to argue for the existence of this cobordism defect.

Along these lines, the main idea will be to first show that for 6D $\mathcal{N} = (2,0)$ SCFTs with a semiclassical holographic dual, the gravity dual admits a discrete symmetry which we shall interpret as a charge conjugation symmetry in the 6D SCFT. As such, there must exist a corresponding codimension-one topological symmetry operator. Proceeding back from the M-theory realization to the F-theory realization, this amounts to a complementary expectation that there must exist a corresponding object in type IIB which implements this symmetry operator; this is nothing but the R7-brane.

To proceed, recall that there are well-known holographic duals for some of the 6D SCFTs just considered. For example, for the A-type $\mathcal{N} = (2,0)$ theories, we can start from N coincident M5-branes in flat space, we reach the gravity dual given by M-theory on $\mathrm{AdS}_7 \times S^4$ with N units of 4-form flux through the S^4 (see e.g., [192]). Similar considerations hold for the D-type theories, where the holographic dual is $\mathrm{AdS}_7 \times \mathbb{RP}^4$.

All the states, operators and symmetries that we found above, including the charge conjugation symmetry, must be apparent in the holographic dual. In this picture, the string defects obtained from D3-branes wrapping noncompact 2-cycles are represented by M2-branes attached to the boundary of the holographic dual. The charge conjugation symmetry is implemented in terms of the Pin^+ symmetry of M-theory [172,173,193–195], under which the M-theory 3-form C_3 transforms as a pseudo-3-form. What this means is that, in a compactification of the form $\mathrm{AdS}_7 \times X_4$, a reflection of an AdS_7 coordinate is not a symmetry of the theory, because the G_4 flux threading X_4 flips sign (and thus changes the vacuum), but a reflection on X_4 (if there is such a symmetry available) will flip both G_4 and the sign of the volume form, being a symmetry of the theory. Indeed, there are M-theory backgrounds which are holographically dual to $\mathcal{N} = (2,0)$ theories in the large N limit of the A- and D-type theories. These involve an X_4 which is either S^4 or $\mathbb{RP}^4$, and both preserve discrete symmetries which in the 6D SCFT specify a charge conjugation which squares to $+1$ in the 6D SCFT.[12]

[12]To be even more concrete, let us illustrate how some examples of such reflections are implemented on S^4 and $\mathbb{RP}^4$. Starting with an S^4 of radius L, we view it as the real hypersurface $(x_1)^2 + (x_2)^2 + (x_3)^2 + (x_4)^2 + (x_5)^2 = L^2$ in $\mathbb{R}^5$. The reflection $(x_1, x_2, x_3, x_4, x_5) \mapsto (-x_1, x_2, x_3, x_4, x_5)$ induces a corresponding reflection on the S^4. Other reflections are obtained by performing a rotation on the S^4. We reach $\mathbb{RP}^4$ by quotienting S^4 by the antipodal map $(x_1, x_2, x_3, x_4, x_5) \mapsto (-x_1, -x_2, -x_3, -x_4, -x_5)$. This still retains a $\mathbb{Z}_{\not 2}$ symmetry given by reflection of one of the ambient $\mathbb{R}^5$ coordinates, so this descends to a charge conjugation symmetry of the D-type theory.

In fact, at this point, one may very well flip the logic. Using the fact that the $\mathcal{N} = (2, 0)$ theory has a charge conjugation symmetry that squares to $+1$, we predict the existence of the R7-brane as the object that realizes the corresponding topological operator in the type IIB description. The R7-brane was originally described in [149,164] as a consequence of the cobordism conjecture, but from this perspective, its existence is required by holography and the standard type IIB description of the $\mathcal{N} = (2, 0)$ theory. In short, one can use concrete holographic constructions to provide evidence for some of the nonsupersymmetric objects predicted by the cobordism conjecture!

The considerations just presented also apply to many other situations, including beyond the AdS/CFT correspondence. For example, the holographic dual of a little string theory is (when it exists), flat space with a linear dilaton profile [196]. In such situations one can consider discrete reflection-type symmetries of the "internal" directions. This also applies to the near horizon limits of various black (and gray) objects in gravity. In short, the existence of a discrete symmetry in a holographic (but not necessarily AdS) dual provides evidence for a corresponding topological symmetry operator which must be implemented by a suitable object.

III. COMPACTIFICATION AND FURTHER REFLECTIONS

Starting from the 6D $\mathcal{N} = (2, 0)$ theories, one reaches a range of 4D SCFTs with $\mathcal{N} \geq 1$ supersymmetry by compactifying further on a genus g Riemann surface with punctures (see e.g., [197–199]).

It is natural to ask whether the R7-brane still implements a charge conjugation topological operator in this compactified theory. Although at first it would seem that the answer is always affirmative, since one can just wrap the 6D topological defect on the Riemann surface, additional ingredients such as a nontrivial flavor or R-symmetry bundle can still end up breaking the charge conjugation symmetry of the parent 6D theory. In such situations, one might still have a charge conjugation symmetry but it will have to be combined with additional discrete symmetry actions.

One should expect to have a charge conjugation symmetry in many cases. For example, this is the case for 4D $\mathcal{N} = 2$ supersymmetric theories. The question is whether the charge conjugation symmetry thus obtained in four dimensions can be directly traced back to the 6D C that squares to $+1$ and that we described above. When the reduction is on T^2, to produce an $\mathcal{N} = 4$ theory, this is automatically the case, and more generally, any toroidal compactification of the 6D $\mathcal{N} = (2, 0)$ SCFT will inherit a charge conjugation symmetry. However, when the compactification is on another genus $g \neq 1$ Riemann surface, the nontrivial R-symmetry bundle used to implement a partial topological twist of the theory will generically break the charge conjugation symmetry, and the same will happen when punctures are included. Moreover, in the case of 4D $\mathcal{N} = 1$ theories, the presence of background curvatures / flavor fluxes will generically lead to a chiral spectrum and broken charge conjugation symmetry (for example, a 6D hypermultiplet in the presence of a background flavor flux will descend to a 4D Weyl fermion).

We now briefly comment on spacetime reflection symmetries. Unlike ordinary symmetries, spacetime symmetries (and in particular, reflections) are not captured by simple topological operators. The only meaning of a reflection in a QFT is that the QFT makes sense on nonorientable manifolds (see [200] for a recent discussion of this point). Nonorientability is detected by the first Stiefel-Whitney class w_1; if we transport any operator along a closed path in the $\mathbb{Z}_2$ cycle dual to w_1, it will come back "reflected" to the starting point. One can take the point of view that this is because in going around the cycle it "crossed" a topological defect inducing a reflection (see [200] for a detailed exposition of this point), but such notions can be misleading since one cannot "insert" the operator in any orientable manifold. In cases where both charge conjugation and reflection symmetries are present, one may construct, in the restricted sense described above, a time-reversal operator. This provides a top down route to implementing various time-reversal symmetry defects of the sort considered in [98].

A. Other brane systems

So far, our discussion has primarily focused on the case of supersymmetric quantum field theories (SQFTs) engineered purely from singular background geometries. One can also consider D-brane probes of a singularity, and ask whether the R7-brane introduces a charge conjugation operation in this setting as well. In some cases, we find that the R7-brane does not implement a charge conjugation symmetry operator, and so we instead seek an alternative, which we explicitly provide in various cases.

It is instructive to observe that not all R7-branes can be introduced as topological operators in such constructions. For example, precisely because the F_L-brane acts via $|\mathrm{D}p\rangle \to |\overline{\mathrm{D}p}\rangle$, this leads to a rather dramatic jump in the asymptotic profile of the corresponding RR flux at the boundary of the background spacetime. Placing the F_L-brane at infinity then leads to a large backreaction in which the RR flux jumps from N to $-N$. See Fig. 3 for a depiction in the case of D3-branes.

The Ω-brane introduces no such issues for D1- and D5-branes, but again sends $|\mathrm{D3}\rangle \to |\overline{\mathrm{D3}}\rangle$ and $|\mathrm{D7}\rangle \to |\overline{\mathrm{D7}}\rangle$. As such, we conclude that a charge conjugation operator may be realized in the D1- and D5-brane gauge theories via Ω-branes, but not in these other systems. Lastly, one can also consider the S-dual brane configurations, and in such situations the roles of the F_L- and Ω-brane are reversed.

As an illustrative example, consider type IIB on $\mathbb{R}^{5,1} \times \mathbb{C}^2$ with N D5-branes filling the first factor. In this system,

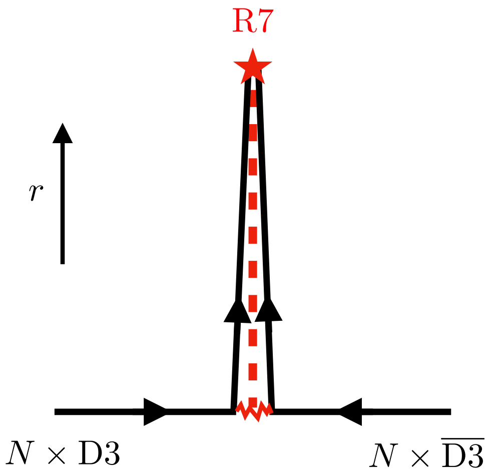

FIG. 3. Depiction of N D3-branes (at $r = 0$) in the presence of an R7-brane (at $r = \infty$). Because the R7-brane sends D3-branes to anti-D3-branes, there is a large jump in the flux and a number of D3-branes extend out from the D3 to the R7-brane at infinity. The jump in the flux emanates from the branch cut (dashed red). In this case the R7-brane does not produce a topological operator due to the significant change to the QFT sector. Rather, it becomes a nonsupersymmetric interface between $\mathcal{N} = 4$ SYM to itself.

we have Wilson line defects as obtained from F1-strings which run along the radial direction of $\mathbb{C}^2 = \mathrm{Cone}(S^3)$, and 't Hooft "membranes," from D3-branes which wrap the same radial direction and fill a three-dimensional subspace of $\mathbb{R}^{5,1}$. The topological operator which implements charge conjugation is given by an Ω-brane wrapped on the boundary $S^3 = \partial\mathbb{C}^2$. Indeed, observe that both the F1-string and D3-brane are conjugated to their antibrane counterparts upon passing through the corresponding topological defect. Wrapping on a T^2 and T-dualizing, we get a 4D gauge theory on the worldvolume of a D3-brane. In this setting, the wrapped D3-brane descends to a D1-brane, namely the 't Hooft line defect of the 4D theory. Observe also that T-duality must act nontrivially on the wrapped R7-brane to realize charge conjugation in this new theory.

1. D3-brane stack

Recently it was shown that for D3-brane probes of geometry, wrapping 7-branes with a constant axiodilaton profile "at infinity" provides a natural way to implement and unify various approaches to the duality defects of [64,65] from a top-down vantage point [165]. A natural candidate for a charge conjugation operator for a stack of N D3-branes realizing an $\mathcal{N} = 4$ $\mathfrak{su}(N)$ gauge theory is the I_0^* 7-brane[13] wrapped along the boundary S^5 and a codimension-one manifold in the D3 world volume. An important feature of the I_0^* 7-brane compared with other constant axiodilaton 7-branes is that it does not fix a specific value of the axiodilaton.[14]

The monodromy matrix for this 7-brane is given by

$$C \equiv \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \in GL(2,\mathbb{Z}), \qquad (3.1)$$

which in particular sends F1- and D1-strings to their antistring counterparts. Since an F1-/D1-string stretching from infinity and ending on the D3 stack is a fundamental Wilson/'t Hooft line we see that (3.1) indeed specifies a charge conjugation. The directions of the various branes in this scenario are as follows:

		0	1	2	3	4	5	6	7	8	9
QFT world volume	D3	×	×	×	×						
Defect	F1 or D1		×			×					
Symmetry operator	I_0^*7-brane	×	×	×			×	×	×	×	×

, (3.2)

where the directions "0, …, 3" represent the D3 world volume, the "4" direction is the radial direction of the transverse $\mathbb{C}^3$, and "5, …, 9," represent the asymptotic S^5 boundary. Similar to Sec. II A, we denote the $M_3 \times S^5$ as the total world volume of the 7-brane which produces a topological charge conjugation symmetry operator $\mathcal{U}_{I_0^*}(M_3)$. A key feature that differentiates this charge conjugation operator from those engineered from R7-branes is that C in (3.1) lifts to an order-four element $\hat{C}$ in $GL^+(2,\mathbb{Z})$ which satisfies $\hat{C}^2 = (-1)^F$, whereas the lift of the R7 monodromy will be an order-two element which squares to the identity due to the Pin^+ condition. For an explicit presentation of generators and relations of $GL^+(2,\mathbb{Z})$ see [74].

Since $\mathcal{N} = 4$ SYM can be obtained from dimensional reduction of the 6D (2,0) A_{N-1} theory on T^2 [201], we expect to have a charge conjugation operator which squares to $+1$. To construct it in the D3-brane system, one may combine the charge conjugation action C defined above with any order-four element of the $SU(4)$ R-symmetry group. The resulting operator, which we will call $\hat{\mathsf{C}}$, will act

[13] In perturbative string language, this is a collection of 4 D7-branes coincident with an $O7^-$ plane.

[14] The Weierstrass model for an I_0^* singularity is $y^2 = x^3 + f_0 z^2 x + g_0 z^3$. Tuning f_0 and g_0, one can reach any desired value of the axiodilaton.

on F1- and D1-strings as above, while not commuting with the R-symmetry; these are precisely the properties of the 6D charge-conjugation operator that we discussed previously.

To summarize then, the I_0^* on $M_3 \times S^5$ engineers the operator,

$$\mathcal{U}_{I_0^*}(M_3) = \hat{\mathsf{C}} \cdot \mathrm{TFT}_3, \tag{3.3}$$

where $\hat{\mathsf{C}}^2 = (-1)^F$ and TFT_3 is a 3D TFT living on the world volume of the topological operator. From the WZ term on the I_0^* world volume,

$$S_{WZ,I_0^*} \supset \int_{M_3 \times S^5} C_4 \mathrm{Tr} F^2_{\mathfrak{so}(8)}, \tag{3.4}$$

we find that the TFT is simply a level N Chern-Simons theory with gauge algebra $\mathfrak{so}(8)$. As in the case of $\mathcal{U}_\Omega$, we similarly obtain a Hanany-Witten effect whereby a topological surface operator attaches to a line operator after dragging it through $\mathcal{U}_{I_0^*}(M_3)$.

Finally, note that clearly these remarks generalize straightforwardly to constructing charge conjugation operators of SCFTs engineered from D3-brane probes of a Calabi-Yau twofold singularity. The nontrivial boundary topology can generally cause the fusion rules to become far richer as the bevy of terms in the Wess-Zumino action of the I_0^* 7-brane other than (3.4) will also have nontrivial Kaluza-Klein (KK)-reductions.[15]

B. Symmetry operators from other nonsupersymmetric branes

We now comment on how various nonsupersymmetric branes in heterotic and type I string theories can be used to construct topological symmetry operators for various field theories and LSTs. The type I nonsupersymmetric branes were first discussed long ago (see e.g., [202,203] for reviews and [204–206] for recent discussions of these branes from a world sheet point-of-view) and admit a KO-theory classification which is roughly equivalent to the topological configurations of the gauge field associated to nontrivial homotopy groups $\pi_*(SO(32))$ [207]. Meanwhile, nonsupersymmetric branes in heterotic string theories were recently discovered[16] in [208,210]. While our presentation is not exhaustive, our aim is to highlight some of the minimal settings in which these branes play a role as symmetry operators. These will be broadly applicable to geometric and brane engineering of QFTs or LSTs since these branes do not act on Ramond-Ramond (RR) p-form potentials nor on the Neveu-Schwarz-Neveu-Schwarz (NSNS) 2-form, and thus will not cause a large back-reaction as we saw in Fig. 3. As for the nonsupersymmetric branes not mentioned in this subsection, which include the heterotic 4-brane and type I D8-brane, we leave the exploration of their utility as symmetry operators for future work. Again, this section can also be read "backwards," in the sense that the fact that the symmetry operators must exist in the corresponding world volume theories provides indirect evidence for the existence of the corresponding nonsupersymmetric branes in the dual quantum theory of gravity.

1. $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ heterotic 7-brane

The heterotic 7-brane introduced in [208] is characterized by having a monodromy that exchanges the two E_8 factors of the gauge group. In other words, there is a nontrivial Wilson line for the $\mathbb{Z}_2$ outer automorphism factor in $(E_8 \times E_8) \rtimes \mathbb{Z}_2$. Wrapping this 7-brane along the asymptotic spatial directions would then be a 0-form symmetry that exchanges the two E_8 factors in a flavor group associated to some localized degrees of freedom.

A natural candidate for a physical system that may realize this 7-brane as a symmetry operator are small heterotic instantons arranged such that the instanton numbers are the same for both E_8 factors. In heterotic M-theory language,[17] this amounts to considering the same number N of parallel M5-branes arranged symmetrically between the two E_8 walls. As in [211,212], we can consider a gravitational decouping limit to isolate these 6D degrees of freedom such that the size of the interval between the two E_8 walls remains fixed but is much larger than the ten-dimensional Planck length, which engineers a 6D LST. If we consider N M5-branes, this engineers a rank-N E-string LST whose tensor branch is captured in the dual F-theory geometry as follows (where the number denote the self-intersection numbers of 2-cycles in the dual F-theory geometry):

$$\text{Rank-}N\text{ E-string LST}: \; [E_8]1, \underbrace{2, 2, \ldots, 2}_{N-2}, 1[E_8]. \tag{3.5}$$

Of the N compact curves, it is only possible to blow down $N-1$ of them with the volume of the remaining curve corresponding with the intrinsic length scale of the LST. The nonsupersymmetric 7-brane then engineers a 0-form symmetry exchanging the E_8 flavor factors only for a

[15]See for instance Sec. 5 of [165] which studied the dimensional reduction of various type IIB 7-branes on S^5/Γ in order to calculate the fusion of duality defects for 4D $\mathcal{N}=1$ SCFTs engineered from D3-branes probing $\mathbb{C}^3/\Gamma$. From that point of view, charge conjugation can be seen as a special case of a duality defect.

[16]The authors of [208] point out that the non-Bogomol'nyi-Prasad-Sommerfield (BPS) 0-brane they discuss is an endpoint for the Spin(32)/$\mathbb{Z}_2$ heterotic string, as initially proposed in [209].

[17]In this duality frame, the nonsupersymmetric 7-brane uplifts to pure geometry and is associated with reflection along the interval direction.

subregion in the LST tensor branch that respects this symmetry. For example, if we take N to be even and are at a tensor branch location such that $N/2$ M5-branes are at one E_8 wall and $N/2$ at the other, then the 7-brane is indeed a symmetry operator. Under the renormalization group flow to the IR we have

$$(\text{Rank-}N\,\text{E-string LST}) \to (\text{Rank-}N/2\,\text{E-string SCFT}) \oplus (\text{Rank-}N/2\,\text{E-string SCFT}),$$

where the right-hand side is a direct sum of two identical Rank-$N/2$ E-string SCFTs and the 0-form symmetry in the IR simply exchanges these two factors.

Similar remarks equally hold if we take four of the spatial directions of the $(E_8 \times E_8) \rtimes \mathbb{Z}_2$ heterotic string theory to be an ADE singularity $\mathbb{C}^2/\Gamma_{ADE}$ and consider small instanton probes thereof [213–215]. These are known as orbi-instanton LSTs, and on a partial tensor branch are characterized by the F-theory geometry

$$\text{Orbi-instanton Rank-}N\,\text{E-string LST:}\ [E_8]\overset{\mathfrak{g}_{ADE}}{1},\underbrace{\overset{\mathfrak{g}_{ADE}}{2},\overset{\mathfrak{g}_{ADE}}{2},\ldots,\overset{\mathfrak{g}_{ADE}}{2}}_{N-2},\overset{\mathfrak{g}_{ADE}}{1}[E_8], \tag{3.6}$$

where the notation $\overset{\mathfrak{g}_{ADE}}{n}$ denotes a $(-n)$-curve with a 7-brane hosting gauge degrees of freedom with Lie algebra $\mathfrak{g}_{ADE}$ wrapping it.

Finally, we mention that this 7-brane would engineer a symmetry operator on a 2D $\mathcal{N} = (0,1)$ SCFT associated to the heterotic string itself. This 0-form symmetry of course acts as an outer automorphism on the momentum lattice associated the internal left-moving T^{16} geometry.

2. 7-brane of type I string and 6-brane of heterotic string

Another set of nonsupersymmetric branes that can easily be interpreted in terms of symmetry operators are the $\mathbb{Z}_2$-valued 7-brane in type I string theory and the $\mathbb{Z}_2$-valued 6-brane in heterotic $\mathrm{Spin}(32)/\mathbb{Z}_2$ string theory. The former is associated with a $\mathrm{Spin}(32)/\mathbb{Z}_2$ gauge bundle such that we have a nontrivial Wilson line along the transverse angular S^1 direction. In particular, the nonsupersymmetric 0-brane of type I (which is S-dual to the massive spinor state in perturbative heterotic string theory) is a $\mathrm{Spin}(32)/\mathbb{Z}_2$ spinor state which has a nontrivial monodromy around this 7-brane [207,216]. In other words, the 7-brane is characterized by a Wilson line in the center of $\mathrm{Spin}(32)/\mathbb{Z}_2$ and winding around the bounding S^1 transverse to the 7-brane. As for the heterotic 6-brane, this is characterized by a nontrivial integral of the second Stiefel-Whitney class, $\int_{S^2} w_2$, along an S^2 that surrounds it.

We can realize both of these as symmetry operators for 6D $\mathcal{N} = (1,0)$ SCFTs considered in [217,218] (for a recent review see [219]) that arise in the low-energy limit of $[\mathrm{Spin}(32)/\mathbb{Z}_2]$-heterotic/type I small instantons probing an ADE singularity. A key property of these SCFTs is that they possess a $\mathrm{Spin}(32)/\mathbb{Z}_2$ flavor symmetry. Wrapping the 7-brane or 6-brane on the entire asymptotic boundary S^3/Γ_{ADE} leads to a $\mathbb{Z}_2$-valued 0-form symmetry (this is the $\mathbb{Z}_2$ flavor center symmetry operator) and $\mathbb{Z}_2$-valued 1-form symmetry operator respectively.[18] Backgrounds for these 0- and 1-form symmetries are simply associated with nontrivial Wilson line and w_2 for the $\mathrm{Spin}(32)/\mathbb{Z}_2$ flavor backgrounds in the dual field theory.[19]

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[18]Notice that when a 6-brane wraps the entire asymptotic boundary, we engineer a codimension-2 topological operator in the world volume of the 6D SCFT which is why it is a 1-form symmetry.

[19]6D SCFTs with nontrivial w_2 for the flavor bundle along compact directions has recently led to the construction of new 4D $\mathcal{N} = 2$ SCFTs [96,220], see also [221].

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