Propagation of gravitational waves in Einstein-Gauss-Bonnet gravity for cosmological and spherically symmetric spacetimes

Shin'ichi Nojiri,[1,2,3,*] S. D. Odintsov,[3,4,†] and V. K. Oikonomou[5,‡]

[1]Department of Physics, Nagoya University, Nagoya 464-8602, Japan
[2]Kobayashi-Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Nagoya 464-8602, Japan
[3]Institute of Space Sciences (ICE, CSIC), C. Can Magrans s/n, 08193 Barcelona, Spain
[4]ICREA, Passeig Luis Companys, 23, 08010 Barcelona, Spain
[5]Department of Physics, Aristotle University of Thessaloniki, Thessaloniki 54124, Greece

(Received 24 October 2023; accepted 24 January 2024; published 21 February 2024)

In this work, we examine the propagation of gravitational waves in cosmological and astrophysical spacetimes in the context of Einstein-Gauss-Bonnet gravity, in view of the GW170817 event. The perspective we approach the problem with is to obtain a theory which can produce a gravitational wave speed that is equal to that of light in the vacuum, or at least the speed can be compatible with the constraints imposed by the GW170817 event. As we show, in the context of Einstein-Gauss-Bonnet gravity, the propagation speed of gravity waves in cosmological spacetimes can be compatible with the GW170817 event, and we reconstruct some viable models. However, the propagation of gravity waves in spherically symmetric spacetimes violates the GW170817 constraints, thus it is impossible for the gravitational wave that propagates in a spherically symmetric spacetime to have a propagating speed which is equal to that of light in the vacuum. The same conclusion applies to the Einstein-Gauss-Bonnet theory with two scalars. We discuss the possible implications of our results on spherically symmetric spacetimes.

DOI: 10.1103/PhysRevD.109.044046

I. INTRODUCTION

During the next decade, the focus of modern theoretical physics and cosmology will be entirely on stage four cosmic microwave background [1,2] and gravitational wave experiments [3–11]. Both these experiments will shed light on the fundamental question of whether inflation ever occurred. Even recently, the inflationary scenario has been considerably constrained, since the NANOGrav 2023 stochastic gravitational wave background observation [12] requires a strongly blue-tilted inflationary era in order to consistently describe the signal [13], if standard postinflationary cosmological scenarios occurred. Thus theories that can yield a mild blue-tilted era can be important phenomenologically. In this line of research, string-inspired theories of gravity like Einstein-Gauss-Bonnet theories may play an important phenomenological role and for an important stream of reviews and research articles on this topic, see, for example, [14–64] and references therein. The Einstein-Gauss-Bonnet theories were severely restricted by the GW170817 neutron star merger event [65–67] which indicated that the speed of the gravitational waves should nearly coincide with that of the light in the vacuum. This GW170817 event imposed a severe constraint on the form of the scalar coupling factor of the scalar field on the Gauss-Bonnet invariant, which is a function often denoted by $\xi(\phi)$, and several scenarios were developed for the construction of a GW170817-compatible Einstein-Gauss-Bonnet theory [42,68–70]. The constraint is, however, valid only in the Friedmann-Lemaître-Robertson-Walker (FLRW) Universe background and it has been shown that any constraint cannot be satisfied around the static and spherically symmetric spacetime, which includes black holes, stellar objects, and wormholes [59].

In this paper, we again show that it is impossible to obtain a model where the propagating speed of the gravitational wave coincides with that of light in spherically symmetric spacetimes. We also consider the gravitational wave speed in cosmological spacetime and a scenario in which the Gauss-Bonnet coupling function $\xi(\phi)$ asymptotically approaches a constant in the late Universe when the speed of the gravitational wave has been observed, although the Gauss-Bonnet coupling may play important roles in the early Universe. This cosmological scenario is a possible description of the early Universe, and it is diverse compared to some previous approaches [42,68–70]. Although from a theoretical point of view, this scenario basically indicates that for some reason the graviton changes its mass at late times, becoming entirely massless, it is nevertheless a possibility that might be examined.

*nojiri@gravity.phys.nagoya-u.ac.jp
†odintsov@ice.csic.es
‡voikonomou@gapps.auth.gr, v.k.oikonomou1979@gmail.com

We construct a realistic model of Einstein-Gauss-Bonnet gravity, where the coupling function rapidly goes to a constant in the late Universe. The model could describe the whole evolutionary history of the Universe, including inflation, reheating, the late accelerating expansion, and so on. We firstly consider a standard Einstein-Gauss-Bonnet gravity coupled with one scalar field ϕ, and we clarify the condition that the propagating speed of the gravitational waves is equal to that of light and we show that the matter fluids do not affect the propagation speed. Since the propagating speed of the gravitational waves cannot coincide with that of light in a nontrivial spherically symmetric background, as shown in [59], we consider the scenario that the Gauss-Bonnet coupling function $\xi(\phi)$ goes to a constant in the late Universe and the propagating speed of the gravitational wave approaches that of light. We construct a more realistic model by using this scenario. The model describes both the inflationary era in the early Universe and the accelerating expansion of the present Universe, without introducing the parameters without hierarchy. We also estimate the speed of the gravitational wave in the epochs of inflation and at the end of the inflationary. We also discuss the reheating era and estimate the temperature and the propagating speed of the gravitational waves in this epoch. In Sec. III, we investigate the propagation of the gravitational wave in the background of a spherically symmetric spacetime. First, we consider the spherically symmetric and also time-dependent spacetime but also static and spherically symmetric spacetimes, and we show that the condition is not satisfied in the nontrivial spacetime including black holes, stellar objects, wormholes, etc. After that, we estimate the propagating speed of the gravitational wave inside the stellar objects. We give a general constraint when we require that the observational results in the GW170817 event should be applied inside the stellar objects. Also, we consider a model of the Einstein-Gauss-Bonnet gravity coupled with two scalars, in a spherically symmetric but time-dependent background. We show, however, that it is impossible to obtain a model where the propagating speed of the gravitational wave coincides with that of light. The last section is devoted to the summary and discussion.

II. SCALAR-EINSTEIN-GAUSS-BONNET GRAVITY

First, we consider the Einstein-Gauss-Bonnet gravity, with the action of the theory being given by[1]

$$S_{\phi\chi} = \int d^4x\sqrt{-g}\left\{\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \xi(\phi)\mathcal{G} + \mathcal{L}_{\rm matter}\right\}, \tag{1}$$

where $V(\phi)$ is the potential for ϕ, $\xi(\phi)$ is also a function of ϕ, and finally $\mathcal{L}_{\rm matter}$ denotes the Lagrangian density of the matter perfect fluids. Furthermore, $\mathcal{G}$ is the Gauss-Bonnet invariant defined by

$$\mathcal{G} = R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\rho\sigma}R^{\alpha\beta\rho\sigma}. \tag{2}$$

By the variation of the action (84) with respect to the metric $g_{\mu\nu}$, we obtain

$$\begin{aligned}0 = {}& \frac{1}{2\kappa^2}\left(-R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R\right) + \frac{1}{2}g_{\mu\nu}\left\{-\frac{1}{2}\partial_\rho\phi\partial^\rho\phi - V(\phi)\right\} + \frac{1}{2}\partial_\mu\phi\partial_\nu\phi \\ & - 2(\nabla_\mu\nabla_\nu\xi(\phi,\chi))R + 2g_{\mu\nu}(\nabla^2\xi(\phi,\chi))R + 4(\nabla_\rho\nabla_\mu\xi(\phi,\chi))R_\nu{}^\rho + 4(\nabla_\rho\nabla_\nu\xi(\phi,\chi))R_\mu{}^\rho \\ & - 4(\nabla^2\xi(\phi,\chi))R_{\mu\nu} - 4g_{\mu\nu}(\nabla_\rho\nabla_\sigma\xi(\phi,\chi))R^{\rho\sigma} + 4(\nabla^\rho\nabla^\sigma\xi(\phi,\chi))R_{\mu\rho\nu\sigma} + \frac{1}{2}T_{\mathrm{matter}\,\mu\nu},\end{aligned} \tag{3}$$

and the field equation for the scalar field is obtained by varying the action with respect to ϕ, and it is given by

$$0 = \nabla^\mu\partial_\mu\phi - V' - \xi'\mathcal{G}. \tag{4}$$

In (3), $T_{\mathrm{matter}\,\mu\nu}$ is the energy-momentum tensor of the perfect matter fluids, which obeys the continuity equation.

A. Gravitational waves in Einstein-Gauss-Bonnet gravity

In this subsection, we consider the condition that the propagating speed of the gravitational waves is equal to that of light in vacuum, and we show that matter is irrelevant to the speed as long as the matter minimally couples with gravity.

[1] In [71], a model similar to (1) has been studied and claimed that a scalar-Gauss-Bonnet coupling could induce tachyonic instabilities in perturbations during accelerated epochs. In the model, although the cosmological term with a cosmological constant appears in the paper, the model does not include the potential for the scalar field as $V(\phi)$ in (1). In the model (1), the effective potential is given by

$$V_{\rm eff}(\phi) = V(\phi) + \xi(\phi)\mathcal{G} = V(\phi) + 24(H^3H' + H^4)\xi(\phi).$$

In the last equality, we have assumed the FLRW Universe (14), $H \equiv \frac{1}{a}\frac{da}{dt}$. Therefore as long as $V''_{\rm eff}(\phi) > 0$, which corresponds to the effective mass of the scalar field, the tachyon does not appear. The condition $V''_{\rm eff}(\phi) > 0$ could correspond to the slow-roll condition and we can construct the model to satisfy the condition $V''_{\rm eff}(\phi) > 0$.

For the general variation of the metric,

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}, \tag{5}$$

we have the following formulas in leading order in terms of $h_{\mu\nu}$:

$$\begin{aligned}
\delta\Gamma^{\kappa}_{\mu\nu} &= \frac{1}{2} g^{\kappa\lambda}(\nabla_\mu h_{\nu\lambda} + \nabla_\nu h_{\mu\lambda} - \nabla_\lambda h_{\mu\nu}),\\
\delta R^{\mu}{}_{\nu\lambda\sigma} &= \nabla_\lambda \delta\Gamma^{\mu}_{\sigma\nu} - \nabla_\sigma \delta\Gamma^{\mu}_{\lambda\nu},\\
\delta R_{\mu\nu\lambda\sigma} &= \frac{1}{2}[\nabla_\lambda\nabla_\nu h_{\sigma\mu} - \nabla_\lambda\nabla_\mu h_{\sigma\nu} - \nabla_\sigma\nabla_\nu h_{\lambda\mu} + \nabla_\sigma\nabla_\mu h_{\lambda\nu} + h_{\mu\rho}R^{\rho}{}_{\nu\lambda\sigma} - h_{\nu\rho}R^{\rho}{}_{\mu\lambda\sigma}],\\
\delta R_{\mu\nu} &= \frac{1}{2}[\nabla^\rho(\nabla_\mu h_{\nu\rho} + \nabla_\nu h_{\mu\rho}) - \nabla^2 h_{\mu\nu} - \nabla_\mu\nabla_\nu(g^{\rho\lambda}h_{\rho\lambda})]\\
&= \frac{1}{2}[\nabla_\mu\nabla^\rho h_{\nu\rho} + \nabla_\nu\nabla^\rho h_{\mu\rho} - \nabla^2 h_{\mu\nu} - \nabla_\mu\nabla_\nu(g^{\rho\lambda}h_{\rho\lambda}) - 2R^{\lambda}{}_{\nu}{}^{\rho}{}_{\mu}h_{\lambda\rho} + R^{\rho}{}_{\mu}h_{\rho\nu} + R^{\rho}{}_{\mu}h_{\rho\nu}],\\
\delta R &= -h_{\mu\nu}R^{\mu\nu} + \nabla^\mu\nabla^\nu h_{\mu\nu} - \nabla^2(g^{\mu\nu}h_{\mu\nu}).
\end{aligned} \tag{6}$$

Then the variation of (3) is given by

$$\begin{aligned}
0 = {}& \left[\frac{1}{4\kappa^2}R + \frac{1}{2}\left\{-\frac{1}{2}\partial_\rho\phi\partial^\rho\phi - V\right\} - 4(\nabla_\rho\nabla_\sigma\xi)R^{\rho\sigma}\right]h_{\mu\nu}\\
&+ \left[-\frac{1}{4}g_{\mu\nu}\partial^\tau\phi\partial^\eta\phi - 2g_{\mu\nu}(\nabla^\tau\nabla^\eta\xi)R - 4(\nabla^\tau\nabla_\mu\xi)R^{\eta}_{v} - 4(\nabla^\tau\nabla_\nu\xi)R^{\eta}_{\mu} + 4(\nabla^\tau\nabla^\eta\xi)R_{\mu\nu}\right.\\
&\left. + 4g_{\mu\nu}(\nabla^\tau\nabla_\sigma\xi)R^{\eta\sigma} + 4g_{\mu\nu}(\nabla_\rho\nabla^\tau\xi)R^{\rho\eta} - 4(\nabla^\tau\nabla^\sigma\xi)R_{\mu}{}^{\eta}{}_{\nu\sigma} - 4(\nabla^\rho\nabla^\tau\xi)R_{\mu\rho\nu}{}^{\eta}\right]h_{\tau\eta}\\
&+ \frac{1}{2}\{2\delta_\mu{}^\eta\delta_\nu{}^\zeta(\nabla_\kappa\xi)R - 2g_{\mu\nu}g^{\eta\zeta}(\nabla_\kappa\xi)R - 4\delta_\rho{}^\eta\delta_\mu{}^\zeta(\nabla_\kappa\xi)R_\nu{}^\rho - 4\delta_\rho{}^\eta\delta_\nu{}^\zeta(\nabla_\kappa\xi)R_\mu{}^\rho\\
&+ 4g^{\eta\zeta}(\nabla_\kappa\xi)R_{\mu\nu} + 4g_{\mu\nu}\delta_\rho{}^\eta\delta_\sigma{}^\zeta(\nabla_\kappa\xi)R^{\rho\sigma} - 4g^{\rho\eta}g^{\sigma\zeta}(\nabla_\kappa\xi)R_{\mu\rho\nu\sigma}\}g^{\kappa\lambda}(\nabla_\eta h_{\zeta\lambda} + \nabla_\zeta h_{\eta\lambda} - \nabla_\lambda h_{\eta\zeta})\\
&+ \left\{\frac{1}{4\kappa^2}g_{\mu\nu} - 2(\nabla_\mu\nabla_\nu\xi) + 2g_{\mu\nu}(\nabla^2\xi)\right\}\{-h_{\mu\nu}R^{\mu\nu} + \nabla^\mu\nabla^\nu h_{\mu\nu} - \nabla^2(g^{\mu\nu}h_{\mu\nu})\}\\
&+ \frac{1}{2}\left\{\left(-\frac{1}{2\kappa^2} - 4\nabla^2\xi\right)\delta^\tau{}_\mu\delta^\eta{}_\nu + 4(\nabla_\rho\nabla_\mu\xi)\delta^\eta{}_\nu g^{\rho\tau} + 4(\nabla_\rho\nabla_\nu\xi)\delta^\tau{}_\mu g^{\rho\eta} - 4g_{\mu\nu}\nabla^\tau\nabla^\eta\xi\right\}\\
&\times\{\nabla_\tau\nabla^\phi h_{\eta\phi} + \nabla_\eta\nabla^\phi h_{\tau\phi} - \nabla^2 h_{\tau\eta} - \nabla_\tau\nabla_\eta(g^{\phi\lambda}h_{\phi\lambda}) - 2R^{\lambda}{}_{\eta}{}^{\phi}{}_{\tau}h_{\lambda\phi} + R^{\phi}{}_{\tau}h_{\phi\eta} + R^{\phi}{}_{\tau}h_{\phi\eta}\}\\
&+ 2(\nabla^\rho\nabla^\sigma\xi)\{\nabla_\nu\nabla_\rho h_{\sigma\mu} - \nabla_\nu\nabla_\mu h_{\sigma\rho} - \nabla_\sigma\nabla_\rho h_{\nu\mu} + \nabla_\sigma\nabla_\mu h_{\nu\rho} + h_{\mu\phi}R^{\phi}{}_{\rho\nu\sigma} - h_{\rho\phi}R^{\phi}{}_{\mu\nu\sigma}\}\\
&+ \frac{1}{2}\frac{\partial T_{\mathrm{matter}\,\mu\nu}}{\partial g_{\tau\eta}}h_{\tau\eta},
\end{aligned} \tag{7}$$

where we have assumed that the perfect matter fluids are minimally coupled with gravity. We now choose a condition to fix the gauge as follows:

$$0 = \nabla^\mu h_{\mu\nu}. \tag{8}$$

Since we are interested in the massless spin-two tensor mode, we also impose the traceless condition,

$$0 = g^{\mu\nu}h_{\mu\nu}. \tag{9}$$

We do not consider the perturbation of the scalar mode in the metric like the trace part, which may couple with the scalar field ϕ (see [72] for example) because we are now interested in the massless and spin-two mode, which corresponds to the usual gravitational wave. As long as we consider the leading order of the perturbation, the massless spin-two mode does not mix with the scalar mode, which is a massive spin-zero mode although the second-order perturbation of the scalar field plays the role of the source of the gravitational wave. The traceless condition (9) makes the massless spin-two mode decouple with the massive spin-zero mode.

Then Eq. (7) is reduced as follows:

$$\begin{aligned}
0 = {}& \left[\frac{1}{4\kappa^2}R + \frac{1}{2}\left\{-\frac{1}{2}\partial_\rho\phi\partial^\rho\phi - V\right\} - 4(\nabla_\rho\nabla_\sigma\xi)R^{\rho\sigma}\right]h_{\mu\nu} \\
&+ \left[\frac{1}{4}g_{\mu\nu}\{-A\partial^\tau\phi\partial^\eta\phi - B(\partial^\tau\phi\partial^\eta\chi + \partial^\eta\phi\partial^\tau\chi) - C\partial^\tau\chi\partial^\eta\chi\}\right. \\
&- 2g_{\mu\nu}(\nabla^\tau\nabla^\eta\xi)R - 4(\nabla^\tau\nabla_\mu\xi)R_\nu{}^\eta - 4(\nabla^\tau\nabla_\nu\xi)R_\mu{}^\eta + 4(\nabla^\tau\nabla^\eta\xi)R_{\mu\nu} \\
&\left.+ 4g_{\mu\nu}(\nabla^\tau\nabla_\sigma\xi)R^{\eta\sigma} + 4g_{\mu\nu}(\nabla_\rho\nabla^\tau\xi)R^{\rho\eta} - 4(\nabla^\tau\nabla^\sigma\xi)R_\mu{}^\eta{}_{\nu\sigma} - 4(\nabla^\rho\nabla^\tau\xi)R_{\mu\rho\nu}{}^\eta\right]h_{\tau\eta} \\
&+ \frac{1}{2}\{2\delta_\mu{}^\eta\delta_\nu{}^\zeta(\nabla_\kappa\xi)R - 4\delta_\rho{}^\eta\delta_\mu{}^\zeta(\nabla_\kappa\xi)R_\nu{}^\rho - 4\delta_\rho{}^\eta\delta_\nu{}^\zeta(\nabla_\kappa\xi)R_\mu^\rho \\
&+ 4g_{\mu\nu}\delta_\rho{}^\eta\delta_\sigma{}^\zeta(\nabla_\kappa\xi)R^{\rho\sigma} - 4g^{\rho\eta}g^{\sigma\zeta}(\nabla_\kappa\xi)R_{\mu\rho\nu\sigma}\}g^{\kappa\lambda}(\nabla_\eta h_{\zeta\lambda} + \nabla_\zeta h_{\eta\lambda} - \nabla_\lambda h_{\eta\zeta}) \\
&- \left\{\frac{1}{4\kappa^2}g_{\mu\nu} - 2(\nabla_\mu\nabla_\nu\xi) + 2g_{\mu\nu}(\nabla^2\xi)\right\}R^{\mu\nu}h_{\mu\nu} \\
&+ \frac{1}{2}\left\{\left(-\frac{1}{2\kappa^2} - 4\nabla^2\xi\right)\delta^\tau{}_\mu\delta^\eta{}_\nu + 4(\nabla_\rho\nabla_\mu\xi)\delta^\eta{}_\nu g^{\rho\tau} + 4(\nabla_\rho\nabla_\nu\xi)\delta^\tau{}_\mu g^{\rho\eta} - 4g_{\mu\nu}\nabla^\tau\nabla^\eta\xi\right\} \\
&\times\{-\nabla^2 h_{\tau\eta} - 2R^\lambda{}_\eta{}^\phi{}_\tau h_{\lambda\phi} + R^\phi{}_\tau h_{\phi\eta} + R^\phi{}_\tau h_{\phi\eta}\} \\
&+ 2(\nabla^\rho\nabla^\sigma\xi)\{\nabla_\nu\nabla_\rho h_{\sigma\mu} - \nabla_\nu\nabla_\mu h_{\sigma\rho} - \nabla_\sigma\nabla_\rho h_{\nu\mu} + \nabla_\sigma\nabla_\mu h_{\nu\rho} + h_{\mu\phi}R^\phi{}_{\rho\nu\sigma} - h_{\rho\phi}R^\phi{}_{\mu\nu\sigma}\} + \frac{1}{2}\frac{\partial T_{\mathrm{matter}\,\mu\nu}}{\partial g_{\tau\eta}}h_{\tau\eta}. \qquad (10)
\end{aligned}$$

The observation of GW170817 gives the constraint on the propagating speed c_{GW} of the gravitational wave as follows:

$$\left|\frac{c_{\mathrm{GW}}{}^2}{c^2} - 1\right| < 6\times 10^{-15}, \qquad (11)$$

where c denotes the speed of light. In order to investigate if the propagating speed c_{GW} of the gravitational wave $h_{\mu\nu}$ could be different from that of the light c, we only need to check the parts including the second derivatives of $h_{\mu\nu}$,

$$\begin{aligned}
I_{\mu\nu} &\equiv I^{(1)}_{\mu\nu} + I^{(2)}_{\mu\nu}, \\
I^{(1)}_{\mu\nu} &\equiv \frac{1}{2}\left\{\left(-\frac{1}{2\kappa^2} - 4\nabla^2\xi\right)\delta^\tau{}_\mu\delta^\eta{}_\nu + 4(\nabla_\rho\nabla_\mu\xi)\delta^\eta{}_\nu g^{\rho\tau} + 4(\nabla_\rho\nabla_\nu\xi)\delta^\tau{}_\mu g^{\rho\eta} - 4g_{\mu\nu}\nabla^\tau\nabla^\eta\xi\right\}\nabla^2 h_{\tau\eta}, \\
I^{(2)}_{\mu\nu} &\equiv 2(\nabla^\rho\nabla^\sigma\xi)\{\nabla_\nu\nabla_\rho h_{\sigma\mu} - \nabla_\nu\nabla_\mu h_{\sigma\rho} - \nabla_\sigma\nabla_\rho h_{\nu\mu} + \nabla_\sigma\nabla_\mu h_{\nu\rho}\}. \qquad (12)
\end{aligned}$$

Since we are assuming that the matter fluids minimally couple with gravity, any contribution of the matter fluids does not couple with any derivative of $h_{\mu\nu}$, and the contribution does not appear in $I_{\mu\nu}$. In other words, matter is not relevant to the propagating speed of the gravitational wave. We should note that $I^{(1)}_{\mu\nu}$ does not change the speed of the gravitational wave from the speed of light. On the other hand, $I^{(2)}_{\mu\nu}$ changes the speed of the gravitational wave from that of the light in general, which may violate the constraint (11). If $\nabla_\mu\nabla^\nu\xi$ is proportional to the metric $g_{\mu\nu}$,

$$\nabla_\mu\nabla^\nu\xi = \frac{1}{4}g_{\mu\nu}\nabla^2\xi, \qquad (13)$$

then $I^{(2)}_{\mu\nu}$ does not change the speed of the gravitational wave from that of light. We should note that ξ is a function specifying the model. Equation (13) is a condition for models so that the propagating speed of the gravitational wave coincides with that of light.

As long as we consider the FLRW Universe, we can find the solution of Eq. (13) as explicitly given in (18). As shown in [59], the condition Eq. (13) cannot be satisfied in more general background like nontrivial spherically symmetric spacetime. Then instead of considering nontrivial solution of Eq. (13), we will consider the scenario where ξ goes to a constant or vanishes in the late Universe as the gravitational waves have been detected and $I^{(2)}_{ij}$ can be neglected in the late Universe.

B. Method for reconstructing realistic models of early and late-time cosmic expansion

In this subsection, we will focus on the scenario that the Gauss-Bonnet coupling function $\xi(\phi)$ goes to a constant in the late-time Universe and also that the propagating speed of the gravitational wave approaches that of light. In this line of research, we construct a realistic model of cosmic expansion. Both the inflationary era in the early Universe and the accelerating expansion of the present Universe can be described in a unified way in this model, without introducing various parameters with different scales. The speed of the gravitational wave in the epochs of the inflation and at the end of the inflation are also estimated.

We consider the FLRW universe with a flat spatial section, the line element of which is given by

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} (dx^i)^2. \tag{14}$$

Here t is the cosmic time, and $a(t)$ denotes the scale factor. We often use $H = \frac{\dot{a}}{a}$ which is the Hubble rate. Now we have

$$\begin{aligned}
&\Gamma^t_{ij} = a^2 H \delta_{ij}, \qquad \Gamma^i_{jt} = \Gamma^i_{tj} = H\delta^i{}_j, \\
&R_{itjt} = -(\dot{H} + H^2) a^2 h_{ij}, \qquad R_{ijkl} = a^4 H^2 (\delta_{ik}\delta_{lj} - \delta_{il}\delta_{kj}), \\
&R_{tt} = -3(\dot{H} + H^2), \qquad R_{ij} = a^2(\dot{H} + 3H^2)\delta_{ij}, \\
&R = 6\dot{H} + 12H^2, \qquad \text{other components} = 0,
\end{aligned} \tag{15}$$

therefore if ξ only depends on the cosmic time t, we obtain

$$\begin{aligned}
&\nabla_t \nabla_t \xi = \ddot{\xi}, \qquad \nabla_i \nabla_j \xi = -a^2 H \delta_{ij} \dot{\xi}, \\
&\nabla_t \nabla_i \xi = \nabla_i \nabla_t \xi = 0, \qquad \nabla^2 \xi = -\ddot{\xi} - 3H\dot{\xi}.
\end{aligned} \tag{16}$$

Then Eq. (13) becomes a second-order ordinary differential equation with respect to the cosmological time t as follows:

$$\ddot{\xi} = H\dot{\xi}, \tag{17}$$

whose solution is

$$\dot{\xi} = \xi_0 + \xi_1 \int dt a(t). \tag{18}$$

Because the differential equation (17) is second order, the solution includes two constants of the integration ξ_0 and ξ_1. In the case $\xi_1 = 0$, ξ becomes a constant and therefore the Gauss-Bonnet term becomes a total derivative term and the term does not contribute to any equation. We should note that the condition $\xi_1 = 0$ is not an initial condition or something else but the condition $\xi_1 = 0$ is the condition defining the model. Instead of choosing the condition $\xi_1 = 0$, which makes the Gauss-Bonnet term trivial, if we choose the condition $\xi_1 \neq 0$, ξ is given by a function of the cosmological time t. Furthermore, if we use the relation $H = \frac{dN}{dt}$, the function ξ can be expressed as a function of the e-foldings N. By using the relation between the scalar field ϕ and the e-foldings N in (26), which appears later, we can determine ξ as a function of the scalar field ϕ, which specifies the model.

We should note, however, that the propagating speed of the gravitational wave cannot be equal to that of light in the nontrivial spherically symmetric background, as it was shown in [59].

Since the propagating speed of the gravitational wave cannot be equal to the speed of light near black holes or stellar objects, we consider the scenario that $I^{(2)}_{ij}$ can be neglected in the late Universe. This requires that ξ goes to a constant or vanishes in the late Universe, which is a special solution of (13) corresponding to $\xi_1 = 0$ in (18). In this solution, the Gauss-Bonnet term in the action (1) becomes a total derivative and does not give any contribution to the expansion of the Universe, although the term may become necessary in the early Universe. If this scenario is realized, then the theory reduces to the scalar-tensor theory in the late Universe.

The equations corresponding to the FLRW equations have the following forms, which are given by Eq. (3):

$$\begin{aligned}
0 &= -\frac{3}{\kappa^2} H^2 + \frac{1}{2}\dot{\phi}^2 + V(\phi) + 24H^3 \frac{d\xi(\phi(t))}{dt}, \\
0 &= \frac{1}{\kappa^2}(2\dot{H} + 3H^2) + \frac{1}{2}\dot{\phi}^2 - V(\phi) - 8H^2 \frac{d^2\xi(\phi(t))}{dt^2} \\
&\quad - 16H\dot{H}\frac{d\xi(\phi(t))}{dt} - 16H^3 \frac{d\xi(\phi(t))}{dt}, \\
0 &= \ddot{\phi} + 3H\dot{\phi} + V'(\phi) + \xi'(\phi)\mathcal{G}.
\end{aligned} \tag{19}$$

The third equation in (19) can be obtained by combining the first and second equations and therefore we forgot the third equation in the following. By using the e-foldings number N defined by $a = a_0 \mathrm{e}^N$ instead of the cosmic time t, we now rewrite (19) as follows:

$$\begin{aligned}
0 &= -\frac{3}{\kappa^2} H^2 + \frac{1}{2} H^2 \phi'(N)^2 + V(\phi) + 24H^4 \frac{d\xi(\phi(N))}{dN}, \\
0 &= \frac{1}{\kappa^2}\left(2H\frac{dH}{dN} + 3H^2\right) + \frac{1}{2}H^2\phi'(N)^2 - V(\phi) \\
&\quad - 8H^4 \frac{d^2\xi(\phi(t))}{dN^2} - 24H^3 \frac{dH}{dN}\frac{d\xi(\phi(N))}{dN} \\
&\quad - 16H^4 \frac{d\xi(\phi(t))}{dN}.
\end{aligned} \tag{20}$$

We should note $\frac{d}{dt} = H\frac{d}{dN}$ and therefore $\frac{d^2}{dt^2} = H^2 \frac{d^2}{dN^2} + \frac{dH}{dN}\frac{d}{dN}$. By deleting $V(\phi)$ in (20), we obtain

$$\begin{aligned}
0 &= \frac{2}{\kappa^2} H'(N) + H(N)\phi'(N)^2 - 8H(N)^3 \frac{d^2\xi(\phi(t))}{dN^2} \\
&\quad - 24H(N)^2 \frac{dH}{dN}\frac{d\xi(\phi(N))}{dN} + 8H(N)^3 \frac{d\xi(\phi(t))}{dN} \\
&= \frac{2}{\kappa^2} H'(N) + H(N)\phi'(N)^2 \\
&\quad - 8\mathrm{e}^N \frac{d}{dN}\left(\mathrm{e}^{-N} H(N)^3 \frac{d\xi(\phi(t))}{dN}\right),
\end{aligned} \tag{21}$$

which can be integrated with respect to $\xi(N)$ and we obtain

$$\xi(\phi(N)) = \frac{1}{8}\int^N dN_1 \frac{e^{N_1}}{H(N_1)^3}\int^{N_1}\frac{dN_2}{e^{N_2}} \times \left(\frac{2}{\kappa^2}H'(N_2) + H(N_2)\phi'(N_2)^2\right). \quad (22)$$

By substituting Eq. (22) into the first equation in (20), we find

$$V(\phi(N)) = \frac{3}{\kappa^2}H(N)^2 - \frac{1}{2}H(N)^2\phi'(N)^2 - 3e^N H(N) \times \int^N \frac{dN_1}{e^{N_1}}\left(\frac{2}{\kappa^2}H'(N_1) + H(N_1)\phi'(N_1)^2\right). \quad (23)$$

Equations (22) and (23) and tell that by using functions $h(N)$ and $f(\phi)$, if $\xi(\phi)$ and $V(\phi)$ are given by

$$V(\phi) = \frac{3}{\kappa^2}h(f(\phi))^2 - \frac{h(f(\phi))^2}{2f'(\phi)^2} - 3h(f(\phi))e^{f(\phi)} \times \int^\phi d\phi_1 f'(\phi_1)e^{-f(\phi_1)}\left(\frac{2}{\kappa^2}h'(f(\phi_1)) + \frac{h'(f(\phi_1))}{f'(\phi_1)^2}\right), \quad (24)$$

$$\xi(\phi) = \frac{1}{8}\int^\phi d\phi_1 \frac{f'(\phi_1)e^{f(\phi_1)}}{h(f(\phi_1))^3}\int^{\phi_1} d\phi_2 f'(\phi_2)e^{-f(\phi_2)} \times \left(\frac{2}{\kappa^2}h'(f(\phi_2)) + \frac{h(f(\phi_2))}{f'(\phi_2)^2}\right), \quad (25)$$

a solution of the equations in (19) is given by

$$\phi = f^{-1}(N) \quad (N = f(\phi)), \qquad H = h(N). \quad (26)$$

Therefore we obtain a general Einstein-Gauss-Bonnet model realizing the time evolution of H given by an arbitrary function $h(N)$ as in (26). We often solve the model with given potential, etc. but here, we have considered the solution $H = h(N)$ first and we have constructed the model that realizes the given solution $H = h(N)$. We should also note that the time evolution is determined only by one function $h(N)$ but in the action (1), there appear two functions $V(\phi)$ and $\xi(\phi)$. There is one additional functional degree of freedom in the model compared with the evolution. We can separate the two functional degrees of freedom in the action into the function $h(N)$ relevant to the evolution of H and the function $f(\phi)$ irrelevant to the evolution. Therefore if we change $f(\phi)$, the functional form of $\xi(\phi)$ changes but, of course, the functional form of $V(\phi)$ also changes. The changes of $\xi(\phi)$ and $V(\phi)$ compensate with each other and the time evolution of the expansion of the Universe given by $H = h(N)$ does not change.

We should note again that the history of the expansion of the Universe is determined only by the function $h(N)$ and does not depend on the choice of $f(\phi)$. By using this indefiniteness of the choice of $f(\phi)$, we consider the possibility that ξ goes to a constant or vanishes in the late Universe. The possibility can be satisfied if

$$0 \sim \frac{2}{\kappa^2}H'(N) + H(N)\phi'(N)^2, \quad (27)$$

which can be solved as

$$\phi(N) = f^{-1}(N) \sim \int dN\sqrt{-\frac{2H'(N)}{\kappa^2 H(N)}}. \quad (28)$$

The above expression can be valid as long as $H'(N) < 0$, which corresponds to the case that the effective equation of state parameter, which is defined by

$$w_{\mathrm{eff}} \equiv -1 - \frac{2H'}{3H}, \quad (29)$$

and is greater than -1. Compared with the past Universe, in the late Universe, the Hubble rate H goes to a constant, and we expect that H become asymptotically constant in the future, that is, the spacetime becomes an asymptotically de Sitter spacetime. This feature indicates that if the scalar field ϕ goes to a constant in the late Universe, the condition (28) is satisfied. This is natural because ξ is a function of ϕ, if ϕ goes to a constant, then $\xi = \xi(\phi)$ becomes a constant as long as ξ is not a singular function.

For example, we may assume,

$$\xi = \xi_0(1 - e^{-\xi_1 N}). \quad (30)$$

Here ξ_0 and ξ_1 are constants and we assume ξ_1 is positive. Then ξ rapidly goes to a constant $\xi \to \xi_0$ when N becomes large. And therefore, the Einstein-Gauss-Bonnet gravity transits to the standard scalar-tensor theory. The cosmic time of the transition can be adjusted by fine-tuning the parameter ξ_1. For example, if we choose $1/\xi_1 \lesssim 60$, $\xi(\phi)$ becomes almost constant in the epoch of the reheating. Equation (22) indicates that

$$\phi'^2 = -\frac{2H'}{\kappa^2 H} + 8\xi_0\xi_1\{2HH' - (\xi_1 + 1)H^2\}e^{-\xi_1 N}. \quad (31)$$

The second term decreases rapidly due to the factor $e^{-\xi_1 N}$ and the first term vanishes, and therefore ϕ also goes to a constant consistently if the spacetime becomes asymptotically a de Sitter spacetime.

We may estimate the propagating speed c_{GW} of the gravitational waves. We now consider the plain wave $h_{ij} \propto \mathrm{Re}(e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{x}})$. Under the condition (8) and (9), by using (16), Eq. (12) gives the following dispersion relation for high energy gravitational waves,

$$0 = \frac{1}{2}\left(-\frac{1}{2\kappa^2} + 4(\ddot{\xi} + 3H\dot{\xi})\right)\left(\omega^2 - \frac{k^2}{a^2}\right) + 2\ddot{\xi}\omega^2. \tag{32}$$

Here $k^2 = \boldsymbol{k}\cdot\boldsymbol{k}$. Equation (32) shows that the

$$c_{\mathrm{GW}}{}^2 = \frac{1 - 8\kappa^2(\ddot{\xi} + 3H\dot{\xi})}{1 - 8\kappa^2(2\ddot{\xi} + 3H\dot{\xi})}c^2 \sim (1 + 8\kappa^2\ddot{\xi})c^2. \tag{33}$$

Now the speed of light c is given by $c^2 = \frac{1}{a^2}$. We also we assumed $|\kappa^2\ddot{\xi}| \ll 1$. Equation (33) shows that if $\ddot{\xi} > 0$ ($\ddot{\xi} < 0$), then the propagating speed of the gravitational wave is larger (smaller) than that of light. The observation of GW170817 in (11) gives the following constraint:

$$|8\kappa^2\ddot{\xi}| < 6\times 10^{-15}. \tag{34}$$

Especially in the case of (30), we obtain

$$|8\kappa^2\xi_0(\xi_1 HH' - \xi_1{}^2H^2)\mathrm{e}^{-\xi_1 N}| < 6\times 10^{-15}. \tag{35}$$

In the present Universe, where the speed of the gravitational wave was measured, we may assume $N = 120$–140.

We consider the following model in terms of e-foldings number N, which satisfies the above conditions,

$$H = h(N) = H_0(1 + \alpha N^\beta)^\gamma. \tag{36}$$

We construct this model to describe the whole history of the Universe, that is, inflation, matter-dominant epoch, and the accelerating expansion of the present Universe. When N is small, we find $H \sim H_0(1 + \alpha\gamma N^\beta)$ and when N is large, $H \sim H_1\alpha^\gamma N^{\beta\gamma}$. Therefore H_1 and α should be positive so that H is positive. We also require $\beta > 0$ and $\gamma < 0$ so that H is a monotonically decreasing function.

The effective equation of state parameter is given by

$$w_{\mathrm{eff}} = -1 - \frac{2\alpha\beta\gamma N^{\beta-1}}{3(1 + \alpha N^\beta)}, \tag{37}$$

which goes to -1 when $N \to 0$ or $N \to \infty$, We should note that $N \to 0$ corresponds to the early Universe and $N \to \infty$ corresponds to the present or future Universe. Therefore because the effective equation of state parameter w_{eff} goes to -1 when $N \to 0$ and when $N \to \infty$, the model (36) describes both the inflation and the accelerating expansion in the late Universe.

We now check if the model (36) also describes the matter-dominated area. When $w_{\mathrm{eff}} = -\frac{1}{3}$, we find

$$-\alpha\beta\gamma N^{\beta-1} = 1 + \alpha N^\beta. \tag{38}$$

Let the two solutions of (38) $N = N_1$ and $N = N_2$ ($0 < N_1 < N_2$). Then the period where $N < N_1$ corresponds to the inflation in the early Universe and the period where $N > N_2$ to the accelerating expansion in the present Universe. During $N_1 < N < N_2$, we expect w_{eff} goes to vanish at least if we include the contribution of the matter that is dust.

The parameters α and γ are given in terms of β, N_1, and N_2, as follows:

$$\alpha = \frac{N_2{}^{\beta-1} - N_1{}^{\beta-1}}{(N_1N_2)^{\beta-1}(N_2 - N_1)}, \qquad \gamma = -\frac{N_2{}^\beta - N_1{}^\beta}{\beta(N_2{}^{\beta-1} - N_1{}^{\beta-1})}. \tag{39}$$

We should note that α and γ are positive as long as $\beta > 1$ as we required. We now estimate the parameters α, β, and γ in order to obtain realistic models compatible with the constraint on the gravitational wave speed. Let the beginning of the inflation correspond to $N = 0$. Then the end of the inflation corresponds to $N = N_1 = 60$–70 and the recombination (clear up of the Universe) to $N = 120$–140. The redshift of the recombination is $z = 1100$. Because $1 + z = 1/a$, where a is the scale factor, we obtain $N_0 - N = \ln(1 + z)$, where N_0 is the redshift of the present Universe. We note $\ln 1,100 \sim 7$. The redshift corresponding to the beginning of the accelerating expansion of the late Universe is approximately 0.4 and $\ln 1.4 \sim 0.3$. Therefore $N_2 \sim 2N_1$. In order to estimate α and β in (39), we assume $N_2 = 2N_1 = \mathcal{O}(10^2)$. Then we find

$$\alpha = \frac{1 - 2^{1-\beta}}{2^{-\beta}N_2{}^\beta}, \qquad \gamma = -\frac{(1 - 2^{-\beta})N_2}{\beta(1 - 2^{1-\beta})}. \tag{40}$$

In the early Universe, where $N \to 0$, Eq. (36) has the following form:

$$H \sim H_0(1 + \alpha\gamma N^\beta). \tag{41}$$

Therefore, H_0 corresponds to the scale of inflation and we now choose $H_0 \sim 10^{14}$ GeV $= 10^{23}$ eV. On the other hand, when N is large ($N \sim 10^2$), which corresponds to the period of the accelerating expansion of the present Universe, we find

$$H = H_0\alpha^\gamma N^{\beta\gamma}, \tag{42}$$

which requires $\alpha^\gamma N^{\beta\gamma} \sim 10^{-56}$ because $H \sim 10^{-33}$ eV. Because $N \sim N_2$, by using (40), we find

$$(2^\beta - 2)^{-\frac{(1-2^{-\beta})N_2}{\beta(1-2^{1-\beta})}} \sim 10^{-56}. \tag{43}$$

When $\beta \to 1$, we find $(2^\beta - 2)^{-\frac{(1-2^{-\beta})N_2}{\beta(1-2^{1-\beta})}} \to 0$. On the other hand, when $\beta = 2$, we find $(2^\beta - 2)^{-\frac{(1-2^{-\beta})N_2}{\beta(1-2^{1-\beta})}} = 2^{-\frac{3}{4}N_2} \sim 10^{-22} \gg 10^{-56}$. Therefore, there is a solution β for Eq. (43) when $1 < \beta < 2$. Then Eq. (40) shows that

$\alpha \sim O(10^{-(2-4)})$ and $\gamma \sim \mathcal{O}(10^2)$, and therefore these parameters are not too small or large.

C. Gravitational waves during the inflationary era

We now consider the propagating speed of the gravitational wave during the inflationary era. The expression of the propagating speed of the gravitational wave in (33) is valid and at the beginning of the inflation $N \sim 0$, we find

$$8\kappa^2\ddot{\xi} = 8\kappa^2\xi_0\xi_1(\dot{H} - \xi H^2)e^{-\xi_1 N} \sim -8\kappa^2{\xi_0}^2\xi_1{H_0}^2. \tag{44}$$

Here we have assumed that Eqs. (30) and (36) hold true. On the other hand, at the end of the inflation, we obtain

$${c_{\mathrm{GW}}}^2 \sim (1 - 8\kappa^2\xi_0{H_0}^2(1+\alpha{N_1}^\beta)^{2\gamma}(\xi_1 + {\xi_1}^2)e^{-\xi_1 N_1})c^2. \tag{45}$$

Here we have used (36) and (38) for $N = N_1$.

The gravitational wave generated in the epoch of inflation has not been detected. Therefore there still be the possibility that the propagating speed of the gravitational wave might be significantly different from the speed of light. As a working hypothesis, we now assume that the speed of the gravitational wave is smaller by 10% than that of light during inflation. Then Eq. (33) shows

$$8\kappa^2{\xi_0}^2\xi_1{H_0}^2 \sim \frac{1}{10}. \tag{46}$$

If we assume $\xi_1 = \mathcal{O}(1)$, then the factor $e^{-\xi_1 N}$ in (45) is very small,

$$e^{-\xi_1 N} \sim e^{-60} \fallingdotseq 8.8 \times 10^{-27}. \tag{47}$$

Therefore, we expect that the difference between the speed of the gravitational wave and that of light can be neglected after the inflationary era, including the epoch of the reheating, a scenario that we discuss in the next subsection.

D. Reheating scenario

In this subsection, we estimate the temperature and the propagating speed of the gravitational wave in the epoch of reheating. We expect that the inflationary will end when $N = N_1$, which is one of the solutions of Eq. (38). So far, we have neglected the contributions from the matter fluids. After $N = N_1$, if the scalar field ϕ couples with matter, it could affect the reheating era and the evolution of H deviates from that in Eq. (36). In order to investigate the behavior of the scalar field dynamics, we expand the quantity around $N = N_1$ as follows:

$$N = N_1 + \delta N. \tag{48}$$

Then by using (30) and (36), we find

$$\begin{aligned} \xi' &\sim \xi_0\xi_1 e^{-\xi_1 N_1}(1 - \xi_1\delta N), \\ H &\sim H_0(1+\alpha{N_1}^\beta)^\gamma(1-\delta N), \\ H' &\sim \alpha\beta\gamma H_0(1+\alpha{N_1}^\beta)^{\gamma-1}{N_1}^{\beta-1} \\ &\quad \times \left\{1 + \left(-\frac{\gamma-1}{\gamma} + \beta - 1\right)\frac{\delta N}{N_1}\right\}. \end{aligned} \tag{49}$$

Here we have used Eq. (38) with $N = N_1$ and the scalar potential reads

$$\begin{aligned} V \sim V_0 - &\left\{\frac{6}{\kappa^2}\left(1 + \frac{\alpha\beta\gamma}{(1+\alpha{N_1}^\beta)^\gamma}\right) + \frac{2{\phi_0}^2 e^{-\frac{2N_1}{N_0}}}{N_0}\right\} \\ &\times H_0^2(1+\alpha{N_1}^\beta)^{2\gamma}\delta N, \end{aligned} \tag{50}$$

where V_0 and ξ_1 are constants of integration, which can be determined by using (19), which gives when $N = N_1$,

$$\begin{aligned} 0 &= -\left(\frac{3}{\kappa^2} - \frac{{\phi_0}^2 e^{-\frac{2N_1}{N_0}}}{2N_0}\right)H_0^2(1+\alpha{N_1}^\beta)^{2\gamma} + V_0 \\ &\quad + 24H_0^4(1+\alpha{N_1}^\beta)^{4\gamma}\xi_0\xi_1 e^{-\xi_1 N_1}, \\ 0 &= \frac{1}{\kappa^2}\left(1 - \frac{\alpha\beta\gamma{N_1}^{\beta-1}}{1+\alpha{N_1}^\beta}\right)H_0^2(1+\alpha{N_1}^\beta)^{2\gamma} - V_0 \\ &\quad - H_0^4(1+\alpha{N_1}^\beta)^{4\gamma}\left(16 + \frac{24\alpha\beta\gamma{N_1}^{\beta-1}}{1+\alpha{N_1}^\beta}\right)\xi_0\xi_1 e^{-\xi_1 N_1}, \end{aligned} \tag{51}$$

that is,

$$\begin{aligned} \xi_0\xi_1 e^{-\xi_1 N_1} &= \left\{\frac{1}{\kappa^2}\left(2 + \frac{\alpha\beta\gamma{N_1}^{\beta-1}}{1+\alpha{N_1}^\beta}\right) - \frac{{\phi_0}^2 e^{-\frac{2N_1}{N_0}}}{2N_0}\right\} \\ &\quad \times H_0^{-2}(1+\alpha{N_1}^\beta)^{-2\gamma}\left(8 - \frac{24\alpha\beta\gamma{N_1}^{\beta-1}}{1+\alpha{N_1}^\beta}\right)^{-1}, \\ V_0 &= \frac{1}{\kappa^2}\left(1 - \frac{\alpha\beta\gamma{N_1}^{\beta-1}}{1+\alpha{N_1}^\beta}\right)H_0^2(1+\alpha{N_1}^\beta)^{2\gamma} \\ &\quad - H_0^2(1+\alpha{N_1}^\beta)^{2\gamma}\left(16 + \frac{24\alpha\beta\gamma{N_1}^{\beta-1}}{1+\alpha{N_1}^\beta}\right) \\ &\quad \times \left\{\frac{1}{\kappa^2}\left(2 + \frac{\alpha\beta\gamma{N_1}^{\beta-1}}{1+\alpha{N_1}^\beta}\right) - \frac{{\phi_0}^2 e^{-\frac{2N_1}{N_0}}}{2N_0}\right\} \\ &\quad \times \left(8 - \frac{24\alpha\beta\gamma{N_1}^{\beta-1}}{1+\alpha{N_1}^\beta}\right)^{-1}. \end{aligned} \tag{52}$$

We now estimate the reheating temperature T_{re}. The effective energy density at the end of the inflationary era is given by

$$\rho_{\mathrm{eff}} = \frac{3}{\kappa^2}H(N_1)^2 = \frac{3}{\kappa^2}{H_0}^2(1+\alpha{N_1}^\beta)^{2\gamma}. \tag{53}$$

We now assume that all the energy density is transformed into radiation. The Stefan-Boltzmann law indicates that

$$\frac{3}{\kappa^2}{H_0}^2(1+\alpha {N_1}^\beta)^{2\gamma} = \left(\frac{\pi^2 g_{\mathrm{re}}}{30}\right){T_{\mathrm{re}}}^4. \tag{54}$$

Here g_{re} denotes the number of the massless degrees of freedom when the reheating era occurred. Then we obtain

$$T_{\mathrm{re}} = \frac{3}{\kappa^2}\sqrt{H_0}(1+\alpha {N_1}^\beta)^{\frac{\gamma}{2}}\left(\frac{30}{\pi^2 g_{\mathrm{re}}}\right)^{\frac{1}{4}}. \tag{55}$$

In the epoch of the reheating, by using (33), we find the propagating speed of the gravitational wave as follows:

$$\begin{aligned} {c_{\mathrm{GW}}}^2 \sim \Big(&1+8\kappa^2\xi_0 {H_0}^2(1+\alpha {N_{\mathrm{rh}}}^\beta)^{2\gamma-1}(\xi_1\alpha\beta\gamma {N_{\mathrm{rh}}}^{\beta-1} \\ &- {\xi_1}^2(1+\alpha {N_{\mathrm{rh}}}^\beta))\mathrm{e}^{-\xi_1 N_{\mathrm{rh}}}\Big)c^2, \end{aligned} \tag{56}$$

where N_{rh} is the e-folding number corresponding to the reheating. So from the above relation, we have a concrete idea on the behavior of the gravitational wave speed during the reheating era, which is nontrivial, as expected, and somewhat model dependent.

III. PROPAGATION OF GRAVITATIONAL WAVES IN SPHERICALLY SYMMETRIC SPACETIME

In this section, we consider the propagation of the gravitational waves in a spherically symmetric spacetime background. First, we consider the spherically symmetric and also time-dependent spacetime. The spacetime includes both the static spherically symmetric spacetime and the FLRW spacetime as special cases. We show that the condition (13) cannot be satisfied in the nontrivial but general spherically symmetric spacetime. After that, we estimate the deviation of the propagating speed of the gravitational wave from the speed of light inside a stellar object.

A. Spherically symmetric time-dependent spacetime

In this subsection, we show that the condition (13) cannot be satisfied in the nontrivial but general spherically symmetric spacetime. The most general form of the spherically symmetric and time-dependent spacetime is given by

$$\begin{aligned} ds^2 = &-\mathcal{A}(\tau,\rho)d\tau^2 + 2\mathcal{B}(\tau,\rho)d\tau d\rho + \mathcal{C}(\tau,\rho)d\rho^2 \\ &+ \mathcal{D}(\tau,\rho)(d\theta^2+\sin^2\theta d\phi^2). \end{aligned} \tag{57}$$

We should note that the spatially flat FLRW universe is a special class of the above spacetime. We define the radial coordinate r by

$$r^2 \equiv \mathcal{D}(\tau,\rho), \tag{58}$$

assuming $\mathcal{D}(\tau,\rho)$ is positive. In principle, Eq. (58) can be solved with respect to ρ as $\rho=\rho(\tau,r)$. Then the metric in (57) can be rewritten as

$$\begin{aligned} ds^2 = &\left\{-\mathcal{A}(\tau,\rho(\tau,r)) + 2\mathcal{B}(\tau,\rho(\tau,r))\frac{\partial\rho}{\partial\tau}\right\}d\tau^2 + 2\mathcal{B}(\tau,\rho(\tau,r))\frac{\partial\rho}{\partial r}d\tau dr \\ &+ \mathcal{C}(\tau,\rho(\tau,r))\left(\frac{\partial\rho}{\partial r}\right)^2 dr^2 + r^2(d\theta^2+\sin^2\theta d\phi^2). \end{aligned} \tag{59}$$

Furthermore, we introduce a new time coordinate t as $\tau=\tau(t,r)$. Then the metric in (59) can be further rewritten as

$$\begin{aligned} ds^2 = &\left\{-\mathcal{A}(\tau(t,r),\rho(\tau(t,r),r)) + 2\mathcal{B}(\tau(t,r),\rho(\tau(t,r),r))\frac{\partial\rho(\tau(t,r),r)}{\partial\tau}\right\}\left(\frac{\partial\tau(t,r)}{\partial t}\right)^2 dt^2 \\ &+ 2\left[\mathcal{B}(\tau(t,r),\rho(\tau(t,r),r))\frac{\partial\rho(\tau(t,r),r)}{\partial r}\frac{\partial\tau(t,r)}{\partial t}\right. \\ &\left.+ \left\{-\mathcal{A}(\tau(t,r),\rho(\tau(t,r),r)) + 2\mathcal{B}(\tau(t,r),\rho(\tau(t,r),r))\frac{\partial\rho(\tau(t,r),r)}{\partial\tau}\right\}\frac{\partial\tau(t,r)}{\partial t}\frac{\partial\tau(t,r)}{\partial r}\right]dtdr \\ &+ \left[\mathcal{C}(\tau,\rho(\tau,r))\left(\frac{\partial\rho}{\partial r}\right)^2 + \mathcal{B}(\tau(t,r),\rho(\tau(t,r),r))\frac{\partial\rho(\tau(t,r),r)}{\partial r}\frac{\partial\tau(t,r)}{\partial r}\right. \\ &\left.\times\left\{-\mathcal{A}(\tau(t,r),\rho(\tau(t,r),r)) + 2\mathcal{B}(\tau(t,r),\rho(\tau(t,r),r))\frac{\partial\rho(\tau(t,r),r)}{\partial\tau}\right\}\left(\frac{\partial\tau(t,r)}{\partial r}\right)^2\right]dr^2 \\ &+ r^2(d\theta^2+\sin^2\theta d\phi^2). \end{aligned} \tag{60}$$

We can choose the time-coordinate t so that

$$0 = \mathcal{B}(\tau(t,r),\rho(\tau(t,r),r))\frac{\partial\rho(\tau(t,r),r)}{\partial r}\frac{\partial\tau(t,r)}{\partial t} + \left\{-\mathcal{A}(\tau(t,r),\rho(\tau(t,r),r)) + 2\mathcal{B}(\tau(t,r),\rho(\tau(t,r),r))\frac{\partial\rho(\tau(t,r),r)}{\partial\tau}\right\}\frac{\partial\tau(t,r)}{\partial t}\frac{\partial\tau(t,r)}{\partial r}. \tag{61}$$

Then finally, the metric has the following form:

$$\begin{aligned} ds^2 &= -\mathrm{e}^{2\nu(r,t)}dt^2 + \mathrm{e}^{2\lambda(r,t)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \\ -\mathrm{e}^{2\nu(r,t)} &\equiv \left\{-\mathcal{A}(\tau(t,r),\rho(\tau(t,r),r)) + 2\mathcal{B}(\tau(t,r),\rho(\tau(t,r),r))\frac{\partial\rho(\tau(t,r),r)}{\partial\tau}\right\}\left(\frac{\partial\tau(t,r)}{\partial t}\right)^2, \\ \mathrm{e}^{2\lambda(r,t)} &\equiv \mathcal{C}(\tau,\rho(\tau,r))\left(\frac{\partial\rho}{\partial r}\right)^2 + \mathcal{B}(\tau(t,r),\rho(\tau(t,r),r))\frac{\partial\rho(\tau(t,r),r)}{\partial r}\frac{\partial\tau(t,r)}{\partial r} \\ &\quad\times\left\{-\mathcal{A}(\tau(t,r),\rho(\tau(t,r),r)) + 2\mathcal{B}(\tau(t,r),\rho(\tau(t,r),r))\frac{\partial\rho(\tau(t,r),r)}{\partial\tau}\right\}\left(\frac{\partial\tau(t,r)}{\partial r}\right)^2. \end{aligned} \tag{62}$$

We define the metric $\tilde{g}_{ij}$ of the unit sphere by $\sum_{i,j=1,2}\tilde{g}_{ij}dx^i dx^j = d\theta^2 + \sin^2\theta d\phi^2$. For the metric (62), the nonvanishing connections are the following:

$$\begin{aligned} &\Gamma^t_{tt} = \dot{\nu}, \quad \Gamma^r_{tt} = \mathrm{e}^{-2\lambda+2\nu}\nu', \quad \Gamma^t_{tr} = \Gamma^t_{rt} = \nu', \quad \Gamma^t_{rr} = \mathrm{e}^{2\lambda-2\nu}\dot{\lambda}, \quad \Gamma^r_{tr} = \Gamma^r_{rt} = \dot{\lambda}, \quad \Gamma^r_{rr} = \lambda', \\ &\Gamma^i_{jk} = \tilde{\Gamma}^i_{jk}, \quad \Gamma^r_{ij} = -\mathrm{e}^{-2\lambda}r\tilde{g}_{ij}, \quad \Gamma^i_{rj} = \Gamma^i_{jr} = \frac{1}{r}\delta^i{}_j. \end{aligned} \tag{63}$$

Here $\tilde{\Gamma}^i_{jk}$ is the connection given by $\tilde{g}_{ij}$. Since

$$R^\lambda{}_{\mu\rho\nu} = -\Gamma^\lambda_{\mu\rho,\nu} + \Gamma^\lambda_{\mu\nu,\rho} - \Gamma^\eta_{\mu\rho}\Gamma^\lambda_{\nu\eta} + \Gamma^\eta_{\mu\nu}\Gamma^\lambda_{\rho\eta}, \tag{64}$$

we find that

$$\begin{aligned} &R_{rtrt} = -\mathrm{e}^{2\lambda}\{\ddot{\lambda} + (\dot{\lambda} - \dot{\nu})\dot{\lambda}\} + \mathrm{e}^{2\nu}\{\nu'' + (\nu' - \lambda')\nu'\}, \quad R_{titj} = r\nu'\mathrm{e}^{2(\nu-\lambda)}\tilde{g}_{ij}, \\ &R_{rirj} = \lambda' r\tilde{g}_{ij}, \quad R_{tirj} = \dot{\lambda}r\tilde{g}_{ij}, \quad R_{ijkl} = (1 - \mathrm{e}^{-2\lambda})r^2(\tilde{g}_{ik}\tilde{g}_{jl} - \tilde{g}_{il}\tilde{g}_{jk}), \\ &R_{tt} = -\{\ddot{\lambda} + (\dot{\lambda} - \dot{\nu})\dot{\lambda}\} + \mathrm{e}^{2(\nu-\lambda)}\left\{\nu'' + (\nu' - \lambda')\nu' + \frac{2\nu'}{r}\right\}, \\ &R_{rr} = \mathrm{e}^{-2(\nu-\lambda)}\{\ddot{\lambda} + (\dot{\lambda} - \dot{\nu})\dot{\lambda}\} - \{\nu'' + (\nu' - \lambda')\nu'\} + \frac{2\lambda'}{r}, \\ &R_{tr} = \frac{2\dot{\lambda}}{r}, \quad R_{ij} = [1 + \{-1 - r(\nu' - \lambda')\}\mathrm{e}^{-2\lambda}]\tilde{g}_{ij}, \\ &R = 2\mathrm{e}^{-2\nu}\{\ddot{\lambda} + (\dot{\lambda} - \dot{\nu})\dot{\lambda}\} + \mathrm{e}^{-2\lambda}\left[-2\nu'' - 2(\nu' - \lambda')\nu' - \frac{4(\nu' - \lambda')}{r} + \frac{2\mathrm{e}^{2\lambda} - 2}{r^2}\right]. \end{aligned} \tag{65}$$

By assuming that ξ only depends on r and t because we are considering spherically symmetric spacetime, we find

$$\begin{aligned} &\nabla_t\nabla_t\xi = \partial_t{}^2\xi - \dot{\nu}\partial_t\xi - \mathrm{e}^{-2\lambda+2\nu}\nu'\partial_r\xi, \quad \nabla_r\nabla_r\xi = \partial_r{}^2\xi - \mathrm{e}^{2\lambda-2\nu}\dot{\lambda}\partial_t\xi - \lambda'\partial_r\xi, \\ &\nabla_i\nabla_j\xi = \mathrm{e}^{-2\lambda}r\tilde{g}_{ij}\partial_r\xi, \quad \nabla_r\nabla_t\xi = \nabla_t\nabla_r\xi = \partial_r\partial_t\xi - \nu'\partial_t\xi - \dot{\lambda}\partial_r\xi, \\ &\nabla_t\nabla_i\xi = \nabla_i\nabla_t\xi = \nabla_r\nabla_i\xi = \nabla_i\nabla_r\xi = 0, \\ &\nabla^2\xi = -\mathrm{e}^{-2\nu}(\partial_t{}^2\xi - \dot{\nu}\partial_t\xi - \mathrm{e}^{-2\lambda+2\nu}\nu'\partial_r\xi) + \mathrm{e}^{-2\lambda}(\partial_r{}^2\xi - \mathrm{e}^{2\lambda-2\nu}\dot{\lambda}\partial_t\xi - \lambda'\partial_r\xi) + \frac{2\mathrm{e}^{-2\lambda}}{r}\partial_r\xi. \end{aligned} \tag{66}$$

Then the condition (13) gives

$$\begin{aligned}
0 &= 3\mathrm{e}^{-2\nu}(\partial_t{}^2\xi - \dot\nu\partial_t\xi - \mathrm{e}^{-2\lambda+2\nu}\nu'\partial_r\xi) + \mathrm{e}^{-2\lambda}(\partial_r{}^2\xi - \mathrm{e}^{2\lambda-2\nu}\dot\lambda\partial_t\xi - \lambda'\partial_r\xi) + \frac{2\mathrm{e}^{-2\lambda}}{r}\partial_r\xi,\\
0 &= -\mathrm{e}^{-2\nu}(\partial_t{}^2\xi - \dot\nu\partial_t\xi - \mathrm{e}^{-2\lambda+2\nu}\nu'\partial_r\xi) - 3\mathrm{e}^{-2\lambda}(\partial_r{}^2\xi - \mathrm{e}^{2\lambda-2\nu}\dot\lambda\partial_t\xi - \lambda'\partial_r\xi) + \frac{2\mathrm{e}^{-2\lambda}}{r}\partial_r\xi,\\
0 &= \partial_r\partial_t\xi - \nu'\partial_t\xi - \dot\lambda\partial_r\xi.
\end{aligned} \tag{67}$$

By combining the first and second equations in (67), we obtain

$$0 = \mathrm{e}^{\nu+\lambda}\{\mathrm{e}^{-2\nu}\partial_t(\mathrm{e}^{-\nu-\lambda}\partial_t\xi) + \mathrm{e}^{-2\lambda}\partial_r(\mathrm{e}^{-\nu-\lambda}\partial_r\xi)\}. \tag{68}$$

$$0 = -(\partial_r{}^2\xi - \mathrm{e}^{2\lambda-2\nu}\dot\lambda\partial_t\xi - \lambda'\partial_r\xi) + \frac{1}{r}\partial_r\xi, \tag{69}$$

$$0 = (\partial_t{}^2\xi - \dot\nu\partial_t\xi - \mathrm{e}^{-2\lambda+2\nu}\nu'\partial_r\xi) + \frac{\mathrm{e}^{-2\lambda+2\nu}}{r}\partial_r\xi. \tag{70}$$

For simplicity, we consider the case that the spacetime is static, that is, ν and λ do not depend on time coordinate t. Then the equations in (67) reduce to

$$\begin{aligned}
0 &= 3\mathrm{e}^{-2\nu}(\partial_t{}^2\xi - \mathrm{e}^{-2\lambda+2\nu}\nu'\partial_r\xi) + \mathrm{e}^{-2\lambda}(\partial_r{}^2\xi - \lambda'\partial_r\xi) + \frac{2\mathrm{e}^{-2\lambda}}{r}\partial_r\xi,\\
0 &= -\mathrm{e}^{-2\nu}(\partial_t{}^2\xi - \mathrm{e}^{-2\lambda+2\nu}\nu'\partial_r\xi) - 3\mathrm{e}^{-2\lambda}(\partial_r{}^2\xi - \lambda'\partial_r\xi) + \frac{2\mathrm{e}^{-2\lambda}}{r}\partial_r\xi,\\
0 &= \partial_r\partial_t\xi - \nu'\partial_t\xi,
\end{aligned} \tag{71}$$

and Eqs. (69) and (70) reduce to

$$0 = -(\partial_r{}^2\xi - \lambda'\partial_r\xi) + \frac{1}{r}\partial_r\xi, \tag{72}$$

$$0 = \partial_t{}^2\xi - \mathrm{e}^{-2\lambda+2\nu}\nu'\partial_r\xi + \frac{\mathrm{e}^{-2\lambda+2\nu}}{r}\partial_r\xi. \tag{73}$$

The general solution of the last equation in (71) is given by

$$\xi(t,r) = \xi_{(t)}(t)\mathrm{e}^{\nu(r)} + \xi_{(r)}(r). \tag{74}$$

Here $\xi_{(t)}$ and $\xi_{(r)}$ are arbitrary functions of t and r, respectively. By substituting (74) into (72), we obtain

$$0 = -(\nu'' + \nu'^2)\xi_{(t)}\mathrm{e}^{\nu} - \xi''_{(r)} + \left(\lambda' + \frac{1}{r}\right)(\nu'\xi_{(t)}\mathrm{e}^{\nu} + \xi'_{(r)}), \tag{75}$$

which gives

$$0 = -\nu'' + \nu'^2 + \left(\lambda' + \frac{1}{r}\right)\nu', \qquad 0 = -\xi''_{(r)} + \left(\lambda' + \frac{1}{r}\right)\xi'_{(r)}. \tag{76}$$

The first equation in (76) gives a nontrivial relation for the spacetime geometry,

$$0 = -\ln\frac{\nu'}{\nu'_0 r} + \nu + \lambda. \tag{77}$$

Here ν'_0 is an integration constant. On the other hand, the second equation in (76) can be solved as follows:

$$\xi'_{(r)} = \xi_0 r\mathrm{e}^{\lambda}. \tag{78}$$

Here ξ_0 is an integration constant. By substituting Eqs. (74) and (78) into (73) and by using Eq. (77), we obtain

$$0 = \ddot\xi_{(t)}\mathrm{e}^{\nu} - \left(\nu' - \frac{1}{r}\right)\frac{\nu_0'^2}{\nu'^2}\mathrm{e}^{4\nu}(\nu'\xi_{(t)}\mathrm{e}^{\nu} + \xi'_{(r)}), \tag{79}$$

which gives

$$0 = \ddot\xi_{(t)}, \qquad \nu' = \frac{1}{r}, \tag{80}$$

which yields

$$\xi_{(t)} = \xi_1, \qquad 0 = \nu'\xi_1\mathrm{e}^{\nu} + \xi'_{(r)}, \tag{81}$$

where ξ_1 is a constant. The second equation in (80) gives

$$\nu = \ln \frac{r}{r_0}, \tag{82}$$

where r_0 is a constant. On the other hand, when Eq. (81) is satisfied, Eq. (74) indicates that ξ does not depend on the time coordinate t. Then Eq. (73) yields

$$\nu' = \frac{1}{r}, \tag{83}$$

which gives (82), again. Equation (82) indicates that there is no horizon and therefore there is no solution for the black hole when the speed of the propagating speed exactly coincides with that of the light, even if we include two scalar fields ϕ and χ in addition to matter. Equation (83) also prohibits more general but nontrivial spherically symmetric spacetime, including the stellar configuration and wormholes.

In the next subsection, we try to solve the problem of the propagating speed of gravitational waves, by considering a model of Einstein-Gauss-Bonnet gravity coupled with two scalar fields. As we will show we reobtain the condition (13) again, and therefore the propagating speed of the gravitational wave does not coincide with that of light.

IV. TWO-SCALAR EINSTEIN-GAUSS-BONNET GRAVITY

Since the propagating speed of the gravitational waves cannot be equal to that of light in the nontrivial spherically symmetric background as we have shown in [59], we consider the model including two scalar fields ϕ and χ to investigate if the problem could be solved or not.

The action with two scalar fields is given by

$$S_{\phi\chi} = \int d^4x\sqrt{-g}\left\{\frac{R}{2\kappa^2} - \frac{1}{2}A(\phi,\chi)\partial_\mu\phi\partial^\mu\phi - B(\phi,\chi)\partial_\mu\phi\partial^\mu\chi - \frac{1}{2}C(\phi,\chi)\partial_\mu\chi\partial^\mu\chi - V(\phi,\chi) - \xi(\phi,\chi)\mathcal{G} + \mathcal{L}_{\mathrm{matter}}\right\}. \tag{84}$$

Here $V(\phi,\chi)$ is the potential for ϕ and χ and $\xi(\phi,\chi)$ is also a function of ϕ and χ. By varying the action (84) with respect to the metric $g_{\mu\nu}$, we obtain

$$\begin{aligned}
0 = {} & \frac{1}{2\kappa^2}\left(-R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R\right) \\
& + \frac{1}{2}g_{\mu\nu}\left\{-\frac{1}{2}A(\phi,\chi)\partial_\rho\phi\partial^\rho\phi - B(\phi,\chi)\partial_\rho\phi\partial^\rho\chi - \frac{1}{2}C(\phi,\chi)\partial_\rho\chi\partial^\rho\chi - V(\phi,\chi)\right\} \\
& + \frac{1}{2}\{A(\phi,\chi)\partial_\mu\phi\partial_\nu\phi + B(\phi,\chi)(\partial_\mu\phi\partial_\nu\chi + \partial_\nu\phi\partial_\mu\chi) + C(\phi,\chi)\partial_\mu\chi\partial_\nu\chi\} \\
& - 2(\nabla_\mu\nabla_\nu\xi(\phi,\chi))R + 2g_{\mu\nu}(\nabla^2\xi(\phi,\chi))R + 4(\nabla_\rho\nabla_\mu\xi(\phi,\chi))R_\nu{}^\rho + 4(\nabla_\rho\nabla_\nu\xi(\phi,\chi))R_\mu{}^\rho \\
& - 4(\nabla^2\xi(\phi,\chi))R_{\mu\nu} - 4g_{\mu\nu}(\nabla_\rho\nabla_\sigma\xi(\phi,\chi))R^{\rho\sigma} + 4(\nabla^\rho\nabla^\sigma\xi(\phi,\chi))R_{\mu\rho\nu\sigma} + \frac{1}{2}T_{\mathrm{matter}\,\mu\nu},
\end{aligned} \tag{85}$$

and the field equation for the scalar field is obtained by varying the action with respect to ϕ and χ, and it is equal to

$$\begin{aligned}
0 &= \frac{1}{2}A_\phi\partial_\mu\phi\partial^\mu\phi + A\nabla^\mu\partial_\mu\phi + A_\chi\partial_\mu\phi\partial^\mu\chi + \left(B_\chi - \frac{1}{2}C_\phi\right)\partial_\mu\chi\partial^\mu\chi + B\nabla^\mu\partial_\mu\chi - V_\phi - \xi_\phi\mathcal{G}, \\
0 &= \left(-\frac{1}{2}A_\chi + B_\phi\right)\partial_\mu\phi\partial^\mu\phi + B\nabla^\mu\partial_\mu\phi + \frac{1}{2}C_\chi\partial_\mu\chi\partial^\mu\chi + C\nabla^\mu\partial_\mu\chi + C_\phi\partial_\mu\phi\partial^\mu\chi - V_\chi - \xi_\chi\mathcal{G}.
\end{aligned} \tag{86}$$

Here $A_\phi = \partial A(\phi,\chi)/\partial\phi$, and similar notation is used in other functions. Also in (85), $T_{\mathrm{matter}\,\mu\nu}$ is the energy-momentum tensor of perfect matter fluids. We should note that the field equations in (86) are nothing but the Bianchi identities. We again consider the equation that describes the gravitational waves and we obtain the condition that the propagating speed of the gravitational wave is equal to that of light.

By considering the variation of the metric in (5), we obtain the equation describing the propagation of the gravitational waves as follows:

$$
\begin{aligned}
0 = &\left[\frac{1}{4\kappa^2}R + \frac{1}{2}\left\{-\frac{1}{2}A\partial_\rho\phi\partial^\rho\phi - B\partial_\rho\phi\partial^\rho\chi - \frac{1}{2}C\partial_\rho\chi\partial^\rho\chi - V\right\} - 4(\nabla_\rho\nabla_\sigma\xi)R^{\rho\sigma}\right]h_{\mu\nu} \\
&+ \left[\frac{1}{4}g_{\mu\nu}\{-A\partial^\tau\phi\partial^\eta\phi - B(\partial^\tau\phi\partial^\eta\chi + \partial^\eta\phi\partial^\tau\chi) - C\partial^\tau\chi\partial^\eta\chi\}\right. \\
&- 2g_{\mu\nu}(\nabla^\tau\nabla^\eta\xi)R - 4(\nabla^\tau\nabla_\mu\xi)R_\nu{}^\eta - 4(\nabla^\tau\nabla_\nu\xi)R_\mu{}^\eta + 4(\nabla^\tau\nabla^\eta\xi)R_{\mu\nu} \\
&\left.+ 4g_{\mu\nu}(\nabla^\tau\nabla_\sigma\xi)R^{\eta\sigma} + 4g_{\mu\nu}(\nabla_\rho\nabla^\tau\xi)R^{\rho\eta} - 4(\nabla^\tau\nabla^\sigma\xi)R_\mu{}^\eta{}_{\nu\sigma} - 4(\nabla^\rho\nabla^\tau\xi)R_{\mu\rho\nu}{}^\eta\right]h_{\tau\eta} \\
&+ \frac{1}{2}\{2\delta_\mu{}^\eta\delta_\nu{}^\zeta(\nabla_\kappa\xi)R - 2g_{\mu\nu}g^{\eta\zeta}(\nabla_\kappa\xi)R - 4\delta_\rho{}^\eta\delta_\mu{}^\zeta(\nabla_\kappa\xi)R_\nu{}^\rho - 4\delta_\rho{}^\eta\delta_\nu{}^\zeta(\nabla_\kappa\xi)R_\mu{}^\rho \\
&+ 4g^{\eta\zeta}(\nabla_\kappa\xi)R_{\mu\nu} + 4g_{\mu\nu}\delta_\rho{}^\eta\delta_\sigma{}^\zeta(\nabla_\kappa\xi)R^{\rho\sigma} - 4g^{\rho\eta}g^{\sigma\zeta}(\nabla_\kappa\xi)R_{\mu\rho\nu\sigma}\}g^{\kappa\lambda}(\nabla_\eta h_{\zeta\lambda} + \nabla_\zeta h_{\eta\lambda} - \nabla_\lambda h_{\eta\zeta}) \\
&+ \left\{\frac{1}{4\kappa^2}g_{\mu\nu} - 2(\nabla_\mu\nabla_\nu\xi) + 2g_{\mu\nu}(\nabla^2\xi)\right\}\{-h_{\mu\nu}R^{\mu\nu} + \nabla^\mu\nabla^\nu h_{\mu\nu} - \nabla^2(g^{\mu\nu}h_{\mu\nu})\} \\
&+ \frac{1}{2}\left\{\left(-\frac{1}{2\kappa^2} - 4\nabla^2\xi\right)\delta^\tau{}_\mu\delta^\eta{}_\nu + 4(\nabla_\rho\nabla_\mu\xi)\delta^\eta{}_\nu g^{\rho\tau} + 4(\nabla_\rho\nabla_\nu\xi)\delta^\tau{}_\mu g^{\rho\eta} - 4g_{\mu\nu}\nabla^\tau\nabla^\eta\xi\right\} \\
&\times\{\nabla_\tau\nabla^\phi h_{\eta\phi} + \nabla_\eta\nabla^\phi h_{\tau\phi} - \nabla^2 h_{\tau\eta} - \nabla_\tau\nabla_\eta(g^{\phi\lambda}h_{\phi\lambda}) - 2R^\lambda{}_\eta{}^\phi{}_\tau h_{\lambda\phi} + R^\phi{}_\tau h_{\phi\eta} + R^\phi{}_\tau h_{\phi\eta}\} \\
&+ 2(\nabla^\rho\nabla^\sigma\xi)\{\nabla_\nu\nabla_\rho h_{\sigma\mu} - \nabla_\nu\nabla_\mu h_{\sigma\rho} - \nabla_\sigma\nabla_\rho h_{\nu\mu} + \nabla_\sigma\nabla_\mu h_{\nu\rho} + h_{\mu\phi}R^\phi{}_{\rho\nu\sigma} - h_{\rho\phi}R^\phi{}_{\mu\nu\sigma}\} + \frac{1}{2}\frac{\partial T_{\text{matter}\,\mu\nu}}{\partial g_{\tau\eta}}h_{\tau\eta}. \qquad (87)
\end{aligned}
$$

Here we have assumed that the matter fluids minimally couple with gravity, once more. By choosing the conditions in (8) and (9), we can reduce Eq. (87) as follows:

$$
\begin{aligned}
0 = &\left[\frac{1}{4\kappa^2}R + \frac{1}{2}\left\{-\frac{1}{2}A\partial_\rho\phi\partial^\rho\phi - B\partial_\rho\phi\partial^\rho\chi - \frac{1}{2}C\partial_\rho\chi\partial^\rho\chi - V\right\} - 4(\nabla_\rho\nabla_\sigma\xi)R^{\rho\sigma}\right]h_{\mu\nu} \\
&+ \left[\frac{1}{4}g_{\mu\nu}\{-A\partial^\tau\phi\partial^\eta\phi - B(\partial^\tau\phi\partial^\eta\chi + \partial^\eta\phi\partial^\tau\chi) - C\partial^\tau\chi\partial^\eta\chi\}\right. \\
&- 2g_{\mu\nu}(\nabla^\tau\nabla^\eta\xi)R - 4(\nabla^\tau\nabla_\mu\xi)R_\nu{}^\eta - 4(\nabla^\tau\nabla_\nu\xi)R^\eta_\mu + 4(\nabla^\tau\nabla^\eta\xi)R_{\mu\nu} \\
&\left.+ 4g_{\mu\nu}(\nabla^\tau\nabla_\sigma\xi)R^{\eta\sigma} + 4g_{\mu\nu}(\nabla_\rho\nabla^\tau\xi)R^{\rho\eta} - 4(\nabla^\tau\nabla^\sigma\xi)R_\mu{}^\eta{}_{\nu\sigma} - 4(\nabla^\rho\nabla^\tau\xi)R^\eta_{\mu\rho\nu}\right\}h_{\tau\eta} \\
&+ \frac{1}{2}\{2\delta_\mu{}^\eta\delta_\nu{}^\zeta(\nabla_\kappa\xi)R - 4\delta_\rho{}^\eta\delta_\mu{}^\zeta(\nabla_\kappa\xi)R^\rho_\nu - 4\delta_\rho{}^\eta\delta_\nu{}^\zeta(\nabla_\kappa\xi)R^\rho_\mu \\
&+ 4g_{\mu\nu}\delta_\rho{}^\eta\delta_\sigma{}^\zeta(\nabla_\kappa\xi)R^{\rho\sigma} - 4g^{\rho\eta}g^{\sigma\zeta}(\nabla_\kappa\xi)R_{\mu\rho\nu\sigma}\}g^{\kappa\lambda}(\nabla_\eta h_{\zeta\lambda} + \nabla_\zeta h_{\eta\lambda} - \nabla_\lambda h_{\eta\zeta}) \\
&- \left\{\frac{1}{4\kappa^2}g_{\mu\nu} - 2(\nabla_\mu\nabla_\nu\xi) + 2g_{\mu\nu}(\nabla^2\xi)\right\}R^{\mu\nu}h_{\mu\nu} \\
&+ \frac{1}{2}\left\{\left(-\frac{1}{2\kappa^2} - 4\nabla^2\xi\right)\delta^\tau{}_\mu\delta^\eta{}_\nu + 4(\nabla_\rho\nabla_\mu\xi)\delta^\eta{}_\nu g^{\rho\tau} + 4(\nabla_\rho\nabla_\nu\xi)\delta^\tau{}_\mu g^{\rho\eta} - 4g_{\mu\nu}\nabla^\tau\nabla^\eta\xi\right\} \\
&\times\{-\nabla^2 h_{\tau\eta} - 2R^\lambda{}_\eta{}^\phi{}_\tau h_{\lambda\phi} + R^\phi{}_\tau h_{\phi\eta} + R^\phi{}_\tau h_{\phi\eta}\} \\
&+ 2(\nabla^\rho\nabla^\sigma\xi)\{\nabla_\nu\nabla_\rho h_{\sigma\mu} - \nabla_\nu\nabla_\mu h_{\sigma\rho} - \nabla_\sigma\nabla_\rho h_{\nu\mu} + \nabla_\sigma\nabla_\mu h_{\nu\rho} + h_{\mu\phi}R^\phi{}_{\rho\nu\sigma} - h_{\rho\phi}R^\phi{}_{\mu\nu\sigma}\} + \frac{1}{2}\frac{\partial T_{\text{matter}\,\mu\nu}}{\partial g_{\tau\eta}}h_{\tau\eta}. \qquad (88)
\end{aligned}
$$

In order to investigate the propagating speed c_{GW} of the gravitational wave $h_{\mu\nu}$, we check the parts including the second derivatives of $h_{\mu\nu}$ and we reobtain (13), although ξ is now a function of the two scalar fields ϕ and χ, $\xi = \xi(\phi, \chi)$. Therefore even in the case of Einstein-Gauss-Bonnet gravity coupled with two scalars, it is impossible to obtain a model for which the propagating speed of the gravitational wave coincides with that of light.

A. Propagating speed of gravitational wave inside stellar objects

We have shown that the propagating speed of gravitational waves in a nontrivial spherically symmetric spacetime is always different from the speed of light, even in the Einstein-Gauss-Bonnet gravity with two scalar fields (84). In this subsection, we estimate the shift of the gravitational wave speed in stellar objects in the context of Einstein-Gauss-Bonnet gravity with only one scalar field ϕ in (1).

For the metric given by Eq. (62), the (t,t)-, (r,r)-, the angular components of Eq. (85) and the equation for the scalar field ϕ, have the following forms:

$$-4r^2 e^{2\lambda}\kappa^2\rho = -16(1-e^{-2\lambda})\xi'' - 4\{-4(1-3e^{-2\lambda})\xi' + r\}\lambda' + 2 + r^2\phi'^2 + 2e^{2\lambda}(Vr^2 - 1), \tag{89}$$

$$4r^2 e^{2\lambda}\kappa^2 p = 4\{-4(1-3e^{-2\lambda})\xi' + r\}\nu' + 2 - r^2\phi'^2 - 2e^{-2\lambda} + 2e^{2\lambda}Vr^2, \tag{90}$$

$$8re^{2\lambda}\kappa^2 p = 2(r + 8\xi' e^{-2\lambda})(\nu'' + \nu'^2) + 16\xi''\nu' e^{-2\lambda} + \{-2(r + 24\xi' e^{-2\lambda})\lambda' + 2\}\nu' - 2\lambda' + r(\phi'^2 + 2e^{2\lambda}V), \tag{91}$$

$$0 = -8\xi'(e^{-2\lambda} - 1)(\nu'' + 2\nu'^2) + \phi'\phi'' r^2 - 8\nu'\xi'\{\nu'(1 - e^{-2\lambda}) - \lambda'(3e^{-2\lambda} - 1)\} + r(\nu' r + 2 - \lambda' r)\phi'^2 - e^{2\lambda}V' r^2. \tag{92}$$

Here ρ is the energy density and p is the pressure of matter, which we assume to be a perfect fluid and satisfies an equation of state, $p = p(\rho)$. The energy density ρ and the pressure p satisfy the following conservation law:

$$0 = \nabla^\mu T_{\mu r} = \nu'(\rho + p) + \frac{dp}{dr}. \tag{93}$$

The conservation law is also derived from Eqs. (89)–(92). Here we have assumed that ρ and p depend only on the radial coordinate r. Other components of the conservation law are trivially satisfied. If the equation of state $\rho = \rho(p)$ is given, then Eq. (93) can be integrated as follows:

$$\nu = -\int^r dr \frac{\frac{dp}{dr}}{\rho + p} = -\int^{p(r)} \frac{dp}{\rho(p) + p}. \tag{94}$$

Because Eq. (93) and therefore (94), can be obtained from Eqs. (89)–(92), as long as we use (94), we forget one equation in Eqs. (89)–(92). In the following, we do not use Eq. (92). Inside the compact stellar object, we can use Eq. (94) but outside the stellar object, we cannot use Eq. (94). Instead of using Eq. (94), we may assume the profile of $\nu = \nu(r)$ so that $\nu(r)$ and $\nu'(r)$ are continuous at the surface of the compact stellar object.

By combining Eqs. (89) and (90), we obtain

$$V = \kappa^2(-\rho + p) + \frac{e^{-2\lambda}}{r^2}\{-4(e^{-2\lambda} - 1)\xi'' - 4(1 - 3e^{-2\lambda})(\lambda' - \nu')\xi' + e^{2\lambda} - 1\} + \frac{e^{-2\lambda}}{r}(\lambda' - \nu'), \tag{95}$$

$$\phi' = \pm\sqrt{-2e^{2\lambda}\kappa^2(\rho + p) - \frac{8}{r^2}\{(e^{-2\lambda} - 1)\xi'' + (1 - 3e^{-2\lambda})(\lambda' + \nu')\xi'\} + \frac{2}{r}(\lambda' + \nu')}. \tag{96}$$

Furthermore, the combination of Eqs. (89) and (91) gives

$$0 = -8\{e^{-2\lambda}(\nu' r - 1) + 1\}\xi'' - 8e^{-2\lambda}\{r(\nu'' + \nu'^2 - 3\nu'\lambda') + \lambda'(3 - e^{2\lambda})\}\xi' - r^2(\nu'' + \nu'^2 - \nu'\lambda') - 2r(\nu' + \lambda') - e^{2\lambda} + 1 - 2\kappa^2 r^2 e^{2\lambda}(\rho + p), \tag{97}$$

which can be regarded as a differential equation for ξ' and therefore for ξ if $\nu = \nu(r)$, $\lambda = \lambda(r)$, $\rho = \rho(r)$, and $p = p(r)$ given, the solution is

$$\xi(r) = -\frac{1}{8}\int\left[\int \frac{e^{2\lambda}\{e^{2\lambda} + r^2(\nu'' + \nu'^2 - \nu'\lambda') + r(\nu' + \lambda') - 1 - 2\kappa^2 r^2 e^{2\lambda}(\rho + p)\}}{U(\nu' r - 1 + e^{2\lambda})} dr + c_1\right] U dr + c_2,$$
$$U(r) \equiv \exp\left\{-\int \frac{r(\nu'' + \nu'^2) + \lambda'(3 - e^{2\lambda} - 3\nu' r)}{\nu' r - 1 + e^{2\lambda}} dr\right\}, \tag{98}$$

where c_1 and c_2 are integration constants. We may properly assume the profile of $\nu = \nu(r)$ and $\lambda = \lambda(r)$. Therefore, by using (98), we find the r dependence of ξ, $\xi = \xi(r)$ and by using Eqs. (95) and (96), we find the r dependencies of V and ϕ, $V = V(r)$, and $\phi = \phi(r)$. By solving $\phi = \phi(r)$ with respect to r, $r = r(\phi)$, we find ξ and V as functions of ϕ, $\xi(\phi) = \xi(r(\phi))$, $V(\phi) = V(r(\phi))$, which realize the model which has a solution given by $\nu = \nu(r)$ and $\lambda = \lambda(r)$. We should note, however, that the expression of ϕ in (96) gives the following constraint:

$$-2e^{2\lambda}\kappa^2(\rho + p) - \frac{8}{r^2}\{(e^{-2\lambda} - 1)\xi'' + (1 - 3e^{-2\lambda})(\lambda' + \nu')\xi'\} + \frac{2}{r}(\lambda' + \nu') \geq 0, \tag{99}$$

so that the ghost could be avoided. If Eq. (99) is not satisfied, then the scalar field ϕ becomes pure imaginary. We may define a new real scalar field ζ by $\phi = i\zeta$ $(i^2 = -1)$ but because the coefficient in front of the kinetic term of ζ becomes negative, ζ is rendered a ghost. The existence of the ghost generates the negative norm states in the quantum theory and therefore the theory becomes inconsistent.

When we consider compact stellar objects like neutron stars, we often consider the following equation of state:

(1) Energy polytrope

$$p = K\rho^{1+\frac{1}{n}}, \tag{100}$$

with constants K and n. It is known that for the neutron stars, n could take the value $0.5 \leq n \leq 1$.

(2) Mass polytrope

$$\rho = \rho_m + Np, \qquad p = K_m \rho_m^{1+\frac{1}{n_m}}, \tag{101}$$

where ρ_m is rest mass energy density and K_m, N are constants.

Now let us study the case of the energy polytrope (100), in detail, in which we can rewrite the equation of state as follows,

$$\rho = \tilde{K} p^{(1+\frac{1}{\tilde{n}})}, \qquad \tilde{K} \equiv K^{-\frac{1}{1+\frac{1}{n}}}, \qquad \tilde{n} \equiv \frac{1}{\frac{1}{1+\frac{1}{n}} - 1} = -1 - n. \tag{102}$$

For the energy polytrope, Eq. (94) takes the following form:

$$\begin{aligned} \nu &= -\int^{p(r)} \frac{dp}{\tilde{K} p^{1+\frac{1}{\tilde{n}}} + p} = \frac{c}{2} + \tilde{n} \ln\left(1 + \tilde{K}^{-1} p^{-\frac{1}{\tilde{n}}}\right) \\ &= \frac{c}{2} - (1+n) \ln\left(1 + K\rho^{\frac{1}{n}}\right), \end{aligned} \tag{103}$$

where c is an integration constant. Similarly, in the case of mass polytrope (101), we obtain

$$\nu = \frac{\tilde{c}}{2} + \ln\left(1 - K_m \rho^{\frac{1}{n_m}}\right), \tag{104}$$

where $\tilde{c}$ is an integration constant.

We now consider the energy polytrope in Eq. (94) and investigate the behavior of the solution in the region around the center of the stellar object. In order to avoid a conical singularity at the center of the stellar object, we require the following behavior of ρ near the center of the star:

$$\rho \sim \rho_0 + \rho_2 r^2, \qquad \lambda = \lambda_2 r^2, \tag{105}$$

where ρ_0, ρ_2, and λ_2 are constants. We should note that when $r \to 0$, we need to require $\lambda, \lambda' \to 0$ in order to avoid the conical singularity. Then Eqs. (100) and (103) give

$$\begin{aligned} &p \sim p_0 + p_2 r^2, \qquad p_0 \equiv K{\rho_0}^{1+\frac{1}{n}}, \qquad p_2 \equiv K{\rho_0}^{1+\frac{1}{n}}\left(1 + \frac{1}{n}\right)\frac{\rho_2}{\rho_0}, \\ &\nu \sim \nu_0 + \nu_2 r^2, \qquad \nu_0 \equiv \frac{c}{2} - (1+n)\ln\left(1 + K{\rho_0}^{\frac{1}{n}}\right), \qquad \nu_2 \equiv -\left(1 + \frac{1}{n}\right)\frac{K{\rho_0}^{\frac{1}{n}-1}\rho_2}{1 + K{\rho_0}^{\frac{1}{n}}}. \end{aligned} \tag{106}$$

Therefore, by using (98), we obtain

$$\xi'(r) = \xi_1 + 2\xi_2 r, \qquad \xi_1 \equiv \frac{c_1}{8}, \qquad \xi_2 \equiv -\frac{2(\nu_2 + \lambda_2) + \kappa^2\left(\rho_0 + K{\rho_0}^{1+\frac{1}{n}}\right)}{8(\nu_2 + \lambda_2)}. \tag{107}$$

In order to avoid the conical singularity, we need to require $\xi_1 = 0$.

For simplicity, we assume

$$h_{ij} = \frac{\mathrm{Re}\left(\mathrm{e}^{-i\omega t + ikr}\right) h^{(0)}_{ij}}{r} \quad \left(i, j = \theta, \phi, \sum_i h^{(0)i}_{i} = 0\right), \qquad \text{other components} = 0, \tag{108}$$

where $h^{(0)}_{ij}$s are constants corresponding to the polarization. At the center of the stellar object, by using (66), we find

$$\begin{aligned} &\nabla_t\nabla_t\xi = \nabla_i\nabla_j\xi = \nabla_r\nabla_t\xi = \nabla_t\nabla_r\xi = \nabla_t\nabla_i\xi = \nabla_i\nabla_t\xi = \nabla_r\nabla_i\xi = \nabla_i\nabla_r\xi = 0, \\ &\nabla_r\nabla_r\xi = 2\xi_2, \qquad \nabla^i\nabla^j\xi = 2\xi_2\tilde{g}^{ij}, \qquad \nabla^2\xi = 6\xi_2. \end{aligned} \tag{109}$$

Then when the energy of the gravitational wave is large, by using (12), we find the following dispersion relation:

$$0 = -\frac{1}{2}\left(\frac{1}{2\kappa^2} + 24\xi_2\right)e^{-2\nu_0}\omega^2 + \frac{1}{2}\left(\frac{1}{2\kappa^2} + 16\xi_2\right)\xi_2 k^2, \tag{110}$$

which indicates that the propagating speed $c_{\rm GW}$ of the gravitational wave is given by

$$c_{\rm GW}{}^2 = \left(\frac{1 + 32\kappa^2\xi_2}{1 + 48\kappa^2\xi_2}\right)c^2. \tag{111}$$

We should note that the speed of light c is now given by $c^2 = e^{2\nu_0}$. If $|\kappa^2\xi_2| \ll 1$, then Eq. (112) is approximated as

$$c_{\rm GW}{}^2 \sim (1 - 16\kappa^2\xi_2)c^2. \tag{112}$$

Therefore if $\xi_2 > 0$ ($\xi_2 < 0$), the propagating speed of the gravitational wave becomes larger (smaller) than that of light.

The GW170817 event in (11) gives a strong constraint on the parameter ξ_2 as follows:

$$|16\kappa^2\xi_2| < 6 \times 10^{-15}. \tag{113}$$

In the limit $\xi_2 \to 0$, ξ becomes almost constant near the center of the stellar objects and Eq. (107) indicates that

$$0 = 2(\nu_2 + \lambda_2) + \kappa^2(\rho_0 + K\rho_0{}^{1+\frac{1}{n}}), \tag{114}$$

which is consistent with Einstein's gravity with $\xi(\phi) = 0$. In fact, Eq. (114) is obtained from Eq. (97) by putting $\xi(\phi) = 0$ and by using (105) and (106) at the center.

V. SUMMARY AND DISCUSSION

In this paper, we have investigated the propagating speed of the gravitational waves in the spherically symmetric spacetime and cosmological spacetimes of the FLRW form, which are solutions of Einstein-Gauss-Bonnet gravity. We have found, that there is no possibility that the speed could coincide with that of light in spherically symmetric backgrounds. We estimated the shift of the propagating speed inside stellar objects and in several epochs like during the inflation, the end of inflation, the reheating, and late time era in the framework of the Einstein-Gauss-Bonnet gravity coupled with one scalar field. In order not to conflict with the GW170817 observations [65–67], we have proposed a scenario that Einstein-Gauss-Bonnet gravity reduces to the standard scalar-tensor theory in late times by requiring that the Gauss-Bonnet coupling $\xi(\phi)$ of the scalar field in the action (1) and also the scalar field ϕ goes to a constant in the late time era, although the Gauss-Bonnet coupling may play important and nontrivial roles in the early Universe. We constructed a rather realistic model that could satisfy the above requirement. An interesting point could be that the model would describe both the inflationary era in the early Universe and the accelerating expansion of the late Universe without introducing parameters with so large a hierarchy.

What could happen when the propagating speed of the gravitational wave is different from that of light in the early Universe? For the fixed frequency, the wavelength becomes longer (shorter) if the speed is larger (smaller) than the light speed. Usually, the wave with a longer wavelength generates higher output. Therefore if the speed is larger (smaller), the primordial gravitational wave becomes more (less) abundant. Another point is a cosmological horizon. For gravity-related fluctuations, the cosmological horizon becomes larger (smaller) if the speed of the gravitational wave becomes larger (smaller) than those of other modes including light and scalar fields. Therefore the tensor mode and so-called B-mode polarization could be affected.

In the case of the stellar object, we have estimated the propagating speed of the gravitational wave at the center of the stellar object, in order to avoid any ambiguities. If the Gauss-Bonnet coupling $\xi(\phi)$ becomes almost constant, most of the gravitational waves propagate at the speed of light. Such a behavior $\xi(\phi)$ strongly depends on the details of the model. There could be, however, the model where $\xi(\phi)$ could depend on the coordinates even outside of the stellar objects, especially in the case of compact stars like neutron stars. Furthermore, there might be a small portion of the gravitational wave that goes through inside the stellar objects. If the propagating speed of the gravitational wave is larger than that of light, there might be a small signal before the main part of the gravitational wave is observed. If the propagating speed of the gravitational wave is larger than that of light, however, the causality could be violated and therefore it might be prohibited. In this case, there could be some interesting phenomena. For example, some information inside the black hole horizon, which is a null surface, might go through outside the horizon, which may solve the problem of the information paradox of the black hole. Finally, let us note that Einstein-Gauss-Bonnet gravity may lead to finite-time future singularity (for a review see [73]) and it would be of interest to study the gravitational wave speed in Einstein-Gauss-Bonnet theory when the Universe reaches a future finite-time singularity.

ACKNOWLEDGMENTS

This work was partially supported by MICINN (Spain), Project No. PID2019–104397GB-I00 and by the program Unidad de Excelencia Maria de Maeztu CEX2020-001058-M, Spain (S. D. O). S. N. was partly supported by MdM Core visiting professorship at ICE-CSIC, Barcelona.

[1] K. N. Abazajian *et al.*, arXiv:1610.02743.
[2] M. H. Abitbol *et al.* (Simons Observatory Collaboration), Bull. Am. Astron. Soc. **51**, 147 (2019).
[3] S. Hild, M. Abernathy, F. Acernese, P. Amaro-Seoane, N. Andersson, K. Arun, F. Barone, B. Barr, M. Barsuglia, and M. Beker *et al.*, Classical Quantum Gravity **28**, 094013 (2011).
[4] J. Baker, J. Bellovary, P. L. Bender, E. Berti, R. Caldwell, J. Camp, J. W. Conklin, N. Cornish, C. Cutler, and R. DeRosa *et al.*, arXiv:1907.06482.
[5] T. L. Smith and R. Caldwell, Phys. Rev. D **100**, 104055 (2019).
[6] J. Crowder and N. J. Cornish, Phys. Rev. D **72**, 083005 (2005).
[7] T. L. Smith and R. Caldwell, Phys. Rev. D **95**, 044036 (2017).
[8] N. Seto, S. Kawamura, and T. Nakamura, Phys. Rev. Lett. **87**, 221103 (2001).
[9] S. Kawamura, M. Ando, N. Seto, S. Sato, M. Musha, I. Kawano, J. Yokoyama, T. Tanaka, K. Ioka, T. Akutsu *et al.*, Prog. Theor. Exp. Phys. **2021**, 05A105 (2021).
[10] A. Weltman, P. Bull, S. Camera, K. Kelley, H. Padmanabhan, J. Pritchard, A. Raccanelli, S. Riemer-Sørensen, L. Shao S. Andrianomena *et al.*, Publ. Astron. Soc. Aust. **37**, e002 (2020).
[11] P. Auclair *et al.* (LISA Cosmology Working Group), Living Rev. Relativity **26**, 5 (2023).
[12] G. Agazie *et al.* (NANOGrav Collaboration), Astrophys. J. Lett. **951**, L8 (2023).
[13] S. Vagnozzi, J. High Energy Astrophys. **39**, 81 (2023).
[14] S. Nojiri and S. D. Odintsov, Phys. Rep. **505**, 59 (2011).
[15] S. Nojiri, S. D. Odintsov, and V. K. Oikonomou, Phys. Rep. **692**, 1 (2017).
[16] J. c. Hwang and H. Noh, Phys. Rev. D **71**, 063536 (2005).
[17] S. Nojiri, S. D. Odintsov, and M. Sami, Phys. Rev. D **74**, 046004 (2006).
[18] S. Nojiri, S. D. Odintsov, and M. Sasaki, Phys. Rev. D **71**, 123509 (2005).
[19] M. Satoh, S. Kanno, and J. Soda, Phys. Rev. D **77**, 023526 (2008).
[20] Z. Yi, Y. Gong, and M. Sabir, Phys. Rev. D **98**, 083521 (2018).
[21] Z. K. Guo and D. J. Schwarz, Phys. Rev. D **80**, 063523 (2009).
[22] P. X. Jiang, J. W. Hu, and Z. K. Guo, Phys. Rev. D **88**, 123508 (2013).
[23] P. Kanti, R. Gannouji, and N. Dadhich, Phys. Rev. D **92**, 041302 (2015).
[24] C. van de Bruck, K. Dimopoulos, C. Longden, and C. Owen, arXiv:1707.06839.
[25] P. Kanti, J. Rizos, and K. Tamvakis, Phys. Rev. D **59**, 083512 (1999).
[26] E. O. Pozdeeva, M. R. Gangopadhyay, M. Sami, A. V. Toporensky, and S. Y. Vernov, Phys. Rev. D **102**, 043525 (2020).
[27] E. O. Pozdeeva and S. Y. Vernov, Eur. Phys. J. C **81**, 633 (2021).
[28] S. Koh, B. H. Lee, W. Lee, and G. Tumurtushaa, Phys. Rev. D **90**, 063527 (2014).
[29] B. Bayarsaikhan, S. Koh, E. Tsedenbaljir, and G. Tumurtushaa, J. Cosmol. Astropart. Phys. 11 (2020) 057.
[30] M. De Laurentis, M. Paolella, and S. Capozziello, Phys. Rev. D **91**, 083531 (2015).
[31] S. Chervon, I. Fomin, V. Yurov, and A. Yurov, *Scalar Field Cosmology* (World Scientific, Singapore, 2019).
[32] K. Nozari and N. Rashidi, Phys. Rev. D **95**, 123518 (2017).
[33] S. D. Odintsov and V. K. Oikonomou, Phys. Rev. D **98**, 044039 (2018).
[34] S. Kawai, M. a. Sakagami, and J. Soda, Phys. Lett. B **437**, 284 (1998).
[35] Z. Yi and Y. Gong, Universe **5**, 200 (2019).
[36] C. van de Bruck, K. Dimopoulos, and C. Longden, Phys. Rev. D **94**, 023506 (2016).
[37] B. Kleihaus, J. Kunz, and P. Kanti, Phys. Lett. B **804**, 135401 (2020).
[38] A. Bakopoulos, P. Kanti, and N. Pappas, Phys. Rev. D **101**, 044026 (2020).
[39] K. i. Maeda, N. Ohta, and R. Wakebe, Eur. Phys. J. C **72**, 1949 (2012).
[40] A. Bakopoulos, P. Kanti, and N. Pappas, Phys. Rev. D **101**, 084059 (2020).
[41] W. Y. Ai, Commun. Theor. Phys. **72**, 095402 (2020).
[42] S. D. Odintsov, V. K. Oikonomou, and F. P. Fronimos, Ann. Phys. (Amsterdam) **420**, 168250 (2020).
[43] V. K. Oikonomou and F. P. Fronimos, Classical Quantum Gravity **38**, 035013 (2021).
[44] S. D. Odintsov and V. K. Oikonomou, Phys. Lett. B **805**, 135437 (2020).
[45] S. D. Odintsov, V. K. Oikonomou, F. P. Fronimos, and S. A. Venikoudis, Phys. Dark Universe **30**, 100718 (2020).
[46] S. A. Venikoudis and F. P. Fronimos, Eur. Phys. J. Plus **136**, 308 (2021).
[47] S. B. Kong, H. Abdusattar, Y. Yin, and Y. P. Hu, Eur. Phys. J. C **82**, 1047 (2022).
[48] R. Easther and K. i. Maeda, Phys. Rev. D **54**, 7252 (1996).
[49] I. Antoniadis, J. Rizos, and K. Tamvakis, Nucl. Phys. **B415**, 497 (1994).
[50] I. Antoniadis, C. Bachas, J. R. Ellis, and D. V. Nanopoulos, Phys. Lett. B **257**, 278 (1991).
[51] P. Kanti, N. Mavromatos, J. Rizos, K. Tamvakis, and E. Winstanley, Phys. Rev. D **54**, 5049 (1996).
[52] P. Kanti, N. Mavromatos, J. Rizos, K. Tamvakis, and E. Winstanley, Phys. Rev. D **57**, 6255 (1998).
[53] D. A. Easson, T. Manton, and A. Svesko, J. Cosmol. Astropart. Phys. 10 (2020) 026.
[54] N. Rashidi and K. Nozari, Astrophys. J. **890**, 58 (2020).
[55] S. D. Odintsov, V. K. Oikonomou, and F. P. Fronimos, Phys. Rev. D **107**, 084007 (2023).
[56] S. D. Odintsov and T. Paul, Phys. Dark Universe **42**, 101263 (2023).
[57] S. D. Odintsov, V. K. Oikonomou, I. Giannakoudi, F. P. Fronimos, and E. C. Lymperiadou, Symmetry **15**, 1701 (2023).
[58] V. K. Oikonomou, Astropart. Phys. **141**, 102718 (2022).
[59] S. Nojiri and S. D. Odintsov, Nucl. Phys. **B998**, 116423 (2024).
[60] J. J. Terente Díaz, K. Dimopoulos, M. Karčiauskas, and A. Racioppi, arXiv:2310.08128.
[61] S. Kawai and J. Kim, Phys. Rev. D **108**, 103537 (2023).
[62] S. Kawai and J. Kim, Phys. Rev. D **104**, 083545 (2021).

[63] S. Kawai and J. Kim, Phys. Lett. B **789**, 145 (2019).

[64] S. Choudhury, arXiv:2307.03249.

[65] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Phys. Rev. Lett. **119**, 161101 (2017).

[66] B. P. Abbott *et al.* (LIGO Scientific, Virgo, Fermi-GBM and INTEGRAL Collaborations), Astrophys. J. Lett. **848**, L13 (2017).

[67] B. P. Abbott *et al.*, Astrophys. J. **848**, L12 (2017).

[68] S. D. Odintsov, V. K. Oikonomou, and F. P. Fronimos, Nucl. Phys. **B958**, 115135 (2020).

[69] V. K. Oikonomou, Classical Quantum Gravity **38**, 195025 (2021).

[70] V. K. Oikonomou, P. D. Katzanis, and I. C. Papadimitriou, Classical Quantum Gravity **39**, 095008 (2022).

[71] T. Anson, E. Babichev, C. Charmousis, and S. Ramazanov, J. Cosmol. Astropart. Phys. 06 (2019) 023.

[72] C. Dalang, P. Fleury, and L. Lombriser, Phys. Rev. D **103**, 064075 (2021).

[73] J. de Haro, S. Nojiri, S. D. Odintsov, V. K. Oikonomou, and S. Pan, Phys. Rep. **1034**, 1 (2023).