Tidal deformation of dynamical horizons in binary black hole mergers and its imprint on gravitational radiation

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In this work, I study the tidal deformation of black holes in binary mergers in the strong field regime. A different approach to the problem of tidal deformability of black holes is taken using the *source* multipole moments of their dynamical horizons and numerical relativity, instead of the *field* multipole moments of the gravitational field at infinity. I compute these source multipole moments in the inspiral phase of binary black hole mergers, uncover several interesting new features in the evolution of the deformations of the dynamical horizon geometry, and characterize how nonspinning black holes deform. Owing to the mutual tidal interactions, I describe how the dynamical horizons of the two black holes deform and steadily acquire various multipole moments that would otherwise vanish when the horizons are isolated. Out of these, the dominant deformation is shown to be quadrupolar. I further show that their evolution has a familiar chirplike behavior. I also find that these deformations encode detailed information about the dynamics of the binary black hole system. Particularly, the dominant quadrupolar deformation is shown to be strongly correlated with the gravitational field of the system at future null infinity. Therefore, the gravitational waves carried away from the system contain imprints of the geometrical structure of the dynamical horizons in the strong-field regime. Thus, although causally disconnected from observers, these correlations may present us with a novel way to probe the strong field structure of gravitational fields in astrophysical scenarios. The results here may be important in the strong field tests of black holes and general relativity, for the no-hair conjecture in the strong field, dynamical regimes, and in astrophysical contexts especially when the black holes are close to the merger.

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I. INTRODUCTION

Numerous binary black hole mergers have been observed to date starting with the first detection in 2015 [1–9]. The parameters of these binary systems, including the masses and spins of the individual black holes, can be inferred from the observed data [10]. These observations have been used to extract information about the parameters of the astrophysical systems. The observations have so far been found to be consistent with standard general relativity [11–13]. One of the aims of this work is to understand if the gravitational radiation received from binary black hole systems can provide information about the dynamics in the strong field regime.

The dynamics of the gravitational field in the far field regime, far away from the horizons of the merging black holes is fairly well understood. In the static and stationary scenarios, the multipole moments of the gravitational field have been studied at infinity using the field multipole moments [14,15]. In dynamical scenarios like a BBH merger, the multipole moments of gravitational radiation are relevant [16–20]. Post-Newtonian and perturbative techniques are frequently used to match the PN source in the near-field regime and radiative multipole moments in the far field regime [21], and construct post-Newtonian waveforms for astrophysical systems. These further enable the construction of templates for the detection of gravitational waves. They have also been used to devise parametrized tests of the early inspiral phase using gravitational wave observations [22–29].

In a binary black hole merger scenario, the black holes are under the mutual influence of the gravitational interactions of the companion and are thus tidally coupled. An important question in astrophysical relativity is whether the black holes tidally deform. In the regime where the effect of the companion can be treated perturbatively, these tidal effects can be calculated analytically [30–37]. However, as the black holes close-in toward the merger, these techniques can no longer be applied as the assumptions break down in this regime.

Starting with our previous work [38], I aim to understand the deformations of the black holes using an

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alternative approach that continues to be valid through the inspiral and merger. During these phases, the horizons of the black holes are tidally coupled and thus are expected to deform under mutual gravitational influences. Instead of studying the gravitational field of the black hole in the asymptotic region, I study the deformations of the dynamical horizons of black holes in the strong field regime using numerical relativity. This would allow us to go beyond the existing calculations, closer to the merger where linear perturbation theory is not sufficient to describe the strong field dynamics.

In the strong field regime, the physical boundaries of the black hole regions are described by their dynamical horizons [39] instead of the more familiar event horizons that have severe limitations. First, event horizons are null surfaces of black holes and cannot be defined, e.g., when the future null infinity is incomplete [40]. They are also teleological in nature. To locate event horizons, one would require knowledge of the entire future of the spacetime. Due to their teleological nature, event horizons can form and grow in empty regions of spacetime in anticipation of matter/energy that would fall in the distant future. In dynamical scenarios, there is also no known way to construct preferred foliations of the event horizons. Furthermore, event horizons can also have nonsmooth features [41].

On the other hand. dynamical horizons are spacelike and are located inside the event horizons and are outside of the domain of outer communications. Isolated horizons have proven to be very useful in describing black holes in equilibrium (in an otherwise dynamical spacetime) and dynamical horizons in dynamical scenarios when there is a flux of matter/energy into the black hole [38,39,42–49]. They are quasilocal and free from the teleological nature of event horizons.

The nonlinear dynamics of a binary black hole system are responsible for the gravitational radiation that escapes to future null infinity and is seen by us. The spacetime region around the detectors that record these waves (in the wave zone) has low curvatures compared to those in the vicinity of the black holes, allowing the gravitational field in the wave zone to be studied using perturbative techniques. On the other hand, the strong field region can only be described using numerical relativity, as all approximation schemes fail, even more so as the black holes close in toward the merger. The use of numerical relativity, as I propose here, opens up the possibility of indirectly inferring the properties of the gravitational fields at the boundaries of the black holes in the strong field regime of binary black hole mergers. This allows us to envisage probing the fundamental properties of black holes using gravitational wave observations.

Due to the severe limitations of analytical methods and the nonlinear structure of the field equations of general relativity, it has been challenging to understand the dynamics of the gravitational fields in the strong field regime. Furthermore, the fact that the dynamical horizons lie inside the event horizon and are causally disconnected from observations adds to the difficulties. Despite these limitations, several authors have conjectured and found evidence for a correlation between the weak field dynamics and the strong field dynamics of the black holes in head-on collisions or postmerger scenarios [46,48,50]. In binary black hole systems, the horizons of the black holes exist in a tidal environment due to their companion, possess an influx of energy and momentum and are hence dynamical. In [49,51], I discovered that a part of the gravitational field at the dynamical horizons of black holes in the inspiral, merger, and ringdown are strongly correlated with the gravitational radiation received at future null infinity, using numerical simulations in full general relativity. This was the first work to study and find evidence for the applicability of this conjecture in the inspiral dynamics of binary black hole mergers.

In our previous work [38], the axisymmetric deformations of the dynamical horizons in binary black hole scenarios were studied, in which I proposed an alternative approach to quantifying the strong field tidal deformability of black holes using the source multipole moments of dynamical horizons [38]. It was found that, unlike in the existing approach using the field multipole moments, the dynamical horizons of nonspinning black holes in the strong field regime deform. They can be associated with tidal deformability coefficients (Love numbers) that are universal and independent of the parameters of the system. Also, I showed that the tidal coefficients associated with the dynamical horizons can be computed, using a small set of numerical relativity simulations of nonspinning binary black hole mergers.

In this work, I further add to the understanding of the physics of deformations of the dynamical horizons and the strong field dynamics of black holes in binary black hole coalescence. In particular, I extend our previous results in [38,49] by computing and describing for the first time, the spectrum of deformations of the dynamical horizons in BBH mergers and their relationship with the dynamics at future null infinity.

II. BASIC NOTIONS

To study the deformations of the dynamical horizons of the black holes in a BBH merger scenario, I use their source multipole moments. The generalized source multipole moments are defined at a dynamical horizon $\mathcal{H}$, [39,52] obtained by a time evolution of marginally trapped surfaces. As mentioned earlier, a dynamical horizon is located inside the event horizon, which marks the boundary of a four-dimensional trapped space-time region.

Additionally, to study the correlations between the strong and weak field dynamics, I need the multipolar structure of the gravitational field, described by the news function, at future null infinity $\mathcal{I}^+$. The future null infinity

$\mathcal{I}^+$ is an invariantly defined null surface and is the end point of future null-geodesics which escape to infinity [17,53].

For convenience, I consider $\mathcal{H}$ and $\mathcal{I}^+$ to be foliated by 2-surfaces $\mathcal{S}$ of spherical topology. For the former, I obtain a marginally trapped surface and for the latter, I approximate it by large coordinate spheres in the numerical domain, extrapolated to infinity. For every cross section $\mathcal{S}$, I assign outgoing and ingoing directions. Denote the outgoing future-directed null vector normal to S by n^+, and the ingoing null normal as n^- satisfying $n^+ \cdot n^- = -1$. Let x be a complex null vector tangent to $\mathcal{S}$ satisfying $x \cdot \bar{x} = 1$ (the over-bar denotes complex conjugation), and $n^+ \cdot x = n^- \cdot x = 0$.

A. Dynamics at $\mathcal{I}^+$

In the weak-field regime of a BBH merger, the vacuum spacetime geometry is completely described by the Weyl tensor C_{abcd}. In particular, outgoing transverse radiation is described by the Weyl tensor component [54]

$$\Psi_4 = C_{abcd} n^{-a} \bar{x}^b n^{-c} \bar{x}^d. \tag{1}$$

Ψ_4 can be expanded in spin-weighted spherical harmonics ${}_{-2}Y_{\ell,m}$ of spin weight -2 [55]. Let $\Psi_4^{(\ell,m)}$ be the mode component with $\ell \geq 2$ and $-m \leq \ell \leq m$. The (ℓ, m) component of the news function $\mathcal{N}^{(\ell,m)}$ and its polarizations $\mathcal{N}_{+,\times}$ are defined as [17]

$$\mathcal{N}^{(\ell,m)}(u) = \mathcal{N}_+^{(\ell,m)} + i\mathcal{N}_\times^{(\ell,m)} = \int_{-\infty}^{u} \Psi_4^{(\ell,m)} du. \tag{2}$$

The outgoing energy flux is related to the integral of $|\mathcal{N}|^2$ over all angles. In a numerical spacetime, it is in principle possible to extract [56] or evolve Ψ_4 going out all the way to $\mathcal{I}^+$ [57] to reduce systematic errors and gauge changes. I shall however follow the common approach of calculating Ψ_4 on a sphere at a finite radial coordinate r and then extrapolating it to $\mathcal{I}^+$ in postprocessing using modules I developed in PYTHON [38,49,58]. I will now describe this procedure.

First, I extrapolate the waveform to $\mathcal{I}^+$ by implementing an improved second-order perturbative procedure that also includes corrections for the angular momentum of the system up to the first order (detailed in Eq. [29] of [59]). All time integrations are handled using fixed frequency integration technique [60] and differentiations using an 11-point finite difference stencil. For all the simulations, I use the Ψ_4 gravitational waveform data computed at large coordinate spheres of radius $r = 500M$ from the center of the spatial domain for extrapolation. Second, I transform the waveform to the center of mass frame. I compute the coordinate center of mass motion of the BBH system (see Ref. [61] for details). I then use the methods described in [62] and implemented in [58] to translate and boost the gravitational waveform Ψ_4 that had been extrapolated to $\mathcal{I}^+$. In particular, for the translation transformation, correction terms up to the leading order are retained. These transformations are implemented using a staggered angular grid on a sphere with a size of (121 by 240).

After these transformations, our waveforms are found to agree quite well with those of the RIT [63] and SXS catalogue [64], with phase evolution deviating at most by 1% during the inspiral phase.

Given the waveforms in the center of mass frame at $\mathcal{I}^+$, I compute the news using the integral in the previous equation (2), which is carried out over the simulation time t/M instead of the retarded time coordinate u. The lower limit in the integral is not $-\infty$ but the earliest time available in the signals received at $r = 500M$. The news function is then a function of time, starting from the earliest time available. A further time integration of $\mathcal{N}$ would yield the gravitational wave strain.

B. Dynamics at $\mathcal{H}$

On the black hole, the basic object here is the dynamical horizon $\mathcal{H}$ whose three-metric is denoted by q_{ab}, and its foliations by marginally outer trapped surface (MOTS) denoted by $\mathcal{S}$. This is a closed spacelike 2-surface with vanishing outgoing expansion Θ_+:

$$\Theta_+ = q^{ab} \nabla_a n_b^+ = 0. \tag{3}$$

Its scalar curvature is denoted by $\tilde{\mathcal{R}}$, the two-metric by $\tilde{q}_{ab}$, and its extrinsic curvature by $\tilde{K}_{ab}$.

The shear of the dynamical horizon, which represents part of the infalling radiation at the horizon and the tidal coupling is denoted by $\sigma = x^a \bar{x}^b \nabla_a n_b^+$. In a previous work, it was this quantity that was of primary interest. They were shown to be strongly correlated with the outgoing gravitational radiation [49] in the inspiral, merger and ringdown phases of nonspinning BBH mergers. In this work, it is the source mass multipole moments that are of primary interest. I will continue to use the techniques developed previously, improved and detailed here to correlate the source mass multipole moments of the dynamical horizons with the outgoing gravitational radiation.

At $\mathcal{H}$, two sets of source multipole moments can be defined; the mass $\mathcal{M}_{l,m}$ and angular momentum multipoles $\mathcal{J}_{lm}$). Although defined on the foliations of the three-dimensional dynamical horizon, these can be used to invariantly reconstruct the horizon geometry [45].

These moments were first defined for isolated horizons [44] and extended to axisymmetric [43] and nonaxisymmetric dynamical horizons [45]. They can be used to study the intrinsic geometry of dynamical horizons, and have been used in predictions of the antikick in binary black hole mergers [65], the study of the no-hair conjecture in general astrophysical environments [47], and for studying tidal deformations of black holes [38,47,66]. In a more recent work [38], the axisymmetric tidal deformations of dynamical horizons in binary black mergers were studied by using the source multipole moments of the dynamical horizon in binary black hole scenarios.

I now describe how these moments have been computed in this work.[1] Consider a time slice of a numerical simulation in which a MOTS $\mathcal{S}$ embedded in the three-dimensional spatial hypersurface Σ has been located. First, a preferred coordinate system on the leaves S of the dynamical horizon is constructed using an appropriately defined axial vector field φ^a. I use the method of Killing transport to compute the axial field [43,44]. This method can be used to find an approximate symmetry field with desired properties on the MOTS from an initial guess. Here, approximate symmetry is not a requirement and the method is merely used to obtain a favorable axial field on every MOTS. E.g, this method yields a vector field that coincides with the direction field ϕ of the Boyer-Lindquist co-ordinates, which is also its Killing field, when the horizon is isolated e.g., when the separation between the black holes is large compared to their horizon sizes, and at late times on the common dynamical horizon when it asymptotes to an axisymmetric Kerr isolated horizon. Thus, even though this vector field φ^a is not an exact Killing field on the MOTS, it furnishes a field that can be used to conveniently establish a coordinate system on the dynamical horizon.

Once the axial field is defined and computed, an invariant coordinate ζ (analogous to the polar coordinate variable $\cos(\theta)$ of the Boyer-Lindquist coordinates) can be defined as:

$$\tilde{D}_a \zeta = \frac{1}{R_S^2} \tilde{\epsilon}_{ab} \varphi^b \tag{4}$$

Here $\tilde{\mathcal{D}}_a$ is the derivative operator and $\tilde{\epsilon}_{ab}$ the volume form on $\mathcal{S}$ compatible with the two-metric $\tilde{q}_{ab}$, and $R_{\mathcal{S}}$ is the areal radius of $\mathcal{S}$. The freedom to add a constant to ζ is removed by requiring that $\oint_S \zeta d^2S = 0$. In order to compute the source multipole moments, one then needs to construct a scalar harmonic basis on each $\mathcal{S}$. This is defined by mapping $\mathcal{S}$ to a unit sphere and using the spherical harmonic functions $\mathcal{Y}_{\ell m}$ of that unit sphere whose axial field is φ and polar angle $\cos(\theta) = \zeta$. Using these ingredients, one can define the mass multipole moments as:

$$\mathcal{M}_{\ell,m} = \frac{M_{\mathcal{S}} R_{\mathcal{S}}^l}{8\pi} \sqrt{(2l+1)\frac{(\ell-m)!}{(\ell+m)!}} \oint_S \tilde{\mathcal{R}} \mathcal{Y}_{\ell m} d^2S, \tag{5}$$

Similarly, the spin-multipole moments can be defined as:

$$\mathcal{J}_{\ell m} = \frac{R_{\mathcal{S}}^{\ell+1}}{8\pi} \sqrt{(2\ell+1)\frac{(\ell-m)!}{(\ell+m)!}} \oint_S \tilde{\epsilon}^{ab} K_{bc} r^c_{-1} \mathcal{Y}_{\ell m} d^2S, \tag{6}$$

Expressed in functional form, $\mathcal{Y}_{\ell m}(\zeta,\phi) = P_\ell^m(\zeta) e^{-im\phi}$. Here K_{ab} is the extrinsic curvature of Σ, P_ℓ^m are the associated Legendre polynomials corresponding to the eigenfunctions of the Laplacian on the unit round sphere, and ϕ is an affine coordinate on the integral curves of the vector field φ^a. ${}_{-1}\mathcal{Y}_{\ell m}$ are the spin weight -1 spherical harmonics on $\mathcal{S}$.

In this notation, $\mathcal{M}_{0,0}$ is the mass $M_{\mathcal{S}}$ of the slice $\mathcal{S}$ of the dynamical horizon, and $\mathcal{J}_{10}$ is its angular momentum $J_{\mathcal{S}}$. $n!$ denotes the factorial of an integer n. These moments are defined for all positive $\ell \geq |s|$, and for each ℓ, the azimuthal mode number m takes integer values ranging from $(-\ell, \ell)$. It is to be noted that modes for which $m \neq 0$ are complex in general and I denote their strength as absolute magnitude or a quadratic sum $|\mathcal{M}_{\ell,m}| = \sqrt{\mathrm{Re}(\mathcal{M}_{\ell,m})^2 + \mathrm{Im}(\mathcal{M}_{\ell,m})^2}$.

In this work, I use a small set of three numerical simulations of nonspinning binary black holes to study the deformations. Considering that this work deals with nonspinning binary black hole mergers, I focus on the source mass-multipole moments and address the spin-multipoles in a future work.

The choice of conventions for defining the spherical harmonics are as follows. For a given mode (ℓ, m),

$$\oint \mathcal{Y}_{\ell m} \mathcal{Y}^*_{\ell m} d^2S = 4\pi. \tag{7}$$

The Condon-Shortley phase has not been included in the definition of our harmonics.

Given that these are not exact eigenfunctions of the Laplacian operator on $\mathcal{S}$, the harmonics from different modes may not exactly be orthogonal to each other. I.e., for modes $(\ell, m) \neq (\ell', m')$,

$$\oint \mathcal{Y}_{\ell m} \mathcal{Y}^*_{\ell' m'} \neq 0 \tag{8}$$

However, I found that the departure from orthogonality was minimal of the order of $\approxeq 10^{-3}$, and can be safely neglected for the purposes of this study. Also, a simplifying assumption, I use the grid coordinate angle ϕ in place of the affine coordinate on the integral curves of φ^a.

I then compute the mass dipole and the quadrupole moments($\ell = 1$, $\ell = 2$), $|m| \leq \ell$, and study their evolution through the inspiral, merger, and ringdown phases of BBH mergers. These correspond to the source mass multipole moments of the individual dynamical horizons of the black

[1]A more elaborate procedure proposed in [45] is reserved for a future work.

holes during the inspiral phase, and that of the common horizon in the postmerger phase.

I then study their relationship with the dynamics of the binary black hole system and the gravitational news function at $\mathcal{I}^+$.

III. THE NUMERICAL SIMULATIONS

Our numerical simulations are performed using the publicly available Einstein Toolkit framework [67,68]. The initial data is generated based on the puncture approach [69,70], which is then evolved through BSSNOK formulation [71–73] using the $1 + \log$ slicing and Γ-driver shift conditions. Gravitational waveforms are extracted [74] on coordinate spheres at various radii between $100M$ to $500M$. The computational grid set-up is based on the multipatch approach using Llama [75] and Carpet modules, along with adaptive mesh refinement (AMR). The various horizons are located using the method described in [76,77]. General quasilocal physical quantities are computed on the horizons following [42,43]. The framework to compute the generalized multipole moments in Eq. (5) does not exist in the QuasiLocalMeasures thorn of the Einstein Toolkit. They have been newly computed here in postprocessing in PYTHON starting with the data from numerical relativity simulations.

I consider nonspinning binary black holes on quasicircular orbits with varying mass ratios $q = M_2/M_1$, where $M_{1,2}$ are the component horizon masses ($M_1 \geq M_2$). I use the GW150914 parameter file available from [78] as a template. For each of the simulations, as input parameters I provide initial separation between the two punctures D, mass ratio q and the radial and azimuthal linear momenta p_r, p_ϕ respectively, while keeping the total physical horizon masses $M = M_1 + M_2 = 1$ fixed in our units. Parameters are listed in Table I. I compute the corresponding initial locations, the x, y, z components of linear momentum for both black holes and grid refinement levels, etc., before generating the initial data and evolving it. I chose three nonspinning cases with mass-ratios $q = 0.6$, 0.7 and 1 for the purposes of this study, based on the initial parameters listed in [79,80]. For computing the quasilocal quantities, I use a uniform angular grid of size (36, 74) on each of the dynamical horizons. This grid resolution allows us to safely study multipole moments of up to $\ell = 2$. I also carry out a convergence test of the horizon data by simulating at two different grid resolutions of the horizon.

Our simulations agree very well with the catalog simulations [63], with merger time discrepancies of less than a few per cent. The results presented here are the main and general features that were seen across the simulations $q = 1$, 0.6, and 0.7 described in Table I. Sometimes when I refer to one simulation, for concreteness, I would be referring to the $q = 0.6$ case unless mentioned otherwise. The outer common horizon of the $q = 0.6$ configuration appears at $t = 1656.045M$, which I designate as merger time. When the common horizon is found, it has an areal radius of $R_c = 1.708M$. 3D visualizations were created using *VisIt* [81].

TABLE I. Initial parameters for nonspinning binary black holes with quasicircular orbits. $q = M_2/M_1$ is the mass ratio, D is the initial separation between the two holes, and p_r and p_ϕ are the linear momenta in the radial and azimuthal directions respectively.

q	D/M	p_r/M	p_ϕ/M
1	9.5332	0	0.09932
0.6	11.5	-5.46×10^{-04}	0.08206
0.7	12.0	-5.07×10^{-04}	0.08246

IV. RESULTS

To compute the generalized source multipole moments, I track the two apparent horizons of the individual black holes at every time step using AHFinderDirect [76]. Then, using the spacetime data on the apparent horizon at every time step and the developed packages, I compute in postprocessing the generalized source mass-multipole moments. I then use the package WAVEFORMTOOLS [58], which has been developed to carry out data analysis with numerical relativity data, to analyze the output. I describe the most important results here.

A. The spectrum of deformation and its evolution

I first discuss the relative strengths of the various multipole moments $|\mathcal{M}_{l,m}|$. For both the black holes across all of the simulations, among all the moments at $\ell = 1$, 2 multipolar order, the multipole moment $\mathcal{M}_{2,\pm2}$ was found to have the largest and mostly monotonically increasing amplitude in the inspiral phase, followed by $\mathcal{M}_{1,\pm1}$ and $\mathcal{M}_{2,0}$. Note that for an isolated Kerr horizon, apart from its mass, only $\ell = 2$, $m = 0$ mass moment is nonzero up to the $\ell = 2$ quadrupolar order). This can be seen in the Figs. 1–4, and the movie [82] in which the evolution of the 2D-Ricci scalars of the dynamical horizons in the inspiral phase have been visualized. In Fig. 1, a snapshot of the movie at one point in its orbit is presented. The multipolar deformations of the horizon geometries are found to be mutual and are dependent on the location of the black holes in the binary system. The dominant quadrupolar, i.e., $\ell = 2$, $m = 2$ pattern can be clearly seen in the movie.

The nonaxisymmetric multipole moments (i.e., $m \neq 0$) of the individual dynamical horizons of the black holes are oscillatory in nature. It was found that the multipole moments $\mathcal{M}_{2,2}$ of the two black holes were in-phase with

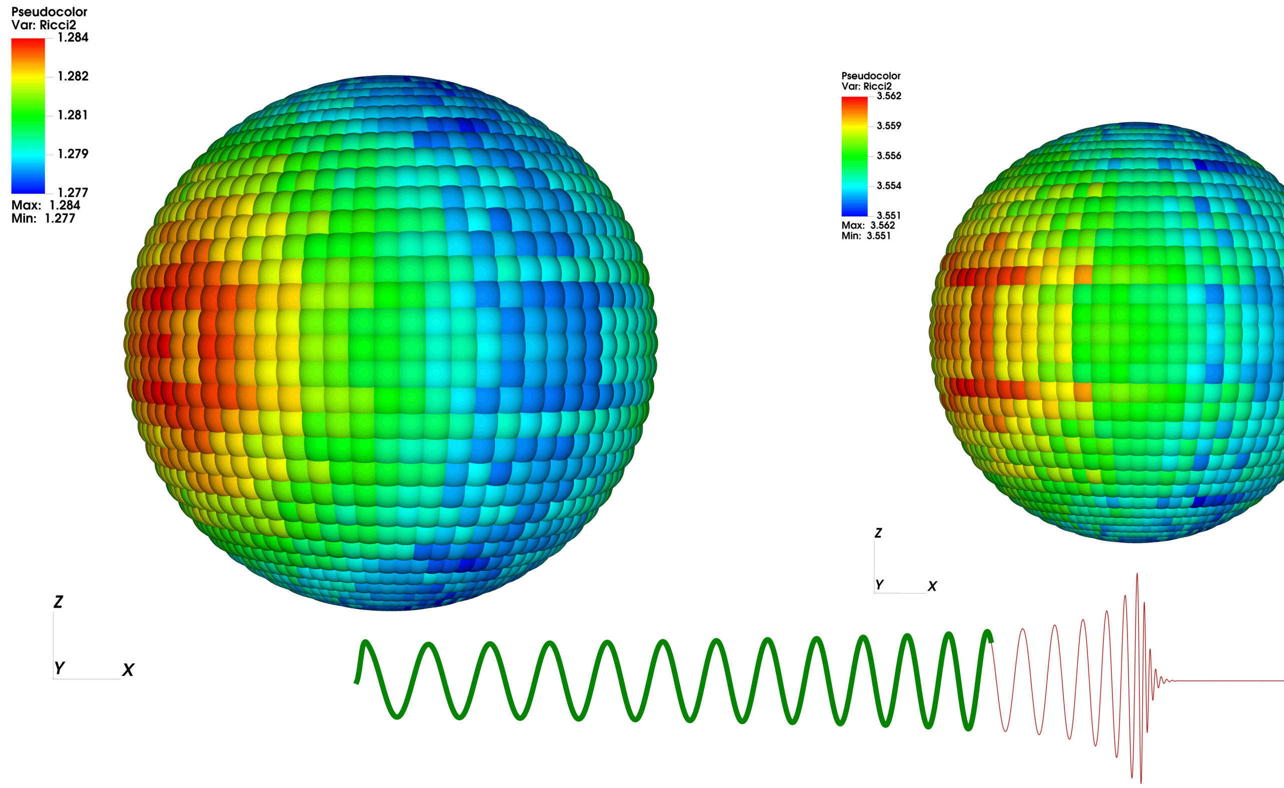


FIG. 1. The deformation of the dynamical horizons of the black holes in the inspiral can be directly visualized in terms of the 2-Ricci scalars $\tilde{\mathcal{R}}$ of their respective two-dimensional slices $\mathcal{S}$. Here the 2-Ricci scalars of $\mathcal{S}$ are visualized for the $q = 0.6$ system when the black holes are at a separation of $d \approx 7.75M$ (about 67.4% of the initial separation), approximately 5 orbits after the start of the simulation, as shown by the thick green line in the waveform plot below the figure. The total number of orbits before the merger is around 9 and the corresponding waveform cycles in the simulation are shown by the thin red line. The more massive black hole $BH1$ is on the left. The values of the Ricci scalar are shown on the color bars to the left of each black hole. A movie for $q = 0.6$ can be viewed here [82]. This movie shows that the deformation of the horizon geometry is mutual for both the horizons due to their tidal interactions, and the Ricci scalar distribution patterns face each other at all points in the orbit. The dominant quadrupolar structure can be seen, which is reflected in the numerical values of the strengths of the multipole moments in Figs. 2–4.

each other while the moments $\mathcal{M}_{1,\pm1}$ differed by a phase of π radians.

B. Relationship with the orbital dynamics and $\mathcal{I}^+$

I found the multipole moments of the dynamical horizons of the two black holes to be strongly correlated with each other. Furthermore, the dominant multipole moment $\mathcal{M}_{2,2}$ of the dynamical horizons was found to be strongly correlated with the dominant $(\ell = 2, m = 2)$ mode of the gravitational wave strain extracted at a very large distance $r = 100M$ from the system. The movie [82] aids in the visualization of some of these results.

In Fig. 5, I plot the time derivative of the multipole moments $\mathcal{M}_{1,1}$ and $\mathcal{M}_{2,2}$ vs the news function of the gravitational waves emitted from the system (Eq. 2, suitably normalized and aligned in time and phase. The time shift was found to be $101.3M$, approximately consistent with the light travel time corresponding to the extraction radius for Ψ_4. It was found that the quadrupole mass moment $\mathcal{M}_{2,\pm2}$ encodes accurate information about the phasing of the gravitational waveform from the system whereas the dipole moment $\mathcal{M}_{1,\pm1}$ reflects the orbital phasing of the system and is correlated with the linear momentum of the dynamical horizon. Thus like with the waveform received at $\mathcal{I}^+$, these deformations encode accurate information about the evolution of the binary system, and the gravitational waveform itself contains information about the deformation.

To demonstrate this, I show that the parameters of the binary system can be recovered using the multipole moments $\mathcal{M}_{2,2}$ and a standard least squares figure of merit, using a procedure detailed in [49]. To carry this out, a template bank of gravitational wave strain in the mass-ratio, chirp-mass parameter space was constructed using the well-known phenomenological waveform model IMRPHenomPv2. It is found that the parameters of the binary system could be estimated quite accurately (with an error of 0.12% in the mass ratio and 0.01% in the chirp-mass of the binary system). Therefore the dynamical horizon carries accurate information on the dynamics of the system and can be used to extract

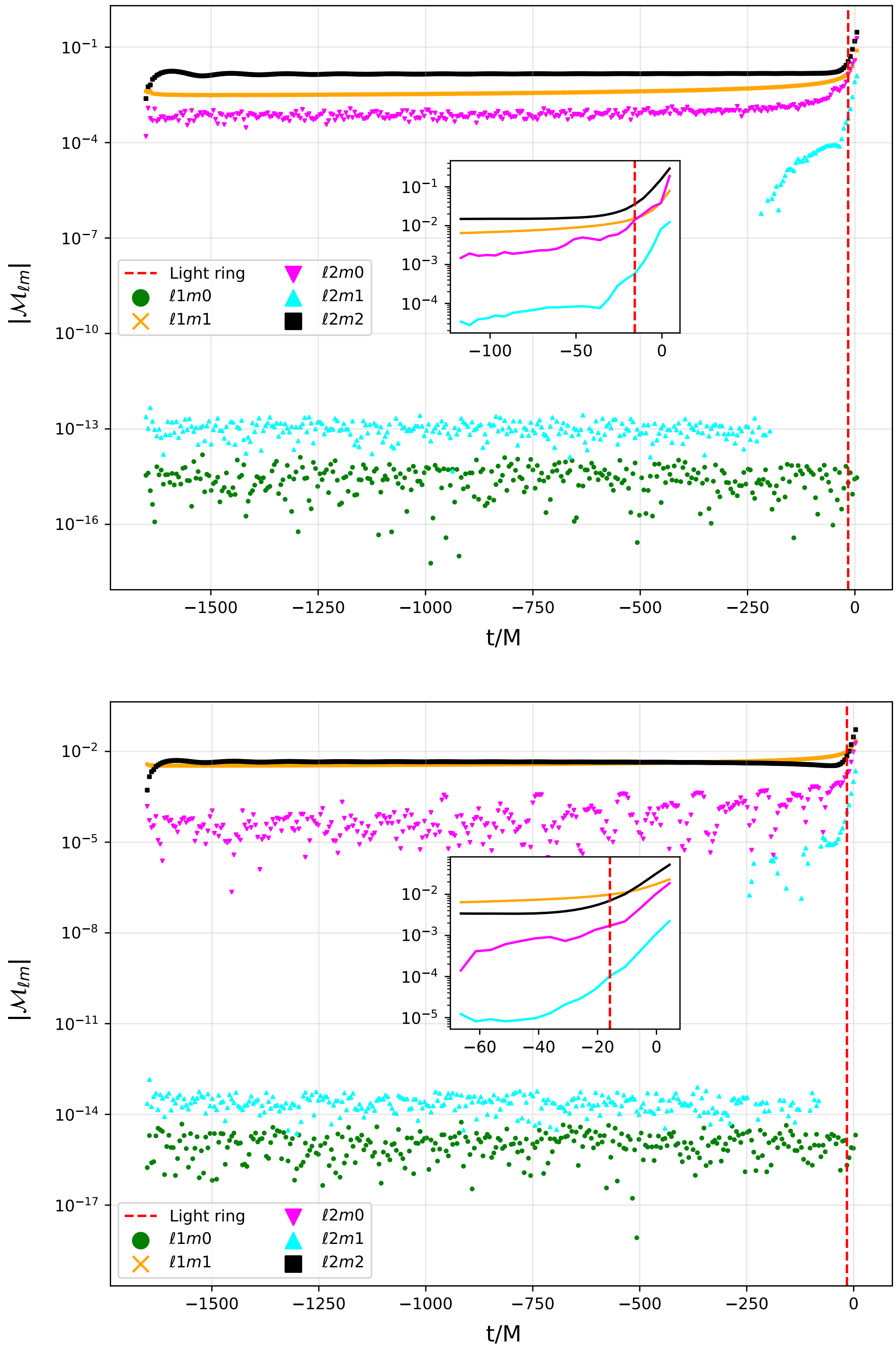


FIG. 2. The variation in time of the strengths (absolute magnitudes) of the multipole moments $\ell = 1$, $\ell = 2$ $|m| < \ell$ for the larger black hole (BH1, left) and smaller black hole (BH2, right), for mass ratios $q = 0.6$ (1 and 2). Here, the values below 10^{-8} depict the numerical noise floors of the respective moments. The time of crossing of the light ring of the system is denoted as a dotted line in red. It can be seen that the quadrupolar moment $\ell = 2$, $m = 2$ has the largest strength for the majority of the inspiral phase. Toward the merger, the dynamical horizons are strongly deformed as the multipole moments sharply acquire greater strengths as the black holes cross the light ring of the system.

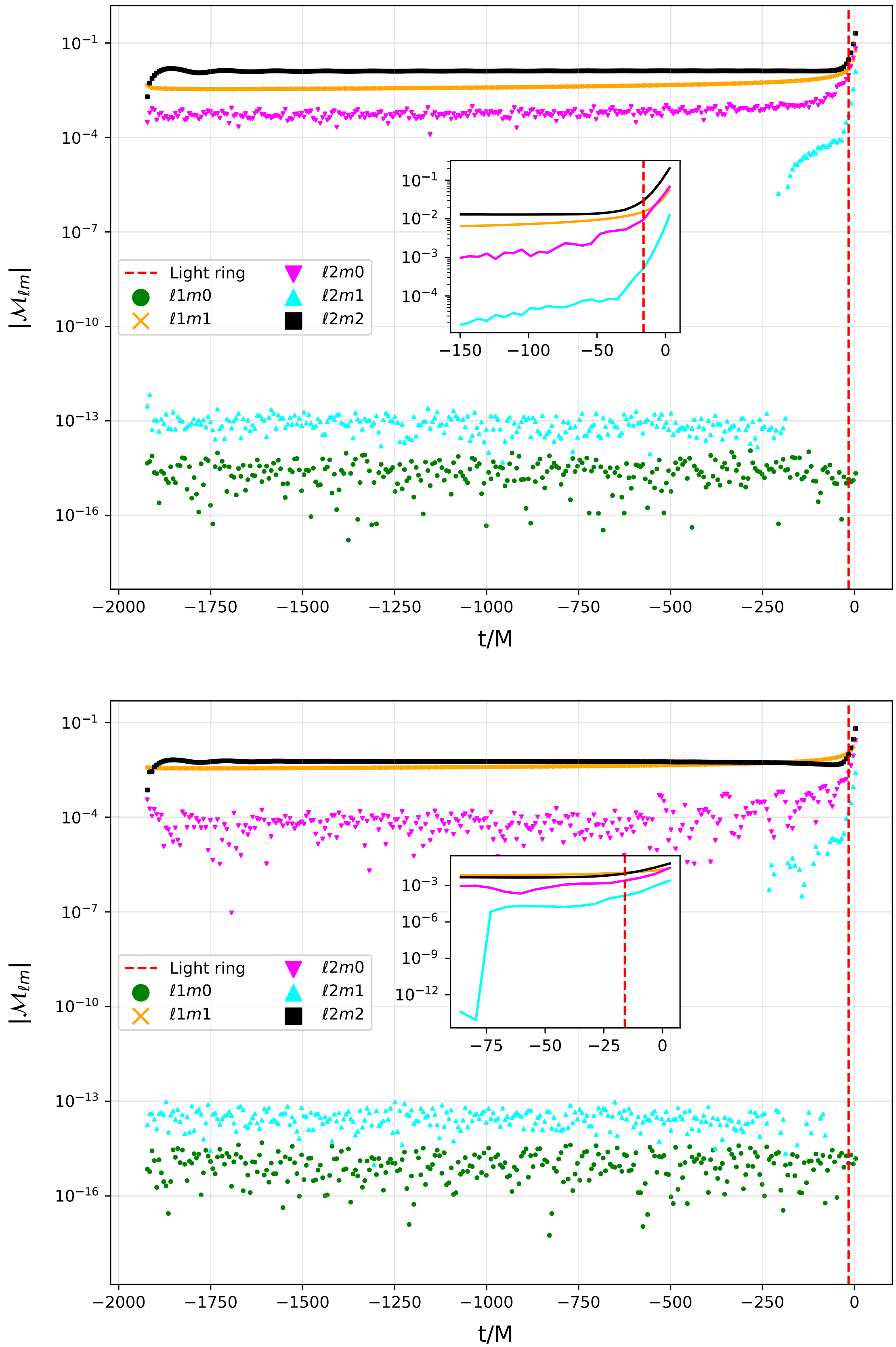


FIG. 3. Similar to Fig. 2, but for $q = 0.7$. Please refer to the caption of Fig. 2 for a description.

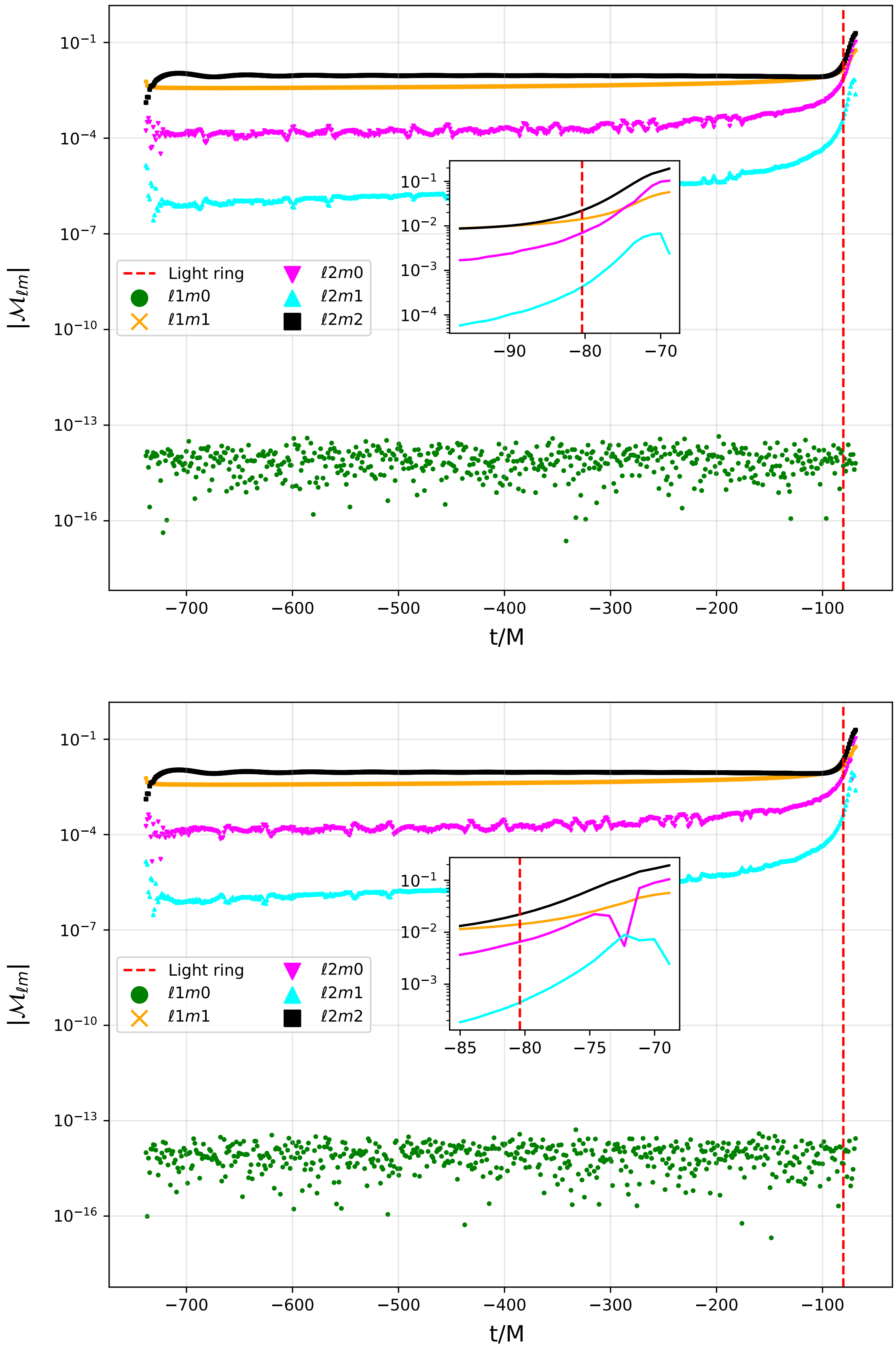
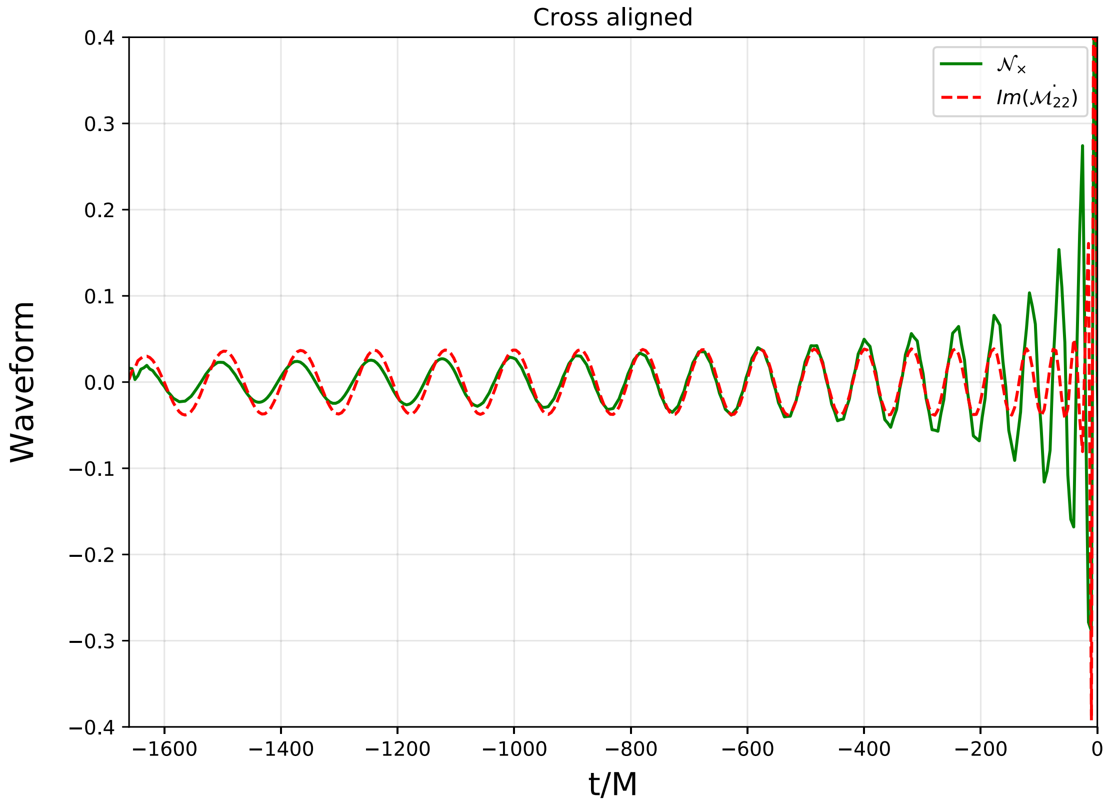


FIG. 4. Similar to Fig. 2, but for $q = 0.1$. Please refer to the caption of Fig. 2 for a description.



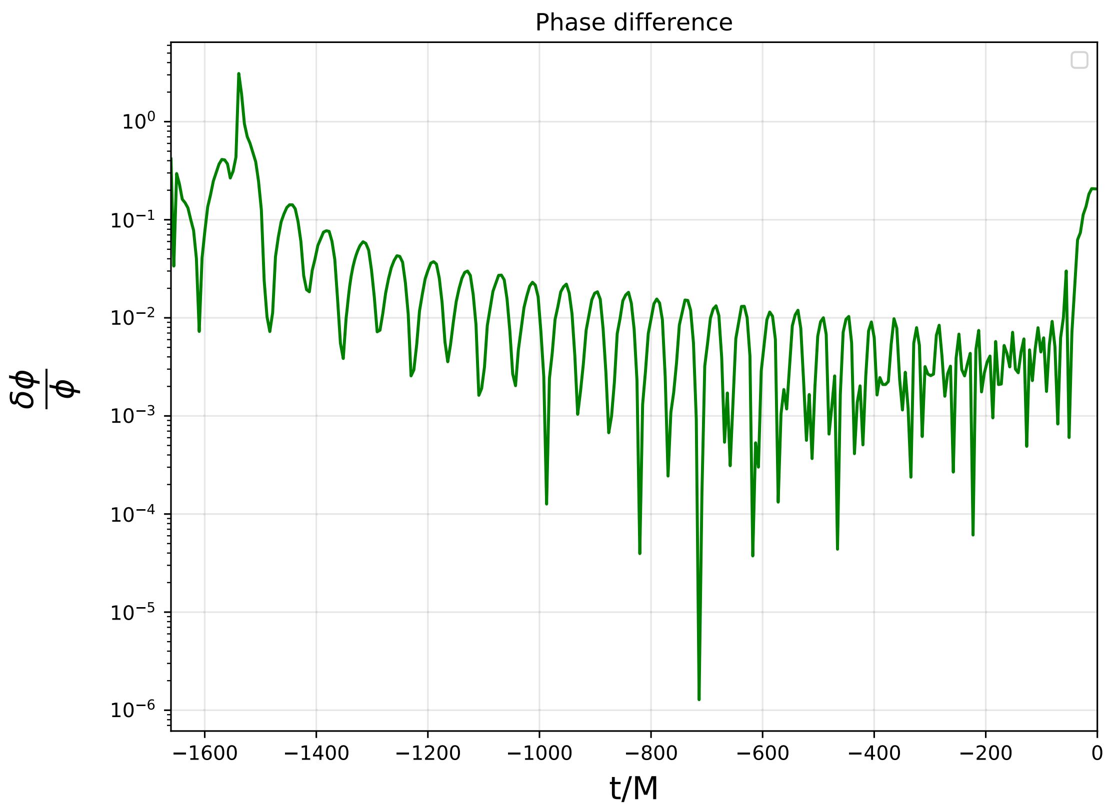


FIG. 5. Left: the coaligned imaginary parts of the $\ell = 2$, $m = 2$ quadrupole moment and outgoing news waveforms for the $q = 0.6$ system. Note that the phasing of the multipole moment closely follows that of the gravitational news waveform. Right: the deviation in the phase evolution of multipole moment $\mathcal{M}_{2,2}$ from the news function.

information about the binary black hole system like their masses, velocities, orbital angular momentum, etc.

These results also show that the source multipole moments are strongly correlated with the multipole moments of the gravitational field at null infinity [17–20].

C. Generic behavior

The evolution of the strengths of the multipole moments was also found to display a generic behavior. By means of maximizing a least-squares figure of merit, I found that the evolution of the multipole moments of both the black holes and across the two simulations can be described by a generic tidal expansion of the form:

$$\frac{\delta|\mathcal{M}_{l,m}|}{M_{\mathcal{H}}^{l+1}} = \sum_{i=3}^{\infty} \frac{a_i}{d^i} \tag{9}$$

where $M_{\mathcal{H}}$ is the mass of the black hole that is being discussed, and d is a measure of the distance of separation between the holes. In particular, the dipole moment was found to be well described by the above expansion that includes terms up to the fourth order in $1/d$ and the quadrupole moment required terms up to the sixth order (see Fig. 6). These are consistent with the results of [38]. These moments can therefore be used to study the tidal deformability of black holes and compute their corresponding Love numbers in a manner described there, which is reserved for another study. Apart from the real and imaginary parts of the moments $\mathcal{M}_{2,\pm2}$, their magnitudes also display an oscillatory behavior as shown in Fig. 6. These oscillations are decaying with time, and exist in the multipole moments of both the dynamical horizons. To understand this further, I decompose the evolution of the magnitude of the moment into a nonoscillatory portion that changes secularly, as described by fits to Eq. (9), and an oscillatory part. This was done by first fitting the multipole moment strength to Eq. (9) (call this the nonoscillatory part), and then computing the normal distance of each data point from the best-fit curve (the oscillatory part). The fit was carried out on the data up to $t = 1000M$. I found that the oscillatory part can be described by a superposition of power-law damped sinusoids of the form:

$$\frac{|\mathcal{M}_{2,2}|}{M_{\mathcal{H}}^3} = \sum_i A_i t^{(-\gamma_i)} \sin(\omega_i t + \phi_i) \tag{10}$$

with power law indices $\gamma = 1.47$ and 2.29, respectively. The periods of oscillations of these modes were found to be at $T_1 = 161.56M$ and $T_2 = 124.71M$ respectively. It is worth noting that the latter is close to half the average orbital period (i.e., twice the orbital frequency) in the domain considered. These values may be expected to be dependent on the mass ratio of the system. As this feature was observed in all three of the simulations $q = 1, 0.6, 0.7$, we are led to speculate if these correspond to dynamic tides or waves traveling on the horizon due to residual eccentricities and perturbations about the quasicircular inspiral, which decay in a quasinormal manner as the simulations proceed in time, i.e., these could be quasinormal like modes excited on the tidally coupled individual dynamical horizons in the inspiral phase or are mere

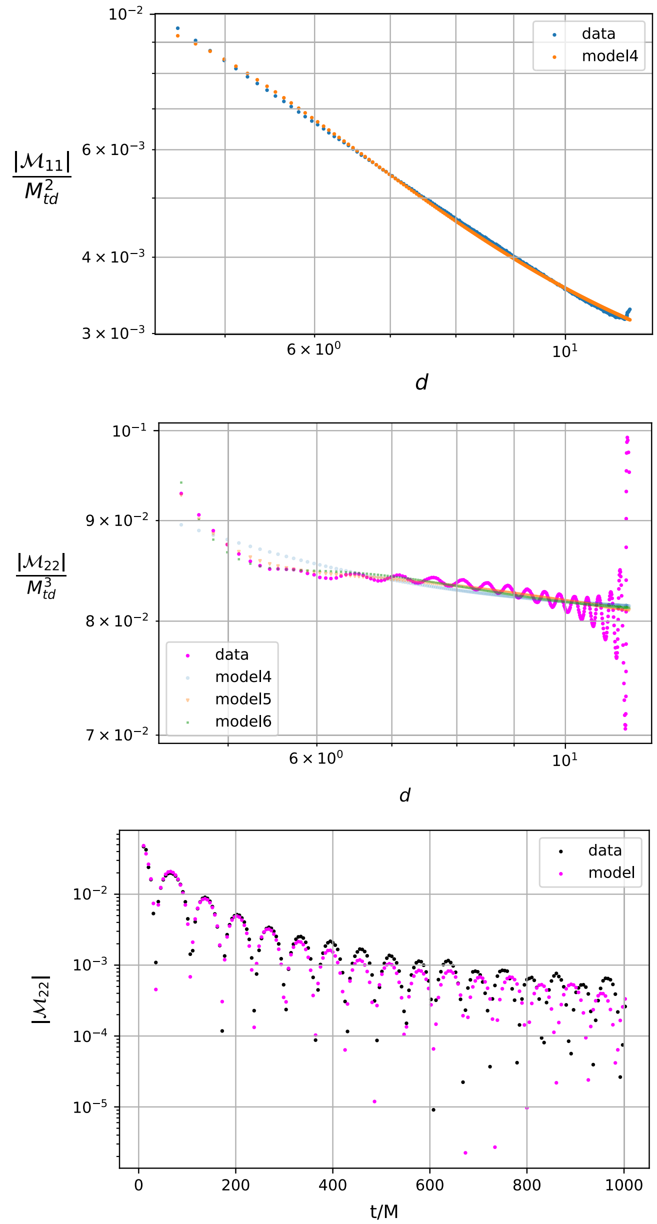


FIG. 6. Top: The fits of the $\ell = 1$, $m = \pm 1$ multipole moments of the larger black hole of the $q = 0.6$ system to the relation in Eq. (9), with two terms: one at the third and the other at the fourth order in the inverse separation d, i.e., $1/d^3$ and $1/d^4$. Middle: the fits of the $\ell = 2$, $m = \pm 2$ multipole moments to the relation in Eq. (9), with terms up to fourth, fifth, and sixth orders in the inverse of the separation d. Bottom: the isolated oscillations of the overall amplitude of the multipole moment $|\mathcal{M}_{2,2}|$ seen in the figure in the middle. Please see Sec. IV C for discussion.

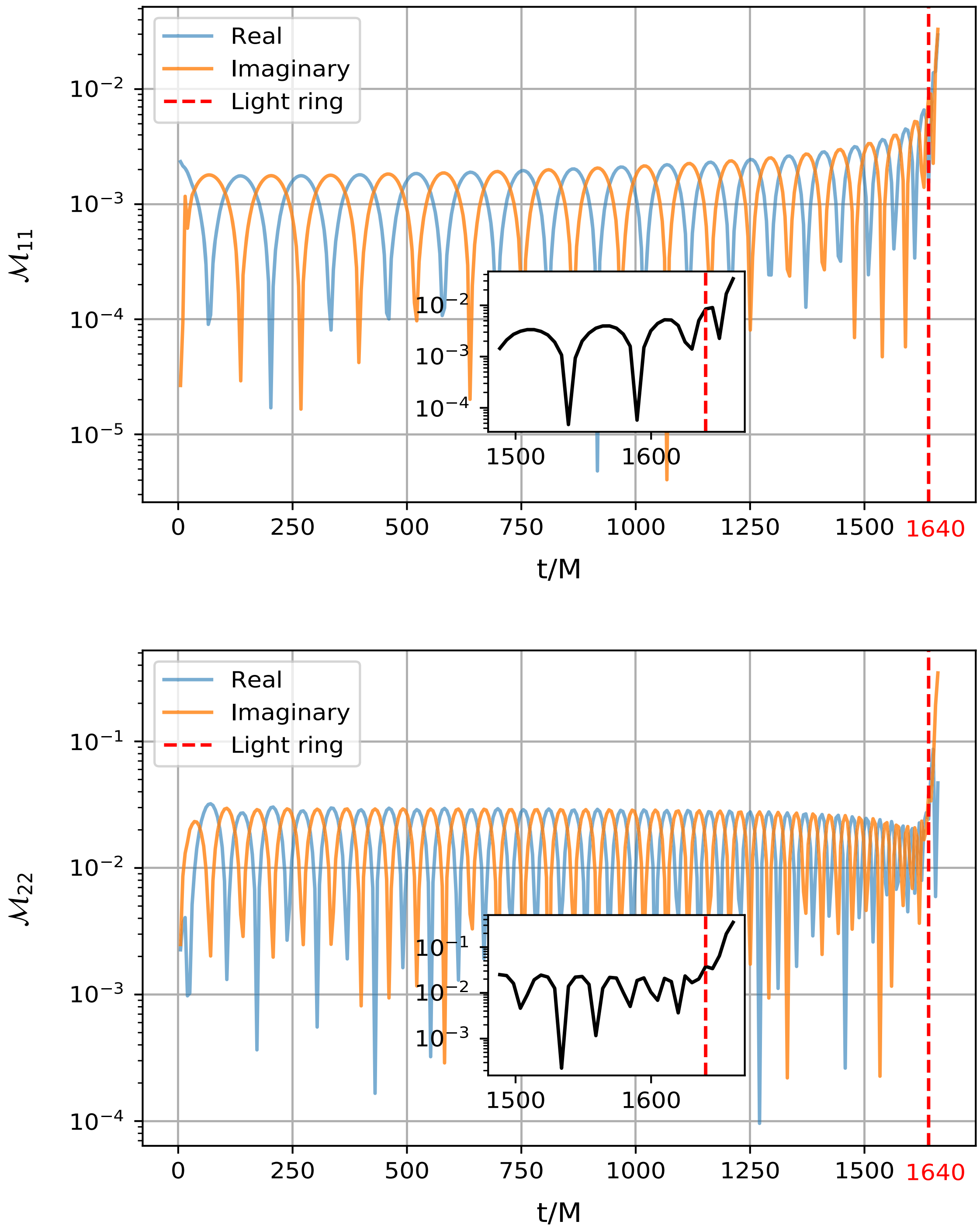


FIG. 7. The time evolution of the $\ell = 1$, $m = 1$ (top) and $\ell = 2$, $m = 2$ (bottom) mass multipole moments of the dynamical horizons for the larger black hole of the $q = 0.6$ system. The $\ell = 1$, $m = \pm 1$ moments are identical. The red line shows the temporal location of the light ring $r \approx 2.856$. The close-up of these plots shows that the location where the steep increase in the growth pattern of the strength of the multipole moments occurs is approximately aligned with the epoch of light ring crossing by the system.

numerical artefacts. Further study is necessary to confirm this.

D. Plunge

The multipole moments were found to qualitatively display two distinct behaviors in the pre-merger phase, as seen in Figs. 2–4 and 7. While the majority of the portion of the evolution of these moments reflected the adiabatic, secular dynamics of the system, their behavior was seen to go through a steeper increase in strength with the seizure of oscillations closer to the merger. I found that this epoch is very close to the time of crossing of the light ring of the binary black hole system. This was confirmed by estimating the light ring radius r_L of the system using the adiabatic re-summed 1PN Hamiltonian [83]:

$$r^3 - 3r^2 + 5\nu = 0 \tag{11}$$

where $\nu = M_1 M_2/(M_1 + M_2)^2$ is the symmetric mass ratio and then locating the time at which the separation reached this value.

The multipole moments ($\ell = 2$, $m = \pm 1$) which were closer to their numerical noise floor $\sim 10^{-5}$ for the majority of the inspiral phase, were found to slowly gain in magnitude as the black holes approach each other. At these times closer to the epoch of the crossing of the light ring, the dynamical horizons will be boosted to high velocities and are expected to significantly deform under each other's influences. Thus, during these last moments of the inspiral of the individual horizons of the black holes, the various multipole moments that were previously very small can be seen to grow to comparable strengths.

E. Postmerger

The physical details of the product of the merger in astrophysical and even numerical simulations, although expected to be Kerr, is a nontrivial question and an important question in itself. From an astrophysical perspective, this is akin to the test of the no-hair theorem and general relativity. From a numerical relativity perspective, the remnant black hole is a product of the long-term evolution of the Einstein field equations, which are highly dynamical and nonlinear in nature. The numerical simulation began with the initial data of two black holes well separated. The initial data of the system is not an exact solution to the astrophysical system, and the deviations are radiated away in the form of junk radiation in the early inspiral. Therefore, it is useful to devise tests of the long-term evolution of numerical simulations themselves.

The source multipole moments of the common dynamical horizon formed provide an ideal avenue to test the dynamical horizon formed in the merger process. This is because the dynamical horizon formed is in the strong field regime, which is expected to asymptote to a Kerr isolated horizon at late times. The multipolar structure of the Kerr isolated horizon is known in a gauge-independent fashion, and thus any departures from these values would be useful for quantifying deviations from the expected Kerr nature of the remnant.

In this section, I present a preliminary analysis of the multipole moments of the outer common dynamical horizon in the postmerger phase, which is formed late into the inspiral phase, since its formation. This would also serve as a direct strong-field, long-term test of the numerical simulations run here.

As the two black holes cross the light ring of the system, common envelopes surrounding the individual black hole horizons form. The outer common horizon when formed is a spinning and highly distorted dynamical horizon of the remnant, i.e., with gravitational "hairs," that is expected to eventually settle down to the Kerr isolated horizon. Thus, one expects the multipole moments of the common horizon formed to be significantly different from that of an isolated Kerr horizon.

The dynamical horizon then proceeds to equilibrium by absorbing radiation and losing hairs, i.e., the multipole moments acquired at the merger. In this process, the source multipole moments at the common dynamical horizon are expected to decay to the corresponding Kerr isolated horizon values at late times.

I compute the $\ell = 1, 2, |m| \leq \ell$ mass multipole moments and plot their strengths in the left panel of Fig. 8. This figure shows (although less accurately) that the deformations acquired by the common dynamical horizon decay exponentially with time, proceeding toward a Kerr isolated horizon. Furthermore, the decay of these multipole moments was found to be consistent with quasinormal decay. The damping rate of the strengths of the moments $|\mathcal{M}_{\ell,m}|$ was found to be close to the theoretical estimate of a Kerr black hole with the same mass and spin as that of the remnant. E.g, damping rate of $\mathcal{M}_{2,2}$ was found to be consistent with the $n = 0$, $\ell = 2$, $m = 2$ mode with a deviation of 2.74%. Since the remnant black hole is spinning, the coordinate system established on the dynamical horizon can also rotate along with it and thus real part of the quasinormal frequencies of one mode can only be estimated relative to another without further transformations. Therefore the estimation of the real part of the quasinormal frequencies requires more care and better resolution, which will be carried out in future work. For an isolated Kerr horizon in the rest frame, the only nonzero mass multipole moment at $\ell = 1, 2$ order is $\mathcal{M}_{2,0}$. The moment $\mathcal{M}_{2,0}$ was found to approach the value of the corresponding isolated horizon of a Kerr black hole with a final deviation of 2.73% from the expected theoretical estimate. This is shown in Fig. 8. The strengths of the moments $\mathcal{M}_{1,\pm 1}$ are nonzero due to the boost from the final kick of the horizon formed. However, it can be seen from the same figure that the moment $\mathcal{M}_{2,\pm 2}$, which is expected to go to zero at late times instead here decays to the order of 10^{-4}. I suspect that this could be due to systematic errors arising from the boost, rotation of the coordinate system on the common horizon, its limited grid resolution, or the accuracy of the numerical simulations themselves. This could also be due to the fact that the Lie transport method succeeds in finding the Kerr symmetry field. This is to be investigated further. As expected from reflection symmetry, the mass multipole moment $\mathcal{M}_{1,0}$ is practically zero.

The quasinormal decay of the deformations observed here is interesting because the dynamical horizon is present in the strong field regime and adds continuity to the conjecture that the strong field and weak field dynamics

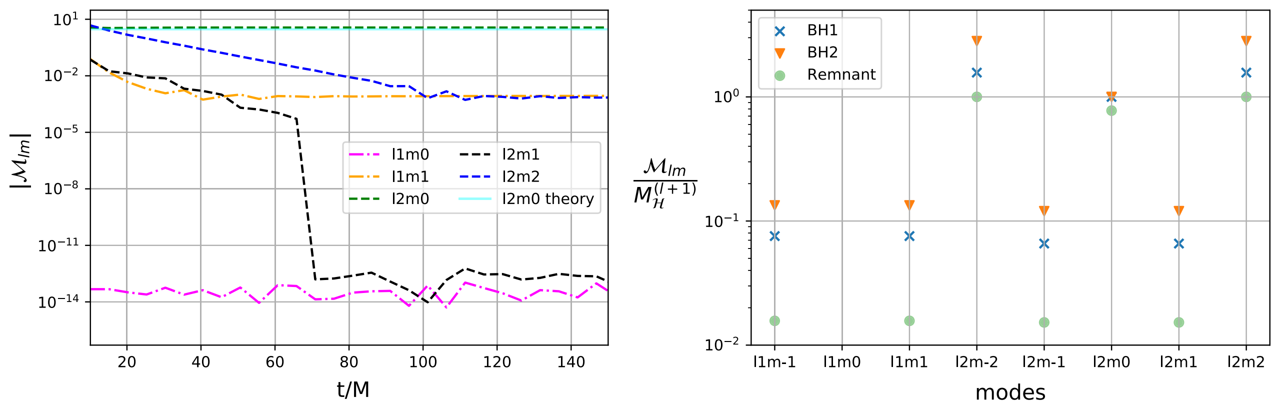


FIG. 8. Left: the time evolution the $\ell = 1, 2$ and $m \le |\ell|$ multipole moments of the outer common dynamical horizon. Here, their strengths (absolute magnitudes) are plotted against time. Values $\sim 10^{-13}$ are below numerical accuracy and represent the noise floors of the moments. The theoretical value of the $\mathcal{M}_{2,0}$ moment of a Kerr isolated horizon of the same mass and spin is plotted in cyan and agrees with the corresponding asymptotic value of the outer common dynamical horizon. Right: the strengths of the multipole moments of the individual horizons of the black holes moments just before the formation of the common outer horizon of the system, and that of the outer common dynamical horizon just when it is formed. The moment $\ell = 1$, $m = 0$ is below machine precision as expected for all the three MOTS.

are strongly correlated to the postmerger phase. Note that, although there have been various studies that suggest this correlation in the postmerger phase, this is one of the first to compute the nonaxisymmetric source multipole moments of the common dynamical horizon in a binary black hole merger scenario and study its evolution.

What decides the deformed state of the common horizon once it is formed? The initial configuration of the parent black holes is expected to decide the deformed state of the common horizon once it is formed. Interestingly, I found that the relative strengths of the mass multipole moments of each of the two individual horizons of the black holes just before the formation of the common dynamical horizon are similar to that of the common dynamical horizon when it is formed (more so for the horizon of the more massive black hole). This can be seen in the right panel of Fig. 8. Thus the deformed state of the individual horizons just before the common horizon appears plays a role in the setting up of the initial state and conditions of the common dynamical horizon when it is formed for the postmerger dynamics. The common horizon thus formed roughly inherits the multipolar structure of the horizon geometry of the black holes at the end of the inspiral phase and then loses them as it absorbs gravitational radiation while settling down to equilibrium at late times in the postmerger phase.

V. SUMMARY

The horizons of black holes in a binary environment are not isolated; their horizon geometries are dynamical and distorted due to their mutual tidal interactions, and details of the strong field dynamics of the system. These distortions are captured by the source multipole moments of the dynamical horizons. These source multipole moments were computed and studied for the first time for axisymmetric deformations of the individual dynamical horizons of black holes in the inspiral phase of binary black hole merger scenarios in [38]. Using the source moments, it was shown that a set of tidal coefficients can be associated with the dynamical horizons that are universal and independent of the parameters of the system. In this work, extending the previous, I define and show how to compute the general nonaxisymmetric deformations of the dynamical horizons due to the tidal interactions using numerical relativity data of nonspinning binary black hole mergers. These multipole moments have been computed and studied here for the first time for the dynamical horizons of the component black holes in binary black hole mergers.

The deformations of the dynamical horizons computed via the defined generalized source multipole moments were found to display interesting features. Firstly, the deformations were found to be mutual for both the dynamical horizons. Furthermore, the dominant structure of the deformation was shown to be quadrupolar in nature.

Secondly, the deformations of the dynamical horizons were found to encode detailed information about the dynamics of the system. They are oscillatory in nature and their phasing is strongly correlated with the orbital dynamics of the binary black hole system. The $\ell = 1$ dipolar deformations were found to be strongly correlated with the linear momentum of the black holes. The multipole moments quantifying these deformations were increasing in magnitude as the black holes closed in toward the merger. Using the approach to strong field tidal deformability of black holes in [38], it was found that the behavior of these multipole moments can be modeled all the way up to the merger using more and more terms in the tidal expansion. Thus, the source multipole moments show a universal behavior.

An interesting feature was found in the strength of the $\ell - 2$, $m = 2$ mode excited on the two individual dynamical horizons. Over and above the secular increase in the strength of the deformation component, it also displayed an oscillatory behavior that can be described by a quasinormal-like damped sinusoid with a power-law decay through the inspiral phase. This leads us to speculate whether this feature is describing dynamical disturbances on the dynamical horizon due to residual eccentricities in the binary black hole system.

While the strengths of the deformations increased, it was found that their oscillatory behavior decreased and appeared to cease, accompanied by a sharp increase in their magnitude, around the same time the black holes crossed the light ring of the system, thus carrying an imprint of the plunge phase of the system.

Note that the dynamical horizons lie inside the event horizons in the strong field regime, outside of the domain of outer communications, while the future null infinity lies in the asymptotic, weak-field regime. The curvature scales at these two regions are entirely different. Thus, there is no reason *a priori* to expect strong correlations between the deformations of the dynamical horizons and the outgoing gravitational radiation, especially in the inspiral phase. However, motivated by the familiar chirplike behavior seen in the multipole moments and the correlations with the orbital dynamics of the system, their relationship with the outgoing gravitational radiation from the system was studied and found that they were strongly correlated. In particular, the quadrupolar deformations of the dynamical horizons of both the black holes were found to be strongly correlated with each other and with the quadrupolar mode of the news of outgoing gravitational radiation at infinity. How strong are these correlations? To answer this, I carry out parameter estimation using a template bank of News waveforms constructed using IMRPhenomPv2, with input waveform as the deformations. It was shown that the parameters of the nonspinning binary black hole system can be fully recovered.

These correlations could potentially allow us to probe the strong field dynamical regime using gravitational wave observations. The correlations of the source mass multipole moments with the dynamics of the system allow the analogous, indirect interpretation that the changing source multipole moments are related to the emission of gravitational waves from the system. I also expect these correlations to extend to all multipolar orders, which will be studied in future work. Owing to these and the previous results in [49], we can conclude that the multipole moments of a dynamical horizon could be strongly correlated with the dynamics of the system. It thus is also strongly correlated with the shear of its outgoing null normal, and the outgoing gravitational radiation emitted from the system. These results urge us to extend the existing conjecture between the strong field and weak field dynamics and state the following conjecture: *In a dynamical scenario involving binary black holes, the source multipole moments associated with charges of the dynamical horizon will be correlated with the multipolar structure of the Bondi flux of the outgoing gravitational radiation from the system received at future null infinity* $\mathcal{I}^+$.

Potential applications of these results are numerous. The nonaxisymmetric tidal deformability coefficients associated with the generalized multipole moments can be estimated following [38]. Together with the strong correlations with the outgoing gravitational radiation, these results show clearly that the dynamical horizons of nonspinning black holes also deform, and have the potential to affect the outgoing gravitational radiation from the system. This opens up the interesting possibility of observing the strong field structure of gravitational fields and tests of general relativity using the tidal deformability of black holes. Work is currently underway in understanding the effect of these deformations of the dynamical horizons on the outgoing gravitational radiation which could help us in probing the nature of the compact object.

The deformations and the approach presented here are valid and more relevant to understanding the deformability of black holes closer to the merger where perturbative techniques cannot be applied. The strong correlations discovered here could be used to probe and test further important aspects of the no-hair conjecture and probe the possible nonstandard structure of gravity (e.g., quantum gravitational corrections) in the strong field regime. In particular, together with the approach described in the previous work [38], the correlations found here suggest the possibility of directly calculating the source multipole moments of dynamical horizons and the associated tidal coefficients using the gravitational wave observations, allowing to test GR.

In the modeling of gravitational waveforms from binary black hole systems (e.g., in the post-Newtonian and effective one-body approach), the deformation of the dynamical horizons is not usually taken into account. As shown here, the dynamical horizons are strongly deformed in the late inspiral phase, and this can potentially have effects on the late-inspiral, plunge waveforms. The models described here can potentially be used to inform the post-Newtonian calculation to improve them.

The results presented also allow for the following simple interpretation: In a binary black hole scenario, the individual dynamical horizon geometries of black holes gain a structure i.e., "gravitational hairs," away from their isolated Kerr geometries in the inspiral phase through the mutual tidal interactions, and lose them in postmerger dynamics of the common horizon.

Certain aspects of the calculations here can be improved. On the numerical side, the computation of the source multipole moments requires a choice of an axial vector field on the horizon. In this work, this has been achieved by using the method of Killing transport, which could lead to inaccuracies in the construction of a preferred coordinate system on the MOTS at times close to the merger. This has been corroborated by the increasing nonorthogonality of the spherical harmonic basis functions observed closer to the merger. Work is in progress on this front. It is to be noted that some of the limitations of the methods used here in the postmerger phase have been alleviated in a more recent study [84], which implements the procedure discussed in [39] to construct the vector fields and appeared consecutive to [85].

Furthermore, although the correlations in phasing are tight, the amplitudes of the deformations were seen to increasingly fall behind toward the merger. Accurate descriptions of the merger phase would require a better choice of the axial vector field. Future works could also extend in this direction.

Furthermore, the results presented here were found to be the same in the qualitative details for the three numerical simulations carried out here. It would be of interest to verify the validity of these results in a wider range of parameters of BBH mergers, including spinning black hole binaries, which are currently underway.

Finally, the ringdown physics of the deformations of the common dynamical horizon, although it was found to qualitatively agree with quasinormal behavior, would require more accurate simulations and warrants a separate publication. In particular, the studies presented here could be expanded to include the spin multipole moments.

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